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INVESTMENT, DEPRECIATION AND CAPITAL UTILIZATION

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ABSTRACT

The purpose of this paper is to analyze the determinants of capital durability and utilization and their interdependence with investment decisions. The approach is based on the view that the flow of undepreciated capital is an output to be used in future production. At each date capital and non-capital inputs are combined to produce current output and the capital inputs to be used for future production. Thus capital accumulation occurs in a joint product context as two kinds of output are produced, one type for current sale and one type for future production.

Another issue investigated in this paper concerns the allocation of resources within a firm between installing and utilizing capital and labor training activities. Often this problem is ignored in the theory of investment, not only because depreciation is exogenous, but also due to the treatment of labor as a variable factor of production. However, it is well recognized that firms cannot costlessly adjust labor. Thus the second purpose of this paper is to analyze the intertemporal relationship between the durability of capital and the growth rate of labor.

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1. Introduction*

The analysis of investment has generally ignored the interdependence between capital depreciation, utilization, and accumulation. Utilization is usually assumed to be costless (and hence there is no incentive for a firm to retain idle capital), while depreciation is often assumed to be constant. Recently, investment theory has been extended to account for costly utilization. A. Abel [1981] and J. Bernstein [1983] have characterized the determination of utilization by the trade-off between output expansion and a higher wage bill.[1] In this framework, the wage rate varies with the utilization rate, but decisions on capital utilization were not forward looking or did not involve an intertemporal dimension because depreciation was assumed to be constant. In other words, the lifetime of capital was unaffected by the rate at which the factor was utilized.

The purpose of this paper is to analyze the determinants of capital depreciation and utilization and their interdpendence with investment decisions. Research on depreciation and utilization by K. Smith (1970), P. Taubman and M. Wilkinson (1970) and W. Oi (1981) has emphasized the dependence of depreciation on the utilization rate. This rate was determined by balancing the increase in current output against the increase in depreciations costs. However, decisions affecting capital depreciation influence not only current but also future production through their effect on investment demand. In this paper we also incorporate into the theory of investment a more general view of capital utilization. The approach, first developed by J. Hicks (1946), E. Malinvaud (1953), and later by C. Bliss (1975) and E. Diewert (1980), characterizes the flow of undepreciated capital as a current output to be used as an input in the future. At each date, capital and non-capital inputs are combined to produce current output and the capital inputs to be used for future production. Thus capital accumulation occurs in a joint product context, as two kinds of output are produced: one type for current sale and one type for future production. Epstein and Denny (1980) estimated a short-run model incorporating undepreciated capital as an output. Their interest was in the estimation of short-run factor demand and output supply functions and not with the dynamics and comparative dynamics associated with choices relating to capital depreciation, utilization, and accumulation.

The second major purpose of this paper is to model the stylized facts obtained by M. Foss (1981) and the estimation results due to M.I. Nadiri and S. Rosen (1969). First, Foss found that, as the wage rate increased, the rate of capital utilization increased while the growth rates of capital and labor declined. Second, as product demand grew the growth rates of capital and labor along with the rate of utilization increased. Third, Nadiri and Rosen estimated that the capital utilization rate exhibited a dynamic adjustment process. They found that the utilization rate not only interacts with the rates of capital and labor accumulation but can indeed be characterized by a flexible accelerator adjustment process. This process was the same type as estimated for capital and labor investment. In this model these three results are established. The rate of capital utilization adjustment is governed by a flexible accelerator. In response to unanticipated supply side shocks, which relate to changes in wage and interest rates, there is counter movement between the utilization rate on the one side and capital and labor growth rates on the other side. As unanticipated demand side shocks occur, there is comovement in the rates of utilization, capital and labor investment. Moreover, these results are obtained in this model both in long-run equilibrium and along the dynamic adjustment path.

In Section 2 of this paper the model is developed and the nature of the short-run equilibrium is established. The dynamic properties and the steady state are analyzed in the third section. In part 4 the comparative steady state and dynamic results are obtained for unanticipated changes in product demand and input supply conditions. Lastly, we summarize and conclude the paper.

2. The Model and Short-Run Equilibrium

A production process is represented by

$$(1) \qquad y(t) = F(K_N(t),\ L(t),\ K_0(t))$$

where $y(t)$ is the output quantity, $K_N(t)$ and $L(t)$ are the quantities of the capital and labor inputs respectively, $K_0(t)$ is the quantity of the capital output, F is the twice continuously differentiable function which is homogeneous of degree one, with positive and diminishing marginal products in the two inputs while increases in the capital output decrease output at a decreasing rate. Thus $F_N > 0$, $F_L > 0$, $F_0 < 0$, $F_{NN} < 0$, $F_{LL} < 0$, $F_{00} < 0$.

The inputs $K_N(t)$ and $L(t)$ are combined to produce the joint products $y(t)$ and $K_0(t)$. The former output is produced for current sale and the latter is to be used for future production. The variable $y(t)$ can be referred to as the final product or output in the current period, while $K_0(t)$ represents an intermediate product which is used in production in the next period. The endogeneity of capital utilization is captured indirectly through the selection of the capital output. The choice regarding capital available for future production reflects decisions on the utilization and also the maintenance or reparation of the capital input in current production. The specific process delimiting utilization and reparation is embedded or internal to the production process and is captured by the production function.

There are two ways in which capital becomes available for future production: internal investment (which is nonutilization or reparation) and external investment (acquisition). This implies that capital accumulates according to

$$(2) \qquad \dot{K}_N(t) = I_N(t) + K_O(t) - K_N(t), \qquad K_N(0) = K_N^o > 0$$

where $I_N(t)$ is gross investment in capital. Equation (2) generalizes the standard formulation of capital accumulation. This can be seen by noting that the depreciation rate is $(K_N(t) - K_O(t))/K_N(t) = \delta(t)$. Thus, equation (2) can be rewritten as $\dot{K}_N(t) = I_N(t) - \delta(t)K_N(t)$. If $\delta(t)$ is time invariant and exogenous, then depreciation occurs in the usual manner. The depreciation rate represents the net outcome regarding the implied decisions on the capital utilization and reparation rates. In this model it is assumed that capital output is nonnegative and does not exceed the capital input. Hence $0 \leq \delta(t) \leq 1$, which implies that the reparation rate never exceeds the utilization rate. The depreciation rate can thereby be considered as synonymous with the net utilization rate.

