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A SIMPLE PROOF THAT FUTURES MARKETS ARE ALMOST ALWAYS INFORMATIONALLY INEFFICIENT

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ABSTRACT

Previous work which showed that prices could aggregate perfectly the diverse information of traders depended critically on the assumption that all agents had constant absolute risk utility. We show that either all agents must have constant absolute risk aversion utility, or all must have constant relative aversion in order for the strong form of the efficient market hypothesis to hold generically.

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1. Introduction

The notion that stock markets, commodity markets, and futures markets are informationally efficient is widely-accepted. When different traders have disparate information, it is clear that a single price cannot reveal the information of each of the participants; but it is still possible that markets can aggregate this information, and that prices can reveal all of the relevant information. (More precisely, prices can be sufficient statistics. For example, knowing the futures price, one may be able to make as good a forecast of the spot price as one could make knowing the futures price and all of the other information of the various market participants.)

In an earlier paper, Grossman and Stiglitz (1976) constructed a simple model of a futures market that was informationally efficient. In their model, there was a large number of farmers, each of whom had perfect information about his own crop (but no direct information about the crops of others). The equilibrium price on the futures market aggregated this disparate information perfectly, and hence the futures price was a sufficient statistic for predicting the spot price.

The Grossman-Stiglitz model had several special properties. Chief among them was the assumption that all producers had the same constant absolute risk aversion utility function. This meant that the aggregate supply of futures only depended on the estimate of aggregate output: the distribution of output across farmers had no effect on the aggregate supply of futures, and hence it had no effect on the equilibrium futures price.

In general, one might expect that the distribution of crops across producers would affect the supply of futures. For instance, the supply of futures might be high because all farmers know that they are going to have a large crop, and want to divest themselves of the associated price risk; or it may be that the aggregate crop is small, but with a larger dispersion of crop sizes. In this latter case, it may be that farmers with small outputs are more risk averse, and take a more hedged position, which outweighs the reduction in hedging by the farmers with large crops, who are less risk averse.

The general observation is that the price on the futures market is a function of the aggregate level of output and the dispersion of crop sizes across farmers. The fact that the futures market is not efficient generically in the model here indicates that the futures price cannot be fully revealing in general.[1,2,3]

This paper determines necessary and sufficient conditions for prices on futures markets to be sufficient statistics for spot market prices.[4] We employ a simple model of trade on a futures market to show that there are only two classes of utility functions for which markets are efficient in general: constant absolute risk aversion and constant relative risk aversion. The restrictions to these two families is important because of the structure of market demand that they generate.

<u>The structure of the paper.</u> The remainder of the paper is divided into three sections. In the first, we set up the basic

model. In the second, we consider the case of ex ante identical farmers. It is shown there that the only utility functions for which futures prices are sufficient statistics are those for which the risk tolerance function is linear in wealth. If prices are to be sufficient statistics in more general models, the utility functions must belong to this class.

In the subsequent section, we extend the model by allowing for heterogeneity among farmers. We show that while the constant absolute risk aversion utility function still works, the constant relative risk aversion utility function no longer gives the efficiency result when farmers differ in their respective coefficients of relative risk aversion.

2. The Model

There are m farmers indexed by $i = 1, 2, \ldots, m$. The farmers are risk-averse von Neumann-Morgenstern expected utility of wealth maximizers. There is no motive for pure speculation, so only farmers trade on the futures market. At time t=0, each farmer sees his crop, q_i. At t=1, a round of trading takes place on the futures market, allowing the farmers to insure themselves by buying or selling claims on the commodity. At time t=2, the spot market opens.

Assume that spot market demand is linear:

$$\tilde{p}_s = a - bQ + \tilde{\theta}.$$

$\tilde{p}_s$ is the spot price, $\tilde{\theta}$ is a demand shock, and $Q = \sum_{i=1}^{m} q_i$ is the total crop. Farmers are price takers, and there is no storage technology here, so the entire crop will be sold on the spot market.

