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AN ANALYSIS OF POSTWAR U.S. CONSUMPTION AND SAVING: Part I
The Model and Aggregation

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ABSTRACT

A new empirical analysis of aggregate United States consumption and saving for the period 1947-80 is presented. The model is based on the theory of exact aggregation. It recognizes explicitly that households with different characteristics may be heterogeneous in their behavior and that aggregate behavior may depend on the changing composition of households by characteristics and therefore may not be adequately portrayed by a representative consumer, but otherwise it imposes minimal assumptions on household behavior. The model integrates longitudinal and cross-sectional microeconomic data on household characteristics with the traditional aggregate time-series data. Various hypotheses on consumption, such as age independence, proportionality to wealth, and price independence, are tested and rejected. Strong evidence of relative price effects and a systematic variation of aggregate consumption with changing age distribution of wealth in the economy is found. Especially important is the substantial estimated difference in the shares of wealth consumed between households headed by persons born prior to and those born after 1939. One important lesson from this study is that modeling the aggregate U.S. economy as a representative consumer may give rise to misleading results.

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1. Introduction

The consumption and saving behavior of an economy reveals much about the nature of its society, since it reflects its values, institutions, incentives, and demographics. However, a large number of conceptual and empirical issues cloud analyses and interpretations of postwar U.S. consumption and saving. It is by now well known that, as conventionally measured, the postwar U.S. saving rate is low by international standards, and that it has fallen since the 1950s and 1960s. Simultaneously, the average annual rate of growth in real consumption rose from 2.74% in the period 1950-1962 to 3.45% in the period 1963-80, an increase of 25%. Had consumption continued growing at its slower early postwar pace, annual real consumption would have been at least 10% less by 1980 than in fact occurred. The difference amounts to approximately 8% of GNP (or approximately double the annual federal budget deficit). The cumulative value of this extra consumption from 1963 to 1980 rivals the value of output lost in all postwar recessions combined. What were the proximate causes of these changes?

Among the more interesting and important empirical questions are the extent to which changes in relative prices (e.g., real after-tax rates of return to saving) and demographics (as reflected by the shares of wealth held by households headed by persons of different ages and vintages) affect aggregate consumption. These issues are important as an explanation of recent economic history, and possibly as a guide to understanding the factors that may affect the future course of consumption and saving and the implications of alternative economic policies. The effects of public debt and Social Security on the level of private saving, the "interest elasticity" of saving, and hence the optimal tax treatment of saving, are not only current policy issues but have been the subject of much analytical and empirical research in the past decade.

Since the aggregate saving rate is the net result of adding the saving done by all households doing positive saving and subtracting the dissaving of all households dissaving, analyses of aggregate consumption and saving must also come to grips with the problem of aggregation to the extent that the households are heterogeneous (see Gorman (1953), Lau (1982), and Jorgenson, Lau, and Stoker (1980, 1982)).

Much recent research on saving behavior has focused on tests of the leading models of saving behavior such as the lifecycle hypothesis (LCH) or the intergenerational altruism model (IGA), also known as the Ricardian equivalence hypothesis. This is not our purpose here. Rather, it is to identify, analyze and account for empirical regularities, if any, in the postwar consumption and saving behavior of U.S. households. We do this by merging longitudinal and cross-sectional microeconomic data on various household characteristics, such as age cross-tabulated by income (or wealth), with aggregate time-series data such as those on consumption, income, wealth, prices, wage rates and interest rates. In the process, we also develop measures of and trends in aggregate household wealth and its composition, including the shares of wealth held by households headed by persons of different ages. We then analyze the share of aggregate wealth consumed in current purchases of goods and services and also of leisure.[1] We do this in a model that imposes minimal behavioral assumptions on the households. We maintain that household budget constraints are identically satisfied and households have "no money illusion", but we do not

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1. Leisure per person per year is measured as the difference between the maximum of hours available, defined to be 4400 hours per year, and the actual number of hours worked.

assume utility maximization on the part of the households, nor any functional restrictions on the structure of household demands, such as inter- and/or intra-temporal separability (which have been prevalent in the tests of the lifecycle and intergenerational altruism models).

Instead, we ask the straightforward questions: Can the integration of these longitudinal and cross-sectional microeconomic data with the aggregate time series data, subject to our minimal behavioral restrictions and "exact aggregation" conditions, provide an adequate empirical model of postwar U.S. consumption and saving? And if so, can the model be used to assess the importance of various factors affecting aggregate consumption and saving in the postwar period and to account for the growth of aggregate consumption on the basis of these factors? One might view this work as an integration of traditional aggregate time series consumption function estimation and growth accounting, but incorporating microeconomic data, paying attention to the development of age-specific household balance sheets, and applying the theory of exact aggregation.[2] We test for the effects on aggregate consumption of some potentially important factors such as the age distribution of wealth, relative prices, and vintage.[3]

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2. We also include leisure, which is generally omitted from most studies of aggregate consumption, in our study.

3. A cautious interpretation would be that our results provide us a good explanation of the aggregate consumption and saving behavior and thus represent a useful reconstruction of recent economic history. A less cautious interpretation, the plausibility of which we leave to the reader, is that the results can be given a structural interpretation which bears on some of the issues raised above. While we are sacrificing some of the advantages associated with assuming an imposed structure such as utility maximization by a representative consumer with intertemporally additive and stationary preferences, we gain substantial flexibility at the expense of some added caution in interpreting the results.

The results themselves are striking: we find substantial age effects on consumption and leisure, which suggest that policies which shift resources among age groups are likely to affect aggregate consumption and saving; the "interest elasticity" of aggregate saving[4] is -0.5 in 1972 and 0.5 in 1980 with wealth held constant, but essentially zero in 1972 and 0.1 in 1980 when one includes the Summers' effect, i.e., the revaluation of human wealth when real after-tax rates of return change. We find a significant vintage effect, i.e., households headed by persons born since 1939 consume a much larger fraction of their wealth than persons born prior to 1939 at <u>the same age</u>, other things being equal. We also present formal tests of various hypotheses concerning aggregate consumption and leisure: unitary wealth elasticity; proportionality of expenditures to wealth; intertemporal separability of household demand functions; and the absence of real interest rate and relative price effects. All of these hypotheses, with the exception of intertemporal separability, can be rejected at the 1-percent level of significance. Each of these results is individually quite interesting and important. Together, they supplement previous empirical research and we believe add an important new insight to the understanding of aggregate consumption and saving behavior in the postwar United States.

Our paper is organized as follows: Section 2 presents a brief review of the recent literature on the determinants of aggregate consumption and saving.

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4. The "interest elasticity" of saving varies considerably across age cohorts and across households with different ratios of nonhuman to human wealth. These elasticities are evaluated at the 1972 and 1980 values of the independent variables respectively. See Section 4 below.

It is not meant to be exhaustive, but to enable the reader to place the novel aspects of our research in perspective, with respect to both their strengths and potential limitations. We discuss issues of aggregation, age effects, relative price effects, estimates of the intertemporal elasticity of substitution and/or interest elasticity of saving, etc. We find that various approaches have both strengths and weaknesses, and that this is inherently true of virtually any approach to analyzing the aggregate data. Our conclusion is that while our approach is not free of its own limitations, it does add considerable novelty in terms of the construction and treatment of the data and the specification of the econometric model of aggregate time series consumption behavior.

Section 3 presents the basic structure, rationale, and maintained hypotheses of the model. We discuss in some detail the theory of exact aggregation and our use of it.[5] Also presented are the comparative static effects on consumption, leisure and saving of changes in factors such as the prices of consumption and leisure, wealth and real after-tax rate of interest. A discussion of the limitations of the model, including its partial equilibrium nature, is given. Readers not interested in the technical details of the derivations may skip to Section 4.

Section 4 presents our empirical results. We begin with a brief description of the data, which include aggregate data taken from relatively traditional sources and our integration of various longitudinal and cross-sectional age-specific individual household data with the aggregate data.

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5. For a detailed discussion of the theory of exact aggregation, see Lau (1988).

We report the model specification, parameter estimates, and the fit of the econometric results. In the aggregate, the model fits the data quite well by the usual statistical criteria. We also discuss the results of the statistical tests. We interpret the estimates of some important parameters, such as the interest elasticity of saving, defined in various ways, and for alternative values of other variables.

We also compare our formulations and results to other typical aggregate time series results. This is important not only to clarify the nature of our results, but also to facilitate the comparison of our results to those of the more traditional approach which analyzes aggregate consumption as a function of aggregate income and possibly nonhuman wealth, since we analyze the shares of aggregate wealth consumed, where wealth includes human and nonhuman wealth, as a function of aggregate wealth and other variables.

We also discuss the implications of the estimated age profile of consumption and saving. It follows a pattern that is consistent with a weak form of the "hump saving" theory of Harrod (1948) and the insight of the lifecycle hypothesis that the propensity to consume varies with age. We also highlight the vintage effect, in which households headed by persons born since 1939 have a substantially higher consumption, and hence lower saving, propensity than those born prior to 1939, other things being equal. Among other implications, these results suggest that the typical "representative consumer" models estimated on aggregate time-series data may be quite misleading. We discuss the implications of this demographic feature of saving behavior for the future of aggregate saving in the United States, and pose some puzzles related thereto.

Section 5 provides an interpretation of our results. We decompose the growth in aggregate real consumption in the United States for the period 1950-1980 into components corresponding to the various factors affecting the growth of consumption, such as population and household growth, and changes in the average real wealth per household, the age distribution of wealth, real after-tax wage rates, real after-tax rates of return, female labor force participation rates, the unemployment rate, the share of wealth held by households headed by persons born prior to 1939, etc. These decompositions reveal interesting features of the factors associated with the postwar growth of aggregate real consumption in the United States and, in addition, illuminate the comparison of real consumption growth over different time periods in which the trends in these factors may differ substantially.

Section 6 presents a discussion of our results in the context of various analytical, empirical, and policy issues. We discuss the basic conclusions of our research in relation to other previous research, including both the advantages and limitations of these new results and an agenda for future research along these lines to complement other approaches to research on aggregate consumption and saving behavior.

A brief discussion of our data and methods is presented in the Appendix.[6]

2. A Brief Review of Research on Consumption and Saving

In the last fifteen years, there has been an explosion of research on consumption and saving behavior. This research has attempted to achieve several goals, explored several different types of data and employed alternative methodologies. It cannot be our purpose here to survey any of these lines of research fully. Further, while the research ought to be complementary among the

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6. A more detailed discussion of the data is available from the authors on request.

different lines, often the work within a particular methodological approach either ignores or is just unsympathetic to the results of research of other approaches. Without commenting further on this issue, we turn to a brief discussion of each of these lines of research.

First, there has been a substantial explosion of research along what might be termed extension of traditional time series aggregate consumption function estimation. Whether or not the lifecycle hypothesis of Modigliani and Brumberg (1954) is maintained in the specification of the consumption relation, the basic approach has been an attempt to expand the traditional set of factors thought to influence aggregate consumption in a given period, and to derive better measures of, or proxies for, the relevant variables such as permanent income, wealth, the government budget deficit, or other fiscal and monetary variables. Particularly influential has been a series of studies of the effect of expected future Social Security benefits on saving, beginning with Feldstein (1974). While the debate is hardly over, we believe that a sensible reading of the evidence would be that each dollar increase in expected future Social Security benefits results in a decrease in private saving of about twenty-five to fifty cents. The interested reader should consult Feldstein and Barro (1978), Darby (1979), Leimer and Lesnoy (1982), Bernheim (1987) and the methodological criticism of Auerbach and Kotlikoff (1983).

