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LEARNING AND THE VALUE OF THE FIRM

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ABSTRACT

The paper studies under what conditions the value of the firm occasionally increases for a while before it suddenly drops, like a "bubble". We consider the environment where the trend of net cash flow from a firm's production depends on uncertain quality of a manager, and the manager is occasionally replaced by a new manager. People know whether the manager is replaced, but they do not know the exact quality of the manager so that they gradually learn about it. We show that, if the current manager is good, the value of the firm tends to increase more rapidly than the net cash flow because people become more and more optimistic about the current manager, until the optimism disappears with sudden retire of the manager. The value of the firm appears to contain a bubble because the value gradually deviates from the present value of the current net cash flow until the deviation disappears. We extend the basic model to allow the firm to replace unsuccessful managers endogenously, and show that the value of the firm more frequently deviates upward from the present value of the current net cash flow than downward.

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I. Introduction

Stock prices appear to be very volatile, and some researchers suspect that business psychology may play an important role. One candidate of such extrinsic psychology is a speculative bubble, which pushes the stock price above the fundamental value implied by expectations of future dividends until the bubble crashes. However, conditions for the existence of a rational bubble are often very restrictive. (See, for example, Tirole [1982].) This paper takes an alternative approach and asks under what conditions on the fundamentals the value of the firm behaves *like* a bubble, that is, the value of the firm tends to increase for a while before it suddenly drops.

In the environment we consider, the change of the net cash flow from a firm's production depends on uncertain quality of a manager and noise, and the manager is occasionally replaced by a new manager. People know whether the manager is replaced, but they, including the manager himself, do not know exactly how good the manager is; they gradually learn about it by observing the changes in net cash flow under the current manager. We can show that, if the current manager is good, people gradually become more and more optimistic about the manager, and that the value of the firm tends to increase more rapidly than the net cash flow, until the optimism disappears with sudden retire of the manager. The value of the firm appears to contain a bubble, because the value gradually deviates from the present value of the current net cash flow and then the deviation disappears. While this bubble-like deviation of the value of the firm can be both upward and downward in the basic model, we later allow the firm to replace the unsuccessful manager endogenously with some fixed cost, in addition to the exogenous stochastic retirement (e.g., sudden death) of the current manager. Then it can be shown that the downward movement in the value of the firm will not last as long as upward movement on average, because the firm will fire the current manager as soon as it finds him not worth continuing. The value of the firm more frequently deviates upward from the present value of current net cash flow than downward, and that the value of the firm tends to increase over time because of the truncation of relatively inferior managers. A similar story may be applied to replacement of investment projects rather than managers.[1]

There have been previous attempts to apply stochastic regime shifts and learning to explain the abnormal behavior of asset prices. For example, Flood and Garber [1980a] develop a model with a regime switch to explain the movement of the price level; Flood and Garber [1980b], Hamilton and Whiteman [1985] and Flood and Hodrick [1986] discuss identification and the observational similarity between a bubble and fundamentals when the underlying process of fundamentals may change in future. Also, Wang [1989] develops a continuous-time dynamic asset pricing model with informed investors, uninformed investors and noise traders, showing that the uninformed investors' learning about the gradually changing trend of the dividends increases volatility and risk-premium of the stock returns, and Barsky and DeLong [1989] empirically examines how much learning explains volatility of stock prices. In the literature of foreign exchange rates, the "peso problem" is extensively analyzed, where the seeming deviation of the exchange rate from fundamentals can be explained by the anticipation of a regime shift in future. Lewis [1989a,b] and Tabellini [1988], for example, use learning to explain the apparent deviation and volatility.[2] The value-added in the present paper is analyzing the time series properties of the value of a firm under recurrent shifts in the trend of the net cash flow rather than a once-and-for-all shift and, also, to explain the shifts that are partly endogenous.

Section II develops the basic model where replacement of managers and shifts in trend are exogenous. Section III compares with the alternative specifications, one without learning and the other misspecified so that shifts are ignored. Section IV extends the basic model to allow for endogenous replacement of managers by the firm.

