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Abstract

This paper introduces a new way to measure competition based on firms' profits. Within a general model, we derive conditions under which this measure is monotone in competition, where competition can be intensified both through a fall in entry barriers and through more aggressive interaction between players. The measure is shown to be more robust theoretically than the price cost margin. This allows for an empirical test of the problems associated with the price cost margin as a measure of competition.

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1. Introduction

A question often asked in both economic policy and research is how the intensity of competition evolves over time in a certain sector. To illustrate, a competition authority may want to monitor an industry so that it can intervene when competition slackens. Alternatively, there may have been a policy change in an industry (e.g. abolishing a minimum price or breaking up a large incumbent firm) with the goal of intensifying competition in the industry. Afterwards policy makers want to check whether the policy change had the desired effect. In economic research, there are empirical papers trying to identify the effect of competition on firms' efficiency (Nickell

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(1996)), on firms' innovative activity (Aghion et. al. (2002) and references therein) and the effects of competition on wage levels (Nickell (1999) for an overview) and wage inequality (Guadalupe (2003)). The question is how should competition be measured for these purposes.

The price cost margin (PCM) is widely used as a measure of competition for these purposes. However, the theoretical foundations of PCM as a competition measure are not very robust. Theoretical papers like Amir (2002), Bulow and Klemperer (1999), Rosentahl (1980) and Stiglitz (1989) present models where more intense competition leads to higher PCM instead of lower margins. We believe that there are two reasons why PCM is still such a popular empirical measure of competition. First, we do not know how important these theoretical counterexamples are in practice. Is it the case that in 20% of an economy's industries the structure is such that more competition would lead to higher PCM or is this only the case in 1% of the industries? In the former case there would be big problems for the empirical papers mentioned above which use PCM as a measure of competition. In the latter case, the theoretical counterexamples do not seem to pose acute problems for empirical research. As long as there is no evidence that the theoretical counterexamples are important empirically, one would expect that PCM remains a popular competition measure. The second reason for the popularity of PCM is that the data needed to get a reasonable estimate of PCM is available in most datasets.[1]

The idea of the current paper is to develop a competition measure that is both theoretically robust and does not pose more stringent data requirements than PCM. This new measure can then be estimated in the same datasets as where PCM is estimated. This allows a comparison between the new measure and PCM for a number of industries over time together. If in 99% of the industries the two measures indicate the same development in intensity of competition over time, this would indicate that the theoretical counterexamples cited above are not particularly

[1]Sometimes PCM is defended as measure of competition with reference to its interpretation as a welfare measure (prices closer to marginal costs lead to higher welfare). However, as shown by Amir (2002) there is, in general, no simple relation between PCM and welfare. The same is true for the measure introduced here: there is no simple relation with welfare.

relevant in practice. However, if in 20% of the cases the two measures diverged then one should be more careful in using PCM as a measure of competition in empirical research and policy analysis.

The measure we introduce in this paper is called relative profit differences (RPD). It is defined as follows. Let $\pi(n)$ denote the variable profit level of a firm with efficiency level $n \in \mathbb{R}_+$ (more details follow below on how variable profits and efficiency are defined). Consider three firms with different efficiency levels, $n'' > n' > n$, and calculate the following variable $\frac{\pi(n'')-\pi(n)}{\pi(n')-\pi(n)}$. Then more intense competition (brought about by either lower entry costs or more aggressive interaction among existing firms) raises this variable for a broad set of models. More precisely, in any model where a rise in competition reallocates output from less efficient to more efficient firms it is the case that more intense competition raises $\frac{\pi(n'')-\pi(n)}{\pi(n')-\pi(n)}$. Since this output reallocation effect is a general feature of more intense competition, RPD is a rather robust measure of competition from a theoretical point of view. Moreover, we show that the output reallocation effect is a natural necessary condition for PCM to be decreasing in intensity of competition, but it is not sufficient.

The intuition for RPD is related to the relative profits measure ($\pi(n')/\pi(n)$ is increasing in intensity of competition for $n' > n$) introduced by Boone (2000). The intuition for the relative profits measure is that in a more competitive industry, firms are punished more harshly for being inefficient. However, Boone (2000) analyzes the relative profits measure in a number of specific examples, not in a general framework as we use here. Next, as explained below, it is harder to derive sufficient conditions for the relative profits measure to be monotone in intensity of competition because of a level effect. This level effect is removed by working with profit differences instead of profit levels.

The intuition why RPD is increasing in intensity of competition can be stated as follows. As the industry becomes more competitive, the most efficient firm n'' gains more relative to a less efficient firm n than firm n' does (with $n'' > n' > n$). Think, for instance, of a homogenous good market where firms produce with constant marginal costs. If these firms compete in quantities (Cournot), one would find (if n is close enough to n'') that $\pi(n'') > \pi(n') > \pi(n) > 0$. If competition is intensified by a switch to Bertrand competition, the profit levels satisfy: $\pi(n'') > \pi(n') = \pi(n) = 0$. Hence the rise in competition raises $\pi(n'') - \pi(n)$ relative to $\pi(n') - \pi(n)$.

Recent papers measuring PCM include the following. First, Graddy (1995), Genesove and Mullin (1998) and Wolfram (1999) estimate the elasticity adjusted PCM. This yields the conduct (or conjectural variation) parameter, which can be interpreted as a measure of competition. This approach has been criticised by Corts (1999) who shows that, in general, efficient collusion cannot be distinguished from Cournot competition using the elasticity adjusted PCM. Second, Berry, Levinsohn and Pakes (1995) and Goldberg (1995) estimate both the demand and cost side of the automobile market. Their models can be used to simulate the effects of trade or merger policies on the industry. Using their estimates, one can also derive firms' PCMs. Nevo (2001) uses the same methods to estimate PCMs for firms in the ready-to-eat cereal industry. He does this under three different models of firm conduct and then compares the outcomes with (crude) direct observations of PCM. In this way he is able to identify the conduct model that explains best the observed values of PCM. As we argue below, in these papers one would also have been able to derive RPD, which has a more robust relation with intensity of competition.

This paper is organized as follows. The next section introduces the model and the way that more intense competition is identified in this general set up using the (generalized) output reallocation effect. Section 3 shows that RPD is increasing in competition and discusses which type of data are needed to estimate RPD in practice. Section 4 compares RPD and PCM and argues that both require similar data to be estimated. Further, we show that whereas the output reallocation effect is sufficient for RPD to be monotone in competition, it is only a necessary condition for PCM to be decreasing in competition, which explains the theoretical counterexamples. Finally, section 5 concludes. The proofs of results can be found in the appendix.

2. The model

The aim of this section is to introduce a fairly general model of firms competing in a market. To keep things general we do not impose either Bertrand or Cournot competition. We simply assume that each firm n chooses a vector of strategic variables $a_n \in \mathbb{R}^K$. This choice leads to output vector $q(a_n, Q, \theta) \in \mathbb{R}_+^L$ for firm n where Q aggregates actions chosen by the firms in the industry that affect firm n's output (see below) and θ is a parameter that affects the aggressiveness with which firms interact in the market. For instance, θ could be related to the substitution elasticity between goods from different producers or it could denote whether firms play Cournot or Bertrand competition. Further, the choices of the strategic variables also lead to a vector of prices $p(a_n, P, \theta) \in \mathbb{R}_+^L$ for firm n's products, where P aggregates actions chosen by the firms in the industry that affect n's prices. We assume that Q and P take the following form

$$\begin{aligned} Q &= \int \zeta(a_n)\, dn \\ P &= \int \xi(a_n)\, dn \end{aligned}$$

for some functions $\zeta(.)$ and $\xi(.)$ where we integrate over all firms in the industry.[2] To illustrate, consider the case where demand is derived from a CES utility function $\left(\int x_n^\theta dn\right)^{\frac{1}{\theta}}$ where each firm n produces one product and consumers spend an amount Y in this industry. Then firm n faces demand of the form $x_n = p_n^{\frac{-1}{1-\theta}} \frac{Y}{\int p_j^{\frac{\theta}{1-\theta}} dj}$. In the notation used here, we get $p(a_n, P, \theta) = p_n$ and $q(a_n, Q, \theta) = p_n^{\frac{-1}{1-\theta}} \frac{Y}{Q}$ where $Q = \int p_j^{\frac{\theta}{1-\theta}} dj$.

[2] We allow $\zeta(.)$ and $\xi(.)$ to be vectors. In that case the integration is done for each vector element separately to obtain the vectors Q and P.

Finally, we specify the costs of production for firm n as $C\left(q\left(a_n, Q, \theta\right), n\right)$. We say that $n \in \mathbb{R}_+$ measures a firm's efficiency level because of the following assumption.

Assumption 1: For a given output vector $q \in \mathbb{R}_+^L$ we assume that

$$\begin{aligned} \frac{\partial C\left(q,n\right)}{\partial q_l} &> 0 \\ \frac{\partial C\left(q,n\right)}{\partial n} &\leq 0 \\ \frac{\partial\left(\frac{\partial C(q,n)}{\partial q_l}\right)}{\partial n} &\leq 0 \end{aligned}$$

for each $l \in \{1, 2, ..., L\}$, where the last inequality is strict for at least one combination of q and l.

