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ABSTRACT

Ongoing international financial integration has greatly increased foreign asset holdings across countries, enhancing the scope for a "valuation channel" of external adjustment (i.e., the changes in a country's net foreign asset position due to exchange rate and asset price changes). We examine this channel of adjustment in a dynamic stochastic general equilibrium model with international equity trading in incomplete asset markets. We show that the risk-sharing properties of international equity trading are tied to the distribution of income between labor income and profits when equities are defined as claims to firm profits in a production economy. For a given level of international financial integration (measured by the size of gross foreign asset positions), the quantitative importance of the valuation channel of external adjustment depends on features of the international transmission mechanism such as the size of financial frictions, substitutability across goods, and the persistence of shocks. Finally, moving from less to more international financial integration, risk sharing through asset markets increases, and valuation changes are larger, but their relative importance in net foreign asset dynamics is smaller.

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1 Introduction

The cumulative total of the U.S. current account deficits between 1990 and 2004 would have been sufficient to increase net foreign liabilities to 33 percent of GDP. Yet, U.S. net foreign liabilities increased to only about 23 percent of GDP over the same period (Figure 1). The difference is due to valuation effects, namely, the effects of asset price and exchange rate movements on the stock of gross assets and liabilities of the United States and the rest of the world. The experience of the United States shows that external adjustment – i.e., changes in a country's net foreign asset position – can take place not only through changes in quantity and price of goods and services, but also through changes in asset prices, as argued by Gourinchas and Rey (2005), Obstfeld (2004), and Lane and Milesi-Ferretti (2006).

This paper explores the valuation channel of external adjustment in a two-country dynamic stochastic general equilibrium (DSGE) model with international equity trading. Specifically, we study the determinants of the valuation channel and its relative importance in external adjustment, and we illustrate its working. In the process, we explore the risk sharing implications of international trade in equity.

We introduce two-way international equity trading in an otherwise standard two-country, DSGE model with monopolistic competition and incomplete asset markets. To focus on the consumption and equity holding behavior of households, we consider a very simple production structure. Output is produced using only labor, subject to country-wide productivity shocks, and the labor supply is inelastic. However, product differentiation across countries ensures that the consumption value of a country's output depends on its relative price, which is endogenous to the conditions of the economy. Monopolistic competition, based on product differentiation within countries, generates non-zero profits and firm values, which are essential for the asset dynamics we focus on.

Markets are incomplete across countries. Specifically, we assume that households can hold non-contingent bonds and shares in firms, but only the latter are traded internationally. Thus, unlike most of the literature, we focus on equity rather than bond trading as the key mechanism for international consumption smoothing and risk sharing. Equity trades are subject to convex financial intermediation costs. As in models with bond trading, these costs ensure uniqueness of the deterministic steady state and stationary responses to temporary shocks.[1] The structure of costs enables us to determine endogenously the international distribution of wealth and the composition of country equity holdings in and outside the steady state. This is a convenient feature of our model that we exploit in calibration exercises.

[1] As Schmitt-Grohé and Uribe (2003) point out, absent any stationarity-inducing device, once the model is log-linearized, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded.

Several qualitative results are unaffected if the costs are removed. Importantly, even in the absence of financial frictions, decentralized international trade in shares does not yield perfect consumption risk sharing when the economy starts from a steady-state equity allocation that differs from the social planner's optimum. The reason is that international sharing is limited to the profit fraction of income, while labor cannot move across countries in response to shocks. So the extent of risk sharing in our model is crucially affected by the distribution of income between profits and labor income, which, in turn, is determined by the substitutability across individual product varieties in consumer preferences. A world social planner would select the equity allocation – conditional on income distribution – that spans all uncertainty and achieves perfect risk sharing with constant equity holdings, as in Heathcote and Perri (2004) and references therein. But this equity allocation is only one of infinitely many possible steady-state market allocations in the absence of financial fees, and it has the empirically unappealing consequence that valuation changes become uniquely responsible for changes in net foreign assets.[2]

Our main results on the role of the valuation channel of adjustment are as follows. In our theoretical analysis, we confirm that net foreign asset dynamics comprise both movements in the current account and valuation changes. The magnitude and significance of the valuation channel depend on initial asset positions and features of the economy such as the magnitude of financial frictions, substitutability across goods, and the persistence of shocks. The degree of substitutability between home and foreign goods, however, has no effect on the relative share of valuation change and the current account in net foreign asset changes. Importantly, although changes in asset valuation and the current account constitute two different channels of shock transmission to net foreign assets, the role of equity markets for consumption dynamics is fully summarized by changes in asset quantities. According to our quantitative analysis, moving from less to more international financial integration – i.e., moving from a steady state with home bias in equity to one with larger, fully symmetric gross foreign asset positions – risk sharing through asset markets increases, and valuation changes are larger, but their relative importance in net foreign asset dynamics is smaller. Larger trade imbalances and a smaller contribution of the income balance are more important determinants of net foreign asset dynamics with more integrated, but still incomplete, international asset markets.

Our work is closely related to that of Kim (2002); Tille (2005); Blanchard, Giavazzi, and Sa (2005); and Devereux and Saito (2005). All these contributions focus on the role of the exchange rate in the valuation channel. We focus on the role of equity return differentials.[3] Unlike Tille (2005), we study also situations in which steady-state net foreign assets differ from zero. Blanchard

[2] We show that properly designed financial fees can decentralize the planner's allocation as unique market outcome.

[3] Tille (2005) documents that return differentials other than exchange rate movements are quantitatively at least as important as the latter for valuation effects, and they are more important in 8 of the last 15 years.

et al (2005) set up a traditional portfolio balance model with imperfect asset substitutability along the lines of Kouri (1982) and discuss valuation effects caused by exchange rate movements. We develop a general equilibrium model which investigates the interaction between valuation driven by equity prices, equities as claims to firm profits, and the transmission of shocks via the terms of trade. Kim (2002) focuses on the consequences of revaluation of nominal asset prices, while changes in nominal prices have no real effect in our model. Devereux and Saito (2005) build a finance model with no production.

Emphasis on macroeconomic dynamics also distinguishes our model from finance models of international equity trading, such as Adler and Dumas (1983). Pavlova and Rigobon (2003) give more prominence to macroeconomic dynamics, but without modeling firm decisions explicitly. Our model differs from earlier DSGE models of international real business cycles (RBCs) in that we incorporate differentiated goods and monopolistic competition to have positive prices of shares in firm profits. Moreover, many international RBC studies do not model equity trading by focusing on complete Arrow-Debreu (or Arrow) asset markets that make all other assets redundant (for instance, Backus, Kehoe, and Kydland, 1992).

Several other papers aim at explaining home bias in portfolios or international financial integration by allowing for international equity trading in standard international macroeconomic models (Engel and Matsumoto, 2005; Heathcote and Perri, 2004; and Kollmann, 2005), whereas we focus on the role of valuation in net foreign asset adjustment.[4] For simplicity, we solve our model through standard log-linearization, but another strand of literature studies alternative solution techniques for incomplete markets models with multiple assets (Devereux and Sutherland, 2006, Evans and Hnatkovska, 2006, and Tille and van Wincoop, 2006).[5] While the focus of this paper and most related literature is positive, Benigno (2006) provides a normative analysis of valuation effects and their consequences for optimal monetary policy, and Ghironi and Lee (2006) explore the role of monetary policy in a sticky-price version of the model developed here.[6]

The rest of the paper is organized as follows. Section 2 spells out the structure of the model. Section 3 studies the valuation channel in two cases that can be solved analytically in log-linear form. Section 4 explores the risk sharing implications of trade in equity. Section 5 illustrates the working of the valuation channel by means of numerical examples and discusses the quantitative performance of the model. Section 6 concludes. Technical details are in appendixes.

[4] Kollmann (2005) studies changes in foreign equity holdings in an effectively complete market, in which equities are defined as claims to country-specific endowments of goods. He finds that changes in net holdings of foreign equity are a key source of current account movements.

[5] The impulse responses generated by solving our model via second-order approximation are not significantly different from those we report.

[6] See also Obstfeld (2006) for a survey and discussion of recent literature on international risk sharing and portfolio models.

2 The Model

There are two countries, home and foreign, populated by infinitely lived, atomistic households. World population equals the continuum $[0, 1]$. Home households are indexed by $j \in [0, a)$; foreign households are indexed by $j^* \in [a, 1]$.

There is a continuum of monopolistically competitive firms on $[0, 1]$, each producing a differentiated good. Home firms are indexed by $z \in [0, a)$; foreign firms are indexed by $z^* \in [a, 1]$.

2.1 Households

The representative home household j supplies one unit of labor inelastically in each period in a competitive home labor market for the nominal wage rate W_t. As customary, we denote consumption with C and the consumer price index (CPI) with P. Money serves the sole role of unit of account, since we assume that prices and wages are flexible. Therefore, we adopt a cashless specification following Woodford (2003).

The preferences of the representative home household j are:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(C_s^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{1}$$

with $0 < \beta < 1$ and $\sigma > 0$. The representative foreign household j^* maximizes a similar utility function and supplies one unit of labor inelastically in each period in a competitive foreign labor market.

The consumption basket C aggregates sub-baskets of individual home and foreign goods in a CES fashion:

$$C_t^j = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^j\right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^j\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{2}$$

where $\omega > 0$ is the elasticity of substitution between home and foreign goods. The consumption sub-baskets C_H and C_F aggregate individual home and foreign goods, respectively, in a Dixit-Stiglitz fashion with elasticity of substitution $\theta > 1$:

$$C_{Ht}^j = \left[\left(\frac{1}{a}\right)^{\frac{1}{\theta}} \int_0^a \left(c_t^j(z)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \quad C_{Ft}^j = \left[\left(\frac{1}{1-a}\right)^{\frac{1}{\theta}} \int_a^1 \left(c_t^j(z^*)\right)^{\frac{\theta-1}{\theta}} dz^*\right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

This structure of consumption preferences implies:

$$P_t = \left[aP_{Ht}^{1-\omega} + (1-a)P_{Ft}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

where P_H (P_F) is the price sub-index for home (foreign)-produced goods – both expressed in units of the home currency. Letting $p_t(z)$ be the home currency price of good z, we have:

$$P_{Ht} = \left(\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}, \quad P_{Ft} = \left(\frac{1}{1-a} \int_a^1 p_t(z^*)^{1-\theta} dz^* \right)^{\frac{1}{1-\theta}}.$$

We assume that the law of one price holds for each individual good: $p_t(z) = \mathcal{E}_t p_t^*(z)$, where $\mathcal{E}_t$ is the nominal exchange rate (the domestic-currency price of a unit of foreign currency) and $p_t^*(z)$ is the foreign currency price of good z.

Consumption preferences are identical across countries. This assumption and the law of one price imply that consumption-based PPP holds: $P_t = \mathcal{E}_t P_t^*$, where P_t^* is the foreign CPI.

Agents in each country can hold domestic, non-contingent bonds denominated in units of the domestic currency, and shares in domestic and foreign firms. Different from most literature, we assume that shares, and not bonds, are traded across countries for international risk sharing and consumption smoothing purposes.

Omitting identifiers for households, firms, and countries, we use x_{t+1} to denote holdings of shares in firms entering period $t+1$, V_t to denote the nominal price of shares during period t, D_t to denote nominal dividends, and B_{t+1} to denote nominal bond holdings entering period $t+1$. Households pay quadratic financial transaction fees to domestic financial intermediaries when they hold share positions that differ from zero. As in models with international trading in uncontingent bonds, these fees are a technical device to pin down equity holdings in the deterministic steady state and ensure reversion to this position after temporary shocks.[7] Table 1 summarizes the details of our notation when agent and country identifiers are taken into account. The budget constraint of home household j is:

$$\begin{aligned} & B_{t+1}^j + \int_0^a V_t^z x_{t+1}^{zj} dz + \mathcal{E}_t \int_a^1 V_t^{z^*} x_{t+1}^{z^*j} dz^* + \\ & + \frac{\gamma_x}{2} \int_0^a V_t^z \left(x_{t+1}^{zj} \right)^2 dz + \mathcal{E}_t \frac{\gamma_{x^*}}{2} \int_a^1 V_t^{z^*} \left(x_{t+1}^{z^*j} \right)^2 dz^* + P_t C_t^j \\ = \; & (1+i_t) B_t^j + \int_0^a (V_t^z + D_t^z) x_t^{zj} dz + \mathcal{E}_t \int_a^1 \left(V_t^{z^*} + D_t^{z^*} \right) x_t^{z^*j} dz^* + W_t + P_t T_t^j, \end{aligned} \tag{4}$$

where i_{t+1} is the nominal interest rate on holdings of bonds between t and $t+1$, T_t^j is a lump-sum transfer from financial intermediaries, and the γ's are positive parameters.

[7] Cooley and Quadrini (1999) model limited participation in financial markets by assuming similar transaction costs. Since aggregate bond holdings are zero in equilibrium in each country, financial fees on bond transactions are omitted without loss of generality. Bodenstein (2006) argues that costs of adjusting asset holdings of the form we employ (and other techniques explored in the literature) fail to pin down a unique steady state unless substitutability between home and foreign goods in consumption is sufficiently low. The specification of consumption baskets in our model (with share parameters a and $1-a$) ensures that there is a unique steady state regardless of the value of ω.

The foreign household's budget constraint is similar. We allow the scaling parameters of financial fees (γ_x, γ_{x^*}, γ_x^*, $\gamma_{x^*}^*$) to differ across countries and across assets. This has implications for the steady state of the model, which we exploit in analytical and numerical exercises below. The financial transaction fees in the budget constraint are rebated to households in equilibrium.[8] Thus, the lump-sum rebate of financial intermediation fees to household j is:

$$T_t^j = \frac{\gamma_x}{2} \int_0^a \frac{V_t^z}{P_t} \left(x_{t+1}^{zj} \right)^2 dz + \frac{\gamma_{x^*}}{2} \int_a^1 \frac{\mathcal{E}_t V_t^{z^*}}{P_t} \left(x_{t+1}^{z^*j} \right)^2 dz^*. \tag{5}$$

First-Order Conditions Home household j maximizes (1) subject to (4) taking the financial fee transfer as given. The first-order conditions with respect to B_{t+1}^j (the domestic bond), x_{t+1}^{zj} (share of home firm), and $x_{t+1}^{z^*j}$ (share of foreign firm) are, respectively:

$$\left(C_t^j \right)^{-\frac{1}{\sigma}} = \beta \left(1 + i_{t+1} \right) E_t \left[\frac{P_t}{P_{t+1}} \left(C_{t+1}^j \right)^{-\frac{1}{\sigma}} \right], \tag{6}$$

$$\left(C_t^j \right)^{-\frac{1}{\sigma}} V_t^z \left(1 + \gamma_x x_{t+1}^{zj} \right) = \beta E_t \left[\left(C_{t+1}^j \right)^{-\frac{1}{\sigma}} \left(V_{t+1}^z + D_{t+1}^z \right) \frac{P_t}{P_{t+1}} \right], \tag{7}$$

$$\left(C_t^j \right)^{-\frac{1}{\sigma}} V_t^{z^*} \left(1 + \gamma_{x^*} x_{t+1}^{z^*j} \right) = \beta E_t \left[\left(C_{t+1}^j \right)^{-\frac{1}{\sigma}} \left(V_{t+1}^{z^*} + D_{t+1}^{z^*} \right) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \frac{P_t}{P_{t+1}} \right]. \tag{8}$$

We omit transversality conditions. Similar Euler equations and transversality conditions hold for the foreign household.

