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ABSTRACT

We present evidence that many households have only a vague notion of what they are spending on various consumption items. We then develop a life-cycle model that captures this "absent-mindedness". The model generates precautionary spending, whereby absent-minded agents tend to consume more than attentive ones. The model also predicts fluctuations over time in the level of attention, and thereby sheds new light on the sharp reduction in consumption both at retirement, and in cyclical downturns. Finally, we find patterns of attention in the data that are consistent with those predicted by the model.

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1 Introduction

Many of us are profoundly ignorant about how much we spend, and on what we spend it. We develop a life cycle model of an “absent-minded” consumer that captures this fact, as well as the uncertainty reducing role of activities such as budgeting. The model generates a form of “precautionary spending” in which absent-minded agents tend to consume more than attentive ones. The model generates realistic variations in the level of attention over time, thereby shedding new light on the sharp reductions in consumption both at retirement, and in cyclical downturns. Finally, the model suggests intriguing analogies between control problems of the sort encountered by our absent-minded consumer and the more familiar problems of self control analyzed by Strotz [1956], Laibson [1997], and Gul and Pesendorfer [2001].

The first order of business is to empirically confirm rampant consumer ignorance. Evidence on this subject, and on the uncertainty reducing role of budgetary attention, is presented in section 2. Given that existing data are inadequate, our key findings derive from a survey that we designed for a sample of TIAA-CREF clients. We asked respondents not only to estimate their total outlays in the last year on various different categories of spending, but also to provide confidence intervals around these estimates. As detailed in section 2, the degree of uncertainty is remarkably high. The data reveal also that those who monitor their spending more closely are less uncertain about past spending.

In section 3 we develop a life cycle model of income and consumption consistent with the facts of section 2. Following Piccione and Rubinstein [1997], we treat consumers who do not monitor as absent-minded, uncertain as to their precise location in the consumption-savings decision tree.¹ In sections 4 and 5, we fully characterize a simple three-period version of the model and track the incentive to monitor over the life cycle. We show that those in their relatively wealthy middle years have the lowest incentive to monitor, and are most likely to be absent-minded. In contrast, the incentive to monitor is particularly high for those who are retired, since they have time on their hands, declining wealth, and less flexibility to adjust to unforeseen spending by working harder in the future. The data confirm that this pattern is indeed

¹Absent-mindedness is a form of imperfect memory. There are obvious connections to other models of imperfect memory, including Dow [1991], Mullianathan [1998], Bernheim and Thomadsen [2002], and Wilson [2002]. Sims [2002] considers the implications of information processing constraints, which may also be related.

prevalent, with a sharp increase in monitoring behaviors at retirement.

Fluctuations over time in the level of monitoring are of most interest if they correlate with changes in the level of spending. Our model suggests just such a link, with inattention inducing a form of “precautionary spending”.² Agents may be willing to consume more on average in order to ensure that their absent-mindedness does not produce damaging under-consumption.³ In combination with our finding of increased monitoring in retirement, this suggests a novel explanation for the sharp decline in consumption at retirement identified by Mariger [1987], Banks, Blundel and Tanner [1998], and Bernheim, Skinner and Weinberg [2001]. Retirement is associated with diminished absent-mindedness, and reduced spending results.

In section 6 we present three applications of our model to topics other than life-cycle consumption. First, we consider cyclical down-turns. If reductions in income lead to increases in monitoring, then the accompanying “belt-tightening” will exacerbate the fall in consumption. Second, we consider analogies between the control problems of the type we model, and self control problems. Both problems create incentives to keep assets in illiquid form, and to restrict the use of credit cards. However the motivations for these behaviors are entirely different. In the case of self control problems, holding assets in illiquid form and restricting credit card constitute valuable commitment devices. In the case of control problems, consumers value illiquidity as an error-reduction device, while restrictions on credit card use increase reliance on cash, which in turn increases the flow of information concerning their volume of spending. Finally, we relate our new findings to prior work concerning the impact of financial planning on wealth accumulation (Lusardi [1999] and Ameriks, Caplin and Leahy [2003a]).

In technical terms, the key ingredient in our model is imperfect *control* and uncertain consumption flows, rather than imperfect *memory* per se. In the final part of section 6 we note that such imperfections in control and uncertain consumption flows may arise for reasons other than forgetfulness. These effects may arise in search markets, in which consumers are only

²In precautionary savings models, the convexity of marginal utility implies that future uncertainty raises the expected marginal utility of future consumption and hence the desire to save. In our formulation of absent-mindedness a similar phenomenon occurs, only it is current rather than future utility that is uncertain.

³In our model, a coefficient of relative risk aversion greater than unity is sufficient for the consumption of the absent-minded consumer to exceed on average the consumption of an attentive consumer.

imperfectly aware of product characteristics (e.g. when purchasing a car). Multi-person households may be especially prone to problems of this sort, to the extent that communication and the alignment of interests are imperfect. Developing a general model to capture the many such imperfections of control is an important open task.

2 Uncertain Consumption: Evidence

2.1 Measuring Uncertainty

It is folk wisdom among financial planners that most of us do not keep track of our purchases. Yet there is little prior research that confirms or quantifies the resulting ignorance. What little evidence exists concerns biases in memory due to forgetfulness. Politz [1958] surreptitiously recorded the supermarket expenditures of a sample of shoppers and interviewed them that evening to see if they could recall their expenditures. He found that on average consumers recalled 91% of the items purchased and 94% of their total expenditures. In a study of home improvement expenditures, Neter and Waksberg [1964], found that between a quarter and a half of all jobs are forgotten within three months. More recently a number of studies have compared diary and recall surveys of consumption data.⁴ The general conclusions of this literature are that diaries are significantly more accurate and that recall has a bias that varies with type of product. Regular purchases, such as “how much do you spend on food at home?”, are more easily recalled. Occasional purchases, such as “how much did you spend on clothing in the last 3 months?”, are not.

While the above findings show that individuals are forgetful, they are silent concerning subjective *uncertainty*. People may be wrong, yet convinced that they are correct. Hence the first requirement of our research is to fill in the gap in our knowledge concerning perceived uncertainty. Fortunately we were able to include questions on this subject in a recent survey of TIAA-CREF participants.⁵ The survey was mailed out in March 2003

⁴See Battistin, Miniaci, and Weber [2000], Battistin [2003], and Browning, Crossley and Weber [2003].

⁵See Ameriks, Caplin, and Leahy [2003a,b] for details concerning survey participants. The main differences between this sample and the population at large is that this sample tends to be wealthier and more highly educated.

to approximately 2500 participant households. We received 1632 responses. We asked participants to provide several estimates of how much they spent during calendar year 2002 on several categories of consumption goods, as follows:

- How much did your household spend in the calendar year 2002 on each of the following categories? Please provide three estimates:
 - A LOW estimate, where you are 90% sure that the correct answer is above this number
 - your CLOSEST estimate of the most accurate answer
 - a HIGH estimate, where you are 90% sure that the correct answer is below this number

The specific categories chosen were: food and drink at home; clothing; telephone; and overall total amount of money spent using credit cards. Food was chosen as the prototypical frequent purchase, clothing as the prototypical infrequent purchase. Telephone spending was included because it is in principle easily measured by monthly bills. Total credit card spending was included to see how an increase in scale impacted the degree of uncertainty.