The definition of $\delta(t)$ enables the production function to be written as $y(t) = F[K_N(t), L(t), (1-\delta(t))K_N(t)]$, where $\delta(t)K_N(t)$ is the depreciated or net utilized capital. In this model, following Nadiri and Rosen (1969), and Taubman and Wilkinson (1970), it is assumed that the marginal product of capital $(F_N > 0)$ is not necessarily equal to the marginal product of utilized capital $(-F_0 > 0)$.[2]

As emphasized by Oi (1962), Nadiri and Rosen (1969), Abel (1981) and Bernstein (1983), labor is also treated as a quasi-fixed factor in this model, but because the focus is on capital utilization and depreciation, we assume that

$$(3) \qquad \dot{L}(t) = I_L(t) - \mu L(t), \qquad L(0) = L^{\circ} > 0.$$

$I_L(t)$ is gross investment in labor, and $0 \leq \mu \leq 1$ is the fixed rate of labor departure, reflecting in a simple way quits, retirements, firings, and layoffs.[3]

The distinction between capital stock and flow decisions can be noted from equations (1) and (2). At any time, the capital stock to be used in current production is predetermined. This means that there exists a given bundle of capital services which is embedded in the stock of capital. The flow of services from the capital stock actually used or capital utilization is selected and combined with labor services to produce current output. The choice on utilization is captured indirectly through the decision on the capital output or the flow of capital services available for production in the next period. The additions to the stock of capital consist of newly acquired capital (or gross investment) and the difference between the stock of capital available for future production and the amount that was available for current production.

There are adjustment costs associated with the quasi-fixed factors, which are internal to and separable from the production process (see R. Lucas (1967), J. Gould (1968), A. Treadway (1971), D. Mortensen (1973) and L. Epstein (1982)). These costs affect the flow of funds which can be represented as

$$(4) \quad V(t) = p(t)y(t) - w(t)L(t) - C(I_N(t)/K_N(t))I_N(t) - D(I_L(t)/L(t))I_L(t)$$

where $V(t)$ is the flow of funds, $p(t)$ is the product price, $w(t)$ is the wage for labor, C is the twice-continuously differentiable unit capital adjustment cost function with $C(0) = p_N(t)$, $C' > 0$ for $I_N(t) > 0$ and $p_N(t)$ is the exogenous purchase price of capital. In addition, total capital adjustment costs are strictly convex in $I_N(t)$. The unit labor adjustment cost function, D, has the same properties as the unit capital adjustment cost function except $D(0) = 0$. The adjustment costs for capital and labor are internal but separable from the production technology and arise from the installation of capital and labor into the production process.[4]

The objective is to maximize the present value of the flow of funds, which is discounted by the interest rate r, subject to equations (1)-(3). Capital output and gross investment in capital and labor are selected in order to carry out this program.[5] The Hamiltonian is

$$(5) \quad H = pLf(k,(1-\delta)k) - wL - C(I_N/K_N)I_N - D(I_L/L)I_L + q_1(I_N-\delta K_N) + q_2(I_L-\mu L),$$

where $y = Lf(k,(1-\delta)k)$ is derived from equation (1) using the homogeneity condition on the technology, $k = K_N/L$ is the capital intensity and f_i $i = 1,2$ are the derivative of the production function defined in intensive form. In addition, q_1 is the capital investment shadow or demand price and q_2 is the labor investment shadow price. These variables also represent the price of installed or unutilized capital and the price of integrating labor.

The first order and canonical conditions are,

(6.1) $$\partial H/\partial K_0 = pf_2 + q_1 = 0$$

(6.2) $$\partial H/\partial I_N = -C'I_N/K_N - C(I_N/K_N) + q_1 = 0$$

(6.3) $$\partial H/\partial I_L = -D'I_L/L - D(I_L/L) + q_2 = 0$$

(6.4) $$\dot{k} = k(I_N/K_N - \delta - I_L/L + \mu)$$

(6.5) $$\dot{q}_1 = (r+\delta)q_1 - p(f_1 + (1-\delta)f_2) - C'(I_N/K_N)^2$$

(6.6) $$\dot{q}_2 = (r+\mu)q_2 - p(f(k,(1-\delta)k) - kf_1 - (1-\delta)kf_2) - D'(I_L/L)^2 + w.$$

To understand the implications of the equilibrium conditions, consider the short run or temporary equilibrium.[6] This equilibrium is defined for given k, q_1 and q_2, by equations (6.1)-(6.3). First equation (6.1) shows the determination of the allocation of the given stocks of capital and labor between current output and capital output. This is illustrated in (K_0,y) space in Figure 1. The slope of the product transformation curve is $f_2 < 0$, and since f is strictly concave, the curve is also strictly concave. The slope of the isorevenue

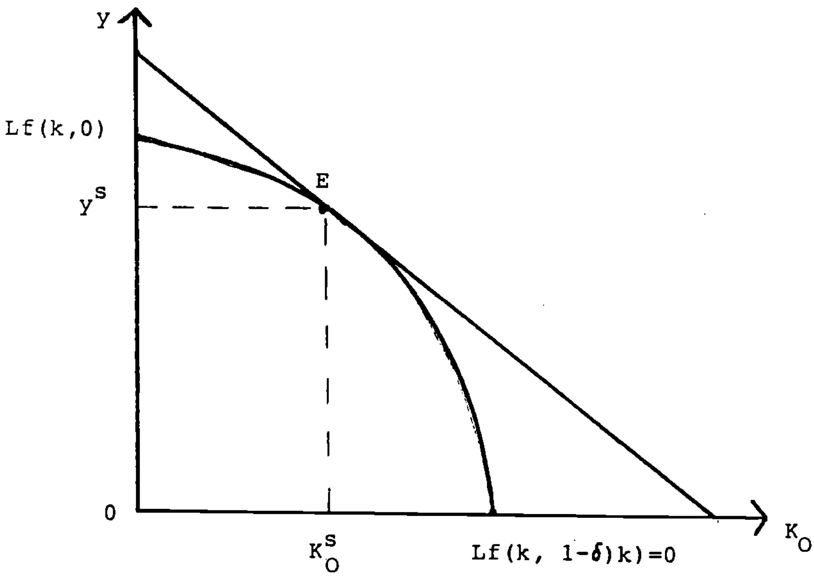

Figure 1. Short Run Equilibrium for Current Output and Capital Output

line in Figure 1 is $-q_1/p$. Given p, q_1, K_N, and L equation (6.1) represents the condition for short-run revenue maximization by choosing y and K_0 subject to the technology.[7] The trade-off is between current output and future output, manifested by the stock of unused capital available for future production. In equilibrium the relative marginal cost $(-f_2)$ is equal to the relative product price (q_1/p). Moreover, since the equilibrium magnitudes depend on the shadow price q_1, the allocation decision is forward looking, as this price equals the present value of the marginal benefits from installed capital or capital available for future production. This means that the utilization of capital embodied in the selection of K_0 is an investment decision.

