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ABSTRACT

This study develops a model of endogenous growth based on increasing returns due to firms' technology choices. Particular attention is paid to the implications of these choices, combined with the substitution of capital for labor, on economic growth in a general equilibrium model in which the R&D sector produces machines to be used for the sector producing final goods. We show that incorporating oligopolistic competition in the sector producing finals goods into a general equilibrium model with endogenous technology choice is tractable, and we explore the equilibrium path analytically. The model illustrates a novel manner in which sustained per capita growth of consumption can be achieved -- through continuous adoption of new technologies featuring the substitution between capital and labor. Further insights of the model are that during the growth process, the size of firms producing final goods increases over time, the real interest rate is constant, and the real wage rate increases over time.

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Introduction

Economic growth over the past 1000 years can be viewed as sporadic. While growth rates show signs of both temporal and spatial cycles, over the past 200 years OECD countries have fared much better than others (see, e.g., Boltho and Toniolo's (1999) Table 1). Within OECD countries, productivities also differ. In his celebrated article, Young (1928) argues the reason that productivity was higher in the US than the UK lies in differences in production technologies between the two countries:

> "It would be wasteful to make a hammer to drive a single nail: it would be better to use whatever awkward implement lies conveniently at hand. It would be wasteful to furnish a factory with an elaborate equipment of specially constructed jigs, gauges, lathes, drills, presses and conveyors to build a hundred automobiles; it would be better to rely mostly upon tools and machines of standard types, so as to make a relatively larger use of directly-applied and a relatively smaller use of indirectly-applied labor. Mr. Ford's methods would be absurdly uneconomical if his output were very small, and would be unprofitable even if his output were what many other manufacturers of automobiles would call large." (Young, 1928, p.530).

Young's argument contains at least three notable elements. First, the existence of fixed costs of production ("an elaborate equipment of specially constructed jigs, gauges, lathes, drills, presses and conveyors") is highlighted. Second, firms in different countries may use different technologies—Ford Company in the US used more specialized equipment than its counterparts in the UK. Finally, a firm's technology is affected by its level of output. Since Ford Company had a higher level of output, it chose more specialized technologies. One might find it difficult to imagine that firms use the same production technology over an extended time period, however. In addition, mechanization of production in which capital substitutes for labor in production is an important feature of modern production.[1]

This paper explores the implications of firms' technology choices and the substitution of capital for labor on economic growth. We analyze two sectors: the R&D sector combines ideas and final goods to produce machines to be used for the sector producing final goods. We show that incorporating oligopolistic competition in the sector producing final goods into a general equilibrium model with endogenous technology choice is tractable, and we explore the equilibrium path analytically.

In our framework, positive growth is generated by increasing returns to scale arising from fixed costs of production.[2] In each period, a firm chooses the level of output and production technology optimally. A more advanced technology is modeled as a technology with a higher fixed cost and a lower marginal cost of production. The amount of capital accumulates over time and the amount of labor is fixed. Thus, as firms continually adopt more advanced technologies suitable for larger scale production, sustained growth is possible. Therefore, on the equilibrium growth path our model also provides an explanation for the increase of firm size over time. As increasing returns to scale is viewed as an important source of long-run growth, it is important to study how a firm's scale of production changes over time.

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[1] In an important aspect, modernalization means mechanization as shown vividly in the famous movies of Charles Chaplin. The daily language "use money to earn money" may be formulated alternatively as "use capital through machines to earn money and accumulate more money through capital accumulation." While countries in the developed world are able to use money to earn money, the developing countries may have to rely on labor to make money.

[2] With the existence of fixed costs, the market structure is imperfect competition, where firms producing the same product engage in Cournot competition. Our modeling of oligopolistic competition allows us to study firm-level behavior while maintaining consistency with the structure of many industries. For example, Pindyck and Rubinfeld (2005, p. 441) note that "oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, aluminum, petrochemicals, electrical equipment, and computers."

The model highlights the important role played by capital accumulation in the growth process. An important feature of modern production technologies is the large-scale usage of capital goods. Maddison (1982, p54) demonstrates the close relationship between economic performance and capital over the long run. In his sample of seven developed countries from 1820-1978, UK had the slowest growth in capital and the slowest productivity while Japanese capital stock and productivity showed the fastest growth. Likewise, the productivity leadership of the US since 1890 is reflected in its superior level of capital.[3]

In our model, capital accumulation is associated with investment in equipment, which is a fixed cost of production.[4] In neoclassical models, capital is usually modeled as a marginal cost of production with diminishing marginal return. With diminishing marginal return to capital, Lucas (1990, 2002) puzzles over why capital does not flow from capital rich countries to countries with much lower levels of capital. King and Rebelo (1993) argue that the real interest rate will be implausibly too high at the initial stage of growth if transitional dynamics in the neoclassical model is used to explain countries' different growth performance.

