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**NONPARAMETRIC BOUNDS IN THE PRESENCE OF
ITEM NONRESPONSE, UNFOLDING BRACKETS,
AND ANCHORING**

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Discussion paper

Nonparametric Bounds in the Presence of Item Nonresponse, Unfolding Brackets, and Anchoring

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Abstract

Household surveys often suffer from nonresponse on variables such as income, savings or wealth. Recent work by Manski shows how bounds on conditional quantiles of the variable of interest can be derived, allowing for any type of nonrandom item nonresponse. The width between these bounds can be reduced using follow up questions in the form of unfolding brackets for initial item nonrespondents. Recent evidence, however, suggests that such a design is vulnerable to anchoring effects. In this paper Manski's bounds are extended to incorporate the information provided by the bracket respondents allowing for different forms of anchoring. The new bounds are applied to earnings in the 1996 wave of the Health and Retirement Survey. The results show that the categorical questions can be useful to increase precision of the bounds, even if anchoring is allowed for.

Key words: unfolding bracket design, anchoring effects, item nonresponse, bounding intervals.

JEL codes: C14, C42, C81, D31

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1 Introduction

Household surveys are often plagued by item nonresponse on variables of interest like income, savings or the amount of wealth. For example, in the 1996 wave of the Health and Retirement Survey (HRS), a US panel often used to study socio-economic behavior of the elderly, 12.4% of those who say they have some earnings, do not give the amount of these earnings. Questions on amounts of certain types of wealth often lead to even larger nonresponse rates. Manski (1989, 1994, 1995) shows how bounds on conditional quantiles of the variable of interest can be derived, allowing for any type of nonrandom response behavior. Manski's framework is intuitively appealing, easy to apply, and very flexible, but has the drawback that the resulting bounds are often too wide to draw meaningful economic conclusions. In Manski's framework the precision with which features of the distribution of the variable of interest (such as its quantiles) can be determined, i.e., the width between the bounds, depends on the probability of nonresponse. If item nonresponse is substantial, the approach cannot lead to accurate estimates of the parameters of interest without additional information or additional assumptions.

Including follow-up questions in the form of unfolding brackets for initial item nonrespondents is an effective way to reduce complete item nonresponse. In the HRS example given above, 73% of the initial nonrespondents answer the question whether or not their earnings exceed \$25,000, and most of these also answer a second question on either \$50,000 (if the first answer was 'yes') or \$5,000 (if the first answer was 'no'). Recent evidence, however, suggests that the follow-up design that is used here leads to an "anchoring effect," a phenomenon well documented in the psychological literature: the distribution of the categorical answers is affected by the amounts in the questions ("bids" become "anchors"). Experimental studies have shown that even an anchor that is arbitrary and uninformative can have large effects on the responses (see, for example, Jacowitz and Kahneman (1995)). Using a special survey with randomized initial bids, Hurd et al. (1998) show that the distribution is biased towards the categories close to the initial bid. They estimate a parametric model to capture the anchoring phenomenon. Their results confirm that the anchoring effect can bias the conclusions on the parameters of interest if not properly accounted for. Alternative parametric models for anchoring are introduced by Cameron and Quiggin (1994) and Herriges and Shogren (1996).

This paper extends the approach by Manski, incorporating the information provided by the bracket respondents. From the existing anchoring models, three different nonparametric assumptions on the anchoring effects are derived, which are used to construct bounds allowing for anchoring. These bounds are compared to the bounds that do not allow for anchoring, i.e., bounds based on the assumption that the bracket information is always correct. Thus, the main goal of the paper is to use the information provided in follow-up unfolding bracket questions, allowing for nonrandom response behavior as well as anchoring.

The bounds are applied to earnings in the 1996 wave of the Health and Retirement Survey. The results show that the categorical questions can be useful to increase precision of the bounds, even if anchoring is allowed for. They also help to improve the power of statistical tests for equality of earnings quantiles in subpopulations. This is shown by comparing bounds for respondents with low and high education levels. The bounds that take account of bracket information are able to detect differences that are not revealed by the bounds based upon full respondents' information only.

The remainder of this paper is organized as follows. Section 2 discusses the problems associated with item nonresponse in economic surveys and compares different ways to deal with such problems. Section 3 derives bounding intervals using the unfolding bracket questions information, accounting and not accounting for anchoring effects. Section 4 describes the HRS data used in the empirical work. Section 5 presents the empirical results. Section 6 concludes.

2 Item Nonresponse in Household Surveys

We analyze item nonresponse on one specific variable of interest and do not consider problems such as unit nonresponse or nonresponse on conditioning variables.¹ The problem of item nonresponse is often associated with questions on exact amounts of variables such as income, expenditure, or wealth. Unless item nonresponse is completely random, the sample of (item) respondents is not representative for the population of interest. This can affect the estimates of parameters describing the distribution of the variable of interest, such as the conditional mean or conditional quantiles given some covariates.

There are several ways to handle this problem. The first is to use as many covariates (X) as possible and to assume that, conditional on X , the response process is independent of the variable of interest. This makes it possible to use parametric or nonparametric regression techniques to impute values for nonrespondents, leading to, for example, the hot-deck imputation approach. The key assumption of this approach is that item nonrespondents are not systematically different from respondents with the same values of X . See Rao and Shao (1992) for an overview of hot-deck imputation and Juster and Smith (1997) for an application and the use of bracket response information in this context.

Since the seminal work by Heckman (for example, Heckman, 1979), the common view in many economic examples is that the assumption of random item nonresponse conditional on observed X is often unrealistic and may lead to serious selection bias. Heckman proposed to use a selection model instead. This is a joint limited dependent variable model of response behavior

¹See Horowitz and Manski (1998) and Manski and Tamer (2000) for bounding intervals in the more general case of incomplete information on outcomes and regressors.

and the variable of interest, conditional on covariates. See, for example, the survey of Vella (1998). Parametric and semiparametric selection models avoid the assumption that item nonresponse is random conditional on X , but require alternative assumptions such as a single index assumption or independence between covariates and error terms.

A new approach to deal with nonrandom item nonresponse was introduced by Manski (1989, 1990). It makes no assumptions on the response process and uses the concept of identification up to a bounding interval. Manski (1989) shows that in the presence of item nonresponse, the sampling process alone does not fully identify most features of the conditional distribution of a variable Y given a vector of covariates X . In many cases, however, lower and upper bounds for the feature of interest (such as a value of the distribution function of Y given X) can be derived. Manski calls these bounds “worst case bounds.” Manski (1994, 1995) shows how these bounds can be tightened by adding nonparametric assumptions on monotonicity of the relation between Y and response behavior, or exclusion restrictions on the conditional distribution of Y . See Lee and Melenberg (1998) for an empirical application. Manski (1990), Manski et al. (1992), and Lechner (1999) apply the bounds to analyze treatment effects.

The problem of item nonresponse can be reduced at the data collection level by, for example, carefully designed surveys, careful coding of responses by the interviewer, reducing question ambiguity, guaranteeing privacy protection, giving respondents the opportunity to consult tax files, etc. A more direct method to reduce item nonresponse is to include categorical questions to obtain partial information from initial nonrespondents. This is often motivated by the argument that cognitive factors such as confidentiality or the belief that the interviewer requires a perfectly precise answer, can make people reluctant to answer open-ended questions (see, for example, Juster and Smith (1997)).

Two types of categorical questions are common. In some surveys, initial nonrespondents are routed to a range card categorical question, where they are asked to choose the category that contains the amount (Y) from a given set of categories. The alternative is unfolding brackets. This is used in well-known US longitudinal studies such as the Panel Study of Income Dynamics (PSID), the Health and Retirement Survey (HRS), and the Asset and Health Dynamics Among the Oldest Old (AHEAD). In an unfolding brackets design, those who initially answer the open question with ‘don’t know’ or ‘refuse’, are asked a question such as ‘*is the amount \$B or more?*’, with possible answers ‘yes’, ‘no’, ‘don’t know’, and ‘refuse’. They typically get two or three such consecutive questions, with changing bids SB : a ‘yes’ is followed by a larger bid and a ‘no’ by a smaller bid. Those who answer ‘don’t know’ or ‘refuse’ on the first bid are full nonrespondents. The others are called bracket respondents. They are referred to as complete or incomplete bracket respondents, depending on whether they answer all the bracket questions presented to them by ‘yes’ or ‘no’, or end with a ‘don’t know’ or ‘refuse’ answer. An unfolding bracket design is much

easier to use in a telephone interview than a range card question. Moreover, unfolding brackets can elicit partial information even if the sequence is not completed, while a range card question might lead to one simple ‘don’t know’ or ‘refuse’.

A problem with unfolding brackets questions is the phenomenon of anchoring (see Jacowitz and Kahneman (1995), Rabin (1998), and Hurd et al. (1998)). A psychological explanation for anchoring effects is that the bid creates a fictitious belief in the respondent’s mind: faced with a question related to an unknown quantity, the respondent treats the question as a problem solving situation, and the given bid is used as a cue to solve the problem. This can result in responses that are influenced by the design of the unfolding sequence. Hurd et al. (1998) formulate a parametric model which can explain observed anchoring patterns in their data. This model will be discussed in detail in Section 3.3. Hurd et al. (1998) estimate their model using experimental data in which respondents are randomly assigned to different starting bids of an unfolding bracket sequence. They find strong evidence of anchoring effects. Other parametric models for anchoring effects are introduced by Cameron and Quiggin (1994) and Herriges and Shogren (1996).

