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## CORPORATE DEMAND FOR LIQUIDITY

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#### **ABSTRACT**

This paper proposes a theory of corporate liquidity demand and provides new evidence on corporate cash policies. Firms have access to valuable investment opportunities, but potentially cannot fund them with the use of external finance. Firms that are financially unconstrained can undertake all positive NPV projects regardless of their cash position, so their cash positions are irrelevant. In contrast, firms facing financial constraints have an optimal cash position determined by the value of today's investments relative to the expected value of future investments. The model predicts that constrained firms will save a positive fraction of incremental cash flows, while unconstrained firms will not. We also consider the impact of Jensen (1986) style overinvestment on the model's equilibrium, and derive conditions under which overinvestment affects corporate cash policies. We test the model's implications on a large sample of publicly-traded manufacturing firms over the 1981-2000 period, and find that firms classified as financially constrained save a positive fraction of their cash flows, while firms classified as unconstrained do not. Moreover, constrained firms save a higher fraction of cash inflows during recessions. These results are robust to the use of alternative proxies for financial constraints, and to several changes in the empirical specification. We also find weak evidence consistent with our agency-based model of corporate liquidity.

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### I Introduction

One of the most important decisions a financial manager makes is how liquid a firm's balance sheet should be. Given an inflow of cash to the firm, a manager can choose to reinvest the cash in physical assets, to distribute the cash to investors, or to keep the cash inside the firm. In fact, managers choose to hold a substantial portion of their assets in the form of cash and liquid securities; in 1999, for the 25 nonfinancial companies in the Dow Jones index, the average ratio of cash and equivalent securities to annual capital expenditures was 227%, and for 11 of the 25 the ratio was at least 97%. The financial press has been critical of these large cash holdings, and suggests that they are a manifestation of agency problems. However, the difficulty with these sorts of criticisms is that they are made without a sense of what cash holdings would be in the absence of agency problems.

As Keynes (1936) originally discussed, the major advantage of a liquid balance sheet is that it allows firms to make value-increasing investments when they occur. However, Keynes also pointed out that this advantage is limited by the extent to which firms have access to capital markets (p. 196). We present a model that formalizes this intuition. In it, a firm whose access to capital markets is limited by the nature of its assets, may anticipate facing financing constraints when undertaking investments in the future. Cash holdings are valuable because they increase the likelihood that the firm will be able to fund those investments. However, increasing cash is also costly for such a firm because it decreases the quantity of current investments that the firm can make. In other words, cash yields a lower return than that associated with the firm's physical investments precisely because the firm foregoes current positive NPV projects in order to hold cash. In contrast to a firm facing constrained access to the capital markets, an unconstrained firm (i.e., a firm that is able to invest in all of its positive NPV projects) has no use for cash, but also faces no cost of holding cash.

Our model contains a number of empirical predictions for corporate cash policies. The cleanest of those predictions concerns a firm's propensity to save cash out of cash inflows, which we refer to as the *cash flow sensitivity of cash*. Our model implies that a firm's cash flow sensitivity of cash depends on the extent to which the firm faces financing constraints: firms that are financially unconstrained should not have a systematic propensity to save cash, while firms that are constrained should have a positive cash flow sensitivity of cash. We also study whether these optimal cash policy

<sup>&</sup>lt;sup>1</sup>See, for example, "What to do with all that cash?", Business Week, Nov/20/2000, and "Time pressure on six continents", Financial Times, Jan/22/2002.

implications remain when the firm can hedge against future cash flows. In a framework similar to that of Froot et al. (1993), we analyze both hedging and cash policies. Unlike these authors, we assume that the same frictions that make firms financially constrained also constrain their ability to hedge. Although the analysis of cash policies becomes more involved when firms are allowed to hedge, the main implications of our model continue to hold.

We also study the implications of agency arguments such as Jensen (1986) in the context of our model. We do so because of the common view that large cash positions are a manifestation of agency problems, and also because evidence from Blanchard et al. (1994), Harford (1999), and Lie (2000) suggests that incremental cash is likely to be used on value-reducing investments, consistent with the story that managers' utilities are increasing with the quantity of the firm's assets. Accordingly, we model a situation in which an overinvestment-prone manager potentially distorts his firm's cash policies. Because the manager derives utility from value-reducing investments in addition to valueincreasing ones, he will save a portion of cash inflows to the firm that can exceed the amount of savings needed to fund the first-best level of investment. Such policies ensure the manager's ability to undertake all the investments he desires in the future, even if he does not have access to capital markets. Perhaps the most interesting implication of the agency model of liquidity is that the effect of overinvestment tendencies on firm's cash policies will be most pronounced for firms that are relatively unconstrained in the capital markets, but which do not have sufficient free cash flow to fund the manager's desired investment. Intuitively, the agency problem turns a firm that would be unconstrained if it invested at the first best level into one that is effectively constrained because of the extra investment its manager would like to undertake.

We evaluate the implications of our theory on a sample of manufacturing firms between 1981 and 2000. Because the main predictions of our model concern differences between constrained and unconstrained firms, we classify firms by the nature of their financial constraints using five alternative approaches suggested by the literature. For each classification scheme, we estimate the cash flow sensitivity of cash for both the constrained and the unconstrained firm subsamples. We find that, under each of the five classification schemes, the cash flow sensitivity of cash is close to and not statistically different from zero for the unconstrained firms, but positive and significantly different from zero for the constrained firms. This finding is consistent with the implications of our baseline (no agency) model. We further test the rationale of our theory by examining the empirical

behavior of firms' propensity to save cash out of cash inflows over the business cycle. We find that the cash flow sensitivity of cash of financially constrained firms is negatively associated with the level of aggregate demand (i.e., constrained firms save more in recessions), while unconstrained firms display no change in their cash policies over the cycle. Our model is consistent with this pattern of changes in liquidity demand over the business cycle because aggregate demand fluctuations work as exogenous shocks affecting both the size of current cash flows as well as the relative attractiveness of current investment vis-a-vis future investment.

We also empirically assess the extent to which agency considerations affect the decision to retain cash flows. To do so, we follow much of the literature in presuming that stock ownership and options help improve managers' incentives. Given this assumption, our model implies that the cash flow sensitivity of cash should be related to managerial compensation packages, but only for those firms that: a) have easy access to capital markets, and b) do not have large stockpiles of cash ("free cash flow"). We find that the extent to which this relationship is supported in the data depends on the measure of financial constraints we use. When we use dividend policy and size to measure financial constraints, the coefficient on cash flow interacted with ownership of stock and options is negative and statically significant for financially unconstrained firms with low free cash flow, and not significant for all other firms. These results are consistent with our theory, and suggest that financially unconstrained firms whose managers are likely to have little or no incentives to adopt value-maximizing policies seem to manage firm liquidity as if they were financially constrained. The results are less compelling, however, when we use alternative measures of financial constraints (such as bond and commercial paper ratings). In all, we interpret these findings as providing at least weak evidence that agency problems at the margin can induce managers to hold excessive cash, which accords with our agency view of liquidity.<sup>2</sup>

We are by no means the first ones to consider the issue of corporate liquidity and its effects on investment. Besides Keynes (1936), the idea that firms may underinvest because of insufficient liquidity and imperfect capital markets has been considered in several classic papers in the corporate finance literature, such as Jensen and Meckling (1976), Myers (1977), and Myers and Majluf (1984). More recently, Kim et al. (1998) present a model of cash holdings for firms which face costly external financing. In their model, firms trade off an (exogenously assumed) lower return of holding liquid

<sup>&</sup>lt;sup>2</sup>This contrasts with Opler et al. (1999) who find little support for the effect of agency on corporate cash policies.

assets and the benefit of relaxing financial constraints in the future. They use their model to derive implications for the optimal level of cash holdings, arguing that firms which face higher costs of external funds and have higher variance in future cash flows should hold more cash.<sup>3</sup>

A number of recent empirical studies examine the cross-section of cash reserves, and the factors that appear to be associated with higher holdings of cash.<sup>4</sup> These papers find that the levels of cash tend to be positively associated with future investment opportunities, business risk, and negatively associated with proxies for the cost of external finance and with the level of protection of outside investors. While these studies focus on differences in the *level* of cash across firms, our paper examines differences in the *sensitivity* of cash holdings to incremental changes in cash flow, and the extent to which they are affected by the firm's financial status. We do so because our analysis suggests that the theory has much clearer predictions about firms' marginal propensity to save/disburse out of cash flow innovations than about the amount of cash in their balance sheets.<sup>5</sup>

The remainder of the paper proceeds as follows. Section II introduces a theory of corporate liquidity demand. Section III presents the empirical tests of our theory's main implications. Section IV is a brief conclusion.

# II A Simple Theory of Liquidity Demand

## A The Basic Model: Cash as a Storage Technology

The first step of our analysis is to model corporate demand for liquid assets as a means of ensuring the ability to invest in the future. In an imperfect capital market, saving for future expenditures might be valuable if the firm anticipates rising financing costs or if the firm anticipates that the future investment opportunities will be particularly profitable. Our basic model is a simple representation of a dynamic problem in which the firm has both present and future investment opportunities. The key feature of the model is that cash flows from current assets might not be

<sup>&</sup>lt;sup>3</sup>Other related papers are John (1993), who studies the link between liquidity and financial distress costs, Baskin (1987), who examines the strategic value of cash in games of product-market competition, and Acharya et al. (2002), who consider the effect of optimal cash policies on corporate credit spreads.

<sup>&</sup>lt;sup>4</sup>An incomplete list of papers includes Kim et. al. (1998), Opler et al. (1999), Pinkowitz and Williamson (2001), Faulkender (2002), Ozkan and Ozkan (2002), and Dittmar et al. (2002). Opler et al. further examine the persistence of cash holdings, and characterize what firms do with "excess" cash.

<sup>&</sup>lt;sup>5</sup>The strategy of analyzing corporate policies by looking at cross-sectional differences in cash flow sensitivities has been used in the empirical literature largely initiated by Fazzari et al. (1988). While that literature has focused on corporate policies such as working capital (Fazzari and Petersen (1993) and Calomiris et al. (1995)), and inventory demand (Carpenter et al. (1994) and Kashyap et al. (1994)), it has remained silent on the issue of liquidity demand.

sufficient to fund all positive NPV projects, in the present and in the future. Hoarding cash may therefore facilitate future investments. Of course, another way the firm can plan for the funding of future investments is by hedging against future earnings. Alternatively, the firm may also adjust its dividend policies or its borrowing. In all, our framework considers four components of financial policy: liquidity management, hedging, dividend payments, and borrowing.

#### A.1 Structure

The model has three dates, 0, 1, and 2. At time 0, the firm is an ongoing concern whose cash flow from current operations is  $c_0$ . At that date, the firm has the option to invest in a long term project that requires  $I_0$  today and pays off  $F(I_0)$  at time 2. Additionally, the firm expects to have access to another investment opportunity at time 1. If the firm invests  $I_1$  at time 1, the technology produces  $G(I_1)$  at time 2. The production functions  $F(\cdot)$  and  $G(\cdot)$  have standard properties, i.e., are increasing, concave, and continuously differentiable. The firm also has existing assets which will produce a cash flow equal to  $c_1$  in period 1. With probability p, the time 1 cash flow is high, equal to  $c_1^H$ , and with probability (1-p), equal to  $c_1^L < c_1^H$ . We assume that the discount rate is 1, everyone is risk neutral, and the cost of investment goods equals 1. Finally, the investments  $I_0$  and  $I_1$  can be liquidated at the final date, generating a payoff equal to  $q(I_0 + I_1)$ , where we assume that q < 1. Define the total cash flows from investments as  $f(I_0) \equiv F(I_0) + qI_0$ , and  $g(I_1) \equiv G(I_1) + qI_1$ .