Table 1 produces basic information on subjective uncertainty concerning past spending for each of the our four categories. Our measure of individual uncertainty is the 90-10 confidence interval as a percent of total spending. The first column of table 1 displays the average value of this uncertainty measure for the 943 households who provided reasonable information on all categories.⁶ The second column records the standard deviation of this measure, while the third displays the share of spending on the category relative to labor income for the 470 households in which one member was working full time.

The answers reveal great uncertainty in all categories. For food and drink at home, the average confidence interval was 45% of total spending on that category. For clothing, it was about 75%. The greater uncertainty for clothing is consistent with the results of the recall-diary literature: clothing is a more infrequent purchase. It is notable also that uncertainty is equally high

⁶We deleted three observations. In two cases subjective uncertainty concerning food and drink at home was greater than 9 and in one case telephone uncertainty was equal to 13. These responses were 25 to 35 standard deviations above the next highest responses.

for telephone and credit card spending, despite the availability of monthly telephone bills, and despite the fact that credit card spending comprises about 30% of labor income. The magnitude of uncertainty is all the more striking in light of the standard finding in the psychology literature that people tend to report confidence intervals that are far too narrow.⁷ We interpret these results as strong motivation for a model in which agents are not fully aware of their level of spending.

We can compare our reported expenditures as a share of income to data from the Consumer Expenditure Survey. Between 1998 and 2001, spending on food at home has comprised roughly 8% of total expenditures, whereas spending on “apparel and services” comprised roughly 5% (BLS [2003]). Our numbers are comparable.⁸

The standard deviation numbers in Table 1 indicate that the level of uncertainty varies greatly over the sample. In the next subsection we look to see if any of this variation is endogenous.

2.2 Impacting Uncertainty

It seems intuitively clear that consumers can adopt measures, such as monitoring their spending, that will reduce their uncertainty concerning how much they spend and on what they spend it. Data from our survey confirm the endogeneity of the level of uncertainty. We asked households to report on a 1 to 6 scale (6=agree strongly, 5 agree, 4 agree somewhat, 3=disagree somewhat, 2=disagree, 1=disagree strongly) the extent to which they agreed with two statements characterizing the intensity with which they monitor their spending:

- Please indicate the extent to which you agree that each of the following statements describes your household (or just you, if you are a single person household). In my household we:
 - Regularly set advance budgets for our spending.

⁷In a typical experiment, individuals are asked to answer 10 questions in which they are most unlikely to have a point estimate of the correct answer (e.g. the length in miles of the Amazon river). They are then asked to provide a 90% confidence interval for the correct answer. For the typical respondent, the truth will lie inside the stated confidence interval only in 3-4 of the 10 cases.

⁸Differences may be due to differences in sample, systematic errors in memory, or in differences between labor income and total expenditure.

- Regularly keep track of our actual spending.

Our hypothesis was that an increase in either of these two measures would diminish subjective uncertainty. Table 2 presents results from eight regressions, in which we ran uncertainty measures for each of our four categories separately on the answers to the budgeting question and the tracking question, as well as the age of the respondent. The sample comprises the 929 households who provided complete data on all of the uncertainty and attention measures. Standard errors are in parentheses.

In line with our hypothesis, all of the coefficients in table 2 are negative, and many are significant at the 5% level, suggesting that attention reduces uncertainty. The effects of advance budgeting on food and clothing uncertainty are significant at the 1% level. These effects are also large. To appreciate the scale of the uncertainty reduction, note first that each of the coefficients in table 2 should be read in conjunction with the coefficients in table 1. For example, table 1 shows that the average measure of uncertainty is 0.45. The -0.22 coefficient in the first row and column of table 2 therefore implies that a unit increase in budgeting reduces the level of uncertainty by more than 4.5% of its initial level. Given also that a one standard deviation increase in budgeting corresponds to an increase in the budgeting measure of 1.4, we conclude that such an increase is associated on average with a 7% decline in uncertainty concerning food consumption. The proportionate reduction in uncertainty concerning clothing consumption induced by a one standard deviation increase in budgeting is likewise 7%.

While the endogeneity of subjective uncertainty is clear from Table 2, there are several reasons to believe that these coefficients understate the magnitude of the effects. One reason for this is that measurement error is likely to bias our estimates towards zero. Fortunately, we have available a simple method to reduce the extent of the measurement error, since we asked a variant of the budgeting question on a previous survey. We therefore have two responses to the budgeting question for 830 of the participants in our sample, which allows us to control for measurement error by using the prior response to instrument for the current response. When we do this the estimated coefficients on food and clothing uncertainty rise to .031 and .047 respectively. A second reason that the impact of monitoring on uncertainty may be larger than reported is that while we ran linear regressions, the data appear to favor the use of concave transformations of the uncertainty measure. For example, when we regress the square root the our measure of

uncertainty on the level of age and advance budgeting, all of the coefficients are significant at the 5% level, and the effects of tracking one’s expenditure on food, clothes and credit card uncertainty are all significant at the 1% level.⁹ The final reason why the impact of monitoring on uncertainty may be larger than indicated in table 2 is reverse causation. Since greater uncertainty creates an incentive for increased attention, these feedback effects will tend to reduce measured coefficients in Table 2.

3 The General Model

In this section we first lay out the general model, and then prove that it is coherent, in the sense that optimal policies always exist.

3.1 Model Structure

The critical modeling issue is how to capture consumers’ uncertainty concerning their spending behavior. Our strategy is to follow Piccione and Rubinstein [1997] and link these problems to agents’ confusion concerning their location in a decision tree. Piccione and Rubinstein analyze the optimal strategy of an “absent-minded” driver who must turn right after a certain number of blocks, and who knows that she will at some point lose track of her precise location. They make the strong simplifying assumption that the driver is completely without memory, seeing all corners as identical. We adopt both their concept of absent-mindedness, and also the simplification of complete forgetfulness. In each discrete period of life, $t \in \{1, \dots, T\}$, our consumers receive a sequence of consumption opportunities. If they pay monitoring costs of m (for simplicity, these costs are measured directly in terms of utility), they will have complete recall of the number of such opportunities previously taken in this period. If they fail to invest in monitoring, they will completely lose track of any previously taken opportunities.

In modeling imperfect memory, one needs to remove indirect tracking devices (e.g. a driver with a GPS always knows her position). In the case

⁹We also estimated an ordered probit relating centiles of the uncertainty measure to dummy variables for the different levels of budgeting and tracking. In almost all cases the coefficients on the attention dummies were diminishing in the level of attention. The zero restrictions on the attention dummies were rejected at greater than .1% significance in most cases.

of consumption spending, the trick is to separate the flow of purchases in any given period from the flow of utility-giving consumption. Absent this separation, agents may refer to an internal “hedonomitor” to instantly gauge marginal utility and infer from it their prior level of spending. The modeling device that accomplishes this is to introduce two distinct states, H and S . Here H refers to home, and $S = \{1, 2, \dots, \infty\}$ refers to a street with a countably infinite set of shops, indexed by i , that contain goods for the consumer to purchase. Consumers begin each period at home. In order to make purchases for the period, they must move to the shopping street and enter stores in increasing order from $i = 1$. In each store entered, the consumer orders x units of the single consumption good for later delivery at home. In order to carry out the act of consumption, the consumer has to make the irreversible decision to return home and await delivery of that period’s purchases. The utility from any given stream of final consumption is defined by a standard additively separable utility function, with period utility $u(c)$ and discount factor β . We assume that $u(c)$ is real valued except possibly at $c = 0$. In addition $u(c)$ is increasing and weakly concave, with $\lim_{c \rightarrow \infty} u'(c) = 0$.