That is, higher production levels lead to higher costs. Further, higher n firms produce the same output vector q with (weakly) lower costs C and (weakly) lower marginal costs for each product l. We assume that the efficiency distribution in the industry is given. In particular, we assume that n has an atomless distribution on the interval $[n_0, n_1]$ with density function $f\left(.\right)$ and distribution function $F\left(.\right)$. Although this distribution is exogenously given, the firms that are active in equilibrium is endogenously determined, as discussed below. The essential assumption here is that efficiency can be captured by a one dimensional variable n. This assumption is not innocuous and will be discussed further below.

Using this set up, consider the following two stage game. In the first stage, firms decide simultaneously and independently whether or not to enter. Let's normalize actions a_n in such a way that a firm n that does not enter has $a_n = 0$ (while firms that do enter have $a_n \neq 0$). If a firm of type n enters it pays an entry cost $\gamma\left(n\right)$, where γ is a continuous function of efficiency n. In the second stage firms know which firms entered in the first stage and all firms that entered choose simultaneously and independently their action vectors a_n.[3] We define an equilibrium of this game as follows.

[3]To simplify notation, we assume that all firms with efficiency n choose the same action a_n in equilibrium.

Definition 1 *The set of actions* $\{\hat{a}_n\}_{n\in[n_0,n_1]}$ *denotes a pure strategy equilibrium if the following conditions are satisfied*

$$\max_{a_n}\left\{p\left(a_n,\hat{P},\theta\right)^T q\left(a_n,\hat{Q},\theta\right)-C\left(q\left(a_n,\hat{Q},\theta\right),n\right)\right\}-\gamma(n)<0 \textit{ implies } \hat{a}_n=0$$

where $p(.)^T$ *denotes the transpose of the column vector* $p(.)$ *and*

$$\left\{p\left(\hat{a}_n,\hat{P},\theta\right)^T q\left(\hat{a}_n,\hat{Q},\theta\right)-C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),n\right)\right\}-\gamma(n)\geq 0 \textit{ for } \hat{a}_n\neq 0$$

further

$$\hat{a}_n=\arg\max_{a_n}\left\{p\left(a_n,\hat{P},\theta\right)^T q\left(a_n,\hat{Q},\theta\right)-C\left(q\left(a_n,\hat{Q},\theta\right),n\right)\right\}$$

with

$$\begin{aligned}\hat{Q} &= \int_{n_0}^{n_1}\zeta(\hat{a}_n)f(n)\,dn\\ \hat{P} &= \int_{n_0}^{n_1}\xi(\hat{a}_n)f(n)\,dn\end{aligned}$$

Thus firm n stays out of the market if it cannot recoup its entry cost $\gamma(n)$. Firms that enter choose action a_n to maximize their (after entry) profits. In other words, we consider a subgame perfect equilibrium here. Finally, we require the equilibrium to be consistent in the sense that the aggregate variables $\hat{Q}$ and $\hat{P}$ follow from the equilibrium actions $\hat{a}_n$.

The following lemma derives an intuitive property of this equilibrium. If two firms n^* and n with $n^*>n$ both enter and produce positive output levels, then n^* produces (weakly) more than n and n^* is (weakly) more profitable.

Lemma 1 *Consider two firms* n^* *and* $n<n^*$ *that both produce positive output levels in equilibrium (i.e.* $\hat{a}_{n^*},\hat{a}_n\neq 0$*). Then*

$$q\left(\hat{a}_{n^*},\hat{Q},\theta\right)\geq q\left(\hat{a}_n,\hat{Q},\theta\right)$$

and

$$
\begin{aligned}
& p\left(\hat{a}_{n^*}, \hat{P}, \theta\right)^T q\left(\hat{a}_{n^*}, \hat{Q}, \theta\right) - C\left(q\left(\hat{a}_{n^*}, \hat{Q}, \theta\right), n^*\right) \\
\geq\ & p\left(\hat{a}_{n}, \hat{P}, \theta\right)^T q\left(\hat{a}_{n}, \hat{Q}, \theta\right) - C\left(q\left(\hat{a}_{n}, \hat{Q}, \theta\right), n\right)
\end{aligned}
$$

We allow the entry cost γ to vary with a firm's efficiency level, $\gamma(n)$. It may be the case that more efficient firms face lower entry costs, $\gamma'(n) < 0$, because these firms are more efficient in both entry and production. But we also allow for the case where more efficient firms pay a higher entry cost to realize their cost advantage, $\gamma'(n) > 0$. For instance, this could reflect investments in R&D to develop a better production technology, investing more in capital or building a bigger factory to reap advantages of economies of scale. Thus an important distinction between $C(q, n)$ and $\gamma(n)$ is that $C(q, n)$ is weakly decreasing in n (for given q) while the sign of $\gamma'(n)$ is unrestricted.

The case with $\gamma'(n) > 0$ is also interesting as it allows for the *selection effect* of competition. In particular, more aggressive interaction between firms may lead to entry by more efficient firms at the expense of less efficient rivals.

Example 1 *Consider an industry with two firms producing perfect substitutes where the demand curve is given by* $p = 1 - q_1 - q_2$.[4] *Firm* i *produces with cost function* $\frac{q_i}{n_i}$ *and faces entry cost* γ_i. *Assume that* $n_1 > n_2$ *and* $n_2 > \frac{2n_1}{1+n_1}$. *If both firms enter in Cournot equilibrium, price, output and profits equal* $p^C = \frac{1+\frac{1}{n_1}+\frac{1}{n_2}}{3}, q_i^C = \frac{1-\frac{2}{n_i}+\frac{1}{n_{-i}}}{3}, \pi_i^C = \left(\frac{1-\frac{2}{n_i}+\frac{1}{n_{-i}}}{3}\right)^2$. *Similarly, in Bertrand equilibrium* $p^B = \frac{1}{n_2}, q_1^B = 1 - \frac{1}{n_2}, q_2^B = 0, \pi_1^B = \left(\frac{1}{n_2} - \frac{1}{n_1}\right)\left(1 - \frac{1}{n_2}\right), \pi_2^B = 0$. *Consider the case where* $n_1 = 100, \gamma_1 = 0.2, n_2 = 3, \gamma_2 = 0$. *Then we find that* $\pi_1^C = 0.19 < 0.2$ *and hence firm* 1 *does not enter. However, with Bertrand competition* $\pi_1^B = 0.22 > 0.2$. *Hence more intense competition makes it possible in this case for the more efficient firm to recoup its*

[4] Although the theory is developed for a continuum of firms, we use examples with discrete firms for two reasons. First, such examples are often easier to verify. Second, it illustrates that the results derived here do not crucially depend on the choice to model firms as a continuum, although this assumption simplifies the analytical exposition.

entry cost. In this sense, the switch from Cournot to Bertrand competition selects the more efficient firms into the industry at the expense of less efficient ones.

Assuming that the second stage equilibrium in definition 1 is unique, we can write the following reduced form expressions for firm n's equilibrium variable profits and output levels

$$\pi\left(n,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right) \equiv \left[\begin{array}{c} p\left(\hat{a}_n,\hat{P},\theta\right)^{T} q\left(\hat{a}_n,\hat{Q},\theta\right) \\ -C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),n\right)\end{array}\right] \quad (1)$$

$$q\left(n,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right) \equiv q\left(\hat{a}_n,\hat{Q},\theta\right) \quad (2)$$

where $\{n_{w0}^{i},n_{w1}^{i}\}_{i=1}^{I}$ denotes the intervals of firms that enter the market. Since we allow for the possibility that the entry cost γ rises with n we cannot exclude the case where firm n enters while a more efficient firm $n' > n$ stays out of the market as it cannot recoup its entry costs. Hence in equilibrium there are $I \geq 1$ intervals of firms that enter the market. Put differently, we let $\cup_{i=1}^{I}[n_{w0}^{i},n_{w1}^{i}]$ denote the set of firms that enter the market in equilibrium. Clearly, the bounds n_{w0}^{i} and n_{w1}^{i} depend on the aggressiveness of interaction θ but this is supressed to ease notation. Figure 1 gives an illustration of these equilibrium intervals of active firms $\{n_{w0}^{i},n_{w1}^{i}\}_{i=1}^{2}$.

In this framework we consider two ways in which competition can be intensified. First, an across the board reduction in entry costs $-d\gamma > 0$ (more formally, $d\gamma(n) = d\gamma < 0$ for all types $n \in [n_0,n_1]$) and second more aggressive interaction between players, parametrized as $d\theta > 0$. The key to the analysis is the following way in which more intense competition is identified in this general framework.