II. Model

Economy consists of a representative agent and many firms producing a homogeneous product. Consider the net cash flow from the firm's production activity as sales of the product minus costs of employing factors of production and purchasing investment goods. Suppose for now that the net cash flow y_t follows the exogenous process

(1) $$y_t = y_{t-1} + x_t$$

(2) $$x_t = \mu_t + \zeta_t$$

(3) $$\mu_t = (1-\delta_t)\mu_{t-1} + \delta_t\varepsilon_t$$

where ζ_t, ε_t and δ_t are mutually independent, and identically and independently distributed over time. ζ_t and ε_t are normally distributed with zero mean and variances σ_y^2 and σ_μ^2. The value of δ_t is equal to 0 with probability π and equal to 1 with probability $1-\pi$. For instance, imagine that the change in the net cash flow of the firm depends on uncertain quality of the manager (trend component) μ_t and noise ζ_t.[3] If the quality of the manger is good, μ_t is positive and the net cash flow tends to increase, while the cash flow tends to decrease with a bad manger. With probability π, the previous manager stays (δ_t=0) and μ_t is unchanged, while with probability $1-\pi$, the manager is replaced (δ_t=1) and new random μ_t is realized. Although later we are going to analyze how the firm replaces unsuccessful managers, we assume for now that the replacement of the manager is exogenous and that the probability of the replacement is independent of the history of the performance.

The representative agent is risk-neutral with a constant discount factor β.

He observes the net cash flow and whether the manager is replaced or not, but does not observe the quality of managers nor the noise. In other words, he observes the history of y_t, x_t and δ_t, but not μ_t, ζ_t nor ε_t. In fact, we assume the only information the agent has is the history of y_t, x_t and δ_t, and general knowledge about the structure of the economy. There is no tax and all the values are measured in terms of the homogeneous product. Define the value of the firm's productive capital as the sum of the value of equity and debt net of the value of monetary assets held by the firm. Under rational expectations, the value of the firm P_t at the beginning of period t satisfies

$$\text{(4a)} \qquad P_t = y_t + \beta E_t[P_{t+1}] = E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau y_{t+\tau}\right] + b_t$$

$$\text{(4b)} \qquad b_t = \beta E_t[b_{t+1}]$$

where $E_t[\cdot]$ is the conditional expectations operator, conditional upon information at the period t, which includes the cash flow in period t, y_t and whether a shift occurs or not, δ_t.[4] The first term in the right hand side of equation (4a) is the expected present value of the future net cash flow which is often called fundamentals. The second term b_t represents the effect of extrinsic expectations about future value of the firm, called a speculative bubble.

In order to concentrate on the fundamentals, we assume there is no speculative bubble:

$$\text{(5)} \qquad b_t = 0, \quad \text{for all t.}$$

Then, because $E_t(x_{t+\tau}) = E_t(\mu_{t+\tau}) = \pi^\tau E_t(\mu_t)$, and $E_t(y_{t+\tau}) = y_t + E_t[\Sigma_{j=1}^{\tau} x_{t+j}]$, we have

$$(6) \qquad P_t = (1-\beta)^{-1}y_t + \frac{\beta\pi}{(1-\beta)(1-\beta\pi)} m_t,$$

where $m_t \equiv E_t(\mu_t)$. The first term is the present value of the net cash flow without the trend, and the second term is the effect of expected quality of the manager.

In order to formulate expectations about the quality of the manager, we assume the agent is a Bayesian learner.[5] Suppose that the current manager came to the firm at period s, while the current period is t, where $s \le t$. Since there was no replacement of the manager between period s and period t, the agent's prior probability density function of μ_t is the same as μ_s, which is also the same as ε_s

$$(7) \qquad \xi(\mu_t) \propto \exp\,[-(\alpha_\mu/2)\,\mu_t^2],$$

where $\alpha_\mu = 1/\sigma_\mu^2$, and $\propto$ means that the both sides are proportional. The likelihood function f of the changes in the net cash flow under the current manager $(x_s, x_{s+1}, \ldots, x_t)$ is

$$(8) \quad f(x_s,\ldots,x_t|\mu_t) \propto \exp\left\{ -(\alpha_y/2)\left[n_t(\bar{x}_t-\mu_t)^2 + \sum_{\tau=s}^{t} (x_\tau-\bar{x}_t)^2 \right] \right\}$$