In the case of capital being a fixed cost of production, we show that different ratios of capital and labor across countries can be absorbed in different technologies chosen by such countries. As a result, the return to capital can be identical even though

[3] The importance of capital accumulation in the growth process is also demonstrated by other studies. For example, Jorgenson et al. (1987) conclude that growth in capital inputs is the most important source of growth for the US. Young (1995) argues that growth of capital plays an important role for the growth of Hong Kong, Singapore, South Korea, and Taiwan.

[4] De Long and Summers (1991) demonstrate that over 1960-1985 each extra percent of GDP invested in equipment is associated with an increase in GDP growth of one third of a percentage point per year. By focusing on developing economies, De Long and Summers (1993) find that there is a very strong growth-equipment association. Jones (1994) finds that machinery appears to be the most important component of capital and there is a strong negative relationship between growth and machinery price.

countries have very different ratios of capital to labor: there is no incentive for capital to flow from a country with a high ratio of capital to labor to a country with a low ratio of capital to labor. Thus, our model captures the important role of capital accumulation in the growth process, and it also produces key insights.

Emphasizing the role of capital accumulation does not necessarily imply that we view that technological progress is not important in the growth process. In our model, development of new technologies leads to capital accumulation. New technologies need to be embodied in machines. Complementarity between technological progress and capital accumulation is also discussed in Aghion and Howitt (1998, Chapter 3).

The remainder of our paper is organized as follows. In Section 2, we set up the model, which is a representative firm and consumer framework. In Section 3, the maximization conditions and market clearing conditions together define an equilibrium. Section 4 studies the steady-state growth path, where the growth rate of consumption is expressed as a function of exogenous parameters. Section 5 discusses why sustained growth is possible in this model, and summarizes the relationship between the growth mechanism in this paper to those in the literature. Section 6 studies the growth rate of consumption in a social optimum, where we emphasize that the growth rate of consumption in a market equilibrium is lower than the social optimum. Section 7 discusses possible extensions of the model.

2. Setup of the model

We focus on a closed economy with continuous time and constant population. To simplify notation, we suppress the time indices of variables when there is no confusion from doing so. There are L workers, who are also consumers. A worker lives indefinitely, has no preference for leisure, and supplies one unit of labor in each period inelastically. There are two sectors of production: the manufacturing sector producing final goods for consumption and the R&D sector. There is a continuum of final products in the economy with a total mass of one, indexed by a number $\varpi \in [0,1]$.

Let ρ denote the subjective discount rate and $c(\varpi)$ denote a consumer's quantity of consumption of product ϖ. A consumer's discounted utility is specified as

$$\int_0^\infty U_t e^{-\rho t} dt, \tag{1}$$

$$U_t = \int_0^1 \left(\frac{c(\varpi)^{1-\sigma} - 1}{1-\sigma} \right) d\varpi, \ \sigma > 1.\text{[5]} \tag{2}$$

A consumer chooses the quantities of consumption of different final goods to maximize utility. For this type of utility function, the absolute value of a consumer's elasticity of demand is equal to $1/\sigma$.

Each final product is produced by multiple firms and the number of firms producing the same product is denoted by m. Because there is an infinite number of final products, an individual firm's market power in the labor market is zero. In each period, firms producing final goods take the wage rate and the interest rate as given, and make decisions on the production technology and the quantity of production. The R&D sector produces new designs and combines them with final goods to produce machines, which are utilized by firms producing final goods.

We focus on symmetric equilibria, thus the number of firms producing each final product is identical. Firms producing final goods have the same level of production

[5] The assumption that $\sigma > 1$ guarantees that there are at least two firms producing the same product.

technology and the same quantity of each good is produced. In addition, all consumers have the same consumption bundle.

For the production of each final product, we assume an infinite number of technologies, indexed by n, with a higher level of n denoting a higher level of capital and a lower level of labor. The level of technology at time zero is normalized to one, thus $n \in [1, \infty)$. Capital cost arises from machine purchases, therefore it is a fixed cost of production. The marginal cost of production accrues from hiring workers. Let $f(n)$ denote the fixed cost of production, and let $\beta(n)$ denote the marginal cost of production associated with technology level n. We assume that $f'(n) > 0$, and $\beta'(n) < 0$. That is, the capital cost of production increases with n, and the labor cost of production decreases with n.