Alternatively, equation (6.1) can be viewed as the short run solution for the depreciation or the net utilization rate δ, which depends on the capital intensity, the product price and the price of installed or unutilized capital. For an increase in q_1, the marginal value of unutilized capital rises, and as a consequence, the net utilization rate falls. This, of course, implies that current output decreases. The converse occurs for an increase in the product price p.[8] Lastly, an increase in the capital intensity generates the following effect on δ, from equation (6.1),

$$(7) \qquad \partial\delta/\partial k = (f_{21} + f_{22}(1-\delta))/f_{22}k.$$

The sign of the right side of (7) depends on f_{21}, since $f_{22} < 0$. It is assumed that $f_{21} \leq 0$. The reasonableness of this assumption can be noted from $-f_1/f_2 > 0$ which is the marginal product of capital in the production of capital for future use. Generally it is assumed that marginal products diminish. Therefore it is assumed that as the capital input (K_N) increases, the marginal product of capital decreases in the production of the capital output (K_0). A sufficient condition for this marginal product to diminish is that $f_{21} \leq 0$. Hence the right side of (7) is positive.[9]

Summarizing the results from (6.1),

$$\text{(8)} \qquad \delta = \Gamma(k, q_1, p), \quad \Gamma_1 > 0, \quad \Gamma_2 < 0, \quad \Gamma_3 > 0.$$

The gross investment decisions for both capital and labor are forward looking. From (6.2) an increase in the marginal value of capital investment raises the rate of capital investment,

$$\text{(9)} \qquad I_N/K_N = X_k(q_1), \quad X_k' = 1/(C''(I_N/K_N) + 2C') > 0.$$

Similarly for labor investment,

$$\text{(10)} \qquad I_L/L = X_\ell(q_2), \quad X_\ell' = 1/(D''(I_L/L) + 2D') > 0.$$

The firm utilizes and invests in capital until the marginal cost of producing capital output through reparation (or nonutilization) equals the marginal cost of purchasing and installing capital (see equations (6.1) and (6.2)). The equality between these marginal costs points out that there are indeed two forms of capital investment in this model. One type of investment can be considered internal through reparation (or nonutilization) and the other can be considered external through acquisition.

3. The Dynamics and the Steady State

Capital and labor are accumulated so that dynamic behavior occurs in this model. The purpose of this section is to characterize the dynamic adjustment paths of the rates of capital and labor investment along with the path of the capital utilization rate. First the capital intensity growth rate is determined by substituting equations (8)-(10) into (6.4) so that,

$$\dot{k}/k = X_k(q_1) - \Gamma(k,q_1,p) - X_\ell(q_2) + \mu. \tag{11}$$

The capital intensity growth rate depends on the investment shadow prices and, unlike the situation with exogenous capital depreciation, also depends on the capital intensity itself. The growth rate is a decreasing function of the capital intensity because as the latter increases, diminishing marginal productivities of labor and capital cause there to be less of a need for further increases in the capital intensity. Thus

$$\partial(\dot{k}/k)/\partial k = -\Gamma_1 < 0. \tag{12.1}$$

An increase in the marginal value of capital leads to more capital investment (both internal and external), thereby causing an increase in the capital intensity growth rate by

$$\partial(\dot{k}/k)/\partial q_1 = X_k' - \Gamma_2 > 0. \tag{12.2}$$

Lastly, since an increase in the marginal value of labor investment increases the labor growth rate, then the capital intensity growth rate decreases by

$$(12.3) \qquad \partial(\dot{k}/k)/\partial q_2 = -X'_{\ell} < 0.$$

Because the capital intensity is changing through time, the marginal values of capital and labor investment exhibit intertemporal movement. Substituting equations (8) and (9) into (6.5), the dynamic path of the price of installed or unutilized capital is given by

$$(13) \qquad \dot{q}_1 = (r + \Gamma(k, q_1,p))q_1 - p(f_1(k, (1-\Gamma(k,q_1,p))k)$$

$$+ (1 - \Gamma(k,q_1,p))f_2(k, (1 - \Gamma(k,q_1,p))k)$$

$$- C'(X_k(q_1))(X_k(q_1))^2.$$

From (13) it can be seen that the capital stock is chosen such that the opportunity cost of funds or the interest rate equals the rate of return on capital. The latter consists of three elements: the value of the marginal product of capital net of depreciation, the decline in installation costs arising from having a larger stock of capital and capital gains associated with the installed capital.

When the price of installed or unutilized capital increases, a capital gain must occur in order to keep the rate of return on capital equal to the interest rate. Indeed,

$$(14.1) \qquad \partial\dot{q}_1/\partial q_1 = r + \delta + pk\Gamma_2(f_{12} + (1-\delta)f_{22}) - I_N/K_N > 0.$$

The right side of (14.1) is positive because $r+\delta > I_N/K_N$ in order for the flow of funds to be finite and $pk\Gamma_2(f_{12} + (1-\delta)f_{22}) > 0$.

An increase in the capital intensity causes a decrease in the value of the marginal product of capital. To retain the equality between the interest rate and rate of return, a capital gain must occur. Thus differentiating (13) with respect to the capital intensity and making use of equation (7) yields

$$(14.2) \qquad \partial\dot{q}_1/\partial k = -p(f_{11}f_{22} - f_{12}^2)/f_{22} > 0.$$

Next, consider the path of the price of integrating labor in the production process.

$$(15) \quad \dot{q}_2 = (r + \mu)q_2 - p\Big[f(k,(1-\Gamma(k,q_1,p))k) - f_1(k,1-\Gamma(k,q_1,p))k)k$$

$$- f_2(k,1-\Gamma(k,q_1,p))k)(1-\Gamma(k,q_1,p))k\Big] - D'(X_\ell(q_2))(X_\ell(q_2))^2 + w.$$

The interest rate is equated to the rate of return on labor. The latter consists of three elements: the value of the marginal product of labor net of departures, the reduction in adjustment costs due to a larger labor force, and the capital gains net of the wage rate paid to the workers.