The interest in the effects of fiscal policy on consumption was given added impetus by the modern restatement of the Ricardian equivalence hypothesis by Barro (1974). This model, also known as the intergenerational altruism model, has the striking time series implication that changes in the age distribution of resources should not affect aggregate consumption, conditional on aggregate resources, because transfers among cohorts will result in exactly offsetting consumption and saving behavior so as to maintain aggregate consumption constant. Moreover, under certain conditions, a shift from tax to debt finance, given the level of government spending, will not affect consumption or national saving as government borrowing would be offset by increased private saving in anticipation of increased future tax liabilities. Numerous time series consumption function studies of this phenomenon and attempted tests of the Ricardian equivalence hypothesis are surveyed and criticized in Barth, et al (1984) and Bernheim (1987). Two important papers are those of Feldstein (1982) and Kormendi (1983) which disaggregate fiscal variables and measure them in several ways. Boskin (1987) extends the measurement of government deficits and debt in several important ways and incorporates these in estimates of consumption functions and in analyses of the effect of deficits on the composition of GNP. While the results of these studies are not uniform, and are subject to some methodological criticisms (see especially the discussion in Bernheim (1987)), the bulk of the research results tends to reject a strict interpretation of the Ricardian equivalence hypothesis. Most studies also reject complete Keynesian myopia and estimate that future taxes are partially anticipated. Perhaps a quasi-consensus estimate is that a one-dollar debt for tax substitution would increase consumption about thirty to forty cents (see Boskin (1988)).

A more direct test, less susceptible to some of the criticisms of the traditional time series consumption functions, is performed by Boskin and Kotlikoff (1985). They build a finite approximation to an intergenerationally altruistic infinitely-lived optimal consumption program and test whether the age distribution of resources affects consumption. One of the striking implications of the Ricardian equivalence hypothesis is that the age distribution of resources should not affect aggregate consumption, given the aggregate level of resources. Boskin and Kotlikoff (1985) reject this implication of the Ricardian equivalence hypothesis.

Of course, the lifecycle hypothesis and the Ricardian equivalence hypothesis, while conflicting theories of aggregate consumption and saving behavior with drastically different implications for the efficacy of fiscal policy, do not exhaust the potential set of possibilities. The strict lifecycle hypothesis with an expected average propensity to consume over the lifecycle of unity -- i.e., no planned bequests -- is also usually rejected by the data in two types of tests. It is important to note that it is usually this strict form of the lifecycle hypothesis, not the potential insight of consumption smoothing or the age distribution of resources affecting aggregate consumption (with or without a planned bequest motive), and therefore, the potential for fiscal policy to affect consumption and national saving, that is being tested. A weaker form of the lifecycle hypothesis with planned bequests or some convex combination of the lifecycle hypothesis and other models of saving could still leave some role for fiscal policy.

The two types of tests of the lifecycle hypothesis are based on the saving or dissaving behavior of the elderly and existence or non-existence of "forward-looking" behavior by consumers in time series studies. Basically, the first type of test attempts to see how wealth varies with age. The strict form of the lifecycle hypothesis suggests that the elderly should be dissaving. Mirer (1979), Darby (1979), David and Menchik (1981), Danziger, et al (1982), and Kurz (1984) all report results from cross-section data that the elderly seem not to dissave, and in fact, may continue to save. This empirical finding has questioned the applicability of the strict form of the lifecycle hypothesis. Related studies attempting to examine consumption and earnings paths of households to see if aggregate saving can account for a substantial fraction of the capital stock also typically reveal that there is a large unexplained residual (see Kotlikoff and Summers (1981)).[7]

Bernheim (1984) and Diamond and Hausman (1984) use longitudinal data to examine the extent to which the elderly save or dissave. Both studies find that the elderly do dissave after retirement, although the extent of dissaving is not large in all cases. Hurd (1987) presents an interesting analysis in which the retired elderly in the sample dissave, and therefore, the wealth-age relationship of the elderly is consistent with the strict lifecycle hypothesis. An interesting test for a bequest motive is whether the saving of the elderly who have living children differs from those who do not. Hurd finds no evidence for a bequest motive via differential saving of those who have living heirs.

The second type of test stems from the pioneering work of Hall (1978) and Sargent (1978). These studies use the estimated Euler equations derived from the first-order conditions for optimal consumption behavior under uncertainty. As noted by Hall, this suggests that consumption should evolve as a random walk or that changes in consumption should not be predictable. Hall (1986) further observes that "the empirical work testing this proposition says, in sum, that consumption is fairly close to a random walk, but certain variables have enough predictive power that the hypothesis is rejected in formal statistical tests." These studies estimate the parameters of a stochastic difference equation for consumption, in which the influence of wealth and income on consumption should be zero. The basic question is often interpreted as whether there is an excess sensitivity of consumption to income which cannot be explained by people fully

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7. This study contained a mathematical error which when corrected would increase the fraction of the capital stock which can be accounted for by lifecycle saving from 20% to 50% -- still far less than the total.

rationally optimizing over a long time horizon. Among the more influential papers in this tradition are Flavin (1981), Hayashi (1982, 1985), and the microeconomic data exploration of Hall and Mishkin (1982), which concludes that about four-fifths of consumers could be modeled as if they were maximizing over a long time horizon, whereas one-fifth could not. The traditional interpretation of these results is a test for liquidity constraints (rather than following rules of thumb or some other interpretation).

The Euler equation approach involves several important advances, especially the ability to circumvent the thorny issue of measuring permanent income. However, most of these studies assume rather strict maintained hypotheses, e.g., maximization of a utility function (usually taken to be intertemporally additive and stationary), that the econometrician can specify the information set available to consumers at each point in time, or more precisely, that innovations in information can be accurately measured and modeled. For example, the response to a change in fiscal policy may depend upon the entire previous history of certain variables, as these may determine the subjective probability distribution of future fiscal decisions as seen by the consumer. Even more importantly, in aggregate time series studies, it is usually assumed that the economy can be modeled as a single representative consumer. This notion has come increasingly into disrepute for a variety of reasons. One is the vast array of empirical results suggesting substantial heterogeneity in saving behavior; the theoretical underpinnings, e.g., of the lifecycle model, suggest that differences in age may matter; another is the studies finding a substantial fraction of the general population liquidity constrained (Hall and Mishkin (1982)), and of the elderly liquidity constrained (Hurd and Boskin (1984)). Some analyses are beginning to explore the ramifications of heterogeneity, e.g., in wages or wage prospects and their implications for liquidity constraints (Hubbard and Judd (1987), for example). The influence of this offshoot of the rational expectations hypothesis from macroeconomic theory is substantial, but the empirical usefulness of the aggregate time series studies is clouded by the aggregation issue; when there are two or more types of consumers, the estimation procedure breaks down and the simple interpretation given to the results is inappropriate.

There have also been several attempts to examine the theoretical underpinnings of the lifecycle hypothesis and their implications for aggregate consumption and employ various advances in technique in reestimating aggregate consumption relations. Particularly important are the papers of Blinder and Deaton (1985) and Deaton (1986). These papers indicate that aggregation is important to issues of interpretation of the effects of various variables on aggregate consumption, such as interest rates; that distinguishing between fiscal actions perceived as temporary rather than permanent is important; and that model specification may have much to do with estimates of the degree of tax discounting.

Much emphasis has been placed on the degree of intertemporal substitutability or in more traditional terminology, the "interest elasticity" of saving. Boskin (1978), Summers (1981, 1982, 1984), Hansen and Singleton (1983) and Hall (1985) are the most often quoted studies, and come to rather different conclusions based on their different methodologies and data. In the traditional aggregate time series framework, small and statistically significant positive interest elasticities of saving are found in Boskin (1978), somewhat larger ones by Summers (1982). Howrey and Hymans (1980) criticize some of this work although they focus on only a very small component of personal saving. The issue of the degree of intertemporal substitutability is important to real business cycle theory and has been a subject of much debate. Hansen and Singleton (1983) estimate substantial intertemporal substitutability, whereas Hall (1985) questions this conclusion and finds little evidence of intertemporal substitutability. Whether one wants to accept all of the maintained hypotheses in estimating the intertemporal substitutability parameters (such as utility maximization itself, or the usual intertemporally additive and stationary functional form) and the methodology used is an open question. Problems also arise in the interpretation of studies estimating interest elasticities of consumption in structural consumption functions, since careful attention is not always paid to a precise definition of the conceptual experiment under consideration, such as what is presumed to be constant or allowed to vary.

Perhaps a tentative summary of recent research on consumption and saving would include the following:

(1) No single model of consumption behavior, for example, the strict lifecycle hypothesis or the Ricardian equivalence hypothesis, is sufficient to explain aggregate consumption fully. Both theories are strongly rejected in studies based on aggregate time series data, and the former is strongly rejected in most studies of the dissaving behavior of the elderly.[8]

(2) There is substantial heterogeneity among consumers. This heterogeneity may be a function of age, the steepness of earnings and desired consumption profiles leading to liquidity constraints, or a host of social, psychological, environmental, historical, and economic variables.

(3) The age distribution of resources, given their aggregate level,

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8. See, however, the important exception of the recent findings of Hurd (1987).

appears to affect aggregate consumption.[9]

While substantial controversy still surrounds these issues, recent methodological, data, and measurement advances hopefully will allow us to improve our understanding of them. This paper is one attempt to do so. Our results complement the strands of research described above without necessarily maintaining the standard hypotheses contained within each of them.

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9. See, for example, Boskin and Kotlikoff (1985).

3. The Model

3.1 Specification of the Variables

We begin by considering the determinants of the values of consumption (including both goods and services) and leisure expenditures of the current period, tth, appropriately imputed as necessary, of an individual, say the ith, household. Real consumption expenditure of the ith household in the tth period, C_{it}, is assumed to be a function of the spot prices of consumption, p_t, and leisure (or equivalently the after-tax spot wage rate) w_{it}, of the current, tth, period and the forward prices of consumption, $p^*_{t,t+t'}$, and leisure, $w^*_{it,t+t'}$, of future, t+t'th, periods in the current period, total wealth W_{it}, consisting of human (HW_{it}) and nonhuman wealth (NHW_{it}) of the ith household in the current period and the value of its set of demographic attributes, A_{it}, which includes variables representing the age of the head of the household, in the current period. The individual household's consumption expenditure function for the current period may thus be written as:

$$(3.1) \quad p_t C_{it} = f_{ic}(p_t, p^*_{t,t+1}, \ldots, p^*_{t,t+T}; w_{it}, w^*_{it,t+1}, \ldots, w^*_{it,t+T}; W_{it}, A_{it}), \quad \forall i,t$$

where T, a positive integer, is the length of the planning horizon, which is set sufficiently large so that it may be taken to be independent of the value of the set of attributes A_{it} and t itself. (Of course, it is entirely possible, depending, for example, on the age of the head of the household, which is included in the set of attributes, A_{it}, that the forward prices of sufficiently distant future periods may have no effect on the values of the consumption and leisure expenditures of the current period.) As specified in equation (3.1), all households are assumed to face identical spot and forward prices of consumption but are allowed to have individual household-specific consumption expenditure functions, as well as individual household-specific spot and forward prices of leisure, wealth and set of attributes in each period. In addition, the consumption expenditure functions are assumed to be stationary with respect to time (but not necessarily with respect to age). What this means is that the consumption expenditure of a household headed by a person aged 45, for example, will be the same in each period if the current and forward prices, wealth and set of other attributes remain unchanged. Thus, changing individual household tastes are ruled out in our specification except insofar as they are embodied in the changing value of the set of attributes over time.

Similarly, the ith household's leisure expenditure function for the current, tth, period may be written as:

$$(3.2)\quad w_{it}Z_{it} = f_{iz}\ (p_t, p^*_{t,t+1}, \ldots, p^*_{t,t+T};\ w_{it}, w^*_{it,t+1}, \ldots, w^*_{it,t+T};\ W_{it}, A_{it}), \forall i,t.$$

3.2 Relationship Between Spot and Forward Prices

Futures markets are far from complete and forward prices of consumption and leisure, to the extent that they exist meaningfully for an individual household in its household decision-making process, are not generally directly observable. It must nevertheless be true that the consumption expenditure of an individual household in the current period depends in general not only on the prices of the current period, but also on the expected prices of the future periods (or more properly speaking, the (possibly subjective) joint distribution of prices of future periods). As these future (spot and forward) prices are not generally observable, some assumptions on the expectations of the individual households are necessary in order that equations (3.1) and (3.2) may be made operational. We make the following simplifying assumptions with respect to the current and future prices of consumption and leisure:

(1) The expected spot prices of consumption of all future periods,

$p_{t+t'}$'s, taken at and conditional on the spot price of consumption of the current period, are assumed to grow geometrically at a constant rate equal to π_t, which may be referred to as the rate of expected inflation.

$$(3.3) \quad p_{t+t'} = p_t \, (1 + \pi_t)^{t'},$$

where $p_{t+t'}$ is the expected spot price of consumption of the (t+t')th period and π_t is the expected constant rate of increase of the spot price of consumption per period, taken at and conditional on the spot price of consumption of the current period (as well as the spot prices of consumption of past periods).