The results of Hurd et al. (1998) and others imply that answers to unfolding bracket questions may often be incorrect. They also imply that unfolding bracket questions may not give the same answers as range card questions. In the next section, Manski’s worst case bounds are extended to account for unfolding bracket questions. Nonparametric versions of the assumptions underlying models for anchoring are then introduced, and the worst case bounds are extended to allow for anchoring under these assumptions.

3 Theoretical framework

3.1 Worst case bounds; no bracket respondents

First, Manski’s (1989) worst case bounds are reviewed for the conditional distribution function of a variable Y at a given $y \in \mathbb{R}$ and given $X=x \in \mathbb{R}^p$. It is assumed that there is neither unit nonresponse, nor item nonresponse on X . Reported (exact) values of Y and X are assumed to be correct; there is no under- or overreporting. Let FR (full response) indicate that Y is observed and let NR indicate (full) nonresponse on Y . $F(y|x)$, the conditional distribution function of Y given $X=x$ in the complete population, can be written as follows:

$$F(y|x) = F(y|x, FR)P(FR|x) + F(y|x, NR)P(NR|x) \quad (1)$$

The assumptions imply that $F(y|x, FR)$ is identified for all x in the support of X , and can be estimated using some nonparametric regression technique. The same holds for the conditional

probabilities $P(FR|x)$ and $P(NR|x)$. If the assumption were added that, conditional on X , response behavior is independent of Y , then all expressions in the right hand side of (1) would be identified, since $F(y|x,FR)=F(y|x,NR)$. This is the assumption of exogenous selection. In general, however, response behavior can be related to Y , and $F(y|x,NR)$ is not identified, so that $F(y|x)$ is not identified either. Without additional assumptions, all that is known about $F(y|x,NR)$ is that it is between 0 and 1. Applying this to (1) gives,

$$F(y|x,FR)P(FR|x) \leq F(y|x) \leq F(y|x,FR)P(FR|x) + P(NR|x) \quad (2)$$

These are Manski's worst case bounds for the distribution function. The difference between the upper and the lower bound is equal to $P(NR|x)$. Thus, a low nonresponse rate leads to narrow and informative bounds. Additional assumptions can tighten the bounds. Examples are monotonicity and exclusion restrictions, see Manski (1994, 1995).

3.2 Partial information from an unfolding bracket sequence

In this paper, the bounds in (2) are extended to incorporate information from a follow-up unfolding bracket sequence. Let $B1$ be the initial bid. This is assumed to be the same for all initial nonrespondents, as is the case in the HRS data. The first bracket question is thus given by

$$\text{Is the amount } \$B1 \text{ or more ?} \quad (3)$$

Individuals can answer 'yes', 'no', or 'don't know'.² Those who answer 'don't know' become full nonrespondents. Those who answer 'yes' get the same question with a new bid $B21$, with $B21 > B1$. If the answer is 'no', the next bid is $B20$, with $B20 < B1$. For many questions in the HRS, this second question is the final bracket question. In some cases a third question is asked, again with a new bid. Our study is limited to the case of two bracket questions, leaving more than two questions as an extension that can be treated along the same lines. For the sake of the exposition, the case where only one bracket question is asked is considered first.

3.3 Bounds and unfolding bracket response: One bracket question

In this case, three types of respondents can be distinguished: full respondents (FR), bracket

² In this study, no distinction is made between the answers 'don't know' and 'refuse'. Both are referred to as 'don't know'.

respondents (*BR*) and (full) nonrespondents (*NR*), so that $F(y/x)$ can be written as

$$F(y/x) = F(y/x,FR)P(FR|x) + F(y/x,BR)P(BR|x) + F(y/x,NR)P(NR|x) \quad (4)$$

Full respondents identify $F(y/x,FR)$, as before. Nonrespondents answer ‘don’t know’ to the initial question and the bracket question and, as before, all that is known about $F(y/x,NR)$ is that it is between 0 and 1. The new issue is what the answers of the bracket respondents say about $F(y/x,BR)$.

Bracket respondents report whether $Y \geq BI$ or not. Define a variable QI by $QI=1$ if the answer to (3) is ‘yes,’ and 0 if it is ‘no’. Then the bracket respondents identify $P(QI=1|x,BR)$. For deriving the bounds, it will be useful to write this as

$$\begin{aligned} P(QI=1|x,BR) &= P(QI=1|Y < BI, x, BR)P(Y < BI|x, BR) + \\ &P(QI=1|Y \geq BI, x, BR)P(Y \geq BI|x, BR) \end{aligned} \quad (5)$$

Not allowing for an Anchoring effect

If there is no anchoring, all bracket respondents answer (3) correctly. This implies that $P(QI=1|Y < BI, x, BR)=0$ and $P(QI=1|x, BR)=P(Y \geq BI|x, BR)$, and thus $P(Y < BI|x, BR)$ is identified by the data on bracket respondents. It leads to the following bounds on $F(y/x, BR)$:

$$\begin{aligned} \text{for } y < BI & \quad 0 \leq F(y|BR, x) \leq P(QI=0|BR, x) \\ \text{for } y \geq BI & \quad P(QI=0|BR, x) \leq F(y|BR, x) \leq 1 \end{aligned} \quad (6)$$

Combining this with the bounds on $F(y/FR, x)$ and $F(y/NR, x)$ yields, for $y < BI$,

$$\begin{aligned} &F(y|FR, x)P(FR|x) \\ &\leq F(y|x) \leq \\ &F(y|FR, x)P(FR|x) + P(QI=0|x, BR)P(BR|x) + P(NR|x) \end{aligned} \quad (7)$$

and for $y \geq BI$,

$$\begin{aligned} &F(y|FR, x)P(FR|x) + P(QI=0|x, BR)P(BR|x) \\ &\leq F(y|x) \leq \\ &F(y|FR, x)P(FR|x) + P(BR|x) + P(NR|x) \end{aligned} \quad (8)$$

The bounds in (7) and (8) are sharper than the worst case bounds in (2) if there are bracket respondents answering ‘yes’ as well as bracket respondents answering ‘no’.

Allowing for an Anchoring Effect

If responses to (3) suffer from anchoring, (6) is no longer valid, since answers to (3) can be wrong and $P(QI=1|Y<BI,x,BR)$ and $P(QI=0|Y>BI,x,BR)$ can be nonzero. In Hurd et al. (1998), QI is based upon comparing Y to $BI+\varepsilon$, where ε is the perception error. Hurd et al. (1998) assume that ε is normally distributed with zero mean and is independent of Y and X . In our nonparametric framework the following weaker distributional assumption with respect to ε is used:

Assumption 1: $QI=1$ if $Y \geq BI + \varepsilon$ and $QI=0$ if $Y < BI + \varepsilon$, where the perception error ε satisfies
For all (x,y) in the support of (X,Y) , $\text{Median}(\varepsilon|X=x,Y=y,BR)=0$.

This assumption implies that the conditional probability that an individual answers question QI correctly is at least 0.5:

$$\begin{aligned} P(QI=1|Y<BI,x,BR) &= P(\varepsilon \leq Y-BI|BI-Y>0,x,BR) \leq 0.5 \\ P(QI=1|Y \geq BI,x,BR) &= P(\varepsilon \leq Y-BI|BI-Y \leq 0,x,BR) \geq 0.5 \end{aligned} \tag{9}$$

Applying (9) to (5) gives:

$$\begin{aligned} P(QI=1|x,BR) &\leq 0.5P(Y<BI|x,BR) + P(Y \geq BI|x,BR) \\ P(QI=1|x,BR) &\geq 0.5P(Y \geq BI|x,BR). \end{aligned} \tag{10}$$

This implies

$$\begin{aligned} P(Y<BI|x,BR) &\leq 2P(QI=0|x,BR) \\ P(Y \geq BI|x,BR) &\leq 2P(QI=1|x,BR). \end{aligned} \tag{11}$$

In other words: the fraction with Y smaller than BI is at most twice the fraction reporting $Y<BI$; the fraction with Y at least BI is at most twice the fraction reporting $Y \geq BI$. Compared to the no-anchoring case, the factor 2 reflects the loss of information due to allowing for anchoring.

The bounds on $F(y/x,BR)$ follow immediately:

$$\begin{aligned}
\text{for } y < BI & \quad 0 \leq F(y|x, BR) \leq 2P(QI=0|x, BR) \\
\text{for } y \geq BI & \quad 1 - 2P(QI=1|x, BR) \leq F(y|x, BR) \leq 1
\end{aligned} \tag{12}$$

This implies either a nontrivial lower bound or a nontrivial upper bound, unless $P(QI=1|x, BR)=0.5$. If $P(QI=1|x, BR)<0.5$, the fraction of bracket respondents with a high value of Y is bounded. This leads to a lower bound on $F(y|BR, x)$. If $P(QI=1|x, BR)>0.5$, not all bracket respondents have a low value of Y . This leads to an upper bound on $F(y|BR, x)$. Replacing (6) by (12) and applying this to (4) straightforwardly leads to bounds on $F(y/x)$:

for $y < BI$

$$\begin{aligned}
& F(y|FR, x)P(FR|x) \\
& \leq F(y|x) \leq \\
& F(y|FR, x)P(FR|x) + \min[1, 2P(QI=0|x, BR)]P(BR|x) + P(NR|x)
\end{aligned} \tag{13}$$

for $y \geq BI$,

$$\begin{aligned}
& F(y|FR, x)P(FR|x) + \max[0, 1 - 2P(QI=1|x, BR)]P(BR|x) \\
& \leq F(y|x) \leq \\
& F(y|x, FR)P(FR|x) + P(BR|x) + P(NR|x)
\end{aligned} \tag{14}$$

These bounds are sharper than Manski's worst case bounds in (2) unless $P(BR/x)=0$ or $P(QI=1|x, BR)=0.5$. On the other hand, they are wider than the bounds in (7)-(8), which were constructed under the stronger assumption of no anchoring.