The assumption that different combinations of capital and labor may be used to produce the same level of goods is important in this paper and deserves further elaboration. Several examples of such substitution exist in modern economies. One such illustration is the technology for word processing. Previous to the invention of typewriters, word processing was a "hands only" chore, with minimal capital used. Then, typewriters were introduced, and subsequently computers became quite popular for word processing. During this process, labor is substituted increasingly with capital. Greenwood and Seshadri (2005, p.1251) show that various appliances such as washing and cleaning appliances may save a four-person family 18.5 hours a week in housework in 1920. Without such time savings, it would be difficult to envision a considerable number of two earner families in the labor force.

The aspect that technologies are embodied in machines is similar to the specification in the vintage capital models. However, in vintage capital models, the substitution between capital and labor is usually not the main focus. More importantly, we do not specify that capital produced at different times have different levels of productivity.

Let x denote a manufacturing firm's production quantity in period t and p denote the price of manufactured goods. A firm's total revenue is px. Let R denote the rental price of a unit of capital services in period t and w denote the wage rate. The firm's cost of purchasing machines is $f(n)R$ and its labor cost is $\beta(n)xw$. Thus, its total cost is $f(n)R + \beta(n)xw$, or $fR + \beta xw$. As a result, its profit in period t is

$$\pi = px - fR - \beta xw. \tag{3}$$

A dot over a variable denotes its time derivative and let δ denote a positive constant. With L_n denoting the amount of labor force employed in the development of new designs, the evolution of the number of new designs is given by

$$\dot{n} = \delta n L_n.[6] \tag{4}$$

In the R&D sector, a firm with a design has monopoly power over the use of this design. A design has to be incorporated into a machine to be useful. Let η denote a positive constant. The cost of incorporating a new design into a unit of machine is η units of final goods. Let I denote the amount of final goods used in machine production.

[6] By changing equation (4) and (5), the "scale effects" that the growth rate increases with the size of population can be eliminated in this model. Whether the scale effect assumption is consistent with empirical evidence remains an open debate. Jones (2005) argues that a weak form of scale effect is consistent with empirical evidence, while a strong form of scale effect is likely not. Barro and Sala-i-Martin (2003) and Aghion and Howitt (2005) provide detailed discussion on scale effects. Since the mechanisms leading to scale effects are well understood in the literature (Barro and Sala-i-Martin, 2003) and whether the scale effect exists is not the focus of this paper, we suppress further discussion of this issue.

The relationship between the amount of final goods used in the production of machines and the number of new designs is given by

$$\dot{n} = \frac{I}{\eta}. \tag{5}$$

3. Equilibrium conditions

We derive the equilibrium conditions in five steps. First, we study a consumer's utility maximization. Let r denote the interest rate. The following familiar Euler equation is necessary for a consumer's utility maximization:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - \rho). \tag{6}$$

Second, we study a manufacturing firm's profit maximization. The solution concept used here is Nash equilibrium. In each period, a firm producing final goods takes other firms' technology and output as given and chooses its own level of technology n and output x simultaneously to maximize profit. Since the initial technology is normalized to one and a firm's output should be positive, for $\otimes$ denoting the Cartesian product, a firm chooses $n \otimes x \in [1, \infty) \otimes (0, \infty)$. A firm chooses the level of technology optimally by taking the derivative of the first order condition with respect to n:

$$- f' R - \beta' x w = 0. \tag{7}$$

The intuition of equation (7) is as follows: a firm's choice of technology depends on the relative price of capital and labor. By adopting a more advanced technology, a firm spends more on the fixed cost of production, which is $f' R$. The benefit arises from the marginal cost of production savings, which is equivalent to $\beta' x w$. In equilibrium, these equate.

From equation (7), the second order condition requires that

$$-f''R-\beta''xw<0. \tag{8}$$

We assume that the second order condition is satisfied.

Firms producing the same product engage in Cournot competition.[7] Thus, a firm also chooses the quantity of production optimally. Taking the derivative of the first order condition with respect to x yields $p+x\dfrac{\partial p}{\partial x}-\beta w=0$.

In equilibrium, a firm producing final goods makes zero profit, a requirement that leads to

$$xp-fR-\beta xw=0. \tag{9}$$

Third, the market for manufactured goods needs to be cleared. Each final product has m firms responsible for its production, and each firm produces x units of output. Thus, the total supply of each final product is mx. The total demand for final goods is the summation of goods used for consumption and goods used in the manufacturing of machines. Equilibrium in the goods market requires that quantity supplied equals quantity demanded:

$$mx=Lc+I. \tag{10}$$

Plugging the value of I from equation (5) into equation (10) leads to

$$mx=Lc+\eta\dot{n}. \tag{11}$$

[7] Oligopolistic competition is also studied by Gali and Zilibotti (1995).