We suppose that the cash flows  $F(I_0)$  and  $G(I_1)$  from the new investments are not verifiable. While the firm cannot pledge those cash flows to outside investors, it can raise external finance by pledging the underlying productive assets as collateral. Following Hart and Moore (1994), the idea is that the liquidation value of "hard" assets is verifiable and if the firm reneges on its debt creditors will seize the physical assets. Assume that the liquidation value of those assets that can be captured by creditors is given by  $(1-\tau)qI$ .  $\tau \in (0,1)$  is a function of factors such as asset tangibility, and the legal environment that dictates the relations between debtors and creditors (see Myers and Rajan (1998)). This parameter is an important element of our theory in that we want to separate the behavior of firms which are financially constrained — and thus need to rely more on cash as a storage technology — from financially unconstrained firms. Clearly, for a high enough  $\tau$ , the firm may pass up positive NPV projects for lack of external financing, and is therefore financially

constrained.<sup>6</sup> In this set up, the firm is only concerned about whether to store cash from time 0 until time 1 as there is no new investment opportunity at time 2. We denote by C the amount of cash the firm chooses to carry from date 0 to date 1.<sup>7</sup> We assume that  $I_0$ ,  $I_1 > 0$ .

As a benchmark case, we solve for the optimal cash policy when the firm can fully hedge future earnings. However, consistent with our assumptions about income contractibility, we also analyze the more interesting situation in which only a fraction  $(1 - \mu)$  of future earnings can be costlessly pledged to external investors. In this case, the firm cannot credibly sign a contract in which it pays more than  $(1 - \mu)c_1^H$  in the high state.<sup>8</sup> Under this richer environment, the underlying source of incomplete contractibility (reflected in both  $\tau$  and  $\mu$ ) caps the firm's ability to transfer resources both across time and across states.<sup>9</sup>

### A.2 Analysis

When the interests of managers and shareholders converge, the objective is to maximize the expected lifetime sum of all dividends subject to several budget and financial constraints. The firm's problem can be written as

$$\max_{C,h,I} \left( d_0 + pd_1^H + (1-p)d_1^L + pd_2^H + (1-p)d_2^L \right) s.t.$$
 (1)

<sup>&</sup>lt;sup>6</sup>In the Hart and Moore framework the optimal contract is most easily interpreted as collateralized debt. Our conclusions, though, do not hinge on any particular element of that framework. So long as constrained firms have limited capacity to issue equity (or if equity issues entail deadweight costs) our theory's intuition will still hold. An alternative framework that allows for equity finance is the moral hazard model of Holmstrom and Tirole (1997), where it is not optimal for firms to issue equity beyond a certain threshold due to private benefits of control.

<sup>&</sup>lt;sup>7</sup>In principle, C can be negative as the firm may not only carry no cash from time 0 to time 1, but also borrow against future earnings  $c_1$ . For practical purposes, one can think of C as a positive quantity. However, we will later show that our main predictions about cash flow sensitivities of cash do not depend on C being necessarily positive.

<sup>&</sup>lt;sup>8</sup>To see how the hedging technology works, consider the case where  $c_1^L = 0$ . Suppose the firm has the same investment opportunities in both states (L and H) at time 1. Because borrowing capacity is limited, the firm might wish to transfer cash flows to state L. This can be accomplished, for example, by selling futures contracts on the asset that produces the time 1 cash flow. In a frictionless world, the firm would be able to fully hedge by selling that asset's entire stream of cash flows in the futures market at time 0 and the firm would have locked in a payoff of  $pc_1^H$  in both states. However, since the amount  $\mu c_1^H$  of cash flows is not contractible, the firm always has the option of walking away with  $\mu c_1^H$  in state H. Thus, a perfectly hedged position through the use of futures contracts can only happen if  $\mu c_1^H \leq pc_1^H$ . If on the other hand  $\mu > p$ , then the hedging policy of the firm will be constrained.

<sup>&</sup>lt;sup>9</sup>One could think of  $\tau$  (the borrowing constraint) and  $\mu$  (the hedging constraint) as highly correlated. In fact, our analysis would yield similar results if we assumed they are equal. We, however, denote those parameters differently so that the effects of hedging on cash policies are made clear in the analysis.

$$d_{0} = c_{0} + B_{0} - I_{0} - C \ge 0$$

$$d_{1}^{S} = c_{1}^{S} + h^{S} + B_{1}^{S} - I_{1}^{S} + C \ge 0, \text{ for } S = H, L$$

$$d_{2}^{S} = f(I_{0}) + g(I_{1}^{S}) - B_{0} - B_{1}^{S}, \text{ for } S = H, L$$

$$B_{0} \le (1 - \tau)qI_{0}$$

$$B_{1}^{S} \le (1 - \tau)qI_{1}^{S}, \text{ for } S = H, L$$

$$ph^{H} + (1 - p)h^{L} = 0$$

$$-h^{H} \le (1 - \mu)c_{1}^{H}$$

The first two constraints restrict dividends (d) to be non-negative in periods 0 and 1.  $B_0$  and  $B_1$  are the amounts of collateralized borrowing. Debt obligations are repaid at the time when the assets they help finance generate cash flows, and their face values are constrained by the liquidation value of those assets.  $^{10}$   $h^H$  and  $h^L$  are the hedging payments. The hedging strategies we focus on typically give  $h^H < 0$  and  $h^L > 0$ . If the firm uses futures contracts, for example, we should think of  $c_1^S + h^S$  as the futures payoff in state S. The firm sells futures at a price equal to the expected future spot value, and thus increases cash flows in state L at the expense of reducing cash flows in state H. If the hedge ratio is equal to 1, then the firm is fully hedged and  $c_1^H + h^H = c_1^L + h^L = E_0[c_1]$ . When the firm faces capital market imperfections not all future cash flows can be used for hedging purposes, and the hedge ratio may be less than 1. Finally, note that the fair hedging constraint  $ph^H + (1-p)h^L = 0$  defines  $h^H$  as a function of  $h^L$ :

$$h^H = \frac{(p-1)}{p} h^L.$$

We refer to  $h^H$  as the firm's "hedging policy". This policy is constrained by the fact that the firm cannot commit to pay out more than  $(1 - \mu)c_1^H$  in state H.

**A.2.1 First best solution** The firm is financially unconstrained if it is able to invest at the first best levels. The first best investment levels at times 0 and 1 ( $I_0^{FB}$  and  $I_1^{FB}$ ) are defined by:

$$f'(I_0^{FB}) = 1$$
  
 $g'(I_1^{FB,S}) = 1$ , for  $S = H, L$ .

<sup>&</sup>lt;sup>10</sup>This formulation implies that our financial constraint is a quantity constraint. Alternatively, we could study a model in which firms face an increasing (deadweight) cost of external finance. As we will argue later, in our analysis all constrained firms have a similar propensity to save cash (irrespective of how tight the constraint is), suggesting that our results do not hinge on the formulation based on quantities.

When the firm is unconstrained its investment policy satisfies all the dividend, hedging, and borrowing constraints above for some financial policy  $(B_0, B_1^S, C, h^H)$ . Except for a case when the constraints are exactly binding at the first best solution, the financial policy of an unconstrained firm will not matter. In particular, if a firm j is financially unconstrained, then its financial policy  $(B_{0j}, B_{1j}, C_j, h_j^H)$  can be replaced by an entirely different financial policy  $(\hat{B}_{0j}, \hat{B}_{1j}, \hat{C}_j, \hat{h}_j^H)$  with no implications for firm value. Consequently, there is no optimal cash hoarding policy for a financially unconstrained firm. To see the intuition, suppose the firm increases its cash holdings by a small amount  $\Delta C$ . Would that policy entail any costs? The answer is no. The firm can compensate for  $\Delta C$  by paying a smaller dividend today. Are there benefits to the increase in cash holdings? The answer is also no. The firm is already investing at the first best level at time 1, and an increase in cash is a zero NPV project since the firm foregoes paying a dividend today for a dividend tomorrow that is discounted at the market rate of return.

**A.2.2 Constrained solution** In the context of our model, it is easy to operationalize the notion of financial constraints; in particular, we define a firm to be financially constrained if its investment policy is distorted from the first-best because of capital market frictions, i.e., if  $(I_0^*, I_1^*) < (I_0^{FB}, I_1^{FB})$ . For a financially constrained firm, holding cash entails both costs and benefits. A financially constrained firm cannot undertake all of its positive NPV projects, so holding cash is costly because it requires sacrificing some valuable investment projects today. The benefits of cash occur because it allows the firm to finance future projects that might arise. Firms will choose cash holdings to trade off these costs and benefits of cash, both of which are generated by the same underlying reason (capital market imperfections). As a result of these countervailing effects, financial constraints will give rise to an optimal cash policy  $C^*$ . This is in stark contrast with the "irrelevance of liquidity" result that holds for financially unconstrained firms.

In order to characterize the optimal cash holding of a constrained firm we have to determine the optimal investment and financial policies of that firm. If the firm is financially constrained, it will not be optimal to pay any dividends at times 0 and 1. Furthermore, borrowing capacity will be exhausted in both periods and in both states at time 1. This must be the case, since foregoing a dividend payment or borrowing an additional unit is a zero NPV project, and, recall, the constrained firm is foregoing positive NPV projects. Using these facts, we can write the firm's

optimization problem as follows:<sup>11</sup>

$$\max_{C,h^L} f\left(\frac{c_0 - C}{1 - q + \tau q}\right) + pg\left(\frac{c_1^H - \frac{1 - p}{p}h^L + C}{1 - q + \tau q}\right) + (1 - p)g\left(\frac{c_1^L + h^L + C}{1 - q + \tau q}\right) s.t.$$
(2)
$$h^L \le (1 - \mu)\frac{p}{1 - p}c_1^H$$

Notice that we have collapsed the fair hedging condition and the hedging constraint into one equation. Moreover, note that the hedging constraint  $h^L \leq (1-\mu)\frac{p}{1-p}c_1^H$  need not bind. This happens, for example, when the firm can transfer enough cash flows to state L such that there is no constrained hedging demand. Now the firm's problem reduces to an optimization in C and  $h^L$ , constrained by the maximum hedging available. As suggested above, we consider solving this problem both with and without hedging constraints. To economize on notation, define  $\lambda \equiv 1-q+\tau q$ .

Unconstrained hedging: Suppose the hedging constraint does not bind. Because hedging is fairly priced the firm can eliminate its cash flow risk. This implies that the optimal amount of hedging is given by  $h^L = p(c_1^H - c_1^L)$ , which gives similar cash flows in both states (equal to  $E_0[c_1]$ ). It is easy to see that the hedging constraint will not bind so long as  $(1-p)(c_1^H - c_1^L) \leq (1-\mu)c_1^H$ .

