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TAXATION, INFLATION, AND MONETARY POLICY

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Taxation, Inflation and Monetary Policy

by

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1. Introduction

The concept of taxable income which is widely accepted by economists is that of total accretion (see e.g. Simons (1938, p.49) and Musgrave (1965, p.165)). According to this concept, all accretions to wealth are included, in whatever form or from whatever source they are received. As a matter of principle it is obvious that accretion as an index of equality should be measured in real terms. The value of net worth at the end of the period must be deflated in order to determine the change in net worth. The application of this principle calls for the exclusion from taxable income of capital gains that reflect an increase in the price level, and for allowance for losses suffered from the holding of claims (such as money).

In favor of disregarding capital gains that do not reflect real income increases, it has sometimes been argued that wage increases in inflationary times also reflect gains that are not real and that, accordingly, comprehensive money income is an appropriate tax base (Groves (1959)). In reply, it has been convincingly demonstrated by Diamond (1974) that a comprehensive tax base would discriminate against taxpayers whose income has a sizeable profit component. The reason is that inflation affects capital and labor incomes differently, resulting in a greater percentage increase in income from capital than from income from labor. He has further shown that a deduction of the inflation rate times the value of assets will preserve the relative contributions to the tax base of the two sources of income. Thus, equity consideration calls for such an <u>inflation-exclusion</u>, to be available to all type of investment. But there are obvious difficulties in providing exclusion to all assets, an example being money holdings (cash). Since cash earns zero money income, the application of an inflation deduction with full loss-offset would call for <u>tax credits</u> for holders of cash. The accounting and book keeping that would be involved seem prohibitive. Diamond has therefore suggested to provide an exclusion only, allowing no tax credits. However, the provision of an inflation-exclusion to some assets only, introduces efficiency considerations into this issue. Thus, partial allowance for inflation may lead to a waste of resources used to convert assets from those without to those with such an exclusion provision. Given that application of the principle with full loss offset to all assets is impracticable, we may therefore wish to consider provision of only a partial inflation-exclusion to assets for which it is feasible. The problem is examined in this paper by means of a simple model of anticipated inflation, in which individuals may invest either in assets for which full or partial inflation-exclusion is provided, or in cash, for which no loss offset is allowed.

Among other issues, we shall examine the short and long run effects of taxation and of the provision of an inflation deduction on the rate of inflation and on the level of savings.

We do not discuss the long-run optimum tax and deduction rates, because it turns out that for a given tax revenue, these instruments are perfect substitutes, i.e. their relative size does not affect the equilibrium configuration.

Our analysis leads us, however, to discuss the optimality of monetary policy in the long-run. There seems to be a remarkable agreement among most writers (for example, Phelps (1965), Samuelson (1968) and (1969), Friedman (1969)) that the laissez faire system does not lead automatically to optimality of the quantity of real balances. The argument is that the cost of holding money for the individual is necessarily different from the cost to society. While the cost to the individual is measured by the (money) rate of interest, the cost to society is ordinarily assumed to be (practically) zero. The suggested recipe for optimality is therefore to satiate the economy with money, i.e., to increase the quantity of real balances to a point where their marginal utility (or product) is zero. In an optimum the cost of holding money should be driven down to zero. Following

Phelps we call this situation "full liquidity".

It seems, however, that the foregoing reasoning has overlooked an important point. Specifically, although there is no direct cost of providing nominal money bills there may be an economic cost of providing the economy with real money balances. This cost results from the fact that an increased quantity of real money can be absorbed in the economy only through the market mechanism, and this may lead to changes in the real part of the system. Thus, for example, if the quantity of real balances increases, consumers may feel better off and increase their consumption expenditures which will reduce the accumulation of physical capital. It is then possible that the cost involved in the expansion of real money may outweigh the gains before full liquidity is reached.

The model considered here is one of imperfect control, where the government operates a relatively small number of basic instruments which are not capable of eliminating the social cost of money. In analyzing this problem we shall assume that the utility functions of the private and government sectors are identical and that the difference between the two sectors (in off-steady state situations) arises from better foresight on the part of the government. In particular, the private sector will be assumed to behave according to (changing) "static-expectations", even under dynamic conditions, while the government will be assumed to have a full evaluation of the future.

In the foregoing model money cannot be made neutral in the short run because of inadequacy of the instruments. We cannot, therefore, expect to have full liquidity in the short run if the economy is off-steady-state. However, even under steady-state conditions, money is not neutral except in the absence of imperfections created by, say, taxes. In principle, there may be a stationary optimum (or optima) where the optimum quantity of money is short of full liquidity because of the real social cost of increasing real balances. We shall illustrate this by an example which is in itself not very realistic but is indicative of similar situations which may arise under realistic conditions. In the foregoing example we shall assume that the rate of monetary expansion is the only instrument available to the government, and that the utility function is of the additive logarithmic form.

The upshot of the foregoing statements is that a policy offull liquidity is not necessarily optimal in the short run or in the long run if there is imperfect foresight and imperfect control in off-study state situations.