Definition 2 *The effects of $d\theta$ and $d(-\gamma)$ in the equilibrium above are as follows. The expression*

$$\frac{d\ln\left(-\frac{\partial C\left(q\left(n,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),n\right)}{\partial n}\right)}{d\theta} \quad (3)$$

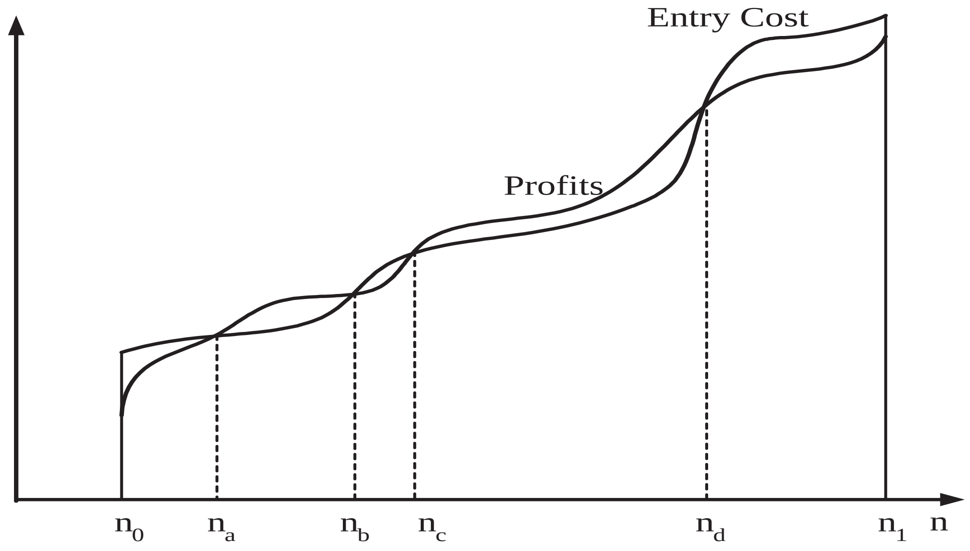

Figure 1: *Entry costs* $\gamma(n)$ *and profits* $\pi(n, \{n^i_{w0}, n^i_{w1}\}^2_{i=1}, \theta)$ *as a function of efficiency* n*, with* $n^1_{w0} = n_a, n^1_{w1} = n_b, n^2_{w0} = n_c,$ *and* $n^2_{w1} = n_d$*.*

is increasing in n*, where the effect of* θ *is partial in the sense that* $\{n^i_{w0}, n^i_{w1}\}^I_{i=1}$ *is here taken as given. And the expression*

$$\frac{d \ln \left(-\frac{\partial C\left(q\left(n, \left\{n^i_{w0}, n^i_{w1}\right\}^I_{i=1}, \theta\right), n\right)}{\partial n} \right)}{d\left(-\gamma\right)} \tag{4}$$

is increasing in n*.*

Although these conditions do not look intuitive at first sight, we view them as a generalization of the output reallocation effect to the case where $q\,(., n)$ is a vector.[5] In the case where firms produce homogenous goods, Boone (2000) and Vickers (1995) identify a rise in competition as a parameter change that raises output of a firm relative to a less efficient firm. Put differently, a rise in θ (or fall in γ) raises $\frac{q(n^*)}{q(n)}$ for $n^* > n$. In words, if more intense competition reduces (raises) firms' output levels, the fall (rise) in output is bigger (smaller) for less efficient

[5]As we will show below these conditions are also natural candidates for necessary conditions to get the result that more intense competition leads to lower PCM. However, in that case the conditions are not sufficient.

firms. Alternatively, the *output reallocation effect* can be stated as:

$$\frac{d \ln q\left(n\right)}{d\theta} \text{ and } \frac{d \ln q\left(n\right)}{d\left(-\gamma\right)} \text{ are increasing in } n \tag{5}$$

Note that the output reallocation effect does not assume anything about the output levels of firms (only about relative output). This is important since we know that a change from Cournot to Bertrand competition tends to raise output of efficient firms, while it reduces output for inefficient firms. Thus there is no direct relation between intensity of competition and a firm's output level. Also, entry by new firms (as a result of a reduction in entry barriers) can both reduce every incumbent firm's output level and increase firms' output levels. See Amir and Lambson (2000) for details.

The reason why we look at the partial effect of θ, for given firms $\{n_{w0}^i, n_{w1}^i\}_{i=1}^I$ that participate in the market, is the well known 'tupsy turvy' result. In the case where firms produce differentiated goods, it may be the case that there are twenty firms under Cournot competition while there are sixteen firms under Bertrand competition. The reason is that Bertrand competition leads to lower rents and hence fewer firms enter in equilibrium. To avoid having to resolve this ambiguity (more aggressive interaction but smaller number of players), we consider the change in θ for a given set of firms in the market. It is clear that a switch from Cournot to Bertrand competition with given number of firms in the market is a rise in competition. Only in this clear cut case do we require the reallocation effect to hold.

If goods are not perfect substitutes, $\frac{q(n^*)}{q(n)}$ is not well defined ('dividing apples by oranges'). Taking this into account and allowing each firm to produce a number of products, it becomes clear that the reallocation effect has to be expressed in money terms. In principle, there are two ways to do that: costs $C\left(q, n\right)$ and revenues $p^T q$. The disadvantage of using revenues is that prices p will be affected by θ as well as output q. To illustrate, intensifying competition by making goods closer substitutes directly affects firms' demand functions and prices irrespective of a change in firms' output levels. Hence costs $C\left(q, n\right)$ seem a more natural choice here as it allows for the isolation of the effect of competition θ and γ on output q.

To gain further intuition for definition 2, note that the conditions above can also be stated as follows. Consider two firms n^{**} and $n^* < n^{**}$. Then the reduction in costs due to a small rise in efficiency $dn > 0$ for firm n^{**} relative to n^* is

$$\frac{-\left.\frac{\partial C\left(q\left(n,\left\{n_{w0}^i,n_{w1}^i\right\}_{i=1}^I,\theta\right),n\right)}{\partial n}\right|_{n=n^{**}}}{-\left.\frac{\partial C\left(q\left(n,\left\{n_{w0}^i,n_{w1}^i\right\}_{i=1}^I,\theta\right),n\right)}{\partial n}\right|_{n=n^{*}}}$$

The conditions above say that a rise in competition raises this ratio. That is, more intense competition leads to a bigger fall in costs (due to the efficiency gain $dn > 0$) for the high efficiency firm n^{**} as compared to the less efficient firm n^*.[6] This makes sense. More intense competition tends to marginalize inefficient firms by reducing their output levels. Therefore their costs become less dependent on their efficiency level. Consider the switch from Cournot to Bertrand competition in example 1. In that example, we find that $-\frac{\partial C(q,n)}{\partial n} = \frac{q(n_i)}{n_i^2}$ for $i = 1, 2$. With Bertrand competition, a small change in the efficiency level of the inefficient firm has no effect on its costs. It does not produce anyway and hence $\frac{\partial C(q,n)}{\partial n} = 0$. While under Cournot competition, the same change in efficiency of the inefficient firm does affect its cost level. For the efficient firm, the effect of its efficiency level on its costs is bigger under Bertrand competition than under Cournot because its output level is bigger under Bertrand. Hence, the ratio $\frac{-\frac{\partial C(q,n_i)}{\partial n_i}}{-\frac{\partial C\left(q,n_j\right)}{\partial n_j}} = \frac{q(n_i)}{q(n_j)}\frac{n_j^2}{n_i^2}$ (with $n_i > n_j$) goes up with a switch from Cournot to Bertrand competition.

The next two examples illustrate this reallocation effect further.

Example 2 *Consider an industry where each firm i produces only one product, faces a demand*

[6] In other words, if the model would allow for firms investing in R&D to improve their efficiency n, we would see the following effect. More intense competition raises R&D investments of firms relative to less efficient firms. This is in line with results found by Aghion et. al. (2002).

curve of the form

$$p(q_i, q_{-i}) = a - bq_i - d\sum_{j\neq i} q_j$$

and has constant marginal costs $\frac{1}{n_i}$*. Then firm* i *chooses output* q_i *which solves*

$$\max_{q\geq 0}\{(a - bq - d\sum_{j\neq i} q_j)q - \frac{1}{n_i}q\}$$

where we assume that $a > \frac{1}{n_i} > 0$ *and* $0 < d \leq b$*. Then the first order condition for a Cournot Nash equilibrium can be written as*

$$a - 2bq_i - d\sum_{j\neq i} q_j - \frac{1}{n_i} = 0 \tag{6}$$

Assuming N firms produce positive output levels, one can solve the N first order conditions (6). This yields

$$q(n_i) = \frac{\left(\frac{2b}{d} - 1\right)a - \left(\frac{2b}{d} + N - 1\right)\frac{1}{n_i} + \sum_{j=1}^{N}\frac{1}{n_j}}{(2b + d(N-1))\left(\frac{2b}{d} - 1\right)} \tag{7}$$