Alternative Models for Anchoring

Although the model Hurd et al. (1998) use can explain the anchoring phenomena in their data, it may not be the intuitively most appealing way to model anchoring, and it seems worthwhile to consider some alternative anchoring models. Herriges and Shogren (1996) allow for anchoring in follow-up questions only, implying the no-anchoring assumption in (6) and (7) for the one bracket question case. The model of Cameron and Quiggin (1994) is specifically designed for two bracket questions. It is straightforward, however, to show that this model is equivalent to the parametric Hurd et al. (1998) model for the case of two bracket questions, although the interpretation of Cameron and Quiggin is different.

The motivation of the Hurd et al. (1998) model stems from Green et al. (1998) and Jacowitz and Kahneman (1995). These studies find that, if a high anchor is used, respondents too often report that the amount exceeds the anchor. In terms of our notation this would mean

$P(QI=1) \geq P(Y \geq BI)$ if BI is large. Jacowitz and Kahneman (1995) report that this finding is not symmetric for their case study, and could well be reversed if the amounts have a natural upper instead of lower bound. An operational version of the phenomenon discussed by Jacowitz and Kahneman for one bracket question would be

$$\begin{aligned} P(QI=1|x, BR) &\geq P(Y \geq BI|x, BR) \quad \text{if } P(QI=1|x, BR) \leq 0.5 \\ P(QI=1|x, BR) &\leq P(Y \geq BI|x, BR) \quad \text{if } P(QI=1|x, BR) \geq 0.5 \end{aligned} \tag{15}$$

Here ‘ BI is large’ is specified as ‘at most half of the respondents report an amount of at least BI .’ It is easily shown that (15) is stronger than (11), and that (15) is satisfied in the parametric model of Hurd et al. (1998). The underlying intuition is that adding noise to BI before comparing it to Y , increases the tail probabilities in the distribution of the difference.

Constructing bounds on $P(Y \geq BI|x, BR)$ from (15) is straightforward. If $P(QI=1|x, BR) \leq 0.5$, the first inequality in (15) leads to an upper bound; if $P(QI=1|x, BR) \geq 0.5$, the other inequality leads to a lower bound. A practical problem with estimating these bounds arises if the estimate of $P(QI=1|x, BR)$ in a given sample is not significantly different from 0.5.

Finally, a robust finding in the literature is that dichotomous questions usually shift the distribution to the right, compared to open-ended questions. This is particularly so if there is a clear lower bound but no obvious upper bound to the amounts in question. In the willingness-to-pay (WTP) literature where the amounts are subjective (reflecting, for example, how much respondents would be willing to pay for some public good), this phenomenon is known as *yea-saying*. Green et al. (1998) find evidence of yea-saying for objective quantities rather than WTP data. Yea-saying implies an asymmetric inequality between reported and true fractions:

$$P(QI=1|x, BR) \geq P(Y \geq BI|x, BR) \tag{16}$$

This immediately gives an upper bound on $P(Y \geq BI|x, BR)$.¹

3.4 Two unfolding bracket questions

With two unfolding bracket questions, those who answer ‘yes’ to question (3) are given a second

¹A common test on yea-saying is to compare the estimated distribution for the open-ended respondents with the (upper and lower bound of the) distribution function for the bracket respondents. In absence of selectivity effects, yea-saying would imply that the latter distribution is to the right of the former. In the present framework, however, selectivity effects can play a role, and this test is not a test on yea-saying only.

question with bid $B21 > B1$, and those who answer ‘no’ get a second question with bid $B20 < B1$. Again, they can answer ‘yes’, ‘no’ or ‘don’t know’. In this subsection it is assumed that every bracket respondent answers the second question with ‘yes’ or ‘no’. The ‘don’t know’ answers will be considered in the next subsection.

Not allowing for an anchoring effect

If the assumption is made that those who answer the bracket questions do this correctly, then, for each bracket respondent, it is known whether Y is in $[0, B20]$, $[B20, B1]$, $[B1, B21]$, or $[B21, \infty]$. The information is the same as the information provided by a range card question with the same four categories.² Bounds on $F(y|x)$ for this case are a straightforward generalization of the bounds in (7) and (8). Denoting the category containing y by $[L(y), U(y))$ (for example, for $B20 \leq y < B1$, $L(y)=B20$ and $U(y)=B1$), the bounds on $F(y|BR,x)$ are given by

$$F(L(y)|BR,x) \leq F(y|BR,x) \leq F(U(y)|BR,x) \quad (17)$$

Combined with (4), this gives bounds on $F(y/x)$ similar to the one bracket question case.

Allowing for anchoring

Similar to $Q1$, define dummy variables $Q20$ and $Q21$ for those who answer the second bracket question on $B20$ and $B21$, i.e., those with $Q1=0$ and $Q1=1$, respectively. Thus, $Q20=1$ if the respondent reports that the amount is at least $B20$, etc. On top of $P(Q1=1|x, BR)$ two other probabilities are now also directly identified by the data: $P(Q20=1|Q1=0,x, BR)$ and $P(Q21=1|Q1=1,x, BR)$. To derive the bounds, Assumption 1 needs to be generalized. Again, the starting point is Hurd et al. (1998). Their model assumes that the answers $Q1$, $Q20$ and $Q21$ to the three bracket questions are based upon comparing Y with $B1 + \epsilon_1$, with $B20 + \epsilon_{2,0}$, and with $B21 + \epsilon_{2,1}$. The perception errors ϵ_1 , $\epsilon_{2,0}$ and $\epsilon_{2,1}$ are assumed to be independent of each other and of X and Y , and normally distributed with zero means. The anchoring effects in the data are captured if $\epsilon_{2,0}$ and $\epsilon_{2,1}$ have smaller variances than ϵ_1 . The following extension of Assumption 1 is a nonparametric, less restrictive, version of the Hurd et al. assumptions:

Assumption 2: $Q1=1$ if $Y \geq B1 + \epsilon_1$ and $Q1=0$ if $Y < B1 + \epsilon_1$; $Q20=1$ if $Y \geq B20 + \epsilon_{2,0}$ and $Q20=0$ if $Y < B20 + \epsilon_{2,0}$; $Q21=1$ if $Y \geq B21 + \epsilon_{2,1}$ and $Q21=0$ if $Y < B21 + \epsilon_{2,1}$ where the perception errors ϵ_1 , $\epsilon_{2,0}$

²See Vazquez et al. (1999) for an application of Manski bounds incorporating information from follow-up range card questions.

and $\varepsilon_{2,1}$ satisfy:

For all (x,y) in the support of (X,Y) : $\text{Median}[\varepsilon_1|Y=y,X=x,BR]=0$;

$\text{Median}[\varepsilon_{2,0}|Y=y,X=x,BR,QI=0]=0$; $\text{Median}[\varepsilon_{2,1}|Y=y,X=x,BR,QI=1]=0$.

A stronger version of this assumption that may be more natural is the assumption that each of the three perception errors has median zero, is independent of being a bracket respondent or not, and is independent of Y,X , and the other two error terms.

Assumption 2 is weaker than the assumptions of Hurd et al (1998). It implies that each bracket question is answered correctly with probability at least 0.5:

$$\begin{aligned}
P(QI=1|Y<B1,x,BR) &\leq 0.5; & P(QI=1|Y\geq B1,x,BR) &\geq 0.5 \\
P(Q20=1|Y<B20,QI=0,x,BR) &\leq 0.5; & P(Q20=1|Y\geq B20,QI=0,x,BR) &\geq 0.5 \\
P(Q21=1|Y<B21,QI=1,x,BR) &\leq 0.5; & P(Q21=1|Y\geq B21,QI=1,x,BR) &\geq 0.5
\end{aligned} \tag{18}$$

In addition to (11), the implication of (18) for those who answer ‘no’ to the first question, is that

$$\begin{aligned}
P(Y<B20|QI=0,x,BR) &\leq 2P(Q20=0|QI=0,x,BR) \\
P(Y\geq B20|QI=0,x,BR) &\leq 2P(Q20=1|QI=0,x,BR)
\end{aligned} \tag{19}$$

and for those who answer ‘yes’ to the first question, the implication is that

$$\begin{aligned}
P(Y<B21|QI=1,x,BR) &\leq 2P(Q21=0|QI=1,x,BR) \\
P(Y\geq B21|QI=1,x,BR) &\leq 2P(Q21=1|QI=1,x,BR)
\end{aligned} \tag{20}$$

Assumption 2 and the bounds in (11), (19) and (20) can be used to derive bounds on the distribution function for bracket respondents. See Appendix A1 for derivations and the results. To illustrate, one example is presented here: the upper bound on $P(Y<B20/x,BR)$ is given by:

$$P(Y\leq B20|x,BR) \leq [1, 2P(Q20=0|QI=0,x,BR)]\min[1, 2P(QI=0|x,BR)] \tag{21}$$

If many people say their income exceeds $B1$ (i.e., $P(QI=0/x,BR)$ is low), this limits the maximum number of people whose income is lower than $B20$. If the majority of those who report that their

income is lower than $B1$ report that their income is higher than $B20$, this also limits the maximum number of people with income below $B20$.