We now discuss the precise structure of the overall shopping strategy. At the start of each period, the first decision that consumers must make is whether or not to pay that period’s monitoring costs of $m > 0$. Those who pay this cost are standard consumers, continuously aware of their prior spending in the period. For this reason, their shopping strategies condition on the precise store in which they are located. Formally, a strategy for a consumer who monitors is a function $\eta_t : S \rightarrow [0, 1]$ specifying for each $i \in S$ the probability of returning home rather than continuing to the next store conditional on being in this store. In contrast, those who choose not to monitor are assumed to be completely unable to distinguish among stores. This implies that all of the locations in S are elements of the same information set. These consumers must follow the same (possibly random) strategy at each location, $\psi_t : \{S\} \rightarrow [0, 1]$.

As Piccione and Rubinstein stress, there is one delicate issue concerning the nature of the strategic options available to absent-minded agents. This issue is whether or not they can commit to a shopping strategy while at home, or rather have the flexibility to adjust the strategy on the street. In our model we assume that commitment is possible, searching for what Piccione and Rubinstein call the *ex ante* optimal strategy. This solution concept is supported by the experimental results of Huck and Müller [2001], and is equivalent to what Piccione and Rubinstein call the modified multi-self

equilibrium, which involves identifying each visit to an information set with a different self and analyzing the resulting intrapersonal game.¹⁰ In addition, using this notion of equilibrium greatly simplifies the analysis, since it allows us to use dynamic programming to solve the model.

Once the period's monitoring decision is made and the shopping strategy set, consumers go onto the shopping street and buy commodities according to the prespecified strategy. At the end of the shopping day, the consumer returns home and awaits deliveries. We assume that a consumer who visited precisely N stores before returning home is committed to consuming the entire amount in the given period, $c_t = Nx$. Given that there is no bound to the number of stores that may be visited, the issue arises as to whether or not the consumer can afford all that has been purchased. To resolve this issue, we assume that each individual has an unbounded endowment of labor that can be used to generate income at the end of each period.¹¹ Moreover, we assume that consumers cannot borrow against future human capital. If consumption in any period exceeds financial resources, the consumer must work at the end of the period to pay off the excess consumption. Of course, consumers may also choose to put in additional work to accumulate resources for future consumption. We assume that the disutility of labor in each period is described by a function $v(l)$ with $v(0) = 0$, $v'(l) > 0$ and $v''(l) \geq 0$. To ensure positive consumption we assume that $v'(0) < u'(0)$.

To close the model we derive the budget constraints. The price of the consumption good is assumed to be constant over time, and is normalized to 1. We let $w_t > 0$ be the real wage per unit of labor for an individual in the t^{th} period of life. We allow the productivity of labor to be period specific to allow for a reasonable life-cycle pattern of earning power, and in particular for the possible fall in productivity in later life. We assume that the agent begins with initial wealth $W_1 \geq 0$. The real interest rate between periods is assumed to be constant at r . Letting W_t denote wealth at the beginning of period t , wealth evolves according to the following equation:

$$W_{t+1} = (1 + r)(W_t + w_t l_t - c_t). \quad (1)$$

¹⁰The alternative solution concept discussed by Piccione and Rubinstein involves looking for a fixed point: a strategy from which the shopper would not choose to deviate while on the street.

¹¹More intricate alternatives are to assume that purchases can be returned with some penalty cost, or that some agency such as a credit card company can cut the consumer off once consumption has exceeded resources. One might also allow the consumer to declare bankruptcy if consumption turns out to be too high.

Our assumptions on borrowing and on the supply of labor imply that $W_t \geq 0$ in all periods.

Before specifying value functions and proving that optimal strategies exist, we make one last simplifying assumption. We consider the limit of the model as x , the quantity available in each store, approaches zero. This smooths out the discreteness in the model and transform sums over S into integrals.¹² This means that the strategies available to the absent-minded consumer comprise all exponential distributions of final consumption, defined by the specific hazard rate chosen. A fixed probability of stopping at every point in S translates in the limit as x approaches zero into an exponential density of stopping times, pe^{-pc} , where p is the fixed rate at which the consumer chooses to quit shopping and return home.

3.2 Existence of an Optimum

In order to prove that our maximization problem has a well-defined solution, we first write the model recursively. Let $V_t(W_t)$ denote the value of an optimal policy for an agent with wealth $W_t \geq 0$ at the start of period t . Since the first choice that the agent makes is whether or not to monitor, we can express V_t as the maximum of two value functions:

$$V_t(W_t) = \max \{V_t^M(W_t), V_t^A(W_t)\}$$

where the superscripts M and A refer respectively to monitoring and to being absent-minded.

Given that the marginal utility of consumption is positive, the only question in the final period for a consumer who monitors concerns the optimal amount of work to put in:

$$V_T^M(W_T) = \max_{l \geq 0} u(W + w_T l) - v(l) - m.$$

The only decision that must be made in the final period by an absent-minded consumer concerns the stopping rate that maximizes expected utility, where the expectation is taken with respect to the exponential distribution:

$$V_T^A(W_T) = \max_p p \int_0^\infty e^{-pc} u(c) dc - p \int_{W_T}^\infty e^{-pc} \frac{1}{w_T} v(c - W_T) dc$$

¹²Setting up the model with discrete S simplifies the definition of a strategy as a mapping into probabilities in $[0, 1]$.

The first term is the expected utility from consumption. The second term is the expected cost. The cost involves only the labor needed to finance consumption in excess of W_T . Any unused wealth in the final period is lost.

In periods $t < T$ the value function conditional on monitoring satisfies the equation:

$$V_t^M(W_t) = \max_{c,l} [u(c) - v(l) + \beta V_{t+1}(W_{t+1})] - m,$$

with W_{t+1} as defined in the budget constraint (1). Note that since there is no uncertainty when the agent monitors, the timing of the consumption and the labor choice decisions is irrelevant. The value function for the absent-minded consumer satisfies,

$$V_t^A(W_t) = \max_p p \int_0^\infty e^{-pc} \left[\max_{l > \min\{0, c - W_t\}} u(c) - v(l) - \beta V_{t+1}(W_{t+1}) \right] dc.$$

Again W_{t+1} is as defined in the budget constraint.(1). Note that the constraint on period t labor effort ensures that $W_{t+1} \geq 0$.

With the value functions in place, we are in a position to establish Proposition 1, which concerns existence of an optimal policy. The proof is in Appendix 1.

Proposition 1 There exists an optimal policy.

4 A Special Case

In order to understand fluctuations in the monitoring incentive over the course of the life cycle, we set up and solve a simple stylized three period model. The assumptions are presented in section 4.1, and the solution characterized in section 4.2. The economic implications are discussed in the next section.