If the optimal hedge is feasible, the optimal cash policy will be determined by:

$$f'\left(\frac{c_0 - C}{\lambda}\right) = g'\left(\frac{E_0\left[c_1\right] + C}{\lambda}\right) \tag{3}$$

The left-hand side of Eq. (3) is the marginal *cost* of increasing cash holdings. If the firm hoards cash it sacrifices valuable (positive NPV) current investment opportunities.<sup>13</sup> The right-hand side of Eq. (3) is the marginal *benefit* of hoarding cash under financial constraints. By holding more cash the firm is able to relax the constraints on its ability to invest in the future.

How much of its current cash flow will a constrained firm save? This can be calculated from the derivative  $\frac{\partial C}{\partial c_0}$ , which we define as the cash flow sensitivity of cash. As we illustrate below, the cash flow sensitivity of cash is a very useful concept in that it reveals a dimension of corporate liquidity policy that is suitable for empirical analysis. The interpretation resembles that of the cash flow sensitivity of investment used in the financial constraint literature (Fazzari et al. (1988)).

<sup>&</sup>lt;sup>11</sup>We replace the binding constraints in the objective function and eliminate the terms that are constant.

 $<sup>^{12}</sup>$ This is just a traditional "full-insurance" result. In order to check it, one can take the derivative of the objective function with respect to  $h^L$  and set it equal to zero.

<sup>&</sup>lt;sup>13</sup>These opportunities are valuable precisely because financial constraints force the marginal productivity of investment to be higher than the opportunity cost of capital.

The cash flow sensitivity of cash is given by:

$$\frac{\partial C}{\partial c_0} = \frac{f''(I_0)}{f''(I_0) + g''(I_1)} > 0$$

This sensitivity is positive, indicating that if a financially constrained firm gets a positive cash flow innovation this period, it will optimally allocate the extra cash across time, saving some resources for future investments.

Importantly, note that the optimal cash holdings bear no obvious relationship with borrowing capacity (the parameter  $\tau$ ). This can be seen by examining the derivative:

$$\frac{\partial C}{\partial \tau} = \frac{-q(c_0 - C)f''(I_0) + q(E_0[c_1] + C)g''(I_1)}{\tau(f''(I_0) + g''(I_1))},$$

which cannot be signed. The intuition is that higher debt capacity relaxes financial constraints both today and in the future, yielding a similar effect on the marginal value of cash across time. An Example: A simple example can show in a more intuitive way the empirically testable implications of our model. Consider parametrizing the production functions  $f(\cdot)$  and  $g(\cdot)$  as follows:

$$f(x) = A \ln(x)$$
 and  $g(y) = B \ln(y)$  (4)

This parametrization assumes that while the concavity of the production function is the same in periods 0 and 1, the marginal productivity of investment may change over time. With these restrictions, it is a straightforward task to derive an explicit formula for C:

$$C^* = \frac{\delta c_0 - E_0[c_1]}{1 + \delta},\tag{5}$$

where  $\delta \equiv \frac{B}{A} > 0$ . The cash flow sensitivity of cash is then given by  $\frac{\delta}{1+\delta}$ , <sup>15</sup> where the parameter  $\delta$  can be interpreted as a measure of the importance of future growth opportunities vis-a-vis current opportunities. Eq. (5) shows that C is increasing in  $\delta$  (i.e.,  $\frac{\partial C}{\partial \delta} > 0$ ), which agrees with the intuition that a financially constrained firm will hoard more cash today if future investment opportunities are more profitable. Note also that cash flow sensitivity of cash, given by  $\frac{\delta}{1+\delta}$ , is independent of the parameter  $\tau$ . Since the optimal cash policy is determined by an intertemporal trade-off, a change

<sup>&</sup>lt;sup>14</sup>Similar results will hold for a more general Cobb-Douglas specification for the production function, namely  $f(x) = Ax^{\alpha}$  and  $g(x) = Bx^{\alpha}$ . The important assumption is that the degree of concavity of the functions f and g is the same. Given this, the particular value of  $\alpha$  is immaterial. We use the  $\ln(\cdot)$  specification because it simplifies the algebra and economizes on notation.

<sup>&</sup>lt;sup>15</sup>Notice that the sensitivity does not depend on whether the cash balance  $C^*$  is positive or negative. Eq. (5) also makes it clear that the sign of  $C^*$  depends on the the size of current versus expected future cash flows.

in borrowing capacity does not matter if the firm is already constrained. This analysis in turn establishes a precise and monotonic empirical relationship between financial constraints and the cash flow sensitivity of cash: constrained firms should display positive cash—cash flow sensitivities, while unconstrained firms should not have a systematic propensity to save cash. For the purpose of empirical testing, the *degree* of the financial constraints will not matter for already *constrained* firms.<sup>16</sup> Thus, the issue of non-monotonicity in the relationship between financial constraints and cash policies is not a first-order concern for empirical analysis in this context.<sup>17</sup> We explore these properties of our theory in the tests of Section III.

Constrained hedging: If the hedging constraint binds, the amount of hedging is given by  $h^L = (1 - \mu) \frac{p}{1-p} c_1^H$ . In this case, the optimal cash policy is determined by:

$$\max_{C} f\left(\frac{c_0 - C}{\lambda}\right) + pg\left(\frac{\mu c_1^H + C}{\lambda}\right) + (1 - p)g\left(\frac{c_1^L + (1 - \mu)\frac{p}{1 - p}c_1^H + C}{\lambda}\right) \tag{6}$$

or

$$f'\left(\frac{c_0 - C}{\lambda}\right) = E_0[g'(I_1)]. \tag{7}$$

The only difference from the previous Eq. (3) is that marginal productivity now varies across states because of the constraint on hedging. The comparative statics are more involved with constrained hedging, but our previous results remain. In the appendix, we use our previous parametrization to derive the following results for hedge-constrained firms:

• If time 0 cash flow increases, then the firm hoards more cash (i.e., the cash flow sensitivity of cash is positive):

$$\frac{\partial C}{\partial c_0} > 0$$

• If future investment opportunities are more profitable than the current investment opportunities ( $\delta$  is high), then the firm hoards more cash:

$$\frac{\partial C}{\partial \delta} > 0$$

<sup>&</sup>lt;sup>16</sup>Notice that the cash flow sensitivity of cash is similar for all constrained firms, irrespective of how constrained they are. This suggests that our conclusions do not hinge on our formulation of financial constraints as quantity constraints. If the financial constraint manifests itself in terms of increasing costs of external finance, a change in the cost would relax constraints by a similar amount today and in the future, generating very similar implications.

<sup>&</sup>lt;sup>17</sup>This result is important because it essentially avoids the theoretical critique advanced by Kaplan and Zingales (1997, 2000) regarding the traditional interpretation of investment-cash flow sensitivities. See also the discussion in Fazzari et al. (2000), Povel and Raith (2001), and Almeida and Campello (2002).

Also similarly to the case of perfect hedging, borrowing capacity will not affect cash policies for firms which are constrained. However, we now have an additional implication related to changes in the hedging constraint:

• Firms which can hedge less hoard more cash:

$$\frac{\partial C}{\partial \mu} > 0$$

The intuition for this last result is as follows. The cost of limited hedging is the difference in the marginal value of funds across states in the future. The marginal value is higher in the state where the firm has lower cash flows. An increase in  $\mu$  increases funds in state H, but decreases funds in state L. This increases the difference in the marginal value of funds across states, and causes the firm to increase cash hoarding so as to rebalance the future marginal value in the two states.

In sum, the main implications of the basic model are still true when hedging is constrained. While there is no optimal cash policy if the firm is financially unconstrained, the cash flow sensitivity of cash is positive for constrained firms. Moreover, conditional on the fact that a firm is financially constrained, borrowing capacity has no additional effect on the optimal cash policy. Thus, cash flow sensitivities of cash should be monotonically increasing in financial constraints. We state this result in the form of a proposition.

**Proposition 1** The cash flow sensitivity of cash of financially unconstrained and financially constrained firms have the following properties:

$$\frac{\partial C}{\partial c_0} = 0 \text{ for financially unconstrained firms}$$

$$\frac{\partial C}{\partial c_0} > 0 \text{ for financially constrained firms}$$
(8)

This is the main implication of the basic model that we test in the empirical section below.<sup>18</sup> Two additional implications of the model are that constrained firms should hoard more cash if they have more valuable future investment opportunities, and that cash and hedging are substitutes for firms that are hedge-constrained. While we can empirically examine the first of these two additional implications, data availability precludes us from studying the latter implication in this paper.

To be precise, when we say that  $\frac{\partial C}{\partial c_0} = 0$  for financially unconstrained firms we do not mean to say that the sensitivity must be always equal to zero in an economic sense. Rather, we imply that the cash policy of unconstrained firms is undetermined, and thus their cash flow sensitivity should not be *statistically* different from zero.

### B Agency Problems: Overinvestment Tendencies

Any model in which those in charge of running the day-to-day operations of the firm (managers) have objectives that are different from those who own the firm (shareholders) can be seen as an agency model. For practical purposes, the more interesting types of agency problems are those in which managers take actions that reduce shareholders' wealth. Within this class of problems, most of the research in corporate finance has investigated one type of agency problem: the overinvestment problem (see Stein (2001) for a review). In this subsection, we build on our basic model of liquidity demand and study how overinvestment-prone managers handle corporate liquidity.

There are alternative ways of modeling managers' tendency towards overinvestment. As in Hart and Moore (1995), we analyze a model in which managers enjoy private benefits that increase with a firm's investment. This assumption is plausible because executives' salaries are typically increasing in firm size, and because non-pecuniary benefits of control are likely to be more valuable in larger firms. Implicitly, we are assuming that no feasible incentive contract, corporate governance system, or other external threats can make managers internalize the full value consequences of inefficient investment decisions (see Jensen (1993) or Stein (2001) for more discussion).

A very simple way to introduce this type agency problem in our model is to assume that managers make investment and financing decisions so as to maximize the following utility function:

$$U^{M} = (1 + \theta) \left[ f(I_0) + pg(I_1^{H}) + (1 - p)g(I_1^{L}) \right], \text{ where } \theta \ge 0,$$
(9)

where  $\theta$  is interpreted as a measure of the residual amount of agency problems which remains after all feasible corrective mechanisms have been applied.<sup>19</sup> Notice that the maximization program of the previous subsection (Eq. (1)) is naturally nested in Eq. (9). In other words, our basic model is a special case of Eq. (9) which obtains when there is no overinvestment problem (i.e., when  $\theta = 0$ ).

It is straightforward to verify that the particular agency problem we consider — a tendency to overinvest — has no first-order effect on the cash policy of financially constrained firms. Intuitively,

$$f^{M}(x) = (1+\theta)f(x)$$
$$q^{M}(x) = (1+\theta)q(x)$$

A broader class of utilities would also lead to our main conclusions about cash management in the presence of overinvestment tendencies.

<sup>&</sup>lt;sup>19</sup>We borrow this formulation from Stein (2001) for ease of exposition. This is equivalent to assuming that managers apply the following transformation to the investment functions  $f(\cdot)$  and  $g(\cdot)$ :

a positive  $\theta$  uniformly raises the marginal productivity of all investment opportunities (current and future), thus the trade-off which determines optimal cash is the same irrespective of the value of  $\theta$ .

