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INTERTEMPORAL CONSTRAINTS, SHADOW PRICES, AND FINANCIAL ASSET VALUES

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ABSTRACT

The conditions under which the unobserved shadow price of capital can be equated to the financial value of the firm have been developed in an important paper by Hayashi (1982). Employing a more powerful analytic method, this paper reexamines the shadow price-asset value relation in a model with a general set of intertemporal constraints. For a model with one capital good, a general relation between shadow prices and asset values is derived, and restrictive assumptions implicit in previous work are highlighted. Of particular importance is the relation between the marginal and average survival rates of capital, and the critical role of geometric depreciation. The impact of a discrete-time framework in specifying and interpreting econometric models is also explored.

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INTERTEMPORAL CONSTRAINTS, SHADOW PRICES, AND FINANCIAL ASSET VALUES

I. Introduction

In dynamic models of the firm, intertemporal constraints are represented by distributed lag and convex technologies. In the presence of the former constraint (e.g., capital accumulation), shadow prices play the critical role in intertemporal allocation decisions, but are not generally observed by applied econometricians. This problem is likely to occur when the model also includes convex constraints (e.g., adjustment costs), and has been addressed in one of two ways. The first solution specifies and estimates the underlying stochastic process for the components of the unobservable, and then computes the shadow price from forecasts of its components. This class of solutions includes the two-step procedure of Abel and Blanchard (1984), the maximum likelihood estimator of Hansen and Sargent (1980), and the Euler equation technique of Hansen and Singleton (1982); in the latter case, the forecasting equations are determined by the choice of instruments. These methods arose in response to the critique of Lucas (1976), who argued that "any change in policy will systematically alter the structure of econometric models" (p. 41). Under this view, changes in policy are identified as changes in the parameters governing policy outcomes, and these parameters will generally affect forecasts of economic variables. Only if one maintains that the sample period contains no changes in policy or non-policy factors affecting the posited stochastic process will the forecasting solution to the unobserved shadow price problem be valid.

The second class of solutions, which will be the focus of this paper, relates financial asset values to the unobserved shadow prices, and possesses the substantial advantage that estimation can proceed even in an unstable stochastic environment. In an important paper, Hayashi (1982) developed the conditions under which the shadow price of capital could be equated to the value of the firm as assessed on asset markets, thus providing a formal theoretical foundation for the Tobin's (1969) Q model of investment.[1] Limited by the analytic method, however, this work did not recognize the significance of a number of implicit assumptions concerning distributed lag constraints. Employing the techniques introduced by Kamien and Muller (1976), the current study will be able to highlight the importance of these distributed lag constraints on the shadow price-asset value relation.

A general discrete-time model of the firm with one capital good but two distributed lags is developed and analyzed in Section II. The relation between shadow prices and asset values are drawn in Section III and, when restrictive assumptions concerning the distributed lag technology are relaxed, the usefulness of asset price data is severely compromised. The conditions under which asset prices remain useful to the applied econometrician and the impact of a discrete-time framework on the specification and interpretation of econometric models are also explored.

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[1] See Chirinko (1987) for a survey of Q models utilized in the study of business fixed investment. Hasbrouck (1985) has used Q to study takeovers; Lindenberg and Ross (1981), Salinger (1984), and Smirlock, Gilligan and Marshall (1984) to examine the relationship between firm rents and market structure

II. Conditions Characterizing An Optimum

In this section, a general model of the firm containing both distributed lag and convex constraints is developed. The firm chooses labor inputs and one capital input to maximize its end of period equity value (V_0), defined as the discounted sum of net revenues ($\pi[K_{t-1}]$),

$$V_0 = \sum_{t=1}^{\infty} \theta_{1,t} \; \pi[K_{t-1}], \qquad (1a)$$

$$\theta_{1,t} = \prod_{s=1}^{t} (1+\rho_s)^{-1}, \qquad (1b)$$

where $\pi[\cdot]$ is defined with respect to the optimal level of the variable labor inputs, $\theta_{1,t}$ is the discount factor between periods 1 and t, and ρ_s is the one-period discount rate. Net revenues depend positively on the capital stock available at the beginning of the current period (K_{t-1}; i.e., at the end of the previous period), and become available to the firm at the end of the current period. For notational convenience, all relative prices are assumed constant, and taxes are omitted.