4.1 The Model

Our life cycle assumptions allow for possible liquidity constraints in early life, and a realistic pattern of labor productivity over the life cycle. In addition, we make simplifying assumptions concerning the rate of return on savings in relation to the discount rate.

Assumption A1: $W_1 = 0$.

Assumption A2: Productivity is highest in the middle period of life: $w_2 > w_1, w_3$.

Assumption A3: The rate of return on savings is insufficient to compensate for discounting and the productivity increase between periods 1 and 2, yet strong enough to so compensate in the final period:

$$\frac{w_3}{w_2} < \beta(1+r) < \frac{w_2}{w_1}.$$

The first two assumptions are self-explanatory. With respect to the final assumption, given that the wage is seen as falling significantly in the last period of life, the left hand inequality is naturally met. The right hand inequality is more substantive. It sets a lower bound for the rate of growth in real wages between the early years and the middle years of life to ensure that the early years of life are not spent saving up for the typically more prosperous middle years. This condition is likely to be met in any reasonable parametrization, at least provided the real interest rate is not too high.

We make two final assumptions on $u(c)$ and $v(l)$ to simplify algebraic derivations. We assume that utility is of the constant relative risk aversion variety, with an additional restriction to ensure the existence of all relevant integrals.¹³ We assume also that the disutility of labor is constant. This allows us to focus our analysis on how absent-mindedness impacts the marginal utility of current consumption, rather than having to deal simultaneously with the impact on future consumption. Generalizations of these assumptions are discussed in section 5.

Assumption A4: Utility is CRRA with $\sigma \in (0, 2)$:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}.$$

Assumption A5: The marginal disutility of labor is constant:

$$v(l) = l.$$

¹³It is a trivial matter to relax this assumption. However, in order to do this we have to add uninteresting new parameters to the model that serve only to obscure the main points.

With these assumptions in place, we are ready to characterize the optimal pattern of monitoring, consumption, and wealth accumulation over the life cycle.

4.2 The Solution

Proposition 2 presents the complete formal solution to the model. Note that our solution for the optimal shopping rate when absent-minded is stated in terms of the corresponding level of expected consumption. Note also that we present a strict inequality to characterize the region in which there is monitoring. When the opposite inequality holds, the unique solution is to be absent-minded, while if there is equality the consumer is indifferent between the two strategies.

Proposition 2: With Assumptions A1-A5, the solution to the model is as follows:

1. *The decision to monitor.* The consumer will monitor in periods 1 and 2 if,

$$\left(\frac{\sigma}{1-\sigma} - \frac{A}{1-\sigma} + 1 \right) > mw_t^{\frac{\sigma-1}{\sigma}},$$

where $A = \int_0^\infty z^{1-\sigma} e^{-z} dz$. The consumer will monitor in period 3 if

$$\left[\frac{\sigma}{1-\sigma} - (\ln \beta(1+r)w_2 - \ln w_3)^{\frac{\sigma-1}{\sigma}} \left(\frac{A}{1-\sigma} - 1 \right) \right] > m [\beta(1+r)w_2]^{\frac{\sigma-1}{\sigma}}.$$

2. *Consumption when monitoring.* The optimal level of consumption in period t of a consumer who monitors is,

$$c_t^M = \alpha_t^\sigma,$$

where $\alpha_1 = w_1$, $\alpha_2 = w_2$, and $\alpha_3 = w_2\beta(1+r)$.

3. *Consumption when absent-minded.* In periods 1 and 2, the absent-minded consumer chooses an expected level of consumption equal to,

$$c_t^A = \alpha_t^\sigma / A.$$

In period 3, the expected level of consumption is

$$c_3^A = \frac{\alpha_3^\sigma}{A \ln \left(\frac{\beta(1+r)w_2}{w_3} \right)}.$$

4. *Labor supply.* In period 1, labor supply is equal to realized consumption divided by the wage rate: $l_1 = c_1/w_1$. In period 2, $l_2 = (c_2 + s_2)/w_2$ where s_2 denotes saving in period 2 for period 3 consumption. In period 3, $l_3 = \max\{0, [c_3 - (1+r)s_2]/w_3\}$.
5. *Saving.* Saving is only positive in period 2, and depends on the monitoring decision in period 3. Should the consumer monitor in period 3, $s_2^M = c_3^M/(1+r)$. Should the consumer be absent-minded, $s_2^A = c_3^A/(1+r)$.

The proof of the proposition is straightforward. With assumptions A1-A3, it is optimal for the individual to work in period 1 just enough to pay for consumption in that period. Given zero initial wealth and the liquidity constraints, the consumer must pay for period 1 consumption with period 1 labor. Given the wage profile, it is strictly optimal to pay for period 2 consumption with period 2 labor. This explains the absence of saving in period 1. The period 1 decisions therefore have no impact on later decisions.

Assumptions A2 and A3 and the linear marginal disutility of labor imply that it is optimal to finance some of period 3 consumption with period 2 labor, and that the amount saved in period 2 is independent of period 2 consumption. This allows us to separate period 2 monitoring and consumption decisions from the period 3 decisions. In this way, we break the three period model into a series of three static optimization problems.

To translate these comments into a full characterization of the optimal policy, we begin by analyzing the consumption of a consumer who monitors. Given the functional form assumptions A4-A5, the consumer who monitors in the first and second periods chooses consumption to maximize:

$$\frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{c}{w_t}.$$

Given zero initial wealth in both periods, the cost of a unit of consumption is $1/w_t$ units of labor. The optimal consumption levels c_1^M and c_2^M follow immediately.

A consumer who monitors in the third period can anticipate this choice in period 2. Given A2-A5, it is optimal to accumulate resources in period 2 sufficient to finance all of period 3 consumption. It takes $c/[w_2(1+r)]$ units of labor in period 2 to purchase c units of period 3 consumption. The consumer therefore maximizes:

$$\frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{c}{w_2\beta(1+r)}.$$

The choices c_3^M and s_2^M follow immediately.

The optimal policy of the absent-minded consumer is a bit more complicated, but with the functional form assumptions A3-A4, this problem takes on a simple form. In periods 1 and 2, the consumer chooses the stopping rate to maximize:

$$\max_p p \int_0^\infty \left[\frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{c}{w_t} \right] e^{-pc} dc.$$

We can simplify this integral in two ways. First, note that associated with a policy p is an expected level of consumption \bar{c} :

$$\bar{c} \equiv p \int_0^\infty c e^{-pc} dc = \frac{1}{p}.$$

Second, we apply a change of variable: $z = pc$. With these two amendments, the maximization problem becomes

$$\max_{\bar{c}} \frac{A\bar{c}^{1-\sigma} - 1}{1 - \sigma} - \alpha_t \bar{c}.$$

The only change from the standard model is the presence of $A = \int_0^\infty z^{1-\sigma} e^{-z} dz$.¹⁴ The optimal choices c_1^A and c_2^A follow immediately.