The equity choices of the representative household in our model depend on time-varying expected return differentials adjusted for financial intermediation costs. In the log-linear solution of the model around a deterministic steady state, choices are not affected by standard risk diversification motives captured by conditional second moments of asset returns and consumption. Thus, our log-linearized setup does not help to explain the contribution of conditional risk diversification to portfolio changes over time. Nevertheless, this does not imply that there is no role for risk premia in the model. As illustrated by Lettau (2003), it remains possible in a log-linearized framework to define average premia based on unconditional second moments. Although we do not pursue the exercise here, it would be possible to address the risk diversification properties of our model – or extensions to a wider menu of assets – from an *ex ante* perspective and compare them to the data along the lines of Lettau's exercise.

[8] We think about the financial intermediaries in the model as local, perfectly competitive firms owned by home households. There is no cross-border ownership of these firms.

2.2 Firms

The representative, monopolistically competitive, home firm z produces output with linear technology using labor as the only input:

$$Y_t^{Sz} = Z_t L_t^z, \tag{9}$$

where Z_t is the aggregate stochastic home productivity. The assumptions that labor is supplied inelastically and is the only factor of production imply that output of each country's sub-basket of goods is exogenously determined by productivity. Importantly, however, each country's GDP in units of the world consumption basket is endogenous, as it depends on the relative price of the country's output in terms of consumption, which is determined by the pricing decisions of firms.

Home firm z faces demand for its output given by:

$$Y_t^{Dz} = \left(\frac{p_t(z)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} Y_t^W = \left(RP_t^z\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} Y_t^W, \tag{10}$$

where $RP_t^z \equiv \frac{p_t(z)}{P_t}$ is the price of good z in units of the world consumption basket, $RP_t \equiv \frac{P_{Ht}}{P_t}$ is the price of the home sub-basket of goods in units of the world consumption basket, and Y_t^W is aggregate world demand of the consumption basket.

Firm profit maximization results in the pricing equation:

$$RP_t^z = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t}, \tag{11}$$

where $w_t \equiv W_t/P_t$. Since $RP_t^z = RP_t$ at an optimum, labor demand is determined by

$$L_t^z = L_t = RP_t^{-\omega} \frac{Y_t^W}{Z_t}. \tag{12}$$

2.3 Relative Prices, GDP, and Income Distribution

We relegate aggregate equilibrium conditions for household behavior to an appendix and focus here on the determination of some key variables in our model.

Define home aggregate per capita GDP in units of consumption as $y_t \equiv aRP_t Y_t^z/a = RP_t Z_t$ (where we used the equilibrium condition $aL_t^z/a = L_t^z = 1$) and world aggregate per capita GDP as $y_t^W \equiv ay_t + (1-a)\, y_t^* = aRP_t Z_t + (1-a)\, RP_t^* Z_t^*$. Market clearing in aggregate per capita terms requires $aL_t^z/a = L_t = 1 = RP_t^{-\omega} y_t^W / Z_t$, and similarly in the foreign economy. We thus have a system of two equations in two unknowns that pins down home and foreign relative prices:

$$1 = RP_t^{-\omega} \frac{aRP_t Z_t + (1-a)\, RP_t^* Z_t^*}{Z_t}, \tag{13}$$

$$1 = \left(RP_t^*\right)^{-\omega} \frac{aRP_t Z_t + (1-a)\, RP_t^* Z_t^*}{Z_t^*}. \tag{14}$$

PPP implies that the real exchange rate is equal to one in all periods. The terms of trade between representative home and foreign goods, instead, change over time and are given by

$$TOT_t = \frac{p_t(z)}{\mathcal{E}_t p_t^*(z^*)} = \frac{P_{H,t}}{\mathcal{E}_t P_{F,t}^*} = \frac{RP_t}{RP_t^*} = \left(\frac{Z_t^*}{Z_t}\right)^{\frac{1}{\omega}}. \tag{15}$$

A positive productivity shock in the home economy causes the terms of trade to deteriorate as increased supply of home goods lowers their relative price. Note that, when $\omega = 1$, the terms of trade move one-for-one with the productivity differential, as in Cole and Obstfeld (1991) and Corsetti and Pesenti (2001).

Given the path of RP_t implied by the system (13)-(14), the real wage that clears the labor market is in turn determined by:

$$w_t = \frac{(\theta-1) RP_t Z_t}{\theta} = \frac{(\theta-1) y_t}{\theta}. \tag{16}$$

In a perfectly competitive environment in which $\theta \to \infty$, all GDP per capita would be distributed to domestic labor in the form of wage income. In a monopolistically competitive environment with constant markups, a share $1/\theta$ of GDP is distributed as profits:

$$d_t = y_t - w_t = \frac{1}{\theta} y_t. \tag{17}$$

The distribution of GDP between wages and profits will be an important determinant of the properties of our model as we discuss below.

2.4 Net Foreign Assets

Denote aggregate per capita home holdings of home (foreign) equity entering period $t+1$ with x_{t+1} (x_{t+1}^*). Similarly, denote aggregate per capita foreign holdings of home (foreign) equity with x_{*t+1} (x_{*t+1}^*).[9] Equilibrium aggregate per capita real home assets entering $t+1$ are thus given by $v_t x_{t+1} + v_t^* x_{t+1}^*$, where $v_t \equiv V_t/P_t$ and $v_t^* = V_t^*/P_t^*$. Home aggregate per capita net foreign assets entering $t+1$ (nfa_{t+1}) are obtained by netting out the values of home holdings of home shares ($v_t x_{t+1}$) and foreign holdings of home shares (adjusted for the population ratio, $[(1-a)/a] v_t x_{*t+1}$). Thus,

$$nfa_{t+1} \equiv v_t^* x_{t+1}^* - \frac{1-a}{a} v_t x_{*t+1}. \tag{18}$$

Foreign net foreign assets satisfy the market clearing condition:

$$a(nfa_t) + (1-a) nfa_t^* = 0. \tag{19}$$

[9] Details on the computation of these shares are in an appendix.

2.5 Steady-State Net Foreign Assets and Equity Returns

We present the details of the solution for the steady state of the model in appendix. Here, we report the main results on steady-state net foreign assets, and equity returns. We denote steady-state levels of variables by dropping the time subscript.

Steady-state home net foreign assets are:

$$nfa = \frac{\beta(1-a)}{\theta}\left(\frac{\gamma^*_{x^*}}{\Gamma_1} - \frac{\gamma_x}{\Gamma_2}\right),$$

where:

$$\begin{aligned}\Gamma_1 &= (1-\beta)\left[(1-a)\gamma_{x^*} + a\gamma^*_{x^*}\right] + \gamma_{x^*}\gamma^*_{x^*},\\ \Gamma_2 &= (1-\beta)\left[(1-a)\gamma_x + a\gamma^*_x\right] + \gamma_x\gamma^*_x.\end{aligned}$$

To gain intuition on this expression, observe that, for $\beta \to 1$, we have:

$$nfa \to \frac{(1-a)}{\theta}\left(\frac{1}{\gamma_{x^*}} - \frac{1}{\gamma^*_x}\right).$$

Home net foreign assets are lower the higher the intermediation cost faced by home agents in the market for foreign equity (γ_{x^*}), and the lower the intermediation cost faced by foreign agents in the market for home equity (γ^*_x). The net foreign asset position is zero if $\gamma_{x^*} = \gamma^*_x$, with gross positions of equal value but opposite sign, proportional to the cost facing a household when buying equity abroad. If $\beta \neq 1$, the net foreign asset position depends on all financial fee scale parameters, reflecting the relative convenience of the two equities for home and foreign households in the two markets.

The difference in steady-state rates of return on equity is:

$$\frac{d}{v} - \frac{d^*}{v^*} = \frac{1}{\beta(1-a)}\left(\frac{\gamma_x\gamma^*_x}{\gamma_x + \gamma^*_x\frac{a}{1-a}} - \frac{\gamma_{x^*}\gamma^*_{x^*}}{\gamma_{x^*} + \gamma^*_{x^*}\frac{a}{1-a}}\right). \quad (20)$$

Even if $\gamma_{x^*} = \gamma^*_x$, the equity return differential may be different from zero. To be zero, it requires equal intermediation costs across home and foreign equity within each country, with potentially different costs across countries for the same equity ($\gamma_x = \gamma_{x^*}$ and $\gamma^*_x = \gamma^*_{x^*}$), or equal costs across countries for the same equity, with potentially different costs across home and foreign equity within each country (if $\gamma_x = \gamma^*_x$ and $\gamma_{x^*} = \gamma^*_{x^*}$).[10]

[10] It is possible to verify that the model would feature a unique steady state even if household discount factors (β's) were different across countries. Financial fees would prevent the outcome in which the most patient country owns all the world equity from arising. Details are available on request.

3 Valuation Changes and the Transmission of Shocks

In this section we provide a decomposition of changes in net foreign assets into valuation changes and the current account, with valuation changes and the current account further decomposed into their components.[11] We then analyze the determinants of valuation changes and the transmission of relative productivity shocks in two special cases of our model that can be solved analytically in log-linear form.

3.1 Valuation Changes and the Current Account

Assume for simplicity that the home and foreign economies have equal size ($a = 1/2$) and the steady state of the model is such that $v = v^*$ and $x^* = x_*$. (Throughout, we assume that structural parameters are such that the symmetry properties we appeal to are satisfied.) Log-linearizing (18) and denoting percent deviations from steady-state levels with a hat yields:

$$\widehat{nfa}_{t+1} = (\hat{v}_t^* - \hat{v}_t) + \left(\hat{x}_{t+1}^* - \hat{x}_{*t+1}\right), \tag{21}$$

where $\widehat{nfa}_{t+1} \equiv \mathsf{d}nfa_{t+1}/(vx)$, reflecting the fact that $nfa = 0$ when $vx_* = v^*x^*$. The change in net foreign assets is then written as:

$$\widehat{nfa}_{t+1} - \widehat{nfa}_t = \left[\left(\hat{v}_t^* - \hat{v}_{t-1}^*\right) - (\hat{v}_t - \hat{v}_{t-1})\right] + \left[\left(\hat{x}_{t+1}^* - \hat{x}_t^*\right) - (\hat{x}_{*t+1} - \hat{x}_{*t})\right]. \tag{22}$$

The first square bracket on the right-hand side of (22) is the valuation change on the existing stock of net foreign assets due to changes in real equity prices. Changes in real equity values, in turn, can be decomposed into changes in their nominal determinants (nominal equity prices, the price level, and the exchange rate). Specifically, $\hat{v}_t = \hat{V}_t - \hat{P}_t$ and $\hat{v}_t^* = \hat{V}_t^* - \hat{P}_t^* = \hat{V}_t^* - \left(\hat{P}_t - \widehat{\mathcal{E}}_t\right)$. This decomposition allows us to highlight a difference between our model with equity trading and the more familiar framework with international trade in bonds. In our economy, the nominal components of real equity prices have no independent effect on real equity values (and thus net foreign assets) because all prices involved are fully flexible. Furthermore, changes in real equity prices are the only source of valuation effects, since the real exchange rate is constant owing to PPP. In an economy with bond trading, the nominal interest rate between $t-1$ and t is predetermined relative to the price level at t, resulting in a valuation effect of nominal price movements on outstanding bond positions via unexpected movements of *ex post* real interest rates under fully flexible goods prices (Tille, 2005, and Benigno, 2006).

[11] As measured in the balance of payments, the current account does not include capital gains on investments, while the international investment position incorporates them. This component of investment income, however, enters the textbook definition of total asset return.

The second square bracket in (22) is the change in net foreign assets due to purchases and sales of assets and liabilities, i.e., portfolio rebalancing. This portfolio rebalancing term corresponds to the current account balance that comprises the income and trade balance. To see this, assume further that the steady state is such that $x = x^* = 1/2$ and $d = d^*$. Log-linearizing the equilibrium budget constraint in aggregate per capita terms and imposing the log-linear asset market equilibrium condition $\hat{x}_{t+1} = -\hat{x}_{*t+1}$, we obtain:

$$\left(\hat{x}^*_{t+1} - \hat{x}^*_t\right) - \left(\hat{x}_{*t+1} - \hat{x}_{*t}\right) = \frac{d}{v}\left[\left(\hat{d}^*_t + \hat{x}^*_t\right) - \left(\hat{d}_t + \hat{x}_{*t}\right)\right] + 2\left(\frac{d}{v}\hat{d}_t + \frac{w}{v}\hat{w}_t - \frac{C}{v}\hat{C}_t\right). \tag{23}$$

The first term on the right-hand side is the dividend income flow from net foreign assets accumulated in the previous period, while the second term is the trade balance.[12] Thus, the portfolio rebalancing term in equation (22) is the current account balance.

As a corollary of the equations above, it is evident that valuation changes play a role in the adjustment of net foreign assets in response to shocks in our model whenever the gross equity positions of a country differ from zero. That is, except in the case where gross (and thus net) foreign asset positions are in zero balance for all countries.[13]

The analog to (23) in the foreign economy is:

$$-\left(\hat{x}^*_{t+1} - \hat{x}^*_t\right) + \left(\hat{x}_{*t+1} - \hat{x}_{*t}\right) = \frac{d}{v}\left[-\left(\hat{d}^*_t + \hat{x}^*_t\right) + \left(\hat{d}_t + \hat{x}_{*t}\right)\right] + 2\left(\frac{d}{v}\hat{d}^*_t + \frac{w}{v}\hat{w}^*_t - \frac{C}{v}\hat{C}^*_t\right), \tag{24}$$

where we used the fact that $x = x^* = 1/2$ implies $x_* = x^*_* = 1/2$ via market clearing.

Subtracting (24) from (23) and using a superscript D to denote cross-country differences (home minus foreign) yields:

$$\hat{x}^D_{t+1} = \left(1 + \frac{d}{v}\right)\hat{x}^D_t + \frac{w}{v}\hat{w}^D_t - \frac{C}{v}\hat{C}^D_t, \tag{25}$$

where $\hat{x}^D_{t+1} = \hat{x}^*_{t+1} - \hat{x}_{*t+1}$ measures home's net cross-border share holdings. Notice the resemblance between (25) and the standard, log-linear law of motion for net foreign bond holdings in the more familiar framework. In our model, the steady-state gross return on share holdings replaces the steady-state gross interest rate.

[12] The "2" that normalizes the trade balance originates from the fact that, with equal country size, asset market equilibrium requires $x_{t+1} + x_{*t+1} = 1$. In the symmetric steady state of this example, this implies $x = x_* = 1/2$. Division of both sides by vx in the log-linearization of the budget constraint results in the presence of the 2.

[13] Note that net foreign asset dynamics can also be decomposed to total asset return and trade balance, where the total return comprises valuation change and investment income. This decomposition is less dependent on the accounting convention of the balance of payments statistics or corporate policies on dividend payouts. Our model assumes that all profits are distributed as dividends, but this assumption does not apply to foreign direct investment, for which profits are fully distributed and then reinvested by the accounting convention of the balance of payments statistics. As noted by Obstfeld (2004), if companies choose to retain profits internally, this can reduce the magnitude of current account variation in (23) and enhance the relative role of valuation changes in external adjustment.

3.2 Valuation and Transmission around a Symmetric Steady State

We now complete the solution of the log-linear model for the case of a fully symmetric steady state in which everything is identical across two equal-sized countries (so that, in particular, $x = x^* = x_* = x_*^* = 1/2$).