where $n_t = t-s+1$, $\alpha_y = 1/\sigma_y^2$ and $\bar{x}_t = (1/n_t)\,(\Sigma_{\tau=s}^{t}\, x_\tau)$. It then follows that the posterior probability density function for μ_t given the changes in the net cash flow $(x_s,\ldots,x_t)$ is proportional to (7) times (8) as

$$(9) \quad \xi(\mu_t|x_s,\ldots,x_t) \propto \exp\left[-[(\alpha_\mu+n_t\alpha_y)/2]\left(\mu_t - (n_t\alpha_y\bar{x}_t)/(\alpha_\mu+n_t\alpha_y) \right)^2 \right]$$

Therefore the posterior distribution of μ_t is normal with mean m_t and variance

$(\alpha_\mu + n_t\alpha_y)^{-1}$, where

$$(10) \quad m_t \equiv E_t[\mu_t] = (n_t\alpha_y\bar{x}_t)/(\alpha_\mu + n_t\alpha_y) = \alpha_y(y_t - y_{t-n_t})/(\alpha_\mu + n_t\alpha_y).$$

Note that n_t, y_t and y_{t-n_t} are the only information necessary for the distribution of manager's quality μ_t and future net cash flow $y_{t+\tau}$ conditional on the information in period t.

This yields an explicit expression for the value of the firm at period t

$$(11) \qquad P_t = (1-\beta)^{-1}y_t + \frac{\beta\pi\alpha_y}{(1-\beta)(1-\beta\pi)(\alpha_\mu + n_t\alpha_y)}(y_t - y_{t-n_t}).$$

The first term may be interpreted as the present value of an infinite stream of the current net cash flow from production. The second term reflects the learning about the current manager. Figure 1 illustrates the behavior of the value of the firm in this model. When the underlying unobserved quality of manager is good and μ_t is positive, not only the first term tends to increase, but also the second term tends to increase over time because the agent tends to raise the assessment of the quality of the current manager with more favorable observations.[6] The second term appears to behave like a bubble, because the absolute value of the second term tends to increase over time and then suddenly crashes. Both terms of (11) are, however, fundamentals in the sense that they come from intrinsic expectations of the future net cash flow from production, rather than extrinsic beliefs about the value of the firm in future.[7]

We also know from (11) that the sensitivity of P_t to $y_t - y_{t-n_t}$ increases with $\alpha_y/\alpha_\mu = \sigma_\mu^2/\sigma_y^2$ and π. If change in the net cash flow x_t reflects more managerial quality than noise, so that α_y/α_μ is large, then P_t responds substantially to a change in the net cash flow, because such changes are very informative for guessing the underlying managerial quality. If manager is replaced infrequently, so that π is close to unity, then the current manager is more likely to stay and has more impact on the expected present value of the future net cash flow.

III. Comparison to Two Alternatives

In order to evaluate the effects of learning, consider the case in which the quality of manager μ_t is directly observable. Then the value of the firm is

$$(12) \qquad P_t = (1-\beta)^{-1} y_t + \frac{\beta\pi}{(1-\beta)(1-\beta\pi)} \mu_t .$$

Since the second term is constant under the same manager, it does not look like a bubble. Recurring trend shifts with replacement of managers may increase the volatility of the value of the firm, and the firm value will deviate from the present value of current net cash flow. The deviation, however, will not tend to grow over time and then crash, unless we include learning about the unknown managerial quality, as in (11). (See Figure 2.)