There is a competitive rational expectations equilibrium in the futures market. This requires first that the futures market clears at t=1:

$$\sum_{i=1}^{m} x_i = 0.$$

Equilibrium also requires that farmer i selects his supply x_i to maximize expected utility, conditional on p_f and q_i:

$$x_i = \arg\max_x E[U(p_f x + \tilde{p}_s(q_i - x)) | p_f, q_i].$$

In sum, plans are optimal with decisions conditioned on the equilibrium futures prices p_f, and one's own output.[5]

3. Conditions on Preferences

We proceed first by getting conditions on utility that are necessary for the futures market to be efficient. The spot price $\tilde{p}_s$ depends on individual crop sizes only through total supply Q. For p_f to be fully revealing, it must depend on Q alone, and not on the individual q_i values. In other words, p_f must be invariant to the division of Q among the farmers. An implication is:

Theorem 1: In the one-period model, a necessary and sufficient condition for efficiency on the futures market, when farmers have identical utility, is that utility be of the linear risk tolerance class:

$$U'(W)/U''(W) = a+bW.$$

Proof: The proof is in Appendix A.

The method of proof is to show that, in order for the futures price to be fully revealing, the supply of futures must be a linear function of the farmer's output. An implication is that the utility function must come from the linear risk tolerance (hyperbolic absolute risk aversion) class.

The class of linear risk tolerance utility functions can be divided into two subclasses. The analysis that follows will focus on the following representative members of the linear risk tolerance class:

1. constant absolute risk aversion (cara): $U(W) = -e^{-\delta W}$;

2. constant relative risk version (crra): $U(W) = (\alpha+W)^{1-\delta}$.

The entire linear risk tolerance family is composed of affine transformations of the two representative utility functions (and also $U(W) = \log(\alpha+W)$). For example, functions of the form $U(W) = \alpha - \beta e^{-\delta W}$ are members. Note also that the class labelled "constant relative risk aversion" represents a broadening of that class to include utility functions that exhibit constant relative risk aversion with respect to a translated origin.

The necessity of linear supply curves for futures is intuitive. Suppose that endowments are altered slightly so that one trader's crop rises while another's drops by the same amount. For instance, let q_1 become $q_1+\Delta$, and let q_2 become $q_2-\Delta$. Since the aggregate output is not altered by this transfer, the net quantity supplied at the price p_f must not change if the price is to be fully revealing of Q. In particular, the one farmer's increased supply must just be matched by the other farmer's decreased supply. In order for this neutrality result to hold for all pairs of farmers and all transfers, each farmer must have a supply curve with the same constant slope.

The following section examines the extent to which the results generalize to cases where individuals have different utility functions.

4. Differences in Risk Aversion

We now ask what happens when farmers have utility functions within the same class, but with different parameter values. Since our interest is in determining those utility functions for which the futures market is always efficient, it is sufficient to examine linear risk tolerance utility functions only. We show:

Theorem 2: Prices are fully revealing if and only if all farmers have constant absolute risk aversion utility, or all farmers have constant relative risk aversion utility, with the same coefficient of relative risk aversion for each farmer.

Proof: The proof is in Appendix B.

Theorem 1 has shown that linear risk tolerance is necessary. The proof of Theorem 2 shows directly that the utility functions listed above do imply a linear supply of futures, with the same slope for each farmer. It is then shown that if farmers all have constant relative risk aversion utility, but differ in their coefficients of relative risk aversion, then the futures price is not fully revealing in general.

In summation, the only two families that achieve efficiency generically are affine transformations of the following two representative utility functions:

1. $U_i(W) = -e^{-\delta_i W}$;
2. $U_i(W) = (\alpha_i + W)^{1-\delta}$.

CONCLUSION

The insight that prices convey information to market participants and non-participants alike is an important one. It is clear that, in general, markets cannot be informationally efficient, in the sense that prices convey <u>all</u> of the information of the informed to the uninformed. Still, it is possible that prices might convey all of the relevant information, in the sense that futures prices might be a sufficient statistic for spot prices.