In order to retain the equilibrium condition (15) when the price of integrating labor increases, a capital gain must accrue and when the capital intensity increases, a capital loss must occur. Thus

$$(16.1) \qquad \partial\dot{q}_2/\partial q_2 = r + \mu - I_L/L > 0$$

and

$$(16.2) \qquad \partial\dot{q}_2/\partial k = pk(f_{11}f_{22} - f_{12}^2)/f_{22} < 0.$$

Moreover, the price of integrating labor depends on the price of installed or unutilized capital since the marginal product of labor is affected by capital utilization. Differentiating (15) with respect to the price of unutilized capital yields

$$(16.3) \qquad \partial\dot{q}_2/\partial q_1 = -pk^2\Gamma_2(f_{12} + f_{22}(1-\delta)) < 0.$$

An increase in the price of unutilized capital lowers the depreciation rate and thereby increases the marginal product of labor. In order for the interest rate to remain equal to the rate of return on labor, a capital loss must occur.

The properties of the time paths of the capital intensity and the prices of unutilized capital and integrating labor have been analyzed. Hence the dynamic path and long-run equilibrium can now be characterized. The long-run equilibrium or steady state, defined for $\dot{k} = \dot{q}_1 = \dot{q}_2 = 0$ can be illustrated in a four quadrant diagram. Figure 2 shows the steady

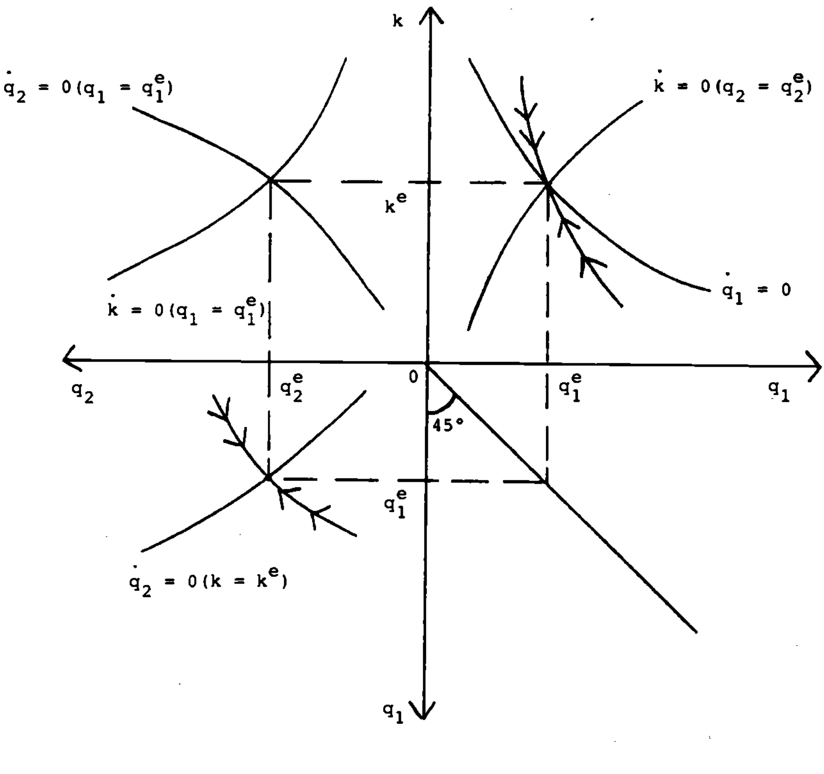

Figure 2. The Steady State and the Dynamic Paths

state in the following manner. First, since the $\dot{k} = 0$ locus in (q_1,k) space depends on q_2, in this quadrant the locus must be defined for the steady state value of q_2 which is q_2^*. Similarly, $\dot{k} = 0$ in the (q_2,k) quadrant and $\dot{q}_2 = 0$ in (q_2,k) must be drawn for the steady state value of q_1, q_1^*. In (q_2,q_1) space, the $\dot{q}_2 = 0$ curve is consistent with the steady state capital intensity, k^*. Second, the curves must be drawn such that their intersections form a rectangle. The two properties together, one relating to the position of each locus, and the other to the position of the intersections, permit the illustration of the steady state.

Not only does the steady state exist (from the properties of the production and adjustment cost functions), but it is unique. Uniqueness can be demonstrated from Figure 3. Suppose point A represents another steady state value of q_1, q_1^1. By construction, $q_1^1 > q_1^*$. The higher price means that in (q_2,k) space the $\dot{q}_2 = 0$ locus shifts down and to the left (by equation (16.3)) and the $\dot{k} = 0$ locus shifts up and to the left (by equation (12.2)). The new curves intersect such that $q_2 = q_2^1 > q_2^*$. The higher q_2 causes the $\dot{k} = 0$ curve in (q_1,k) space to shift down and to the right (by equation (12.3)) so that capital intensity decreases to $k^1 < k^*$. But the decrease in capital intensity shifts the $\dot{q}_2 = 0$ locus down and to the right in (q_2,q_1) space (by equation (16.2)). Hence with q_1^1 and k^1 the price of integrating labor in the production process is q_2^2 and not q_2^1. This means that there is only a single rectangle consistent with the various curves and intersections and therefore there is a unique steady state.