The firm can increase its capital stock only indirectly through the placement of new orders (O_t) and, in maximizing (1), faces two distributed lag constraints.[2] First, the quantity of delivered capital goods (D_t) is determined through the delivery lag by current and past orders. Second, following delivery, capital depreciates over time, and the capital stock depends on a fixed set of survival weights.

[2] The model can be expanded easily to include additional distributed lag constraints - for example, the gestation lag between deliveries and increments to the productive capital stock and the expenditure lag between orders and payments.

These two constraints are represented by the following linear technologies,[3]

$$D_t = \sum_{t-b}^{s=t} \beta_{t-s} O_s, \qquad 0 \le \beta_u \le 1 \quad u=0,b \qquad (2a)$$

$$K_t = \sum_{t-d}^{s=t} \delta_{t-s} D_s, \qquad 0 \le \delta_u \le 1 \quad u=0,d \qquad (2b)$$

where b is the length of the delivery lag, d is the length of capital's useful life, and the β's and δ's represent the delivery and capital depreciation technologies, respectively. In this discrete-time framework, deliveries and net revenues (including expenditures) are received and the capital stock is altered at the end of the period. Current deliveries thus have no effect on current net revenues, and do not begin depreciating until the following period (i.e., $\delta_0=1$).

In addition to the distributed lag constraints (2), the firm faces two convex constraints representing the adjustment costs associated with placing orders and incorporating new capital into the production process.[4] These costs are represented in the model as negative arguments in the net revenue function.

In calculating the necessary conditions for an optimum, we could substitute for D_t and K_{t-1} with (2) and then differentiate (1) with respect to new orders. Even with relatively simple technologies, the resulting computations can prove complex, and economic interpretations can be obscure. A preferred alternative, introduced

[3] In general, these technologies may depend on the control and state variables or on time. Consideration of these more general representations usually leads to models that are empirically intractable, and would thus obfuscate the main results in this paper.
[4] Adjustment costs were analyzed initially in the models of Eisner and Strotz (1963) and Lucas (1967), and are an intergal element in the Tobin's Q investment model (Abel, 1979; Hayashi, 1982).

by Kamien and Muller (1976), is to append the constraints to (1) with multipliers ϕ_t and λ_t, and form the Lagrangian,

$$\begin{aligned} \pounds = & \sum_{t=1}^{\infty} \theta_{1,t} \{\pi[K_{t-1}, O_t, D_t] - \phi_t D_t - \lambda_t K_t\} \\ & + \sum_{t=1}^{\infty} \theta_{1,t} \ \phi_t \sum_{s=1}^{t} \beta_{t-s} O_s \\ & + \sum_{t=1}^{\infty} \theta_{1,t} \ \lambda_t \sum_{s=1}^{t} \delta_{t-s} D_s \\ & + \sum_{t=1}^{\infty} \theta_{1,t} \{\phi_t D_{0,t} + \lambda_t K_{0,t}\}. \end{aligned} \qquad (3)$$

The sums for the delivery and depreciation lags have been split into two parts to separate variables that depend on current decisions and those that are predetermined (though not necessarily constant) from the beginning of period 1 onward. These predetermined variables are defined as follows,

$$D_{0,t} = \sum_{t-b}^{s=0} \beta_{t-s} O_s, \qquad 0 \leq t \leq b \qquad (4a)$$

$$K_{0,t} = \sum_{t-d}^{s=0} \delta_{t-s} D_s. \qquad 0 \leq t \leq d \qquad (4b)$$

Since $D_{0,t}$ and $K_{0,t}$ represent precommittments made prior to period 1, they will not affect optimal choices over the planning horizon, but will be important in linking the financial value of the firm to the shadow price of capital.