(2) The expected after-tax spot prices of leisure of a household with a standard set of attributes, A_0, of all future periods, $w_{0(t+t')}$'s, taken at and conditional on its spot price of leisure of the current period, w_{0t}, are assumed to grow geometrically at a constant rate equal to π_t;

$$(3.4) \quad w_{0(t+t')} = w_{0t} \, (1 + \pi_t)^{t'}.$$

For example, suppose the standard household is one headed by a 45-year old white male person, then the expected spot wage rates for a 45-year old white male person for all future periods, taken at and conditional on the spot wage rate of the current period, are assumed to be equal to the current spot wage rate times a factor which grows geometrically at a constant rate equal to π_t per period. Thus, the expected real after-tax spot prices of leisure are assumed to be constant.

(3) For a household with a set of attributes, A_{it}, different from A_0, the spot price of leisure in the current period is given by:

$$w_{it} = h(A_{it})/h(A_0) \cdot w_{0t};$$

where $h(\cdot)$ is a known function which is independent of time. Without loss of generality $h(A_0)$ can be taken to be unity. Holding all components of the set of attributes other than age constant, then $h(\cdot)$ considered as a function of age gives precisely the age profile of spot wage rates.

The spot price of leisure in any period, t'th, whether current or future, is given by:

$$(3.5) \qquad w_{it'} = h(A_{it'})\, w_{0t'}$$

where $A_{it'}$ is the value of the set of attributes of the ith household in the t'th period.

The expected spot prices of leisure of all future periods of a household with a sequence of values of its set of attributes $(A_{it}, A_{i,t+1}, \ldots, A_{i,t+T})$ are equal to a sequence formed by the products of $h(A_{i,t+t'})$'s and the expected spot prices of leisure of the standard household in the t+t'th period, and hence, by assumption (2), equal to $h(A_{i,t+t'})\, w_{0t}\, (1 + \pi_t)^{t'}$.

(4) The expected nominal after-tax interest rate for all future periods, $i_{t+t'}$, $t'=1,\ldots T$, taken at and conditional on the nominal after-tax interest rate of the current period, is fixed and equal to the nominal after-tax interest rate of the current period, i_t, which is identical for all households;

(5) The joint distribution of the spot prices of consumption and leisure (of the standard household), the nominal interest rate and the rate of inflation for all future periods in the current period, conditional on the spot prices of consumption and leisure, the spot nominal interest rate and the rate of inflation in the current period are known to all individual

households;

(6) The moments of the joint distribution of the spot prices of consumption and leisure, the nominal interest rate and the rate of inflation for all future periods of order greater than or equal to the second, conditional on the spot prices of consumption, leisure and the spot nominal interest rate and the rate of inflation in the current period, are fixed constants known to all individual households.

Note that the means of the conditional joint distribution in any current period are assumed to be equal to the spot values of the current period. Thus the means are allowed to change over time even though the higher-order moments are assumed to be fixed constants.

Given these assumptions, and the possibility of arbitrage across current and future periods, the forward prices of consumption are given by:

(3.6)
$$
\begin{aligned}
p^*_{t,t+t'} &= \frac{p_{t+t'}}{(1+i_t)^{t'}} \\
&= \frac{p_t(1+\pi_t)^{t'}}{(1+i_t)^{t'}} \\
&\cong p_t/(1+i_t-\pi_t)^{t'} \text{ [10]} \\
&= p_t/(1+r_t)^{t'}, \qquad \forall\, t',\ t' \geq 1;
\end{aligned}
$$

where $r_t \equiv i_t - \pi_t$ is the real after-tax rate of interest in the tth period. Taking logarithms,

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10. This relationship is exact in continuous time.

(3.7) $$\ell np^*_{t,t+t'} = \ell np_t - t'\ell n(1+r_t)\ , \qquad \forall\ t',\ t' \geq 1$$

Note that equation (3.6) is merely an intertemporal efficiency condition and does not, in itself, imply that the individual household behaves as if it maximizes a household utility function.

Similarly, given these assumptions, and the possibility of intertemporal borrowing and lending on the part of the individual household, the forward prices of leisure are simply given by:

(3.8) $$w^*_{it,t+t'} = \frac{w_{0t}h(A_{i,t+t'})}{(1+r_t)^{t'}}, \qquad \forall\ t',\ t' \geq 1;\ \text{or}$$

(3.9) $$\ell n\ w^*_{it,t+t'} = \ell n\ w_{0t} - t'\ell n(1+r_t) + \ell nh(A_{i,t+t'})\ , \quad \forall t',\ t' \geq 1.$$

For an individual household with a set of attributes equal to A_{it} in the current period, the sequence of $A_{i,t+t'}$'s is not arbitrary. It is reasonable, in fact even likely, that $A_{i,t+t'}$ is expressible as a function of A_{it} and t', known to the individual households:

(3.10) $$A_{i,t+t'} = g(A_{it},t'),\ t' = 1,\ldots,T.$$

Equations (3.9) and (3.10) in turn imply that

(3.11) $$\ell nw^*_{it,t+t'} = \ell nw_{0t} - t'\ell n(1+r_t) + \ell nh^*(A_{it},t'),\ \forall t',t'\geq 1,$$

where $h^*(A_{it},t') \equiv h(g(A_{it},t'))$, $\forall t',t'\geq 1$, and $h^*(A_{it},t')$ considered as a function of t' represents the stationary time profile of future expected spot prices of leisure faced by the head of the ith individual household with set of attributes

A_{it} in the tth period. Note that equation (3.8) is also merely an intertemporal efficiency condition for the ith individual household and does not, in itself, imply that the ith individual household maximizes a household utility function.

Moreover, under these assumptions, the ratio of the spot prices of leisure facing the heads of the ith and jth individual households in the current period depend only on the values of the sets of attributes of the ith and jth individual households in the current period. Thus,

$$\frac{w_{it}}{w_{jt}} = \frac{h(A_{it})}{h(A_{jt})}, \qquad \forall\ i,j;\ \forall\ t\ .$$

By substituting equations (3.5), (3.7) and (3.11) into equation (3.1), we obtain:

$$(3.12) \quad p_t C_{it} = f^*_{ic}(p_t, r_t, 1, 2, \ldots, T;\ w_{0t}, A_{it}, 1, 2, \ldots, T;\ W_{it}, A_{it}), \qquad \forall i,t.$$

A similar substitution may be made for equation (3.2). Since the integers 1,...,T are constants, the individual household consumption and leisure expenditure functions may be simplified into:

$$(3.13) \qquad p_t C_{it} = f^*_{ic}(p_t,\ w_{0t},\ r_t,\ W_{it},\ A_{it})\ , \qquad \forall\ i,t\ ;$$

and

$$(3.14) \qquad w_{it} Z_{it} = f^*_{iz}(p_t,\ w_{0t},\ r_t,\ W_{it},\ A_{it})\ , \qquad \forall\ i,t\ ;$$

where all the constants, including T, the length of the maximum planning horizon, being constant over i and t, are suppressed.[11] (Alternatively, one may assume T_i, the maximum planning horizon of the ith individual household, to be a function of A_{it}, resulting in the same consumption and expenditure functions as equations (3.13) and (3.14).) $f^*_{ic}(.)$ and $f^*_{iz}(.)$ can, of course, be directly derived from $f_{ic}(.)$ and $f_{iz}(.)$ under the assumptions on forward prices of consumption and leisure. Note that it is not necessary to take into account "planned" bequests, if any, explicitly, as they are functions of the same variables as those on the right-hand sides of equations (3.13) and (3.14).[12]

3.3 Aggregation

If data on the consumption and leisure expenditures as well as the wealth and values of the set of attributes of individual households (or groups of households with approximately identical wealth and set of attributes) are available on a time-series basis, then one can estimate the consumption and leisure expenditure functions in equations (3.13) and (3.14) directly after specifying parametric functional forms for $f^*_{ic}(.)$ and $f^*_{iz}(.)$ and stochastic disturbance terms for the system of two equations. (All the variables included in the consumption and leisure expenditure functions are observable.) Unfortunately, such disaggregated data are not generally available on a time-

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11. Similarly, the higher-order moments of the conditional joint distribution of prices and interest rates, being constants, may also be suppressed even if they affect current consumption and leisure expenditures.

12. We note that alternative assumptions on expectations can be consistent with this model so long as they result in stationary household consumption and leisure expenditure functions of the form in equations (3.13) and (3.14).

series basis. What are more generally available are data on aggregate consumption and leisure expenditures for the whole economy, data on the joint distribution of individual household wealth and attributes, and the total number of individual households, N_t, in the economy in the current period. What is needed is thus a model in which variations in aggregate consumption and leisure expenditures can be explained by variations in the spot prices of consumption and leisure, the real interest rate, the joint distribution of individual household wealth and attributes and the total number of individual households. Variations in the joint distribution of individual household wealth and attributes may be measured through variables that may be considered as "statistics" of the joint distribution. Examples of such variables include average wealth per household, average age of heads of individual households, and the variance of wealth over individual households. If the joint distribution of individual household wealth and attributes changes, the values of these variables may be expected to change.

The aggregate consumption and leisure expenditure functions are thus expected to take the form:

$$\sum_{i=1}^{N_t} p_t C_{it} = F_c(p_t, w_{0t}, r_t, N_t, S_{1t}, S_{2t}, \ldots, S_{nt}) \tag{3.15}$$

$$\sum_{i=1}^{N_t} w_{it} Z_{it} = F_z(p_t, w_{0t}, r_t, N_t, S_{1t}, S_{2t}, \ldots, S_{nt}) \tag{3.16}$$

where N_t is the total number of individual households in the current, tth, period, S_{it}, $i = 1, \ldots, n$, are the values of the n variables -- "statistics" -- reflecting the joint distribution of individual household wealth and attributes of the current period.

The S_{it}'s, $i = 1, \ldots, n$, are each functions of $W_{1t}, \ldots, W_{N_t t}, A_{1t}, \ldots, A_{N_t t}$ as well as N_t. Moreover, each such function should remain unchanged with respect to a simple renumbering of the individual households. In other words, each such function must be symmetric with respect to the indexes or equivalently, invariant to a permutation of the indexes 1 through N_t. Thus, for example,

$$(3.17) \qquad S_{1t}(W_{1t}, \ldots, W_{N_t t}; A_{1t}, \ldots, A_{N_t t}; N_t)$$

$$= S_{1t}(W_{2t}, W_{3t}, \ldots, W_{N_t t}, W_{1t}; A_{2t}, A_{3t}, \ldots, A_{N_t t}, A_{1t}; N_t)$$

Furthermore, even if not all of $W_{1t}, \ldots, W_{N_t t}, A_{1t}, \ldots, A_{N_t t}$ are observed, the values of $S_{it}(.)$'s can in general be estimated from an appropriate sample of $W_{1t}, \ldots, W_{N_t t}, A_{1t}, \ldots, A_{N_t t}$.