Now assume that because of a fall in entry cost γ *an additional firm* $N + 1$ *with constant marginal costs* $\frac{1}{n_{N+1}}$ *can enter the industry. Then it is routine to verify that*

$$\frac{-\frac{\partial C(q,n_i)}{\partial n_i}}{-\frac{\partial C(q,n_j)}{\partial n_j}} = \frac{q(n_i)}{q(n_j)}\frac{n_j^2}{n_i^2}$$

increases after entry for $n_i > n_j$ $(i, j \neq N + 1)$*.*

Example 3 *Consider an Hotelling beach of length 1 with consumers distributed uniformly over the beach with density 1. Firm* 1 *is located on the far left of the beach and firm* 2 *on the far right. Firm* i *has constant marginal costs* $\frac{1}{n_i}$ $(i = 1, 2)$*. A consumer at position* $x \in \langle 0, 1\rangle$ *who buys a product from firm* 1 *incurs a linear travel cost* tx*, and if she buys from firm* 2 *she incurs travel cost* $t(1-x)$*. Assume that each consumer buys one and only one product and that he buys from the firm with the lowest overall cost. Then demand for firm* i *equals* $q_i(p_i, p_j; t) = \frac{1}{2} + \frac{p_j - p_i}{2t}$*. As travel costs decrease, consumers are more inclined to buy from the cheapest firm rather than* *the closest one. So as travel costs decrease, firms' monopoly power is reduced and competition is more intense. Parametrizing competition as* $\theta = \frac{1}{t}$*, the Nash equilibrium output levels equal respectively*

$$q_i = \frac{1}{6}(3 + \theta(\frac{1}{n_j} - \frac{1}{n_i})) \tag{8}$$

Clearly, we find that in increase in θ *raises* $\frac{-\frac{\partial C(q,n_i)}{\partial n_i}}{-\frac{\partial C\left(q,n_j\right)}{\partial n_j}} = \frac{3+\theta(\frac{1}{n_j}-\frac{1}{n_i})}{3-\theta(\frac{1}{n_j}-\frac{1}{n_i})}\frac{n_j^2}{n_i^2}$ *for* $n_i > n_j$*.*

In these simple examples with constant marginal costs, we see that the condition on $\frac{d\ln\left(-\frac{\partial C(q,n)}{\partial n}\right)}{d\theta}$ and $\frac{d\ln\left(-\frac{\partial C(q,n)}{\partial n}\right)}{d(-\gamma)}$ actually boils down to the output reallocation effect. That is, $\frac{d\ln\left(-\frac{\partial C(q,n)}{\partial n}\right)}{d\theta}$ and $\frac{d\ln\left(-\frac{\partial C(q,n)}{\partial n}\right)}{d(-\gamma)}$ increasing in n is equivalent to $\frac{d\ln q(n)}{d\theta}$ and $\frac{d\ln q(n)}{d(-\gamma)}$ increasing in n. This is true for more general cost functions as well, as the next lemma illustrates.

Lemma 2 *Consider the case where a firm produces only one product,* $L = 1$*. Assume that the cost function* $C(q,n)$ *can be written as*

$$C(q,n) = \int_n^{+\infty} \omega(t)\, e^{\int_{q_0}^{q} \frac{\phi(t,x)}{x}dx} dt \tag{9}$$

with $q_0, \omega(.), \phi(.,.), \frac{\partial\phi(n,q)}{\partial n}, \frac{\partial\phi(n,q)}{\partial q} \geq 0$*. Then the output reallocation effect, that is*

$$\frac{d\ln\left[q\left(n, \{n_{w0}^i, n_{w1}^i\}_{i=1}^I, \theta\right)\right]}{d\theta}$$

is increasing in n*, is sufficient for (3) to hold. The same is true for* $d(-\gamma)$*.*

Although the cost function in equation (9) looks nonstandard, it is quite general. It is, for instance, routine to verify that $\omega(n) = \frac{1}{n^2}$ and $\phi(n,q) = \frac{\sum_{m=1}^{M}\gamma_m m q^m}{\sum_{m=0}^{M}\gamma_m q^m}$ lead to a cost function of the form $C(n,q) = \frac{\Gamma}{n}\left(\sum_{m=0}^{M}\gamma_m q^m\right)$ for some constant $\Gamma > 0$ and $M+1$ scalars γ_m. Indeed, the case considered in the example above with $C(n,q) = \frac{q}{n}$ is a special case of this cost function.

Finally, to get some intuition for the multi-product case, consider the case where the cost function $C(q,n)$ can be written as $C(q,n) = \omega(n)\phi(q)$ with $\omega : \mathbb{R}_+ \to \mathbb{R}_+, \omega'(.) < 0$ and

$\phi : \mathbb{R}_+^L \rightarrow \mathbb{R}_+$ an increasing function of the output vector q. Then it is routine to verify that the following two conditions are sufficient for definition 2 to hold. The elasticity $\frac{\partial \phi(q)}{\partial q_l} \frac{q_l}{\phi(q)}$ is nondecreasing in q_l and the output reallocation effect ($\frac{d \ln(q_l)}{d\theta}$ and $\frac{d \ln(q_l)}{d(-\gamma)}$ are increasing in n) holds at the product level for each product l.

3. New measure of competition

This section introduces relative profits differences, RPD, as a measure of competition and discusses under which conditions this measure can be estimated using firm level panel data. Broadly speaking, the better one is able to separate fixed and variable costs in the data, the more robust the competition measure will be that one can estimate.

The innovation of this paper is to measure intensity of competition in an industry by estimating the following variable

$$\frac{\pi\left(n^{**}, \{n_{w0}^i, n_{w1}^i\}_{i=1}^I, \theta\right) - \pi\left(n, \{n_{w0}^i, n_{w1}^i\}_{i=1}^I, \theta\right)}{\pi\left(n^{*}, \{n_{w0}^i, n_{w1}^i\}_{i=1}^I, \theta\right) - \pi\left(n, \{n_{w0}^i, n_{w1}^i\}_{i=1}^I, \theta\right)} > 0 \tag{10}$$

for any three firms with $n^{**} > n^* > n$, where $\pi(.)$ is defined in equation (1). The following theorem shows why this is a robust measure of competition.

Theorem 1 *An increase in competition raises the expression in equation (10) for any three firms with $n^{**} > n^* > n$. That is,*

$$\frac{d\left(\frac{\pi\left(n^{**}, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right) - \pi\left(n, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right)}{\pi\left(n^{*}, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right) - \pi\left(n, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right)}\right)}{d\theta} > 0$$

where the effect of θ is partial, i.e. taking $\{n_{w0}^i, n_{w1}^i\}_{i=1}^I$ as given, and

$$\frac{d\left(\frac{\pi\left(n^{**}, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right) - \pi\left(n, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right)}{\pi\left(n^{*}, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right) - \pi\left(n, \left\{n_{w0}^i, n_{w1}^i\right\}_{i=1}^I, \theta\right)}\right)}{d\left(-\gamma\right)} > 0$$

To illustrate this result, consider the example in figure 2. This is based on example 2 with $a = 20, b = 2, N = 20$ and firm $i \in \{1, 2, ..., 20\}$ has constant marginal costs equal to $\frac{i}{10}$ (hence efficiency of i equals $n_i = \frac{10}{i}$). Figure 2 has firm n's normalized efficiency level $\frac{n-\underline{n}}{\bar{n}-\underline{n}}$ on the horizontal axis and n's normalized profits $\frac{\pi(n,\theta)-\pi(\underline{n},\theta)}{\pi(\bar{n},\theta)-\pi(\underline{n},\theta)}$ (note that this is the inverse of the expression in (10) to avoid dividing by zero for $n = \underline{n}$) on the vertical axis with $\underline{n} \leq n \leq \bar{n}$ ($\underline{n} = 1, \bar{n} = 10$) and where $\pi(n, \theta)$ is used as a shorthand for $\pi\left(n, \{n^i_{w0}, n^i_{w1}\}^I_{i=1}, \theta\right)$. This relation is increasing (more efficient firms make higher profits π). The more competitive the industry, the more this curve is pulled into the corner at bottom-right. This is illustrated in the graph for the case where competition is intensified by making goods closer substitutes (d increases from 0.1 to 2). Further, with Bertrand competition, homogenous goods and constant marginal costs one finds that the curve is flat and equal to zero for all $n \in [\underline{n}, \bar{n}\rangle$ and equal to 1 at $n = \bar{n}$. This corresponds to perfect competition. Competition can now be measured as the area under this curve. The smaller this area, the more intense competition is (note that because of the normalizations used on the axes, this area under the curve lies between 0 and 1). In particular, in the Bertrand equilibrium just mentioned, the area under the curve equals 0.

Note that one does not need to observe all firms in an industry to make a graph like the one in figure 2. Indeed figure 2 also just uses a subset of the firms ($i \in \{1, ..., 10\}$). The reason is that the result in theorem 1 holds for any three firms. This is in contrast to concentration measures which make no sense if not all firms in the industry are observed.