From the utility function (2) and equation (10), the elasticity of demand faced by a manufacturing firm is $\frac{\sigma x}{mx - I}$. Combining this result with a manufacturing firm's optimal choice of output leads to

$$p\left(1 - \frac{\sigma x}{mx - I}\right) - \beta w = 0\,. \tag{12}$$

Fourth, the labor market needs to be in equilibrium. The return to labor in the manufacturing sector is w. Let p_n denote the price of a new design. From equation (4), the return for labor in the R&D sector is $p_n \delta n$. Since a worker may work in either sector, the return from the two sectors is equal:

$$w = p_n \delta n\,. \tag{13}$$

The total supply of labor is L and the demand for labor is the sum of labor demand in R&D and manufacturing. The amount of labor in the R&D sector is L_n and the amount of labor in the manufacturing sector is $m\beta x$. Labor market equilibrium requires that labor demand equal labor supply:

$$L_n + m\beta x = L\,. \tag{14}$$

Finally, we study the equilibrium condition in the R&D sector. For simplicity, we assume that machines do not depreciate. An R&D firm is able to sell m machine units at a price of R. An R&D firm's revenue in each period is mR and therefore the firm's total revenue is $\frac{mR}{r}$.[8] Its cost is the sum of the cost of producing a design and the cost of incorporating this design into a machine. Thus, the profit of the R&D firm is

[8] Here we treat the interest rate, the price of machines, and the number of manufacturing firms as constant over time. This point is verified later in the balanced growth path presented in Section 4.

$\pi_R = \dfrac{mR}{r} - p_n - m\eta\ p$. With free entry and exit into the R&D sector, a R&D firm makes zero profit:

$$\pi_R = \frac{mR}{r} - p_n - m\eta\ p = 0\,. \qquad (15)$$

With the equilibrium conditions established, we now study the evolution of the economy. For the remainder of the paper, the price of final goods in each period is normalized to one: $p \equiv 1$. From equations (11) and (14), the number of R&D workers is therefore:

$$L_n = (1-\beta c)L - \beta\eta L\dot{n}\,. \qquad (16)$$

Plugging this equation into equation (4), the evolution of new designs is given by

$$\frac{\dot{n}}{n} = \frac{\delta(1-\beta c)L}{1+\delta\eta\beta n}\,. \qquad (17)$$

Since the price of final goods is equal to one, from equation (12), the number of firms producing each final product can be expressed as

$$m = \frac{\sigma(Lc+\eta\dot{n})}{(1-\beta w)Lc}\,. \qquad (18)$$

Plugging the value of R from equation (9), the value of mx from equation (11), and the value of m from equation (18) into equation (15), the interest rate is given by

$$r = \frac{\delta nLc(Lc+\eta\dot{n})}{f[wLc(1-\beta w)+\sigma\delta n\eta(Lc+\eta\dot{n})]}\,. \qquad (19)$$

Likewise, plugging the value of R from equation (7) into equation (9), the wage rate is given by

$$w = \frac{f'}{\beta f' - f \beta'}. \tag{20}$$

Plugging the value of $\dot{n}$ from equation (17), the value of interest rate from equation (19), and the wage rate from equation (20) into equation (6), the evolution of the per capita consumption is given by

$$\frac{\dot{c}}{c} = \frac{nfLc(c+\eta\delta n)(\beta')^2}{\sigma\delta\eta n(c+\eta\delta n)(f\beta' - \beta f')^2 - cff'\beta'(1+\delta\beta\eta n)} - \frac{\rho}{\sigma}. \tag{21}$$

Equations (17) and (21) define the evolution of $\dot{n}$ and $\dot{c}$ as functions of n, c, and exogenous variables. In the next section, we place restrictions on the cost functions in the sector producing final goods to derive the balanced growth rate.

4. Balanced growth path

In this section, we derive the balanced growth rate and study its properties. In a balanced growth path, the growth rate of per capita consumption is the same as the growth rate of new designs. For g denoting this common growth rate, we have $g = \frac{\dot{c}}{c} = \frac{\dot{n}}{n}$.

A balanced growth path may not exist for general cost functions. Equations (17) and (21) provide some hint about the type of cost functions for which a balanced growth path exists. In a steady state with balanced growth, the right-hand sides of equations (17) and (21) should be constants. From the evolution of new designs equation (17), the marginal cost should decrease at the same rate as the rate that new designs increase. From the evolution of per capita consumption equation (21), the fixed cost should increase at the same rate as the number of new designs increased. Thus, the relationship between the fixed cost of producing final products and the level of technology is specified as

$$f(n) = n\,. \tag{22a}$$

Let ψ denote a positive constant. The relationship between the marginal cost of producing final output and the level of technology is specified as

$$\beta(n) = \psi / n\,. \tag{22b}$$

With this combination of fixed and marginal costs, the unique balanced growth rate will be derived. As discussed in Section 5, these cost functions are useful in demonstrating the existence of a balanced growth path. However, they are not necessary for showing the feasibility of sustained per capita output growth driven by continuous adoption of new technologies and substitution of capital for labor.