2. The Model

2.1 The Individual's Optimum Conditions

We assume that the individuals in the economy can be represented by one family. This family lives forever and the number of its members (L) increases by n per-cent per unit of time so that $L_t = L_0 e^{nt}$. The utility function of the representative unit is given by

$$U = \int_0^\infty e^{-\delta t} u(c_t, m_t) dt \tag{1}$$

where c_t is physical consumption per-capita planned for time t and m_t is the amount of real balances per-capita at time t. The latter are related to nominal balances (M) by

$$m_t = \frac{M_t}{P_t L_t} \tag{2}$$

where P_t is the expected price level of the physical good in terms of nominal money. The instantaneous utility function, $u(\ )$, is assumed to have the usual concavity properties of neo-classical production functions. In addition we have a positive subjective discount rate δ.

Let w_t and r_t be the expected real wage rate and real rate of interest at time t. The money rate of interest (i) is related to the real rate of interest by $i_t = r_t + p_t$, where $p_t = \frac{\dot{P}_t}{P_t}$ is the expected rate of change of the price level.

The stock of capital per-capita (k) owned by the individuals is rented to firms and yields the money rate of interest i.

The material wealth of the individuals is given by $a = k + m$. To this we must add their human welath (h) which consists of the present value of the wage stream. The latter is given by

$$(3) \qquad h_u = \int_u^{\infty} w_s e^{n(s-u)} e^{-\int_u^s r_x d_x} = \int_u^{\infty} w_s e^{-\int_u^s \rho_x dx}$$

where u denotes the individual's planning time and where $\rho_x = r_x - n$. If w_s converges and ρ_x converges to a positive constant (as along steady states) then h_u is well-defined.

Finally, we add the present value of real government transfers (g) which is given by the same expression as (3) with transfers at time s, v_s, substituted for w_s. Again if v_s converges to a constant, then g is well-defined.

Denote the total wealth of the individual by y_u, so that $y_u = a_u + h_u + g_u$. Then feasibility of his consumption plan can be taken to mean that $y_u \geq 0$ for all u along the plan horizon. Since $\dot{a}_u = \rho_u a_u - i_u m_u - c_u + w_u + v_u$, $\dot{h}_u = \rho_u h_u - w_u$ and $\dot{g}_u = \rho_u g_u - v_u$, we have

$$(4) \qquad \dot{y}_u = \rho_u y_u - (c_u + i_u m_u) = \rho_u y_u - \bar{c}_u$$

where we use the fact that money does not earn any interest and it is subject to a "depreciation" rate p. The value of <u>total</u> consumption, $\bar{c} = c + im$, includes both ordinary consumption and consumption of services of liquid assets. Solving (4) we obtain

(5) $$y_u = e^{\int_0^u \rho_x ds}\left(y_0 - \int_0^u \bar{c}_s e^{-\int_0^s \rho_x dx} ds\right)$$

for some initial date $t = 0$. It follows that feasibility implies

(6) $$y_0 \geqq \int_0^u \bar{c}_s e^{-\int_0^u \rho_x dx} ds \quad \text{for all} \quad u \geqq 0$$

meaning that the present value of consumption for any u cannot exceed initial wealth.

Denote the present value of consumption in (6) by z_u. We may assume that $i \geqq 0$. Since both c and m are non-negative we have $\bar{c} \geqq 0$. It follows that $\dot{z}_u \geqq 0$. Noting that z_u is bounded from above and non-decreasing we know that there exists $\lim\limits_{u \to \infty} z_u = z$ (say). From (6), $z \leq y_0$. However, if $z < y_0$, then it is obviously possible to construct an alternative feasible consumption path which has more consumption than the original one over some finite interval, and no less consumption elsewhere. Since $u_c = \frac{\partial u}{\partial c} > 0$, the original path cannot be a candidate for an optimal one. We may thus define an <u>efficient program</u> as one which satisfies $z = y_0$, i.e. a program which satisfies the budget constraint with equality. By (5), then, an efficient program satisfies

(7) $$\lim_{u \to \infty} y_u e^{-\int_0^u \rho_s ds} = 0$$

meaning that the present value of 'terminal' net worth is zero.

The consumer's optimum problem is to maximize (1) subject to (4) and the non-negativity conditions on y, c and m. Assuming an interior solution, the Euler-Conditions for the optimal paths c_u, m_u and y_u are given by

$$\frac{u_m(c_u,m_u)}{u_c(c_u,m_u)} = i_u \tag{8}$$

$$\frac{\dot{u}_c(c_u,m_u)}{u_c(c_u,m_u)} = \rho_u - \delta \tag{9}$$

We may solve the differential equation (9) to obtain

$$\hat{u}_{c,u} = \hat{u}_{c,0}\, e^{-\int_0^u (\rho_s - \delta)ds} \tag{10}$$

where $\hat{u}_{c,u}$ is the value of u_c corresponding to the Euler-path at time u. Combining (7) and (10) we obtain the transversality condition

$$\lim_{u\to\infty} e^{-\delta u}\, \hat{u}_{c,u} y_u = 0 \tag{11}$$

which holds for any efficient path of y_u. It can be shown by standard methods that a feasible path which satisfies (4), (8), (9) and (11) is optimal, i.e. it maximizes (1).