Substituting equations (3.13) and (3.14) into equations (3.15) and (3.16), we obtain:

$$(3.18) \qquad \sum_{i=1}^{N_t} f^*_{ic}(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$= F_c(p_t, w_{0t}, r_t, N_t, S_{1t}, \ldots, S_{nt})$$

$$(3.19) \qquad \sum_{i=1}^{N_t} f^*_{iz}(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$= F_z(p_t, w_{0t}, r_t, N_t, S_{1t}, \ldots, S_{nt})$$

If equations (3.18) and (3.19) are to hold identically, that is, for all p_t, w_{0t}, r_t and all joint distributions of W_{it}'s and A_{it}'s, and $N_t > n$, it can be shown that[13]

$$(3.20) \qquad f^*_{ic} = f^*_c(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$+ f^{**}_{ic}(p_t, w_{0t}, r_t), \; i = 1, 2, \ldots, N_t;$$

$$(3.21) \qquad f^*_{iz} = f^*_z(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$+ f^{**}_{iz}(p_t, w_{0t}, r_t), \; i = 1, 2, \ldots, N_t;$$

In other words, the consumption and leisure expenditure functions of individual households with identical wealths and attributes are each identical up to the addition of a function independent of wealth and attributes. If it were further assumed that the individual consumption and leisure expenditures are nonnegative and aggregate consumption and leisure expenditures are both zero when aggregate wealth is zero, the individual consumption and leisure expenditure functions may be simplified to:

$$(3.22) \qquad f^*_{ic} = f^*_c(p_t, w_{0t}, r_t, W_{it}, A_{it}) \quad i = 1, \ldots, N_t;$$

and

$$(3.23) \qquad f^*_{iz} = f^*_z(p_t, w_{0t}, r_t, W_{it}, A_{it}) \quad i = 1, \ldots, N_t;$$

where

$$f^*_c(p_t, w_{0t}, r_t, 0, A_{it}) = f^*_z(p_t, w_{0t}, r_t, 0, A_{it}) = 0 \; .$$

It can be further shown under mild regularity conditions that the

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13. See, for example, Jorgenson, Lau and Stoker (1982).

individual consumption and expenditure functions must take the form:

(3.24) $$f^*_c(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$= \sum_{k=1}^{n} h_{ck}(p_t, w_{0t}, r_t)\ g_k(W_{it}, A_{it});$$

and

(3.25) $$f^*_z(p_t, w_{0t}, r_t, W_{it}, A_{it})$$

$$= \sum_{k=1}^{n} h_{zk}(p_t, w_{0t}, r_t)\ g_k(W_{it}, A_{it})$$

where $g_k(0, A_{it}) = 0 \ , \quad k = 1, \ldots n \ .$

There are additional restrictions on the $h_{ck}(.)$, $h_{zk}(.)$ and $g_k(.)$ functions to ensure that the expenditure functions, including planned future expenditures and bequests, sum to total wealth (summability) and that the individual households have no money illusion (zero degree homogeneity). For our analysis we choose the following types of $g_k(W_{it}, A_{it})$ functions:

$$g_1(W_{it}, A_{it}) = W_{it};$$

$$g_2(W_{it}, A_{it}) = W_{it}\ell nW_{it};$$

$$g_k(W_{it}, A_{it}) = A_{i(k-2)t}W_{it}, \quad k=3,\ldots,n;$$

where A_{ijt} is the jth component of the vector of the set of attributes A_{it}.

3.4 Specification of the Individual Expenditure Functions

For each individual household of the jth type, that is, $A_{it} = A_j$, the consumption and leisure expenditure functions are specified parametrically as follows:

$$(3.26) \qquad p_t C_{it} = \left[\frac{\alpha_c + \alpha_{cj} + \beta_{cc}\ell np_t + \beta_{cz}\ell nw_{0t} + \beta_{cr}\ell n(1 + r_t) + \beta_{cW}\ell nW_{it}}{1 + \beta_c^c \ell np_t + \beta_z^c \ell nw_{0t} + \beta_r^c \ell n(1 + r_t)}\right] W_{it}$$

$$(3.27) \qquad w_{it} Z_{it} = \left[\frac{\alpha_z + \alpha_{zj} + \beta_{zc}\ell np_t + \beta_{zz}\ell nw_{0t} + \beta_{zr}\ell n(1 + r_t) + \beta_{zW}\ell nW_{it}}{1 + \beta_c^z \ell np_t + \beta_z^z \ell nw_{0t} + \beta_r^z \ell n(1 + r_t)}\right] W_{it}$$

If it were required that the balance of the total (planned) expenditures, including "planned" bequests, if any, VO_{it}, has the same parametric form; that is:

$$VO_{it} = \left[\frac{\alpha_0 + \alpha_{0j} + \beta_{0c}\ell np_t + \beta_{0z}\ell nw_{0t} + \beta_{0r}\ell n(1 + r_t) + \beta_{0W}\ell nW_{it}}{1 + \beta_c^0 \ell np_t + \beta_z^0 \ell nw_{0t} + \beta_r^0 \ell n(1 + r_t)}\right] W_{it}$$

and that summability holds, that is,

$$p_t C_{it} + w_{it} Z_{it} + VO_{it} = W_{it} ,$$

identically, then it can be shown that the consumption and leisure expenditure functions may both be written in the form:

$$(3.28) \qquad p_t C_{it} = \left[\frac{\alpha_c + \alpha_{cj} + \beta_{cc}\ell np_t + \beta_{cz}\ell nw_{0t} + \beta_{cr}\ell n(1 + r_t) + \beta_{cW}\ell nW_{it}}{1 + \beta_c \ell np_t + \beta_z \ell nw_{0t} + \beta_r \ell n(1 + r_t)} \right] W_{it}$$

$$(3.29) \qquad w_{it} Z_{it} = \left[\frac{\alpha_z + \alpha_{zj} + \beta_{zc}\ell np_t + \beta_{zz}\ell nw_{0t} + \beta_{zr}\ell n(1 + r_t) + \beta_{zW}\ell nW_{it}}{1 + \beta_c \ell np_t + \beta_z \ell nw_{0t} + \beta_r \ell n(1 + r_t)} \right] W_{it}$$

that is, the parameters in the denominators are identical.

If it were required that the consumption and leisure demand functions of each individual household satisfy the zero degree homogeneity restriction, that is, the individual household has "no money illusion," then it can be shown that the following restrictions on the parameters are implied:

$$(3.30) \qquad \beta_{cc} + \beta_{cz} + \beta_{cW} = 0$$

$$(3.31) \qquad \beta_{zc} + \beta_{zz} + \beta_{zW} = 0$$

and

$$(3.32) \qquad \beta_c + \beta_z = 0 \quad .$$

The consumption and leisure expenditure functions may be rewritten in the form:

$$(3.33) \qquad p_t C_{it} = \left[\frac{\alpha_c + \alpha_{cj} + \beta_{cc}\ell np_t + \beta_{cz}\ell nw_{0t} + \beta_{cr}\ell n(1 + r_t) - (\beta_{cc} + \beta_{cz})\ell nW_{it}}{1 + \beta_c(\ell np_t - \ell nw_{0t}) + \beta_r \ell n(1 + r_t)} \right] W_{it}$$

$$(3.34) \qquad w_{it} Z_{it} = \left[\frac{\alpha_z + \alpha_{zj} + \beta_{zc}\ell np_t + \beta_{zz}\ell nw_{0t} + \beta_{zr}\ell n(1 + r_t) - (\beta_{zc} + \beta_{zz})\ell nW_{it}}{1 + \beta_c(\ell np_t - \ell nw_{ot}) + \beta_r \ell n(1 + r_t)} \right] W_{it}$$

In our empirical implementation we maintain the hypotheses of summability and zero degree homogeneity. However, we do not maintain the hypothesis of utility maximization on the part of the individual households, although our specifications in equations (3.33) and (3.34) are not inconsistent with it. The rationale for maintaining the hypothesis of summability is simply that each individual household can make its lifetime consumption, leisure and bequest choices only within its lifetime wealth constraint. The rationale for maintaining the hypothesis of zero degree homogeneity is simply that each individual household should not change its lifetime consumption, leisure and bequest choices if the set of possible choices (represented by all available choices within the lifetime wealth constraint) is unchanged. Note that the two hypotheses do not imply utility maximization of the individual household. The decision-making process within each household can be arbitrary as long as it is stationary over time. For example, one may have the husband and the wife of a household each making decisions on fifty percent of the individual household's total resources. The resulting behavior for the household can be expected to satisfy summability and zero degree homogeneity but not necessarily utility maximization by the individual household as a whole.

3.5 Specific Hypotheses on Consumption Behavior

The specification in equations (3.33) and (3.34) is sufficiently flexible to embed a number of hypotheses found in the literature on consumption behavior. We discuss each of these hypotheses in turn.

(1) Unitary Wealth Elasticity. It is often assumed that individual household consumption (and leisure) expenditure is proportional to individual household wealth. In the context of our specification, this hypothesis implies that:

$$\beta_{cW} = -(\beta_{cc} + \beta_{cz}) = 0$$

and

$$\beta_{zW} = -(\beta_{zc} + \beta_{zz}) = 0 \quad .$$

(2) Proportionality of Expenditures to Wealth. This hypothesis is often identified with Friedman (1957).[14] It implies that consumption (and leisure) expenditure is proportional to wealth, and, in addition, that the factors of proportionality depend only on the rate of interest but not the prices, given wealth. In the context of our specification, this hypothesis implies that:

$$\beta_{cc} + \beta_{cz} = 0$$

$$\beta_{zc} + \beta_{zz} = 0$$

$$\beta_{cc} = 0$$

$$\beta_{zc} = 0$$

$$\beta_{c} = 0$$ [15]

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14. See also Blanchard's (1985) derivation and discussion.

15. Except for the degenerate case in which $\beta_{cc} + \beta_{cz} = 0$ and $\beta_{zc} + \beta_{zz} = 0$;

$$(\alpha_c + \alpha_{cj})\beta_c = \beta_{cc} \;;$$
$$(\alpha_c + \alpha_{cj})\beta_z = \beta_{cz} \;;$$
$$(\alpha_c + \alpha_{cj})\beta_r = \beta_{cr} \;;$$
$$(\alpha_z + \alpha_{zj})\beta_c = \beta_{zc} \;;$$
$$(\alpha_z + \alpha_{zj})\beta_z = \beta_{zz} \;;$$
$$(\alpha_z + \alpha_{zj})\beta_r = \beta_{zr} \;; \quad \forall\, j.$$

For this degenerate case consumption and leisure expenditures are fixed constant proportions of wealth. The values of β parameters, subject to $\beta_{cc} + \beta_{cz} = 0$ and $\beta_{zc} + \beta_{zz} = 0$, are arbitrary but in particular can be chosen so that all of them are identically zero, so that they satisfy the restrictions here. See hypothesis (5), "Complete Price Independence", below.

Obviously, if the hypothesis of unitary wealth elasticity does not hold, the proportionality hypothesis cannot hold.

(3) Intertemporal Separability. Intertemporal separability is a hypothesis that is often maintained when each individual household is assumed to maximize its utility. It implies that the intertemporal utility function of the household has the form,

$$U_{it}(C_{it}, Z_{it}, C_{i(t+1)}, Z_{i(t+1)}, \ldots, C_{i(t+T)}, Z_{i(t+T)})$$

$$= U_{it}(U_t(C_{it}, Z_{it}), U_{t+1}(C_{i(t+1)}, Z_{i(t+1)}), \ldots, U_{t+T}(C_{i(t+T)}, Z_{i(t+T)}))$$

It encompasses as special cases that of intertemporal additivity:

$$U_{it} = \sum_{t'=0}^{T} U_{t+t'}(C_{i(t+t')}, Z_{i(t+t')})$$

and that of intertemporal stationary additivity:

$$U_{it} = \sum_{t'=0}^{T} \delta(t')U_t(C_{i(t+t')}, Z_{i(t+t')})$$ [16]

The principal empirical implication of intertemporal separability is that the relative expenditures on different commodities within the same period are independent of the prices of the commodities in a different period. This is a necessary (but not in general sufficient) condition for

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16. A further possible specialization consists of replacing $U_t(.)$ by $U(.)$.

intertemporal additivity and intertemporal stationary additivity as defined above. This hypothesis is testable, however, even for a household not necessarily assumed to be maximizing utility. Under our assumptions about the relationship between current and expected future spot prices, this hypothesis implies that the relative expenditures of the current period are independent of the real after-tax rate of interest, holding wealth constant. In terms of our specification, this hypothesis implies that:

$$\beta_{cr} = 0$$

and

$$\beta_{zr} = 0.$$ [17]

Note, however, that under this hypothesis, current consumption and leisure expenditures may still be sensitive to the real after-tax rate of interest, only their ratio is insensitive to the real after-tax rate of interest. The hypothesis that we test, as discussed above, is <u>weaker</u> than the usual hypothesis of intertemporal separability of the utility function (and a fortiori weaker than the hypotheses of intertemporal additivity and intertemporal stationary additivity).