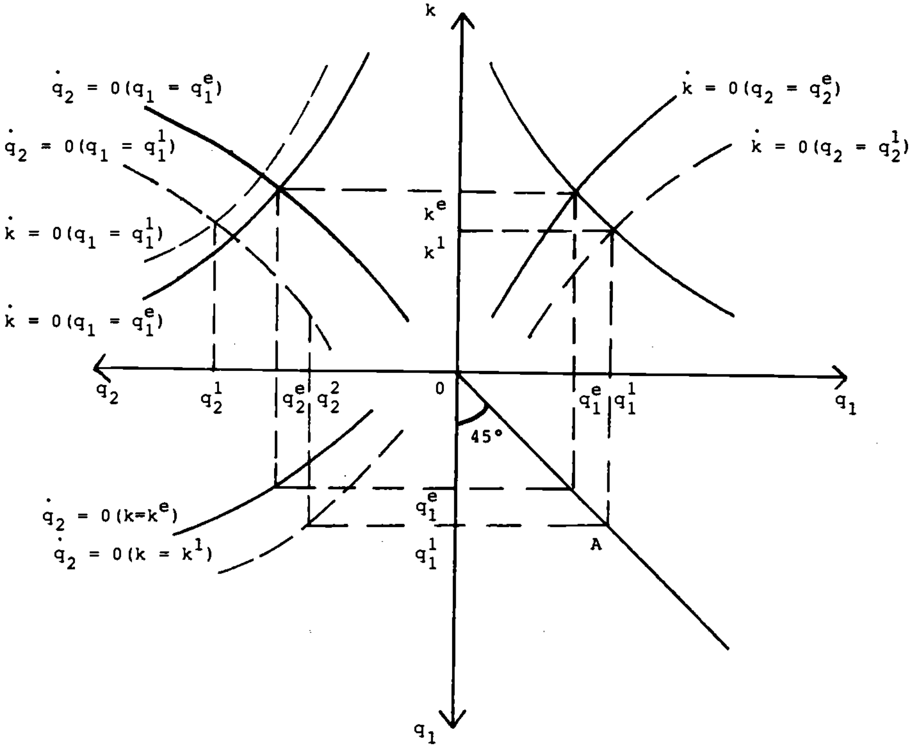

Figure 3. Uniqueness of the Steady State

The stability of the steady state and the dynamic path can be determined from the linearization of equations (11), (13) and (15) around the steady state (k^*, q_1^*, q_2^*). The system is

$$(17) \quad \begin{bmatrix} -k\Gamma_1 & k(X_k' - \Gamma_2) & -kX_\ell' \\ -p(f_{11}f_{22}-f_{12}^2)/f_{22} & r+\delta+pk\Gamma_2(f_{12}+(1-\delta)f_{22})-I_N/K_N & 0 \\ pk(f_{11}f_{22}-f_{12}^2)/f_{22} & -pk^2\Gamma_2(f_{12}+(1-\delta)f_{22}) & r+\mu-I_L/L \end{bmatrix} \begin{bmatrix} k^a \\ q_1^a \\ q_2^a \end{bmatrix} = \begin{bmatrix} \dot{k} \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

where $k^a = k-k^*$, $q_1^a = q_1-q_1^*$ and $q_2^a = q_2-q_2^*$. There are three characteristic roots or eigenvalues which solve equation set (17). Defining the elements in the matrix (17) as $[a_{ij}]$, the roots are solved from the characteristic equation

$$a_{13}[a_{21}a_{32} - a_{31}a_{22} + \lambda a_{31}] + (a_{33}-\lambda)[a_{11}a_{22} - a_{21}a_{12} - \lambda(a_{11} + a_{22}) + \lambda^2] = 0.$$

In addition, since

$$(a_{21}a_{32} - a_{31}a_{22}) = -a_{33}a_{31}.$$

then the characteristic equation can be written as

$$(a_{33}-\lambda)[a_{11}a_{22} - a_{21}a_{12} - a_{13}a_{31} - \lambda(a_{11} + a_{22}) + \lambda^2] = 0.$$

This means that the first root is

$$\lambda_1 = a_{33} = r + \mu - I_L/L > 0.$$

The second and third roots are determined from

$$\lambda^2 - \lambda b + c = 0,$$

where $b = a_{11} + a_{22} = r+\delta - I_N/K_N > 0$ and $c = a_{11}a_{22} - a_{21}a_{12} - a_{13}a_{31} < 0$. Thus

$$\lambda_2, \lambda_3 = [b \pm (b^2 - 4c)^{\frac{1}{2}}]/2.$$

Since $c < 0$, then the roots are real and because $(b^2-4c)^{\frac{1}{2}} > b > 0$, one root is positive and the other is negative. Therefore there are three distinct roots, two positive and one negative. This means that the steady state is a saddle point. In addition, because the roots are real, the path to the steady state does not involve any cycles. The unstable roots are positive and the stable root is negative.

The stable solution to equation set (17) is

(18.1) $$k^* = c\alpha_k e^{\lambda_2 t}$$

(18.2) $$q_i^* = c\alpha_i e^{\lambda_2 t}, \quad i=1,2 \ ,$$

where $\lambda_2 < 0$ is the stable root, c is the arbitrary constant which satisfies the initial condition on capital and labor, and α_k, α_i, $i=1,2$

are the elements of the characteristic vectors. The characteristic vectors are found from

$$[A - \lambda_2 I]\alpha = 0$$

where $\alpha = [\alpha_k \;\; \alpha_1 \;\; \alpha_2]$, A is the matrix in (17) and I is the identity matrix. Now an element in the vector α can be set arbitrarily. It is simplest to set $\alpha_1 = 1$.[10] We then find that

$$\alpha_k = (\lambda_2 - a_{22})/a_{21} < 0$$

$$\alpha_2 = a_{31}/a_{21} < 0.$$

Substituting α_1, α_2 and α_k into equation set (18) and time differentiating yields the dynamic behavior of the firm,

(19.1) $$\dot{k} = \lambda(k - k^e)$$

(19.2) $$\dot{q}_1 = [\lambda a_{21}/(\lambda - a_{22})](k - k^e)$$

(19.3) $$\dot{q}_2 = [\lambda a_{31}/(\lambda - a_{22})](k - k^e),$$

where $\lambda = \lambda_2 < 0$. The shape of the adjustment paths of the capital intensity and the prices of unutilized or installed capital and integrating labor are given by equation set (19) and illustrated in Figure 2. From equation (19.1) $k^o > k^e$ then $\dot{k} < 0$ and k decreases along the path. Simultaneously, from (19.2) since $a_{21} > 0$, $a_{22} > 0$ then $\dot{q}_1 > 0$ and q_1 increases. Thus there is an inverse relationship between k and q_1 along the path. This movement is shown in the (q_1,k) quadrant in Figure 2. Next, from (19.2) with $a_{31} < 0$, and $a_{22} > 0$, then $\dot{q}_2 < 0$ and q_2 decreases. Thus there is a direct relationship between k and q_2 along the path and an inverse relationship between q_1 and q_2. This latter movement is illustrated in the (q_2,q_1) quadrant of Figure 2.

To understand the intuition behind these results, consider an initial situation with insufficient integrated labor relative to installed (or unutilized) capital $(k^o > k^*)$. The marginal value of the integrated labor force in the production process must exceed the long-run magnitude $(q_2^o > q_2^*)$, in order for the firm to increase its labor force. Simultaneously, the marginal value of installed or unutilized capital is below the steady state value $(q_1^o < q_1^*)$ so the firm has less incentive to accumulate capital either through acquisition or reparation (non-utilization).

The results on the prices for unutilized capital and integrated labor imply (from equations (9) and (10)) that $(I_N/K_N)^o < (I_N/K_N)^*$ and $(I_L/L)^o > (I_L/L)^*$. Since the capital intensity decreases to the steady state, the rate of labor investment must exceed the steady state rate, while the converse must occur for the capital investment rate.