To facilitate the calculation of the necessary conditions for an optimum, it will prove convenient to utilize the following transformation,[5]

$$\sum_{t=1}^{\infty} \theta_{1,t}\ \psi_t \sum_{s=1}^{t} \gamma_{t-s}\, X_s \ = \ \sum_{t=1}^{\infty} \theta_{1,t}\ X_t \sum_{s=t}^{t+c} (\theta_{t,s}/\theta_{t,t})\ \gamma_{s-t}\, \psi_s\ , \qquad (5)$$

where $\gamma_{t-s} = \{\beta_{t-s}, \delta_{t-s}\}$,

$c = \{b, d\}$,

$X_t = \{O_t, D_t\}$,

$\psi_t = \{\phi_t, \lambda_t\}$.

With (3) and (5), we construct the following current-value Hamiltonian,

$$H[K_t, D_t, O_t, \lambda_t, \phi_t, t] =$$

$$\theta_{1,t} \{\ \pi[K_{t-1}, D_t, O_t]\ -\ \phi_t D_t\ -\ \lambda_t K_t$$

$$+\ O_t \sum_{s=t}^{t+b} (\theta_{t,s}/\theta_{t,t})\ \beta_{s-t}\ \phi_s \qquad (6)$$

$$+\ D_t \sum_{s=t}^{t+d} (\theta_{t,s}/\theta_{t,t})\ \delta_{s-t}\ \lambda_s\ \}. \qquad t \geq 1$$

The necessary conditions for an optimum are computed by differentiating $H[\cdot]$ with respect to the state (K_t, D_t) and control (O_t) variables,[6]

[5] The transformation follows directly by stating (5) in matrix notation and then transposing the scalar expression.

[6] The necessity of these conditions for an optimum has been established by Kleindorfer, Kleindorfer, and Thompson (1977) and Weitzman and Schmidt (1971).

$$K: \quad \lambda_u = \pi_K[u+1] / (1+\rho_{u+1}), \qquad \pi_K[\cdot] > 0 \qquad (7a)$$

$$D: \quad \phi_s = \sum_{u=s}^{s+d} (\theta_{s,u}/\theta_{s,s})\, \delta_{u-s}\, \lambda_u + \pi_D[s], \qquad \pi_D[\cdot] \leq 0 \qquad (7b)$$

$$O: \quad -\pi_O[t] = \sum_{s=t}^{t+b} (\theta_{t,s}/\theta_{t,t})\, \beta_{s-t}\, \phi_s, \qquad \pi_O[\cdot] \leq -1 \qquad (7c)$$

$$O: \quad \pi_{O,O}(t) \leq 0, \qquad (7d)$$

$$\lim_{t\to\infty} \theta_{1,t}\ \phi_t\ D_t \to 0 \qquad \lim_{t\to\infty} \theta_{1,t}\ \lambda_t\ K_t \to 0 \qquad (7e)$$

These conditions have the following economic interpretations. In (7a), the marginal value of an additional unit of productive capital in a <u>given time period</u> equals next period's marginal product discounted by the cost of capital. This formulation arises because production is affected by the capital stock available at the beginning of the period and net revenues accrue at the end of the period. By weighting these λ's over the useful life of the capital good and subtracting the adjustment costs associated with deliveries, we obtain in (7b) the marginal value of an additional delivery, ϕ_s, the shadow price of new capital. The counterbalancing marginal cost from placing an order is stated on the left side of (7c), and is equated to the sum of shadow prices weighted by delivery lag parameters and discount factors. In addition, the firm's optimal policy is constrained by the intertemporal technologies for deliveries (2a) and capital (2b). In regard to the depreciation technology, it should be noted that the δ's are relatively unrestricted, and an advantage of the model developed here is that, rather than assuming depreciation occurs geometrically, as has been done in most previous studies, alternative patterns, such as straight-line or "one-hoss-shay," can be considered. Throughout we assume that the constraints are always binding and the optimal level of orders is always non-negative. Since the constraints are linear, the Kuhn-Tucker constraint qualifications are satisfied, and the sufficiency of (7) depends on the properties of $\pi[\cdot]$.