Alternative Models for Anchoring

In the previous subsection some alternative assumptions on anchoring for the one bracket question case were discussed. The assumptions following the findings of Jacowitz and Kahneman (1995) basically treat every bracket question separately. In addition to (15), they are, for $k=0,1$:

$$\begin{aligned} P(Q2k=1|x, BR, Q1=k) \geq P(Y \geq B1|x, BR, Q1=k) \quad \text{if} \quad P(Q2k=1|x, BR, Q1=k) \leq 0.5 \\ P(Q2k=1|x, BR, Q1=k) \leq P(Y \geq B1|x, BR, Q1=k) \quad \text{if} \quad P(Q2k=1|x, BR, Q1=k) \geq 0.5 \end{aligned} \quad (22)$$

From these assumptions, bounds can be derived on the distribution function for bracket respondents in a similar way as for the Hurd et al. model. The results depend on whether the bids are ‘small’ or ‘large’. Appendix A2 presents the formulas for the relevant case for our data.

An intuitively appealing way of allowing for anchoring in the second question is provided by Herriges and Shogren (1996). They formulate a simple model which explicitly allows for an effect of the first bid on the respondent’s subjective opinion on the amount Y . The essential feature of their model is that there is no anchoring effect in the first bracket question, but the first bid $B1$ serves as an anchor for the second bid $B2$ (which is either $B20$ or $B21$). Thus, in the second bracket question the respondent does not compare $B2$ to Y , but to $Y^* = (1-g)Y + gB1$. This reflects the intuition behind anchoring: the respondent is uncertain about the true value of Y . The bid $B1$ is taken to be informative about Y , and the respondent’s new estimate Y^* of Y is drawn towards $B1$. Herriges and Shogren (1996) assume that g is a fixed parameter, but also discuss an extension in which g can vary with $B1$. They apply their model to data on willingness to pay for water quality improvement, and find an estimate for g of 0.36, with standard error 0.14. In another application, O’Connor et al. (1999) find a similar significantly positive value of g .

The Herriges and Shogren (1996) model offers an alternative explanation for the shift in the estimated distribution based upon unfolding bracket questions due to the order of the bids, the main finding in Hurd et al. (1998). On the other hand, the Herriges and Shogren model cannot explain the main finding of Jacowitz and Kahneman (1995), since that finding is related to the first bid, for which the Herriges and Shogren (1996) model imposes the no anchoring assumption.

A natural way to relax the Herriges and Shogren (1996) assumptions is to replace them by the following nonparametric assumptions:

$$\begin{aligned}
P(QI=1|x, BR) &= P(Y \geq BI|x, BR) \\
P(Q20=1|x, BR, QI=0) &\geq P(Y \geq B20|x, BR, QI=0) \\
P(Q21=1|x, BR, QI=1) &\leq P(Y \geq B21|x, BR, QI=1)
\end{aligned} \tag{23}$$

The first assumption says that there is no anchoring in the first question, the other two assumptions say that anchoring in the second question is towards BI . These assumptions can be used to derive bounds on the distribution function for bracket respondents in the same way as in the other models. The results are presented in Appendix A3.

3.5 Complete and incomplete bracket respondents

Until now it was assumed that all bracket respondents answered both bracket questions with ‘yes’ or ‘no’. In practice, however, some of them answer ‘don’t know’ to the second bracket question. Thus, there are two types of bracket respondents: those who answer both questions with ‘yes’ or ‘no’ (CBR , complete bracket respondents), and those who answer the first question with ‘yes’ or ‘no,’ but the second question with ‘don’t know’ (IBR , incomplete bracket respondents). We make no assumptions on the relation between response behavior and Y , so that incomplete bracket respondents can be a selective subsample of all bracket respondents.

The conditional distribution function for bracket respondents can be written as follows:

$$F(y|BR,x) = F(y|CBR,x)P(CBR|BR,x) + F(y|IBR,x)P(IBR|BR,x) \tag{24}$$

The probabilities $P(CBR|BR,x)$ and $P(IBR|BR,x)$ are both identified, since it is observed whether bracket respondents are complete or incomplete bracket respondents. Bounds (allowing or not allowing for anchoring) on $F(y|CBR,x)$ can be derived as in Section 3.4, using complete bracket respondents only. Bounds on $F(y|IBR,x)$ can be derived as in Section 3.3, using incomplete bracket respondents only. Combining these bounds and inserting them into (24) leads to bounds on $F(y|BR,x)$. The bounds on $F(y|BR)$ can be combined with $F(y|FR,x)$ and bounds on $F(y|NR,x)$ in the same way as in (13)-(14), yielding bounds on $F(y/x)$.

3.6 Bounds on Quantiles

Distributions of income, savings, etc., are often described in terms of (conditional) quantiles. For $\alpha \in [0,1]$, the α -quantile of the conditional distribution of Y given $X=x$, is defined as

$$q(\alpha, x) = \inf \{y: F(y|x) \geq \alpha\} \quad (25)$$

For $\alpha > 1$, $q(\alpha, x) = \infty$, and for $\alpha < 0$, $q(\alpha, x) = -\infty$. Following Manski (1994), bounds on these quantiles can be derived by ‘inverting’ the bounds on the distribution function. All the bounds in Sections 3.1-3.5 can be written as

$$lb(y, x) \leq F(y|x) \leq ub(y, x) \quad (26)$$

for functions $lb(y, x)$ and $ub(y, x)$ that are nondecreasing in y . Inverting (26) gives the following bounds on the quantiles:

$$\inf \{y: lb(y, x) \geq \alpha\} \geq \inf \{y: F(y|x) \geq \alpha\} \geq \inf \{y: ub(y, x) \geq \alpha\} \quad (27)$$

This is easily illustrated using a graph of the distribution function, with y along the horizontal axis and $F(y|x)$ along the vertical axis. The bounds on the distribution function squeeze $F(y|x)$ in between two curves; the vertical distance between these curves is the width between the bounds (at each given value y of Y). Reading the same graph horizontally gives, for a given probability value $\alpha \in [0, 1]$, a lower and an upper bound on the α -quantile.

4 Data

The data comes from the 1996 wave of the Health and Retirement Survey (HRS). This is a longitudinal study conducted by the University of Michigan for the US National Institute of Aging. It focuses mainly on aspects of health, retirement and economic status of US citizens born between 1931 and 1941, collecting individual and household information from a representative sample of this cohort. The data is collected every two years, starting in the Summer of 1992.

Initially the panel consisted of approximately 7,600 households. The respondents are the household representatives that satisfy the age criteria, and their partners, regardless of their age (second household respondents). This leads to approximately 12,600 individual respondents in the first wave. Each individual answers questions on health and retirement issues. Household representatives also answer questions on past and current income and pension plans (including those of their partner) and questions at the household level, on, for example, housing conditions, household assets, and family structure. If health problems prevent the household representative from responding, someone else (for example, the spouse) will answer on their behalf. All follow

up interviews are conducted over the phone, unless the household has no phone, or health reasons prevent the household representative and the spouse from answering over the phone, in which a face to face interview is held. If respondents die, they are replaced by another household member (if possible). This reduces attrition in the panel at the household level.

The 1996 wave has data from 6,739 households with 10,887 individuals. In 4,148 of these households, two respondents gave interviews. The remaining 2,591 are single respondent households. To get some insight in the nature of the data, Table 1 shows sample statistics for some background variables. The first column refers to the full sample, while the second and third refer to the sub-samples of household representatives and second household respondents, respectively. The statistics show that 51% of the household representatives are women, and 62% of the second household respondents (usually the spouse) are women. There is little difference between educational achievement of household representatives and second household respondents. The shares of Whites, Blacks and Hispanics reflect the ethnic composition of the cohort. About 62% of the respondents participate in the labor market, most of them are employees. Approximately 80% of all households are home owners.

All household representatives are asked to provide information on employment status and earned incomes for themselves and their partners. Initially, each household representative is asked if he or she worked for pay during the last calendar year. Each of the 4,145 who answered ‘yes’ was asked if the earnings during the last calendar year came from self-employment, wages and salaries, or a combination of these sources. The 3,608 individuals who reported that all or some of their earnings came from wages and salaries were asked the following question:

‘About how much wages and salary income did you receive during the last calendar year?’

‘any amount’ (in US dollars)

‘Don’t know’

‘Refuse’

3,160 individuals answered this question with an exact amount in US dollars, ranging from \$ 0,00 to \$350,000, with a mean of \$29,430 and standard deviation \$26,430. The median was \$25,000. The remaining 448 individuals answered ‘don’t know’ (or ‘refuse’), implying a 12.4% initial nonresponse rate. The latter were routed to a sequence of unfolding bracket questions as formulated in (3), with starting bid $BI = \$25,000$. At this initial stage of the unfolding sequence, 119 individuals answered ‘don’t know’ (or ‘refuse’). Thus, the full nonresponse rate is 3.3%. The remaining 329 individuals form the sample of bracket respondents.

The unfolding sequence for the wages and salaries question consists of two steps. Those who answered ‘yes’ to the initial bid of \$25,000 were routed to a second question with bid

$B21 = \$50,000$. Those who answered ‘no’ were routed to a question with bid $B20 = \$5,000$. In each case, the question was the same as that in (3) - only the bid changed. The second question of the unfolding sequence could again be answered with ‘don’t know’ (or ‘refuse’), leading to incomplete bracket respondents (IBR). For the earnings variable considered, 320 individuals completed the sequence of unfolding brackets (CBR). The other 9 bracket respondents are incomplete bracket respondents.