(4) <u>Absence of Interest Rate Effects.</u> This hypothesis implies that

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17. Except for the degenerate case in which

$$(\alpha_c + \alpha_{cj})\beta_{zc} = (\alpha_z + \alpha_{zj})\beta_{cc} \ ;$$
$$(\alpha_c + \alpha_{cj})\beta_{zz} = (\alpha_z + \alpha_{zj})\beta_{cz} \ ;$$
$$(\alpha_c + \alpha_{cj})\beta_{zr} = (\alpha_z + \alpha_{zj})\beta_{cr} \ ; \ \forall \ j.$$

both consumption and leisure expenditures are, given aggregate wealth and its distribution, independent of the real after-tax rate of interest. However, it does not preclude a change in the real rate of interest from affecting consumption and leisure expenditures indirectly through its effect on the revaluation of wealth (the so-called "Summers' effect"). In the context of our specification, this hypothesis implies:

$\beta_{cr} = 0$; $\beta_{zr} = 0$; and $\beta_r = 0$.[18]

Obviously, if the hypothesis of intertemporal separability does not hold, the hypothesis of no interest rate effects cannot hold.

(5) Complete Price Independence. An extreme hypothesis about consumption and leisure behavior is that the consumption and leisure expenditure to wealth ratios are fixed independently of current and expected future prices, holding wealth constant. Such a hypothesis implies that all the parameters are zero except α_c, α_z and α_{cj}'s and α_{zj}'s.[19]

All of these hypotheses imply specifications that are special cases of our basic specification, which imposes only minimal assumptions on individual household behavior (summability and zero degree homogeneity (no money illusion)) and minimal assumptions necessary for exact aggregation. We distinguish among types of households based on the age of the head of household and allow changes in the composition of the households by type to affect aggregate

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18. Except for the degenerate case in footnote 15 above.

19. Except for the degenerate case in footnote 15 above. However, under the conditions of complete price independence, β parameters are arbitrary and can, in particular, be set identically equal to zero.

consumption and leisure expenditures. While it would be interesting to attempt to analyze other attributes as well, we focus in this paper on the attribute which has received the most attention in the analysis of consumption and saving, namely age, as discussed in many of the studies reviewed in Section 2.

3.6 Specification of the Aggregate Expenditure Functions

The aggregate consumption and leisure expenditure functions are obtained by adding up the individual consumption and leisure expenditure functions across the households:

$$(3.35) \quad \sum_i p_t C_{it} = [(\alpha_c + \beta_{cc}\ell n p_t + \beta_{cz}\ell n w_{0t} + \beta_{cr}\ell n(1 + r_t)) \sum_i W_{it} - (\beta_{cc} + \beta_{cz}) \sum_i W_{it}\ell n W_{it} + \sum_j \sum_i \alpha_{cj} D^j_{it} W_{it}] / [1 + \beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1 + r_t)]$$

$$(3.36) \quad \sum_i w_{it} Z_{it} = [(\alpha_z + \beta_{zc}\ell n p_t + \beta_{zz}\ell n w_{0t} + \beta_{zr}\ell n(1 + r_t)) \sum_i W_{it} - (\beta_{zc} + \beta_{zz}) \sum_i W_{it}\ell n W_{it} + \sum_j \sum_i \alpha_{zj} D^j_{it} W_{it}] / [1 + \beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1 + r_t)]$$

where D^j_{it} is a dummy variable which takes the value 1 if the ith household is of the jth type at time t and 0 if the ith household is not of the jth type at time t.

It is possible to estimate equations (3.35) and (3.36) directly. However, we note that both dependent variables, aggregate nominal consumption and leisure expenditures, are likely to show sustained increases over time because of the increase in the number of households (population) and inflation. It is therefore unlikely that additive stochastic disturbance terms are (separately) homoscedastic for each of the two functions. In order to mitigate the possibility of heteroscedasticity, we divide both sides of equations (3.35) and (3.36) by aggregate wealth at time t, obtaining the aggregate consumption expenditure to wealth and leisure expenditure to wealth ratios:

$$(3.37) \qquad \frac{\sum_i p_t C_{it}}{\sum_i W_{it}} = [(\alpha_c + \beta_{cc}\ell n p_t + \beta_{cz}\ell n w_{0t} + \beta_{cr}\ell n(1 + r_t)) - (\beta_{cc} + \beta_{cz}) \sum_i W_{it}\ell n W_{it} / \sum_i W_{it} + \sum_j \sum_i \alpha_{cj} D^j_{it} W_{it} / \sum_i W_{it}] / [1 + \beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1 + r_t)]$$

$$(3.38) \qquad \frac{\sum_i w_{it} Z_{it}}{\sum_i W_{it}} = [(\alpha_z + \beta_{zc}\ell n p_t + \beta_{zz}\ell n w_{0t} + \beta_{zr}\ell n(1 + r_t)) - (\beta_{zc} + \beta_{zz}) \sum_i W_{it}\ell n W_{it} / \sum_i W_{it} + \sum_j \sum_i \alpha_{zj} D^j_{it} W_{it} / \sum_i W_{it}] / [1 + \beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1 + r_t)]$$

where $(\sum_i W_{it}\ell n W_{it})/\sum_i W_{it}$ can be interpreted as "entropy," a measure of the variability of wealth over individual households and $(\sum_i D^j_{it} W_{it})/\sum_i W_{it}$ can be identified as the share of aggregate wealth held by households of the jth type. Equations (3.37) and (3.38) are the specifications used in the estimation.

The variables -- aggregate consumption expenditure to wealth and leisure expenditure to wealth ratios, and the prices of consumption, after-tax wage rates, the after-tax real rate of interest, and aggregate wealth -- are all either available as or can be developed from standard time series data.[20] The "statistics" representing the distribution of wealth by size and by the type of household can be obtained from a time-series of cross-sectional income surveys.

In addition, if the parameters are known, equations (3.37) and (3.38) may be used to predict the effects on aggregate consumption, leisure and saving of alternative potential future paths of evolution of the size and age distribution of total wealth.

Finally, it is important to note that our model focuses entirely on the household side -- the demands for current and future consumption and leisure (or the mirror image, supplies of current and future labor). The households are assumed to behave as price-takers, that is, as if their individual actions do not affect the (possibly subjective) joint distribution of current and future prices. The model leaves unexplained the current and future prices, wealth and its distribution. Thus, it is a partial equilibrium model, reflecting only the equilibrium of the households, but not necessarily the markets for consumption goods, labor and saving.

3.7 Comparative Static Effects

We attempt to estimate the comparative static effects of changes in the current price of consumption, after-tax wage rate, wealth and the real after-tax rate of interest (with human wealth held constant as well as revalued) on the consumption, leisure and value of saving (defined as in the National Income and

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20. See footnote 1 for the calculation of leisure.

Product Accounts)[21] of the individual households. Specifically, we calculate the elasticities of each of the dependent variables with respect to the independent variables (at a given set of values of the latter).

Equations (3.33) and (3.34) may be rewritten as

$$(3.39) \qquad p_t C_{it} = \frac{N_{cit}}{D_t} W_{it}$$

$$(3.40) \qquad w_{it} Z_{it} = \frac{N_{zit}}{D_t} W_{it}$$

The elasticity formulae may be computed as follows:

$$\frac{\partial \ell n C_{it}}{\partial \ell n p_t} = -1 + \frac{\partial \ell n N_{cit}}{\partial \ell n p_t} - \frac{\partial \ell n D_t}{\partial \ell n p_t}$$

$$= -1 + \frac{\beta_{cc}}{N_{cit}} - \frac{\beta_c}{D_t}$$

$$= -1 + \frac{1}{D_t}\left[\frac{\beta_{cc}}{(p_t C_{it}/W_{it})} - \beta_c\right]$$

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21. The NIPA definition of current saving is the difference between current income and current consumption and does not include net changes in wealth due to revaluations.

$$= -1 + \left[\frac{\beta_{cc}}{(p_t C_{it}/W_{it})} - \beta_c\right] \;/\; (1+\beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1+r_t)).$$

In 1972, $\ell n p_t = \ell n w_{0t} = 0$, and the elasticity formula simplifies to:

$$\left.\frac{\partial \ell n C_{it}}{\partial \ell n p_t}\right|_{t=1972} = -1 + \left[\frac{\beta_{cc}}{(p_t C_{it}/W_{it})_{t=1972}} - \beta_c\right] \;/\; (1+\beta_r \ell n(1+r_{1972})).$$

Similarly,

$$\frac{\partial \ell n C_{it}}{\partial \ell n w_{it}} = \left[\frac{\beta_{cz}}{(p_t C_{it}/W_{it})} + \beta_c\right] \;/\; ((1+\beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1+r_t))$$

where we have made use of the fact that $\frac{\partial \ell n w_{0t}}{\partial \ell n w_{it}} = 1$;

$$\left.\frac{\partial \ell n C_{it}}{\partial \ell n w_{it}}\right|_{t=1972} = \left[\frac{\beta_{cz}}{(p_t C_{it}/W_{it})_{t=1972}} + \beta_c\right] \;/\; (1 + \beta_r \ell n(1+r_{1972})).$$

$$\frac{\partial \ell n C_{it}}{\partial \ell n W_{it}} = 1 - \left[\frac{(\beta_{cc} + \beta_{cz})}{(p_t C_{it}/W_{it})}\right] \;/\; (1+\beta_c(\ell n p_t - \ell n w_{0t}) + \beta_r \ell n(1+r_t));$$

$$\left.\frac{\partial \ell n C_{it}}{\partial \ell n W_{it}}\right|_{t=1972} = 1 - \left[\frac{(\beta_{cc}+\beta_{cz})}{(p_t C_{it}/W_{it})_{t=1972}}\right] \;/\; (1+\beta_r \ell n(1+r_{1972})).$$

$$\frac{\partial \ell nC_{it}}{\partial \ell nNHW_{it}} = \frac{\partial \ell nC_{it}}{\partial \ell nW_{it}} \cdot \frac{\partial \ell nW_{it}}{\partial \ell NHW_{it}} = \frac{\partial \ell nC_{it}}{\partial \ell nW_{it}} \cdot \frac{NHW_{it}}{W_{it}}$$

$$= (NHW_{it}/W_{it})(1 - \left[\frac{(\beta_{cc}+\beta_{cz})}{(p_t C_{it}/W_{it})}\right])/(1+\beta_c(\ell np_t - \ell nw_{0t})+\beta_r \ell n(1+r_t));$$

$$\frac{\partial \ell nC_{it}}{\partial \ell nNHW_{it}}\bigg|_{t=1972} = (NHW_{it}/W_{it})_{t=1972}(1 - \left[\frac{(\beta_{cc}+\beta_{cz})}{(p_t C_{it}/W_{it})_{t=1972}}\right]) \ / \ (1+\beta_r \ell n(1+r_{1972})).$$

$$\frac{\partial \ell nC_{it}}{\partial \ell nr_t} = \left[\frac{\beta_{cr}}{(p_t C_{it}/W_{it})} - \beta_r\right] \frac{r_t}{(1+r_t)} \ / \ (1+\beta_c(\ell np_t - \ell nw_{0t}) + \beta_r \ell n(1+r_t));$$

$$\frac{\partial \ell nC_{it}}{\partial \ell nr_t}\bigg|_{t=1972} = \left[\frac{\beta_{cr}}{(p_t C_{it}/W_{it})_{t=1972}} - \beta_r\right]\frac{r_{1972}}{(1+r_{1972})} \ / \ (1 + \beta_r \ell n(1+r_{1972})).$$