If the individual does not monitor in period 3, the consumer chooses savings in period 2 and the stopping rate in period 3 to maximize:

$$\max_{s_2} \left\{ -\frac{s_2}{w_2} + \beta \max_p p \int_0^\infty \left[\frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{1}{w_3} \max\{0, c - (1+r)s_2\} \right] e^{-pc} dc \right\}.$$

¹⁴Note that this integral is well-defined only for $\sigma \in (0, 2)$, which explains the need for assumption A2.

It takes s_2/w_2 units of period 2 labor to accumulate s_2 units of saving. s_2 units of saving finances up to $(1+r)s_2$ units of period 3 consumption. Consumption in excess of this amount must be financed by labor in period 3, which earns a wage w_3 . After a little algebra the problem reduces to:

$$\max_{s_2, \bar{c}} -l + \beta \frac{A\bar{c}^{1-\sigma} - 1}{1 - \sigma} - \beta\alpha_3\bar{c}e^{-(1+r)s_2/\bar{c}}$$

The optimal choices c_3^A and s_2^A follow immediately.

Given the optimal consumption choices, we can easily compute the utility when absent-minded and compare it to the utility under monitoring. The conditions determining the choice to monitor follow immediately, rounding out the solution to the model.

5 Economic Implications

We begin in section 5.1 by comparing the level of consumption in a period of absent-mindedness with that in a period of monitoring. With standard assumptions on preferences, absent-mindedness gives rise to increased consumption. In section 5.2 we consider the pattern of monitoring over the life cycle, and show that the incentive to monitor is highest in the early and late years of life, and lowest in the relatively unconstrained middle years. In section 5.3 we provide some confirmatory evidence from our survey on the life cycle pattern of monitoring, and relate these findings to existing results concerning the fall in consumption at retirement.

5.1 Precautionary Spending

Proposition 3 shows that absent-minded consumers spend more on average than do attentive consumers if and only if their utility is more risk averse than log utility.

Proposition 3: In periods 1 and 2, there is the following relationship between c^M and c^A :

1. If $\sigma \in [0, 1)$ then $c^M < c^A$.
2. If $\sigma = 1$, then $c^M = c^A$.

3. If $\sigma > 1$, then $c^M > c^A$.

Proof: From Proposition 2, we see that c^A is greater than (less than, equal to) c^M as A is greater than (less than, equal to) 1. The value of A , in turn, depends on the coefficient of relative risk aversion. Given the concavity of utility, Jensen's inequality implies:

$$u(Ec) = \frac{(c^A)^{1-\sigma} - 1}{1-\sigma} > \frac{A(c^A)^{1-\sigma} - 1}{1-\sigma} = Eu(c)$$

It follows that A is greater than (less than, equal to) 1 if σ is greater than (less than, equal to) 1. \square

There is a simple intuition behind Proposition 3. Using the change of variable $z = c/\bar{c}$, the first order condition for the absent-minded consumer in periods 1 and 2, $A\bar{c}^{-\sigma} = 1/w$, can be rewritten as,

$$\frac{Ecu'(c)}{Ec} = \frac{1}{w}.$$

Hence the convexity of $cu'(c)$ is a sufficient condition for absent-mindedness to increase consumption. Given CRRA preferences, $cu'(c)$ is convex if $\sigma > 1$, linear if $\sigma = 1$, and concave if $\sigma < 1$. Since almost all empirical work finds $\sigma > 1$, the relevant case is $cu'(c)$ is convex. Absent-mindedness raises the level of consumption: $c^A > c^M$. Whether or not absent-mindedness leads to higher consumption in period 3 is a bit more complicated. As in periods 1 and 2, $\sigma > 1$ tends to increase the marginal utility of consumption in period 3. However since $\beta(1+r)w_2 > w_3$, absent-mindedness also tends to increase the costs of consumption as well. Which effect dominates depends on the parameters of the model.

There is a close relationship between absent-mindedness and precautionary saving. Exactly the same condition, the convexity of $cu'(c)$, arises in the precautionary saving literature when risks are multiplicative (Sandmo, 1970). The main difference between the two literatures is where the uncertainty lies: today and tomorrow. That difference leads to opposite predictions regarding consumption and saving. If $cu'(c)$ is convex, then uncertainty regarding *future* consumption raises the expected marginal utility of future consumption and leads to precautionary saving. With absent-mindedness, uncertainty regarding *current* consumption raises the expected marginal utility of current consumption and leads to precautionary spending. Consumption moves towards the period in which consumption is uncertain.

In a more general model, there would be a more complex connection between absent-mindedness and consumption. Our assumption of a constant marginal disutility of labor removes any effect of consumption realizations today on the resources available for consumption tomorrow. If instead the disutility of labor were strictly convex, absent-mindedness would not only make consumption today more uncertain, it would also make future consumption more uncertain. The resulting incentive to increase precautionary saving would counteract to a certain extent the desire for precautionary spending. This effect of current consumption on future resources will generally be greater nearer to the end of life. As the time horizon increases, consumption mistakes today may be spread over a greater and greater number of periods. The resulting effect on permanent income will therefore be lower.

5.2 Monitoring and the Life Cycle

Proposition 4 describes the incentive to monitor. We focus on the standard case in which $\sigma > 1$ and comment on other possibilities.

Proposition 4 Given $\sigma > 1$ and Assumptions 1-3:

1. If monitoring were costless, then the consumer would always choose to monitor.
2. The incentive to monitor is decreasing in w_t .
3. The incentive to monitor is greater in period 1 than in period 2.

All three statements follow directly from part 1 of Proposition 2. The intuition for the first result is that an attentive consumer could choose to randomize consumption in a way similar to the absent-minded consumer, but because of the strict concavity of utility, would choose not to do so. Absent-mindedness results in mistakes which reduce the utility from consumption (note that this result holds regardless of the value of σ).

The intuition for the second statement is that the loss due to absent-mindedness results from the randomness of consumption and the concavity of utility. An increase in w_t reduces the effective cost of consumption in period t and raises consumption. With $\sigma > 1$, utility becomes less concave

as consumption rises, thereby reducing the loss due to absent-mindedness.¹⁵ The third statement follows directly from the second and the assumptions on w_t .

The monotonicity of the incentive to monitor implies that for each level of m , there is a unique wage $w_t(m)$ such that if $w < w_t(m)$ the agent is absent-minded and for $w > w_t(m)$ the agent monitors. Note that in period 3, this cutoff also depends on the wage level in period 2.

Whether the incentive to monitor in the second period is greater or less than the incentive to monitor in the third period is more complicated. There are two effects. On the one hand, if $\beta(1+r) > 1$, then some consumption in period 3 that is financed by labor in period 2 is cheaper in terms of labor effort than consumption in period 2. This effect would favor absent-mindedness. On the other hand, the possibilities that accumulated saving may go unspent and that consumption in excess of saving must be financed by costly period 3 labor, both raise the effective cost of the absent-minded policy. Which effect dominates is generally ambiguous. However if $\beta(1+r)$ is close to 1 it is easy to show that the second effect dominates: the incentive to monitor in period 3 is less than the incentive to monitor in period 2. Since conventional parameterizations place $\beta(1+r)$ close to 1, we will take this as the benchmark case.