The more interesting result on the influence of agency on cash management happens when the firm is financially unconstrained. For a large  $\theta$ , an unconstrained firm will behave as if it were financially constrained. Intuitively, even when capital markets are perfect, investors will only be willing to give funds to the firm up to a limit determined by the true payoff from investment. Consequently, there exists an "optimal" financial policy of a firm controlled by an overinvestment-prone manager, similarly to what we predict for a financially constrained firm.

In order to see this result, notice that we can write the program solved by a firm facing perfect capital markets that is subject to agency problems as:

$$\max_{C,I,h^L} (1+\theta)f(I_0) - B_0 + p[(1+\theta)g(I_1^H) - B_1^H] + (1-p)[(1+\theta)g(I_1^L) - B_1^L] \ s.t.$$
 (10)

$$I_{0} = c_{0} + B_{0} - C$$

$$I_{1}^{S} = c_{1}^{S} + h^{S} + B_{1}^{S} + C, \text{ for } S = H, L$$

$$B_{0} \leq f(I_{0})$$

$$B_{1}^{S} \leq g(I_{1}^{S}) \text{ for } S = H, L$$

$$\frac{1-p}{p}h^{L} \leq (1-\mu)c_{1}^{H}$$

The firm can borrow up to the true value of the investments  $I_0$  and  $I_1$ . Recall, the functions  $f(\cdot)$  and  $g(\cdot)$  include the cash flows from liquidation  $qI_0$  and  $qI_1$ . And we are implicitly setting  $\tau = 1$ , consistent with the idea that the firm faces perfect capital markets. Clearly, if  $\theta = 0$ , the firm would invest at the first best levels and would not have a systematic cash policy.<sup>20</sup> Let us in turn solve for the optimal investment and cash policies when  $\theta > 0$ .

If managers are able to (over)invest as much as they wish, they would choose the levels of investment to satisfy:

$$f'(\widehat{I}_0) = g'(\widehat{I}_1) = \frac{1}{1+\theta} < 1.$$

$$f(I_0^{FB}) \ge I_0^{FB}$$

and thus the firm can finance the first best level of investment (similarly for  $I_1$ ).

<sup>&</sup>lt;sup>20</sup>Notice that profit maximization implies that if  $I_0^{FB} > 0$ :

The question is whether managers can finance these levels of investment. Consider the case of unconstrained hedging. We can assume with no loss of generality that the firm will have the same cash flow in both states at date 1 (equal to  $E_0[c_1]$ ). Now the condition guaranteeing that the firm is able to finance the investment levels  $\hat{I}_0$  and  $\hat{I}_1$  is that there exists a level of cash  $\hat{C}$  such that:

$$\widehat{I}_0 \leq c_0 + f(\widehat{I}_0) - \widehat{C}$$

$$\widehat{I}_1 \leq E_0[c_1] + f(\widehat{I}_1) + \widehat{C}$$

Summing the two equations we obtain the condition:

$$\widehat{I}_0 - f(\widehat{I}_0) + \widehat{I}_1 - f(\widehat{I}_1) \le c_0 + E_0[c_1]$$
 (11)

The left hand-side of (11) is the negative NPV generated by the projects at the super-optimal scales of production.<sup>21</sup> The right hand-side can be interpreted as the firm's "free cash flow", or total free resources available for investment. Since there is a tendency for overinvestment, shareholder wealth is decreasing in the amount of firm's free cash flows (Jensen (1986)). The expression simply says that overinvestment will be limited by the availability of cash from current operations ( $c_0$  and  $E_0[c_1]$ ). Free cash flow will enable mangers to invest in negative NPV projects even when the market is not willing to finance those projects.

The overinvestment-prone firm will have a uniquely defined cash policy only if  $\hat{I}_0 - f(\hat{I}_0) + \hat{I}_1 - f(\hat{I}_1) > c_0 + E_0[c_1]$ , that is, if total resources available for investment are not too large. If condition (11) is met, then there are multiple cash policies  $\hat{C}$  which allow the firm to invest at the super-optimal levels (unless the condition is satisfied with an exact equality). Similarly to financially unconstrained firms which invest optimally, these firms do not have a well-defined cash policy. In other words, if these firms receive an additional cash inflow (i.e.,  $c_0$  goes up), it is a matter of indifference to such firms whether they save the additional cash or pay dividends.

If condition (11) is not obeyed then the firm behaves as if it were a financially constrained firm. The borrowing constraints will be binding, and the firm will choose optimal cash and investment policies according to:

$$\max_{C,I_0,I_1} \theta f(I_0) + \theta g(I_1) \ s.t.$$

 $<sup>^{21}</sup>$ It is possible that the investment projects are still positive NPV, even at the super-optimal scale. In this model, only the marginal investments above  $I^{FB}$  are necessarily negative NPV. The total NPV of the investment  $\hat{I}$  may be positive or negative. If it is positive, then the firm can always overinvest in this model. However, if the difference between  $\hat{I}$  and  $I^{FB}$  is high enough then the total NPV should also be negative.

$$I_0 = c_0 + f(I_0) - C$$
  
 $I_1 = E_0 [c_1] + g(I_1) + C$ 

The only difference with respect to the constrained firm's optimization problem (recall Eq. (2)) is that since the borrowing constraints are not linear in investment we cannot solve the constraints explicitly for  $I_0$  and  $I_1$ . The intuition, though, is the same. The optimal cash balance  $C^*$  is determined so as to equate the marginal productivity of investment at the two dates (note that  $\theta$  will not matter for this choice):

$$f'[I_0(C^*, c_0)] = g'[I_1(C^*)]$$

where  $I_0(C^*, c_0)$  and  $I_1(C^*)$  represent the optimal investment levels  $(I_0^*, I_1^*)$  as functions of cash and cash flows.<sup>22</sup> It is easy to show that the cash flow sensitivity of cash is positive. Noting that  $\frac{\partial I_0}{\partial c_0} > 0$ ,  $\frac{\partial I_0}{\partial C} < 0$  and  $\frac{\partial I_1}{\partial C} > 0$ , differentiating the fist order condition allows us to write the cash flow sensitivity of cash as:

$$\frac{\partial C^*}{\partial c_0} = \frac{f''(I_0)\frac{\partial I_0}{\partial c_0}}{g''(I_1)\frac{\partial I_1}{\partial C} - f''(I_0)\frac{\partial I_0}{\partial C}} > 0$$

Our analysis formalizes Jensen's (1986) argument about managerial overinvestment tendencies: firms with plenty of free resources in hand (high  $c_0$ ,  $E_0[c_1]$ ) will invest at the level  $\hat{I}_t$ , while firms with less resources will only be able to invest at the lower level,  $I_t^*$ , which is more desirable from the perspective of shareholders — i.e., it is closer to the first best investment level.<sup>23</sup>

This analysis has testable implications for the effect of overinvestment on cash policies. If a firm is underinvesting because of limited access to the capital markets (i.e., is financially constrained), then overinvestment tendencies have no distinct effect on cash policies in general, and on the cash flow sensitivity of cash in particular. This is because overinvestment does not affect a constrained firm's trade-off between foregoing investment opportunities today and increasing investment tomorrow. Tests of the overinvestment hypothesis should thus focus on firms with good access to external funds. But notice that in order for overinvestment to have an effect on cash policies, it is also necessary that the firm does not have too much internal resources (free cash flow). If one

$$I_0^* = c_0 + f(I_0^*) - C^*$$
  
 $I_1^* = E_0[c_1] + g(I_1^*) + C^*$ 

<sup>&</sup>lt;sup>22</sup>Investment levels are determined directly from the binding constraints:

<sup>&</sup>lt;sup>23</sup>Notice that  $I_t^{FB} < I_t^* < \widehat{I}_t$ , with t = 1, 2. The first inequality is true because the constraints cannot be binding for investment levels lower than  $I_t^{FB}$ , and the second follows from the condition in Eq.(11).

is able to empirically identify such firms, then the implication of our overinvestment model is that the cash flow sensitivity of cash should be zero when overinvestment tendencies are not too strong, but positive when the propensity to overinvest is high. In other words, the cash flow sensitivity of cash should increase with empirical proxies for managerial tendencies to overinvest. We re-state the implications of our agency-based liquidity demand model in the form of a proposition.

**Proposition 2** Managerial overinvestment tendencies (captured by  $\theta$ ) will have the following effects on the sensitivity of cash holdings to cash flow:

$$\frac{\partial C}{\partial c_0 \partial \theta} = 0 \text{ for financially constrained firms}$$

$$\frac{\partial C}{\partial c_0 \partial \theta} > 0 \text{ for financially unconstrained firms with limited free cash flow}$$

$$\frac{\partial C}{\partial c_0 \partial \theta} = 0 \text{ for financially unconstrained firms with abundant free cash flow}$$
(12)

Empirically implementing the implications of this model is not a simple task.<sup>24</sup> It requires us to focus on a particular group of firms, and, moreover, requires us to be able to separate financial constraints (limited access to capital markets) from resource constraints (free cash flow). Thus, even when overinvestment tendencies are present, it might not be possible to empirically identify their effect on cash policies. This might help explain why previous papers like Opler et al. (1999) have failed to find strong evidence for the effect of the Jensen's overinvestment problem on firm's optimal cash policies. In the next section, we empirically test our agency-based liquidity model.

# III Empirical Tests

### A Sample

We now test our model's main predictions about the cash flow sensitivity of cash, and its relation to financial constraints and the nature of agency problems inside the firm. To do so, we consider the sample of all manufacturing firms (SICs 2000-3999) over the 1981-2000 period with data available from COMPUSTAT's P/S/T and Research tapes on total assets, sales, and holdings of cash and marketable securities. Because we are interested in relating our findings on cash holdings to agency problems, we require that the sample firms appear in the Standard & Poor's ExecuComp dataset for at least one year. Our final sample contains 1,026 firms yielding 11,135 firm-years.

<sup>&</sup>lt;sup>24</sup> Again, a more precise statement for the proposition above would be that the cash flow sensitivity of cash should not be statistically different than zero for the unconstrained firms with abundant resources, because such firms do not have a systematic cash policy.

### B Measuring the Cash Flow Sensitivity of Cash and Financial Constraints

According to our basic theory, we should expect to find a strong positive relationship between cash flow and changes in cash holdings for financially constrained firms. Unconstrained firms, in contrast, should display no such sensitivity. In order to implement a test of this argument, we need to specify an empirical model relating changes in cash holdings to cash flows, and also to distinguish between financially constrained and unconstrained firms. We tackle each of these issues in turn.

### B.1 An Empirical Model of Cash Flow Retention

To measure the cash flow sensitivity of cash, we estimate a model explaining a firm's decision to change its holdings of cash as a function of its sources and (competing) uses of funds. We borrow insights from the literature on investment—cash flow sensitivities (e.g., Fazzari et al. (1988), Carpenter and Fazzari (1993), Calomiris et al. (1995)) and on cash management (Opler et al. (1999) and Harford (2000)), modeling the annual change in a firm's cash to total assets as a function of cash flows, capital expenditures, acquisitions, changes in non-cash net working capital (NWC), investment opportunities (proxied by Q) and size:

$$\frac{Cash_{i,t} - Cash_{i,t-1}}{Assets_{i,t-1}} = \alpha_0 + \alpha_1 \frac{CashFlow_{i,t}}{Assets_{i,t-1}} + \alpha_2 \frac{Expenditures_{i,t}}{Assets_{i,t-1}} + \alpha_3 \frac{Acquisitions_{i,t}}{Assets_{i,t-1}} + \alpha_4 \frac{NWC_{i,t} - NWC_{i,t-1}}{Assets_{i,t-1}} + \alpha_5 Q_{i,t} + \alpha_6 Ln(Assets_{i,t}) + \mu_i + \varepsilon_{i,t}.$$
(13)

Cash is COMPUSTAT's data item#1. Following Opler et al. (1999), CashFlow is defined as earnings before extraordinary items and depreciation, minus dividends: item #18 + item #14 - item #19 - item #21.<sup>25</sup> We use item #6 for Assets, item #128 for Expenditures, and item #129 for Acquisitions. NWC is receivables, plus inventories minus accounts payable (item#2 + item#3 - item#70). Finally, Q is computed as the market value of assets divided by book assets (item #6 + (item #24 × item #25) - item #60) / (item #6).