Similar elasticities may be computed for the demand for leisure. The elasticity of total expenditures, $(E_{it} \equiv p_t C_{it} + w_{it} Z_{it})$, with respect to wealth and nonhuman wealth may be computed as:

$$\frac{\partial \ell nE_{it}}{\partial \ell nW_{it}} = 1 - \left[\frac{(\beta_{cc}+\beta_{cz}+\beta_{zc}+\beta_{zz})}{(E_{it}/W_{it})}\right] \ / \ (1+\beta_c(\ell np_t - \ell nw_{0t}) + \beta_r \ell n(1+r_t));$$

$$\left.\frac{\partial \ell n E_{it}}{\partial \ell n W_{it}}\right|_{t=1972} = 1 - \left[\frac{(\beta_{cc}+\beta_{cz}+\beta_{zc}+\beta_{zz})}{(E_{it}/W_{it})_{t=1972}}\right] / (1+\beta_r \ell n(1+r_{1972}));$$

$$\frac{\partial \ell n E_{it}}{\partial \ell n NHW_{it}} = (NHW_{it}/W_{it}) \cdot \frac{\partial \ell n E_{it}}{\partial \ell n W_{it}};$$

$$\left.\frac{\partial \ell n E_{it}}{\partial \ell n NHW_{it}}\right|_{t=1972} = (NHW_{it}/W_{it})_{t=1972} \cdot \left.\frac{\partial \ell n E_{it}}{\partial \ell n W_{it}}\right|_{t=1972}$$

Given the estimated parameters and their estimated asymptotic variance-covariance matrix, these elasticities and their corresponding asymptotic standard errors can be estimated, conditional on the values of $\frac{p_t C_{it}}{W_{it}}$, $\frac{w_{it} Z_{it}}{W_{it}}$, $\frac{NHW_{it}}{W_{it}}$ and the other independent variables.

We also compute the elasticities of consumption and leisure demands with respect to the after-tax wage rate and the real after-tax rate of interest under the assumption of full revaluation of human wealth of the household. Nonhuman wealth of the household is assumed to consist entirely of floating rate assets and liabilities and hence to remain unchanged with respect to changes in the real after-tax rate of interest.

$$(3.41)\qquad \frac{d\ell nC}{d\ell nw_{it}} = \frac{\partial\ell nC}{\partial\ell nw_{it}} + \frac{\partial\ell nC}{\partial\ell nW_{it}} \cdot \frac{\partial\ell nW_{it}}{\partial\ell nw_{it}}.$$

But
$$\frac{\partial\ell nW_{it}}{\partial\ell nw_{it}} = \frac{1}{W_{it}}\frac{\partial HW_{it}}{\partial\ell nw_{it}} = \frac{HW_{it}}{W_{it}},$$

where we have made use of the fact that $(\partial\ell nHW_{it})/(\partial\ell nw_{it}) = 1$ under our assumptions on expectations. Hence:

$$(3.42)\qquad \frac{d\ell nC}{d\ell nw_{it}} = \frac{\partial\ell nC}{\partial\ell nw_{it}} + \frac{\partial\ell nC}{\partial\ell nW_{it}} \cdot \frac{HW_{it}}{W_{it}}.$$

Similarly,

$$(3.43)\qquad \frac{d\ell nC}{d\ell nr_{t}} = \frac{\partial\ell nC}{\partial\ell nr_{t}} + \frac{\partial\ell nC}{\partial\ell nW_{it}} \cdot \frac{\partial\ell nW_{it}}{\partial\ell nr_{t}};$$

$$(3.44)\qquad \frac{\partial\ell nW_{it}}{\partial\ell nr_{t}} = \frac{1}{W_{it}}\frac{\partial(HW_{it} + NHW_{it})}{\partial\ell nr_{t}}$$

$$= \frac{1}{W_{it}}\frac{\partial HW_{it}}{\partial\ell nr_{t}} = \frac{HW_{it}}{W_{it}}\frac{\partial\ell nHW_{it}}{\partial\ell nr_{t}}.$$

Thus, equation (3.43) may be rewritten as:

$$\frac{d\ell nC}{d\ell nr_t} = \frac{\partial \ell nC}{\partial \ell nr_t} + \frac{\partial \ell nC}{\partial \ell nW_{it}} \cdot \frac{HW_{it}}{W_{it}} \frac{\partial \ell nHW_{it}}{\partial \ell nr_t} \qquad (3.45)$$

which represents the elasticity of consumption with respect to an increase in the real rate of interest with revaluation of human wealth. The values of $\frac{HW_{it}}{W_{it}}$ and $\frac{\partial \ell nHW_{it}}{\partial \ell nr_t}$ may be calculated numerically for each age-cohort based on its current and expected future wage rates and real rates of interest.

Finally, we attempt to estimate the effects of changes in the independent variables on saving. The saving of the ith household in the tth period, S_{it}, may be defined as the difference between current full income and current expenditure on consumption and leisure:

$$S_{it} = w_{it} \bar{Z}_{it} + (r_t+\pi_t)NHW_{it} - p_tC_{it} - w_{it}Z_{it} \qquad (3.46)$$

where $\bar{Z}_{it}$ is the maximum quantity of leisure of the ith household in the tth period.[22] The elasticity formulae for saving may be computed as follows:[23]

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22. Note that the net change in the value of wealth, including capital gains or losses and transfers, is not included in this NIPA definition of saving. Of course, transfers net to zero for the economy as a whole (except for net unilateral transfers to foreigners).

23. It is assumed that $\partial \ell n\pi_t/\partial \ell np_t = 0$.

$$\frac{\partial \ell n S_{it}}{\partial \ell n p_t} = \frac{1}{S_{it}} \frac{\partial S_{it}}{\partial \ell n p_t} = \frac{p_t C_{it}}{S_{it}} \left(\frac{-\partial \ell n C_t}{\partial \ell n p_t} - 1\right) - \frac{w_{it} Z_{it}}{S_{it}} \frac{\partial \ell n Z_{it}}{\partial \ell n p_t}.$$ 24/

$$\frac{\partial \ell n S_{it}}{\partial \ell n w_{it}} = \left[(w_{it}(\bar{Z}_{it} - Z_{it}))/S_{it}\right]\left[(\partial \ell n(\bar{Z}_{it} - Z_{it}))/\partial \ell n w_{0t} + 1\right]$$

$$- \frac{p_t C_{it}}{S_{it}} \frac{\partial \ell n C_{it}}{\partial \ell n w_{0t}}$$

$$\frac{\partial \ell n S_{it}}{\partial \ell n W_{it}} = \left[- \frac{E_{it}}{S_{it}} \frac{\partial \ell n E_{it}}{\partial \ell n W_{it}} + \frac{(r_t + \pi_t) NHW_{it}}{S_{it}} \right]$$ 25/

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24. S_{it} is allowed to take negative values. If S_{it} is negative,

$\frac{\partial \ell n S_{it}}{\partial \ell n p_t}$ is taken to be $\frac{1}{S_{it}} \frac{\partial S_{it}}{\partial \ell n p_t}$.

The same applies to the other saving elasticities below.

25. It is assumed that the change in wealth results from equal proportional changes in both human and nonhuman wealth.

$$\frac{\partial \ell nS_{it}}{\partial \ell nHW_{it}} = \left[- \frac{E_{it}}{S_{it}} \frac{\partial \ell nE_{it}}{\partial \ell nW_{it}} \cdot \frac{HW_{it}}{W_{it}} \right]$$ [26]

$$\frac{\partial \ell nS_{it}}{\partial \ell nNHW_{it}} = \left[- \frac{E_{it}}{S_{it}} \frac{\partial \ell nE_{it}}{\partial \ell nW_{it}} \frac{NHW_{it}}{W_{it}} + (r+\pi_t) \frac{NHW_{it}}{W_{it}} \right]$$

$$\frac{\partial \ell nS_{it}}{\partial \ell nr_t} = \left[- \frac{E_{it}}{S_{it}} \frac{\partial \ell nE_{it}}{\partial \ell nr_t} + r_t \frac{NHW_{it}}{S_{it}} \right]$$

$$\frac{d\ell nS_{it}}{d\ell nw_{it}} = \frac{\partial \ell nS_{it}}{\partial \ell nw_{it}} + \frac{\partial \ell nS_{it}}{\partial \ell nHW_{it}} \quad (3.47)$$

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26. We note that this effect is expected to be negative for positive saving because the NIPA definition of saving does not include the net change in wealth due to revaluation.

$$(3.48) \qquad \frac{d\ell nS_{it}}{d\ell nr_t} = \frac{\partial \ell nS_{it}}{\partial \ell nr_t} + \frac{\partial \ell nS_{it}}{\partial \ell nHW_{it}} \cdot \frac{\partial \ell nHW_{it}}{\partial \ell nr_t}$$

where the values of $\frac{HW_{it}}{W_{it}}$, $\frac{NHW_{it}}{W_{it}}$ and $\frac{\partial \ell nHW_{it}}{\partial \ell nr_t}$ may be calculated numerically for each age-cohort. We note that, with positive saving, both $\frac{\partial \ell nS_{it}}{\partial \ell nHW_{it}}$ and $\frac{\partial \ell nHW_{it}}{\partial \ell nr_t}$ are expected to be negative. Thus, $\frac{d\ell nS_{it}}{d\ell nr_t} \geq \frac{\partial \ell nS_{it}}{\partial \ell nr_t}$, that is, the interest elasticity of saving with human wealth evaluation is expected to be greater than that without human wealth revaluation. With negative saving, the opposite is true. One can also use these same formulae to calculate the aggregate elasticities. For example, the elasticity of aggregate saving with respect to the real after-tax rate of interest, with human wealth revaluation, may be computed as:

$$(3.49) \qquad \frac{d\ell nS_t}{d\ell nr_t} = \sum_i N_{it}S_{it} \frac{d\ell nS_{it}}{d\ell nr_t} / \sum_i N_{it}S_{it} = \sum_i s_{it} \frac{d\ell nS_{it}}{d\ell nr_t}$$

where $s_{it} \equiv N_{it}S_{it} / \sum_j N_{jt}S_{jt}$ is the share (possibly negative) of aggregate saving accounted for by households of the ith type in the tth period.

4. Empirical Results

4.1 A Brief Description of the Data

As discussed in the introduction and Section 3, we attempt to combine various types of data in order to develop an improved empirical model of aggregate consumption. Primarily, we combine disaggregated data on the age distribution of income and its components and consumption with aggregate time series data on consumption, leisure, wealth, and price variables. We use the rich source of disaggregated information available in the annual Current Population Surveys on income and other variables cross-tabulated by various household characteristics such as age of head of household to build cohort-specific human and nonhuman wealth accounts which aggregate to national wealth. We use additional information from the 1972-73 Consumer Expenditure Survey and the Panel Study of Income Dynamics to estimate how the ratio of household consumption and leisure to wealth varies by the age of the head of household.

The aggregate consumption data come from the National Income and Product Accounts (NIPA), and no attempt is made here to develop a more consistent treatment of the services of consumer durables, as is sometimes done (see, for example, Boskin, Robinson and Huber (1988), Christensen and Jorgenson (1973), or David and Scadding (1974)).

Data on interest rates, wage rates, and price levels are developed from standard sources and their derivations are briefly described in the Appendix, as are our data on female labor force participation and unemployment rates. Our methods for deriving expected inflation and the expected present value of human and nonhuman wealth are also briefly described in the Appendix. For human wealth, we estimate an age-wage profile[27] and discount expected future earnings of each age cohort by the sum of the real after-tax discount rate and mortality probability. For nonhuman wealth, we blow up, for each category of a property income, the sum across all age cohorts so that it conforms to the corresponding NIPA aggregate totals, and then capitalize total property income for each age cohort. We use different discount rates for human and non-human capital as discussed in the Appendix.

Before proceeding to a discussion of the econometric results it is worthwhile to examine the trends in the actual consumption and leisure data. Figure 4.1 presents the aggregate consumption and leisure expenditures for the U.S. economy from 1947 to 1980 in constant 1972 prices. These have grown steadily in the time period under study, with an apparent acceleration in the rate beginning in the early 1960s.