2.2. Static Expectations: Explicit Solutions

An explicit solution of the optimal path can be found with the additional assumption that the consumer expects all the exogeneous variables to remain constant, i.e. static expectations,

$$\rho_u = \rho,\quad p_u = p,\quad w_u = w,\quad v_u = v \quad (\rho = r-n > 0,\quad i = r + p > 0). \tag{12}$$

The budget relation (6), which we have shown to hold with equality, is now given by

$$(13) \qquad a_0 + \frac{w+v}{\rho} = \int_0^\infty (c_u + im_u)\, e^{-\rho u} dn$$

where $a_0 = k_0 + m_0$ is the initial real value of assets per-capita.

Assume further that the utility function $u(\ )$ is of the additive-log form

$$(19) \qquad u(c,m) = \log c + \beta \log m.$$

Under these assumptions (8) and (9) become

$$(8') \qquad \frac{\beta c_u}{m_u} = i$$

$$(9') \qquad \frac{\dot{c}_u}{c_u} = \rho - \delta$$

(9') can be solved to yield $c_u = c_0 e^{(\rho-\delta)u}$. Substituting this solution in (13), using (8'), we may solve for the demand for c_0 and m_0

$$(15) \qquad c_0 = \frac{\delta}{1+\beta}\left(a_0 + \frac{w+r}{\rho}\right)$$

$$(16) \qquad m_0 = \frac{\delta\beta}{(1+\beta)i}\left(a_0 + \frac{w+r}{\rho}\right)$$

Note that the money rate of interest, i, must be positive if m_0 is to be positive and finite. As expected, the demand functions (15) and (16) are seen to have negative price and positive wealth derivatives.

2.3 Macro Relations

The foregoing maximization takes place at an actual time $t = 0$ (to distinguish from the individual's planning time denoted by u). By relating the consumer's expectations to prevailing values of the exogeneous variables, and the latter to the macro variables by means of an aggregate production function and market equilibrium conditions, the dynamic path of the economy can be traced.

The production function in the economy is of the neo-classical type and is given by

$$\dot{k}_t + \lambda k_t + c_t = f(k_t), \quad \lambda = n + \mu \tag{17}$$

where λ is the sum of the rate of population growth and physical depreciation (μ).

Transfer payments to individuals are equal to the government's deficit, which is covered by an increase in the money supply, $v_t = \frac{\dot{M}_t}{P_t L_t}$, or $v_t = \theta_t m_t$ where $\theta_t = \frac{\dot{M}_t}{M_t}$. By definition, the time rate of change in m_t is given by

$$\dot{m}_t = (\theta_t - p_t - n)m_t \tag{18}$$

where p_t is now the actual rate of price change, which implies instantaneous adjustment of expectations or "warranted price expectations"[1]. Competitive equilibrium in labor and capital markets implies

$$\rho_t = f'(k_t) - \lambda, \quad w_t = f(k_t) - k_t f'(k_t) \tag{19}$$

We assume that ρ_t is positive for all t (which entails $r = f' - \mu > f' - \lambda = \rho > 0$). This assumption is justified, as will be seen below, along any equilibrium path.

Using the relations $a = k + m$, $v = \theta m$, (15) and (19), we may express the demand for consumption at time t as

$$c_t = \frac{\delta}{1+\beta}\left(k_t + m_t + \frac{f(k_t) - k_t f'(k_t) + \theta_t m_t}{f'(k_t) - \lambda}\right) \tag{20}$$

Interpreting (16) in macro terms, and assuming that at each moment of time the expectations about the instantaneous rate of price increase (p) are determined so as to be consistent with momentary equilibrium, the condition for instantaneous equilibrium in the money market is

$$m_t = \frac{\delta\beta}{(1+\beta)(f'(k_t)-\mu+p_t)}\left(k_t + m_t + \frac{f(k_t)-k_t f'(k_t)+\theta_t m_t}{f'(k_t)-\lambda}\right) \tag{21}$$

We may solve (21) for p_t, and by substitution of this solution and (20) into equations (17) and (18) obtain the dynamic path of the economy for given initial conditions (k_0, m_0) at $t = 0$, and for any given function θ_t, $t \in [0,\infty]$.

The simplest case is to take θ_t as constant[2], say, θ. It turns out that for <u>any</u> such policy, there exists a unique steady-state of the system, with $\dot{k} = \dot{m} = 0$. Obviously, by (18), in steady-state $p = \theta - n$. It is interesting to note that the system can be shown to be unstable for deviations from steady state. A similar result has been obtained by Uzawa (1966) under somewhat different assumptions, but also with <u>instantaneous adjustment</u> of expectations, which seems to be the crucial assumption for this result.[3]

We shall now introduce into the system an income tax and examine its implications on individual behavior and on the equilibrium configuration.

3. Income Taxation

3.1. Equity and Efficiency Considerations

Since we assume that labor is supplied inelastically, we can concentrate on the effects of taxation on the income from capital.

In the presence of a positive inflation rate, the shortcoming of a comprehensive tax base applied to money income is that it results in a greater percentage increase in taxable income from capital then from labor income. This inequity provides an argument for an adjustment for inflation of the return to capital. Specifically, we shall consider an inflation exclusion, allowing a deduction from taxable income that depends on the inflation rate times the value of capital assets.