Table 1: Sample statistics of some background variables
Means (with standard deviation) and Percentages (with standard error); complete sample

	All Units	Household Representatives	Second Household Respondents
Number of Observations	10,887	6,739	4,148
Age	59.6 (5.62)	60.7 (5.07)	58.6 (6.41)
Percentage males	45 (0.5)	49 (0.6)	38 (0.8)
Education	2.32 (1.02)	2.36 (1.03)	2.25 (0.98)
Percentage home owners	-	79 (0.5)	-
Percentage whites	71 (0.4)	69 (0.6)	76 (0.7)
Percentage Hispanics	9 (0.3)	8 (0.3)	11 (0.5)
Percentage Afro-Americans	16 (0.4)	19 (0.5)	9 (0.4)
Percentage other Races	4 (0.2)	4 (0.3)	4 (0.3)
Percentage working	62 (0.5)	62 (0.6)	64 (0.7)
Working for wage/salary	47 (0.5)	46 (0.6)	50 (0.8)
Self-employed	9 (0.3)	8 (0.3)	10 (0.5)
Both working for wage/salary & self-employed	6 (0.2)	8 (0.3)	4 (0.3)

Explanation: Education: educational achievement on a scale of 1 to 4; 1: has completed primary education (up to the 10th grade in the USA education system), 2: has completed high school (up to the 12th grade); 3: some form of college or post-high school education; 4: has completed at least a first degree at university level.

Table 2 shows some statistics for the sample with nonzero household respondent wages and salaries. Comparing it with Table 1 shows that those who received wages and salaries less often own their home and are, on average, somewhat younger. The subsample of complete bracket respondents contains a larger percentage of females than the other samples. Likewise, complete bracket respondents have lower educational achievement, are less likely to own their

home, and are less often white. The statistics of the incomplete bracket respondents are very different from those of the other groups, but this is based upon only 9 observations.

**Table 2: Sample statistics of some background variables for household respondents who received wages and salaries in the past calendar year
Means (with standard deviation) and Percentages (with standard error);**

	All employed with wages	Full Respondents (FR)	Full Nonrespondents (NR)
Number of Observations	3602	3160	113
Average age	58.6 (4.7)	58.6 (4.7)	59 (4.9)
Percentage Males	50 (0.8)	52 (0.9)	45 (4.7)
Education	2.52 (1.01)	2.6 (1.03)	2.6 (0.99)
% Home owners	73 (0.7)	74 (0.8)	83 (3.5)
% White	72 (0.7)	75 (0.8)	72 (4.2)
% Hispanics	8 (0.5)	7 (0.5)	5 (2.1)
% Afro-American	18 (0.6)	16 (0.7)	21 (3.8)
% Other races	2 (0.2)	2 (0.3)	3 (1.6)

	Complete Bracket Respondents (CBR)	Incomplete Bracket Respondents (IBR)
Number of Observations	320	9
Average age	58.8 (4.7)	55.7 (3.2)
Percentage Males	38 (2.7)	78 (14)
Education	2.2 (1.02)	3.1 (1.01)
% Home owners	65 (2.7)	89 (10.0)
% White	58 (2.8)	78 (14)
% Hispanics	9 (1.6)	0 (0)
% Afro-American	32 (2.6)	12 (11)
% Other races	2 (0.8)	10 (10)

Note: See Table 1 for explanations of the variables.

5 Estimates of the Bounds

This section applies the bounds derived in Section 3 to wages and salaries of the household representative, as described in Section 4. In Section 5.1, there is no conditioning on covariates. In Section 5.2, the bounds are estimated separately for high and low educated wage earners, and the results are used to test for differences in the quantiles for the two education levels. Since this involves conditioning on discrete variables only, these estimates are based upon (sub-)sample fractions and do not require nonparametric smoothing.

The width between point estimates of upper and lower bound reflects the uncertainty due to item nonresponse. Both point estimates and confidence bands are presented, to measure uncertainty due to sampling error. These confidence bands are estimated using a bootstrap method, based on 500 (re-)samples drawn with replacement from the original data. The lower and upper bounds are estimated 500 times, and the confidence bands are formed by the 2.5% and 97.5% percentiles in these 500 estimates, resulting in two-sided 95% confidence bands for both the upper and the lower bound. The figures present the lower confidence band for the lower bound and the upper confidence band for the upper bound. The gap between these reflects both the uncertainty due to sampling error and the uncertainty due to item nonresponse.

5.1 Bounds for all wage earners

If item nonresponse is completely random, the full respondents are a representative sample and the quantiles in the sample of full respondents are consistent estimates of the population quantiles. These estimates are shown in Figure 1a. The solid curve connects the point estimates of the log earnings quantiles, the dashed curves give (point-wise) 95% confidence bands. Bracket respondents and nonrespondents are discarded. Table 3 provides similar information as Figure 1a, giving the point estimates of some selected quantiles for the full respondents and their standard errors, but now in earnings levels instead of logs.

Figure 1b shows the estimates of Manski's (1995) worst case bounds, not using the bracket response information. Bracket respondents are treated as nonrespondents, and the relevant nonresponse rate in this case is 12.4%. The solid curves are the estimated upper and lower bounds, and the dashed curves are the confidence bands. The horizontal distance between the upper and lower bound equals 0.124, the initial nonresponse rate. To make a comparison possible, the confidence bands for the full respondents quantiles depicted in Figure 1 are also included. These are contained in the worst case bounds, since the latter allow for the possibility that nonresponse is completely random. The uncertainty due to nonresponse appears to be much larger than the uncertainty due to finite sampling errors.

Table 4 shows selected point estimates and confidence bands for the worst case bounds. For example, with at least 95% confidence, the median of wages and salaries is between \$19,500

and \$29,900. Due to the high percentage of initial item nonresponse, the difference between upper and lower bound is quite large. This makes it hard to draw meaningful economic conclusions. The width between the curves in Figure 1a was much smaller, but this came at the cost of the strong assumption of random nonresponse.

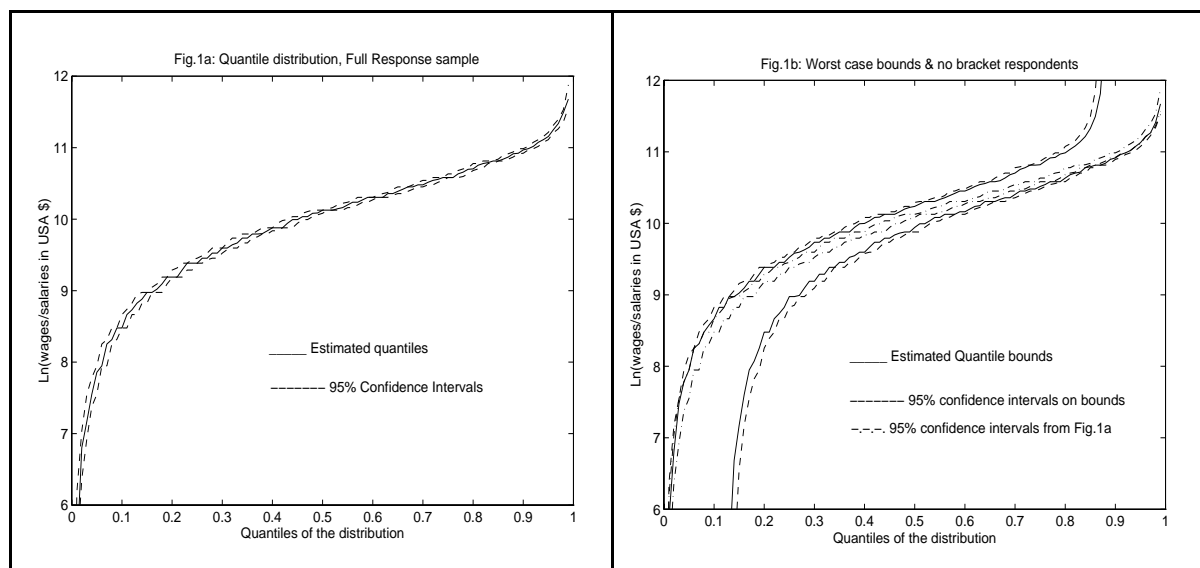


Table 3: Selected quantiles for the full response sample (n=3,160) (cf. Figure 1a).

Quantile	Point estimate	Standard error
25 th Percentile	\$11,900	\$352
40 th Percentile	\$19,500	\$373
50 th Percentile	\$25,000	\$361
60 th Percentile	\$29,900	\$346
75 th Percentile	\$39,400	\$389

Table 4: Worst case bounds not using bracket information (cf. Figure 1b)

Quantile	Confidence band lower bound	Point estimate lower bound	Point estimate Upper bound	Confidence band Upper bound
25 th Percentile	\$5,800	\$7,700	\$13,700	\$14,700
40 th Percentile	\$13,700	\$14,700	\$22,500	\$24,500
50 th Percentile	\$19,500	\$20,800	\$27,900	\$29,900
60 th Percentile	\$25,000	\$26,000	\$34,600	\$37,000
75 th Percentile	\$35,600	\$36,900	\$50,000	\$55,000

Note: Lower bound and upper bound are the lower end of the 95% confidence interval for the lower bound and the upper end of the 95% confidence interval for the upper bound, respectively.

The next step is to incorporate the information provided by the 329 bracket respondents, summarized in Table 5. To illustrate how the assumptions on anchoring affect the bounds, estimates of the bounds for the bracket respondents only are presented first. Figure 2 is based on the assumption of no anchoring (A0 in the figures). Figures 3A1 to 3A3 allow for the three types of anchoring discussed in Section 3, following Hurd et al. (A1 in the figures, (11), (19) and (20) in Section 3), Jacowitz and Kahneman (A2 in the figures, (15) and (22) in Section 3) or Herriges and Shogren (A3 in the figure, (23) in Section 3). In each figure, the confidence bands for the full respondents are also included. The no anchoring assumption A0 is stronger than all three anchoring assumptions, and thus leads to the narrowest bounds. Under the no anchoring assumption, the distribution function for complete bracket respondents is exactly identified at the three bids \$5,000, \$25,000 and \$50,000. Due to the incomplete bracket respondents, however, the upper and lower point estimates for all bracket respondents at \$50,000 are different.