Our theory's predictions concern the change in cash holdings in response to a shock to cash flows, captured by  $\alpha_1$  in Eq. (13). We control for investment expenditures and acquisitions because firms can draw down on cash reserves in a given year in order to pay for investments and acquisitions. We thus expect  $\alpha_2$  and  $\alpha_3$  to be negative. We control for the change in net working capital

 $<sup>^{25}</sup>$ Results are very similar when we measure cash flows before dividends (item #14 + item #18).

because working capital can be a substitute for cash (Opler et al. (1999)),<sup>26</sup> or it may compete for the available pool of funds (Fazzari and Petersen (1993)). Our model also suggests that a constrained firm's cash policy should be influenced by the relative profitability of future investment opportunities vis-a-vis current opportunities. We use Q as an empirical proxy for the relative attractiveness of future investment opportunities. In principle, we would expect the coefficient for  $\alpha_5$  to be positive for constrained firms and indistinguishable from zero for unconstrained firms. We, however, recognize that Q may also capture other factors possibly related to liquidity demand and re-state our priors about  $\alpha_5$ : that coefficient should be larger for constrained firms.<sup>27</sup> Finally, we control for size because there could be economies of scale in cash management (Opler et al. (1999)), consistent with a negative estimate for  $\alpha_6$ .

#### **B.2** Financial Constraints Criteria

Testing the implications of our model requires separating firms according to the financial constraints they face. Unfortunately, the literature is not clear on the best way to identify those constraints. There are a number of plausible approaches to sorting firms into 'constrained' and 'unconstrained' categories. Since we do not have strong priors about which approach is best, we use five alternative schemes to partition our sample:

- Scheme #1: We rank firms based on their average annual dividend payout ratio over the 1981-2000 period and assign to the financially constrained (unconstrained) group those firms in the bottom (top) three deciles of the payout distribution. The intuition that financially constrained firms have significantly lower payout ratios follows from Fazzari et al. (1988), among others. As in Fazzari et al., firms stay in a given group throughout the sample period.
- Scheme #2: We rank firms based on their average real asset size over the 1981-2000 period and assign to the financially constrained (unconstrained) group those firms in the bottom (top) three deciles of the size distribution. This approach resembles Gilchrist and Himmelberg

<sup>&</sup>lt;sup>26</sup>Lines of credit and loan commitments are another potential substitute for cash. Unfortunately, we do not have data on these cash alternatives. We will later include changes in short-term borrowings in our regressions.

 $<sup>^{27}</sup>$ It would be a problem to our tests if the quality of Q as a proxy for future opportunities influencing liquidity demand varied precisely along the lines of the sample partitions we use below. This type of concerns have become a major issue in the related investment–cash flow literature, as the "well-established" evidence of higher cash flow sensitivities of constrained firms has been attributed to measurement errors in Q that happen to be more relevant in samples of constrained firms (see, e.g., Erickson and Whited (2000)). Fortunately to our strategy, this critique implies that, contrary to our hypothesis, we should find  $\alpha_5$  to be smaller in the constrained firm subsample.

(1995), who also distinguish between groups of financially constrained and unconstrained firms on the basis of firm size.

- Scheme #3: We retrieve data on firms' bond ratings and assign to the financially constrained group those firms which never had their public debt rated during our sample period.<sup>28</sup> Financially unconstrained firms are those whose bonds have been rated during the sample period. Related approaches for characterizing financial constraints are used by Whited (1992), Kashyap et al. (1994), and Gilchrist and Himmelberg (1995).
- Scheme #4: We retrieve data on firms' commercial paper ratings and assign to the financially constrained group those firms which never had their issues (if any) rated during our sample period. Firms that issued commercial papers receiving ratings at some point during the sample period are considered unconstrained. This approach follows from the work of Calomiris et al. (1995) on the characteristics of commercial paper issuers.
- Scheme #5: We construct an index of firm financial constraints based on Kaplan and Zingales (1997) and separate firms according to this index as follows. First, using the original data from Kaplan and Zingales, we run a ordered probit regression which models the probability each one of the five categories of financial constraints in their paper as a function of a firm's cash flows, Q, leverage, and dividend payout.<sup>29</sup> Next, the coefficient estimates from this regression are used in our data (employing those author's variable definitions) to construct a linear index of firm financial constraints, which we call "KZ Index":

$$KZ Index = -1.126961 \times CashFlow + 0.2643228 \times Q$$

$$+3.480773 \times Leverage - 35.78327 \times Dividends.$$

$$(14)$$

Firms in the bottom (top) three deciles of the KZ Index ranking are considered financially unconstrained (constrained). In the spirit of Kaplan and Zingales, we allow firms to change their status throughout the sample period by ranking firms on an annual basis. See Lamont et al. (2001) or Baker et al. (2002) for a similar approach.

Table 1 reports the number of firm-years under each of the ten financial constraints categories used in our analysis. According to the dividend payout scheme, for example, there are 2,507 finan-

<sup>&</sup>lt;sup>28</sup>Comprehensive coverage on bond ratings by COMPUSTAT only starts in the mid-1980s.

<sup>&</sup>lt;sup>29</sup>We thank Steve Kaplan and Luigi Zingales for kindly providing access to their data.

cially constrained firm-years and 4,075 financially unconstrained firm-years. More interestingly, the table also displays the cross-correlation among the various classification schemes, illustrating the differences in sampling across the different criteria. Of the 2,507 firm-years considered constrained according to dividends, 1,147 are also constrained according to size, while 452 are unconstrained. Nearly half of the dividend-constrained firm-years are also constrained according to the bond rating criterion, while the other half is unconstrained. Finally, note the remarkable discrepancy in financial constraint categorization provided by the KZ Index and all of the other measures, including dividends. Only 18% (or 453) of the dividend-constrained firms are also constrained according to the KZ Index, while about 23% of those firms are unconstrained according to the KZ Index.

#### B.3 Results

Table 2 presents summary statistics on the level of cash holdings of firms in our sample after separating them into constrained and unconstrained categories. According to the dividend payout, size, and the ratings criteria, unconstrained firms hold on average roughly 10% of their total assets in the form of cash and marketable securities. This figure resembles that of Kim et al. (1998), who report mean (median) holdings of 8.1% (4.7%). Constrained firms, on the other hand, hold far more cash in their balance sheets; on average, some 30% of total assets. Mean and median tests reject equality in the level of cash holdings across groups in all cases. The one classification scheme that yields figures substantially different from the others is the KZ Index, which tends to classify firms that hold a lot cash as unconstrained. Notice that Kaplan and Zingales' presumption is that an unconstrained firm is one that has sufficient internal resources to fund its investments. Our implementation of the KZ Index reflects their premises even though we have expunged cash holdings from their original empirical model.

Table 3 presents the results obtained from the estimation of our baseline regression model (Eq.(13)) within each of the above sample partitions. The model is estimated via OLS with firm fixed-effects and the error structure (estimated via Hubber-White) allows for residual correlation within industry-years. In <u>all</u> cases the set of constrained firms display a statistically significant (at better than 1% test level) positive sensitivity of cash to cash flow, while unconstrained firms show insignificant cash sensitivities. The sensitivity estimates for constrained firms vary between 0.257 and 0.365. The median estimate, which is based on firm size (=0.311), suggests that for each dollar

of additional cash flow (normalized by assets), a constrained firm will save around 1/3 of a dollar, while an unconstrained firm does nothing. Notice that the difference in cash flow sensitivities across constrained and unconstrained firms is statistically significant at the 5% level or better in all cases with only one exception (the KZ Index). These results are fully consistent with the prediction of our basic model.

The Q-sensitivity of cash is always positive and significant for constrained firms, as predicted by our model. This sensitivity is sometimes also positive and significant in the unconstrained sample, but the magnitude of the coefficient is always larger in the constrained sample and the cross-group differences are statistically significant at the 5% level in four of the five cases. The coefficients on the control variables also have the expected signs in our regressions. Investments, acquisitions, and size are negatively correlated with the change in cash. The coefficient on the change in net working capital is generally positive, but not always significant.

#### B.4 Robustness of the baseline results

We now subject our estimates to a number of robustness checks, in order to address potential concerns about model specification and other estimation issues. First, because of the possibility of outliers having undue influence on our results, we re-estimate our models using trimmed data and (alternatively) via quantile regressions. Doing so does not materially affect our findings. In addition we address the issue of coefficient stability over time by partitioning the data according to whether the observations come from the 1980's or the 1990's. Again, our results remain, although cross-sectional differences seem stronger in the first half of the sample.<sup>30</sup>

In addition, we present estimates of the cash flow sensitivity of cash using three alternative empirical specifications in Table  $4.^{31}$  At the top of the table (see first row) we report cash flow sensitivities from a regression model that includes only the primitive elements of our basic theory (CashFlow and Q) in the set of independent variables. The second specification we experiment with adds changes in the ratio of short-term debt to total assets to the right-hand side of our baseline model. We include this extra variable because of the possibility that firms use short-term debt to build cash reserves. The third set of estimates in the table are from an alternative specification

<sup>&</sup>lt;sup>30</sup>Tabulated results for the outlier-robust and subperiod regressions are omitted for brevity, but are available from the authors upon request.

<sup>&</sup>lt;sup>31</sup>Results from a larger set of alternative specifications are available upon request.

in which we move the lag level of cash/assets from the left-hand side to the right-hand side of our baseline model, effectively removing the constraint that lagged cash/assets should have a coefficient of 1. This last approach resembles more closely that in Opler et al. (1999).

In all, there are 15 pairs of constrained/unconstrained estimates in Table 4. In each case, the estimated cash flow sensitivity of cash for the constrained firms is higher (often by a factor of 2 or more) than for comparable unconstrained firms. All of the constrained firm subsamples return a statistically significant coefficient for cash flow, while none of the unconstrained cash flow estimates are significant. The economic magnitude of the cash flow coefficient for the constrained firms is reasonably consistent, varying from 0.21 to 0.35 in most specifications. Overall, Table 4 suggests that our results are robust to changes in model specification.

### B.5 Dynamics of Liquidity Management: Responses to Macroeconomic Shocks

A potential objection to the results presented above arises from the endogeneity of a firm's financial decisions. Since choice variables such as investment in net working capital and fixed assets enter the baseline cash equation, it is possible that the *levels* of the estimates presented in Table 3 are biased. Notice, though, that our main finding concerns the *relative* sizes of coefficients across two subsamples of firms, and is not clear why any potential bias would occur differentially across the two subsamples. In other words, any argument that endogeneity issues are responsible for our results has to explain both why our estimates of cash flow sensitivities of cash are positively biased, and also why these biases are related to our partitions by financial constraints.