Exploiting $\widehat{RP}_t^D = -(1/\omega)\hat{Z}_t^D$ and the definitions of home and foreign GDP's in units of consumption ($y_t = RP_t Z_t$ and $y_t^* = RP_t^* Z_t^*$, respectively), it is immediate to verify that the log-linear GDP differential is proportional to relative productivity:

$$\hat{y}_t^D = \left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D. \tag{26}$$

As expected, there is no GDP differential if $\omega = 1$.

Since dividends and wage income are constant fractions of GDP, it follows immediately that

$$\hat{w}_t^D = \hat{d}_t^D = \left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D. \tag{27}$$

Using the steady-state properties of the model, we may then rewrite the law of motion (25) as:

$$\hat{x}_{t+1}^D = \frac{(1+\gamma)}{\beta}\hat{x}_t^D + \frac{(\theta - 1)(1 - \beta + \gamma)}{\beta}\left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D - \frac{\theta(1 - \beta + \gamma)}{\beta}\hat{C}_t^D, \tag{28}$$

where γ is the scaling parameter of financial frictions, common across equities and countries, and β is the household discount factor.

We show in an appendix that no-arbitrage across different equities implies that expected relative consumption growth is tied to net cross-border share holdings, and relative share valuation reflects expected future share prices and dividends:

$$\hat{C}_t^D = E_t\hat{C}_{t+1}^D + \frac{\sigma\gamma}{1+\gamma}\hat{x}_{t+1}^D. \tag{29}$$

$$\hat{v}_t^D = \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1}^D + \frac{1 - \beta + \gamma}{1+\gamma}E_t\hat{d}_{t+1}^D. \tag{30}$$

Note that, absent financial frictions ($\gamma = 0$), the consumption differential follows a random walk: Any differential at time t is expected to persist at $t+1$. As we show below, consistent with models with bond trading only, the link between expected relative consumption growth and relative cross-border share holdings introduced when $\gamma > 0$ is crucial to deliver stationary responses to temporary shocks.

Equations (27) and (30) allow us to solve for the determinants of relative share prices (and thus the valuation effect). Assuming that home and foreign productivities $\hat{Z}_t$ and $\hat{Z}_t^*$ follow $AR(1)$ processes with common persistence $\phi \in [0, 1)$, we have:

$$\hat{v}_t^D = \eta_{v^D Z^D}\hat{Z}_t^D = \left(\frac{1 - \beta + \gamma}{1 + \gamma - \beta\phi}\right)\left(\frac{\omega - 1}{\omega}\right)\phi\hat{Z}_t^D. \tag{31}$$

The effect of relative productivity shocks on relative share prices depends on the persistence of the shock (ϕ), the elasticity of substitution between home and foreign goods (ω), the size of financial frictions (γ), and the patience of households (β). We assume $0 \leq \gamma < 1$ and $\omega \geq 1$. Combining these assumptions with the restrictions $0 < \beta < 1$ and $0 \leq \phi < 1$, we can conclude that:

$$\frac{\partial \eta_{v^D Z^D}}{\partial \phi} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \omega} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \gamma} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \beta} \leq 0.$$

Relative productivity shocks induce larger changes in relative share valuation the more persistent the shocks, the more substitutable home and foreign goods, the larger financial frictions, and the more impatient households. Notice that purely temporary productivity shocks ($\phi = 0$) have no effect on relative share valuation, because the differential in share prices is determined by its expected future level and expected relative dividends, which are not affected by the shock if this has no persistence.

We note here that some of these results need to be qualified when we abandon the case of a fully symmetric steady-state. Around a non-symmetric steady state, also purely temporary shocks affect relative share valuation, as will be shown in the next sub-section around an extremely non-symmetric steady state. It is also important to keep in mind that the comparative statics above on the effect of changes in the size of financial fees (γ) are performed for unchanged steady state. In other words, changes in γ are such that the symmetry of the steady state appealed to in log-linearization is unaffected. We can then compare shock transmission properties for different size of financial fees around an unchanged steady state. A different, but important question is how changes in the size of financial fees affect dynamics around different steady states when changes in γ's do not leave the steady state unaffected. Since this case becomes algebraically intractable, we will study it by means of numerical exercises in the next section.

No arbitrage around the fully symmetric steady state implies that we can solve for the dynamics of relative consumption and cross-border share holdings independently of the path of $\hat{v}_t^D$, by solving the system of equations (28) and (29). This implies that changes in equity valuation do not provide an independent channel of consumption adjustment in response to shocks.[14] Since optimal asset pricing is the dual of the asset quantity choice in a forward-looking, general equilibrium model, and equity prices are fully forward-looking, the role of asset markets for consumption dynamics is fully summarized by the change in relative equity quantities held at home and abroad. Note that this differs from models with international trade in a risk-free real bond or currency-denominated bonds. Changes in the risk-free real interest rate provide an additional channel of shock transmission to consumption when initial holdings of the real bond are not zero across countries. When currency-denominated bonds are traded, unexpected changes in inflation generate unexpected changes in

[14] Of course, the assumptions of our model imply that equity values have no effect on GDP either.

real returns on existing bond holdings by inducing a difference between *ex ante* and *ex post* real interest rates in the impact period of a shock. The difference in results between equity and bonds stems from the fully forward-looking nature of equity prices versus the predetermined nature of *ex ante* interest rates relative to shocks. The latter results in an independent role of bond valuation in shock transmission.[15]

The solution for $\hat{x}_{t+1}^D$ and $\hat{C}_t^D$ takes the form:

$$\hat{x}_{t+1}^D = \eta_{x^D x^D}\hat{x}_t^D + \eta_{x^D Z^D}\hat{Z}_t^D, \quad (32)$$
$$\hat{C}_t^D = \eta_{C^D x^D}\hat{x}_t^D + \eta_{C^D Z^D}\hat{Z}_t^D. \quad (33)$$

Applying the method of undetermined coefficients, the elasticity $\eta_{x^D x^D}$ solves the quadratic equation:

$$\eta_{x^D x^D}^2 - \left[1 + \frac{1+\gamma}{\beta} + \frac{\theta(1-\beta+\gamma)\sigma\gamma}{\beta(1+\gamma)}\right]\eta_{x^D x^D} + \frac{1+\gamma}{\beta} = 0. \quad (34)$$

If $\gamma = 0$, this equation has solutions 1 and $1/\beta$, and – discarding the explosive solution $1/\beta$ – we are left with the familiar unit root for net cross-border share holdings as in models with bond trading only and no stationarity inducing device. When $\gamma > 0$, there is still an explosive solution larger than 1, and the unit root is "pulled" inside the unit circle, between 0 and 1, ensuring stationary net foreign equity dynamics in response to temporary shocks.

Given the stable root $\eta_{x^D x^D}$, the solutions for the other elasticities are:

$$\eta_{C^D x^D} = \frac{\sigma\gamma}{(1+\gamma)(1-\eta_{x^D x^D})}\eta_{x^D x^D} > 0,$$
$$\eta_{x^D Z^D} = \frac{(\theta-1)(1-\beta+\gamma)}{\beta}\left(\frac{\omega-1}{\omega}\right)\left[1 + \frac{\theta(1-\beta+\gamma)}{\beta(1-\phi)}\left(\eta_{C^D x^D} + \frac{\sigma\gamma}{1+\gamma}\right)\right]^{-1} \geq 0,$$
$$\eta_{C^D Z^D} = \frac{1}{1-\phi}\left(\eta_{C^D x^D} + \frac{\sigma\gamma}{1+\gamma}\right)\eta_{x^D Z^D} \geq 0. \quad (35)$$

Note that our model replicates the result of Cole and Obstfeld (1991) and Corsetti and Pesenti (2001) that the economy mimics complete markets, and there is no movement in net cross-border share holdings nor consumption differential if $\omega = 1$. In that case, the terms of trade move in directly proportional fashion with relative productivity, there is no GDP differential, and $\eta_{x^D Z^D} = \eta_{C^D Z^D} = 0$, ensuring that $\hat{C}_t^D = \hat{x}_{t+1}^D = 0$ in all periods given the initial condition $\hat{x}_t^D = 0$ in the period of a shock.

[15] See Ghironi, İşcan, and Rebucci (2005) on the role of the risk-free real interest rate as an additional state variable in the determination of consumption in the real-bond case. Unexpected changes in *ex post* real interest rates with trade in nominal bonds are central to results in Benigno (2006), Ghironi (2000), and Tille (2005).

We are thus in a position to draw conclusions on the determinants of net foreign asset changes. Using the results above yields:

$$\widehat{nfa}_{t+1}-\widehat{nfa}_t=-\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi}\right)\left(\frac{\omega-1}{\omega}\right)\phi\left(\hat{Z}_t^D-\hat{Z}_{t-1}^D\right)-\left(1-\eta_{x^Dx^D}\right)\hat{x}_t^D+\eta_{x^DZ^D}\hat{Z}_t^D. \quad (36)$$

Of course, given the initial condition $\hat{x}_t^D=\hat{Z}_{t-1}^D=0$, there is no change in net foreign assets if $\omega=1$, since there is no valuation change and $\eta_{x^DZ^D}=0$. The relative contributions of valuation and current account to the change in net foreign assets induced by a relative productivity shock are thus given by:

$$\begin{aligned} VALShare_t &\equiv \frac{-\left(\hat{v}_t^D-\hat{v}_{t-1}^D\right)}{\widehat{nfa}_{t+1}-\widehat{nfa}_t}=\left(1-\frac{\hat{x}_{t+1}^D-\hat{x}_t^D}{\hat{v}_t^D-\hat{v}_{t-1}^D}\right)^{-1}, \\ CAShare_t &\equiv \frac{\hat{x}_{t+1}^D-\hat{x}_t^D}{\widehat{nfa}_{t+1}-\widehat{nfa}_t}=\left[1-\left(\frac{\hat{x}_{t+1}^D-\hat{x}_t^D}{\hat{v}_t^D-\hat{v}_{t-1}^D}\right)^{-1}\right]^{-1}, \end{aligned} \quad (37)$$

where the minus sign at the numerator of $VALShare_t$ follows from the fact that an increase in the relative price of home equity contributes negatively to home's net foreign assets. Note that $VALShare_t+CAShare_t=1$, but $VALShare_t$ and $CAShare_t$ are not individually constrained to being between 0 and 1. For instance, a more than proportional contribution of valuation can offset a negative share of the current account in a given increase in net foreign assets.

The ratio $\left(\hat{x}_{t+1}^D-\hat{x}_t^D\right)/\left(\hat{v}_t^D-\hat{v}_{t-1}^D\right)$ has solution:

$$\frac{\hat{x}_{t+1}^D-\hat{x}_t^D}{\hat{v}_t^D-\hat{v}_{t-1}^D}=\frac{-\left(1-\eta_{x^Dx^D}\right)\hat{x}_t^D+\eta_{x^DZ^D}\hat{Z}_t^D}{\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi}\right)\left(\frac{\omega-1}{\omega}\right)\phi\left(\hat{Z}_t^D-\hat{Z}_{t-1}^D\right)}. \quad (38)$$

The elasticity $\eta_{x^Dx^D}$ from (34) does not depend on substitutability between home and foreign goods (ω). Thus, when evaluating the effect of ω on the relative share of valuation in net foreign asset changes, we may restrict attention to the ratio

$$\frac{\eta_{x^DZ^D}\hat{Z}_t^D}{\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi}\right)\left(\frac{\omega-1}{\omega}\right)\phi\left(\hat{Z}_t^D-\hat{Z}_{t-1}^D\right)}.$$

Inspection of the solution for $\eta_{x^DZ^D}$ in (35) shows that this ratio is independent of ω (because $(\omega-1)/\omega$ appears at both numerator and denominator).[16] Therefore, the degree of substitutability between home and foreign goods has no effect on the relative shares of valuation and the current account in net foreign asset changes. The effect of other parameters – specifically, the size of

[16] This independence from ω applies also to the alternative decomposition of net foreign asset dynamics between the movements in trade balance and total rate of return.

financial frictions, γ – on the relative share of valuation versus the current account in net foreign asset changes cannot be disentangled analytically in such simple fashion. Thus, we evaluate it by means of numerical examples in Section 5. Next, we turn to shock transmission around a non-symmetric steady state.

3.3 A Non-Symmetric Steady State: The Case of Full Cross-Shareholding

Consider now a steady state in which equities issued by each country are wholly owned by residents of the other country (we call this full cross-shareholding). In terms of our notation: $x = 0$, $x_* = 1$, $x_*^* = 0$, and $x^* = 1$. This portfolio allocation arises endogenously by assuming that investing abroad is costless in both the home and the foreign economy (with common friction of size γ for domestic investment). Under this steady-state configuration, equity prices are $v = v^* = \beta / [\theta(1-\beta)]$, but steady-state levels of wages, dividends, consumption, and relative prices are the same as in the previous case.

We show in an appendix that the following system now determines the dynamics of relative equity and the consumption differential:

$$\hat{x}_{t+1}^D = \frac{1}{\beta}\hat{x}_t^D + \frac{(1-\beta)(\theta-2)}{2\beta}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_t^D - \frac{(1-\beta)\theta}{2\beta}\hat{C}_t^D, \tag{39}$$

$$\hat{C}_t^D = E_t\hat{C}_{t+1}^D + \sigma\gamma\hat{x}_{t+1}^D, \tag{40}$$

where we have used $\hat{x}_{t+1}^D = -2\hat{x}_{*t+1}$ that holds under full cross-shareholding.[17] As in the fully symmetric case, and for the same reason, changes in equity valuation do not provide an independent channel of consumption adjustment.

The solution of this system has the same form as (32)-(33). The elasticity $\eta_{x^D x^D}$ now satisfies:

$$\beta\eta_{x^D x^D}^2 - \left[1+\beta+\frac{(1-\beta)\sigma\gamma\theta}{2}\right]\eta_{x^D x^D} + 1 = 0. \tag{41}$$

As before, we select the stable root between 0 and 1 when $\gamma > 0$. The other elasticities are determined by:

$$\begin{aligned}
\eta_{C^D x^D} &= \frac{\sigma\gamma\eta_{x^D x^D}}{1-\eta_{x^D x^D}}, \\
\eta_{x^D Z^D} &= \frac{(1-\beta)(\theta-2)}{2\beta}\left(\frac{\omega-1}{\omega}\right)\left[1+\frac{\theta(1-\beta)\sigma\gamma}{2\beta(1-\eta_{x^D x^D})(1-\phi)}\right]^{-1}, \\
\eta_{C^D Z^D} &= \frac{\sigma\gamma\eta_{x^D Z^D}}{(1-\eta_{x^D x^D})(1-\phi)}.
\end{aligned} \tag{42}$$

The equity price differential now obeys (see the appendix for details):

$$\hat{v}_t^D = \beta E_t\hat{v}_{t+1}^D + (1-\beta)E_t\hat{d}_{t+1}^D - \gamma\hat{x}_{t+1}^D, \tag{43}$$

[17] In this scenario, for analytical tractability, we allow countries to go short in their aggregate equity positions.

where the difference from the symmetric case is due to the non-symmetric steady-state equity holdings. Notice that the dynamics of share holdings now affect relative share valuation. This has the consequence of making relative valuation sensitive to zero-persistence productivity shocks via their effect on share holdings entering the following period.