III. The Unobserved Shadow Price Of Capital And The Financial Value Of The Firm

The above characterization of the firm's optimal policy is not useful in empirical work because it contains variables unobserved by the applied researcher.[7] In the present context, this problem arises because ϕ_t, depends on the path of λ's extending far into the future. In Proposition I, we explore the relation between the unobserved shadow price of capital and the financial value of the firm in a discrete-time framework with two distributed lag constraints.

Proposition I

$$V_0 = \sum_{t=1}^{m} \theta_{1,t} \{\phi_t D_{0,t} + \lambda_t K_{0,t}\} \qquad m = \max[b,d] \qquad (8)$$

if and only if

$\pi[\cdot]$ is homogeneous of degree one in all arguments and the output and factor markets are perfectly competitive.

Derivation

The homogeneity of $\pi[\cdot]$ implies that

$$\theta_{1,t} \{ \pi[t] - \pi_O[t]\, O_t - \pi_D[t]\, D_t - \theta_{t+1,t+1}\, \pi_K[t+1]\, K_t \} = 0. \qquad (9a)$$

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[7] While the firm has been endowed heretofore with perfect foresight, the model can be recast with no loss in generality as one in which the firm maximizes expected profits, where expectations are conditioned on information known when decisions are being undertaken.

Sum over t from 1 to ∞, and substitute (7a), (7b), and (7c),

$$\sum_{t=1}^{\infty} \theta_{1,t} \{ \pi[t] + O_t \sum_{s=t}^{t+b} (\theta_{t,s}/\theta_{t,t}) \beta_{s-t} \phi_s - \phi_t D_t + D_t \sum_{s=t}^{t+d} (\theta_{t,s}/\theta_{t,t}) \delta_{s-t} \lambda_s - \lambda_t K_t \} = 0. \tag{9b}$$

The two inner sum in (9b) can be transformed according to (5),

$$\sum_{t=1}^{\infty} \theta_{1,t} \pi[t] = \sum_{t=1}^{\infty} \theta_{1,t} \{ \phi_t [D_t - \sum_{s=1}^{t} \beta_{t-s} O_s] + \lambda_t [K_t - \sum_{s=1}^{t} \delta_{t-s} D_s] \}. \tag{9c}$$

In (9c), the left side is the value of the firm (1a). On the right side, the first term in braces represents deliveries from orders made prior to the beginning of the planning period ($D_{0,t}$; 4a); a similar interpretation applies to the second braced term in regard to capital ($K_{o,t}$; 4b). Noting that $D_{0,t}$ and $K_{0,t}$ are zero for $t \geq m = \max[b,d]$, we obtain (8). The converse follows directly. //

In (9), the asset value of the firm does not identify the shadow price of capital, as V_0 depends on discount factors, shadow prices, deliveries, and capital stocks extending m periods into the future. The intuition underlying this result is that, since markets are competitive, the firm is unable to earn any economic profits from period 1 onward. The cash flows on the precommitted stocks represent quasi-rents that are measured as the product of anticipated deliveries and existing capital multiplied by their shadow prices. The importance of the constant returns to scale assumption is that these shadow prices are independent of the stocks.[8]