Equity and efficiency considerations both call for the inflation exclusion to be available for any type of investment. This is necessary in order to have the net relative rates of return from various investments independent of the inflation rate. There are, however, some obvious difficulties in applying the inflation exclusion to all assets. The asset that raises the most obvious difficulties is cash. Since cash yields zero monetary gains (although it provides monetary benefits in the form of liquidity services), the application of an inflation deduction on cash holdings would call for tax credits for individuals holding cash. The non-taxation of liquidity benefits could be removed, in principle, by including these imputed benefits, equal (in equilibrium) to the rate of return on other assets times the amount of cash, in taxable income. But the accounting and record keeping that would be involved seem prohibitive.

It is thus reasonable to consider an adjustment for inflation by means of an exclusion only, allowing no tax credit for the failure of cash to yield at least the inflation rate. Allowing an inflation-adjustment only for some assets and none for others clearly involves an efficiency loss, generated by resources used to convert assets from those without to those with the inflation exclusion. It may therefore be appropriate to provide only for partial adjustment of those assets for which exclusion is available. We attempt here to evaluate these considerations relating to the provision of an inflation adjustment, from the point of view of the individual and from the point of view of society.

3.2 Equilibrium Conditions

Suppose that taxable income is defined as the money return on the value of capital minus a fraction $\varepsilon(1 \geq \varepsilon \geq 0)$ of the rate of inflation times the value of capital. Taxable income per-unit of capital is thus $i - \varepsilon p = r + (1-\varepsilon)p$. A full inflation exclusion $(\varepsilon = 1)$ means that only real income is taxable, while no exclusion $(\varepsilon = 0)$ means that all nominal returns are taxable. Let the tax rate be $\tau \geq 0$. For the individual, real after-tax return of a unit of capital per capita is now given by

$$(22) \qquad \bar{\rho} = \rho - \gamma \quad \text{where} \quad \gamma = \tau(i - \varepsilon p).$$

Accordingly, the opportunity cost of cash holdings is now $\bar{i} = i - \gamma$.[4] The individual's optimum conditions are given by (8') and (9') with $\bar{\rho}$ and $\bar{i}$ replacing ρ and i, respectively. Assuming that tax receipts are returned to the individual by lump-sum transfers, the budget constraint, (4), remains unchanged. Under the assumption of static expectations, the solutions for c_0 and m_0 are found to be:

$$c_0 = \frac{(\rho - \bar{\rho} + \delta)\bar{i}}{\bar{i} + i\beta}(a_0 + \frac{w+v}{\rho}) = \frac{(\gamma + \delta)\bar{i}}{\bar{i} + i\beta}(a_0 + \frac{w+v}{\rho}) \tag{15'}$$

$$m_0 = \frac{(\rho - \bar{\rho} + \delta)}{\bar{i} + i\beta}(a_0 + \frac{w+v}{\rho}) = \frac{(\gamma + \delta)}{\bar{i} + i\beta}(a_0 + \frac{w+v}{\rho}) \tag{16'}$$

The previous solutions (15) and (16) are seen to be special cases of (15') and (16') when $\gamma = 0$.

Using the relations $a = k + m$ and $v = \theta m$, and assuming that the demand and supply of money at any time t are equal, (21) becomes

$$m = \frac{(\gamma + \delta)\beta}{\bar{i} + i\beta}(k + m + \frac{f - kf' + \theta m}{f' - \lambda}) \tag{21'}$$

Again, the previous solution, (21), is seen to be a special case of (21') when $\gamma = 0$. Equation (21'), the condition for momentary equilibrium in the money market, can be solved for the rate of price change,

$$p = \frac{\delta\beta x - (1+\beta - (1+\beta x)\tau)(f' - \mu)}{1+\beta - (1-\varepsilon)(1+\beta x)\tau} \equiv \pi(k,m,\xi,\tau,\varepsilon) \tag{23}$$

where $x = 1 + \frac{k}{m} + \frac{\frac{1}{m}(f-kf') + \theta}{f' - \lambda}$.

Using (23) and (15'), consumption at any time t can be expressed by

$$c = \frac{(\gamma + \delta)\bar{i}}{\bar{i} + i\beta}(k + m + \frac{f - kf' + \theta m}{f' - \lambda}) \equiv \hat{c}(k,m,\theta,\tau,\varepsilon) \tag{20'}$$

3.3. Short-Run Effects of Monetary Expansion

In the short-run k and m are constant, and equilibrium is attained by adjustment of the rate of inflation and consumption. Thus, given k and m, the effect on π of an increase in θ can be directly calculated from (23):

$$\pi_\theta = \frac{\beta(\gamma + \delta)}{(1+\beta - (1-\varepsilon)(1+\beta x)\tau)(f'-\lambda)} > 0 \tag{24}$$

where a subscript denotes partial derivative with respect to this variable.