The second alternative is when the true process of the unobserved quality of the manager is (3), but a researcher misspecifies the managerial quality to follow the AR(1) process:

$$(13) \qquad \mu_t = \pi \mu_{t-1} + \varepsilon'_t ,$$

where ε'_t is distributed normally, independently, and identically with mean zero and the variance $(1-\pi^2)\sigma^2_\mu$. The variance and the auto-covariances of the change in the net cash flow x_t are identical for the true system (1,2,3) and the misspecified system (1,2,13):

$$(14) \quad Var(x_t) = \sigma_\mu^2 + \sigma_y^2, \text{ and } Cov(x_t, x_{t+\tau}) = \pi^{|\tau|} \sigma_\mu^2, \text{ for } \tau \neq 0.$$

However, the Bayesian learning process for the unknown trend in the misspecified system is very different from that in the true system. The process of $m_t = E_t[\mu_t]$ can be represented by Kalman filtering

$$(15) \qquad m_t = \pi m_{t-1} + \gamma (x_t - \pi m_{t-1})$$

where (γ, Σ) satisfy $\gamma = \Sigma/(\Sigma+\sigma_y^2)$ and $\Sigma = \pi^2[(1-\gamma)^2\Sigma+\gamma^2\sigma_y^2]+(1-\pi^2)\sigma_\mu^2$. We can think of Σ as the variance of μ_t conditional on time t-1 information, and γ as the weight of the unanticipated change in the net cash flow in revising the expectation of the trend. Then the implied process of the value of the firm deviates systematically from the present value of current net cash flow, but the deviation will not suddenly crash as in (11). See Figure 3 for the comparison. Therefore the researcher who specifies the trend to be AR(1) may find it difficult to explain bubble-like volatile movements in the value of the firm, since the variance-covariance of changes of net cash flow does not reveal the true structure of the net cash flow. If changes in the trend of the net cash flow are not directly observable, the system might become somewhat in between the basic model (11) and the model (6,15), but this is a topic for future investigation.

IV. Replacing Managers.

One difficulty with our basic model is that it predicts both positive and negative deviations of the firm value from the present value of the current net cash flow. But observations about the value of firms appear to suggest more frequent upward deviations followed by crashes than downward deviations followed by sudden upward jumps. Thus, we extend the basic model to allow the firm to shift the trend of the net cash flow by replacing managers endogenously[8].

Suppose that the firm decides whether to keep the current manager or fire him at the end of each period. Even if the firm decides to keep the manager, the manager retires (or dies) with probability $1-\pi$ each period. After the manager either is fired or dies, the firm hires a new manager with a one-time adjustment cost k in the beginning of the following period. Here we assume for simplicity that the adjustment cost does not depend on whether the previous manager was fired or died. All new managers look identical in ex ante, but in ex post the trend of the net cash flow is different depending on a match between his skill and the firm's environment[9]. The net cash flow, apart from the adjustment cost, is characterized by equations (1,2,3). The difference from the previous model is that the manager's quality changes ($\delta_t=1$) when the previous manager is fired in addition to a period he dies exogenously. Also we assume all firms are free to replace managers but that the adjustment cost of a firm increases with the number of firms who replace managers in the economy.

In the steady state, the value of the firm depends on the number of periods the current manager has managed the firm (the age) n_t, the expected quality of the current manager m_t, and current net cash flow y_t. It satisfies Bellman's equation

$$(16)\quad P(n_t,m_t,y_t) \;=\; y_t + \beta\,\mathrm{Max}\left\{ E_t[P(1,m_{t+1},y_{t+1})|\mathrm{new}]-k,\; \pi E_t[P(n_t+1,m_{t+1},y_{t+1})|\mathrm{old}] + (1-\pi)\Big(E_t[P(1,m_{t+1},y_{t+1})|\mathrm{new}]-k\Big) \right\}$$

where $E_t[x|\mathrm{new}]$ means the expectation of x conditional on the information in period t with a new manager, while $E_t[x|\mathrm{old}]$ is the expectation conditional on the current manager continuing. Equation (16) says that the value of the firm at the beginning of period t is equal to the current net cash flow plus the discounted maximized expected value of the firm in the next period with or without replacement. The first term in the braces is the value with firing the current manager and recruiting a new one. The second term is without firing the manager, where with probability π the current manager continues to manage and gets older by one period, and with probability $1-\pi$ he dies and is replaced exogenously.