Although earlier work showed that there were examples in which price was a sufficient statistic, this paper has used simple and intuitive techniques to demonstrate that the parameterizations employed in much of this work are indeed special: the constant absolute risk aversion utility functions, and the constant relative risk aversion utility functions, with the same coefficient of relative risk aversion for all traders, are the only utility functions for which efficiency holds generally. Introducing private information about the demand side further reduces the likelihood that price can be a sufficient statistic (Bray [1981]).

While the introduction of additional markets may result in futures markets conveying more information, the number and kinds of markets that would have to be introduced to resolve the problems discussed here cast doubt on the hypothesis that markets can aggregate and transmit all relevant information. This paper can thus be thought of as providing further support for the view of prices as noisy signals. Under this view, prices convey some, but not all, of the information possessed by market participants. For an exposition of that latter view see Grossman and Stiglitz (1976, 1980) or Kyle (1989).

APPENDIX A

Proof of Theorem 1

A necessary condition for p_f to reveal total output $Q = \sum_{i=1}^{m} q_i$ is that p_f be independent of the distribution of Q across farmers. When facing a price p_f, and knowing that the spot price is distributed as $\tilde{p}_s$, farmer i selects his net supply of futures x_i to maximize:

$$E[U(p_f x_i + \tilde{p}_s(q_i - x_i)) | p_f, q_i].$$

(Henceforth the conditioning on p_f and q_i will be suppressed.)

The first-order condition is:

$$0 = E[U'(\tilde{p}_s q_i + (p_f - \tilde{p}_s)x_i)(p_f - \tilde{p}_s)].$$

By symmetry, if $q_i = \bar{Q}$ for all i, where $\bar{Q} = Q/m$, there will be no trade on the futures market and hence $x_i = 0$ for all i. At the same time, if p_f is fully revealing, one need only look at the case with $q_i = \bar{Q}$ to solve for the equilibrium futures price:

$$p_f = E[U'(\tilde{p}_s \bar{Q}) \cdot \tilde{p}_s] / E[U'(\tilde{p}_s \bar{Q})].$$

Let $z_i = q_i - \bar{Q}$, and let $x(z_i)$ be the desired net supply of futures, given p_f. Now define the function

$$g(z_1, z_2, \ldots, z_{m-1}) = x(z_1) + x(z_2) + \ldots + x(z_{m-1}) + x(-z_1 - z_2 \ldots - z_{m-1}).$$

Since g is merely the sum of the net supplies of futures from each farmer at the price p_f, it must be identically zero if the futures market is efficient. Differentiating with respect to z_i gives

$$0 = x'(z_i) - x'(-z_1 - z_2 \ldots - z_{m-1});$$

$i = 1, 2, \ldots, m-1$. Hence, for $m > 2$, we have

$$x'(z_1) = x'(z_2) = x'(-z_1-z_2 \ldots -z_{m-1}).$$

Moreover, for arbitrary Δ, the following must also hold:

$$x'(z_1+\Delta) = x'(z_2-\Delta) = x'(-z_1-z_2 \ldots -z_{m-1})$$

Thus $x'(z)$ is a constant function, so $x(z)$ is linear in z.

Farmer i's wealth can be written:

$$W_i = p_f x_i + (q_i-x_i)\tilde{p}_s = p_f q_i - (p_f-\tilde{p}_s)(q_i-x_i).$$

Since the farmer is a price-taker on the futures market, we say that he has "certain wealth" $p_f q_i$. Linearity of $x(\cdot)$ shows that each farmer accepts a gamble that is linear in certain wealth, given p_f. Following Stiglitz (1973), utility must be of the linear risk tolerance class

$$U'(W)/U''(W) = a+bW,$$

for the futures market to be efficient.

When the above expression is inverted, it can be rewritten

$$d(\log(U'(W)))/dW = 1/a \qquad b=0.$$

$$= (1/b)d(\log(a+bW))/dW \qquad b\neq 0.$$

The first differential equation characterizes the constant absolute risk aversion class, while the second characterizes the constant relative risk aversion class, with a translated origin.