The short-run change in consumption due to an increase in θ is found from (20'):

$$\hat{c}_\theta = \frac{m}{\beta}(1-\tau(1-\varepsilon))\pi_\theta = \frac{m(1-(1-\varepsilon))(\gamma+\delta)}{(1+\beta - (1-\varepsilon)(1+\beta x)\tau)(f'-\lambda)} > 0. \tag{25}$$

These results are stated in the following:

Proposition 1. In the short-run, an increase in the rate of change of money supply increases the rate of inflation and the level of consumption.

3.4. Short-Run Effects of Taxation

Given k and m, the effects on π of an increase in τ or ε can also be calculated from (23):

$$\pi_\tau = \frac{(1+\beta x)(f'-\mu+p(1-\varepsilon))}{1+\beta - (1-\varepsilon)(1+\beta x)} > 0 \tag{26}$$

and

$$\pi_\varepsilon = \frac{p(1+\beta x)}{1+\beta - (1-\varepsilon)(1+\beta x)} < 0. \tag{27}$$

An increase in the tax rate creates a substitution effect in favor of the demand for real balances, (16'). In order to bring demand into equilibrium with the given supply of money, this increase generates an increase in the (actual and expected) rate of inflation. An increase in the inflation-exclusion rate has exactly the opposite effect:

Proposition 2. In the short-run, an increase in the income tax rate increases, and an increase in the inflation-exclusion rate decreases, the rate of inflation.

Short-run changes in consumption due to an increase in τ or ε are found by differentiating (20'), using (21'),

$$(28) \qquad \hat{c}_\tau = \frac{m}{\beta}\,[(1-\tau(1-\varepsilon))\pi_\tau - r - p(1-\varepsilon)] =$$

using (27)

$$= \frac{(f'-\mu+p(1-\varepsilon))(x-1)m}{1+\beta-(1-\varepsilon)(1+\beta x)} > 0$$

since $x > 1$. Similarly,

$$(29) \qquad \hat{c}_\varepsilon = \frac{m}{\beta}\,[\tau p + (1-\tau(1-\varepsilon))\pi_\varepsilon] =$$

using (27)

$$= -\frac{\tau p(x-1)m}{1+\beta-(1-\varepsilon)(1+\beta x)} < 0$$

We thus have:

Proposition 3. In the short-run, an increase in the income tax rate increases, and an increase in the inflation-exclusion rate decreases, the level of consumption.

Notice that the government's tax revenue is equal to $\gamma k = \tau(k-\varepsilon p)k$, so that a constant k and a given revenue imply a constant γ. Since only γ

appears in the equilibrium equations (20') and (21'), we have:

Proposition 4. Changes in the tax rate and in the inflation-exclusion rate that keep the government's tax revenue constant, do not affect the short-run equilibrium values.

4. Long-Run Effects

4.1. Steady-State

Inserting (20') and (21') into (17) and (18), we obtain the dynamic equations for the economy

$$(30) \qquad \dot{k} = f(k) - \lambda k - c = f(k) - \lambda k - \hat{c}(k,m,\theta,\tau,\varepsilon) \equiv z(k,m,\theta,\tau,\varepsilon)$$

and

$$(31) \qquad \dot{m} = (\theta - p - n)m = (\theta - \pi(k,m,\theta,\tau,\varepsilon) - n)m \equiv s(k,m,\theta,\tau,\varepsilon)$$

Equations (28) and (29) determine the dynamic path of the economy for given initial conditions (k_0, m_0) at $t = 0$, and with any arbitrary policies $(\theta,\tau,\varepsilon)$, for $t \in [0,\infty]$.

Let us examine the steady-state properties of the model for constant values of θ, γ, and ε, and for $\dot{k} = \dot{m} = 0$, i.e.

$$(32) \qquad \begin{aligned} &z(k,m,\theta,\tau,\varepsilon) = 0 \\ &\text{and} \\ &s(k,m,\theta,\tau,\varepsilon) = 0 \end{aligned}$$

It can easily be shown that at $\tau = \varepsilon = 0$, we have for <u>any</u> θ: $f' - \lambda - \delta = 0$, that is the '<u>Modified Golden Rule</u>' path.[6] Denote this path by (k^*, m^*). From the individual's equilibrium condition (8'), it is seen that on this path

$$(33) \qquad m^* = \frac{\beta(f - \lambda k^*)}{\delta+\theta}$$

By differentiating (32) at $\tau = \varepsilon = 0$, using (20') and (23), we obtain:

$$z_k = \delta - \hat{c}_k = \delta - \frac{\delta m}{1+\beta}\,\frac{\partial x}{\partial x} = \frac{\beta\delta}{1+\beta} + \frac{mf''}{1+\beta}\left(\frac{f-\lambda k}{m} + \frac{\theta}{\delta}\right)$$

$$(34)\quad z_m = -\hat{c}_m = -\frac{\delta+\theta}{1+\beta} < 0.$$

$$s_k = -\pi_k = -\frac{\delta\beta}{1+\beta}\,\frac{\partial x}{\partial x} + f'' = -\frac{\delta\beta}{1\ \beta}\left[\frac{1}{m} - \frac{f''}{\delta}\left(\frac{f-\ k}{m} + \frac{\theta}{\delta}\right)\right] + f'' \quad 0$$

$$s_m = -\pi_m = -\frac{\delta\beta}{1+\beta}\,\frac{\partial x}{\partial m} = \frac{\beta(f-\lambda k)}{(1+\beta)m^2} > 0$$