Define $J_n(m) = P(n,m,y)-(1-\beta)^{-1}y$. It measures the deviation of the value of the firm from the present value of the current net cash flow, when the manager is n periods old and has the expected quality m. Then the assumption of free trial of new manager with the upward-sloping adjustment cost function implies that the adjustment cost in equilibrium is equal to the value-added of introducing a new manager

$$(17)\qquad k \;=\; E\,[J_1(m)] \;=\; \int_{-\infty}^{\infty} J_1[(1+\omega)^{-1/2}\sigma_\mu z]\; d\Phi(z)$$

where $\omega = \sigma_y^2/\sigma_\mu^2$ and $\Phi(z)$ is the distribution function of the standard normal distribution. The last equality comes from the learning equation (10) with $n_t=1$. Then Bellman's equation becomes

$$(18)\quad J_n(m) = \text{Max}\left\{ 0,\ \beta\pi\, E_t\left((1-\beta)^{-1}x_{t+1} + J_{n+1}(m_{t+1})\,|\,\text{old}\right) \right\}$$

$$= \text{Max}\left\{ 0,\ \beta\pi\left((1-\beta)^{-1}m + \int_{-\infty}^{\infty} J_{n+1}[m + \sqrt{\omega/((\omega+n)(\omega+n+1))}\ \sigma_\mu z]d\Phi(z)\right) \right\}$$

The second equality comes from the fact that when there is no change in the manager, $m_{t+1} = m_t + \alpha_y(x_{t+1}-m_t)/[\alpha_\mu+(n_t+1)\alpha_y]$ by (10).

The value of the firm in period t with an n_t periods old manager with expected quality m_t and current net cash flow y_t is

$$(19)\qquad P_t = P(n_t, m_t, y_t) = (1-\beta)^{-1}y_t + J_{n_t}(m_t),$$

From (18), we know $J_n(m)$ is a non-decreasing continuous function of m, and non-negative because the firm always has a choice of replacement.[10] Thus the value of the firm is never below the present value of the current net cash flow. Also, from (17), the value of the firm immediately after the replacement of the project is higher than the present value of the current net cash flow by the magnitude k on average.[11] Therefore, the magnitudes of upward jumps in the value of the firm tends to be smaller than downward crashes caused by the exogenous death of a previously successful manager.

Figure 4 illustrates the value of the firm with endogenous replacement of managers. When the net cash flow tends to be increasing with a good manager, the value of the firm tends to increase more rapidly than the present value of the current net cash flow with the appreciation of the manager. The firm chooses to keep him until he dies, and the death of the successful manager causes a drop in the value of the firm. When the net cash flow is decreasing with either a bad manager or a bad luck, the firm will keep the manager as long as the option value ($J_n(m)$) of the current manager is still positive.

But when the evidence becomes unfavorable enough so that the option value becomes no longer positive, the firm will decide to replace him for a new manager. Since bad managers are more frequently fired before die than good managers, good managers tend to stay longer than bad ones, and the value of the firm tends to increase over time.

VI. Conclusion

In this paper we study theoretically conditions under which the value of the firm is not only volatile, but also sometimes increases more rapidly than the underlying net cash flow from production for a while then crashes. We found that the value of the firm appears to contain a bubble when the trend in net cash flow occasionally shifts but is not directly observable so that an agent learns only gradually. One obvious question is how much this type of model can explain empirically aspects of data which are difficult to explain with existing models. We have to overcome a number of problems for such empirical implementations. First, here, we study the relationship between the value of the firm and the net cash flow from production, because our environment incorporates the Modigliani-Miller theorem, which implies the dividend process and the debt-equity ratio are indeterminate. But data on the net cash flows and the value of the firm are inferior relative to data on dividends and equity prices. Second, we should study the relation between the aggregate and the individual values of firms in order to explain fluctuations and growth of the aggregate value of firms. Our hope is that the results in this paper may convince the reader that models like this and its potential generalizations are worth pursuing.

1. Rogoff [1990] independently develops a similar model to analyze political business cycle, i.e., replacement of manager of a country.

2. Froot and Obstfeld [1989] study a model where a stock price bubble depends on the dividend in a nonlinear way. Although our model has similar aspects, it explains where the deviation comes from and how it is related to learning. Also, Hamilton [1988,89] develops an econometric model when researchers do not directly observe the shifts in trend of aggregate variables.

3. In some literature, "trend" includes all of y_t, i.e., the effects of both μ_t and ζ_t. But here, we call only μ_t "trend" or "quality of manager", and call ζ_t "noise".