The results in Proposition 4, describe a life cycle of attention. Attention is high early in life when the cost of consumption is high and the opportunities for borrowing are low. Attention wanes in middle age as resources improve. It rises again in retirement when it is costly to leave resources unspent and additional labor effort yields a low reward.¹⁶

Together the results in Propositions 3 and 4 have implications for consumption over the life-cycle. Greater attention tends to push down consumption early and late in life. absent-mindedness tends to raise consumption in mid-life. Overall, these effects exacerbate the standard hump-shaped life-cycle pattern of consumption.

¹⁵In periods 1 and 2, the loss due to absent-mindedness falls to zero as wages rise to infinity. If $\sigma < 1$, then the effect of wages is reversed. If $\sigma = 1$, then the loss due to absent-mindedness is independent of the level of wages.

¹⁶This life-cycle pattern of attention reflects the evolution of the benefits of attention over the life-cycle. We have assumed that the costs of monitoring are constant. As monitoring effort is relatively costly for the middle aged and the rich, a reasonable modeling of costs should only reinforce this pattern.

5.3 Evidence

Our data allow us to test whether or not the model's predictions concerning the life-cycle pattern of attention hold in our TIAA-CREF sample. We have data on whether the respondent is working full time, working part-time, unemployed, or retired. We also have data on their age and net worth. We regress the answers to our questions concerning advance budgeting and tracking of spending on these variables. The sample includes 1196 respondents for whom we have data on all of the variables included in the regressions. Table 3 presents results.

The principle finding is the large and significant effect of retirement on monitoring behaviors. For example, retirement is associated with an economically and statistically significant .6 increase in the budgeting measure.¹⁷ Other empirical results are also generally consistent with the predictions of the model. In the data, budgeting is inversely related with wealth as measured by financial assets. Since asset-holdings generally rise early in life and fall late in life, this correlation is consistent with the model's U-shaped life-cycle pattern of attention. It is also consistent with the model's positive correlation between attention and the marginal utility of wealth. Finally, attention is inversely related to age, which is consistent with the model's prediction that the young are less absent-minded.

Combining the result of this section concerning the increase in monitoring at retirement with the earlier finding that monitors tend to spend less, our model predicts a discrete decline in consumption at retirement. This is consistent with the evidence. The fact that many households undergo a sharp decline in consumption at retirement was noted by Mariger [1987], Banks, Blundel and Tanner [1998], and Bernheim, Skinner and Weinberg [2001]. Ameriks, Caplin, and Leahy [2003b] show that this decline is largely anticipated prior to retirement, a fact that rules out many alternative explanations that rely on systematic expectational errors.

¹⁷We get similar results for our measures of financial planning behavior. Retirees are more likely to monitor the allocation and performance of their investments and to update their plans for their long-term financial future.

6 Applications and Extensions

6.1 Belt-Tightening in Recessions

Our model of absent-mindedness may have implications for business cycles. The same forces that lead to fluctuations in absent-mindedness over the life-cycle may lead to cyclical fluctuations in attention that exacerbate cyclical movements in consumption. In a cyclical downturn, those with fewer resources and more time, such as the unemployed, may decide to pay greater attention to their spending. This may further dampen their consumption and amplify the effects of the downturn in economic activity.

Short-run fluctuations in monitoring may have implications not only for cyclical fluctuations, but also for the debate on the excess sensitivity of consumption to income. If an increase in income is also associated with an increase in absent-mindedness, then we would expect that the increase in consumption would be somewhat greater than what would be justified under the standard permanent income hypothesis.

6.2 The Analogy with Self Control

The most obvious analogy between our model and models of self control problems is the finding concerning precautionary spending. This is spending over and above the amount that the consumer would ideally choose. Absent-minded consumers who indulge in this form of spending may look like they have self control problems, since they will wish that they has spent less. The source of the over-consumption, however, is a problem of control rather than self control.

Another interesting analogy concerns the desire for constraints. It is well-understood that constraints on spending may be of value to those with self control problems. Our absent-minded consumer does not have a self control problem, but her inability to precisely control spending may nevertheless render such constraints valuable. Suppose that the absent-minded consumer had access to a technology in period 2 (other periods are similar) that would at cost κ cut off consumption as soon as it reached some level \hat{c} . The maximization problem in period 2 would be,

$$\max_{p, \hat{c}} p \int_0^{\hat{c}} e^{-pc} [u(c) - \alpha_2 c] dc + e^{-p\hat{c}} [u(\hat{c}) - \lambda \hat{c} - \kappa]$$

It is clear that the consumer would like to use this technology. The first order condition for \hat{c} sets,

$$u'(\hat{c}) = \alpha_2 - p\kappa.$$

The agent sets the constraint where the loss in utility from consumption equals the cost of invoking the constraint. Of course, the reason that consumers like constraints in this model is not that they limit behavior, but rather that they provide information concerning an already excessive level of spending.

The desire of an absent-minded consumer for information provides a third and final analogy with self control problems. Standard explanations for the impact of credit cards on spending stress their dangers to those with self control problems. They may be similarly problematic for absent-minded agents. The use of cash is potentially very informative about spending. The amount of cash in one's wallet or purse is a signal as to how much has been spent, and trips to the bank provide an opportunity to check one's account balance. Moreover, an agent will tend to run out of cash before spending excessive amounts. In all of these ways the use of cash promotes attentiveness. Credit cards, on the other hand, remove this crutch. They promote absent-mindedness, and may thereby increase spending.¹⁸

6.3 Planning and Wealth Accumulation

Lusardi [2000] and Ameriks, Caplin and Leahy [2003a] show that people who plan for their retirement tend to accumulate greater wealth. This raises the question of why planning matters. A view that planners are more informed about the future works in the wrong direction. If planners faced less future uncertainty then they would have less incentive to save for precautionary reasons. Our model suggests a more promising line of argument concerning the relationship between financial planning and wealth accumulation. If people who engage in financial planning activities are more likely to pay attention to their consumption, then they have less of an incentive to engage in precautionary spending. This increases their saving rate and thereby increases their wealth.¹⁹

¹⁸The role of cash as a substitute for memory is reminiscent of Kocherlakota [1998] analysis of money in a search model.

¹⁹Ameriks, Caplin, Leahy, and Tyler [2003c] suggest an alternative explanation of the link between planning and wealth accumulation. For some, planning may be a means for

6.4 Absent-Mindedness as a Behavioral Error

Our model provides a highly-structured framework in which to analyze a consumer who is aware that she does not have perfect control of her consumption, and makes behavioral errors. In the model, these mistakes arise because of imperfect memory and imperfect monitoring. However similar mistakes may arise for entirely different reasons. For example, suppose that agents differ in their tastes and that the quality of the match between any given individual and any chosen commodity uncertain. Suppose further that one can expend effort searching amongst goods, and that in so doing one can reduce the uncertainty in the match. In this case, search effort plays much the same role as attention does in our model. The greater is the search effort, the less uncertain the corresponding consumption flow.

Another example of mistakes in consumption revolves around imperfect communication within the household. It may take effort for one person to understand the preferences of others. Equally, it may take strong communication for the group as a whole to act efficiently. One may expect many mistakes to be made, the extent of which will diminish with increases in efforts at understanding and communication.