We can obtain the following algebraic expression for the relative contribution of the current account and valuation changes following shocks with no persistence:

$$\frac{\hat{x}_{t+1}^D - \hat{x}_t^D}{\hat{v}_t^D - \hat{v}_{t-1}^D} = -\frac{1-\beta\eta_{x^D x^D}}{\gamma} \quad \forall t \geq 0, \tag{44}$$

where 0 is the time of the shock. The relative contribution of the valuation change in (44) is higher the higher γ (the common financial intermediation cost on domestic shares). When financial intermediation is more costly, valuation changes play a bigger role around the steady state, as portfolio rebalancing entails larger costs.[18] As in the symmetric case, the elasticity of substitution between home and foreign goods, ω, does not affect the relative contribution of valuation change and current account, but it plays a critical role in determining the extent of adjustment via terms of trade movements.

Starting from two different steady states, this section shows that changes in equity values provide a channel for net foreign asset adjustment but, in general equilibrium, do not provide an independent channel of shock transmission to other variables. Therefore, valuation changes do not constitute an independent channel of risk sharing in our model. The next section explores the risk sharing implications of international trade in equity, the nature of asset market incompleteness in our model, and how it affects the role of valuation in net foreign asset adjustment.

4 Equity Trade and Risk Sharing

We begin this section by addressing the consequences of completely removing financial frictions while holding the initial steady-state allocation of equity given at the two alternatives considered in the previous section. This allows us to tie the risk sharing properties of equity trading – conditional on a given steady-state allocation of equity – to the distribution of income between profits and labor income. Next, we ask whether it is possible to achieve complete risk sharing via a proper allocation of equity, conditional on income distribution. We show that such equity allocation exists, and it is the allocation that would be chosen by a world social planner. But it generally differs from the decentralized market equilibrium, even if we show that a properly designed system of

[18] A larger γ also causes $\eta_{x^D x^D}$ to decrease, but the net effect on the relative share of valuation in net foreign asset changes is positive. The relative contribution of valuation changes also increases with $\eta_{x^D x^D}$, which is larger when σ and/or θ become smaller. Lower values of these parameters lead to lower intertemporal substitution (more consumption smoothing) and weaker competition (higher profits and equity prices).

financial fees can decentralize the first-best equity allocation. The difference between planning optimum and market equilibrium is important for our analysis of the valuation channel, because the planner's equity allocation is constant across time and realizations of productivity, implying that changes in equity values become the sole source of net foreign asset movements. The fact that the decentralized market does not reproduce the first-best equilibrium is thus crucial for both price and quantity changes to play an empirically plausible role in net foreign asset dynamics.

4.1 Trade in Risky Assets Revisited

The textbook intuition is that frictionless trade in two equities in an environment with only two shocks – such as the one we are exploring – should reproduce the full insurance allocation of complete asset markets. We ask here whether our model delivers the complete markets equilibrium if there are no financial fees (and $\omega \neq 1$) owing to the ability to trade equity at no cost in the presence of productivity shocks only.

Without financial fees, the decentralized economy features a continuum of possible deterministic steady-state allocations of equity. This is a well known indeterminacy consequence of the absence of a link between equity holdings and consumption growth in the Euler equations for optimal asset holdings.[19] The steady state around which the model is log-linearized is only one of infinitely many possible, chosen as a matter of convenience. We consider the two steady states that were examined in the previous section.

4.1.1 Risk Sharing and Transmission around a Symmetric Steady State

Focus first on the case of a fully symmetric steady state of Section 3.2 and assume that $\gamma = 0$. The elasticity of net foreign equity holdings entering period $t+1$ to their past level ($\eta_{x^D x^D}$) is now equal to 1, and the solution of the model takes the form:

$$\hat{x}_{t+1}^D = \hat{x}_t^D + \eta_{x^D Z^D} \hat{Z}_t^D, \tag{45}$$

$$\hat{C}_t^D = \eta_{C^D x^D} \hat{x}_t^D + \eta_{C^D Z^D} \hat{Z}_t^D. \tag{46}$$

For the solution to replicate complete markets, it must be $\hat{C}_t^D = 0$. In other words, given the initial condition $\hat{x}_t^D = 0$ at the time of a shock, it must be $\eta_{C^D Z^D} = \eta_{x^D Z^D} = 0$. The conjecture (45)-(46) must now be substituted in the system:[20]

$$\begin{aligned} \hat{x}_{t+1}^D &= \frac{1}{\beta}\hat{x}_t^D + \frac{(\theta-1)(1-\beta)}{\beta}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_t^D - \frac{\theta(1-\beta)}{\beta}\hat{C}_t^D, \\ \hat{C}_t^D &= E_t\hat{C}_{t+1}^D. \end{aligned}$$

[19] See Ghironi (2006) and references therein.

[20] The solution for the case $\gamma = 0$ cannot be obtained simply by setting $\gamma = 0$ in (35). Note that the implied expression for $\eta_{C^D x^D}$ would not be defined, as $\eta_{x^D x^D} = 1$ would imply division by 0.

Doing this and applying the method of undetermined coefficients yields:

$$\eta_{C^D x^D} = \frac{1}{\theta}, \quad \eta_{x^D Z^D} = \frac{(\theta-1)(1-\beta)(1-\phi)}{1-\beta\phi}\left(\frac{\omega-1}{\omega}\right), \quad \eta_{C^D Z^D} = \frac{(\theta-1)(1-\beta)}{\theta(1-\beta\phi)}\left(\frac{\omega-1}{\omega}\right).$$

Therefore, the solution does not coincide with the full insurance outcome in which $\hat{C}_t^D = 0$. Relative productivity shocks case a consumption differential on impact, and the consumption differential persists as a consequence of the unit root in net cross-border share holdings.[21]

This result highlights an important property of our model with equity trading. It is well known that if the world economy consists of two countries consuming the same good, with country-specific stochastic endowments of the good, CRRA preferences, and the ability to trade equity in the form of shares in the endowments of the good, frictionless trade in these equities will lead to the complete markets equilibrium. (For instance, see the discussion of this case in Obstfeld and Rogoff's, 1996, textbook.) The same mechanism carries through to the case of two goods and a CES aggregator. But the crucial difference is that our model does not allow trade in equity claims on endowments. Our equity provides claims to profits, with the rest of a country's income going to wages. To put it differently, even with symmetric equity holdings, only part of GDP gets to be shared between home and foreign residents. The wage portion is kept wholly by the residents of each country. As a result, even without financial fees (and thus frictionless trade in two equities in a world with only two shocks), equity trading starting from the symmetric steady-state allocation does not yield complete risk sharing.

This reasoning is confirmed by the results above. For equity trading to result in full insurance around the fully symmetric steady state, all of a country's GDP should be distributed as profit, leaving nothing for wages. The share of dividends in GDP is $1/\theta$, implying that all of GDP goes to shareholders in the limiting case in which $\theta \to 1$ (the maximum possible degree of monopoly power). As one can see from the expressions above, $\eta_{C^D Z^D} \to 0$ in this case, and so does $\eta_{x^D Z^D}$. There is full risk sharing under the initial, symmetric equity allocation, and (given the initial condition $\hat{x}_t^D = 0$ at the time of a shock) the equilibrium is such that $\hat{C}_t^D = \hat{x}_{t+1}^D = 0$ in all periods.

Under this interpretation, we can conclude that a proper definition of equity in a production economy (claims to profit rather than whole output) is sufficient to disturb completeness of the market in the "conventional" case. The deviation from full consumption risk sharing around the symmetric steady state is smaller the higher the degree of monopoly power along the two dimensions that are commonly explored in international macroeconomics: the higher monopoly power of individual producers within a country (the closer θ to 1) and the higher monopoly power of a

[21] It is easy to verify that one obtains the same solution for the case $\gamma = 0$ even if the conjecture for $\hat{x}_{t+1}^D$ is written without imposing the restriction $\eta_{x^D x^D} = 1$. In this case, applying the method of undetermined coefficients simply yields $\eta_{x^D x^D} = 1$ along with the elasticities above.

country over its sub-basket of goods (the closer ω to 1). This result points to a difference between economies with bond trading only and our model with equity trading. In the economy with bond trading, $\omega = 1$ is the only scenario in which incomplete markets reproduce the full consumption insurance of complete markets. Once we allow for international trade in shares issued by firms with monopoly power, full consumption insurance across countries arises also with $\omega \neq 1$ if firms' monopoly power is extreme and long-run equity positions are fully symmetric.

It is important to remark at this point that the risk sharing implications of extreme firm-level monopoly power in our model are conditional on the assumption of a symmetric steady state in which each country owns fifty percent of the other country's equity. As we show below, a different distribution of income between wages and profits, associated with less-than-extreme firm-level monopoly power, is required for equity trade to deliver full consumption insurance with $\omega \neq 1$ when the steady state asset position is different.

As for responses to productivity shocks, the elasticities without financial fees imply that the qualitative direction of responses starting from the symmetric steady state is the same as with $\gamma > 0$ explored in Section 3.2. Responses when $\gamma = 0$ feature a permanent change in net cross-border share holdings and consumption differential, but quantitative differences depend on parameter values.[22]

Finally, we examine the share of valuation in net foreign asset adjustment when there are no financial fees. It is possible to verify that:

$$VALShare_0 = \frac{\phi}{1 - \theta(1 - \phi)}, \quad VALShare_{t \geq 1} = \frac{1}{\theta},$$

with $CAShare_t = 1 - VALShare_t$ and $t = 0$ denoting the time of a shock. Absent financial fees, the unit root in net cross-border share holdings implies that the share of valuation in external adjustment is constant in all periods after the initial one, and is determined by the share of income distributed to profits. If $\theta \to 1$, the share of valuation in net foreign asset changes tends to 1 in all periods, consistent with the fact that there is full risk sharing and no change in net cross-border share holdings.

[22]It is impossible to pin down the response of holdings of individual equities when $\gamma = 0$, but this does not limit our ability to solve for all variables of interest, including the paths of net cross-border share holdings, net foreign assets, and the current account, starting from the symmetric initial position.

4.1.2 Risk Sharing with Full Cross-Shareholding

When each country fully owns the other country's equity in the initial steady state, it is possible to verify that $\gamma = 0$ yields the solution:

$$\begin{aligned} \hat{x}_{t+1}^D &= \hat{x}_t^D + \frac{(1-\beta)(\theta-2)}{2}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_t^D, \\ \hat{C}_t^D &= \frac{2}{\theta}\hat{x}_t^D + \frac{(1-\beta)(\theta-2)}{\theta(1-\phi)}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_t^D. \end{aligned}$$

As before, the equilibrium does not mimic complete markets, and the intuition is the same – sharing is limited only to a portion of GDP. However, assuming $\omega \neq 1$, it is no longer the case that $\theta = 1$ (and thus complete distribution of GDP to profits) is required for full consumption insurance to arise. This now happens when $\theta = 2$, i.e., with a share of dividends in GDP equal to 1/2. With full cross-border shareholding, complete consumption insurance arises when half of GDP is allocated to profits, and the remainder goes to wages. The wage income portion is now necessary to compensate for the effect of full initial cross-border equity ownership on income sharing. Recalling the result above for the case of a symmetric steady-state allocation of equity, we have thus established the general result that, once equities are defined as claims to firm profits in a production economy, the risk sharing properties of international equity trading conditional on a market-determined initial allocation of equity are tied to the distribution of income between profits and wages determined by substitutability across individual product varieties in consumer preferences.[23] In addition, the distribution of income that ensures perfect risk sharing implies that the share of valuation in net foreign asset changes is equal to 1.

4.2 The Planner's Optimum and the Incompleteness of Decentralized Markets

We now turn to the question whether there exists an allocation of equity that delivers full risk sharing conditional on income distribution and the consequences of decentralized markets. The planning optimum for the world economy in our model is the allocation that delivers full risk sharing, with a zero consumption differential in all periods for all realizations of productivity. As we showed above, the assumptions of inelastic labor supply and labor market clearing imply that firm-level monopoly power induces no distortion in the path of output, and thus aggregate world consumption. Therefore, a world social planner who maximizes a weighted average of home and foreign households' welfare in our model would choose the allocation that equates home and foreign consumption to world consumption in all periods. The question then is: Is there an allocation

[23] See Cass and Pavlova (2004) for additional findings on the fragility of the welfare properties of the Lucas Trees model. Ghironi and Lee (2006) explore the normative implications of this result and its consequences for optimal monetary policy in a sticky-price version of the model.

of equity that achieves such goal? The answer is yes: There exists a (constant) allocation of share holdings such that the consumption differential is zero at all points in time regardless of the realizations of Z_t and Z_t^*.

To verify this, use the equilibrium budget constraints for home and foreign, respectively:

$$v_t x_{t+1} + v_t^* x_{t+1}^* + C_t = (v_t + d_t) x_t + (v_t^* + d_t^*) x_t^* + w_t, \tag{47}$$

$$v_t x_{*t+1} + v_t^* x_{*t+1}^* + C_t^* = (v_t + d_t) x_{*t} + (v_t^* + d_t^*) x_{*t}^* + w_{*t}. \tag{48}$$

Subtracting (48) from (47), using equity market equilibrium and the proportionality of dividends and wage incomes to GDP, and rearranging yields:

$$\frac{v_t}{1-a}(x_{t+1} - x_t) + \frac{v_t^*}{1-a}(x_{t+1}^* - x_t^*) + C_t^D$$
$$= \left[\left(\frac{x_t}{1-a} - \frac{a}{1-a}\right)\frac{1}{\theta} + \frac{\theta-1}{\theta}\right] y_t + \left[\left(\frac{x_t^*}{1-a} - 1\right)\frac{1}{\theta} - \frac{\theta-1}{\theta}\right] y_t^*,$$

where $C_t^D \equiv C_t - C_t^*$.

Straightforward substitutions show that $x_{t+1} = x_t = x^P = a - (1-a)(\theta - 1)$ and $x_{t+1}^* = x_t^* = x^{P*} = (1-a)\theta$ imply $C_t^D = 0$ for every possible realization of y_t and y_t^* (i.e., for every possible realization of Z_t and Z_t^* in our model).[24] This is the allocation of share holdings that ensures perfect risk sharing (spanning the two shocks, for all possible realizations) conditional on the distribution of income between labor income and profits (the parameter θ). And this allocation would be chosen by a social planner maximizing aggregate world welfare (hence the superscript P).

In the planning optimum, constant equity holdings are adjusted to reflect income distribution in order to deliver perfect risk sharing: The smaller the share of income distributed as profit (the higher θ), the smaller the share of home equity that home households should hold, and the larger the share of foreign equity. Given $\theta > 1$, $x^P > 0$ if and only if $\theta < 1 + a/(1-a)$. If $a = 1/2$ (symmetric country size), the planner's share allocation implies going short in domestic equity whenever $\theta > 2$ (i.e., whenever less than half of income is distributed as profit). The planner's allocation always requires holding a positive amount of foreign equity ($\theta/2$ if $a = 1/2$).

As anticipated above, constant equity holdings in the first-best equilibrium imply that changes in asset values would be solely responsible for changes in net foreign assets. There would be no need for current account movements.

To compare our results with those of the literature (specifically, Heathcote and Perri, 2004), we note several properties of our model. First, the equity allocation that ensures perfect risk sharing for all realizations of income is feasible also when financial fees are present (because it is obtained by using only the equilibrium budget constraints). Second, with financial fees, the planner's equity

[24] Equity market equilibrium then implies $x_*^P = a\theta$ and $x_*^{*P} = 1 - a\theta$.

allocation is generally different from the market allocation, except in the special case discussed below in which fees are designed appropriately to decentralize the first-best equilibrium. In all other cases, arbitrarily small, strictly positive financial fees ensure that the market equilibrium differs from the planner's equilibrium, resulting in imperfect risk sharing. Finally, without financial fees, the planner's equity allocation is one of infinitely many possible deterministic steady-state market allocations. It is a measure zero set in the continuum of possible steady-state market equilibria. Hence, there is no presumption that the decentralized, steady-state market outcome should coincide with the planning optimum. Whenever it does not, risk sharing is incomplete, as we showed in the previous subsection.