With the specification of costs in equations (22a) and (22b), the profit of a firm producing final goods is $x - nR - \frac{\psi}{n} w x$. The firm's optimal choice of technology leads to

$$R - \frac{\psi}{n^2} w x = 0\,. \tag{23}$$

From equation (23),

$$fR = \beta x w\,. \tag{24}$$

Combining this with the result that a firm producing final goods has a profit of zero, the output of a firm producing final goods is given by

$$x = 2nR\,. \tag{25}$$

From equations (9) and (24), $\beta w = 1/2$. From equation (12), the number of firms producing each final product is given by

$$m = 2\sigma + \frac{I}{x}. \tag{26}$$

From equation (22b), the wage rate is given by

$$w = \frac{n}{2\psi}. \tag{27}$$

From equations (13) and (27), the price of a new design is expressed as

$$p_n = \frac{1}{2\psi\delta}. \tag{28}$$

From equation (6), the growth rate of per capita consumption is given by

$$g = \frac{1}{\sigma}(r - \rho). \tag{29}$$

Plugging equation (28) into equation (15) leads to

$$\frac{mR}{r} = \frac{1}{2\psi\delta} + m\eta. \tag{30}$$

From equation (4), the number of workers employed in the R&D sector is given by

$$L_n = \frac{g}{\delta}. \tag{31}$$

From equations (14) and (30), the interest rate is given by

$$r = \frac{\delta L - g}{1 + 2\psi\eta\delta\, m}. \tag{32}$$

Plugging equation (32) into equation (29) leads to

$$g = \frac{\delta L - \rho(1 + 2\psi\delta\eta m)}{1 + \sigma(1 + 2\psi\delta\eta m)}. \tag{33}$$

The formula of the growth rate in equation (33) is similar to that in Romer with $1 + 2\psi\eta\delta m$ replaced by Λ in Romer (1990, p. S92).

Plugging the value of I from equation (5) and the value of x from equation (14) into equation (26), the number of firms is given by

$$m = \frac{2\sigma(\delta L - g)}{\delta L - g - \eta\psi\delta g}. \tag{34}$$

Define three constants by

$$\theta_1 \equiv (1+\sigma)(1+\eta\psi\delta) + 4\psi\delta\eta\sigma^2,$$

$$\theta_2 \equiv (1+\sigma)\delta L + (\delta L - \rho)(1+\eta\psi\delta) + 4(\sigma\delta L - \rho)\psi\delta\eta\sigma,$$

$$\theta_3 \equiv (\delta L - \rho - 4\psi\delta\eta\sigma\rho)\delta L.$$

The degree of efficiency in the R&D sector as measured by δ should be sufficiently high for the balanced growth rate to be positive. If there is positive growth, the following proposition expresses the steady-state growth rate as a function of exogenous variables.

Proposition 1: The balanced growth rate is given by

$$g_{Market} = \frac{\theta_2 - \sqrt{(\theta_2)^2 - 4\theta_1\theta_3}}{2\theta_1}. \tag{35}$$

Proof: Plugging equation (34) into equation (33) leads to $\theta_1 g^2 - \theta_2 g + \theta_3 = 0$. This leads to equation (35). Q.E.D.

The growth path in equation (35) shows the impact of various factors on the growth rate. First, the growth rate decreases with the cost of incorporating a design into a machine. Second, the growth rate decreases with the discount rate. Third, the growth rate decreases with the elasticity of demand. The intuition behind this result is that a higher elasticity of demand leads to a larger number of firms producing the same product. With the existence of fixed costs of production, a larger number of firms increases the average cost. In addition, for a firm producing final goods, its output decreases with σ. Ceteris paribus, this also increases average cost. Thus, the growth rate decreases with the elasticity of demand.

We have shown that there exists a steady state in which the per capita consumption grows at a constant rate. As capital grows, the real interest rate does not change, and the wage rate increases over time.[9] Thus, capital becomes relatively cheaper than labor, resulting in firms using more capital. It is well recognized that a worker in a developed country has more capital to work with and her productivity is higher than her counterparts in developing countries. In their study of the process of how the West grew rich, Rosenberg and Birdzell (1986, p. 16) write "the main thrust of capitalist development has been toward capital-intensive production." Thus, our result shares a consistency with extant empirical evidence.

The following proposition shows that with internal increasing returns to scale, a firm's output increases over time.

Proposition 2: *A manufacturing firm's output increases during the growth process.*

Proof: From equations (14) and (22b), a firm's output is given by

$$x = \frac{(L - L_n)n}{m\psi}. \tag{36}$$

Since L, L_n, and m are constants in a steady state, a firm's level of output increases over time as it grows at the rate of new designs. Q.E.D.