In the (k,m) plane, the slope of the curve defined by $s(\ ) = 0$ is thus positive:

$$(35)\qquad \left.\frac{dm}{dk}\right|_{s=0} = -\frac{s_k}{s_m} > 0$$

The slope of the curve defined by $z(\ ) = 0$ depends on the sign of z_k. We assume that the first term in z_k, (34), is relatively small so that $z_k < 0$. Under this assumption

$$(36)\qquad \left.\frac{dm}{dk}\right|_{z=0} = -\frac{z_k}{z_m} < 0.$$

By (35) and (36), at $\tau = \varepsilon = 0$, if an equilibrium steady-state exists then it is unique (Figure 1).

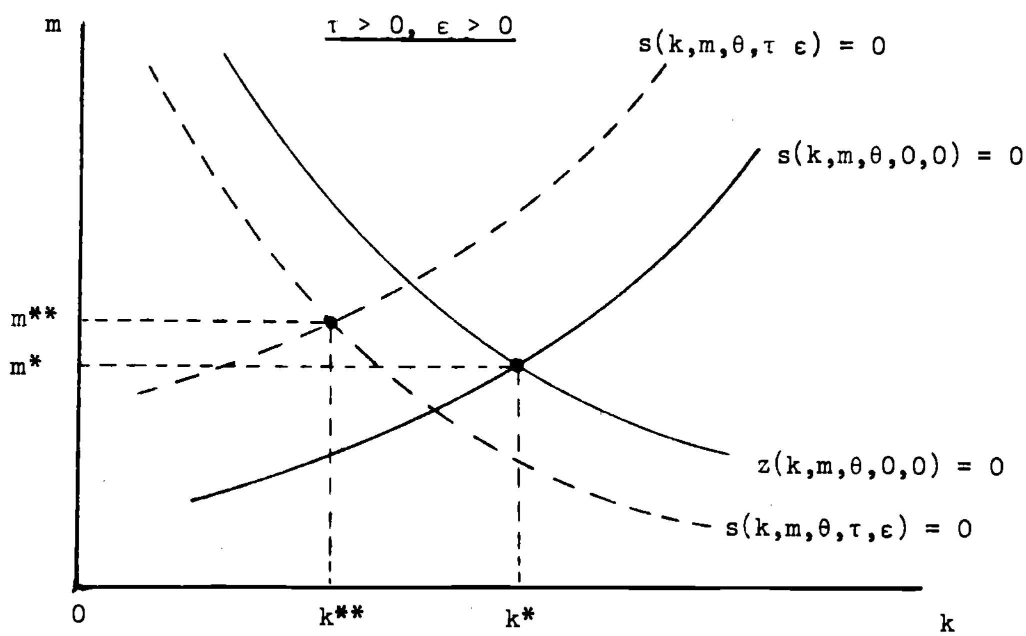

Figure 1

Now, by (32) and (28) $z_\tau = -\hat{c}_\tau < 0$. Since $z_m < 0$ an increase in τ (around $\tau = 0$) requires in order to keep $z = 0$ a decrease in m. By (32) and (26), $s_\tau = -\pi_\tau < 0$. Accordingly, since $s_m > 0$, an increase in τ requires an increase in m.

In Figure 1, the equilibrium steady-state with $\tau > 0$, is denoted by k** and m**. We may conclude that

Proposition 5. In steady-state, an income tax reduces the capital labor ratio but may increase or decrease the amount of real balances.

It is easy to see that an increase in the inflation-exclusion rate has the opposite effect of an increase in the tax rate.

5. The Optimal Monetary Policy

The government has several instruments - monetary (θ) and fiscal $(\tau$ and $\varepsilon)$ - which it can use to achieve certain goals. Let us assume first that there exists a fixed positive tax $\tau > 0$ with a full inflation-exclusion $\varepsilon = 1$ (for a <u>given</u> tax revenue, γk, we have seen that a substitution of τ for ε, does not affect the equilibrium values), and that the government wants to choose θ so as to maximize the welfare of the individuals.

We may derive from (8'), (20') and (30) the modified <u>steady-state</u> relation (33) in the presence of a tax

$$(33') \qquad m = \frac{\beta(f-k)}{(1-\tau)(f'-\mu)+\theta-n}$$

which reduces to (33) when $\tau = 0$ and $k = k^*$. The graph of (33') for a given k ($<k^*$ as was shown above) is given in Figure 2. The reason for the negative relation is that in steady states an increase in θ increases β, which reduces the rate of return and consequently the demand for money. It can also be seen that the economy may "produce" any amount of m provided θ is sufficiently close to $-(1-\tau)(f'-\mu) + n$. The question is whether in these circumstances it is at all possible to have an <u>optimal</u> stationary state with a <u>finite</u> m.

Given the dynamic equations (30) and (31), the government wants to determine the time path of θ_t so as to maximize the value of the utility integral (1) given the initial values of k_0 and m_0.