Table 5: Information provided by bracket respondents

Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	\$50,000 — max	30
		Yes	> \$50,000 ?	No	\$25,000 — \$50,000	86
CBR	>\$25,000 ?					
		No	> \$ 5,000 ?	Yes	\$5,000 — \$25,000	170
				No	\$0 — \$5,000	34
		Yes	> \$50,000 ?	DK	> \$25,000	9
IBR	>\$25,000 ?					
		No	> \$ 5,000 ?	DK	< \$25,000	0

Explanation: CBR=Complete Bracket Response; IBR=Incomplete Bracket Response; DK= Don't know (or refused to answer).

Comparing the bounds for bracket respondents in Figure 2 with the full respondents curves suggests that equality of $P(Y < B/FR)$ and $P(Y < B/BR)$ is rejected for $B = \$25,000$ but maybe not for $B = \$5,000$ or $B = \$50,000$. The results of the corresponding formal (point-wise) tests are reported in Table 6. The null hypothesis would be valid if there was no anchoring and no selective response behavior. Rejecting this hypothesis at \$25,000 and \$50,000 suggests that at least one of these conditions is violated. The fact that the bracket respondents more often report an income below \$25,000 (or \$50,000) than full respondents, suggests that rejecting the null is not due to yea-saying (see Section 3). It could be, for example, that workers with low earnings tend not to know their exact income level and, therefore, answer the bracket questions only.

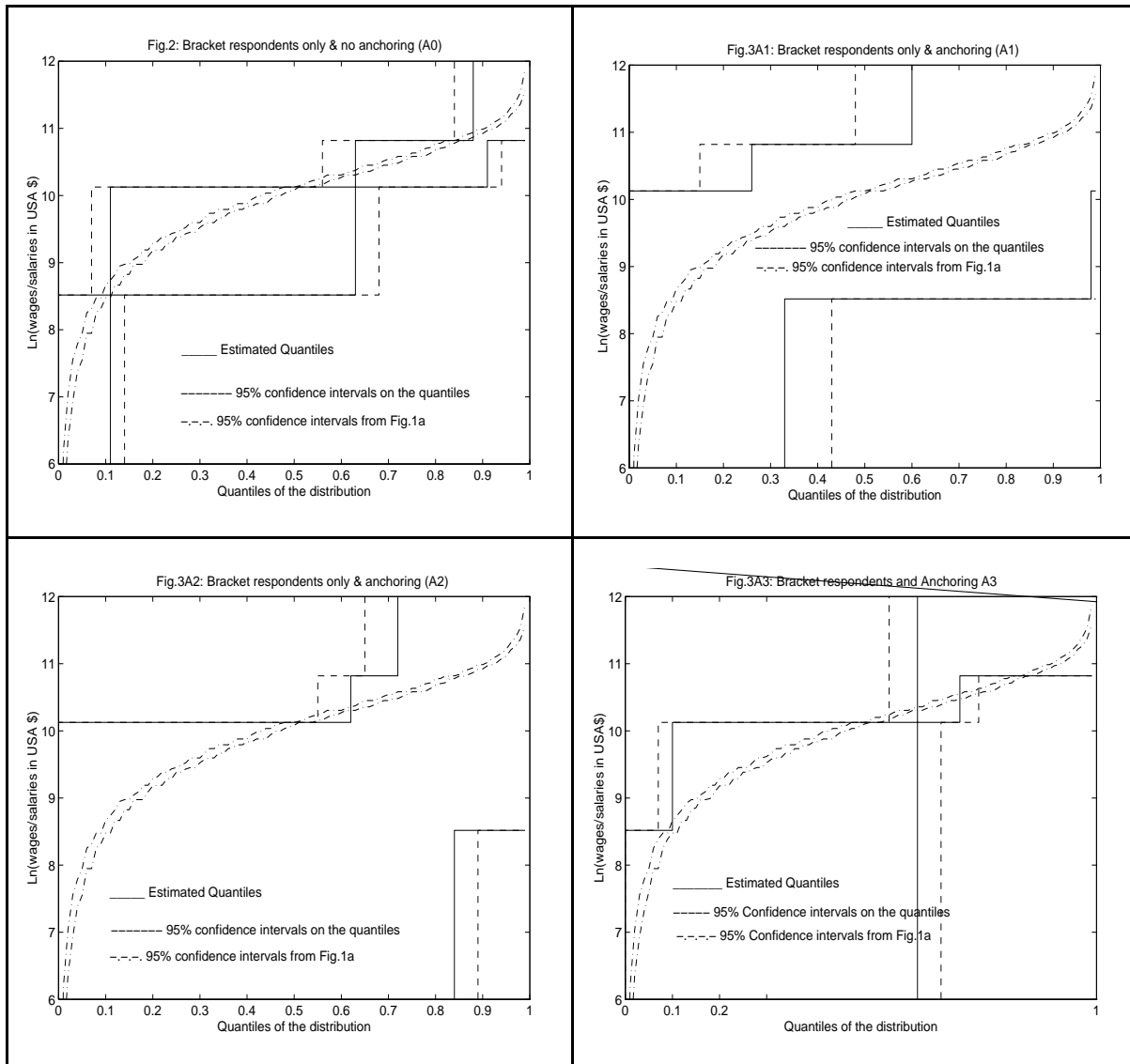
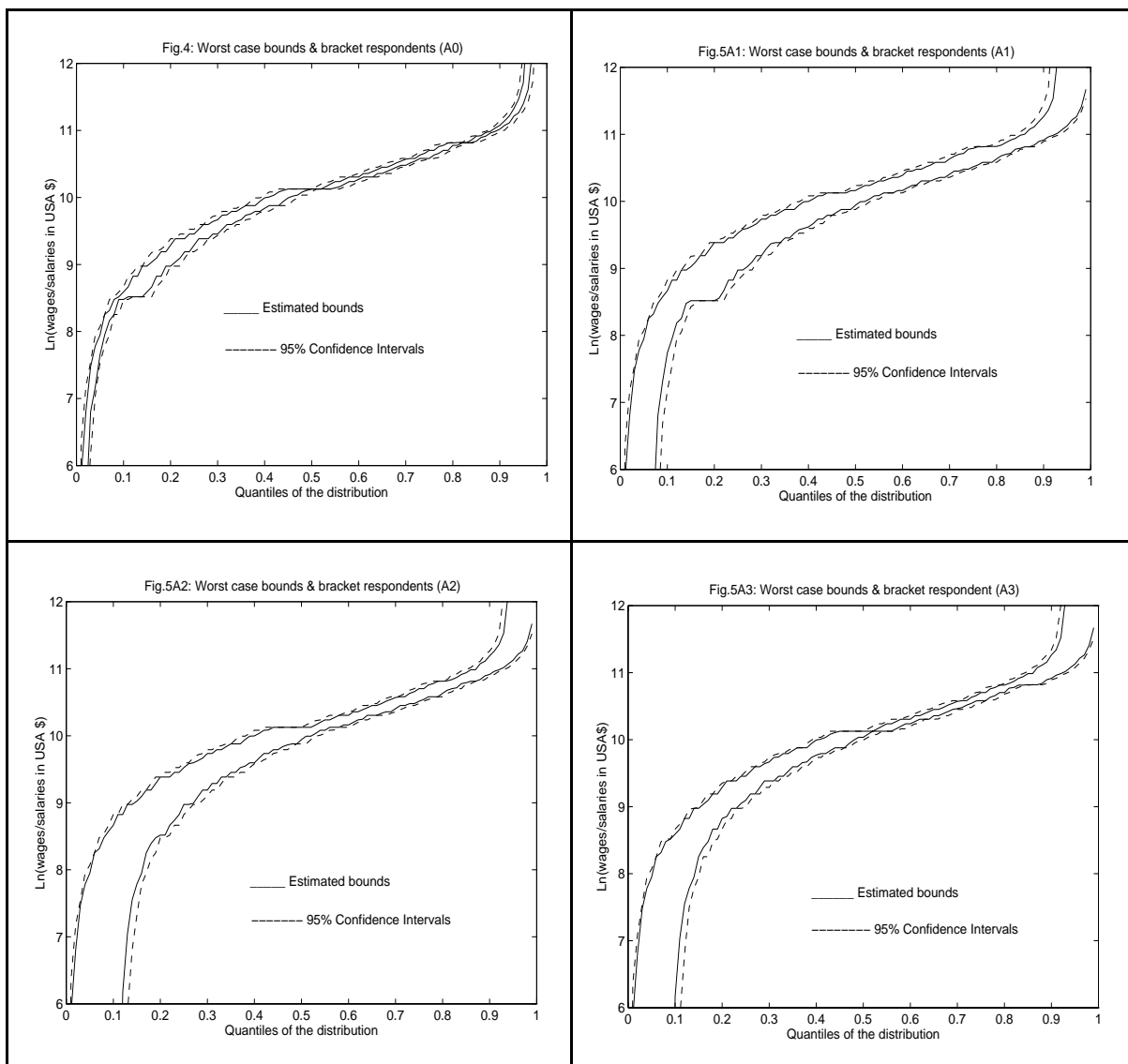


Table 6: Tests for differences between full and bracket respondents

B	P(Y<B FR)	standard error	P(Y<B BR)	standard error	Test Statistic
\$5,000	0.101	0.005	0.104	0.017	-0.11
\$25,000	0.529	0.0083	0.62	0.027	-3.22
\$50,000	0.853	0.006	0.901	0.016	-2.81

Test statistic: difference between point estimates (Full Response – Bracket Response) normalized by its estimated standard error (equal to the square root of the sum of the variances corresponding to the two point estimates); under the hypothesis $P(Y<B|FR)=P(Y<B|BR)$, the test statistic is asymptotically standard normal.