One way of providing independent confirmation of the interpretation we propose (as opposed to the above "endogenous bias story") comes from examining exogenous shocks affecting both firms' ability to generate cash flows as well as the shadow cost of new investment. Those shocks should be economy-wide, simultaneously affecting all firms in the sample at a given point in time and thus providing for cross-sectional contrasts. We find that examining the path of cash flow sensitivity of cash holdings over the business cycle allows for an alternative test of the idea that financial constraints drive significant differences in corporate cash policies. To wit, if our conjecture about those policies are correct, then we should see financially constrained firms saving an even greater proportion of their cash flows during recessions. This should happen because these periods are characterized both by an increase in the marginal attractiveness of future investment when

compared to current investments as well as a decline in current income flows. The cash policy of financially unconstrained firms, on the other hand, should not display such pronounced patterns. In other words, the *responses* of cash flow sensitivity of cash to changes in aggregate demand should be stronger for financially constrained firms. This should happen regardless of the *levels* of those estimates, and the test thus sidesteps concerns with endogeneity biases in the baseline equation.

To implement this test, we use a two-step approach similar to that used by Kashyap and Stein (2000) and Campello (2002). The idea is to relate the sensitivity of cash to cash flow and aggregate demand conditions by combining cross-sectional and times series regressions. The approach sacrifices statistical efficiency, but reduces the likelihood of Type I inference errors; that is, it reduces the odds of concluding that cash holdings respond to cash flow along the lines of out theory when they really don't.<sup>32</sup>

The first step of our procedure consists of estimating the baseline regression model (Eq. (13)) every year separately for groups of financially constrained and unconstrained firms. From each sequence of cross-sectional regressions, we collect the coefficients returned for cash flow (i.e.,  $\alpha_1$ ) and 'stack' them into the vector  $\Psi_t$ , which is then used as the dependent variable in the following (second-stage) time series regression:

$$\Psi_t = \eta + \sum_{k=1}^{2} \phi_k Log(GDP)_{t-k} + \rho Trend + u_t.$$
(15)

We are interested in the impact of aggregate demand, proxied by the real log of GDP, on the sensitivity of cash to cash flow. The economic and the statistical significance of aggregate demand can be gauged from the sum of the coefficients for the lags of GDP,  $\sum \phi_k$ , and from the t-statistics of this sum. We allow for two lags of GDP to account for the fact that macroeconomic movements spread out at different speeds throughout different sectors of the economy.<sup>33</sup> A time trend (Trend) is included to capture secular changes in cash policies. Finally, because movements in aggregate demand and other macroeconomic variables often coincide, in 'multivariate' versions of Eq. (15) we also include current inflation (log CPI) and interest rates (Fed funds rate) to ensure that our findings are not driven by contemporaneous macroeconomic innovations affecting the cost of money.<sup>34</sup>

<sup>&</sup>lt;sup>32</sup>An alternative one-step specification — with Eq. (15) below nested in Eq. (13) — would impose a more constrained parametrization and have more power to reject the null hypothesis of cash policy irrelevance.

<sup>&</sup>lt;sup>33</sup>Not allowing for lagged responses could bias our results if the distribution of financially constrained firms happen to be more concentrated in sectors of the economy that respond more rapidly to changes in demand. Our results are largely insensitive to sensible variations in the lag structure of Eq. (15).

<sup>&</sup>lt;sup>34</sup>These series are from the Bureau of Labor Statistics and from the Federal Reserve (Statistical Release H.15).

The results from the two-stage estimator are summarized in Table 5. The table reports the sum of the coefficients for the two lags of GDP from Eq. (15), along with the t-statistics for the sum. Panel A collects the results for financially constrained firms and Panel B reports results for unconstrained firms. Additional tests for differences between coefficients across groups are reported in the bottom of the table (Panel C). Standard errors for the "difference" coefficients are estimated via a SUR system that combines the two constraint categories (p-values reported).

All of the GDP-response coefficients for the constrained firms displayed in Panel A are negative and statistically significant at the 5% level or better, suggesting that constrained firms' cash policies respond to shocks affecting cash flows and the intertemporal attractiveness of investment along the lines of our theory. In contrast, the response coefficients for the unconstrained firms presented in Panel B display no clear patterns, but with only one exception (the KZ Index), are uniformly lower than those of Panel A. The differences between those sets of coefficients in Panel C suggest that the cash flow sensitivity of cash for financially constrained and unconstrained firms follow markedly different paths over the business cycle. These differences are consistent with the ideas in the model of liquidity management we proposed above.

In order to gauge the economic significance of the estimates in Table 5, consider a scenario in which the GDP falls by 1% over two years. Take two hypothetical firms, one constrained and the other unconstrained according to size.<sup>35</sup> Our multivariate regression estimates suggest that while there is no effect on the propensity of the unconstrained firm to hoard cash, the cash flow sensitivity of the constrained firm increases by around 0.15 over the two years. Given that the overall propensity to save cash flows estimated in Table 3 is around 0.3, the effect of a recession on the propensity to save appears to be substantial.

### C Agency Problems and Cash Holdings

Our model predicts that agency problems will lead otherwise unconstrained firms to display a propensity to store greater portions of cash flows when management has poorly-aligned incentives. This prediction holds only for those firms that become effectively constrained because of the desire of managers to overinvest. If the unconstrained firm has sufficient financial slack so that it can invest up to the manager's preferred point, its cash policy will be observationally equivalent to those

<sup>&</sup>lt;sup>35</sup>We use the size criterion here so as to be consistent with the discussion of the results of the previous table.

of financially unconstrained firms whose managers do not overinvest. In this subsection we examine the main testable implication of our agency model (Proposition 2): that the cash flow sensitivity of cash of financially unconstrained firms should be increasing in proxies for misalignment of CEO incentives when those firms have limited internal resources (i.e., low free cash flow).

In implementing a test of our agency model of liquidity, our first task is finding suitable proxies for: a) agency problems potentially associated with overinvestment tendencies, and b) Jensen's (1986) notion of "free cash flow". We use CEO ownership as our proxy for the extent of agency problems. We do so because of the idea that managerial ownership functions as a partial solution to agency problems between managers and owners and because of studies documenting a relation between firm value and management ownership (see, for example, Mørck et al. (1988) or McConell and Servaes (1990)). Accordingly, increases in ownership potentially lead to less suboptimal managerial behavior, which in our case will translate into a decreased propensity to save cash flows. To integrate this idea into our testing strategy, we estimate an augmented version of our baseline regression model in which we allow for the interaction between CEO ownership and cash flow:

$$\frac{Cash_{i,t} - Cash_{i,t-1}}{Assets_{i,t-1}} = \alpha_0 + \alpha_1 \frac{CashFlow_{i,t}}{Assets_{i,t-1}} + \alpha_2 \frac{Expenditures_{i,t}}{Assets_{i,t-1}} + \alpha_3 \frac{Acquisitions_{i,t}}{Assets_{i,t-1}} + \alpha_4 \frac{NWC_{i,t} - NWC_{i,t-1}}{Assets_{i,t-1}} + \alpha_5 Q_{i,t} + \alpha_6 Ln(Assets_{i,t}) + \alpha_7 CEO\ Ownership_{i,t} + \alpha_8 \left(CEO\ Ownership_{i,t} \times \frac{CashFlow_{i,t}}{Assets_{i,t-1}}\right) + \mu_i + \varepsilon_{i,t}.$$
(16)

Our theory suggests that the coefficient  $\alpha_8$  should be negative for financially unconstrained firms with limited stocks of internal funds, but insignificant for either financially constrained firms or unconstrained firms with abundant idle internal resources.

Our proxy for CEO ownership includes both stock and option holdings, with the data collected from ExecuComp for the 1993-2000 period.<sup>36</sup> To measure the sensitivity of CEO option holdings to firm value, we follow the procedure suggested by Yermack (1995), which uses existing options as a fraction of total shares outstanding as a proxy for incentives coming from option holdings.<sup>37</sup> As a sensitivity check, we replicate our results using only stock holdings to measure CEO ownership with similar results to those reported here.

<sup>&</sup>lt;sup>36</sup>We use the adjustment factor provided by ExecuComp (item AJEX) to adjust CEO ownership for stock splits. <sup>37</sup>As in Aggarwal and Samwick (1999), we measure option holdings using in-the-money vested options (item IN-MONEX in ExecuComp).

Nearly all of the free cash flow measures used in previous work are based on the difference between a firm's cash flow and its current investments, "excess operating cash flow" (e.g., Fenn and Liang (2001), Cristophe (2002), Opler and Titman (1993), and Lang et al. (1991)). We construct two measures of free cash flow using a similar approach. The first such measure is based on the estimate of the difference between firm cash flow and capital expenditures (both scaled by assets) for firms in the same 2-digit SIC industry and in the same year. We denote by "high" ("low") free cash flow firms those firms in the top (bottom) quintiles of the industry-year distributions of the "excess" operating cash flow. In our second measure, we denote a firm as a "high" free cash flow firm when its excess operating cash flow is above the distribution median and its Q is below the median Q. A "low" free cash flow firm has below median excess cash flow and above median Q. 38,39 Rather than directly including our free cash flow measures in the empirical specification, we partition the sample firms according to their relative ranking on those constructed proxies and estimate Eq. (16) separately over those subsamples. This approach more closely relates to the statement of Proposition 2 and facilitates the interpretation of the results — avoiding multiple interaction terms in reported outputs — with little cost to estimation.

Table 6 reports estimates of Eq. (16) across subsamples of financially constrained and unconstrained firms, with the latter group further subdivided into "high" and "low" free cash flow firms. Financial constraints are defined according to firm dividend policies through the 1981-2000 period (scheme #1). Panel A in the table reports the results when the first measure of free cash flow (or excess operating cash flow) is considered and Panel B does similarly for the second measure (which also includes Q).

The first column in each of the panels in Table 6 shows that our main conclusions about the cash policies of financially constrained firms remain for the 1993-2000 subsample and are insensitive to managerial incentive issues. The second and third columns examine the implication of our agency story more directly. The results in those columns indicate that managerial incentives do not

 $<sup>^{38}</sup>$ The only difference from the previous papers is that instead of using the median of the entire distribution of excess cash flow and Q, we use industry-year medians as benchmarks to compute our measures of free cash flow. While adjusting for industry and year effects does not alter the nature of our findings, we believe this to be more appropriate given that ignoring industry and year idiosyncractic differences might lead to biased sampling. Consider, for instance, what happens with firms in durable goods industries. Given a fixed investment schedule, these firms will (jointly) observe a steeper decline in cash flows during recession years relative to all other firms, and thus be systematically labeled as "low" free cash flow in those periods, with the opposite occurring in booms.