That the asset value of the firm is not a sufficient statistic for identifying the shadow price of capital stems from two independent factors. The first is that the financial markets generate only one variable to evaluate the firm's profit possibilities, but these are affected by the two state variables following from the distributed lag constraints Multiple distributed lags arise when production depends on multiple capital stocks with different technologies (Chirinko, 1982; Wildasin, 1984) or, as highlighted in this model, when there is only one capital stock but other constraints impinge on the firm. However, the inadequacy of asset values extends beyond the number of distributed lags, and also involves the nature of the distributed lag parameters. If we assume that capital is delivered immediately upon order (i.e., $\beta_0=1$, $\beta_u=0$, $u=1,b$), $D_{0,t} = 0$, and we obtain the following relation,

$$V_0 = \sum_{t=1}^{d} \theta_{1,t} \lambda_t K_{0,t} . \qquad (10)$$

In the case of only one distributed lag but with unrestricted parameters, (10) indicates that the asset value remains unable to identify the shadow price of capital. This important result is masked when the analysis is conducted in terms of standard capital transition equations that depend on geometric depreciation.

The above considerations lead to the following proposition.

[8] If relative prices were allowed to vary, then the assumption of perfectly competitive markets would be needed to ensure their independence from current decisions.

Proposition II

Given the assumptions in Proposition I and the absence of a delivery lag, the financial value of the firm and the shadow price of capital are related as follows,

$$V_0 = (\phi_1 (1-\delta) K_0) / (1+\rho_1), \tag{11}$$

if and only if

capital depreciates according to a geometric pattern.

Derivation

Rewrite (10) as,

$$V_0 = K_0 \sum_{t=1}^{d} \theta_{1,t} \Delta_t \lambda_t , \tag{12a}$$

$$K_0 = \sum_{t-d}^{s=0} \delta(-s) D_s, \tag{12b}$$

$$\Delta_t = K_{0,t} / K_0. \tag{12c}$$

Note that Δ_t is the percentage of the capital stock from the beginning of the planning period surviving in period t (i.e., the average survival rate) and, in general, depends on the history of delivered capital. Consider the following functional equation,

$$\delta_{t-s} = \delta_t \delta_{-s}, \tag{12d}$$

and its relation to the Δ's in (12a) and the δ's in (7b),

$$\Delta_t = \sum_{t-d}^{s=0} \delta_{t-s} D_s / \sum_{t-d}^{s=0} \delta_{-s} D_s = \delta_t \sum_{t-d}^{s=0} \delta_{-s} D_s / \sum_{t-d}^{s=0} \delta_{-s} D_s = \delta_t \tag{12e}$$

Equation (12d) is Cauchy's functional equation of the exponential function, and all non-trivial solutions to (12d) imply that the δ's follow a geometric pattern (Eichhorn, 1978, Chapter 1.4),[9]

$$\delta_{t-s} = (1-\delta)^{t-s} = (1-\delta)^{t} (1-\delta)^{-s} = \delta_t \delta_{-s} \quad \text{and} \quad d \rightarrow \infty. \tag{12f}$$

Equation (12a) can be written as follows,

$$V_0 = K_0 \sum_{t=1}^{\infty} \theta_{1,t} (1-\delta)^{t} \lambda_t. \tag{12g}$$

By (7b) with $\pi_D[\cdot] = 0$ and (1b), we obtain (11). The converse follows directly. //

The key condition underlying the derivation of Proposition II is that the <u>average</u> survival weights for existing capital (Δ_t) equal the <u>marginal</u> survival weights for new capital (δ_t), which are independent of past acquisitions.[10] Equation (11) states that the value of the firm at the beginning of the period equals the available capital stock multiplied by the marginal gross return per unit of capital adjusted for depreciation.[11] Since returns become available

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[9] The two trivial solutions are $\delta_{t-s}=0$ $\forall t,s$, which is of no economic interest, and $\delta_0=1$ and $\delta_{t-s}=0$ $\forall t,s$ $t\neq s$, which defines a non-durable factor of production.

[10] The relation between geometric depreciation and the constancy of the average replacement rate has been noted by Jorgenson (1974).