Allowing for anchoring widens the bounds. Under the Hurd et al. (1998) assumptions the bracket response data provides some information on $P(Y < y/BR)$ for all y , in the form of either a lower bound or an upper bound. Under the Jacowitz and Kahneman assumptions, the bracket response data does not say anything about $P(Y < y/BR)$ for y between \$5,000 and \$25,000. The figures make clear that the three different assumptions on anchoring are nonnested: none of the three is uniformly more informative than any of the others. Figure 4 and Figures 5A1 - 5A3 combine the bounds for bracket respondents with the information for full respondents, and show the bounds on the quantiles for all respondents in the population. As expected, the bounds under no anchoring are narrower than the bounds allowing for anchoring, and all bounds allowing for anchoring are narrower than the worst case bounds in Figure 1b.



**Table 7: Upper and lower bounds incorporating bracket responses; 95% confidence level
(c.f. Figure 4 and Figures A1 to A3)**

Quantiles	No anchoring (A0)	Anchoring A1	Anchoring A2	Anchoring A3
25 th Percentile				
Lower Bound	\$9,800	\$6,800	\$6,800	\$8,000
Upper Bound	\$13,700	\$14,700	\$14,700	\$13,700
Difference	\$3,900	\$7,900	\$7,900	\$5,700
40 th Percentile				
Lower Bound	\$17,900	\$14,700	\$14,500	\$16,900
Upper Bound	\$22,800	\$23,900	\$23,900	\$22,800
Difference	\$4,900	\$9,200	\$9,400	\$6,000
50 th Percentile				
Lower Bound	\$23,900	\$19,500	\$19,500	\$21,900
Upper Bound	\$25,000	\$27,900	\$25,000	\$25,000
Difference	\$1,100	\$8,400	\$5,500	\$3,100
60 th Percentile				
Lower Bound	\$27,900	\$25,000	\$25,000	\$26,900
Upper Bound	\$31,500	\$34,600	\$31,500	\$31,500
Difference	\$3,600	\$9,600	\$6,500	\$4,600
75 th Percentile				
Lower Bound	\$39,400	\$35,600	\$35,600	\$37,900
Upper Bound	\$45,000	\$49,700	\$46,800	\$48,000
Difference	\$5,600	\$14,100	\$11,200	\$8,100
80 th Percentile				
Lower Bound	\$44,500	\$39,400	\$39,400	\$43,500
Upper Bound	\$50,000	\$52,500	\$50,000	\$51,000
Difference	\$5,500	\$13,100	\$10,600	\$7,500

Note: Lower bound and upper bound are the lower end of the 95% confidence interval for the lower bound and the upper end of the 95% confidence interval for the upper bound, respectively.

More precise information is given in Table 7. For example, under the assumption of random nonresponse, the 95% confidence interval for the 40th percentile is rather precisely determined with width about \$ 1,450 (Table 3). Allowing for nonrandom nonresponse and ignoring bracket information reduces the precision enormously, giving an interval width of \$ 10,800 (Table 4). Allowing for nonrandom nonresponse and using the bracket information gives a precision in between these two: \$ 4,900 if no anchoring effects are allowed for; \$ 9,200, \$ 9,400 and \$ 6,000 under the three types of anchoring (Table 7). The precision under no anchoring is particularly large for the median since the sample median for full-respondents is close to one of the bids (\$

25,000), where, under the no anchoring assumption, the distribution function for bracket respondents is exactly identified.

5.2 Comparing Earnings of the High and Low Educated

Table 8 presents some details on the response behavior of the lower educated (at most high school; levels 1 and 2 in Table 1) and higher educated (more than high school; levels 3 and 4 in Table 2) separately. The latter have a slightly lower initial nonresponse rate than the former. On the other hand, the low educated are more often willing to answer the bracket questions, so that their full nonresponse rate is lower than that of the high educated. Mean and median incomes of full respondents are clearly higher for the higher educated than for the lower educated.

Figures 6a and 7a show the confidence intervals of the quantiles for full respondents with low and high education level. These are consistent estimates for the quantiles of all low and high educated wage and salary earners if, conditional on education level, nonresponse is not related to the level of earnings. This assumption is again quite strong, although conditioning on education level makes it different from the unconditional random nonresponse assumption underlying Figure 1a. Table 9 presents details for some selected quantiles.

Figure 8a compares the 95% confidence bands for the high and low educated full respondents. It suggests that most quantiles are significantly different. This is confirmed by the formal test results presented in the final column of Table 9. Thus if response behavior is random conditional on education level, the quantiles of the high and low educated are significantly different. The issue in the remainder is whether this conclusion can still be drawn if nonrandom response behavior is allowed for.

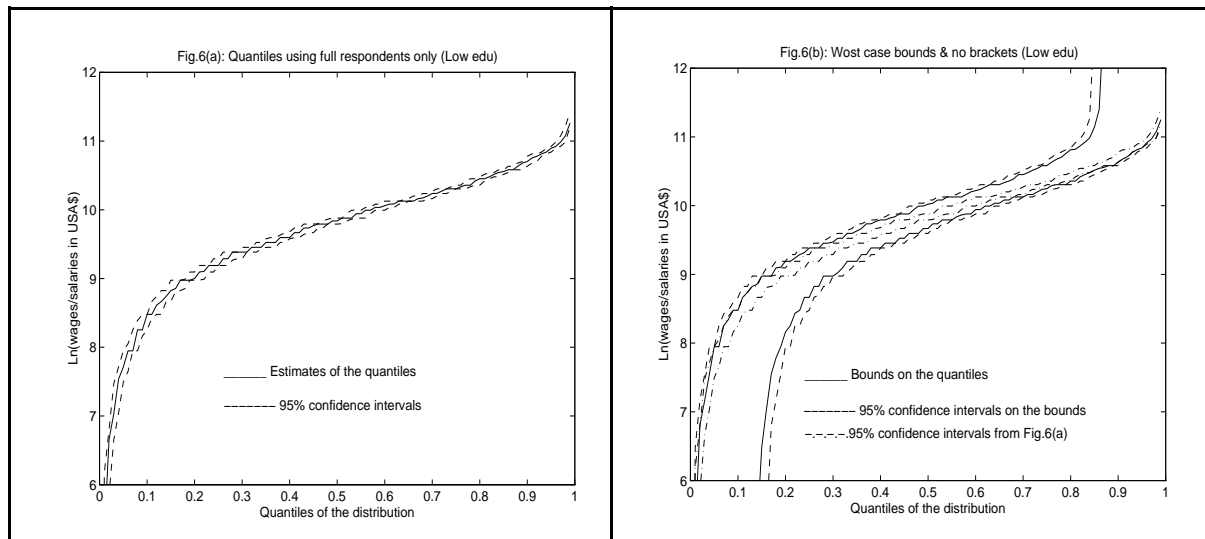
Figures 6b (low educated) and 7b (high educated) present confidence bands for the worst case bounds allowing for nonrandom nonresponse and not using the bracket information. Figure 8b compares these for the two education levels. The latter figure suggests that the null of equal quantiles is rejected only for quantiles in the range from about 0.3 to about 0.8.

Table 10 presents the formal (point-wise) test results of the one-sided null hypothesis that the upper bound of the lower educated is at least as high as the lower bound of the higher educated. The null is rejected for the 40%, 50%, 60% and 75% quantiles, but not for the 20%, 25%, 30% and 80% quantiles. This again illustrates that item nonresponse particularly reduces the information on the quantiles in the tails, where the distribution function is rather flat.

Table 8: Sample statistics and response behavior by education level of household respondent

	All	Low education	High education
Observations in complete sample	6,739	4,110	2,629
Observations with wages and salaries	3,602	1,978	1,624
Number of full respondents	3,160 (88%)	1,713 (86.6%)	1,447 (89.1%)
Mean	\$29,430	\$22,813	\$38,298
Standard Deviation	\$26,430	\$18,080	\$31,765
Median	\$25,000	\$19,000	\$33,000
Initial nonrespondents (number and percentage)	442 (12.3%)	265 (13.4%)	177 (10.9%)
Bracket respondents (number and percentage)	329 (9.1%)	212 (10.7%)	117 (7.2%)
Nonrespondents (number and percentage)	113 (3.1%)	53 (2.3%)	60 (3.7%)

Explanation: Low education: education levels 1 and 2, i.e., at most high school; High education: education levels 3 and 4, i.e., more than high school.