The behavior of the depreciation or the net utilization rate is governed by the movements over time of the capital intensity and the price of unutilized capital (from equation (8)). By time differentiating equation (8) and using equations (19.1) and (19.2), the adjustment path of the net utilization rate is

$$(19.4) \qquad \dot{\delta} = [(\Gamma_1(\lambda - a_{22}) + \Gamma_2 a_{21})\lambda/(\lambda - a_{22})](k - k^*).$$

Thus the adjustment path for the capital utilization rate is a flexible accelerator. This result rigorously establishes the empirical finding obtained by Nadiri and Rosen (1969) that the adjustment path of the capital utilization rate is similar to the paths of capital and labor growth rates which are governed by flexible accelerators. Along the dynamic path, as the capital intensity decreases the net utilization rate declines for two reasons. First, there is the direct effect of the capital intensity on the net utilization rate. A decrease in the capital intensity leads the firm to reallocate resources towards capital output which decreases the rate. Second, there is the indirect effect, which arises because the decrease in capital intensity causes the price of unutilized capital to increase. Since the marginal value of unutilized capital increases, the firm then utilizes less of its capital and so the utilization rate falls. Thus $k^\circ > k^*$ implies that $\delta^\circ > \delta^*$. These results mean that along the dynamic path the net capital utilization rate and the rate of capital investment are inversely related while the net capital utilization rate and the labor investment rate are directly correlated.

4. Comparative Steady State and Dynamics

This section is concerned with the analysis of the effects of unanticipated changes in input supply and product demand conditions on the steady state and dynamic adjustment path. The stable adjustment path can be obtained from equation sets (18) and (19) and it can be written as

$$(20.1) \qquad \dot{k} = \lambda(k - \Lambda(w,r,\mu,p))$$

$$(20.2) \qquad q_i = Q^i(w,r,\mu,p) + \phi_i(k - \Lambda(w,r,\mu,p)), \quad i=1,2,$$

where $\lambda < 0$, $\phi_1 = 1/\alpha_k < 0$, $\phi_2 = \alpha_2/\alpha_k > 0$, $k^* = \Lambda(w,r,\mu,p)$ and $q_i^* = Q^i(w,r,\mu,p)$. Clearly, in order to determine the effects of unanticipated shocks to the dynamic adjustment path, the effects on the steady state must be derived. These results are presented in Table 1 and they will be discussed as we consider each shock to the dynamic adjustment path. The expressions in Table 1 were determined from equations (11), (13) and (15) with $\dot{k} = 0 = \dot{q}_1 = \dot{q}_2$ so that $k = k^*$, $q_1 = q_1^*$ and $q_2 = q_2^*$ and then differentiating the three equations with respect to the wage rate, interest rate, departure rate and product price.

To begin the analysis, from equation (20.1) it is clear that unanticipated changes in the wage rate, interest rate, departure rate, and product price cause the capital intensity to change along the path in a direct and proportional manner to the steady state capital intensity. From the first column of Table 1, the latter increases in response to a higher wage rate. The capital intensity increases in the long-run

TABLE 1

Comparative Steady State Results

Exog. Var.	Endogenous Variable		
	k^*	q_1^*	q_2^*
w	$a_{22}a_{13}/A^* > 0$	$-a_{21}a_{13}/A > 0$	$-(a_{11}a_{22}-a_{21}a_{12})/A < 0$
r	$[q_1(a_{33}a_{12}-a_{13}a_{32})$ $+ q_2a_{13}a_{22}]/A$	$[a_{13}(q_1a_{31}-q_2a_{21})$ $- q_1a_{11}a_{33}]/A < 0$	$[q_1(a_{32}a_{11}-a_{31}a_{12})$ $- q_2(a_{11}a_{22}-a_{12}a_{21})]/A < 0$
μ	$(a_{22}a_{33}+q_2a_{22}a_{13})/A > 0$	$a_{21}(a_{33}-q_2a_{13})/A < 0$	$[(a_{31}a_{22}-a_{21}a_{32})$ $- q_2(a_{11}a_{32}-a_{21}a_{12})]/A$
p	$[(\partial\dot{q}_1/\partial p)(a_{12}a_{33}-a_{32}a_{13})$ $+a_{22}(a_{13}\partial\dot{q}_2/\partial p$ $-a_{33}\partial\dot{k}/\partial p)]/A$	$[a_{21}(a_{33}\partial\dot{k}/\partial p-a_{13}\partial\dot{q}_2/\partial p)$ $-(\partial\dot{q}_1/\partial p)(a_{11}a_{33}-a_{13}a_{31})]/A > 0$	$[a_{11}a_{32}\partial\dot{q}_1/\partial p +$ $(\partial\dot{q}_2/\partial p)(a_{21}a_{12}-a_{11}a_{22})-$ $a_{21}a_{32}\partial\dot{k}/\partial p +a_{31}(a_{22}\partial\dot{k}/\partial p-$ $a_{12}\partial\dot{q}_1/\partial p)]/A > 0$

* $A < 0$ is the determinant of the matrix in (17).

thereby along the adjustment path because the higher wage causes labor to be relatively more expensive than capital. The result on the capital intensity is not as clear when the interest rate rises. An increase in the interest rate causes the rates of return on capital and labor to rise. This means that the prices of installed or unutilized capital and of integrating labor must fall. However, as both prices fall, the capital intensity responds in an ambiguous manner because both the capital and labor rates of investment decline. Next an increase in the labor departure rate causes the capital intensity to increase because the higher departure rate decreases the labor growth rate. Lastly, an unanticipated rise in product demand which is reflected as an increase in the product price generates an ambiguous effect on the capital intensity. The reason is that the higher product price increases the value of the marginal products of capital and labor and thereby their rates of return. In order to restore long-run equilibrium, the prices of installed or unutilized capital and of integrating labor must increase. These price rises, in turn, cause the rates of labor and capital investment to increase and therefore there is an ambiguous effect on the capital intensity.