What type of data is needed to estimate the measure in equation (10)? The data we have in mind is firm or plant level data that specify per firm total revenues, total wage bill (or preferably wage costs split according to production workers (blue collar) and management (white collar), see below), costs of inputs used, energy etc. Data sets like this are available in more and more countries (usually at country's statistical offices where this data forms the basis of the national

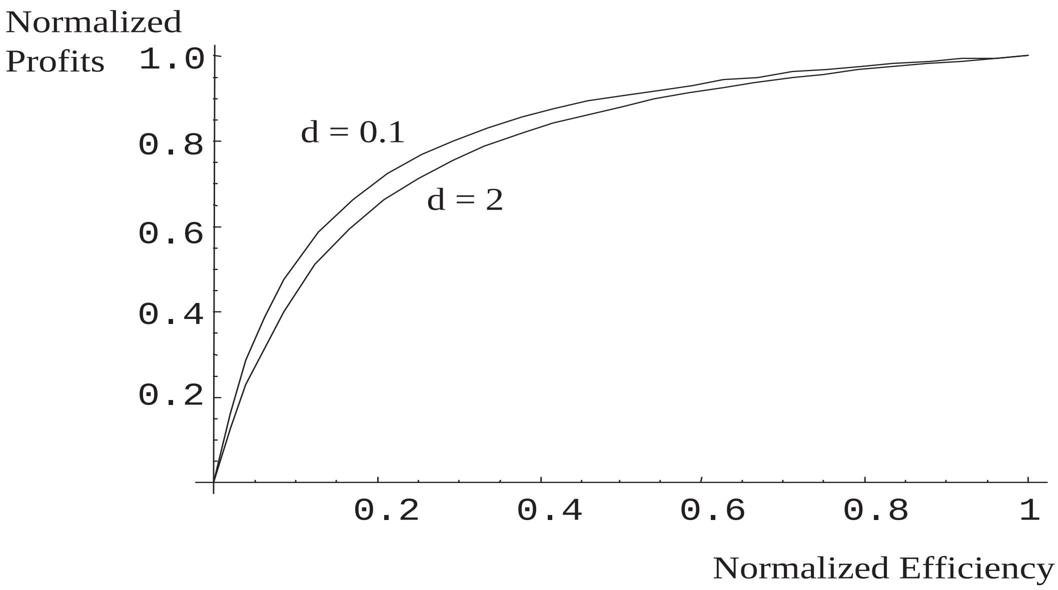

Figure 2: *Firm n's normalized profits* $\frac{\pi(n,\theta)-\pi(\underline{n},\theta)}{\pi(\overline{n},\theta)-\pi(\underline{n},\theta)}$ *as a function of n's normalized efficiency* $\frac{n-\underline{n}}{\overline{n}-\underline{n}}$.

accounts). Examples of papers using such data are Aghion et. al. (2002), Klette (1999), Klette and Griliches (1999) and Lindquist (2001). Further, the data should be available at the four or five digit level such that the one dimensional efficiency assumption is a decent approximation. In particular, the more aggregated the data become, say at the two digit level, the more likely it is that one firm is more efficient in producing one good and another firm more efficient in producing another good within this two digit category. In that case, efficiency is no longer a one dimensional variable. As we will argue below, this one dimensional efficiency assumption is also necessary for the price cost margin as a measure of competition.

Equation (1) defining variable profits $\pi(.)$, states that the costs $C(q,n)$ should be included in calculating profits while $\gamma(n)$ should not be included. Hence $\pi(.)$ equals total revenue for a firm minus costs $C(q,n)$.

The following describes how to decide which cost categories in the data should be included in $C(q, n)$ and which in $\gamma(n)$. First, any costs, like materials and energy, that are viewed as variable costs (i.e. varying with small changes in production) should be included in $C(q, n)$. Second, fixed costs that are seen as being positively correlated with a firm's efficiency level should be included in $\gamma(n)$ because only the costs γ are allowed to be increasing in n (see assumption 1). Examples mentioned above are investments in R&D and capital stocks, where higher investments may lead to lower marginal costs and hence higher efficiency in production. For cost categories in the data that are seen as fixed costs that do not vary with efficiency, it is immaterial whether they are included under $C(q, n)$ or $\gamma(n)$.[7] Finally, with fixed costs that fall with efficiency, one has a choice whether to incorporate them under $C(q, n)$ or $\gamma(n)$. Here the decision should be based on definition 2 and the equilibrium properties of the model one has in mind to describe the sector.

To illustrate this last point, first consider example 3 but suppose that firm i's costs are of the form $\frac{q}{n_i} + \frac{c_0}{n_i}$ for $i = 1, 2$. In that case, both $\frac{-\frac{\partial\left(\frac{q(n_i)}{n_i}\right)}{\partial n_i}}{-\frac{\partial\left(\frac{q(n_j)}{n_j}\right)}{\partial n_j}} = \frac{3+\theta(\frac{1}{n_j}-\frac{1}{n_i})}{3-\theta(\frac{1}{n_j}-\frac{1}{n_i})}\frac{n_j^2}{n_i^2}$ and $\frac{-\frac{\partial\left(\frac{q(n_i)}{n_i}+\frac{c_0}{n_i}\right)}{\partial n_i}}{-\frac{\partial\left(\frac{q(n_j)}{n_j}+\frac{c_0}{n_j}\right)}{\partial n_j}} = \frac{\frac{3+\theta(\frac{1}{n_j}-\frac{1}{n_i})}{6}+c_0}{\frac{3-\theta(\frac{1}{n_j}-\frac{1}{n_i})}{6}+c_0}\frac{n_j^2}{n_i^2}$ are increasing in θ for $n_i > n_j$. Thus in this model one is free to choose whether $\frac{c_0}{n_i}$ is part of $C(q, n)$ or $\gamma(n)$. In other models, however, it may be the case that the conditions in definition 2 only hold when the fixed cost is categorized under $\gamma(n)$. In that case it is essential that these costs are not included in costs $C(q, n)$ nor in profits $\pi(.)$. It appears to be the case that the conditions in definition 2 are more easily satisfied the more categories of fixed costs are included under $\gamma(.)$ and hence the closer $C(0, n)$ gets to 0. In this sense, $C(q, n)$ should ideally include only variable costs and no fixed costs.

If the data allows the researcher to identify different cost categories, variable costs should be calculated as the sum of labour costs (if possible only the costs of (blue collar) produc-

[7]To see this, note that fixed costs that do not vary with n have no effect on the expression $-\frac{\partial C(q,n)}{\partial n}$ (in definition 2) and such fixed costs drop out when considering profit differences $\pi(n^*) - \pi(n)$ (in equation (10)).

tion workers, since (white collar) managers tend to be viewed as fixed costs), material costs, intermediate inputs and energy expenditure. Hence expenditures on or depreciation of R&D, advertisement and capital should not be included in the variable costs nor in profits $\pi(.)$. Since costs of depreciation that are economically relevant (instead of advantagous from a tax point of view) are usually hard to come by, it is actually an advantage that such costs should not be included in the calculation of $\pi(.)$.

However, if the dataset only specifies total costs per firm, the observable profit level is $\pi(n) - \gamma(n)$. In that case, we need additional assumptions for the RPD measure to work. The following proposition formulates conditions for the measure $\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}$ to be monotone in θ and γ.

Proposition 1 *Take three firms with $n^{**} > n^* > n$. Then the assumption that*

$$\frac{\gamma'(t)}{\left(-\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}\right)}$$

is nondecreasing[8] in $t \in \langle n, n^{**}\rangle$ is sufficient for the measure $\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}$ to be monotone in competition ($d\theta > 0$ and $d(-\gamma) > 0$) if either of the following conditions holds for all $t \in \langle n, n^{**}\rangle$:

$$(i)\quad \frac{d\left(-\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}\right)}{d\theta} \geq 0 \text{ and } \gamma'(t) < -\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}$$

$$(ii)\quad \frac{d\left(-\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}\right)}{d\theta} \leq 0 \text{ and } \gamma'(t) > -\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}$$

with similar expressions for $d(-\gamma)$.

[8] The case where $\frac{\gamma'(t)}{\left(-\frac{\partial C\left(q\left(t,\left\{n^i_{w0},n^i_{w1}\right\}^I_{i=1},\theta\right),t\right)}{\partial t}\right)}$ is decreasing in $t \in \langle n, n^{**}\rangle$ is considered in Lemma 4 in the appendix.

In words, the condition prevents the case where $\gamma'(n)$ falls faster than $-\frac{\partial C\left(q\left(n,\left\{n_{w0}^i,n_{w1}^i\right\}_{i=1}^I,\theta\right),n\right)}{\partial n}$ as a function of n. The intuition for this is the following. We know from above that a rise in θ raises $\frac{\pi(n^{**})-\pi(n)}{\pi(n^*)-\pi(n)}$. That is, it raises the difference $\pi(n^{**})-\pi(n)$ more in percentage terms than it raises $\pi(n^*)-\pi(n)$. If $\gamma'(n)$ falls 'too fast', the difference $\gamma(n)-\gamma(n^{**})$ is going to be big and hence $\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]$ is going to be big in absolute value. In case (i), the increase (due to $d\theta>0$) in $\pi(n^{**})-\pi(n)=\int_n^{n^{**}}-\frac{\partial C(.,t)}{\partial t}dt$ (see appendix) becomes too small to raise $\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]$ more in percentage terms than the rise in θ raises $\pi(n^*)-\gamma(n^*)-[\pi(n)-\gamma(n)]$. So we need to exclude this case to be sure that $\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^*)-\gamma(n^*)-[\pi(n)-\gamma(n)]}$ is increasing in competition. A similar intuition applies in case (ii).

Above we have focused on the partial effects of θ, taking the firms active in the market as given. As one would expect, if condition (3) in definition 2 holds for the overall effect of θ (i.e. taking the effect on the active firms into account as well), then the overall effect of a rise in θ is indeed to increase the RPD measure in equation (10).