<sup>&</sup>lt;sup>39</sup>We keep only industry-years with more than ten observations, so that the measure of centrality makes sense.

influence cash management of financially unconstrained firms when they have a wealth of internal resources, but do alter those policies when internal resources are relatively scarce. Consistent with our theory, the results in the third column show that the tendency to hold cash is decreasing in CEO ownership. The fourth column in both panels of Table 6 shows that differences in the influence of ownership on cash policies are significant across groups of high and low free cash flow firms. Indeed, the differences in the coefficient of the ownership—cash flow interaction term ( $\alpha_8$ ) are significant at better than 1% level in both panels. In terms of economic significance, our estimates imply that an increase of 1% on CEO ownership reduces the cash flow sensitivity of cash of unconstrained-low free cash flow firms by a factor between 0.025 and 0.075, depending on the measure of free cash flow used. This effect appears substantial given the point estimates we obtained for the cash flow sensitivity of cash for constrained firms throughout our analysis (around 0.3).

While we interpret the results of Table 6 as consistent with the implications of our agency model, we find that they hold only weakly when we replicate the same experiments using the other four alternative measures of financial constraints. In particular, although the results hold well when we partition on size and also — albeit more weakly — when we use bonds ratings for financial constraint categorization, the same does not occur when we use either commercial paper ratings or the Kaplan-Zingales criteria to classify firms into financial constraint categories (outputs omitted). Overall, we take the empirical findings of this section as suggestive, but not conclusive evidence in support of the agency view of corporate cash management. Further study on the extent which agency problems affect corporate decisions on liquidity appears to be well worth pursuing.

# IV Concluding Remarks

One of Keynes' (1936) many important contributions is his notion that the liquidity of a firm's assets is not predetermined, but rather an important decision firms have to make. He suggested that observed choices regarding liquidity will depend on firms' access to capital markets and the importance of future investments to the firms. This idea was echoed in a recent talk given by Richard Passov, the Treasurer of Pfizer, who claimed that Pfizer keeps cash balances that are larger than its long-term debt precisely because its assets are not easily collateralizable, and that its future investments have a huge value to the company.<sup>40</sup>

<sup>&</sup>lt;sup>40</sup>Richard Passov, address to the National Forum on Corporate Finance, delivered on May 3, 2002 in Austin, Texas.

This paper models this idea formally, and tests it using data from publicly-traded manufacturing firms. In the model, firms have access to investment opportunities, but potentially cannot fund them. As in Froot et al. (1993, 1994), financial decisions are important because they enable firms to undertake valuable investment opportunities. In the model's equilibrium, cash holdings for an unconstrained value-maximizing firm are irrelevant because such a firm can undertake all positive NPV projects regardless of its cash position. However, value-maximizing firms that face financial constraints have an optimal level of cash holdings, which are determined by a first-order condition equating the value of the foregone marginal projects (the marginal cost of cash holdings) to the expected value of additional projects the firm will be able to fund in the future (the marginal benefit of cash holdings). This model contains the testable prediction that an unconstrained firm will not have a systematic propensity to save out of incremental cash inflows, while a constrained firm will save a positive portion of cash flows.

We also analyze the case where, as hypothesized by Jensen (1986), managers have incentives to overinvest, especially from internally-generated funds. In this version of the model, the equilibrium of a constrained firm is unaffected, because the overinvestment effect alters preferences equally in each period, so it does not affect the intertemporal choice between investments. However, a firm that is unconstrained can effectively become constrained given agency considerations if cash flows in each period are not sufficient to the allow the firm to undertake all the investments managers would like to undertake. In this case, an otherwise unconstrained firm will act like a constrained firm in that there will be an "optimal" level of cash holdings, and the firm will display a propensity to save a positive portion of incremental cash inflows.

We examine the empirical implications of this model on a sample of 1,026 publicly-traded manufacturing firms between 1981 and 2000. Using five alternative measures of financial constraints suggested in the literature, we test the hypothesis that the propensity to save from cash inflows is larger for the constrained firms than for the unconstrained ones. Consistent with the value-maximizing model, we find that constrained firms save a positive fraction of cash flows, while unconstrained firms do not display a systematic propensity to save cash flows. Moreover, we find that the propensity of constrained firms to save incremental cash flows is larger in recessions than in good economic times, consistent with the Keynes' notion that when firms are more likely to be financially constrained in the future they are more likely to save cash today.

Finally, we also test the implication of the agency version of the model that financially unconstrained firms whose managers are likely to have little or no incentives to adopt value-maximizing policies manage firm liquidity as if they were financially constrained. The results for this test depend on the particular measure of financial constraints used. We do, nonetheless, find some evidence consistent with the agency view of corporate liquidity management.

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## Appendix: Comparative statics with constrained hedging

We use the same parametrization described in Eq. (4) to write the fist order condition as:

$$(c_0 - C^*)^{-1} = \delta \left[ p \left( \mu c_1^H + C^* \right)^{-1} + (1 - p) \left( c_1^L + (1 - \mu) \frac{p}{1 - p} c_1^H + C^* \right)^{-1} \right]. \tag{17}$$

There is no closed form solution for  $C^*$ , but we can still obtain comparative statics results. Define the function  $F \equiv F(C, c_0, c_1^H, c_1^L, \mu, p)$  as:

$$F(\cdot) = (c_0 - C^*)^{-1} - \delta p \left(\mu c_1^H + C^*\right)^{-1} - \delta (1 - p) \left(c_1^L + (1 - \mu) \frac{p}{1 - p} c_1^H + C^*\right)^{-1}.$$

Notice that this equation is independent of  $\tau$ , and thus similarly to the case of perfect hedging, borrowing capacity will not affect cash policies for firms which are constrained.

With some algebra, we can show that:  $\frac{\partial F}{\partial C} > 0$ ,  $\frac{\partial F}{\partial c_0} < 0$ ,  $\frac{\partial F}{\partial \mu} < 0$ , and  $\frac{\partial F}{\partial \delta} < 0$ . These derivatives yield the following results for hedge-constrained firms:

• If time 0 cash flow increases, then the firm hoards more cash:

$$\frac{\partial C}{\partial c_0} = -\frac{\frac{\partial F}{\partial c_0}}{\frac{\partial F}{\partial C}} > 0$$

• If future investment opportunities are more profitable than the current investment opportunities ( $\delta$  is high), then the firm hoards more cash:

$$\frac{\partial C}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial C}} > 0$$

• Firms which can hedge less hoard more cash:

$$\frac{\partial C}{\partial \mu} = -\frac{\frac{\partial F}{\partial \mu}}{\frac{\partial F}{\partial C}} > 0$$

Table 1: Cross-Classification of Financial Constraint Types

This table displays firm-quarter cross-classification for the various criteria used to categorize firm-quarters as either financially constrained or unconstrained (see text for definitions). The sampled firms include only manufacturers (SICs 2000-3999) in the COMPUSTAT annual industrial tapes. The sample period is 1981 through 2000.

FINANCIAL CONSTRAINTS CRITERIA	Div. I	PAYOUT	FIRM	Size	Bond F	RATINGS	CP R.	ATINGS	KZ I	NDEX
	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
1. Dividend Payout										
Constrained Firms (A)	2,507									
Unconstrained Firms (B)		4,075								
2. Firm Size										
Constrained Firms (A)	1,147	472	$2,\!865$							
Unconstrained Firms (B)	452	2,111		3,793						
3. Bond Ratings										
Constrained Firms (A)	1,252	1,029	2,351	194	4,480					
Unconstrained Firms (B)	1,255	3,046	514	3,599		6,655				
4. Commercial Paper Ratings										
Constrained Firms (A)	2,257	1,946	2,865	1,020	4,339	3,342	7,681			
Unconstrained Firms (B)	250	2,129	0	2,773	141	3,313		3,454		
5. Kaplan-Zingales Index										
Constrained Firms (A)	453	1,218	136	1,796	378	2,321	1,245	$1,\!454$	2,699	
Unconstrained Firms (B)	873	551	1,535	123	1,869	686	2,387	168		2,555

Table 2: Summary Statistics of Cash Holdings

This table displays summary statistics for cash holdings across groups of financially constrained and unconstrained firms (see text for definitions). All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000-3999) and the sample period is 1981 through 2000.

nough 2000.				
Cash Holdings $\left(\frac{Cash_{i,t}}{Assets_{i,t-1}}\right)$	Mean	Median	Std. Dev.	N. Obs
FINANCIAL CONSTRAINTS CRITERIA				
1. DIVIDEND PAYOUT				
Constrained Firms (A)	0.372	0.176	0.539	2,507
Unconstrained Firms (B)	0.112	0.049	0.208	4,075
$p\text{-value }(\mathbf{A} - \mathbf{B} \neq 0)$	0.00	0.00		
2. Firm Size				
Constrained Firms (A)	0.372	0.219	0.502	2,865
Unconstrained Firms (B)	0.096	0.043	0.168	3,793
$p$ -value (A-B $\neq$ 0)	0.00	0.00		
3. Bond Ratings				
Constrained Firms (A)	0.296	0.140	0.452	4,480
Unconstrained Firms (B)	0.135	0.051	0.259	6,655
$p\text{-value }(\mathbf{A} - \mathbf{B} \neq 0)$	0.00	0.00		
4. Commercial Paper Ratings				
Constrained Firms (A)	0.253	0.105	0.412	7,681
Unconstrained Firms (B)	0.081	0.041	0.130	3,454
$p$ -value (A-B $\neq$ 0)	0.00	0.00		
5. Kaplan-Zingales Index				
Constrained Firms (A)	0.068	0.029	0.100	2,699
Unconstrained Firms (B)	0.469	0.303	0.566	2,555
$p$ -value (A-B $\neq$ 0)	0.00	0.00		

Table 3: Baseline Regressions: The Cash Flow Sensitivity of Cash

This table displays results for OLS with firm fixed-effects estimations of the baseline model. All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000-3999) and the sample period is 1981 through 2000. The estimations correct the error structure both for heterosckedasticity and for within-period error correlation using the White-Hubber estimator. t-stats (in parentheses).

Dependent Variable	Independent Variables						$\mathbb{R}^2$
$\frac{Cash_{i,t} - Cash_{i,t-1}}{Assets_{i,t-1}}$	CashFlow	Expenditures	Acquisitions	$\Delta NWCapital$	Q	Ln(Assets)	
FINANCIAL CONSTRAINTS CRITERIA							
1. DIVIDEND PAYOUT							
Constrained Firms (A)	0.3645 (2.73)*	-0.8850 $(-5.88)$ *	-0.3280 $(-2.47)$ *	0.1110 (1.49)	0.0210 (4.71)*	-0.0454 $(-2.19)**$	0.283
Unconstrained Firms (B)	0.0471 (0.86)	$-0.3949 \ (-5.35)^*$	-0.1913 $(-3.68)*$	0.0190 (0.16)	0.0037 $(0.72)$	-0.0075 $(-0.65)$	0.122
$p\text{-value }(\mathbf{A} - \mathbf{B} \neq 0)$	0.01	0.03	0.27	0.55	0.04	0.67	
2. Firm Size							
Constrained Firms (A)	0.3118 (3.03)*	$-0.7674 \ (-4.09)^*$	$-0.6160 \ (-4.72)^*$	$0.1666 \ (1.50)$	0.0264 (7.38)*	-0.0396 $(-2.30)**$	0.264
Unconstrained Firms (B)	0.0324 $(0.49)$	-0.4504 $(-4.26)$ *	-0.1408 $(-3.97)*$	0.0457 $(0.87)$	0.0026 $(0.40)$	-0.0454 $(-3.75)*$	0.234
$p\text{-value }(\mathbf{A} - \mathbf{B} \neq 0)$	0.05	0.19	0.00	0.30	0.00	0.72	
3. Bond Ratings							
Constrained Firms (A)	0.3157 (3.20)*	$-0.8208 \ (-5.06)^*$	-0.5777 $(-4.97)*$	0.2087 $(1.57)$	0.0224 (5.54)*	-0.0571 $(-3.28)*$	0.317
Unconstrained Firms (B)	0.1351 $(1.35)$	-0.4985 $(-4.19)*$	-0.1678 $(-7.08)*$	0.0538 $(1.94)$	0.0117 (2.32)**	$-0.0340 \ (-2.37)**$	0.274
$p\text{-value }(\mathbf{A} \!-\! \mathbf{B} \!\neq\! \! 0)$	0.19	0.19	0.00	0.23	0.01	0.13	