4. This relationship is independent of the debt-equity ratio and dividend policy by the Modigliani-Miller theorem, and can be explained as follows. Assume that the firm issues one type of equity and a one-period riskless discounted bonds. The value of the firm at the beginning of the period is $P_t = p_{St}S_t + B_t$, where p_{St} and S_t are the price and amount of equity at the beginning of period t, and B_t is the value of the firm's net bond position (i.e., the value of the firm's bonds minus bonds of other agents that it holds). The net cash flow from production y_t is related to financial transactions as follows

$$y_t - d_tS_t + (p_{St}-d_t)(S_{t+1}-S_t) + (p_{Bt}B_{t+1} - B_t) = 0,$$

where d_t is the dividend per share and p_{Bt} is the price of the one-period discount bond which pays one unit of output at the beginning of period t+1. This equation says that the excess of dividend payments over the net cash flow from production is financed by either new equity issues at the post-dividend price (the third term) and bond issues net of the bond repayment (the fourth term). Expected utility maximization of the risk-neutral agent under rational expectations implies $p_{St} = d_t+\beta E_t[p_{St+1}]$ and $p_{Bt} = \beta$. Applying these and the definition of the value of the firm to the cash flow constraint, we get the expression in the text. An unfortunate feature of our model, shared with some other literature, is that the value of the firm is negative with positive probability. But we ignore this for now until Section III, where the endogenous replacement virtually overcomes this problem.

5. See DeGroot [1970] for a detailed discussion on Bayesian learning.

6. When the underlying trend μ_t is positive from period $t-n_t+1$, the term $y_t-y_{t-n_t}$ on average grows proportionally with n_t, while the denominator grows less than proportionally with n_t.

7. Equation (11) might provide a rational to technical analysis, although usually shifts in trend are not directly observable in technical analysis.

8. See Jovanovic [1982] and Pakes-Ericson [1988] for the relationship between learning and exits of firms. The subsequent analysis builds from their formulation.

9. Although a good match between a manager and a firm may provide bargaining power to the good manager, we ignore the effect of the manager's compensation on the net cash flow.

10. The firm's choice of whether to continue or intentionally replace the

current manager, can be characterized by the reservation m strategy: continue an n periods old manager if $m \geq \hat{m}(n)$, and fire the manager if $m < \hat{m}(n)$, where $\hat{m}(n)$ solves

$$0 = \hat{m}/(1-\beta) + \int_{-\infty}^{\infty} J_{n+1}[\hat{m} + \sqrt{\omega/((\omega+n)(\omega+n+1))}\, \sigma_{\mu} z] d\Phi(z)$$

We hope to show that $J_n(m) \geq J_{n+1}(m)$ and $\hat{m}(n) \leq \hat{m}(n+1) \leq 0$. In other words, the option value of the current manager is an increasing function of the conditional variance of the expected quality, and thus a decreasing function of the age of the manager. Therefore, the reservation value of m for replacing the manager is non-decreasing with the age of the manager. Another issue of interest is how the slope of the adjustment cost function affects the growth in the long run. If the economy has a limited capacity to adjust so that the adjustment cost quickly becomes large even when a small number of firms replacing the managers, then only a small number of firms replace the managers in equilibrium and average growth will be low.

11. With endogenous replacement of managers, the average increase in the value of the firm is k, but the average equity price change is zero. This is because the value of an additional issue of bonds or equities is equal to the adjustment cost by the cash flow constraint. For example, if the firm issues debts to finance the adjustment cost k, the value of the firm (debts and equities) will increase by k on average, while the equity price will not.