Corollary 1 *If the expression*

$$\frac{d\ln\left(-\frac{\partial C\left(q\left(n,\left\{n_{w0}^i(\theta),n_{w1}^i(\theta)\right\}_{i=1}^{I(\theta)},\theta\right),n\right)}{\partial n}\right)}{d\theta} \tag{11}$$

is increasing in n, where the effect of θ on $\{n_{w0}^i(\theta),n_{w1}^i(\theta)\}_{i=1}^{I(\theta)}$ is taken into account, then the overall effect of an increase in θ on

$$\frac{\pi\left(n^{**},\{n_{w0}^i(\theta),n_{w1}^i(\theta)\}_{i=1}^{I(\theta)},\theta\right)-\pi\left(n,\{n_{w0}^i(\theta),n_{w1}^i(\theta)\}_{i=1}^{I(\theta)},\theta\right)}{\pi\left(n^{*},\{n_{w0}^i(\theta),n_{w1}^i(\theta)\}_{i=1}^{I(\theta)},\theta\right)-\pi\left(n,\{n_{w0}^i(\theta),n_{w1}^i(\theta)\}_{i=1}^{I(\theta)},\theta\right)}$$

for any three active firms with $n^{**}>n^*>n$ is positive:

$$\frac{d\left(\frac{\pi\left(n^{**},\left\{n_{w0}^i(\theta),n_{w1}^i(\theta)\right\}_{i=1}^{I(\theta)},\theta\right)-\pi\left(n,\left\{n_{w0}^i(\theta),n_{w1}^i(\theta)\right\}_{i=1}^{I(\theta)},\theta\right)}{\pi\left(n^{*},\left\{n_{w0}^i(\theta),n_{w1}^i(\theta)\right\}_{i=1}^{I(\theta)},\theta\right)-\pi\left(n,\left\{n_{w0}^i(\theta),n_{w1}^i(\theta)\right\}_{i=1}^{I(\theta)},\theta\right)}\right)}{d\theta}>0$$

4. Discussion

This section compares the RPD and PCM measures of competition. We argue that the data requirements to estimate these two measures are the same. Further, although some of the assumptions made above (like one dimensional efficiency) are not usually mentioned when PCM is used as a measure of competition, we show that these assumptions are needed to interpret a fall in PCM as an increase in competition. Finally, we show that the generalized output reallocation effect in definition 2 is a natural necessary condition for PCM to be monotone in competition, but it is not sufficient. This explains why RPD is a theoretically robust measure of competition while there are counterexamples where a rise in competition leads to higher PCM.

Broadly speaking, there are two ways in the literature to estimate price cost margins. One is to approximate firm i's price cost margin by an expression like (see, for instance, Scherer and Ross (1990:418))

$$\frac{revenues_i - variable\ costs_i}{revenues_i} \tag{12}$$

Using this to calculate PCM requires similar data as one needs to calculate profits $\pi(.)$ in (1) as revenues minus variable costs. An important assumption in the PCM case is that average variable costs can be used as an estimate for marginal costs. This is correct if marginal costs are constant.[9] Note that this assumption is not directly required for estimating π, since $C(q,n)$ is allowed to take any form. However, for the RPD measure we need to rank firms according to their efficiency level. And assuming that marginal costs are constant clearly makes the ranking of firms in terms of efficiency n very simple. In other words, although assuming that marginal costs are constant is, strictly speaking, not needed to estimate RPD, the assumption does make the implementation of RPD a lot simpler.

[9] To see this, consider the one dimensional case where q is a scalar. Then $PCM = \frac{p - \frac{c(q)}{q}}{p}$ only measures the price cost margin if $\frac{c(q)}{q}$ is equal to marginal costs. That is, if variable costs are of the form $c(q) = cq$ and marginal costs are constant.

The other way to estimate price cost margins is to use a structural approach (see Reiss and Wolak (2002) for a survey). In this case, the researcher specifies precisely what the demand function and the cost function $C(q,n)$ look like and what equilibrium is played by the firms. The data are then used to identify the specified demand and cost parameters. From this PCM can be derived.

Note that the RPD measure is a variable that can be estimated in both ways. One can estimate RPD in an analogous way as PCM is estimated in equation (12). But it is also possible to use a structural approach and be more specific about the functional forms of demand and costs $C(q,n)$. To illustrate, table VIII in Berry, Levinsohn and Pakes (1995) contains all the needed information (efficiency n and variable profits π) to calculate RPD. Our paper just offers RPD as a complementary competition measure to PCM and does not take a position on how the measures should be estimated in practice.

When PCM is used as a measure of competition, it is not always explicitly assumed that efficiency is one dimensional nor that the efficiency level can be observed. We argue, using two simple examples, that these assumptions are, in fact, implicitly made once the estimated PCM is interpreted as a measure of competition. The first example shows that information about efficiency is needed if one wants to interpret a higher price cost margin as less intense competition.

Example 4 *Consider the same, homogenous good, industry in two countries A and B. In both countries, demand in the industry takes the form $X(p) = \frac{1}{p}$ where $X(p)$ is the quantity demanded at price p. Assume the most efficient firm in country A produces with constant marginal cost equal to $c_1^A = 1$ and in country B with $c_1^B = 3$. Further, assume that the next efficient firm produces with constant marginal costs $c_2^A = 5$ in country A and with $c_2^B = 6$ in country B. If in both countries, the industry is characterized by Bertrand competition, one finds that the equilibrium price cost margins equal $PCM^A = \frac{4}{5}$ and $PCM^B = \frac{3}{6}$. Clearly, the* *industry is more competitive in country A but PCM (without information on the marginal cost levels of firms) points in the opposite direction as $PCM^A > PCM^B$.*

Indeed, Nevo (2001) compares the PCM generated by his estimated model with a (crude) direct observation of PCM based on accounting cost data to see how the magnitude of PCM should be interpreted in terms of intensity of competition.

The second example shows that in the multiproduct case where efficiency is not one-dimensional, more intense competition can be associated with higher PCM.

Example 5 *Suppose that a researcher observes two firms, 1 and 2, which both can produce two goods, a and b. The data contain only information about aggregate sales and costs of both goods at the firm level. Instead of assuming that efficiency is one dimensional, we assume that it is two dimensional. More precisely we assume that firm 1 is more efficient than 2 in producing good a while 2 is more efficient than 1 in producing b: $n_{1a} = 10, n_{1b} = 2.5, n_{2a} = 2.5$ and $n_{2b} = 10$. The cost function is of the form $C(q,n) = \frac{q}{n}$ for each firm and product combination. Finally, demand for good $i\,(= a, b)$ is of the form $p_i = 1 - q_{1i} - q_{2i}$. PCM for firm $f\,(= 1, 2)$ at the aggregate level is defined as total revenue minus total (variable) costs divided by total revenue*

$$PCM_f = \frac{\left(p_a - \frac{1}{n_{fa}}\right) q_{1a} + \left(p_b - \frac{1}{n_{fb}}\right) q_{fb}}{p_a q_{fa} + p_b q_{fb}}$$

With Cournot competition on both markets one finds $q_{fi}^C = \frac{1 - \frac{2}{n_{fi}} + \frac{1}{n_{-fi}}}{3}, p_i^C = \frac{1 + \frac{1}{n_{1i}} + \frac{1}{n_{2i}}}{3}$ and hence $PCM_1^C = PCM_2^C = 0.68$. Under Bertrand competition each firm only produces the product at which it is most efficient (and hence has the highest PCM) and we find $PCM_1^B = PCM_2^B = \frac{\frac{1}{2.5} - \frac{1}{10}}{\frac{1}{2.5}} = 0.75$. In other words, if at the unit of observation the firms produce more than one product and the researcher wants to use PCM as a measure of competition, he has to assume that efficiency is a one dimensional variable. Put differently, he has to assume, as we did above, that $\frac{\partial C(q,n)}{\partial n} \leq 0$ and $\frac{\partial\left(\frac{\partial C(q,n)}{\partial q_l}\right)}{\partial n} \leq 0$: the more efficient firm is (weakly) more efficient *in the production of all goods. Absent this assumption, an increase in competition can lead to higher PCM.*

As argued so far, the data requirements and assumptions for PCM and RPD to measure competition are similar. The main advantage of the RPD measure is the robust theoretical foundation for the relation between RPD and the intensity of competition. We will now argue that the generalized output reallocation effect in definition 2 is a natural candidate for a necessary condition to get that more intense competition leads to lower PCM. However, this condition is not sufficient which explains theoretical counterexamples where more intense competition leads to higher PCM. This is the sense in which RPD is a theoretically more robust measure of competition than PCM.

Writing PCM as

$$PCM(n) = \frac{p(n)^T q(n) - C(q(n), n)}{p(n)^T q(n)} = \frac{\pi(n)}{\pi(n) + C(q(n), n)}$$

one can show the following result on the effect of competition on PCM.