Sufficiency is now shown. That is, it is shown that the futures price is fully revealing when all farmers have the same linear risk tolerance utility. Look first at constant absolute risk aversion utility:

$$U(W) = -e^{-\delta W}.$$

The first-order condition for farmer i is

$$0 = \delta E[e^{-\delta[\tilde{p}_s q_i + (p_f-\tilde{p}_s)x_i]}(p_f-\tilde{p}_s)].$$

If $q_i-x_i = \bar{Q}$ for all i, then the first-order conditions are identical. Thus x_i is linear in q_i, and the slope is the same for all i. The futures price is clearly equal to

$$p_f = E[e^{-\delta\tilde{p}_s\bar{Q}}\cdot\tilde{p}_s]/E[e^{-\delta\tilde{p}_s\bar{Q}}].$$

Now consider constant relative risk aversion utility:

$$U(W) = (\alpha+W)^{1-\delta}.$$

The first-order condition for farmer i is

$$0 = (1-\delta)E[(\alpha + \tilde{p}_s q_i + (p_f-\tilde{p}_s)x_i)^{-\delta}(p_f-\tilde{p}_s)],$$

This can be written:

$$0 = E[(\alpha + p_f x_i + \tilde{p}_s(q_i-x_i))^{-\delta}(p_f-\tilde{p}_s)].$$

If $\alpha + p_f x_i$ is a constant multiple of (q_i-x_i) for all i, then the first-order conditions are identical for all i. Suppose, then, that

$$\alpha+p_f x_i = \beta(q_i-x_i)$$

for all i. Summing both terms over i gives

$$\sum_{i=1}^{m}(\alpha+p_f x_i) = \beta\sum_{i=1}^{m}(q_i-x_i).$$

Market clearing implies that $\alpha = \beta\bar{Q}$.

The net supply of futures by farmer i is:

$$x_i = \alpha(q_i-\bar{Q})/(p_f\bar{Q}+\alpha);$$

which is linear in q_i (for fixed $\bar{Q}$). Once again, the slope is the same for all i. The equilibrium futures price is

$$p_f = E[\tilde{p}_s(\alpha + \bar{Q}\tilde{p}_s)^{-\delta}]/E[(\alpha + \bar{Q}\tilde{p}_s)^{-\delta}].$$

This completes the proof of sufficiency.

For m = 2, a separate proof of necessity is required. The method of the proof is to assume that both individuals know Q, and then to solve for the equilibrium futures price. If p_f is fully revealing, for each value of Q there must be a unique p_f, independent of the distribution of Q between the two farmers.

As before, let $x(\cdot)$ denote the net supply of futures, given the equilibrium futures price p_f. Now let $q_1 = \frac{Q}{2} + z$ and let $q_2 = \frac{Q}{2} - z$. Market clearing ensures that the following condition holds for all z:

$$x(\tfrac{Q}{2} + z) = -x(\tfrac{Q}{2} - z).$$

It likewise follows that

$$x'(\tfrac{Q}{2} + z) = x'(\tfrac{Q}{2} - z)$$

and

$$x''(\tfrac{Q}{2} + z) = -x''(\tfrac{Q}{2} - z).$$

The above conditions imply that

$$x(\tfrac{Q}{2}) = x''(\tfrac{Q}{2}) = 0.$$

We now return to the first-order conditions to get expressions for these derivatives. For farmer i, the first-order condition is:

$$0 = E[U_i'(\tilde{p}_s q_i + x_i(p_f - \tilde{p}_s))(p_f - \tilde{p}_s)].$$

Thus,

$$\frac{dx_i}{dq_i} = \frac{-E[U''\cdot\tilde{p}_s(p_f-\tilde{p}_s)]}{E[U''\cdot(p_f-\tilde{p}_s)^2]} = 1 - p_f\frac{E[U''\cdot(p_f-\tilde{p}_s)]}{E[U''\cdot(p_f-\tilde{p}_s)^2]}$$