Herein lie similarity to and difference from the literature on risk sharing via equity trade. As in this literature, it is possible in our model with incomplete markets to span uncertainty with only trade in equity conditional on the distribution of income, i.e., to have an equilibrium in which market incompleteness does not "bite." This is the equilibrium that a world social planner would choose. Even abstracting from the evidence against perfect consumption risk sharing (or risk sharing in the form of perfect correlation between consumption differentials and the real exchange rate in models without PPP), the planning optimum has the empirically implausible property that all changes in net foreign assets are entirely driven by valuation. When we remove financial fees and tie risk sharing to income distribution, we assume that the steady-state share allocation is one of the infinitely many possible steady-state market equilibria that differ from the planner's allocation. Conditional on one such initial equilibrium for the decentralized equity market, the distribution of income must satisfy certain conditions for perfect risk sharing to arise.

4.3 Decentralizing the First Best

Is it possible to design a system of taxes and transfers that decentralizes the first-best equity allocation? The answer is yes, and it relies on the observation made above that the planner's optimum is feasible in an economy with financial fees. Given this, it is possible to design a system of taxes and transfers that decentralizes the efficient equilibrium. One can intuitively see that quadratic fees and their lump-sum rebates can operate as a system of distortionary taxes and lump-sum transfers in achieving this purpose. Suppose that the fee-scaling parameters are such that $\gamma_x = \gamma_{x^*} = \gamma_x^* = \gamma_{x^*}^* = \gamma$ and quadratic fees are centered around the planner's allocation ($x^P \equiv a - (1-a)(\theta - 1)$, $x^{*P} \equiv (1-a)\theta$, $x_*^P \equiv a\theta$, and $x_*^{*P} \equiv 1 - a\theta$) instead of zero holdings. We show in appendix that this modification yields $x = x^P$, $x^* = x^{*P}$, $x_* = x_*^P$, $x_*^* = x_*^{*P}$ as unique deterministic steady state for the decentralized economy. Additionally, once the model with the modified fees is log-linearized around this steady state, equity holdings remain constant at the planner's allocation and risk sharing is perfect in response to productivity shocks.

Thus, the efficient equity allocation can be decentralized by designing the proper system of financial taxes and transfers: The fees are the distortionary taxes that modify household incentives appropriately in the Euler equations; the fee rebates are the lump-sum transfers that "balance the budget." Properly designed financial fees are welfare improving, as they implement perfect risk sharing in the decentralized market economy.[25]

A question naturally arises on our modeling choice of centering fees around zero holdings rather than the planning optimum. The motivation is implicit in statements above: The planning optimum implies a degree of risk sharing that is contradicted by empirical evidence. It implies that only valuation matters for net foreign asset movements, clearly at odds with evidence. Quadratic fees centered around zero are then a convenient device to determine a unique steady state among those in which market incompleteness "bites" – letting it be determined by the size of financial frictions rather than the centering of the fee function – and avoid unappealing consequences for second moment computation in the quantitative exercises to which we turn below.

5 The Valuation Channel at Work

In this section, we illustrate the working of the valuation channel of external adjustment and the role of financial frictions (i.e., the γ's) for the share of net foreign asset changes due to valuation effects by means of numerical examples. Before proceeding, we describe briefly the model parametrization and discuss its performance relative to U.S. data in Section 5.1. We do so under two different assumptions on the degree of international financial integration, represented by the different size of the gross foreign asset positions in 1990 – the beginning of the most recent and rapid period of international integration – and the last year for which we have data, 2004 (Figure 2). In the first scenario, which we call "home bias in equity," we set parameters so that steady-state gross foreign assets and liabilities are about 40 percent of annual GDP, approximately as in the 1990 data, while net foreign assets are zero. In the second scenario, which we call "international financial integration," steady-state gross foreign assets and liabilities are about 100 percent of annual GDP, approximately as in the 2004 data. Under this scenario, we allow for the possibility of negative U.S. steady-state net foreign assets at about 25 percent of annual GDP. Then, in Section 5.2, we compare the relative importance of alternative sources of net foreign asset changes and channels of risk sharing in the model in the home bias and integration scenarios. Consistent with Section 3, this comparison assumes zero steady-state net foreign assets in both cases.

[25] One may wonder if it is possible to decentralize the efficient allocation by properly choosing the scaling parameters of fees centered around zero equity holdings. The answer is no. Details are available on request.

5.1 Parameter Values and Model Performance

Most parameter choices are standard, and steady-state productivity levels are normalized to 1 for simplicity. As in Section 3, we assume that countries have equal size ($a = 1/2$). We interpret periods as quarters, and we set relative risk aversion to the standard value of 2 ($\sigma = 0.5$) and the rate of time preference so that the annual real interest rate is approximately 4 percent in steady state ($\beta = 0.99$).

The elasticity of substitution among individual product varieties within each economy, θ, determines the constant degree of market power and the profit share of income in our model. We set $\theta = 6$ to imply a 20 percent markup of prices over marginal cost, as in Rotemberg and Woodford (1992).

The elasticity of substitution between the home and foreign baskets of goods determines the extent of risk sharing through terms of trade changes. Estimates of this elasticity range from values close to 1 in the macro literature to values as high as 12 in the micro, trade literature. We set $\omega = 2$. Higher values deliver more realistic correlations between home and foreign consumption and home and foreign output than those reported below, but they would be less conventional. Lower values, closer to structural estimates based on richer macro models, imply unrealistic terms of trade dynamics and risk sharing properties.[26] Note, however, that this parameter does not affect the share of valuation changes in net foreign asset changes at any time horizon. So we can safely condition on any specific value for our purposes.

The size of financial intermediation costs determines the degree of international integration in equity markets. (As we assumed in Section 3, bonds are not traded internationally.) In the home bias scenario, the assumption is that intermediation costs for home (foreign) agents on foreign (home) shares are larger than costs on home (foreign) shares. Specifically, to match the 1990 data, we set $\gamma_x = \gamma^*_{x^*} = 0.01$ and $\gamma_{x^*} = \gamma^*_x = 0.03$. In the integration scenario with zero net foreign assets, the assumption is that intermediation costs for home (foreign) agents on foreign (home) shares are the same as the costs on home (foreign) shares. Specifically, to match the 2004 data, we set $\gamma^*_{x^*} = \gamma_x = \gamma_{x^*} = \gamma^*_x = 0.01$. Finally, in the integration scenario with negative net foreign assets, the assumption is that the cost for foreign agents on home shares is lower than the cost on foreign shares (i.e., $\gamma^*_{x^*} = 0.01$ and $\gamma^*_x = 0.006$), while the cost for home agents on foreign shares is higher than the cost on home shares (i.e., $\gamma_x = 0.006$ and $\gamma_{x^*} = 0.01$). Thus, interpreting home as the U.S. economy, U.S. shares are cheaper for both U.S. and foreign agents. As a result, the steady-state price of U.S. equity shares is higher than the price of foreign equity shares, while the distribution of equity holdings remains symmetric as in the integration case with zero net foreign

[26] Results not reported in the tables below are available on request.

assets. This assumption induces a negative net foreign asset position of about 25 percent of annual GDP.[27] Table 2 summarizes the steady-state portfolio shares and returns implied by our parameter choices.

Table 3 reports second moments of the U.S. (home) data and the unconditional moments generated by our model. The latter are computed with DYNARE under the assumption that the exogenous (log) productivities in the two countries follow a bivariate $AR(1)$ process with no cross-border spillovers, identical autoregressive parameters equal to 0.9, and uncorrelated innovations with 1 percent quarterly variance.[28] Three features of changes in U.S. net foreign asset data stand out. First, this measure of external balance is much more volatile than the trade balance or the current account balance. Second, while the trade balance and the current account are countercyclical, the change in net foreign assets is essentially a-cyclical. Third, the change in net foreign assets is slightly less persistent than the trade balance and current account.[29] From Table 3, we can also see that U.S. equity prices are highly volatile, positively correlated with foreign equity prices, procyclical, and quite persistent.

With financial integration and steady-state net foreign assets that differ from zero, our model matches qualitatively – albeit not perfectly – the volatility, comovement with GDP, and persistence of changes in U.S. net foreign assets. The model can generate changes in net foreign assets that are more volatile, much less correlated with output, and less persistent than the trade balance and the current account. The model, however, underpredicts equity price volatility and overpredicts equity price comovement across countries. As a result, the matching of the moments of U.S. net foreign asset changes is less satisfactory if the net foreign asset position is zero. Absence of investment in physical capital in the model also generates a procyclical trade balance and current account in contrast to the data.[30]

[27] Note that the configuration of parameters that generates a given net foreign asset position is not unique.

[28] Details on data sources and computation of data moments are in the appendix.

[29] When calculated using Gourinchas and Rey's (2005) quarterly data for 1973:1-2004:1, generously provided to us by Pierre-Olivier Gourinchas, the persistence of net foreign asset changes is much lower (0.09), closer to the persistence generated by our model with non-zero steady-state net foreign assets. The persistence of net foreign asset changes is much smaller than that of the current account or trade balance also at annual frequency (Kollmann, 2005). Gourinchas and Rey's data produce estimates similar to those in Table 3 for the standard deviation of net foreign asset changes (1.11, thus more volatile than current account and trade balance) and the correlation with output (0.02, thus a-cyclical).

[30] We experimented with alternative assumptions on productivity. In the financial integration scenario with zero net foreign assets, using the bivariate productivity process in Baxter (1995) (with persistence 0.995, zero spillovers, 0.73 percent variance of innovations, and 0.19 percent covariance) matches relatively well the key moments of net foreign asset changes and equity price volatility, but grossly overstates GDP and consumption volatility. Changing the $AR(1)$ matrix to that in Backus, Kehoe, and Kydland (1992) (with persistence 0.906 and positive spillovers 0.088) also worsens the performance in terms of volatility of net foreign assets, because productivity spillovers cause equity prices to become excessively correlated across countries. Removing the spillovers from this parametrization allows the model to match GDP and consumption volatility, but results in too little volatility of both net foreign asset changes and equity prices. Details are available on request.

5.2 Home Bias versus Integration and the Role of Valuation

To illustrate the role of valuation in net foreign asset dynamics and the relative importance of alternative risk sharing channels under different degrees of international financial integration, we report selected impulse responses to a productivity shock in the home economy in Figure 3. As mentioned above, we focus on scenarios in which steady-state net foreign assets are zero. We consider a one-percent productivity shock in the home economy with persistence 0.9 at time $t = 0$. Time is on the horizontal axis of each panel in the figure.

Panels A and B compare, in the cases of home bias in equity and international financial integration, respectively, the responses of changes in net foreign assets (DNFAH) and their three components: (i) the valuation change (VAH), the trade balance (TBH), and the income balance (IBH), all in deviation from steady state, in percent of GDP (since steady-state levels of these variables are zero). The valuation change is defined as the difference between the change in net foreign assets and the current account. An increase in VAH thus contributes positively to the change in net foreign assets, as $-\left(\hat{v}_t^D - \hat{v}_{t-1}^D\right)$ in the log-linear setup of Section 3. Panels C and D compare the responses of the terms of trade (TOT), world consumption (CW), and the deviation of each country's consumption from world consumption (CHCW for home and CFCW for foreign), all in percent deviation from steady state. The deviation of each country's consumption from world consumption measures the overall degree of risk sharing generated by the model.

Under both scenarios, a favorable relative productivity shock to the home economy causes the relative price of shares in home equity to increase, and home households to increase their holdings of foreign equity relative to foreign holdings of home equity (i.e., to run a current account surplus) to smooth consumption. The initial current account surplus, however, is smaller than the trade balance surplus, as the income balance goes into deficit in response to the shock. Thus, initially, both the relative increase in the value of home equity and the income balance contribute negatively to the change in home net foreign assets, and adjustment through total equity returns (capital gains and dividends) more than offsets adjustment through the trade balance, determining a negative change in net foreign assets. Given the time profile of the productivity shock, the valuation change (a function of the productivity differential) is negative in all periods after the initial one, while the income (trade) balance swings into surplus (deficit) more gradually. As a result, unlike in a standard bond-only model, the change in net foreign assets is negative on impact, and relatively less positive during the following transition path.

Note that, in our model, the wealth transfer through equity price changes does not take place through violation of any arbitrage condition at the time of the shock, or thereafter. Valuation effects are an equilibrium phenomenon in our model and reflect the forward-looking nature of equity prices, which jump on impact to keep equilibrium in the asset market and then return gradually to steady state. Note also that the responses of world consumption (and thus income) and the terms of trade are identical across scenarios. Therefore, the extent of risk sharing through terms of trade changes is also the same. Further, we know from the theoretical analysis that the share of valuation changes in net foreign asset changes is not affected by the value of ω (although this does affect the absolute magnitude of the valuation change). So the only determinant of any change in the overall degree of risk sharing across scenarios in the model, or in the relative importance of different channels of net foreign asset adjustment and risk sharing, is the different degree of financial integration.

Importantly, as Figure 4 shows, the features of the transmission of a productivity shock that we described thus far are robust to the removal of financial intermediation costs. Figure 4 compares the responses of the model to the same shock around the same steady-state gross foreign asset positions (the integration scenario), with and without financial intermediation costs (Panels B and A, respectively).[31] As we can see, the dynamic response of net foreign assets, the current account, and the valuation change are qualitatively the same in the two cases.[32]

Returning to the comparison between home bias in equity and international financial integration, Figure 3 also shows that inducing larger steady-state gross foreign asset positions, while keeping the degree of risk sharing through terms of trade changes and the income distribution constant (i.e., for given ω and θ), increases risk sharing through asset markets, and affects the split of net foreign asset changes between the current account (or portfolio rebalancing) and valuation changes. To see this, note that the consumption differentials are smaller under financial integration (Figure 3, Panels C and D), the correlation between home and foreign consumptions is larger, and the volatility of consumption is lower (Table 3). Since risk sharing through terms of trade movements is the same across scenarios, we conclude that a higher degree of international financial integration (measured by steady-state gross asset positions) enhances risk sharing through asset markets. Though not large in absolute terms, these effects increase with higher values of ω and may have a significant impact on welfare. Valuation effects are also larger on impact (amplifying net foreign asset variation through this channel) the higher the degree of integration. However, their share of the total change in net foreign assets is smaller than in the case of home bias in equity. This suggests that the relative importance of valuation effects in net foreign asset dynamics decreases as we move from a less to a more integrated economy. Interestingly, going from home bias

[31] The responses in Figure 4 are computed using the analytical solutions in Section 3 and the normalizations defined there for variables with zero steady-state levels. For instance, the deviation of net foreign assets from the steady state is defined as $\widehat{nfa}_{t+1} \equiv dnfa_{t+1}/(vx)$, and the change in net foreign assets is the first difference of this variable (denoted DNFAH in the figure). All γ parameters are zero in the case of no intermediation cost. In this case, net and gross foreign asset positions depend on initial conditions and no longer revert to their initial levels after the shock. As we discussed in Section 3, the consumption differentials and net foreign assets become unit root processes.