Based on our estimated consumption and leisure expenditure equations, we present in Section 5 below the decomposition of the annual growth rate of aggregate consumption into factors presumed to explain it, broken down into two periods, 1950-62 and 1963-80, roughly corresponding to modest and more rapid growth in aggregate consumption, respectively.

Figure 4.2 presents a prelude to the econometric results. We compare the estimated consumption to GNP ratio (derived from the product of our econometrically estimated ratio of consumption to wealth and the ratio of our estimated wealth to GNP) to the actual consumption to GNP ratio for the period

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27. Male age-earnings profiles, controlling for education, race, etc., are quite stable over time in the U.S., and surprisingly similar across countries (see, for example, Smith and Welch (1986) and Psacharopoulos (1981)).

Figure 4.1

Aggregate Consumption
and Leisure Expenditures
(billion, 1972$)

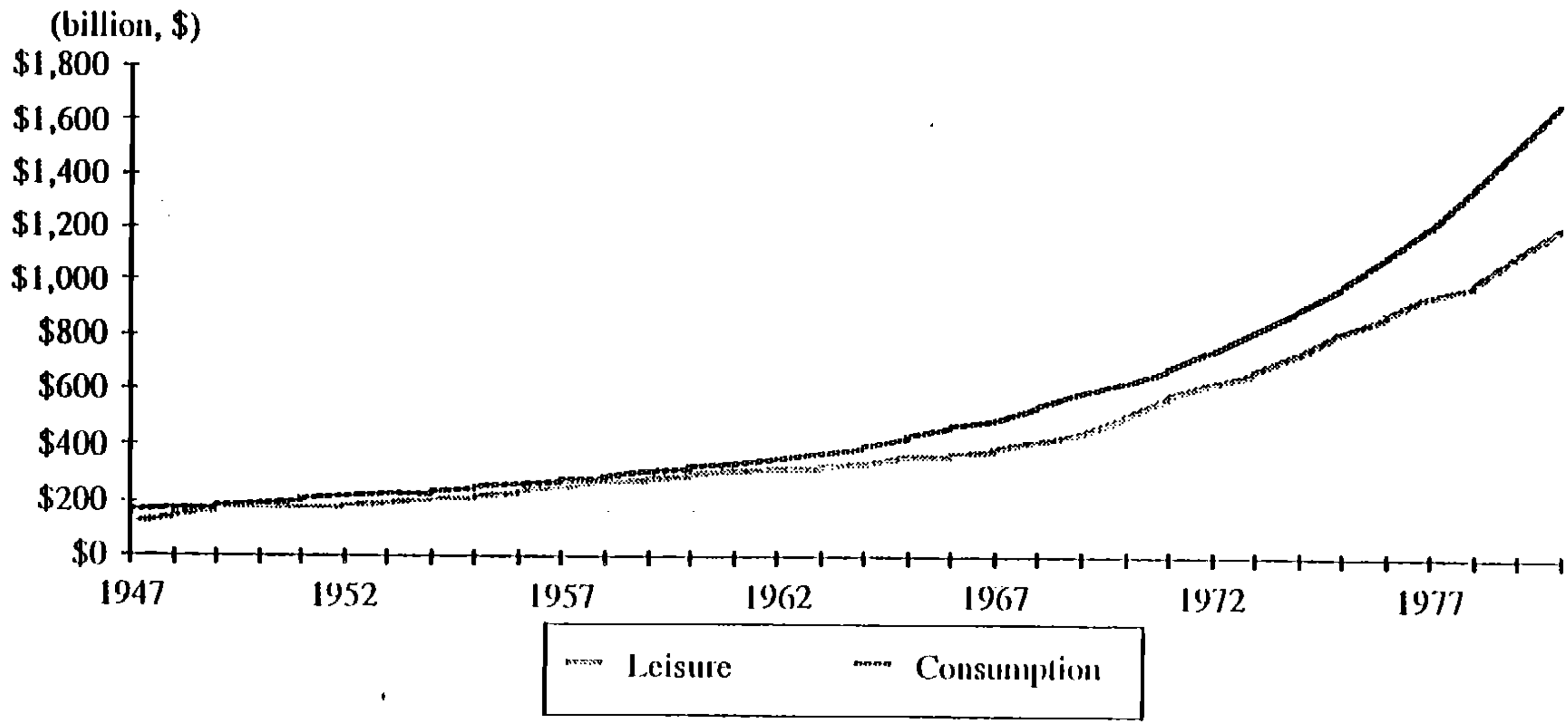
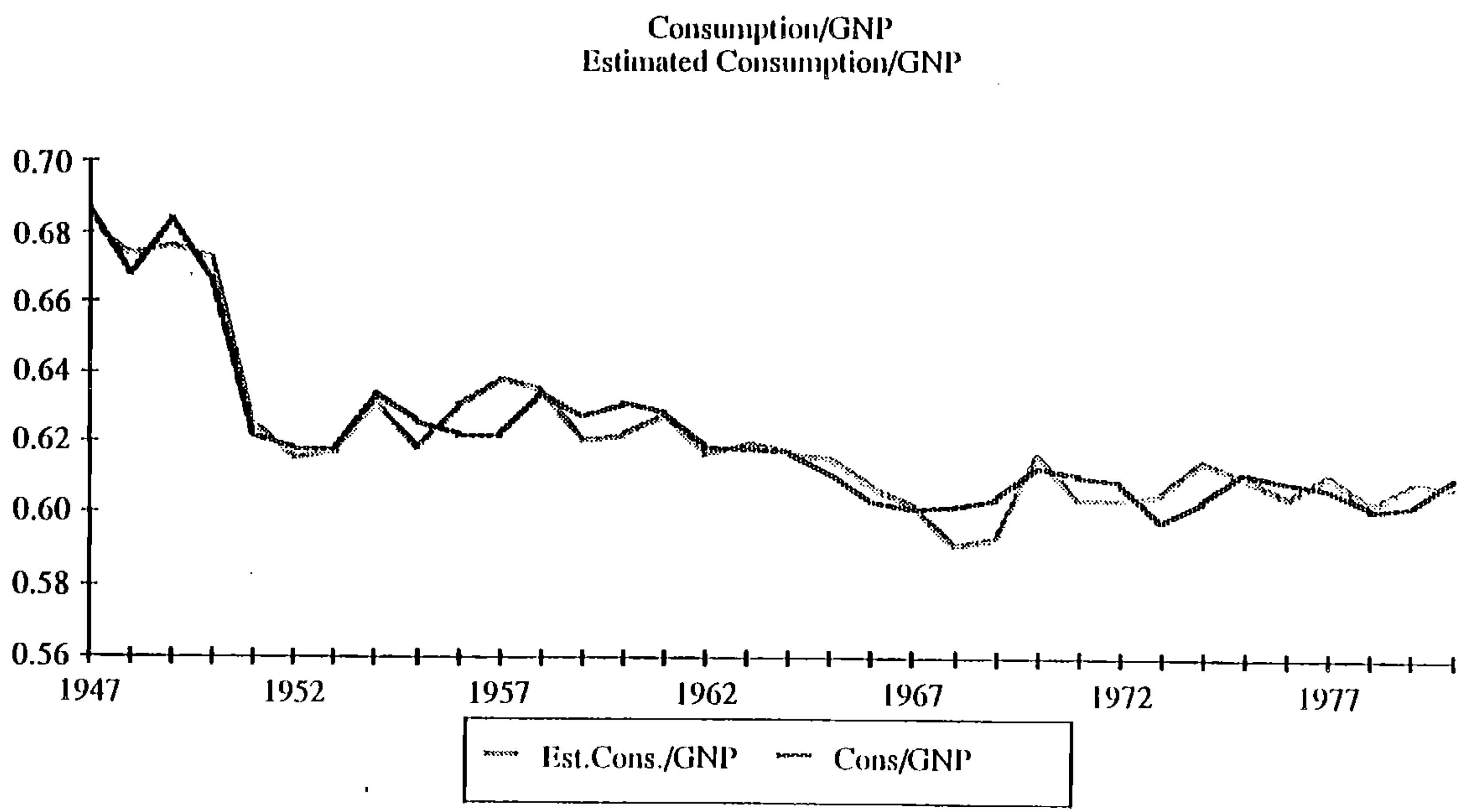


Figure 4.2



Consumption/GNP
Estimated Consumption/GNP
0.70
0.68
0.66
0.64
0.62
0.60
0.58
0.56
1947
1952
1957
1962
1967
1972
1977
Est.Cons./GNP
Cons/GNP

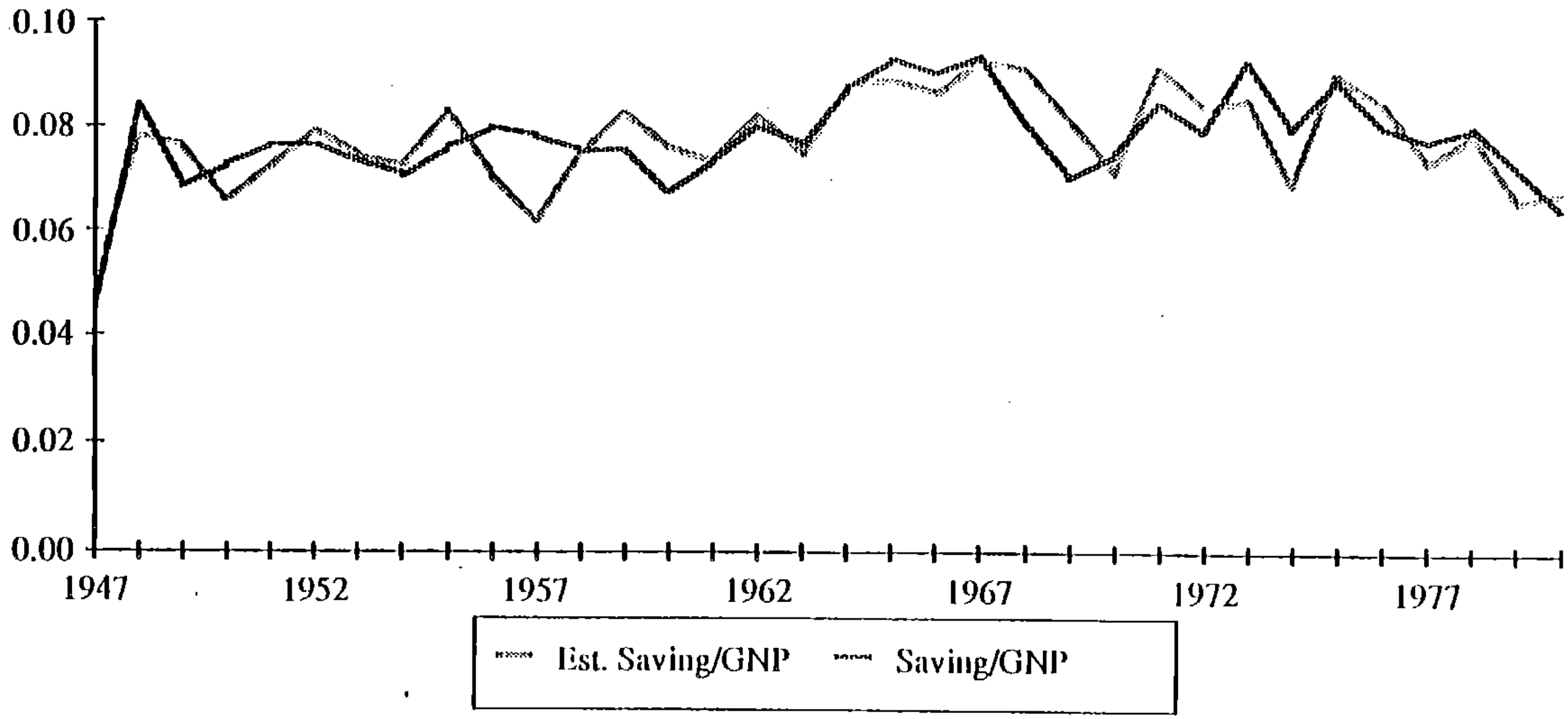
1947-80. In the immediate post-war period, saving, which had been a very small fraction of GNP, rose substantially and this accounts for a five or six percentage point decline in the consumption to GNP ratio from 1947 to 1951. Consumption from the 1950s to the early 1960s was relatively constant as a share of GNP. Beginning in the early 1960s, it fell for several years, and then fluctuated in a narrow band around 0.61 through the end of the 1970s. Our estimated consumption/GNP ratio tracks the actual consumption/GNP ratio quite well. The maximum deviation for any year is about one percent of GNP.