Lemma 3 *The effect of θ on PCM can be written as*

$$sign\left(\frac{dPCM(n)}{d\theta}\right) = sign\left(\begin{array}{c} \frac{-\gamma}{[C(q(n,\theta),n)]^2}\frac{dC(q(n,\theta),n)}{dq}\frac{dq(n,\theta)}{d\theta} + \\ \int_{n_w}^{n} \frac{d\left[\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}{C(q(n,\theta),n)}\right]}{d\theta} dt \end{array}\right)$$

where the effect of θ is partial (as above). Next, assume for notational simplicity that $\cup_{i=1}^{I}[n_{w0}^i, n_{w1}^i] = [n_w, n_1]$. Then the effect of γ on PCM can be written as

$$sign\left(\frac{dPCM(n)}{d\gamma}\right) = sign\left(\begin{array}{c} \frac{1}{C(q(n),n)} - \frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\Big|_{t=n_w}\right)}{C(q(n),n)}\frac{dn_w}{d\gamma} \\ -\frac{\gamma}{[C(q(n,\theta),n)]^2}\frac{dC(q(n,\theta),n)}{dq}\frac{dq(n,\theta)}{d\gamma} + \int_{n_w}^{n} \frac{d\left[\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}{C(q(n,\theta),n)}\right]}{d\gamma} dt \end{array}\right)$$

One case for which one wants the result $\frac{dPCM(n)}{d\theta} < 0$ to hold, is the case where $\gamma = 0$. Hence a natural requirement is $\frac{d\left[\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}{C(q(n,\theta),n)}\right]}{d\theta} < 0$ for $n > t$. For the class of cost functions where

$C(q,n) = \omega(n)\, c(q)$ this condition boils down to the output reallocation effect in definition 2. However, the conditition in definition 2 is not sufficient to get $\frac{dPCM(n)}{d\theta} < 0$ because we cannot exclude the case where more intense competition leads to lower output levels for inefficient firms. Hence $\frac{dq(n,\theta)}{d\theta} < 0$ and $\gamma > 0$ works in the direction of $\frac{dPCM(n)}{d\theta} > 0$ and the output reallocation effect is no longer sufficient.

Similarly, when the entry cost γ is increased one would expect the PCM to go up. A natural conditition here is $\frac{d\left[\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}{C(q(n,\theta),n)}\right]}{d\gamma} > 0$. For cost functions of the form $C(q,n) = \omega(n)\, c(q)$ this again boils down to the reallocation effect in definition 2. Again the condition is not sufficient here, because the other terms in the expression for $sign\left(\frac{dPCM(n)}{d\gamma}\right)$ can go either way.

As with PCM, a necessary condition for the relative profits measure $\frac{\pi(n^{**})}{\pi(n^{*})}$ (introduced by Boone (2000)) to be monotone in competition is the reallocation effect in definition 2. As is routine to verify, this reallocation effect is also sufficient for $\frac{\pi(n^{**})}{\pi(n^{*})}$ to be monotone in θ but not for $\frac{\pi(n^{**})}{\pi(n^{*})}$ to be monotone in γ because of a level effect of $d\gamma$.[10] This level effect drops out when considering differences, which explains why RPD needs less stringent sufficient conditions to be monotone in both γ and θ than relative profits.

	$d = 1.5$	$d = 2.0$
PCM_1	0.99	0.99
PCM_2	0.30	0.23
PCM_3	0.26	0.17
Industry PCM	0.68	0.76
$RPD = \frac{\pi(c_2)-\pi(c_3)}{\pi(c_1)-\pi(c_3)}$	0.04	0.02

Table 1: increasing competition in example 2 by making goods closer substitutes with $c_1 = 0.1; c_2 = 6.5; c_3 = 7; \gamma_1 = \gamma_2 = \gamma_3 = 0; a = 20; b = 2$.

Lemma 3 considers the PCM of an individual firm. However, the question of the paper concerns the measurement of industry competition. This is usually done by calculating the

[10]More precisely, profits can be written as $\pi(n) = \gamma + \int_{n_w}^{n} \pi'(t)\, dt$ where n_w is the least efficient firm to enter. When γ changes this has three effects on $\pi(n)$ (which can potentially go in opposite directions): (a) direct effect of γ, (b) effect of γ on n_w and (c) the effect of γ on firms' conduct which appears in $\pi'(t)$. Only effect (c) is relevant here and this is the only effect of γ that remains when considering $\pi(n^*) - \pi(n) = \int_{n}^{n^*} \pi'(t)\, dt$.

weighted industry average PCM, where the weight of a firm equals its market share in the industry (see, for instance, Wolfram (1999)). Tables 1 and 2 show simulations to illustrate that industry PCM is not monotone in competition while RPD does pick up what happens to competition in these two cases. Both tables work with the framework introduced in example 2 with $a = 20, b = 2, d = 1.5$ and Cournot competition. In table 1 there are three firms with constant marginal costs equal to $c_1 = 0.1, c_2 = 6.5, c_3 = 7$. Competition is intensified by making goods closer substitutes (raising d to $d = 2$). The PCM for firms 2 and 3 falls, but industry PCM goes up as competition is intensified. The reason is the output reallocation effect: as competition is intensified, output is reallocated from firms 2 and 3 to the most efficient firm 1 which is the firm with the highest PCM. This increases the weight of firm 1 in the industry average PCM and thus raises the industry PCM. RPD (defined here as $\frac{\pi(c_2)-\pi(c_3)}{\pi(c_1)-\pi(c_3)}$ which is the only point that changes with d in a graph like figure 2)[11] falls with the rise in d indicating correctly that competition becomes more intense. Table 2 considers the case of a fall in entry costs $d\gamma = -0.1$ for all firms. In particular, it considers the following costs distributions $c_1 = c_2 = 0.1, c_3 = 3, c_4 = c_5 = 6$ and $\gamma_1 = \gamma_2 = 16.5, \gamma_3 = 5, \gamma_4 = \gamma_5 = 0.7$. Before the fall in γ, there is a Cournot equilibrium where firms $1, 3, 4$ and 5 are active. Firm 2 cannot profitably enter in this equilibrium. After the across the board reduction in γ however, this equilibrium is broken and firm 2 can enter at the expense of firm 5. Although PCM falls for each individual firm, the industry average PCM goes up as firm 2 has a higher PCM than firm 5. Again RPD (defined here as $\frac{\pi(c_3)-\pi(c_4)}{\pi(c_1)-\pi(c_4)}$, as above the only relevant point in a graph like figure 2) falls with the fall in entry costs, correctly indicating more intense competition.

[11] That is, with $\frac{c_3-c}{c_3-c_1}$ on the horizontal axis the other two points are $(0, 0)$ and $(1, 1)$.

	$d\gamma = 0$	$d\gamma = -0.1$
PCM_1	0.99	0.98
PCM_2	n.a.*	0.98
PCM_3	0.60	0.55
PCM_4	0.25	0.17
PCM_5	0.25	n.a.*
Industry PCM	0.66	0.81
$RPD = \frac{\pi(c_3)-\pi(c_4)}{\pi(c_1)-\pi(c_4)}$	0.37	0.34

Table 2: increasing competition in example 2 by reducing entry costs ($d\gamma = -0.1$) with $c_1 = c_2 = 0.1; c_3 = 3; c_4 = c_5 = 6; \gamma_1 = \gamma_2 = 16.5;$ $\gamma_3 = 5; \gamma_4 = \gamma_5 = 0.7; a = 20; b = 2; d = 1.5.$
$*$ n.a. = firm is not active in equilibrium

5. Conclusion

This paper started off with the observation that PCM is often used as a measure of competition in empirical research. From a theoretical point of view, however, it is not clear what the relation between PCM and competition actually is. There are a number of theoretical papers where more intense competition leads to higher PCM. At the moment we do not know how relevant these theoretical counterexamples are from an empirical point of view.

To answer this question we have developed a new measure of competition, RPD, which has two properties. First, RPD has a robust theoretical foundation as a measure of competition. It is monotone in competition both when competition becomes more intense through more aggressive interaction between firms and when entry barriers are reduced. Second, the data requirements to estimate RPD are the same as the requirements to estimate PCM. That implies that any firm (or plant) level data set which allows a researcher to estimate PCM should also allow for the estimation of RPD. In this way we can see in which percentage of industries both measures point in the same direction. If it turns out that the measures are congruent for more than 95% of the industries, PCM can be used as a measure of competition in empirical research without much concern for the theoretical counterexamples.

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Appendix A. Proof of results

This appendix contains the proofs of the results in the main text.

Proof of Lemma 1

We use a proof by contradiction. Suppose that $q(n) > q(n^*) > 0$. Let p^*, q^* (p, q) denote the optimal price output pair for firm n^* (n). Then profit maximization by these firms implies that

$$\begin{aligned} p^*q^* - C(q^*, n^*) - [pq - C(q, n^*)] &\geq 0 \\ p^*q^* - C(q^*, n) - [pq - C(q, n)] &\leq 0 \end{aligned}$$

Combining both inequalities we can write

$$\int_{q^*}^{q} \frac{\partial C(x, n^*)}{\partial x} dx - \int_{q^*}^{q} \frac{\partial C(x, n)}{\partial x} dx \geq 0$$

or equivalently

$$\int_{n}^{n^*} \int_{q^*}^{q} \frac{\partial^2 C(x, t)}{\partial x \partial t} dx dt \geq 0$$

However, this contradicts $\frac{\partial^2 C(q,n)}{\partial q \partial n} < 0$ in assumption 1.