[32] With intermediation costs, the current account and the change in net foreign assets become slightly negative several years after the shock to ensure the return of net foreign equity holdings and net foreign assets to the initial steady state.

in equity to international financial integration, a larger trade imbalance and a smaller contribution of income balance dynamics are more important determinants of net foreign asset changes, rather than larger valuation changes. This is seen on impact from Figure 3. The valuation change at the time of the shock is larger in absolute value under integration, but its share of the change in net foreign assets is smaller than under home bias. Even cumulating the response of net foreign assets and its components over the first 40 quarters, we find that the income balance contributes to a smaller build-up of net foreign assets (liabilities) in the home (foreign) economy under integration than home bias (Table 4). Absolute equity price differentials are smaller under financial integration, and equity prices are more correlated across countries (Table 3), consistent with our theoretical analysis. The intuition is that, with portfolio quantities less costly to rebalance, asset prices need to do a smaller job in transmission. Agents are more willing to engage in international trade in equity to smooth consumption, and asset prices play a smaller role in the transmission of productivity shocks.

When we completely remove the financial intermediation costs, while leaving the initial gross foreign asset positions unchanged at the integration scenario, the valuation share of net foreign asset changes remains larger than the (negative) current account share on impact. Lowering the intermediation cost to zero reduces the equity price differential, and hence the size of the valuation change, and the ratio $CAShare_0/VALShare_0$ becomes larger (in absolute value). This happens because it is now relatively easier to adjust asset quantities. But this does not imply that the equilibrium change in cross-border share holdings is necessarily larger when financial fees are absent. In fact, when intermediation costs are zero, the portfolio rebalancing term in equation (23) is smaller, and so is the overall change in net foreign assets. As a result, the valuation share of the initial net foreign asset change ($VALShare_0$) increases, but portfolio rebalancing plays a relatively larger role (the absolute value of $CAShare_0/VALShare_0$ increases), in comparison to the scenario with financial frictions.

Overall, removing financial frictions generates some quantitative differences, but does not affect qualitative results on impact and in the first years after a shock. The ratio of valuation change to current account becomes smaller (in absolute value), consistent with intuition. Differences across scenarios become more pronounced (and some qualitative differences emerge) only several years after a shock, once the mean reversion induced by small financial frictions has had time to play out.[33]

Specifically, $CAShare_t$ and $VALShare_t$ are constant in all periods following the initial one if financial fees are absent, as shown in Section 3. Given our choice of θ, the valuation share of net foreign asset movements after the initial impact is approximately 16.7 percent. When the γ's

[33] Details are available on request.

are set to 0.01, the valuation share increases from 18.8 percent one quarter after the shock to approximately 37 percent four years after. These numbers are not distant from the magnitude range of valuation effects in Gourinchas and Rey (2005) (for the upper end) and Gros (2006) (for the lower end).

6 Conclusions

Ongoing financial integration has greatly increased gross foreign asset holdings, enhancing the scope for a "valuation channel" of external adjustment. We examine this channel of adjustment in a dynamic stochastic general equilibrium model with international equity trading in incomplete asset markets. We show that the decentralized equity market equilibrium when equities are defined as claims to firm profits in a production economy generally does not deliver the perfect risk sharing implied by the planning optimum, unless an appropriate system of financial taxes and transfers is in place. Absent such a system, the risk-sharing properties of international equity trading in decentralized markets are tied to the distribution of income between labor income and profits. Departure from the planning optimum is necessary for both asset quantity and price changes to play a role in net foreign asset adjustment.

We also find that for given steady-state gross foreign asset positions, valuation changes in response to productivity shocks increase with the magnitude of financial frictions, the persistence of shocks, and the degree of substitutability across home and foreign goods – though the latter has no effect on the share of valuation in net foreign asset changes. Increasing the size of steady-state gross asset positions by lowering financial frictions enhances risk sharing through financial markets, but the relative importance of the valuation channel in net foreign asset dynamics decreases.

A Appendix

A.1 Aggregation and Equilibrium Household Behavior

We present here aggregate equilibrium conditions for household behavior, focusing on the home economy. Before doing that, we first define the following notation for equity holdings:

$\int_0^a x_{t+1}^{zj} dz = a x_{t+1}^{zj} \equiv x_{t+1}$: share of home equity held by the representative home household;

$\int_a^1 x_{t+1}^{z^*j} dz^* = (1-a)\, x_{t+1}^{z^*j} \equiv x_{t+1}^*$: share of foreign equity held by the representative home household;

$\int_0^a x_{*t+1}^{zj} dz = a x_{*t+1}^{zj} = x_{*t+1}$ = share of home equity held by the representative foreign household;

$\int_a^1 x_{*t+1}^{z^*j} dz^* = (1-a)\, x_{*t+1}^{z^*j} = x_{*t+1}^*$ = share of foreign equity held by the representative foreign household.

Households Equilibrium in bond markets implies that aggregate per capita bond holdings are zero in each country, since bonds are not traded internationally. Given the notation above, equilibrium in the international market for equities requires:

$$\begin{aligned} ax_{t+1} + (1-a)x_{*t+1} &= a, \\ ax^*_{t+1} + (1-a)x^*_{*t+1} &= 1-a. \end{aligned} \tag{49}$$

Equilibrium versions of household budget constraint and Euler equations in aggregate per capita terms are thus given by:

$$v_t x_{t+1} + v^*_t x^*_{t+1} + C_t = (v_t + d_t)\, x_t + (v^*_t + d^*_t)\, x^*_t + w_t, \tag{50}$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} \frac{1+i_{t+1}}{1+\pi^{CPI}_{t+1}} \right], \tag{51}$$

$$C_t^{-\frac{1}{\sigma}} v_t\, (a + \gamma_x x_{t+1}) = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} a\, (v_{t+1} + d_{t+1}) \right], \tag{52}$$

$$C_t^{-\frac{1}{\sigma}} v^*_t \left(1 - a + \gamma_{x^*} x^*_{t+1}\right) = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} (1-a) \left(v^*_{t+1} + d^*_{t+1}\right) \right], \tag{53}$$

where $v_t \equiv V_t/P_t$, $v^*_t \equiv V^*_t/P^*_t$, $d_t = D_t/P_t$, $d^*_t = D^*_t/P^*_t$, $1 + \pi^{CPI}_{t+1} \equiv P_{t+1}/P_t$, and we used PPP. Similar budget constraint and Euler equations hold abroad.

A.2 Solving for the Steady State

Steady-State Equilibrium Conditions Denoting a product with "·" where necessary for clarity, steady state equilibrium conditions are as follows:

Relative prices:

$$RP^{\omega} Z = aRP \cdot Z + (1-a)\, RP^* Z^*, \tag{54}$$

$$(RP^*)^{\omega} Z^* = aRP \cdot Z + (1-a)\, RP^* Z^*, \tag{55}$$

GDPs:

$$y = RP \cdot Z, \quad y^* = RP^* Z^*, \tag{56}$$

Real wages:

$$w = \frac{\theta - 1}{\theta} RP \cdot Z, \quad w^* = \frac{\theta - 1}{\theta} RP^* Z^*. \tag{57}$$

Real dividends:

$$d = y - w, \quad d^* = y^* - w^*. \tag{58}$$

Budget constraints:

$$vx + v^* x^* = (v + d)\, x + (v^* + d^*)\, x^* + w - C. \tag{59}$$

$$vx_* + v^* x_*^* = (v+d)\, x_* + (v^* + d^*)\, x_*^* + w^* - C^*, \tag{60}$$

Equity market equilibrium:

$$\begin{aligned} ax + (1-a)x_* &= a, & (61) \\ ax^* + (1-a)x_*^* &= 1-a. & (62) \end{aligned}$$

Households' first-order conditions for bond and equity choices (with r denoting the steady-state real interest rate):

$$\begin{aligned} 1 &= \beta\,(1+r)\,, & (63) \\ v\,(a + \gamma_x x) &= \beta a\,(v+d)\,, & (64) \\ v^*\,(1 - a + \gamma_{x^*} x^*) &= \beta\,(1-a)\,(v^* + d^*)\,, & (65) \\ v\,(a + \gamma_x^* x_*) &= \beta a\,(v+d)\,, & (66) \\ v^*\,(1 - a + \gamma_{x^*}^* x_*^*) &= \beta\,(1-a)\,(v^* + d^*)\,. & (67) \end{aligned}$$

Solution Consider the case in which $Z = Z^* = 1$. Then:

$$RP^\omega = aRP + (1-a)\,RP^* = (RP^*)^\omega\,, \tag{68}$$

implying

$$RP = RP^* = 1. \tag{69}$$

It follows that

$$y = y^* = 1, \quad w = w^* = \frac{\theta - 1}{\theta}, \quad d = d^* = \frac{1}{\theta}. \tag{70}$$

From the budget constraints,

$$C = \frac{1}{\theta}\,(x + x^*) + \frac{\theta - 1}{\theta}, \quad C^* = \frac{1}{\theta}\,(x_* + x_*^*) + \frac{\theta - 1}{\theta}. \tag{71}$$

In steady states with $x + x^* = 1$ ($x_* + x_*^* = 1$), with complete home bias or symmetric equity holdings, this simplifies to $C = C^* = 1$.

From the steady-state Euler equations (64)-(67) and asset market equilibrium, equity prices, gross returns on equity holdings, and home and foreign equity positions are, respectively:[34]

$$\begin{aligned} v &= \frac{1}{\theta}\frac{\beta a}{a(1-\beta) + \gamma_x x}, \quad v^* = \frac{1}{\theta}\frac{\beta(1-a)}{(1-a)(1-\beta) + \gamma_{x^*} x^*}, & (72) \\ 1 + \frac{d}{v} &= \frac{1}{\beta a}\,(a + \gamma_x x)\,, \quad 1 + \frac{d^*}{v^*} = \frac{1}{\beta(1-a)}\,(1 - a + \gamma_{x^*} x^*)\,, & (73) \end{aligned}$$

[34] It is easy to verify that the system of equations above leaves steady-state equity holdings (and thus consumption levels) indeterminate if $\gamma_x = \gamma_{x^*} = \gamma_x^* = \gamma_{x^*}^* = 0$.

$$x = \frac{\gamma_x^* \frac{a}{1-a}}{\gamma_x + \gamma_x^* \frac{a}{1-a}}, \quad \text{with } x_* = \frac{a}{1-a}(1-x), \tag{74}$$

$$x^* = \frac{\gamma_{x^*}^*}{\gamma_{x^*} + \gamma_{x^*}^* \frac{a}{1-a}}, \quad \text{with } x_*^* = 1 - \frac{a}{1-a} x^*. \tag{75}$$

A.3 No-Arbitrage in the Symmetric Case

Consider the equilibrium Euler equation for home holdings of foreign equity. In log-linear terms:

$$\frac{1}{\sigma} E_t \left(\hat{C}_{t+1} - \hat{C}_t \right) + \frac{\gamma}{1+\gamma} \hat{x}_{t+1}^* = -\hat{v}_t^* + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1}^* + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}^*.$$

Similarly, the equilibrium, log-linear Euler equation for foreign holdings of home equity is

$$\frac{1}{\sigma} E_t \left(\hat{C}_{t+1}^* - \hat{C}_t^* \right) + \frac{\gamma}{1+\gamma} \hat{x}_{*t+1} = -\hat{v}_t + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1} + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1},$$

and the difference between these equations yields:

$$\frac{1}{\sigma} E_t \left(\hat{C}_{t+1}^D - \hat{C}_t^D \right) + \frac{\gamma}{1+\gamma} \hat{x}_{t+1}^D = \hat{v}_t^D - \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1}^D - \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}^D. \tag{76}$$

Similarly, the log-linear Euler equations for home holdings of home equity and foreign holdings of foreign equity may be written:

$$\begin{aligned} \frac{1}{\sigma} E_t \left(\hat{C}_{t+1} - \hat{C}_t \right) - \frac{\gamma}{1+\gamma} \hat{x}_{*t+1} &= -\hat{v}_t + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1} + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}, \\ \frac{1}{\sigma} E_t \left(\hat{C}_{t+1}^* - \hat{C}_t^* \right) - \frac{\gamma}{1+\gamma} \hat{x}_{t+1}^* &= -\hat{v}_t^* + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1}^* + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}^*, \end{aligned}$$

and their difference implies:

$$\frac{1}{\sigma} E_t \left(\hat{C}_{t+1}^D - \hat{C}_t^D \right) + \frac{\gamma}{1+\gamma} \hat{x}_{t+1}^D = -\hat{v}_t^D + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1}^D + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}^D. \tag{77}$$

Inspection of equations (76) and (77) makes it possible to conclude that, for both equations to hold at the same time, it must be:

$$\frac{1}{\sigma} E_t \left(\hat{C}_{t+1}^D - \hat{C}_t^D \right) + \frac{\gamma}{1+\gamma} \hat{x}_{t+1}^D = 0, \tag{78}$$

$$-\hat{v}_t^D + \frac{\beta}{1+\gamma} E_t \hat{v}_{t+1}^D + \frac{1-\beta+\gamma}{1+\gamma} E_t \hat{d}_{t+1}^D = 0. \tag{79}$$

A.4 No-Arbitrage under Full Cross-Shareholding

Assume $\gamma_{x*} = \gamma_x^* = 0$ and $\gamma_{x*}^* = \gamma_x > 0$. In the implied asymmetric steady state with full cross-shareholding, we have:

$$v = v^* = \frac{1}{\theta}\frac{\beta}{1-\beta}, \quad x = 0 = x_*^*, \quad x_* = 1 = x^*.$$

The equilibrium Euler equation for home holdings of home equity is:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) = \beta E_t(\hat{v}_{t+1}) - \hat{v}_t + 2\gamma\hat{x}_{*t+1} + (1-\beta)E_t(\hat{d}_{t+1}). \tag{80}$$

To write this equation in log-linear form, we used the implication of the equilibrium condition: $\mathsf{d}x_{t+1} = -\hat{x}_{*t+1}x_* = -\hat{x}_{*t+1}$.

The equilibrium Euler equation for home holdings of foreign equity is:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) = \beta E_t(\hat{v}_{t+1}^*) - \hat{v}_t^* + (1-\beta)E_t(\hat{d}_{t+1}^*) \tag{81}$$

This equation has no portfolio choice term because the corresponding equity investment costs (γ's) were assumed to be zero.

Subtracting equations (80) and (81) and using $\hat{x}_{t+1}^D = -2\hat{x}_{*t+1}$ yields equation (43).

A.5 Decentralizing the Planning Optimum through Financial Fee Centering

Suppose that the fee-scaling parameters are such that $\gamma_x = \gamma_{x^*} = \gamma_x^* = \gamma_{x^*}^* = \gamma$ and quadratic fees are centered around the planner's allocation ($x^P \equiv a - (1-a)(\theta - 1)$, $x^{*P} \equiv (1-a)\theta$, $x_*^P \equiv a\theta$, and $x_*^{*P} \equiv 1 - a\theta$) instead of zero holdings. The aggregate Euler equations for equity holdings become:

$$C_t^{-\frac{1}{\sigma}}v_t\left[a + \gamma\left(x_{t+1} - x^P\right)\right] = \beta E_t\left[C_{t+1}^{-\frac{1}{\sigma}}a\left(v_{t+1} + d_{t+1}\right)\right], \tag{82}$$

$$C_t^{-\frac{1}{\sigma}}v_t^*\left[1 - a + \gamma\left(x_{t+1}^* - x^{*P}\right)\right] = \beta E_t\left[C_{t+1}^{-\frac{1}{\sigma}}(1-a)\left(v_{t+1}^* + d_{t+1}^*\right)\right], \tag{83}$$

for home households, and

$$(C_t^*)^{-\frac{1}{\sigma}}v_t\left[a + \gamma\left(x_{*t+1} - x_*^P\right)\right] = \beta^* E_t\left[\left(C_{t+1}^*\right)^{-\frac{1}{\sigma}}a\left(v_{t+1} + d_{t+1}\right)\right], \tag{84}$$

$$(C_t^*)^{-\frac{1}{\sigma}}v_t^*\left[1 - a + \gamma\left(x_{*t+1}^* - x_*^{*P}\right)\right] = \beta^* E_t\left[\left(C_{t+1}^*\right)^{-\frac{1}{\sigma}}(1-a)\left(v_{t+1}^* + d_{t+1}^*\right)\right], \tag{85}$$

for foreign households. Following the same steps as with fees centered around zero equity holdings, steady-state versions of these equations and equity market equilibrium conditions yield $x = x^P$, $x^* = x^{*P}$, $x_* = x_*^P$, $x_*^* = x_*^{*P}$ as unique deterministic steady state for the decentralized economy.