There is considerable empirical evidence that suggests firm size increases over time. One example is the increase of firm size in the agricultural sector. Compared with many manufacturing sectors, the agricultural sector is relatively less concentrated. Even in this sector, Suits (2005, p.16) shows that average acreage per farm in the United States has increased significantly over time. Average acreage per farm in 1880 was 133.7 and it increased to 174.5 in 1940, and 434.0 in 2000. Thus, by replacing perfect competition with oligopolistic competition in the sector producing goods for final consumption, we are able to capture a salient feature of the growth process.

A further interesting issue concerns the distribution of income between labor and capital in the economy. For the value of output px, $\beta x w$ goes to labor and the remainder to capital. Since the price of manufactured goods is normalized to one and βw equals ½, we have

$$\frac{\beta x w}{px} = \frac{1}{2}.$$

Thus, the share of labor income to total output is constant over time. As a result, the share of payment to capital is also constant over time.

5. Discussion and relation to the literature

In this section, we provide an alternative specification of the form of fixed and marginal costs in the sector producing final goods to provide a more thorough understanding of the growth mechanism. The cost functions are chosen to make the presentation and intuition perspicuous, sacrificing generality. We then discuss the relation between this model and other growth models in the literature.

For a and b denoting positive constants, the fixed and marginal costs of producing final goods are specified as

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[9] In Kaldor (1961), a stylized fact about growth is that the interest rate is constant over time.

$$f(n) = n^a , \tag{37a}$$

$$\beta(n) = \psi \, n^{-b} . \tag{37b}$$

Thus, the cost functions in equations (22a) and (22b) in Section 4 are the special case that a and b are restricted to be one.[10]

For a firm producing final goods with output x, the amount of labor l used in production is βx, or $l = \psi \, n^{-b} x$. Thus, the relationship between a firm's output and the amount of labor it uses is given by

$$x = \frac{n^b}{\psi} l . \tag{38}$$

For this firm, the amount of capital k used in production is equal to f, or $k = f(n) = n^a$. Thus, the relationship between the level of technology and capital is given by

$$n = k^{\frac{1}{a}} . \tag{39}$$

Inserting the value of technology from equation (39) into equation (38), output per worker is given by

$$\frac{x}{l} = \frac{1}{\psi} k^{\frac{b}{a}} . \tag{40}$$

Depending on the relative magnitudes of a and b, when the amount of labor is constant while capital accumulates, from equation (40), there are three cases. First, for $b > a$, output per worker grows at an increasing rate. Second, for $b = a$, output per worker grows at a constant rate. Third, for $b < a$, output per worker grows at a

[10] One must take care to check the specification of costs in equations (37a) and (37b) to ensure that the second order condition (8) is satisfied. From (7), the unit cost of capital is $R = -\beta' x w / f'$. Plugging this into (8), $\beta'' f' - f'' \beta' > 0$ is necessary for the second-order condition to be satisfied. For cost functions

decreasing rate. Whenever $b \geq a$, sustained per capita consumption growth is possible. Intuitively, if the rate of increase of fixed costs as measured by a is not larger than the rate of decrease of marginal costs as measured by b, sustained growth is feasible. Thus, the specification of cost functions in Section 4 is useful for the existence of balanced growth path, not necessarily for the feasibility of sustained growth.

Barro and Sala-i-Martin (2003) provide a thoughtful synthesis of the literature on economic growth. In the literature, it has been shown that long-run growth may be a result of R&D spillovers (Romer 1990), externalities in investment in human capital (Lucas 1988), or constant returns to scale to capital as in the AK type growth models (see, e.g., Romer (1986), and Rebelo (1991)).

In Lucas (1988), there are externalities in the accumulation of human capital. In Romer (1990), a firm's R&D generates knowledge that is exploited by other firms. The increasing returns to scale arise from the increased usage of intermediate inputs, which is external to the firm, but internal to the industry. In this paper, sustained growth is achieved through substitution of capital for labor. In Romer (1990), the growth rate is not affected by η, the parameter measuring R&D efficiency. While Romer argues that in a general setup, the impact of this parameter on the growth rate is ambiguous, the growth rate increases with this parameter in our model. The direction of the impact of the discount rate and the elasticity of demand on the growth rate here is the same as in Romer (1990).