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Figure 1 Basic Model

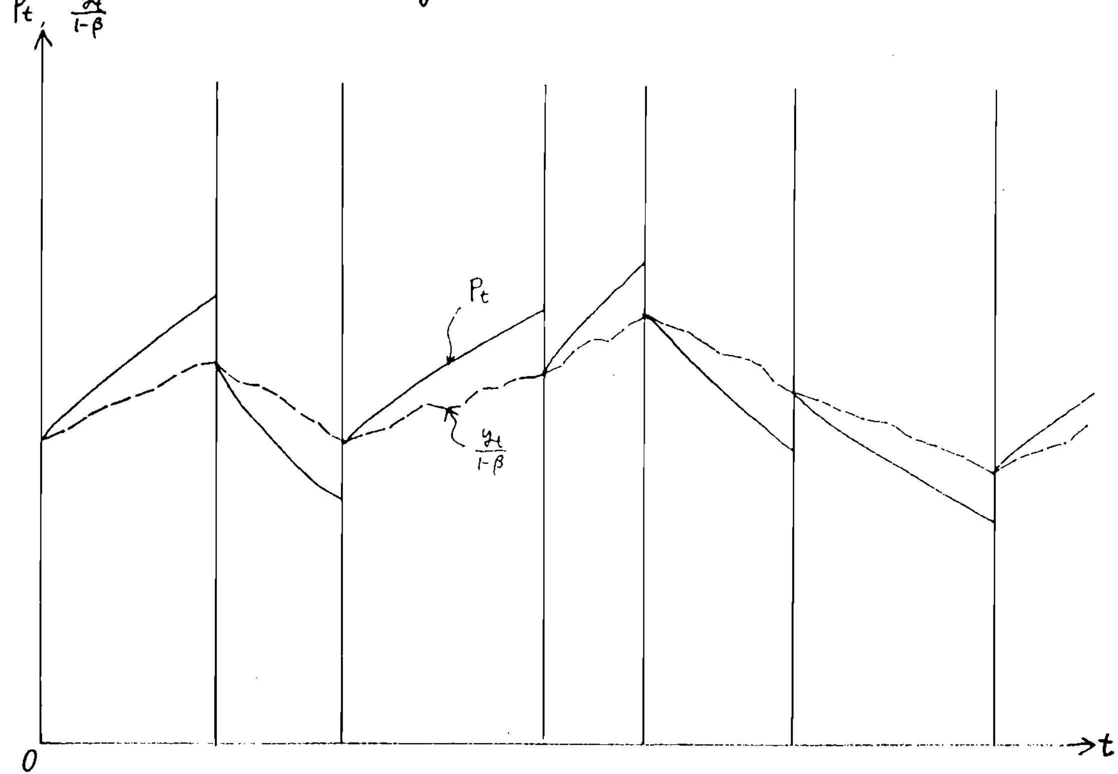

Figure 2 without learning