Let us turn next to the log-linearized model. Log-linear versions of equations (82)-(85) are:

$$\begin{aligned}
\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}-\hat{C}_t\right)+\frac{\gamma}{a}\left[a-(1-a)(\theta-1)\right]\hat{x}_{t+1} &= -\hat{v}_t+\beta E_t\left(\hat{v}_{t+1}\right)+(1-\beta)E_t\left(\hat{d}_{t+1}\right), &(86)\\
\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}-\hat{C}_t\right)+\gamma\theta\hat{x}^*_{t+1} &= -\hat{v}^*_t+\beta E_t\left(\hat{v}^*_{t+1}\right)+(1-\beta)E_t\left(\hat{d}^*_{t+1}\right), &(87)\\
\frac{1}{\sigma}E_t\left(\hat{C}^*_{t+1}-\hat{C}^*_t\right)+\gamma\theta\hat{x}_{*t+1} &= -\hat{v}_t+\beta E_t\left(\hat{v}_{t+1}\right)+(1-\beta)E_t\left(\hat{d}_{t+1}\right), &(88)\\
\frac{1}{\sigma}E_t\left(\hat{C}^*_{t+1}-\hat{C}^*_t\right)+\frac{\gamma}{1-a}(1-a\theta)\hat{x}^*_{*t+1} &= -\hat{v}^*_t+\beta E_t\left(\hat{v}^*_{t+1}\right)+(1-\beta)E_t\left(\hat{d}^*_{t+1}\right), &(89)
\end{aligned}$$

where we used the fact that $v/(v+d)=v^*/(v^*+d^*)=\beta$ and $d/(v+d)=v^*/(v^*+d^*)=1-\beta$ in the planner's steady state.

Log-linear asset market equilibrium conditions require:

$$\hat{x}_{*t+1} = -\frac{a-(1-a)(\theta-1)}{(1-a)\theta}\hat{x}_{t+1}, \tag{90}$$

$$\hat{x}^*_{*t+1} = -\frac{a\theta}{1-a\theta}\hat{x}^*_{t+1}. \tag{91}$$

Subtracting (88) from (86), using (90), and rearranging yields:

$$\frac{1}{\sigma}E_t\left(\hat{C}^D_{t+1}-\hat{C}^D_t\right)+\frac{\gamma\left[a-(1-a)(\theta-1)\right]}{a(1-a)}\hat{x}_{t+1}=0. \tag{92}$$

Similarly, equations (87), (89), and (91) imply:

$$\frac{1}{\sigma}E_t\left(\hat{C}^D_{t+1}-\hat{C}^D_t\right)+\frac{\gamma\theta}{1-a}\hat{x}^*_{t+1}=0. \tag{93}$$

It follows that optimal equity choices by home households are such that:

$$\hat{x}^*_{t+1}=\frac{a-(1-a)(\theta-1)}{a\theta}\hat{x}_{t+1}. \tag{94}$$

Now, log-linearizing home and foreign equilibrium budget constraints around the planner's steady state yields, respectively:

$$\begin{aligned}
&\frac{\beta}{\theta(1-\beta)}\left(x^P\hat{x}_{t+1}+x^{*P}\hat{x}^*_{t+1}\right)+\hat{C}_t\\
=\ &\frac{1}{\theta}\left(x^P\hat{y}_t+x^{*P}\hat{y}^*_t\right)+\frac{1}{\theta(1-\beta)}\left(x^P\hat{x}_t+x^{*P}\hat{x}^*_t\right)+\frac{\theta-1}{\theta}\hat{y}_t, &(95)\\
&\frac{\beta}{\theta(1-\beta)}\left(x^P_*\hat{x}_{*t+1}+x^{*P}_*\hat{x}^*_{*t+1}\right)+\hat{C}^*_t\\
=\ &\frac{1}{\theta}\left(x^P_*\hat{y}_t+x^{*P}_*\hat{y}^*_t\right)+\frac{1}{\theta(1-\beta)}\left(x^P_*\hat{x}_{*t}+x^{*P}_*\hat{x}^*_{*t}\right)+\frac{\theta-1}{\theta}\hat{y}^*_t. &(96)
\end{aligned}$$

Subtracting (96) from (95), using the expressions for steady-state equity holdings, the equity market clearing conditions (90)-(91), and equation (94) at t and $t+1$, and rearranging yields:

$$-\beta\left[\theta\left(1-a\right)-1\right]\left(\hat{x}_{t+1}-\hat{x}_{t}\right)+\theta a\left(1-a\right)\left(1-\beta\right)\hat{C}_{t}^{D}=0. \tag{97}$$

Note that optimal decentralized behavior in a neighborhood of the planner's steady state subject to fees centered around this point implies that the relation between the consumption differential and growth in home holdings of home equity is independent of home and foreign GDP. Equations (92) and (97) constitute a system of two difference equations in two endogenous variables ($\hat{x}_{t+1}$ and $\hat{C}_{t}^{D}$), which can be rewritten as:

$$\begin{bmatrix} E_t\left(\hat{C}_{t+1}^{D}\right) \\ \hat{x}_{t+1} \end{bmatrix} = M \begin{bmatrix} \hat{C}_{t}^{D} \\ \hat{x}_{t} \end{bmatrix},$$

with

$$M \equiv \begin{bmatrix} -\left\{1-\frac{\sigma\gamma\theta(1-\beta)[a-(1-a)(\theta-1)]}{\beta[1-\theta(1-a)]}\right\} & -\frac{\sigma\gamma[a-(1-a)(\theta-1)]}{a(1-a)} \\ -\frac{\theta a(1-a)(1-\beta)}{\beta[1-\theta(1-a)]} & 1 \end{bmatrix}.$$

The characteristic equation for the matrix M is:

$$J\left(e\right)\equiv\beta e^{2}-\sigma\gamma\theta\left(1-\beta\right)e-\beta=0.$$

Graphing the polynomial $J\left(e\right)$ shows that its roots are always one inside and one outside the unit circle. Hence, we can rule out sunspot equilibria, and the system (92)-(97) has unique solution $\hat{x}_{t+1}=\hat{C}_{t}^{D}=0$ in all periods, with initial condition $\hat{x}_{t}=0$. Equation (94) and equity market equilibrium then imply that holdings of all equities remain constant at the planner's steady state.

We have thus proved that a system of quadratic fees centered around the planner's equity allocation and lump-sum rebates decentralize the first-best equilibrium with perfect risk sharing.[35]

A.6 Data

The U.S. variables used in the quantitative analysis are defined and constructed as follows.

CPI: Consumer Price Index, IMF IFS, series code 62064...ZF.

PPI: Producer Price Index, IMF IFS, series code 62063...ZF.

NEER: Nominal Broad Trade-Weighted Exchange Value of the U.S. dollar, Jan. 1997=100, Haver Analytics, series FXTWB@USECON.

[35] Note that, if $\gamma=0$, the characteristic equation for the matrix M has roots 1 and -1. The unitary root signals the existence of the non-stationarity problem associated to the decentralized economy if equities ever move from the steady state. Importantly, absence of an eigenvalue outside the unit circle implies that the decentralized economy would be open to sunspot fluctuations in consumption and equities.

REER: Real Broad Trade-Weighted Exchange Value of the U.S. dollar, Jan. 1997=100, Haver Analytics, series FXTWBC@USECON.

VUS: MSCI US Equity Price Index (in U.S. dollar), Bloomberg, series MXUS.

VEXUS: MSCI World Index through 1987Q4 and MCI All Country World Index from 1988Q1 to 2004Q4 (in U.S. dollar), Bloomberg, series MXWDU and MXWOU.

GDPB$: Gross Domestic Product, Seasonally Adjusted Annual Rates (SAAR), Billion of Dollar, Haver Analytics, series code GDP@USECON.

CAB$: Balance on current account, SAAR, Billions of Dollars, Haver Analytics, series CAB@USECON.

NXB$: Net Exports of Goods and Services, SAAR, Billions of Dollars, Haver Analytics, series XNET@USECON.

NFAB$: Net Foreign Assets, Interpolated linearly from annual data, Billions of Dollars, Lane and Milesi-Ferretti (2006).

Output (GDPH): Real GDP, SAAR, Chained 2000 dollars, Haver Analytics, series GDPH@USECON.

Consumption: (CH): Real Personal Consumption Expenditures, SAAR, Chained 2000 dollars, Haver Analytics, series CH@USECON.

Trabe balance/Output (NX/GDP): NXB$/ GDPB$.

Current account/Output (CA/GDP): CAB$/GDPB$.

NFA change/Output (DNFA/GDP): First difference of NFAB$/GDPB$.

Current transfer/Output (CT/GDP): Current transfer/GDP$.

Income Balance/Output (IB/GDP): Income balance /GDP$.

Valuation Change/Output (VC/GDP): (CAB$-DNFA)/GDP$.

Foreign Equity Price (VF): (VEXUS*REER)/CPI.

Home Equity Price (VH): VUS/CPI.

All variables are percent deviations from HP-filtered trend (with smoothing parameter equal 1600). Variables are transformed in natural logarithm whenever possible. All indices are rebased so that the level in 2000 is 100. All series are quarterly, except NFA, which is interpolated.

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Figure 1. U.S. Net Foreign Assets and Cumulative Current Account