In order to determine the effects of unanticipated product demand and factor supply shocks on the rates of capital and labor investment and the depreciation rate or the net rate of capital utilization, the results presented in Table 1 and equation (20.2) must be combined. First, for an increase in the wage rate using the first row of Table 1 and the values of α_k and α_2,[11] (21.1) $\partial q_1^*/\partial w < (\partial q_1^*/\partial w)\lambda/(\lambda - a_{22}) = \partial q_1/\partial w < 0$

(21.2) $\partial q_2/\partial w = \partial q_2^*/\partial w - \phi_2 \partial k^*/\partial w < \partial q_2^*/\partial w < 0.$

An increase in the wage rate decreases the price of installed capital along the dynamic path but not by as much as the decrease in the steady state price of installed capital. However, along the dynamic path not only does the price of integrated labor decrease but the decrease exceeds that obtained in the steady state.

The results from (21.1) and (21.2) together with the investment demand functions defined by equations (9) and (10) imply that the rate of capital investment declines along the dynamic path in response to an increase in the wage rate but not by as much as the steady state capital investment rate decreases. The labor investment rate also decreases along the adjustment path, but the decrease exceeds the steady state decline in the rate of labor investment as the wage rate increases.

The dynamic path of the net capital utilization rate is also affected by the wage rate. Using equation (8), the results in Table 1 and (21.1),

(21.3) $0 < \partial\delta/\partial w = \Gamma_2(\partial q_1^*/\partial w)\lambda/(\lambda - a_{22}) < \Gamma_2 \partial q_1^*/\partial w < \partial\delta^*/\partial w.$

Thus an increase in the wage rate increases the net capital utilization rate but the increase in the rate is not as great as the increase in the steady state rate. These results establish that along the adjustment path increases in the wage rate cause the rates of capital and labor investment to move in the opposite direction to the utilization rate.

Next, for an increase in the interest rate the price of installed or unutilized capital and the price of integrating labor decrease along the dynamic adjustment path to the lower steady state price. This result can be obtained from the second row of Table 1, equation (20.2), and the value of α_k,[12]

$$(22.1) \quad \partial q_1/\partial r = -q_1[a_{11}a_{33} + \phi_1 a_{33}a_{12}]/A - q_2 a_{13}\lambda\phi_1/A + q_1\phi_1 a_{13}a_{31}[\lambda - a_{33}]/Aa_{21} < 0.$$

$$(22.2) \quad \partial q_2/\partial r = q_1[a_{32}a_{11} + \phi_2 a_{13}a_{32}]/A - q_2[a_{11}a_{22} - a_{21}a_{12} + \phi_2 a_{13}a_{22}]/A - q_1\phi_2 a_{12}[\lambda - a_{22} + a_{33}]/A < 0.$$

The sign of the right side of equations (22.1) and (22.2) imply that the rates of capital and labor investment decline in response to an unanticipated change in the interest rate along the dynamic path. In addition, from (22.1) and equation (8) an increase in the interest rate causes the capital utilization rate to increase since the price of unutilized capital falls along the adjustment path. Thus, as for the wage rate, an unanticipated increase in the interest rate causes the counter movement along the adjustment path between the capital utilization rate and the rates of capital and labor investment.

If there is an unanticipated increase in the rate of labor departures, then from equation (20.2), the third row of Table 1 and the value of α_k,

$$(23.1) \qquad \partial q_1^*/\partial\mu \;<\; (\partial q_1^*/\partial\mu)\lambda/(\lambda - a_{22}) \;=\; \partial q_1/\partial\mu \;<\; 0.$$

However, unlike the price of installed capital, there is an ambiguous effect on the price of integrating labor along the adjustment path for an increase in the labor departure rate.[13] The ambiguity arises because the increase in the departure rate decreases the rate of return on labor and simultaneously decreases the capital intensity and the price of unutilized capital. The latter two effects serve to increase the rate of return on labor. Therefore, although the rate of capital investment decreases along the path in response to an increase in the departure rate, and in addition, this decrease is less than that found in the steady state, it is not possible to unambiguously determine the effect on the path of labor investment.

Increases in the labor departure rate cause the net capital utilization rate to increase. Thus, there is a direct relationship between the two rates. Moreover, the movement in the capital utilization rate in the steady state is more pronounced than that found along the adjustment path. Indeed, from equations (8) and (23.1)

$$(23.2) \qquad 0 \;<\; \partial\delta/\partial\mu \;=\; \Gamma_2\partial q_1^*/\partial\mu(\lambda/(\lambda - a_{22})) \;<\; \Gamma_2\partial q_1^*/\partial\mu \;<\; \partial\delta^*/\partial\mu.$$

Turning to the product demand shocks, suppose that the firm is confronted with an unanticipated increase in product demand along the dynamic adjustment path. This increase implies that there is an increase in the product price. The increase in the product price generates an increase in the price of installed capital such that from (20.2) and the fourth row of Table 1,

$$\partial q_1/\partial p = -(\phi_1 a_{13}\lambda\partial\dot{q}_2/\partial p)/A + \phi_1 a_{33}(a_{22}\partial\dot{k}/\partial p - a_{12}\partial\dot{q}_1/\partial p)/A$$

(24.1)

$$+ a_{33}(a_{21}\partial\dot{k}/\partial p - a_{11}\partial\dot{q}_1/\partial p)/A + (a_{13}(a_{31}+\phi_1 a_{32})\partial\dot{q}_1/\partial p)/A > 0.$$

In addition, the price of integrating labor increases along the path for an increase in the product price,

(24.2)

$$\partial q_2/\partial p = (\partial\dot{q}_1/\partial p)(a_{11}a_{32} + \phi_2 a_{32}a_{13})/A + (\partial\dot{q}_2/\partial p)(a_{21}a_{12} - a_{11}a_{22} - \phi_2 a_{22}a_{13})/A - (\partial\dot{k}/\partial p)a_{21}a_{32})/A + a_{31}(\lambda - a_{22} + a_{33})[-(\partial\dot{q}_1/\partial p)a_{12} + (\partial\dot{k}/\partial p)a_{22}]/A(\lambda - a_{22}) > 0.$$

Thus the prices of installed capital and of integrating labor move in the same direction along the adjustment path. These results also imply that the rates of investment in capital and labor increase and move in the same direction.

There is also a tendency for the capital utilization rate to increase in response to changing product demand conditions. The effect on the net capital utilization rate (using (8) and (24.1)) can be seen from

(24.3)

$$\partial\delta/\partial p = \delta\eta_q(\xi_p - \xi_q)/p\xi_q,$$

where $\eta_q < 0$ is the elasticity of the net utilization rate with respect to the price of unutilized capital, $\xi_p > 0$ is the elasticity of the rate of capital investment with respect to the product price and $\xi_q > 0$ is the elasticity of the rate of capital investment with respect to the price of unutilized capital.[14] An unanticipated increase in the product price will increase capital utilization if the rate of capital investment is relatively more inelastic with respect to the product price compared to the price of unutilized capital. In this situation, the increase in product demand will cause relatively more resources to be devoted to capital utilization and thereby current output will rise compared to capital investment and future output.