[11] This adjustment is needed because existing capital depreciates immediately while new capital begins depreciating the following period. Since ϕ_t is the shadow price for new capital, it must be adjusted for one period of depreciation.

at the end of the period, they are discounted by $(1+\rho_1)$.[12]

These adjustments for depreciation and discounting go unnoticed in continuous-time models, and may have a measurable effect on estimated investment models of the Tobin's Q variety. In this class of models, ϕ_t is the key regressor, and has been equated in previous studies to Tobin's Q, defined as "the ratio of the market value of *existing* capital to its replacement cost" (Hayashi, 1982, p. 214, italics in the original):

$$\phi_t = Q^{cvl} = V_{t-1} / K_{t-1} , \qquad (13a)$$

This conventional specification of Q differs from the modified definition following from the discrete-time framework (Proposition II),[13]

$$\phi_t = Q^{mod} = ((1+\rho_t) V_{t-1}) / ((1-\delta) K_{t-1}) , \qquad (13b)$$

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12 Note that the presence of $(1-\delta)$ and $(1+\rho_1)$ in (11) is independent of the assumption that only capital available at the beginning of the period affects current production. This timing convention is reflected solely in the definitions of $\pi[.]$ (1a) and λ_t (7a). If current deliveries were permitted to affect current production through the capital stock, then $\lambda_t = \pi_K[t]$.

13 Differences between the conventional and modified definitions of Q would disappear if we maintained the unreasonable assumption that the firm receives its cash flows at the beginning of the period and the more palatable assumption that new capital begins depreciating immediately upon delivery. In this model, the latter timing assumption does not comport well with the exclusion of current deliveries from affecting current production (cf., (1a) and (2b)).

and has two implications for the estimated coefficient on ϕ_t. First, this coefficient is usually interpreted as depending solely on the adjustment cost technology.[14] Equation (13b) reveals, however, that it is properly interpreted as a combination of adjustment cost and depreciation parameters. Furthermore, econometric Q models have uniformly generated estimates of the ϕ_t coefficient that imply unreasonably sluggish responses of investment to variations in the economic environment.[15] One explanation for this unsatisfactory result is that Q^{cvl} is excessively volatile relative to the time series for investment. However, insofar as the firm's discount rate and financial market value are negatively correlated, Q^{mod} will be less volatile, and may lead to more reasonably estimated parameters.

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[14] See Hayashi (1982, p. 218) or equation (7c) with separability between the adjustment cost and production technologies.

[15] See the simulations in Summers (1981) in which "only three-fourths of the ultimate adjustment of the capital stock takes place within twenty years" (p. 101).

IV. SUMMARY AND CONCLUSION

This paper has highlighted the importance of distributed lag constraints on the shadow price-asset value relation. In a model with only one capital good, there nonetheless exists a number of distributed lag constraints that lead to additional state variables impairing the usefulness of asset values in empirical work (Proposition I). Even when the model was reduced to one state variable, the unrestricted nature of the distributed lag parameters precluded the firm's financial value from identifying the unobserved shadow price of capital. In order for asset prices to regain their role in empirical work, it was required that the survival weights on capital follow a geometric pattern (Proposition II). With the implicit assumptions concerning distributed lags made clear, these results indicate to the applied researcher the conditions under which the substantial information contained in asset prices can be exploited. If delivery, expenditure, or gestation lags are viewed as important to the problem under study, then asset prices will not prove useful in estimation, and other methods of solving the unobservable expectations problem will have to be utilized.[16] However, these distributed lags will be of less importance at lower frequencies and, thus, the estimation of Q investment models is most likely to be successful when conducted with annual data.

16 Empirical studies that incorporate these distributed lags and are based explicitly on an optimizing framework are rare. See Chirinko (1987, Table II) for a review of the intertemporal constraints used in previous investment studies, and Chirinko (1984) for a model estimating distributed lag parameters with a two-step forecasting method.

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