Figure 4.3 looks at the flip side of consumption, namely saving. It presents the actual NIPA saving to GNP ratio and the estimated saving to GNP ratio derived from our consumption expenditure equation reported below. Again, from a very low rate in the immediate post-war period, saving as a ratio of GNP rose rapidly in the late 1940s, was fairly level around 7% throughout the 1950s, rose to a little over 8% in the early 1970s, and has been on a somewhat downward trend from the early 1970s to 1980 (a trend which continued in the 1980s). Again, since our estimated consumption to GNP ratio reflects actual consumption to GNP quite well, so does our estimated saving to GNP ratio track actual saving to GNP quite well. The maximum deviation is again approximately 1% of GNP (in 1957, at the time considered a bad recession).

We now turn to a detailed discussion of the econometric model and results. These results include estimates of aggregate consumption and leisure expenditure share (of aggregate wealth) equations, tests of various hypotheses concerning aggregate consumption and estimates of parameters of particular economic interest.

Figure 4.3

Saving/GNP
Estimated Saving/GNP



4.2 Econometric Specification and Estimation

We start from the specifications in equations (3.37) and (3.38). We distinguish the types of households by means of the following set of dummy variables:

$$D_{it}^{14} = \begin{cases} 1, & \text{if the age of the head of the ith household is between 14 and 24 years in the tth period;} \\ 0, & \text{otherwise.} \end{cases}$$

$$D_{it}^{25} = \begin{cases} 1, & \text{if the age of the head of the ith household is between 25 and 34 years in the tth period;} \\ 0, & \text{otherwise} \end{cases}$$

$$D_{it}^{35} = \begin{cases} 1, & \text{if the age of the head of the ith household is between 35 and 44 years in the tth period;} \\ 0, & \text{otherwise.} \end{cases}$$

$$D_{it}^{55} = \begin{cases} 1, & \text{if the age of the head of the ith household is between 55 and 64 years in the tth period;} \\ 0, & \text{otherwise.} \end{cases}$$

$$D_{it}^{65} = \begin{cases} 1, & \text{if the age of the head of the ith household is greater than or equal to 65 in the tth period;} \\ 0, & \text{otherwise.} \end{cases}$$

$$D_{it}^{39} = \begin{cases} 1, & \text{if the head of the ith household in the tth period was born prior to 1939;} \\ 0, & \text{otherwise.} \end{cases}$$

We note that because of the presence of the constants α_c and α_z in equations (3.37) and (3.38), it is not necessary to have a dummy variable for households headed by persons in the age-cohort of 45-54 years. We further note that the dummy variable D_{it}^{39} attempts to distinguish between those households headed by persons who experienced the Great Depression and those which were not.

In accordance with equations (3.37) and (3.38), we form variables of the type:

$$\frac{\sum_i D_{it}^j W_{it}}{\sum_i W_{it}}$$

for each j and t. These variables may be interpreted as the share of aggregate wealth held by households headed by persons in the jth age-cohort in the tth period.

In addition, we introduce two non-household specific independent variables which are believed to influence current consumption and leisure decisions. The first variable is the female labor force participation rate (FLPR), in percent.[28] The second variable is the unemployment rate, represented by the natural logarithm of the prime age white male unemployment rate (UE), in percent. These two independent variables, as well as the share of wealth held by the pre-1939 cohort, are presented in Figure 4.4 (normalized to be 1.0 in 1972), and compared against the pure time trend variable. It is readily apparent that these variables move quite independently of one another and of the pure time trend.

Finally, we add stochastic disturbance terms to both the consumption and leisure expenditure share equations. The stochastic disturbance terms are assumed to have a constant variance-covariance matrix over time and are possibly correlated across equations but not across time periods. We use annual data

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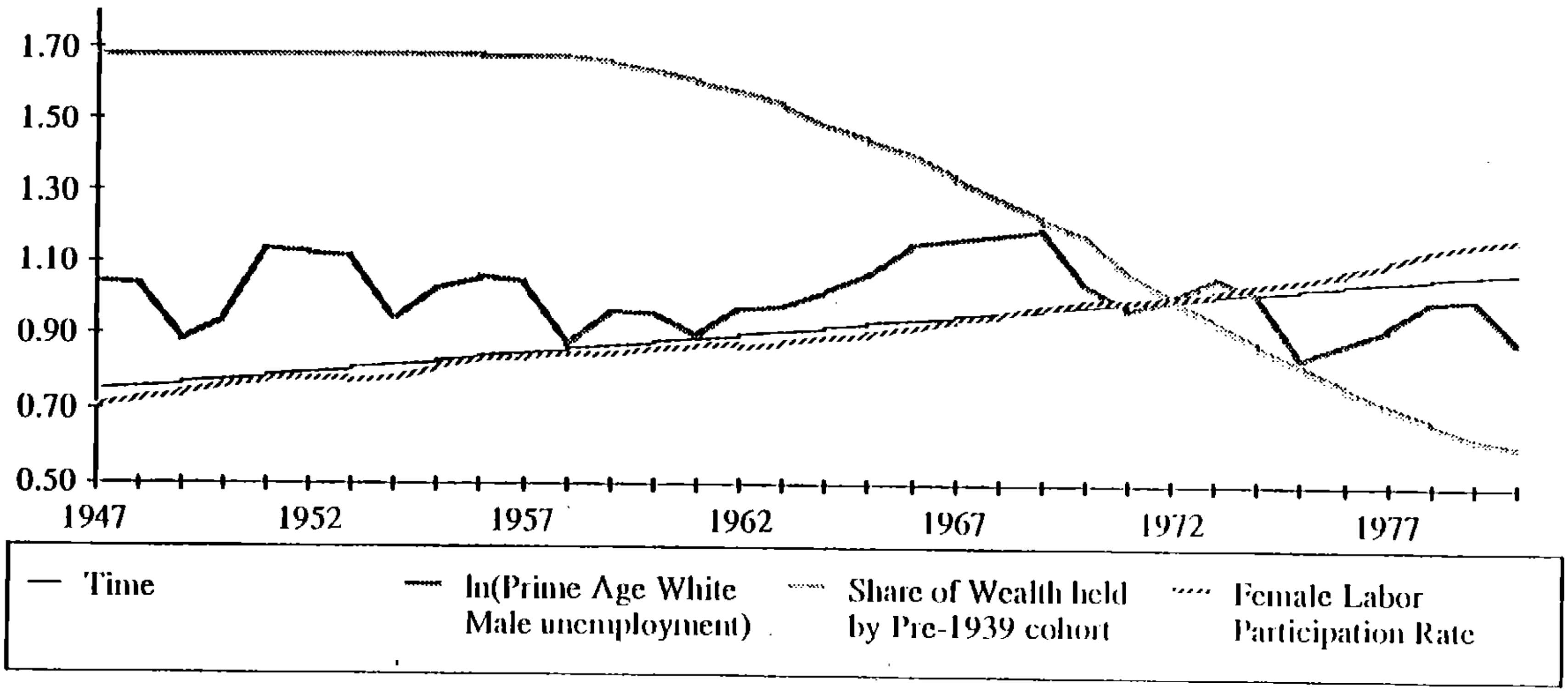
28. For the leisure expenditure equation, it is assumed that the female labor force participation rate has no effect before 1963. The female labor force participation rate accelerated around 1963, as did related household formation and dissolution data, and it roughly marked the beginning of the changing perception of women's roles in society.

Figure 4.4

Additional Regressors Used in Base Case
(normalized 1972 = 1)



from 1947 through 1980 for the estimation. The method of estimation used is maximum likelihood, conditional on the given values of the right-hand-side variables.[29] The final specification takes the form:

$$(4.1)\quad \sum_i p_t C_{it} \Big/ \sum_i W_{it}$$

$$= \Big[(\alpha_c + \beta_{cc}\ln p_t + \beta_{cz}\ln w_{0t} + \beta_{cr}\ln(1+r_t)) - (\beta_{cc}+\beta_{cz})(\sum_i W_{it}\ln W_{it} / \sum_i W_{it})$$

$$+ \alpha_{c14}(\sum_i D^{14}_{it}W_{it} \Big/ \sum_i W_{it}) + \alpha_{c25}(\sum_i D^{25}_{it}W_{it} / \sum_i W_{it})$$

$$+ \alpha_{c35}(\sum_i D^{35}_{it}W_{it} \Big/ \sum_i W_{it}) + \alpha_{c55}(\sum_i D^{55}_{it}W_{it} \Big/ \sum_i W_{it})$$

$$+ \alpha_{c65}(\sum_i D^{65}_{it}W_{it}) \quad + \quad \gamma_{cf}\ FLPR + \gamma_{cu}UE$$

$$+ \gamma_{cv}(\sum_i D^{39}_{it}W_{it} \quad \Big/ \quad \sum_i W_{it}) \Big] \quad \Big/ \quad [1+\beta_c(\ln p_t - \ln w_{0t}) + \beta_r \ln(1+r_t)]$$

$$+ \epsilon_{ct}$$

$$(4.2)\quad \sum_i w_{it}Z_{it} \quad \Big/ \quad \sum_i W_{it}$$

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29. While at the level of the individual household it is reasonable to assume that the household is a price-taker in the markets of consumption, labor and credit, in the aggregate it is not plausible to maintain that the prices are exogenous or predetermined. Thus, it may be preferable to estimate the parameters by a nonlinear instrumental variables approach. This will be attempted at a later date.

$$= \Big[(\alpha_z + \beta_{zc} \ell np_t + \beta_{zz} \ell nw_{0t} + \beta_{zr} \ell n(1+r_t)) - (\beta_{zc} + \beta_{zz}) (\sum_i W_{it} \ell nW_{it} \ / \ \sum_i W_{it})$$

$$+ \alpha_{z14} (\sum_i D^{14}_{it} W_{it} \ / \ \sum_i W_{it}) + \alpha_{z25} (\sum_i D^{25}_{it} W_{it} \ / \ \sum_i W_{it})$$

$$+ \alpha_{z35} (\sum_i D^{35}_{it} W_{it} \ / \ \sum_i W_{it}) + \alpha_{z55} (\sum_i D^{55}_{it} W_{it} \ / \ \sum_i W_{it})$$

$$+ \alpha_{z65} (\sum_i D^{65}_{it} W_{it} \ / \ \sum_i W_{it})$$

$$+ \gamma_{zf} \ FLPR + \gamma_{zu} UE + \gamma_{zv} (\sum_i D^{39}_{it} \ / \ \sum_i W_{it}) \Big] \ / \ [1 + \beta_c (\ell np_t - \ell nw_{0t}) + \beta_r \ell n(1+r_t)]$$

$$+ \epsilon_{zt}$$

A word needs to be said about the estimation of the parameters α_{cj}'s and α_{zj}'s. They are not estimated entirely from the aggregate time-series consumption and leisure to wealth share equations. Instead, the difference between the consumption expenditure-wealth ratios (and the leisure expenditure-wealth ratios) of any two age cohorts in 1972 is assumed to be the same as the difference between the corresponding averages derived from the 1972-73 Consumer Expenditure Survey. However, the absolute levels of the cohort-specific consumption expenditure-wealth and leisure expenditure-wealth ratios are not constrained to be the same as the averages in the 1972-1973 Consumer Expenditure Survey. In addition, these ratios are required to satisfy unimodality constraints, that is, the slopes of their respective age profiles are allowed to change signs only once up to age 64.