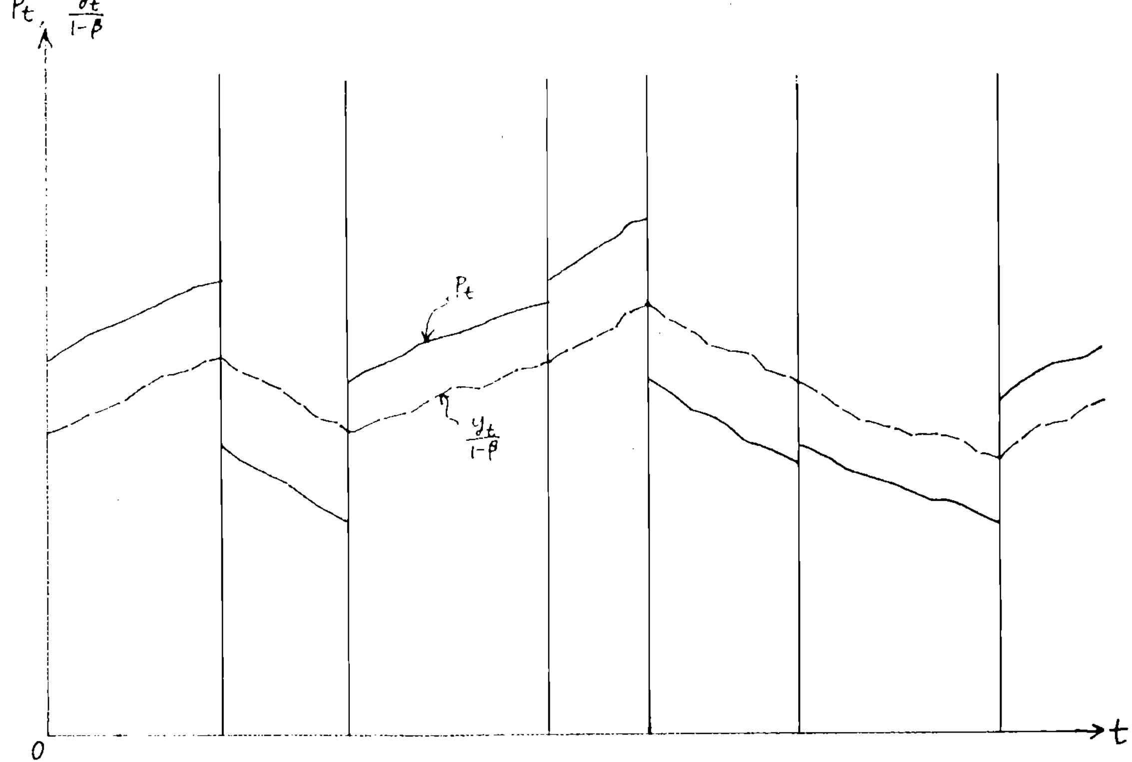

Figure 3 AR(1)

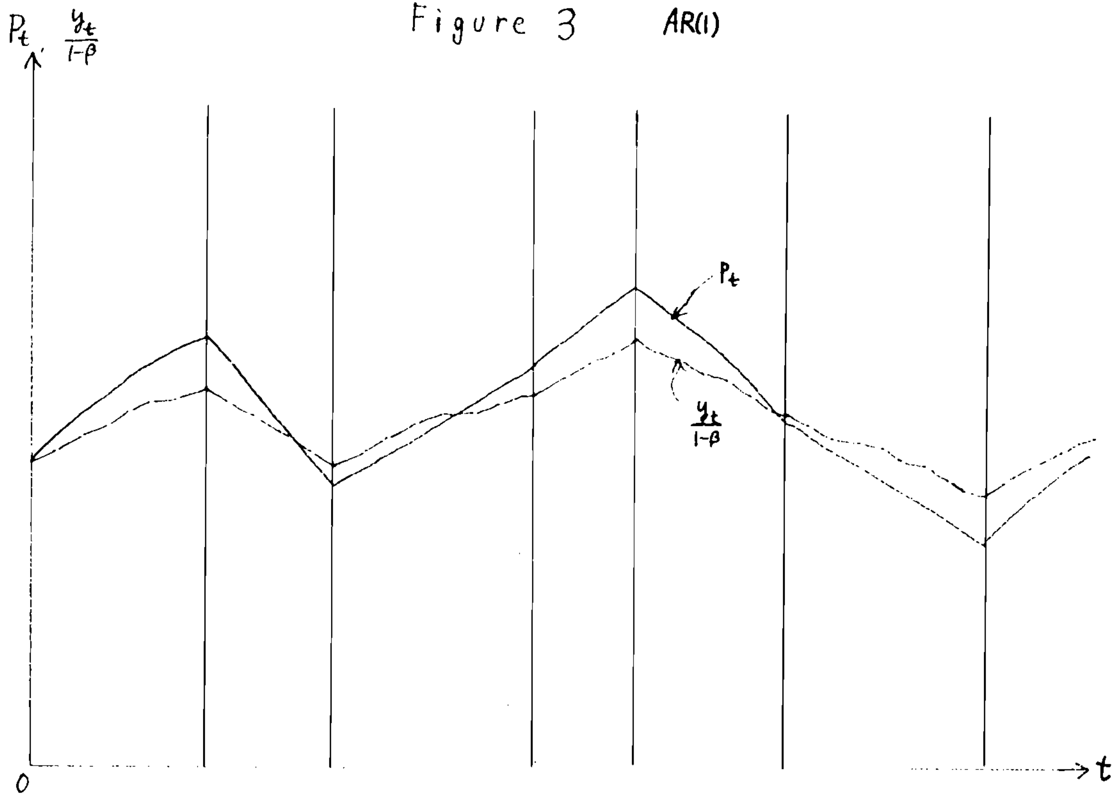

Figure 4 With Firing