We believe that the qualitative results of our paper will survive addition of these and other sources of errors in consumption. The question of how best to structure a general model capturing the many underlying sources of error remains open. One simple way forward would be to suppose that when agents spend x dollars they get $u(x + \varepsilon)$ units of utility. By making the variance of ε declining in the level of attention, one would expect to recover results closely analogous to those we have derived in our model. Developing a work-horse model of this variety is a high priority in future research.

7 Conclusion

We have presented evidence that many consumers know little about their pattern of spending. We developed a life cycle model consistent with this evidence. The model provides new insights into fluctuations in spending over time, and in particular the “belt-tightening” associated with increased budgetary attention.

exerting self-control.

8 Appendix: Proofs

The general structure of the proof of the existence of an optimal policy is standard. There are several complications due to our modeling of absent-mindedness. First, absent-mindedness leads to unbounded choice sets. Labor l and the hazard rate p naturally lie in $[0, \infty]$. As the standard theorems require compact choice sets, we use the bijection $\varphi(x) : [0, 1] \rightarrow [0, \infty]$ where $\varphi(x) = x/(1-x)$ to relate $[0, 1]$ to $[0, \infty]$. A choice of x in $[0, 1]$ implies a choice $\varphi(x)$ in $[0, \infty]$ and visa versa.

Second, in order to apply standard results like the Theorem of the Maximum, we want to insure that the functions that describe the payoff to a choice are real-valued functions. This requires that they not take the values of $\pm\infty$. Natural utility functions, including the one used in the example of section 4, are unbounded as choices approach 0 or ∞ . We can easily introduce a lower bound to an arbitrary payoff $\pi(x)$ without affecting the optimal choice of x , by considering the transformed payoff $\hat{\pi}(x) = \max\{\pi(x), \pi(\bar{x}) - c\}$ where \bar{x} is an arbitrary feasible choice of x and c is an arbitrary strictly positive constant. Choices of x that maximize $\hat{\pi}(x)$ will maximize $\pi(x)$ and visa versa.

Showing that payoff functions are bounded above is more difficult. In standard problems, one considers payoff functions that are the sum of benefits $B(x)$ and costs $C(x)$. The benefit function is assumed to be real valued for $x \in (0, \infty)$, continuous, increasing, and to have marginal benefits that diminishes to zero as x approaches infinity. The cost function is assumed to be real valued $x \in [0, \infty)$, continuous, increasing and to have marginal costs that are in the limit strictly positive. Together these assumptions imply that the benefits eventually outweigh the costs, so that the payoff function is bounded above. The problem that arises in the current case is that the value function is the maximum of the value of being absent-minded and monitoring, and therefore may not be differentiable. Hence we cannot speak of the marginal benefit. Our solution is to show where necessary that the benefit of action is bounded above by a differentiable function whose derivative approaches zero as the relevant choice approaches infinity. With these preliminaries in mind, we turn to the proof.

Proposition 1: There exists an optimal policy.

Proof: The proof is by induction. Consider first period T and assume that the agent chooses to pay attention in period T . Let $g_1(x, W) =$

$u(W + w_T\varphi(x)) - v(\varphi(x))$. For each W , our assumptions on u and v imply that $g_1(x; W)$ is bounded above. It may, however, approach $-\infty$ as x approaches zero or one. Let $g_2(x, W) = \max\{g_1(x, W), g(.5, W) - 1\}$. $g_2(x, W)$ maps $[0, 1] \times [0, \infty)$ into R . Any choice of x that maximizes $g_2(x, W)$, maximizes $g_1(x, W)$. The assumptions on u and v imply that $g_2(x, W)$ is continuous. We can therefore apply the Theorem of the Maximum to $g_2(x, W)$. For each $W \in [0, \infty)$, let $V_T^M(W) = \max_{x \in [0, 1]} g_2(x, W)$ and $G_1(W) = \{x \in [0, 1] : g_2(x, W) = V_T^M(W)\}$. Then $V_T^M(W)$ is a continuous real-valued function, and $G_1(W)$ is a non-empty, compact valued and upper hemi-continuous correspondence.

We will use one more property of $V_T^M(W)$ in order to bound the payoff function in the induction step. Given the assumptions on u and v , $V(W)$ is differentiable. By the envelope theorem, $V_T^{M'}(W) = u'(c_T^M)$. Given the assumptions on u and v , as W approaches infinity, c_T^M approaches infinity. Since $\lim_{c \rightarrow \infty} u'(c) = 0$ by assumption, it follows that $\lim_{W \rightarrow \infty} V_T^{M'}(W) = 0$.

Similar arguments apply if the agent chooses to remain absent-minded in period T . Let $g_3(x, W) = \varphi(x) \int e^{-\varphi(x)c} [u(c) - v(\max\{0, (c - W)/w_T\})] dc$, and let $g_4(x, W) = \max\{g_1(x, W), g(.5) - 1\}$. Given W , $u(c) - v(\max\{0, (c - W)/w_T\})$ is bounded above by assumption. Therefore $g_3(x, W)$ is bounded above for given W . g_4 therefore maps $[0, 1] \times [0, \infty)$ into R . g_4 is continuous, and any choice of x that maximizes $g_4(x, W)$, maximizes $g_3(x, W)$. We apply the Theorem of the Maximum to $g_3(x, W)$. For each $W \in [0, \infty)$, let $V_T^A(W) = \max_{x \in [0, 1]} g_4(x, W)$ and $G_2(W) = \{x \in [0, 1] : g_4(x, W) = V_T^A(W)\}$. Then $V_T^A(W)$ is a continuous real-valued function, and $G_2(W)$ is a non-empty, compact valued and upper hemi-continuous correspondence.

Again we need to discuss the limiting behavior of $V_T^A(W)$. The first order condition for the optimal choice of x is

$$\int e^{-\varphi(x^*)c} u'(c) dc = \frac{1}{w} \int_W^\infty e^{-\varphi(x^*)c} v'((c - W)/w) dc \quad (2)$$

It follows immediately that as W approaches infinity, $\varphi(x^*)$ approaches zero. Now by the envelope theorem, $V_T^{A'}(W) = \frac{\varphi(x^*)}{w} \int_W^\infty e^{-\varphi(x^*)c} v'((c - W)/w) dc$, which differs from the right-hand side of (2) by the presence of the extra c in the integrand. If $W > 1$, $V_T^{A'}(W) < \frac{\varphi(x^*)}{w} \int_W^\infty e^{-\varphi(x^*)c} v'((c - W)/w) dc$, and by equation (2) $V_T^{A'}(W) < \frac{\varphi(x^*)}{w} \int e^{-\varphi(x^*)c} u'(c) dc$. Moreover as $\varphi(x^*)$ approaches zero, $\frac{\varphi(x^*)}{w} \int e^{-\varphi(x^*)c} u'(c) dc$ approaches zero. It follows that $\lim_{W \rightarrow \infty} V_T^{A'}(W) = 0$.

To complete our analysis of period T , note $V_T(W) = \max\{V_T^M(W), V_T^A(W)\}$. Since $V_T^M(W)$ and $V_T^A(W)$ are continuous and real valued, $V_T(W)$ is continuous and real valued. Since $V_T^{M'}(W)$ and $V_T^{A'}(W)$ approach zero as W approaches infinity, $V_T(W)$ is dominated by a real-valued and differentiable function whose derivative approaches zero as W approaches infinity. This completes our analysis of period T .