for i = 1, 2; where we have suppressed the argument of $U''(\cdot)$ and the index i. Now we have

$$\frac{d^2x_i}{dq_i^2} = -p_f\{E[U'''\cdot(p_f-\tilde{p}_s)(\tilde{p}_s+(p_f-\tilde{p}_s)\frac{dx_i}{dq_i})]\cdot$$

$$E[U''\cdot(p_f-\tilde{p}_s)^2] - E[U''(p_f-\tilde{p}_s)]\cdot$$

$$E[U'''(p_f-\tilde{p}_s)^2(\tilde{p}_s+(p_f-\tilde{p}_s)^2 \frac{dx_i}{dq_i})]\}/\{E[U''\cdot(p_f-\tilde{p}_s)^2]\}^2.$$

It was demonstrated above that a necessary condition for futures market efficiency is $x''(\frac{Q}{2}) = 0$ or, equivalently,

$$\left.\frac{d^2x_i}{dq_i^2}\right|_{q_i = Q/2} = 0.$$

Substituting for dx_i/dq_i, the condition at $q_i = Q/2$ becomes

$$0 = 2EU''\tilde{\alpha}\cdot EU''\tilde{\alpha}^2\cdot EU'''\tilde{\alpha}^2 - EU'''\tilde{\alpha}^3\cdot EU''\tilde{\alpha}\cdot EU''\tilde{\alpha}$$

$$-EU'''\tilde{\alpha}\cdot EU''\tilde{\alpha}^2\cdot EU''\tilde{\alpha}^2,$$

where $\tilde{\alpha} = p_f-\tilde{p}_s$. This condition must hold for all possible distributions for $\tilde{\theta}$. In particular, we look at the two-point distribution in which $\tilde{\theta}$ equals Δ or $-\Delta$ with equal probability.

If the condition above is to hold for all Δ, it must hold in a neighborhood of $\Delta = 0$. It is clear, then, that all derivatives must be zero, when evaluated at $\Delta = 0$. Taking six (sic) derivatives with respect to Δ, and evaluating at $\Delta = 0$, gives

$$U''\cdot[2(U''')^2 - U''''\cdot U'' - \frac{(U'')^2U'''}{U'}] = 0.$$

This is precisely the condition that $T'' = 0$, where $T = -U'/U''$ is the measure of absolute risk tolerance. The condition $T'' = 0$ implies

$$U'(W)/U''(W) = a+bW.$$

In these derivations we have made use of the following formulas:

A. The first-order condition is:

$$0 = U'((\bar{p}_s+\Delta)q)\cdot(\bar{p}_s+\Delta-p_f) + U'((\bar{p}_s-\Delta)q)\cdot(\bar{p}_s-\Delta-p_f),$$

where $q = Q/2$. (Henceforth, let $U'(\Delta) = U'((\bar{p}_s+\Delta)q)$, $U'(-\Delta) = U'((\bar{p}_s-\Delta)q)$, etc.) Differentiating the first-order condition totally with respect to Δ gives:

$$\frac{dp_f}{d\Delta} = \frac{qU''(\Delta)(\bar{p}_s+\Delta-p_f) - qU''(-\Delta)(\bar{p}_s-\Delta-p_f) + U'(\Delta) - U'(-\Delta)}{U'(\Delta) + U'(-\Delta)} .$$

Thus

$$\frac{dp_f}{d\Delta} = 0 \text{ at } \Delta = 0.$$

Similarly,

$$\frac{d^2p_f}{d\Delta^2} = \frac{2U''q}{U'} \text{ at } \Delta = 0 .$$

B. At $\Delta = 0$; $EU''\alpha$, $EU''{}'\alpha^2$, etc. $= 0$.

$$\frac{dE\phi\alpha^i}{d\Delta} = 0; \ i = 1, 2, 3; \text{ where } \phi = U'', U''{}'.$$

$$\frac{d^2E\phi\alpha}{d\Delta^2} = 2\phi' q - \frac{2U''\phi q}{U'} .$$

$$\frac{d^2E\phi\alpha^2}{d\Delta^2} = 0. \quad \blacksquare$$

APPENDIX B

Proof of Theorem 2

Write the two candidate families as:

1. cara $\quad U_i(W) = -e^{-\delta_i W}$,

2. crra $\quad U_i(W) = (\alpha_i + W)^{1-\delta_i}$.