The economic meaning of the optimality conditions for the government's program can be best explained by using the <u>Maximum-Principle</u> formulation. The relevant imputed value of the "national-income" at each t is given by

$$H = u(\hat{c}(k,m,\theta,\tau,\varepsilon),m) + q_z z(k,m,\theta,\tau,\varepsilon) + q_s s(k,m,\theta,\tau,\varepsilon) \tag{37}$$

where q_z and q_s are the undiscounted shadow prices of $\dot{k}$ and $\dot{m}$, which should be considered as continuous functions of time. The first order conditions for a maximum yield

$$u_c\hat{c}_k + \dot{q}_z + q_z(z_k-\delta) + q_s s_k = 0 \tag{38}$$

$$u_c\hat{c}_m + u_m + q_z z_m + \dot{q}_s + q_s(s_m-\delta) = 0 \tag{39}$$

$$u_c\hat{c}_\theta + q_z z_\theta + q_s s_\theta = 0 \tag{40}$$

$$\lim_{t\to\infty} e^{-st} q_{z_t} k_t = 0; \quad \lim_{t\to\infty} e^{-st} q_{s_t} m_t = 0 \tag{41}$$

where subscripts denote partial derivatives. The first two equations are concerned with the effects of k and m on H along the optimal path. Along this path the gains and losses associated with a change in k or m must exactly offset each other as stated in these equations. As for (40), it is the condition that it should be maximized with respect to the control variable θ at each moment of time. Indeed in our model we have $H_{\theta\theta} = c_\theta^2 u_{cc} < 0$ for all t. Suppose that s_θ and c_θ are positive (actually, c_θ is positive, and s_θ is positive around k^*, when $\tau = 0$). Then a positive shadow price for money implies $q_z > u_c$. This means that the same physical unit of our good has a higher value in investment than in consumption. This paradox is resolved if we remember that the shift from c to $\dot{k}$ cannot be accomplished directly but only through the instrument θ. Now, if we reduce c through θ it must involve additional loss via the reduction of $\dot{m}$.

Equations (38)-(40), together with the accumulation equations (30) and (31), the "end-conditions" (41) and the initial values k_0 and m_0 determine the optimal time path for the economy. Because of the complexity of the system we cannot analyze its time path by conventional graphical (or other) methods. The most one can hope to do is to analyze the steady-state solution of the system.

We are now interested in examining whether there exists a steady-state, with $\dot{k} = \dot{m} = \dot{q}_z = \dot{q}_s = 0$, which satisfies the first order conditions for an optimal path. From (30) we have the relationships $z_\theta = -\hat{c}_\theta$, $z_m = -\hat{c}_m$, $z_k = f - \lambda - \hat{c}_k$. From (31) we have $s_\theta = m - \pi_\theta m$ $s_m = -\pi_m m$, $s_k = -\pi_k m$. Conditions (38)-(40) thus become

(42) $$(u_c - q_z)\hat{c}_k + q_z(f' - \lambda - \delta) - q_s \pi_k m = 0$$

(43) $$(u_c - q_z)\hat{c}_m + u_m - q_s \pi_m m = 0$$

(44) $$(u_c - q_z)\hat{c} + q_s m - q_s \pi_\theta m = 0$$

Assuming that $\varepsilon = 1$ we have $\pi_k m = \beta\hat{c}_k - (1-\tau)f''m$, $\pi_m m = \beta\hat{c}_m - (1-\tau)(f'-\mu)-$ and $\pi_\theta m = \beta\hat{c}_\theta$. Equations (42)-(44) can then be further simplified

(42') $$(u_c - q_z - \beta q_s)\hat{c}_k + q_z(f' - \lambda - \delta) + q_s(1-\tau)f''m = 0$$

(43') $$(u_c - q_z - \beta q_s)\hat{c}_m + u_m + q_s((1-\tau)f' - \mu) + \theta - n) = 0$$

(44') $$(u_c - q_z - \beta q_s)\hat{c} + q_s m = 0$$

We have seen, (25), that $\hat{c}_\theta > 0$. Hence by (44') $u_c - q_z - \beta q_s < 0$. Substituting in (43') the expressions for $\hat{c}_m$, $u_c = \frac{1}{c} = \frac{1}{f-\lambda k}$ and $u_m = \frac{\beta}{m}$, it can be shown that for small τ and a given k, (43') is a strictly monotone, increasing relation between θ and m.[7]