We now turn to the induction step. Suppose that $V_{t+1}(W)$ is continuous and real valued and there exists a real-valued, differentiable function $F_{t+1}(W) > V_{t+1}(W)$ such that $\lim_{W \rightarrow \infty} F'_{t+1}(W) = 0$.

We first consider the optimal period t policy in the case of attention. We first consider the optimal consumption choice given labor input and then the optimal labor input. Let $I = W + w_t l$ be the resources available given the period t labor decision. Let $g_5(c) = u(c) + \beta V_{t+1}((1 + r_{t+1})(I - c))$. To bound g_5 below, we define $g_6(c) = \max\{g_5(c), g_5(I/2) - 1\}$. For each $I \in [0, \infty)$, let $h_1(I) = \max_{c \in [0, I]} g_6(c)$ and $G_3(I) = \{c \in [0, I] : g_6(c) = h_1(I)\}$. Since g_6 is a continuous, real-valued function and the constraint correspondence is continuous in I , we can use the Theorem of the Maximum to conclude that $h_1(I)$ is continuous and $G_3(W)$ is a non-empty, compact valued and upper hemi-continuous correspondence.

Let $\hat{h}(I) = \max_c u(c) + \beta F((1 + r_{t+1})(I - c))$. $\hat{h}(I) > h_1(I)$. Moreover, $\hat{h}(I)$ is differentiable and $\hat{h}'(I) = u'(\hat{c})$ where $\hat{c} \in \arg \max \hat{h}(I)$. Since $\lim_{W \rightarrow \infty} F'(W) = 0$, c must approach infinity as I approaches infinity. Hence $\lim_{I \rightarrow \infty} \hat{h}'(I) = 0$.

We turn to the labor choice. Let $g_7(x, W) = h_1(W + w\varphi(x)) - v(\varphi(x))$, and let $g_8(x, W) = \max\{g_7(x, W), g_7(.5, W) - 1\}$. Since $h_1(I) < \hat{h}(I)$, and $\lim_{I \rightarrow \infty} \hat{h}'(I) = 0$, and $v'(l) > 0$, we know that for each W , g_7 is bounded above. Hence g_8 is a real valued function. It maps $[0, 1] \times [0, \infty)$ into R . Similar arguments to those above establish that $V_t^M(W) = \max_{x \in [0, 1]} g_8(x, W)$ is continuous and that $G_4(W) = \{x \in [0, 1] : g_8(x, W) = V_t^M(W)\}$ is a non-empty, compact valued and upper hemi-continuous correspondence.

Note that $V_t^M(W) < F^M(W) \equiv \max_x \hat{h}_1(W + w\varphi(x)) - v(\varphi(x))$. Moreover, $\lim_{W \rightarrow \infty} F^{M'}(W) = 0$.

Now suppose that the agent is absent-minded in period t . Again we break down the period. We first consider the optimal labor choice given the realization of consumption and then the optimal stopping strategy. Let $g_9(x; c, W) = \beta V_{t+1}((1 + r_{t+1})(\max\{0, W - c + w\varphi(x)\})) - v(\varphi(x))$ and let $g_{10}(x; c, W) = \max\{g_9(x; c, W), \beta V_{t+1}(0) - 1\}$. The labor choice must pay for consumption. This implies that $\varphi(x) > \min\{0, (c - W)/(c - W + w)\} \equiv \hat{\varphi}$.

Note also that $g_9(x; c, W) < g_{10}(x; c, W) \equiv \beta F((1+r_{t+1})(\max\{0, W-c+w\varphi(x)\})-v(\varphi(x)))$. Since F and v are real valued functions with $\lim_{W \rightarrow \infty} F'(W) = 0$ and $v'(\varphi(x)) > 0$, we know that given c and W , $g_{10}(x; c, W) < \infty$ for all $x \in [0, 1]$. $g_9(x; c, W)$ is therefore a real valued function mapping $[0, 1] \times [0, \infty)$ into R .

Similar arguments to those above establish that $h_2(c, W) = \max_{x \in [\hat{\varphi}, 1]} g_9(x; c, W)$ is continuous and that $G_5(W) = \{x \in [\hat{\varphi}, 1] : g_{10}(x; c, W) = h_2(c, W)\}$ is a non-empty, compact valued and upper hemi-continuous correspondence. Moreover we can define $\hat{h}_2(c, W) = \max_{x \in [\hat{\varphi}, 1]} g_{10}(x; c, W)$. $\hat{h}_2(c, W) > h_2(c, W)$ and $\lim_{W \rightarrow \infty} \hat{h}'_2(c, W) = 0$.

We turn to the choice of stopping strategy. Let $g_{11}(x, W) = \varphi(x) \int e^{-\varphi(x)c} [h_2(c, W)] dc$, and let $g_{12}(x, W) = \max\{g_{11}(x), g_{11}(.5) - 1\}$. Similar arguments to those above establish that $V_t^A(W) = \max_{x \in [0, 1]} g_{12}(x, W) - m$ is continuous and that $G_6(W) = \{x \in [0, 1] : g_{12}(x, W) = V_t^A(W)\}$ is a non-empty, compact valued and upper hemi-continuous correspondence.

Now $V_t^A(W) < \hat{V}_t^A(W) = \max_{x \in [0, 1]} \varphi(x) \int e^{-\varphi(x)c} [\hat{h}_2(c, W)] dc$. Similar arguments as above show $\lim_{W \rightarrow \infty} \hat{V}_t^A(W) = 0$.

$V_t(W) = \max\{V_t^A(W), V_t^M(W)\}$ is therefore continuous and there exists a differentiable function $F_t(W)$ such that $F_t(W) > V_t(W)$ and $\lim_{W \rightarrow \infty} F'_t(W) = 0$. This completes the induction step and the proof of the proposition. \square

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Table 1
Subjective Uncertainty Concerning Past Spending

	mean uncertainty	st. dev. uncertainty	share of income
food and drink at home	.449	.287	.11
clothing	.745	.502	.03
telephone	.425	.308	.02
credit card spending	.520	.414	.29

943 households. Uncertainty is the 90-10 confidence interval divided by the mean.

The share of income is calculated from the subset of fully employed households.

Table 2
Regressions of Subjective Uncertainty on Measures of Attention

	Advance Budget	Track Spending
food and drink at home	-.022** (.007)	-.017* (.007)
clothing	-.036** (.011)	-.027* (.013)
telephone	-.013 (.007)	-.011 (.008)
credit card spending	-.011 (.009)	-.023* (.011)

Standard errors are in parentheses. * indicates 5% level of significance.

** indicates 1% level of significance.

Table 3
Regressions of Measures of Attention on Life-cycle variables

variable	Advance Budget	Track Spending
age	-.015** (.005)	-.009* (.004)
gross financial assets	-.110** (.035)	-.035 (.032)
employment status		
full time	<i>omitted</i>	<i>omitted</i>
retired	.614** (.124)	.357** (.111)
part time	.348 (.198)	.144 (.177)
unemployed	.226 (.286)	.202 (.225)
constant	4.823** (.236)	4.764** (.229)

Standard errors are in parentheses. * indicates 5% level of significance.

** indicates 1% level of significance.