From equation (40), when a and b are equal to unity, per capita output is a linear function of capital. This aspect is similar to AK type models. As discussed in Barro and

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given in equations (37a) and (37b), $\beta'' f' - f'' \beta' > 0$ requires that $a + b > 0$. Since a and b are positive

Sala-i-Martin (2003), for balanced growth to be feasible, the reduced form of the production function of various growth models has the feature that per capita output is a linear function of some factors that may accumulate without an upper bound, such as physical capital, human capital, a combination of human and physical capital, or the number of varieties to produce final goods. In the AK model, the marginal productivity of capital A is treated as a constant and unexplained. In this model, the marginal productivity of capital is measured as the total amount of labor saved, which is the product of the amount of labor saved for each unit of output and the level of output.

There are two important building blocks in this paper: substitution between capital and labor, and the choice of technology. First, Arrow et al. (1961) contains a detailed discussion of the substitution between capital and labor, and includes estimates of the degree of substitution for various industries. The substitution between capital and labor has been explored in Zeira (1998) in which firms adopt technologies employing more capital as the economy grows. There are several significant differences between our paper and Zeira's (1998). In this paper, we solve the general equilibrium model when there is increasing returns to scale at the firm level and firms engage in oligopolistic competition. Importantly, the interest rate is endogenously determined and our model is able to generate sustained growth. Zeira uses a partial equilibrium framework in which the interest rate is exogenously given, production technology exhibits constant returns to scale, and firms engage in perfect competition. Accordingly, Zeira's model cannot generate sustained growth.

Second, economic growth is associated with the continuous adoption of new production technologies. Bencivenga et al. (1995), Parente (1994), and Jovanovic and

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constants, the second order condition (8) is always satisfied.

Nyarko (1996) also study the choice of technologies during the growth process. Their focuses are very different from our contribution.[11] In this paper, the choice of technology is closely related to a firm's level of output.

6. Socially optimal choice of technology

In our model we assume that firms in the R&D and final goods sectors make decisions to maximize profits. In this section, the social optimum growth path is studied. The properties of the social optimum are interesting in their own right. In addition, the social optimum serves as a benchmark to compare to the market outcome.

The social planner chooses the number of workers to be employed in the R&D sector, the level of production technology, and output in the final goods sector optimally to maximize a representative consumer's utility function (1). To make the social optimum comparable with the market outcome, the fixed and marginal costs of production are isomorphic to equations (22a) and (22b) in Section 4. Since there are fixed costs of production, the social planner will not allow more than one firm producing in a given industry. That is, $m = 1$. For $\sigma > 1$, maximization of $\frac{c^{1-\sigma} - 1}{1-\sigma}$ is the same as minimization of $c^{1-\sigma}$, and the social planner faces the following minimization problem.

$$\textit{Minimize:} \qquad \int_0^\infty U_t e^{-\rho t} dt \ , \tag{1}$$

$$\textit{subject to} \qquad \dot{n} = \delta n L_n \, , \tag{4}$$

$$f(n) = n \, , \tag{22a}$$

[11] In Bencivenga et al. (1995), the optimal choice of finance technology is affected by the transaction cost of financial services. When transactions cost decrease, more illiquid capital investment is pursued. Parente (1994) and Jovanovic and Nyarko (1996) study the situation that new technologies may be continuously adopted during the growth process. In their models, there is learning by doing. The potential of learning by doing decreases as a technology is used, which motivates a firm to adopt new technologies.

$$\beta(n) = \psi / n, \tag{22b}$$

$$L_n + \beta x = L, \tag{41}$$

$$\dot{n} = \frac{x - Lc}{\eta}. \tag{42}$$

Equation (41) is similar to equation (14) with the additional restriction that there is only one firm producing each final product. Equation (42) is the equation for the evolution of new machines. Total output of final goods is x, after deducting the amount for consumption Lc, the amount of final goods available for producing machines is $x - Lc$. As each unit of machine needs η units of final goods to be produced, the rate of change of new machines is given by the right-hand side of equation (42).

The following proposition shows the unique optimal growth rate of consumption. In proving this proposition, the social planner's minimization problem is solved by the method of calculus of variations. There are three steps in solving the problem. First, the minimization problem with constraints is transferred into a minimization problem without constraints. Second, it is shown that the necessary conditions for minimization lead to a unique growth path. Third, it is shown that these necessary conditions for optimization are also sufficient for the social optimum.