Table 3 - Continued

	Independent Variables						$R^2$
	CashFlow	Expenditures	Acquisitions	$\Delta NWCapital$	Q	Ln(Assets)	
4. Commercial Paper Ratings							
Constrained Firms (A)	0.2573 $(3.24)*$	$-0.7080 \ (-7.88)^*$	-0.3556 $(-4.88)$ *	0.1735 $(2.06)**$	0.0217 $(4.59)*$	-0.0489 $(-3.35)*$	0.30
Unconstrained Firms (B)	0.0008 (0.01)	$-0.4004 \ (-2.91)^*$	-0.0971 $(-4.38)$ *	-0.0738 $(-2.37)**$	0.0013 $(0.23)$	-0.0259 $(-1.94)$	0.17
$p$ -value (A-B $\neq$ 0)	0.02	0.12	0.01	0.01	0.01	0.07	
5. Kaplan-Zingales Index							
Constrained Firms (A)	0.2758 $(5.60)*$	-0.4686 $(-6.19)*$	-0.1293 $(-3.45)*$	-0.0685 $(-1.49)$	0.0182 $(2.07)**$	-0.0015 $(-0.37)$	0.46
Unconstrained Firms (B)	0.1225 $(0.74)$	-1.2513 $(-4.32)*$	-0.6669 $(-3.38)*$	0.2009 (1.66)	0.0120 (2.42)**	-0.0673 $(-2.26)**$	0.37
$p$ -value (A-B $\neq$ 0)	0.01	0.54	0.00	0.01	0.84	0.00	

Table 4: Robustness: Alternative Specifications of the Baseline Regression Model

This table displays results for OLS with firm fixed-effects estimations of alternative versions of the baseline liquidity model. The reported estimates are the coefficients returned to CashFlow under the different model specifications described in the table (see text for more details). All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000-3999) and the sample period is 1981 through 2000. The estimations correct the error structure both for heterosckedasticity and for within-period error correlation using the White-Hubber estimator. t-stats (in parentheses).

	Financial Constraints Criteria						
	DIV. PAYOUT	FIRM SIZE	BOND RATINGS	CP RATINGS	KZ Index		
1. $CashFlow$ and $Q$ (only) in the R.H.S. of model							
Constrained Firms (A)	0.3339 (2.61)*	0.2390 (2.31)**	0.2337 (2.36)**	0.2130 (3.05)*	0.1008 (3.93)*		
Unconstrained Firms (B)	$-0.0183 \ (-0.41)$	0.0439 $(0.72)$	0.1299 $(1.64)$	$-0.0100 \ (-0.15)$	0.0344 $(0.23)$		
$p$ -value (A-B $\neq$ 0)	0.01	0.09	0.42	0.02	0.22		
2. Changes in short-term debt added to the r.h.s. of baseline model							
Constrained Firms (A)	0.3478 (2.66)*	0.2861 (2.84)*	0.2970 (3.08)*	0.2399 (3.07)*	0.1596 (5.24)*		
Unconstrained Firms (B)	0.0318 $(0.58)$	0.0336 $(0.51)$	0.1275 $(1.27)$	0.0093 $(0.11)$	0.0895 $(0.54)$		
$p$ -value (A-B $\neq$ 0)	0.01	0.07	0.18	0.02	0.67		
3. Dep. var. is $Cash/Assets$ ; lagged $Cash/Assets$ added to the r.h.s. of baseline model							
Constrained Firms (A)	0.3580 $(2.62)*$	0.3107 (3.01)*	0.3319 (3.10)*	0.2749 (3.38)*	0.1788 (6.26)*		
Unconstrained Firms (B)	0.0298 $(0.59)$	$0.0790 \\ (1.30)$	0.1567 $(1.63)$	0.0253 $(0.31)$	0.0685 $(0.44)$		
$p$ -value (A-B $\neq$ 0)	0.02	0.07	0.22	0.01	0.34		

Table 5: Macroeconomic Dynamics: Two-Stage Estimator of the Impact of Aggregate Activity on the Cash Flow Sensitivity of Cash

The dependent variable is the estimated sensitivity of cash holdings to cash flow for firms classified as financially constrained vs. unconstrained. In each estimation, the dependent variable is regressed on two lags of the log of real GDP and a time trend. In the multivariate regressions, current inflation (log CPI) and interest rates (Fed funds rate) are also added. The sampled firms include only manufacturers (SICs 2000-3999) and the sample period is 1981 through 2000. The sum of the coefficients for the two lags of the economic activity measure is shown along with the t-statistic of this sum (in paranthesis). Heteroskedasticity- and autocorrelation-consistent errors are computed with a Newey-West (1987) lag window of size two. The standard errors for the difference of the sum of the two lags of GDP are computed with a SUR system that estimates constraint categories regressions jointly.

	Financial Constraints Criteria						
	DIV. PAYOUT	FIRM SIZE	BOND RATINGS	CP RATINGS	KZ Index		
1. Constrained Firms							
Univariate							
Sum of Lag Coefficients	-16.566	-14.101	-10.791	-7.125	-2.428		
Summation Test $(t\text{-stat})$	(-3.92)*	(-3.08)*	(-4.83)*	(-2.13)**	(-2.42)**		
Multivariate							
Sum of Lag Coefficients	-18.834	-16.188	-11.139	-8.770	-2.409		
Summation Test $(t\text{-stat})$	(-5.21)*	(-5.62)*	(-5.31)*	(-3.53)*	(-2.32)**		
2. Unconstrained Firms							
Univariate							
Sum of Lag Coefficients	11.332	-1.475	-4.003	7.749	-14.984		
Summation Test $(t\text{-stat})$	(2.02)**	(-0.53)	(-0.82)	(1.30)	(-3.56)*		
Multivariate							
Sum of Lag Coefficients	11.502	-0.960	-5.130	7.365	-16.024		
Summation Test $(t\text{-stat})$	(2.54)**	(-0.47)	(-1.51)	(1.39)	(-6.45)*		
3. Diff. Constrained—Unconstrained							
Univariate							
Diff. Coefficient	-27.898	-12.626	-6.788	-14.874	12.556		
Diff. $p$ -value	0.01	0.09	0.47	0.12	0.06		
Multivariate							
Diff. Coefficient	-30.336	-15.228	-6.009	-16.135	13.616		
Diff. $p$ -value	0.00	0.04	0.51	0.09	0.05		

## Table 6: The Cash Flow Sensitivity of Cash the Agency Problem

This table displays results for fixed-effects OLS estimations of the cash flow sensitivity of cash with proxies for agency problems. The baseline regression model is augmented with the inclusion of CEO Ownership (based on stocks + options) as a proxy for CEO incentives, and its interaction with cash flow. Financial contraints are defined based on firm dividend policies throughout the 1981-2000 period. In Panel A, High (Low) free cash flow firms are those in the top (bottom) quintile of the industry-year-normalized distribution of the "excess operating cash flow" measure. This measure is based on the differece between a firm's cash flow and capital expeditures (both scaled by assets) and that of the median firm in the same 2-digit SIC industry in the same year. In Panel B, High (Low) free cash flow firms are those with above (below) industry-year median "excess operating cash flow" and below (above) industry-year median Q. The estimation period is 1993-2000. The estimations correct the error structure both for heterosckedasticity and for within-period error correlation using the White-Hubber estimator. t-stats (in parentheses).

PANEL A: FREE CASH FLOW PROXYED BY (RELATIVE TO INDUSTRY-YEAR) "EXCESS OPERATING CASH FLOW" RANKINGS

	FINANCIALLY CONSTRAINED	Financially U		
Indep. Variables		High Free Cash Flow (A)	Low Free Cash Flow (B)	$\begin{array}{c} p\text{-value} \\ (\mathbf{A} - \mathbf{B} \neq 0) \end{array}$
CashFlow	0.2676 (2.67)*	0.1041 (0.38)	$0.1329 \ (0.52)$	0.85
Expenditures	-1.1997 (-3.04)*	$-0.2301 \ (-0.34)$	$-0.9996 \ (-1.65)$	0.58
Acquisitions	$-0.5053 \ (-5.00)^*$	$0.3661 \\ (0.41)$	-0.2284 $(-1.70)$	0.19
$\Delta NWCapital$	0.0827 $(0.48)$	$0.1764 \ (1.56)$	$0.1079 \ (0.84)$	0.78
Q	0.0179 (2.45)*	0.0186 (3.06)*	0.0155 $(1.19)$	0.82
Ln(Assets)	$0.0428 \ (0.87)$	$-0.0026 \ (-0.09)$	-0.0201 $(-1.31)$	0.42
$CEO\ Ownership$	$0.1116 \ (0.33)$	$-0.5051 \ (-0.93)$	0.9870 (2.27)**	0.04
$CEO\ Ownership \ \times\ CashFlow$	$-0.3900 \ (-0.20)$	4.2166 (1.27)	$-7.0219 \ (-2.58)^*$	0.00
Observations $R^2$	$965 \\ 0.194$	$206 \\ 0.521$	187 0.580	

Table 6 - Continued

Panel B: Free Cash Flow Proxyed by the Joint Distribution of "Excess Operating Cash Flow" and Tobin's Q

	FINANCIALLY CONSTRAINED	FINANCIALLY U		
Indep. Variables		High Free Cash Flow (A)	Low Free Cash Flow (B)	$p -value (A-B \neq 0)$
CashFlow	0.2676 (2.67)*	0.3287 (0.82)	-0.1689 $(-0.57)$	0.18
Expenditures	$-1.1997 \\ (-3.04)*$	$-0.7024 \ (-0.83)$	-0.0338 $(-0.17)$	0.41
Acquisitions	$-0.5053 \ (-5.00)*$	-0.0368 $(-0.41)$	-0.2326 $(-3.38)$ *	0.03
$\Delta NWCapital$	0.0827 $(0.48)$	$-0.1706 \ (-0.97)$	$0.1589 \ (0.97)$	0.44
Q	0.0179 (2.45)*	$-0.0219 \ (-0.97)$	0.0240 (3.25)*	0.01
Ln(Assets)	$0.0428 \ (0.87)$	0.0245 $(1.79)$	$0.0122 \ (0.33)$	0.54
$CEO\ Ownership$	$0.1116 \ (0.33)$	$-1.3643 \ (-1.04)$	$-0.3906 \ (-1.14)$	0.27
$CEO\ Ownership \\ \times\ CashFlow$	$-0.3900 \ (-0.20)$	5.0026 (0.69)	$-2.5261 \\ (-1.87)**$	0.01
Observations $\mathbb{R}^2$	$965 \\ 0.194$	$\frac{202}{0.553}$	$191 \\ 0.242$	