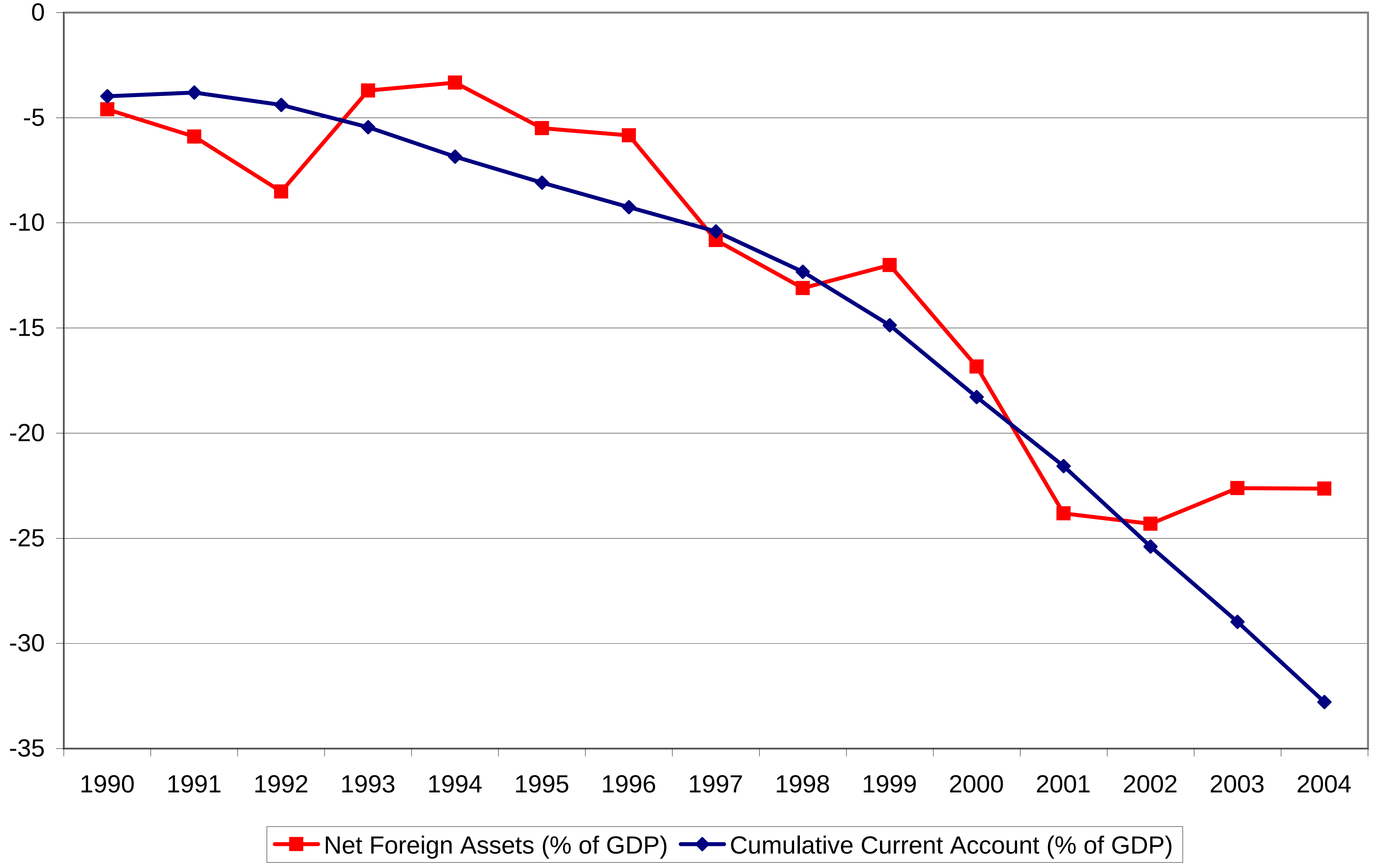

Figure 2. U.S. Gross Foreign Assets and Liabilities

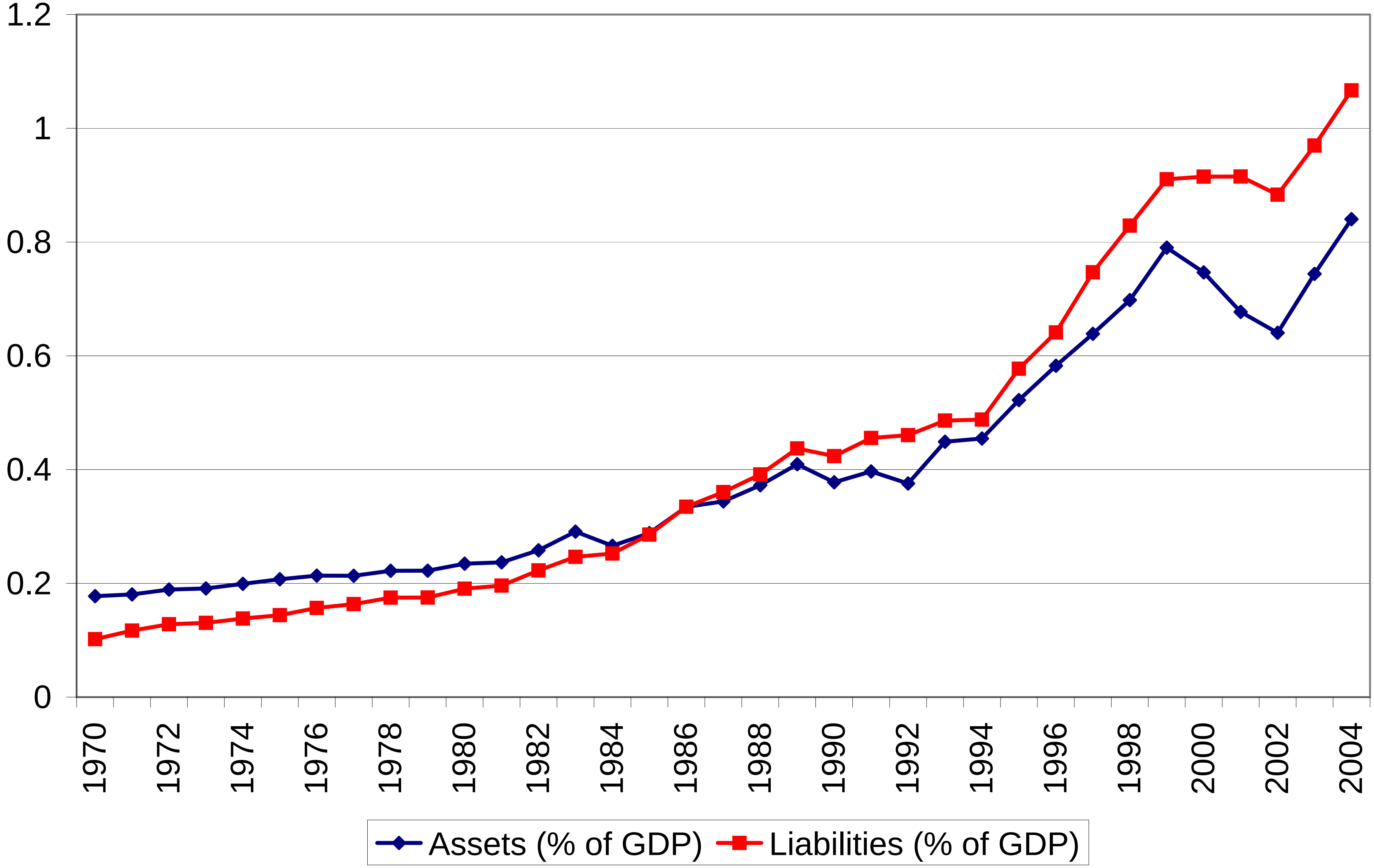

Figure 3. Home Bias in Equity versus International Financial Integration

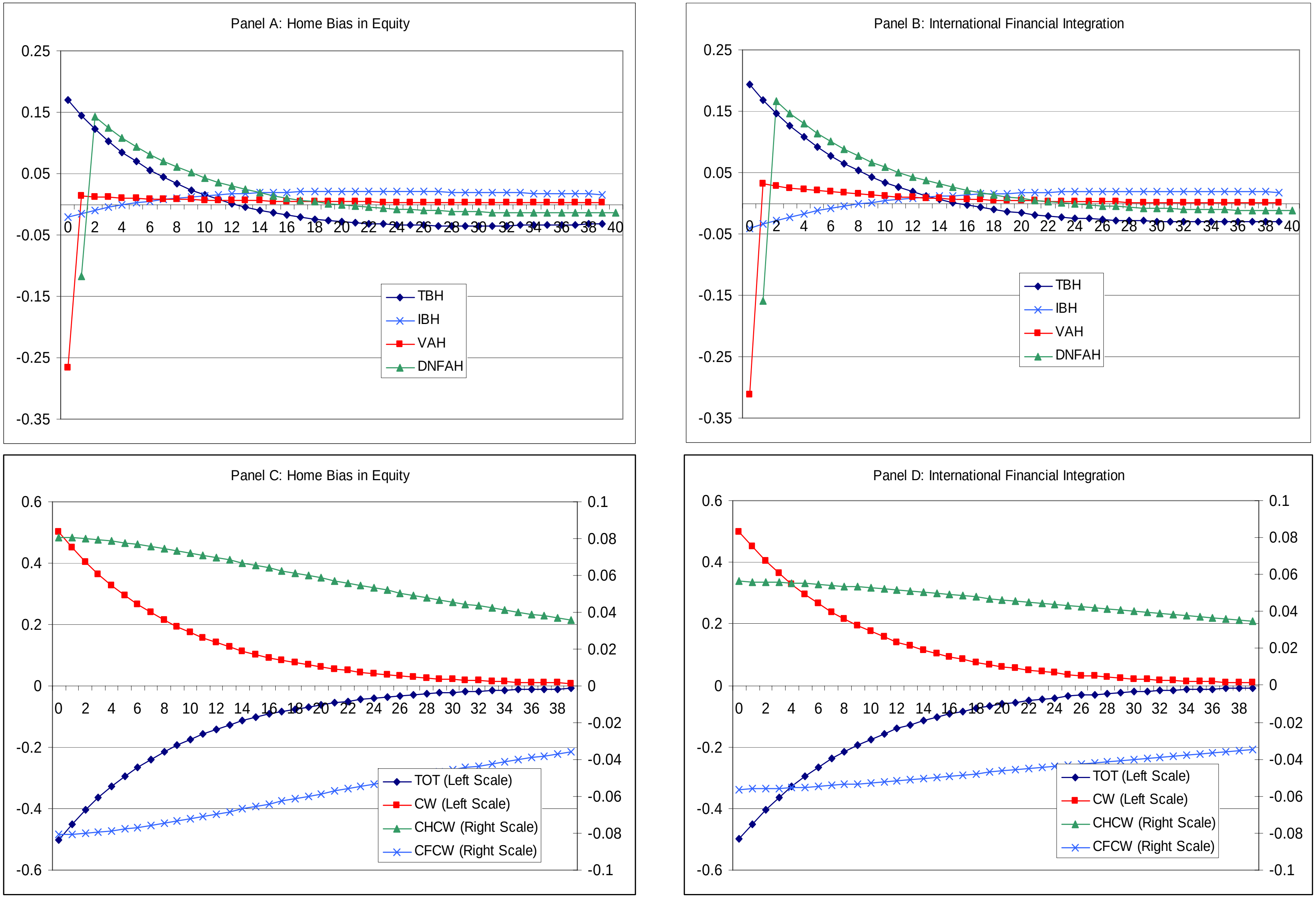

Figure 4. Intermediation Costs and Dynamics

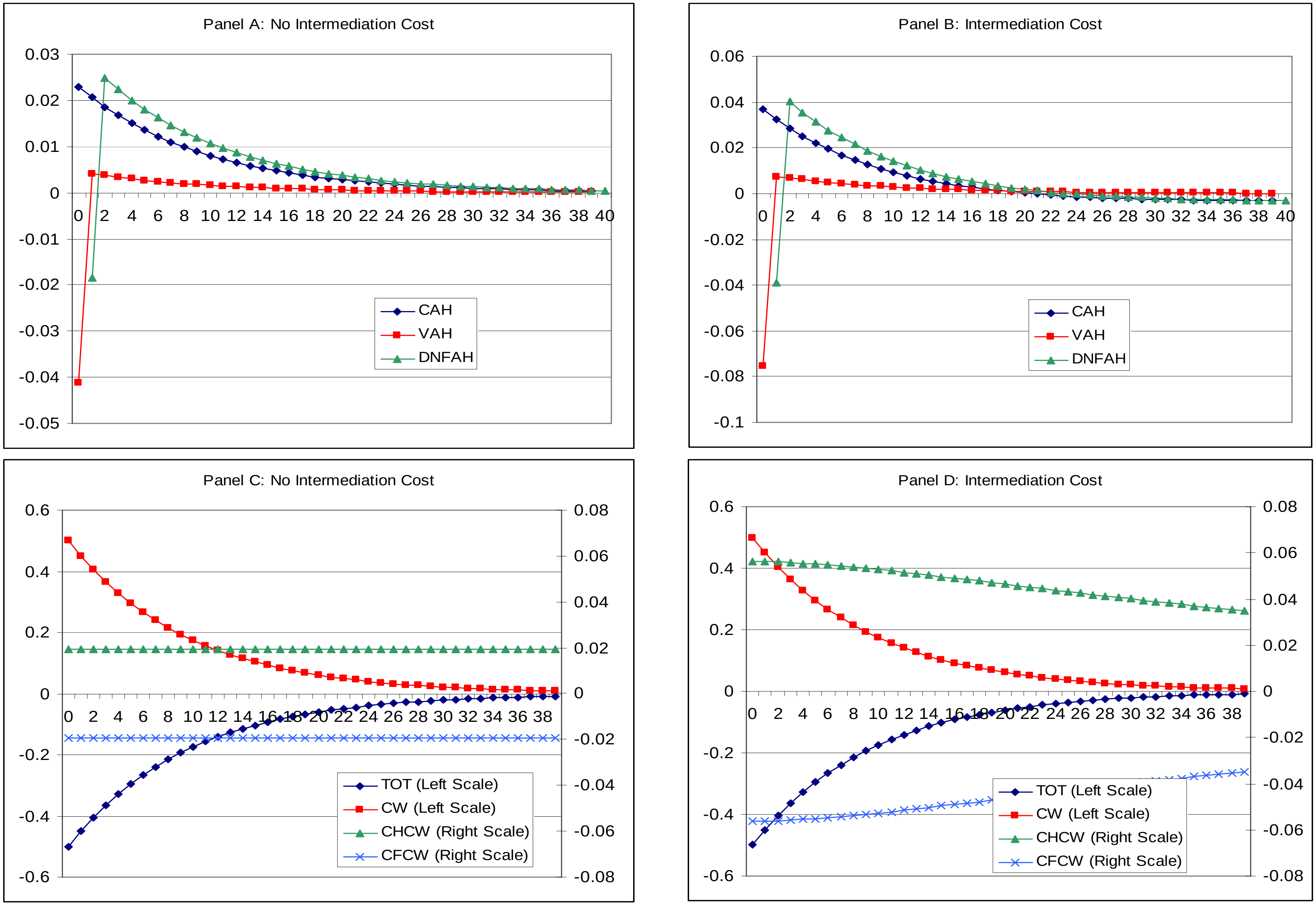

Table 1. Notation Summary

x_{t+1}^{zj} = share of *home* firm z held by home agent j entering period $t+1$.
$x_{t+1}^{z^*j}$ = share of *foreign* firm z^* held by home agent j entering period $t+1$.
x_{*t+1}^{zj} = share of *home* firm z held by foreign agent j^* entering period $t+1$.
$x_{*t+1}^{z^*j}$ = share of *foreign* firm z^* held by foreign agent j^* entering period $t+1$.
V_t^z = price of shares in profits of home firm z starting in period $t+1$.
$V_t^{z^*}$ = price of shares in profits of foreign firm z^* starting in period $t+1$.
D_t^z = dividends paid by home firm z.
$D_t^{z^*}$ = dividends paid by foreign firm z^*.
B_{t+1}^j = stock of *home* bonds held by home agent j entering period $t+1$.
B_{*t+1}^{*j} = stock of *foreign* bonds held by foreign agent j^* entering period $t+1$.
$\frac{\gamma_x}{2}\int_0^a \frac{V_t^i}{P_t}\left(x_{t+1}^{zj}\right)^2 dz$ = *home* intermediation cost of holding shares in home firms.
$\frac{\gamma_{x^*}}{2}\int_a^1 \frac{\mathcal{E}_t V_t^{i^*}}{P_t}\left(x_{t+1}^{z^*j}\right)^2 dz^*$ = *home* intermediation cost of holding shares in foreign firms.
$\frac{\gamma_x^*}{2}\int_0^a \frac{V_t^i}{P_t}\left(x_{*t+1}^{zj}\right)^2 dz$ = *foreign* intermediation cost of holding shares in home firms.
$\frac{\gamma_{x^*}^*}{2}\int_a^1 \frac{\mathcal{E}_t V_t^{i^*}}{P_t}\left(x_{*t+1}^{z^*j}\right)^2 dz^*$ = *foreign* intermediation cost of holding shares in foreign firms.

Table 2. Steady-State Portfolios and Asset Returns

	Home Bias in Equity	International Financial Integration	
		Zero Net Foreign Assets	Non-Zero Net Foreign Assets
Home Holdings of Home Shares	0.75	0.5	0.5
Home Holdings of Foreign Shares	0.25	0.5	0.5
Home Equity Price	6.6	8.3	10.3
Foreign Equity Price	6.6	8.3	8.3
Home Stock Market Capitalization$^{(i)}$	165	206.3	256.3
Home Gross Foreign Assets$^{(i)}$	41	103	103
Home Net Foreign Assets$^{(i)}$	0	0	–25.0
Home Total Assets$^{(i)}$	165	206.3	231.3
Home Equity Return on Home Shares$^{(ii)}$	9.7	7.8	6.2
Home Equity Return on Foreign Shares$^{(ii)}$	9.7	7.8	7.8
Home Real Interest Rate$^{(iii)}$	4.0	4.0	4.0
Home GDP$^{(iv)}$	1	1	1

(i) Percent of Annual GDP; (ii) Annual, Gross of Intermediation Cost, Percent; (iii) Annual, Percent; (iv) Quarterly.

Table 3. Second Moments: Data and Model

	Data, 1973:Q1-2004:Q4[i]	Home Bias in Equity	International Financial Integration	
			Zero Net Foreign Assets	Non-Zero Net Foreign Assets
		Volatility (Standard Deviation, Percent)		
GDP	1.60	1.81	1.81	1.81
Consumption	1.20	1.72	1.70	1.69
Trade Balance/GDP	0.40	0.53	0.62	0.65
Current Account/GDP	0.50	0.43	0.46	0.49
Net Foreign Asset Change/GDP	1.70	0.45	0.55	1.59
Home Equity Price	10.10	2.98	3.00	3.80
		Comovement (Contemporaneous Correlation with GDP)		
Consumption	0.85	0.96	0.94	0.93
Trade Balance/GDP	–0.50	0.31	0.35	0.36
Current Account/GDP	–0.45	0.40	0.42	0.42
Net Foreign Asset Change/GDP	–0.08	0.27	0.27	–0.08
Home Equity Price	0.41	0.94	0.91	0.91
		International Comovement (Contemporaneous Cross-Correlation)		
Home and Foreign GDP[ii]	0.28	0.60	0.60	0.60
Home and Foreign Consumption[ii]	0.15	0.77	0.83	0.84
Home and Foreign Equity Price	0.69	0.96	1.00	1.00
		Persistence (First-Order Autocorrelation)		
GDP	0.88	0.90	0.90	0.90
Consumption	0.88	0.91	0.91	0.91
Trade Balance/GDP	0.77	0.89	0.89	0.90
Current Account/GDP	0.76	0.87	0.88	0.89
Net Foreign Assets Change/GDP	0.68	0.59	0.56	0.04
Home Equity Price	0.82	0.90	0.90	0.90

(*i*) Details in Appendix; (*ii*) Source: Ambler, Cardia, and Zimmermann (2004, Table 1).

Table 4. Cumulative Impulse Responses: Net Foreign Assets and Components$^{(i)}$

	Trade Balance	Total Return$^{(ii)}$	Income Balance	Valuation Change	Net Foreign Assets
	Home Bias in Equity				
As a Share of GDP	0.12	0.47	0.53	–0.07	0.59
As a Share of Net Foreign Assets	0.21	0.79	0.91	–0.11	1
	International Financial Integration				
As a Share of GDP	0.63	0.29	0.30	–0.01	0.92
As a Share of Net Foreign Assets	0.69	0.31	0.32	–0.01	1

(i) Over the First 40 Quarters; (ii) Sum of Income Balance and Valuation Change.