Proposition 3: *The growth rate of consumption in the social optimum is given by*

$$g_{Optimum} = \frac{1}{\sigma}\left(\frac{\delta L}{1+\delta\psi\eta} - \rho\right). \tag{43}$$

Proof: First, plugging equations (4), (22b), and (41) into equation (42) yields

$$c = \frac{n}{\psi} - \left(\frac{1}{\psi\delta L} + \frac{\eta}{L}\right)\dot{n}. \tag{44}$$

To simplify notation, define a constant by

$$\gamma \equiv \frac{1}{\psi\delta L} + \frac{\eta}{L}. \tag{45}$$

For γ defined in equations (45), the social planner faces the following minimization problem.

$$Minimize: \int_0^{\infty} \left(\frac{1}{\psi}n - \gamma \dot{n}\right)^{1-\sigma} e^{-\rho t} dt. \tag{46}$$

Two dots over a variable denote its second order time derivative. Second, for the minimization problem (46), the following Euler equation is necessary,

$$\sigma\gamma^2\psi^2 \ddot{n} + (\rho\psi^2\gamma^2 - \gamma\psi - \sigma\gamma\psi)\dot{n} + (1 - \rho\gamma\psi)n = 0. \tag{47}$$

There are two characteristic roots for equation (47): one is $\frac{1}{\psi\gamma}$, and the other is $\frac{1}{\sigma}\left(\frac{1}{\psi\gamma} - \rho\right)$. For the characteristic root $\frac{1}{\psi\gamma}$, it leads to per capita consumption to zero and is discarded. With the initial condition that the level of technology at time zero is normalized to one, the solution for equation (47) is

$$n = e^{\frac{1}{\sigma}\left(\frac{1}{\psi\gamma} - \rho\right)t}. \tag{48}$$

By inserting equations (45) into equation (48), the social optimal growth rate is identical to equation (43).

Third, define the integrand of (46) as $H \equiv \left(\frac{1}{\psi}n - \gamma \dot{n}\right)^{1-\sigma} e^{-\rho t}$. Differentiation of H leads to

$$\frac{\partial^2 H}{\partial^2 \dot{n}} = \sigma(\sigma - 1)\gamma^2 \left(\frac{1}{\psi}n - \gamma \dot{n}\right)^{-1-\sigma} e^{-\rho t},$$

$$\frac{\partial^2 H}{\partial^2 n} = \frac{\sigma(\sigma-1)}{\psi^2}\left(\frac{1}{\psi}n - \gamma \dot{n}\right)^{-1-\sigma} e^{-\rho t},$$

$$\frac{\partial^2 H}{\partial n \partial \dot{n}} = \frac{\sigma(\sigma-1)\gamma}{\psi}\left(\frac{1}{\psi}n - \gamma \dot{n}\right)^{-1-\sigma} e^{-\rho t}.$$

Thus, $\frac{\partial^2 H}{\partial^2 \dot{n}} > 0$, and $\frac{\partial^2 H}{\partial^2 \dot{n}}\frac{\partial^2 H}{\partial^2 n} - \frac{\partial^2 H}{\partial \dot{n} \partial n}\frac{\partial^2 H}{\partial n \partial \dot{n}} = 0$. As a result, H is convex in $(n, \dot{n})$, which means that the necessary conditions for optimization are also sufficient.

Q.E.D.

This proposition naturally leads to a proposition comparing the growth rate of consumption in a market equilibrium with the social optimum:

Proposition 4: *The growth rate of consumption in a market equilibrium is lower than the social optimum.*

Proof: If the social optimal growth rate is positive, we have $\delta L > \rho(1 + \delta \psi \eta)$. From equation (33), the growth rate in a market economy decreases with the number of firms. From equation (34), the number of firms in a market economy is higher than 2σ. Since $\sigma > 1$, from equations (31) and (43), $g_{market} < g_{Optimum}$. Q.E.D.

The intuition behind Proposition 4 is as follows. As firms have market power, prices charged by firms are higher than the marginal cost of production. Also, there are multiple firms producing the same final product in a market economy. As a result, the market level of output is less than the social optimum. As the market level of output is not optimal, the market level of technology will not be optimal since the choice of technology depends on the output level. As a result, the growth rate of consumption in a market equilibrium is lower than the social optimum.

7. Concluding remarks

This paper explores the implications of firms' technology choices and the substitution of capital for labor on economic growth. We show that considering oligopolistic competition in the sector producing finals goods in a general equilibrium model with the microeconomic feature of firms' technology choice is tractable and we explore the equilibrium path analytically. To produce a given level of output, different combinations of capital and labor may be used. As capital accumulates, technologies employing more capital are adopted. In this model, increasing returns to scale arises from the fixed cost of production embodied in machines and it is internal to a firm. Incorporating fixed costs into the study of economic growth leads to some empirically plausible implications. First, during the growth process, the real interest rate is constant and the real wage rate increases. Second, the output of firms producing final output increases over time. Finally, the reduced form of the production function is similar to that of the AK type models. Here the marginal productivity of capital is measured by the amount of labor saved.

There are some interesting generalizations and extensions of our model. First, to apply this model in analyzing a country's growth over time, a calibration can be performed. Second, this paper studies economic growth in a closed economy. Studying economic growth in an open economy is an interesting avenue for future research. In an open economy model, the interaction between trade and growth can be addressed explicitly.

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