For constant absolute risk aversion utility, the first-order condition for farmer i can be written

$$0 = E[e^{-\delta_i(q_i - x_i)\tilde{p}_s}(p_f - \tilde{p}_s)].$$

Satisfying the first-order conditions for all i simultaneously requires that $\delta_i(q_i - x_i)$ be equated across all farmers. If $\delta_j(q_j - x_j) = k$ for all j, then

$$\sum_{j=1}^{m}(q_j - x_j) = \sum_{j=1}^{m}(k/\delta_j).$$

Hence the supply of futures from farmer i is

$$x_i = q_i - (Q/\delta_i)/\sum_{j=1}^{m}(1/\delta_j);$$

which is linear in q_i, with the same slope for all i.

For constant relative risk aversion utility, consider first the case of $\delta_i = \delta$ for i = 1, 2, ..., m. The first-order condition for farmer i is

$$0 = (1-\delta)\cdot E[(\alpha_i + p_f x_i + \tilde{p}_s(q_i - x_i))^{-\delta}(p_f - \tilde{p}_s)].$$

If $\alpha_i + p_f x_i$ is a constant multiple of $(q_i - x_i)$ for all i, then the first-order conditions are identical. Letting $\alpha_i + p_f x_i = \beta(q_i - x_i)$ for all i, the first-order condition becomes

$$0 = E[(\beta + \tilde{p}_s)^{-\delta}(p_f - \tilde{p}_s)].$$

To solve for β, note that

$$\sum_{i=1}^{m}(\alpha_i+p_f x_i) = \beta\sum_{i=1}^{m}(q_i-x_i).$$

Since the net quantity supplied is zero in equilibrium, this implies

$$\sum_{i=1}^{m}\alpha_i = \beta Q.$$

Thus $\beta = \bar{\alpha}/\bar{Q}$, where $\bar{\alpha} = \sum_{i=1}^{m}\alpha_i/m$.

For farmer i, the net supply is:

$$x_i = (\bar{\alpha}q_i-\alpha_i\bar{Q})/(p_f\bar{Q}+\bar{\alpha});$$

which is linear in q_i, for fixed $\bar{Q}$. Finally, the equilibrium futures price is

$$p_f = E[\tilde{p}_s(\bar{\alpha} + \bar{Q}\tilde{p}_s)^{-\delta}]/E[(\bar{\alpha} + \bar{Q}\tilde{p}_s)^{-\delta}].$$

It is now shown that the efficiency result does not carry over to the case of farmers who differ in their respective coefficients of relative risk aversion. Consider the simplest case:

$$U_i(W) = W^{1-\delta_i}.$$

The first-order condition for farmer i is now

$$0 = E[((\tilde{p}_s q_i + (p_f-\tilde{p}_s)x_i)^{-\delta_i}(p_f-\tilde{p}_s)].$$

There exists a distribution of quantities across farmers such that there is no trade on the futures market, i.e., $x_i = 0$ for i = 1, 2, ..., m. In this case the following must hold for all pairs i and j:

$$p_f = \frac{E[(\tilde{p}_s)^{1-\delta_i}]}{E[(\tilde{p}_s)^{-\delta_i}]} = \frac{E[(\tilde{p}_s)^{1-\delta_j}]}{E[(\tilde{p}_s)^{-\delta_j}]}.$$

Now suppose that $\tilde{p}_s$ has a two-point distribution, with equal probabilities of being $\bar{p}_s+\Delta$ or $\bar{p}_s-\Delta$.