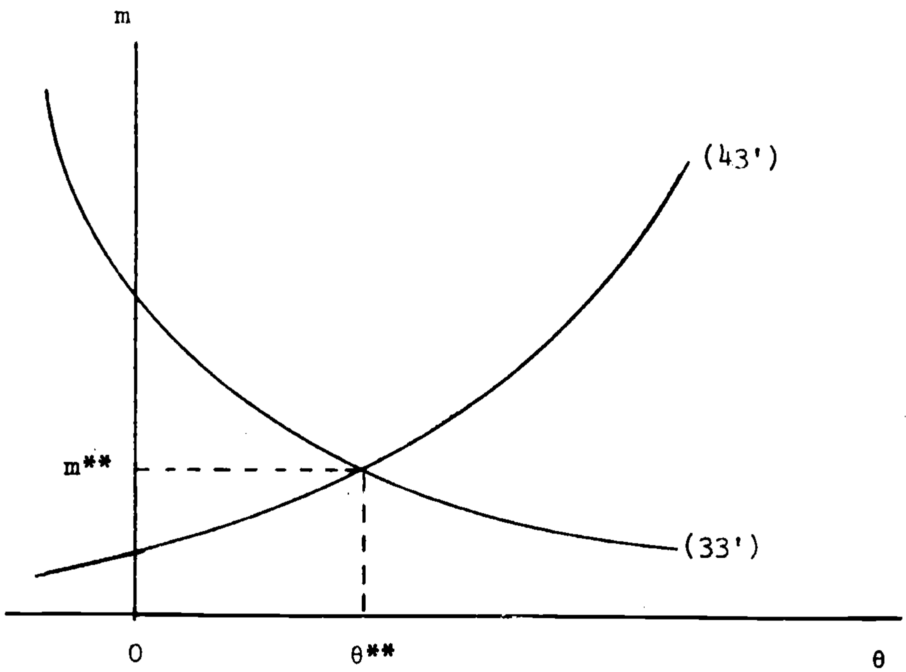

Figure 2

Thus, for given q_z, q_s and k, equations (33') and (43') determine a unique pair (m^{**}, θ^{**}). It remains to show that the system as a whole is consistent, i.e. that all variables can be determined sinultaneously. This can be demonstrated by numerical calculations which we have performed with particular examples.[8]

The important thing to note about the solution displayed in Figure 2, is that it is obtained for a finite m, which implies positive u_m and q_s, in spite of the fact that it is feasible to create any value of m. Thus, the optimum stationary state is short of full liquidity.

It should be noted that when $\tau = 0$, the present model does give full liquidity $(m = \infty$, with $\theta = -\delta)$ as the unique optimum stationary solution. It seems therefore, that for the given expectations' structure, the imperfect control model in the absence of taxes, still tends to the 'bliss' solution. It is only with the additional "imperfection" created by the presence of taxes, that a 'non-bliss' solution becomes optimal. It would be interesting to examine whether other expectation structures could provide for such an optimum even in the absence of taxes. In any case. we have demonstrated that in an economy with "imperfections" of various kinds - due to individuals' erroneous expectations, taxes etc. - the optimum policy may drive the economy to a stationary state short of full liquidity.

Footnotes

1. Note that static expectations imply generally, unfulfilled expectations. However, in steady states $(p_t = p)$ the consumer will have perfect foresight.

2. The more ambitious target of choosing the function θ_t so as to maximize the utility integral, has been examined by Liviatan and Sheshinski (1971).

3. Under the more general assumption of adaptive expectations, it can be shown that for a sufficiently slow speed of adjustment, the system is stable (instantaneous adjustment means, of course, an infinite speed of adjustment).

4. These calculations can be deduced directly from the budget equation, which (in real terms) is now given by

$$c_u + \dot{k}_u + (n+p_u)k_u + \dot{m}_u + (n+p_u)m_u = (i_u - \gamma_u)k_u + w_u + v_u$$

or

$$\dot{y}_u = \bar{\rho}_u y_u - (c_u + \bar{i}_u m_u) .$$

The net return on assets being $\bar{\rho}_u$, while the implied consumption value (liquidity) of a unit of real balances is $\bar{i}_u$.

5. The necessary and sufficient condition for existence of a solution to (21') is that $\frac{(\gamma+\delta)\beta}{\bar{i} + i\beta} < 1$. This condition guarantees that the denominator in (23) is positive.

6. Note that the steady-state level of capital per-capita, k^*, is <u>independent of θ</u>. Thus, monetary policy does not affect, in the long

run, the real part of the system. This dichotomy disappears once taxes are introduced, as seen below.

7. From (20'), $\hat{c}_m = \frac{(\gamma+\delta)\bar{i}}{\bar{i} + i\beta} (1 + \frac{\theta}{f'-\lambda}) > 0$, which is a function of θ only. Now,

$$\hat{c}_{m\theta} = \frac{(\gamma+\delta)\bar{i}}{(\bar{i}+i\beta)(f'-\lambda)} +$$

$$+ \quad (1 + \frac{\theta}{f'-\lambda}) \frac{(\tau(f'-\mu)+\delta)}{((f'-\lambda+\theta)(1+\beta)-\tau(f'-\mu))} (1 - \frac{(f'-\lambda+\theta - \tau(f'-\mu))(1+\beta)}{((f'-\lambda+\theta)-\tau(f'-\mu))(1+\beta)+\tau\beta(f'-\mu)}) > 0$$

For $\gamma \sim 0$, $\hat{c}_{m\theta} \sim \frac{1}{1+\beta}$, and $\hat{c}_{m\theta}$ approaches $\frac{1}{1+\beta}$ from above. Since, by $\hat{c}_\theta \sim \frac{m}{1+}$ and, by (44') $u_c - q_z - \beta q_s \sim -(1+\beta)q_s$, we can see from (43') that the relation between θ and m is positive.

8. Available upon request from the author.

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