To summarize the results, the present model is able to capture the stylized facts of Foss (1981). Unanticipated changes in factor supply conditions generate movements in the rates of capital and labor investment in the same direction. These rates generally decrease. The capital utilization rate increases in response to changes in the supply side conditions and thereby moves in the opposite direction to the rates of investment. Unanticipated changes in product demand conditions, however, cause both rates of investment to increase and there is also the possibility for the capital utilization rate to increase. Thus, unlike changes in the supply side conditions, changes in product demand conditions can generate comovement in all three variables.

5. Conclusion

A model of investment and capital utilization was developed in this paper. The problem of capital utilization was considered in a context of joint products as current output and capital output were determined given the stocks of the quasi-fixed factors. In addition, since capital output forms part of the accumulation process, there were two types of capital investment: internal investment through reparation (or non-utilization) and external investment through acquisition.

In the present model, capital utilization is a forward-looking decision. Capital is utilized and investment occurs until the marginal cost of capital utilization equals the marginal cost of installed capital. The intertemporal nature of the equilibrium arises because the marginal costs of capital utilization and capital installation each equal the present value of the marginal benefits from capital. Hence the decision on capital utilization can be viewed as a trade-off between current and future output production.

A significant feature of the model is that the dynamic adjustment path can be characterized along with the effects on the rates of capital and labor investment and capital utilization from unanticipated changes in factor supply and product demand conditions. It was established that the path of the capital utilization rate can be characterized as a flexible accelerator and is similar in nature to the paths for the rates of capital and labor investment. This result captures the empirical finding of Nadiri and Rosen (1969). Along the adjustment path the rates of investment are inversely related to each other while the capital utilization rate is directly related to the rate of labor investment and inversely related to the capital investment rate. In addition, the model captures the stylized facts obtained by Foss (1981). Unanticipated changes in factor supply conditions (as represented by changes in the wage rate, interest rate and labor departure rate) caused comovement in the rates of capital and labor investment, while the capital utilization rate was inversely related to the rates of investment. This model was also able to capture the feature that unanticipated product demand changes generated comovement in all three variables along the dynamic adjustment path.

NOTES

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1 These models were based on the work of Lucas (1970), Winston and McCoy (1974), and Betancourt and Clague (1978), who focused on rhythmic factor prices (e.g., overtime wage rates) as the costs associated with capital utilization.

2 If as a special case $F_N = -F_0$, so the marginal product of the capital input equals the marginal product of utilized capital, then $y(t) = F(\delta(t)K_N(t), L(t))$. This special case is the way capital utilization is often introduced into the production function. Here it is clearly seen that $\delta(t)$ is the net utilization rate.

3 The results from this model can be generalized to a situation where there are two labor inputs, with one treated as a variable factor of production and the other as a quasi-fixed factor. Also the results apply to the special case where labor is only a variable input.

4 The unit adjustment cost function $C(I_N(t)/K_N(t))$ is composed of the purchase price and the internal cost of installing capital. Thus

$$C(I_N(t)/K_N(t)) = p_N(t) + A(I_N(t)/K_N(t)).$$

Now with $A(0) = 0$ then $C(0) = p_N$. Also $A' > 0$ for $I_N(t) > 0$ so $C' > 0$. Finally, we assume that the total capital adjustment cost,

$A(I_N(t)/K_N(t))I_N(t)$, is strictly convex in $I_N(t)$. We do not assume that unit installation costs are strictly convex. This implies that $C''I_N(t)K_N(t) + 2C' > 0$. Since $C' > 0$ then C'' can be negative but not too negative.

5 We drop the notation (t) for simplicity. In addition, K_0, I_N and I_L are piecewise continuous functions of time, while K_N and L are continuous functions with piecewise continuous first derivatives.

The transversality conditions are $\lim_{t\to\infty} q_i \geq 0$, $i=1, 2$ $\lim_{t\to\infty} q_1K_N = \lim_{t\to\infty} q_2L = 0$. The Legendre-Clebsch conditions imply that the matrix of second order derivatives of the control variables (K_0, I_N, and I_L) is negative definite.

7 To see that short-run revenue is maximized, consider the problem $\max_{(y,K_0)} py + q_1K_0$ subject to $y = Lf(k,(1-\delta)k)$ given p, q_1, L, K_N and recall $(1-\delta) = K_0/K_N$. The first order conditions are $p-\lambda = 0$, $q_1+\lambda f_2 = 0$. Thus $-f_2 = q_1/p$ which is equation (6.1).

8 An increase in q_1 leads to an increase in K_0 and a decrease in y. This can be seen from Figure 1 where the isorevenue line becomes more negatively sloped. The opposite occurs for an increase in p.

9 We could have f_{21} positive but small and still have $\partial K_0/\partial K_N = -f_1/f_2$ decrease as K_N decreases. This is also consistent with the fact that the right side of (7) can be positive when f_{21} is positive and small.

10 In deriving α_2 we used the fact that $a_{21}a_{32} - a_{31}a_{22} = -a_{33}a_{31}$.

11 These results are derived by assuming that the dynamic path is close to the steady state so that the derivatives are evaluated at $k = k^*$. Alternatively, it can be assumed that the elements of the A matrix in (17) are constants. This is the usual assumption to obtain local comparative equilibrium results.

[12] Since the comparative dynamics are local results, near the steady state it is true that $-a_{22} + a_{33} < 0$. This result enabled us to establish that the right side of (22.2) was negative.

[13] To establish that $\partial q_1/\partial p > 0$ we used the fact that $a_{22}\partial\dot{k}/\partial p - a_{12}\partial\dot{q}_1/\partial p > 0$ which is derived from equation (6.1), (6.2) and (6.5) for $\dot{q}_1 = 0$. In addition, we used the results that $\partial\dot{q}_1/\partial p < 0$, $\partial\dot{q}_2/\partial p < 0$ and $\partial\dot{k}/\partial p < 0$.

[14] It is important to recognize that the two capital investment elasticities determining the sign of the right side of equation (24.3) do have empirical content. The model in this paper is consistent with the set of investment models under the generic name "q-model". Indeed, our model can be considered a q-model of investment and capital utilization. Various versions of the q-model of investment have been estimated (see for example Abel and Blanchard (1986)).

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