Define

$$f(\delta) = \frac{E[(\tilde{p}_s)^{1-\delta}]}{E[(\tilde{p}_s)^{-\delta}]} = \frac{(\bar{p}_s+\Delta)^{1-\delta} + (\bar{p}_s-\Delta)^{1-\delta}}{(\bar{p}_s+\Delta)^{-\delta} + (\bar{p}_s-\Delta)^{-\delta}}.$$

The numerator of $f'(\delta)$ is not identically zero. Since $f(\delta)$ is not constant, a fully revealing price cannot exist in general.

It is equally clear that the futures price is not fully revealing in general if some farmers have constant absolute risk aversion utility while others have constant relative risk aversion utility. ∎

REFERENCES

Bray, Margaret, "Futures Trading, Rational Expectations, and the Efficient Markets Hypothesis," Econometrica, 49 (1981), 575-596.

Green, Jerry, "The Non-Existence of Informational Equilibria," Review of Economic Studies, 44 (1977), 451-463.

Grossman, Sanford and Joseph E. Stiglitz, "Information and Competitive Price Systems," American Economic Review, 66 (1976), 246-253.

Grossman, Sanford and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," American Economic Review, 70, (1980) 393-408.

Jordan, J. S., "On the Efficient Markets Hypothesis," Econometrica, 51 (1983), 1325-1343.

Kyle, Albert S., "Informed Speculation with Imperfect Competition," Review of Economic Studies, (1989) forthcoming.

Radner, Roy, "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," Econometrica, 47 (1979), 655-678.

Stiglitz, Joseph E., "Perfect and Imperfect Markets," paper presented to the Econometric Society, New Orleans, 1971.

Stiglitz, Joseph E., "Portfolio Allocations with Many Risky Assets," Cowles Foundation Paper No. 384, 1973.

Tirole, Jean, "On the Possibility of Speculation Under Rational Expectations," Econometrica, 50 (1982), 1163-1181.

ENDNOTES

[1] There is an alternative approach to the question of whether prices are fully revealing, which assumes a finite number of states of nature and a corresponding finite number of markets. This approach is of little economic interest: even limiting ourselves to an economy in which the only random variables are the outputs of each of N farms, and if these can take on M different values, the number of states of nature is M^N, far larger than the number of markets. The most that can be hoped is that prices will be sufficient statistics for conveying information about the limited number of aggregate economic variables of interest. For discussions of this alternative approach, see Green (1977) and Radner (1979), and the works cited there.

[2] Grossman and Stiglitz (1976, 1980) and Stiglitz (1971) present another category of argument against the view that markets are informationally efficient: if they were, individuals would have no incentive to obtain information, and thus the only information which would be reflected in the market is costless information. The issue is not relevant here since we assume that each producer can costlessly obtain information concerning his own output. When Grossman and Stiglitz constructed their model in which prices perfectly aggregated information, they were aware of the special role that the assumption of constant absolute risk aversion played. The present paper grew out of an attempt to show that the utility functions they employed were almost the only utility functions which had the property that the dispersion of crop sizes would have no effect on price.

[3] In the earlier studies, Grossman and Stiglitz also discussed a problem with the existence of equilibrium. In particular, they pointed out that if each farmer had perfect information concerning his own output, and if the market perfectly revealed all the relevant information (i.e., provided a perfect predictor of the future spot price), then individuals' demand for futures would not depend on their own information about their crop size; but then it would be impossible for the futures price to be fully revealing. This existence problem is simply an artifact of the assumption that farmers are perfectly informed concerning their own crops, which comprise the only source of uncertainty.

[4] Jordan (1983) addresses the same questions, using markedly different techniques. Not surprisingly, he arrives at similar answers. The assumptions underlying our analyses are slightly different, and we arrive at our results using only elementary techniques, which make transparent precisely why the class of utility functions for which markets are efficient is so restricted.

[5] To simplify matters we take as given that utility is concave in the region of interest. Second order conditions will therefore not be made explicit.