The proof that $\pi(n^*) \geq \pi(n)$ follows from the observation that n^* can copy what n does and do so at (weakly) lower costs since $n^* > n$. *Q.E.D.*

Proof of Lemma 2

Condition (3) can be written as

$$\frac{d \ln \left(-\frac{\partial C(q,n)}{\partial n} \right)}{d\theta} = \frac{\partial^2 C(q, n)}{\partial n \partial q} \frac{q}{\frac{\partial C(q,n)}{\partial n}} \frac{d \ln q}{d\theta}$$

is increasing in n. If the expression $\frac{\partial^2 C(q,n)}{\partial n \partial q} \frac{q}{\frac{\partial C(q,n)}{\partial n}}$ is non-decreasing in n, then a sufficient condition for $\frac{d \ln\left(-\frac{\partial C(q,n)}{\partial n}\right)}{d\theta}$ to be increasing in n is the output reallocation effect. For the cost function in the lemma, it is the case that

$$\frac{\partial^2 C(q, n)}{\partial n \partial q} \frac{q}{\frac{\partial C(q,n)}{\partial n}} = \phi(n, q)$$

Hence it remains to be shown that $\phi(n,q)$ is non-decreasing in n. This can be written as

$$\frac{d\phi(n,q)}{dn}=\frac{\partial\phi(n,q)}{\partial n}+\frac{\partial\phi(n,q)}{\partial q}\frac{dq(n)}{dn}$$

The assumptions made imply that $\frac{\partial\phi(n,q)}{\partial n},\frac{\partial\phi(n,q)}{\partial q}\geq 0$, and we know from lemma 1 that $\frac{dq(n)}{dn}\geq 0$.

Q.E.D.

Proof of Theorem 1

First note that for any differentiable function π of n it is the case that

$$\pi(n^*)-\pi(n)=\int_n^{n^*}\frac{d\pi(t)}{dt}dt$$

Next note that the envelop theorem applied to

$$\pi\left(n,\{n_{w0}^i,n_{w1}^i\}_{i=1}^I,\theta\right)=\max_{a_n}\left\{p\left(a_n,\hat{P},\theta\right)^T q\left(a_n,\hat{Q},\theta\right)-C\left(q\left(a_n,\hat{Q},\theta\right),n\right)\right\}$$

implies that

$$\frac{d\pi\left(n,\{n_{w0}^i,n_{w1}^i\}_{i=1}^I,\theta\right)}{dn}=-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),n\right)}{\partial n}$$

Hence for any two firms n^* and n that produce positive output levels in equilibrium it is the case that

$$\pi\left(n^*,\left\{n_{w0}^i,n_{w1}^i\right\}_{i=1}^I,\theta\right)-\pi\left(n,\left\{n_{w0}^i,n_{w1}^i\right\}_{i=1}^I,\theta\right)=\int_n^{n^*}-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}dt$$

Therefore we can write the effect of θ on the measure $\frac{\pi^{**}-\pi}{\pi^*-\pi}$ as

$$\frac{d\left(\frac{\int_n^{n^{**}}-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}dt}{\int_n^{n^*}-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}dt}\right)}{d\theta}=\frac{d\left(1+\frac{\int_{n^*}^{n^{**}}\frac{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}}{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}\Big|_{t=n^*}}dt}{\int_n^{n^*}\frac{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}}{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}\Big|_{t=n^*}}dt}\right)}{d\theta}>0$$

because definition 2 implies that

$$\frac{d\left(\frac{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}}{-\frac{\partial C(q(\hat{a}_n,\hat{Q},\theta),t)}{\partial t}\Big|_{t=n^*}}\right)}{d\theta}>0$$

for $t \in \langle n^*, n^{**}]$ and

$$\frac{d\left(\frac{-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}}{-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}\Big|_{t=n^*}}\right)}{d\theta} < 0$$

for $t \in [n, n^*\rangle$. To see this, note that

$$sign\left(\frac{d\left(\frac{-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}}{-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}\Big|_{t=n^*}}\right)}{d\theta}\right) = sign\left(\frac{d\ln\left(-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}\right)}{d\theta} - \frac{d\ln\left(-\frac{\partial C\left(q\left(\hat{a}_n,\hat{Q},\theta\right),t\right)}{\partial t}\Big|_{t=n^*}\right)}{d\theta}\right)$$

The same proof applies to the case with $d\left(-\gamma\right)$ instead of $d\theta$. *Q.E.D.*

Proof of proposition 1

As in the proof of theorem 1, we can write the difference in profits as

$$\pi\left(n^{**}\right) - \gamma\left(n^{**}\right) - \left[\pi\left(n\right) - \gamma\left(n\right)\right] = \int_n^{n^{**}} \left(-\frac{\partial C\left(q\left(t,\theta\right),t\right)}{\partial t} - \gamma'\left(t\right)\right) dt$$

Hence

$$\begin{aligned}
\frac{d\left(\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}\right)}{d\theta} &= \frac{d\left(1+\frac{\int_{n^*}^{n^{**}}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}-\gamma'(t)\right)dt}{\int_{n}^{n^{*}}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}-\gamma'(t)\right)dt}\right)}{d\theta} \\
&= \frac{d\left(1+\frac{\int_{n^*}^{n^{**}}\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}-\gamma'(t)\right)}{\left(-\frac{\partial C(q(n^*,\theta),n^*)}{\partial n^*}-\gamma'(n^*)\right)}dt}{\int_{n}^{n^{*}}\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}-\gamma'(t)\right)}{\left(-\frac{\partial C(q(n^*,\theta),n^*)}{\partial n^*}-\gamma'(n^*)\right)}dt}\right)}{d\theta}
\end{aligned}$$

So we find that $\frac{d\left(\frac{\pi\left(n^{**}\right)-\gamma\left(n^{**}\right)-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}\right)}{d\theta} > 0$ if $\frac{d\left[\frac{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}-\gamma'(t)\right)}{\left(-\frac{\partial C(q(n^*,\theta),n^*)}{\partial n^*}-\gamma'(n^*)\right)}\right]}{d\theta} > 0$ for $t > n^*$.

Under assumption (i) in the proposition it is the case that $-\frac{\partial C(q(t,\theta),t)}{\partial t} - \gamma'\left(t\right) > 0$ and hence we have to prove that

$$\frac{d\ln\left[-\frac{\partial C(q(t,\theta),t)}{\partial t} - \gamma'\left(t\right)\right]}{d\theta}$$

is increasing in t. This can be written as $\frac{d[g(t)h(t)]}{dt} > 0$ with

$$
\begin{aligned}
g(t) &= \frac{d \ln\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}{d\theta} \\
h(t) &= \frac{1}{1 - \frac{\gamma'(t)}{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}}
\end{aligned}
$$

We know from definition 2 that $g'(t) > 0$ and the assumption in the proposition that $\frac{\gamma'(t)}{\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)}$ is nondecreasing in t ensures that $h'(t) \geq 0$. The conditions under (i) then make sure that both $g(.)$ and $h(.)$ are nonnegative and hence we find that $\frac{d[g(t)h(t)]}{dt} = g'(t)h(t) + g(t)h'(t) > 0$.

Under assumption (ii) a similar argument can be proved about monotonicity, but in that case we find $\frac{d\left(\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}\right)}{d\theta} < 0$. *Q.E.D.*

With a similar argument the following result can be proved.

Lemma 4 *Take three firms with $n^{**} > n^{*} > n$. Then the assumption that* $\frac{\gamma'(t)}{\left(-\frac{\partial C\left(q\left(t,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),t\right)}{\partial t}\right)}$ *is decreasing in $t \in \langle n, n^{**}\rangle$ is sufficient for the measure $\frac{\pi(n^{**})-\gamma(n^{**})-[\pi(n)-\gamma(n)]}{\pi(n^{*})-\gamma(n^{*})-[\pi(n)-\gamma(n)]}$ to be monotone in competition ($d\theta > 0$ and $d(-\gamma) > 0$) if either of the following conditions holds for all $t \in \langle n, n^{**}\rangle$:*

$$
\begin{aligned}
(i)\quad \frac{d\left(-\frac{\partial C\left(q\left(t,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),t\right)}{\partial t}\right)}{d\theta} &\leq 0 \text{ and } \gamma'(t) < -\frac{\partial C\left(q\left(t,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),t\right)}{\partial t} \\
(ii)\quad \frac{d\left(-\frac{\partial C\left(q\left(t,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),t\right)}{\partial t}\right)}{d\theta} &\geq 0 \text{ and } \gamma'(t) > -\frac{\partial C\left(q\left(t,\left\{n_{w0}^{i},n_{w1}^{i}\right\}_{i=1}^{I},\theta\right),t\right)}{\partial t}
\end{aligned}
$$

with similar expressions for $d(-\gamma)$.

Proof of Lemma 3

Writing PCM as follows

$$
PCM(n) = \frac{1}{1 + \frac{C(q(n),n)}{\pi(n)}}
$$

we find that $\frac{dPCM(n)}{d\theta} < 0$ if an only if

$$\frac{d\left[\frac{\gamma+\int_{n_w}^{n}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)dt}{C(q(n),n)}\right]}{d\theta} < 0$$

where we have used from the proof of theorem 1 that $\pi\left(n\right) = \gamma + \int_{n_w}^{n}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)dt$. Differentiating $\frac{\gamma+\int_{n_w}^{n}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)dt}{C(q(n),n)}$ with respect to θ (taking n_w as given) we get the expression in the lemma.

Similarly we find that $\frac{dPCM(n)}{d\gamma} > 0$ if and only if

$$\frac{d\left[\frac{\gamma+\int_{n_w}^{n}\left(-\frac{\partial C(q(t,\theta),t)}{\partial t}\right)dt}{C(q(n),n)}\right]}{d\gamma} > 0$$

Evaluating this expression gives the equation in the lemma. *Q.E.D.*