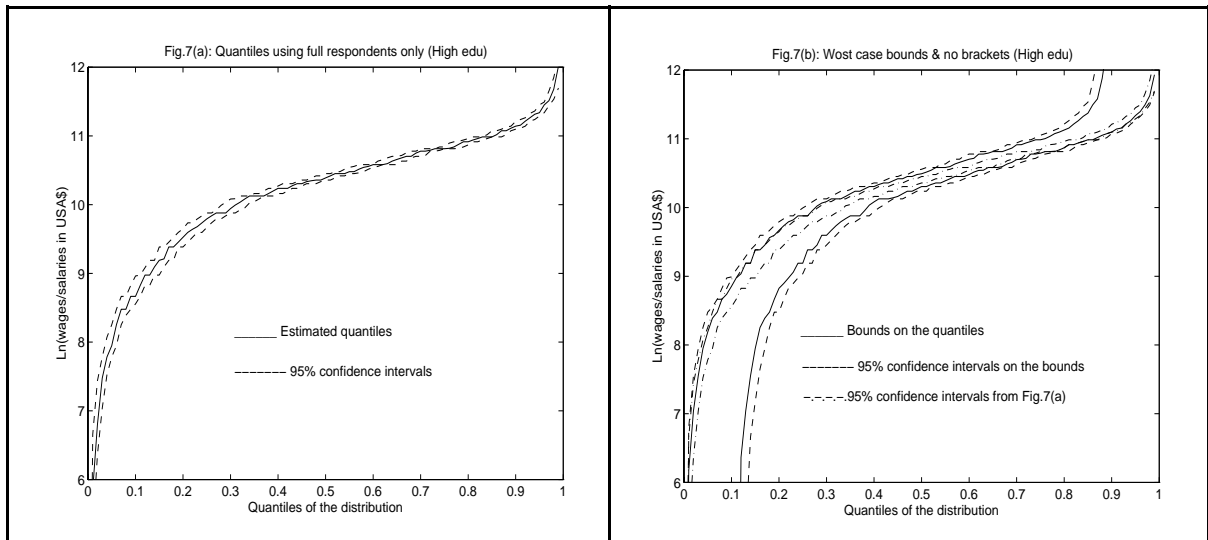


Table 9: Quantiles of full respondents by education level (cf. Figures 6(a) and 7(a)).

Quantile	Low education level (n=1979)		High education level (n=1623)		Test Statistic
	Point estimate	Standard error	Point estimate	Standard error	
25 th Percentile	\$9,800	\$439	\$17,900	\$790	8.96
40 th Percentile	\$14,700	\$542	\$27,900	\$784	13.85
50 th Percentile	\$18,700	\$503	\$32,500	\$923	13.13
60 th Percentile	\$23,400	\$620	\$39,400	\$601	18.53
75 th Percentile	\$29,900	\$537	\$49,700	\$623	24.07
90 th Percentile	\$44,500	\$1,279	\$69,000	\$1,660	12.17

Test statistic: difference between point estimates (High educated – Low educated) normalized by its estimated standard error; under the null that quantiles of high and low educated are equal, the test statistic is asymptotically standard normal.

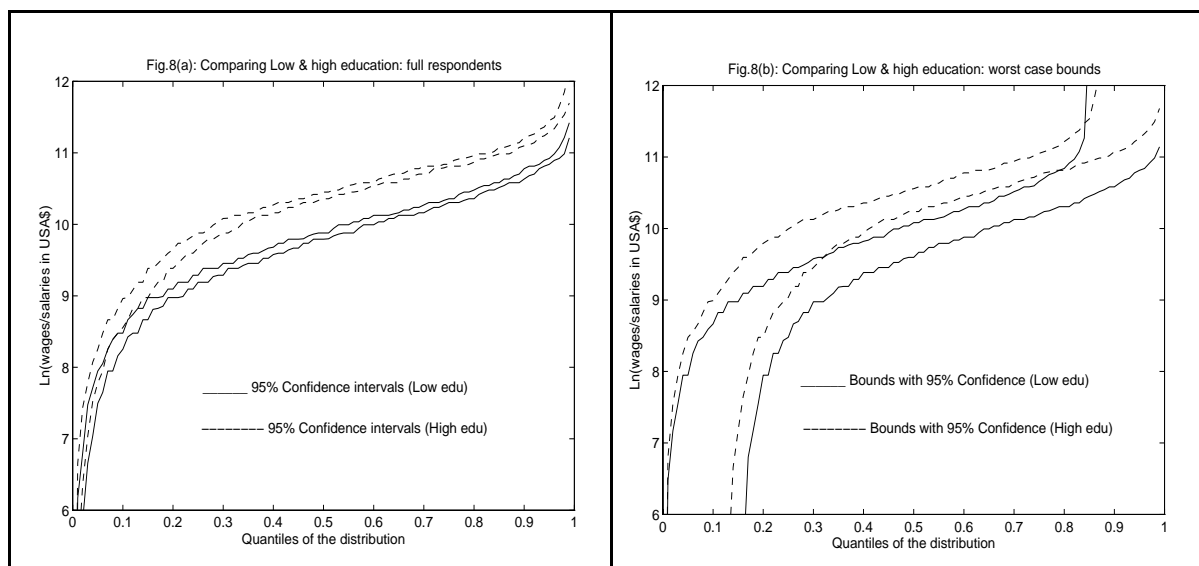


Table 10: Worst case bounds by education level not using bracket responses

Quantile	Low Education		High Education		Test Statistic
	Point estimate	Standard error	Point estimate	Standard error	
20 th Percentile	\$9,800	\$337	\$6,800	\$761	-3.6
25 th Percentile	\$11,900	\$408	\$9,800	\$1,050	-1.87
30 th Percentile	\$13,000	\$505	\$14,700	\$960	1.57
40 th Percentile	\$17,900	\$390	\$23,900	\$1,138	4.99
50 th Percentile	\$22,500	\$572	\$29,900	\$707	8.14
60 th Percentile	\$27,400	\$863	\$35,600	\$885	6.64
75 th Percentile	\$39,400	\$1,189	\$48,600	\$863	6.26
80 th Percentile	\$49,700	\$1,732	\$52,500	\$1,692	1.17

Note: The test statistic is $(Q_H - Q_L)/\Delta$, where Q_H is the lower bound point estimate for the high educated and Q_L is the upper bound point estimate for the low educated, and Δ is the estimated standard deviation of $Q_H - Q_L$. If the assumption that the lower bound of the high educated equals the upper bound of the low educated, the test is asymptotically normal. The null is rejected if the test statistic is larger than 1.645.

Information on bracket response of high and low educated respondents is included in Tables 11 and 12. High educated bracket respondents much more often report that their income exceeds the first bid than low educated bracket respondents. This suggests that using the bracket respondents may lead to more rejections in the tests for equality of quantiles.

Table 11: Bracket responses of the low educated (212 observations)

Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	\$50,000 — max	6
		Yes	> \$50,000 ?	No	\$25,000 — \$50,000	47
CBR	>\$25,000 ?			Yes	\$5,000 — \$25,000	133
		No	> \$ 5,000 ?	No	\$0 — \$5,000	22
		Yes	> \$50,000 ?	DK	> \$25,000	4
IBR	>\$25,000 ?			DK	< \$25,000	0

Note: See Table 5 for explanation.

Table 12: Bracket responses of the high educated (117 observations)

Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	\$50,000 — max	24
		Yes	> \$50,000 ?	No	\$25,000 — \$50,000	39
CBR	>\$25,000 ?			Yes	\$5,000 — \$25,000	37
		No	> \$ 5,000 ?	No	\$0 — \$5,000	12
		Yes	> \$50,000 ?	DK	> \$25,000	5
IBR	>\$25,000 ?			DK	< \$25,000	0

Note: See Table 5 for explanation.

Figures 13 and 14 compare the results for the low and high educated including the bracket information, allowing for selective nonresponse and making two different assumptions about anchoring: no anchoring (A0) and anchoring following Hurd et al. (1998) (A1). The results for the other two forms of anchoring lead to similar conclusions. The formal tests (of the null hypothesis that the upper bound of the lower educated is at least as high as the lower bound of the higher educated, against the alternative that this is not the case) are presented in Table 13. Under the no anchoring assumption, the differences between the quantiles in this table are all significant. Allowing for anchoring reduces some of the significance levels, and the lowest quantiles are no longer significantly different. But, in general, the point-wise tests reject much more often and with higher significance levels than if the bracket information is not used at all.

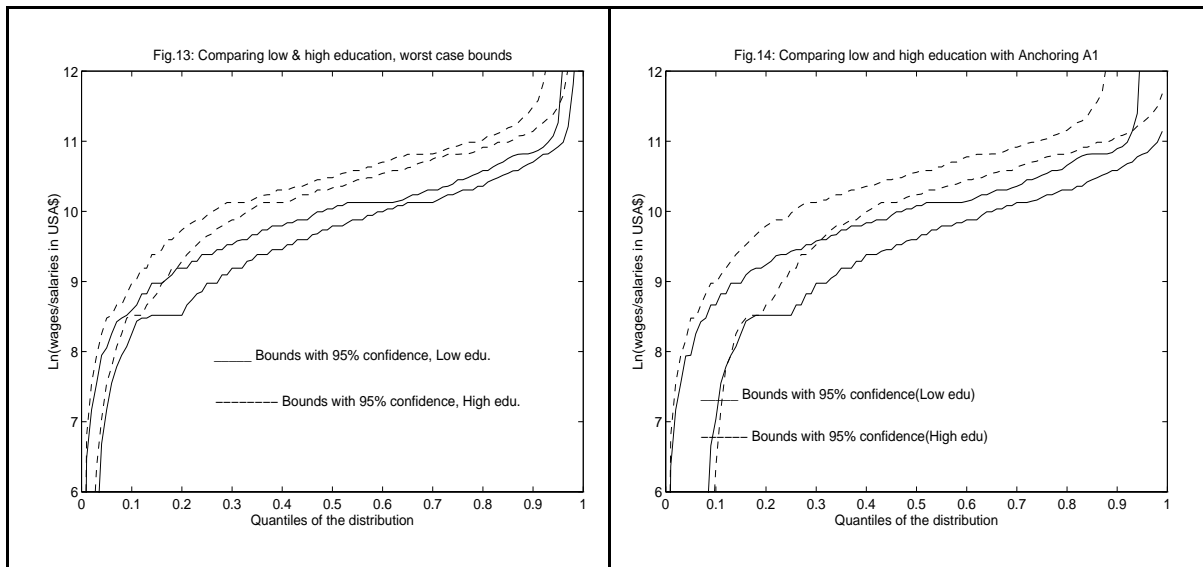


Table 13: differences between earnings quantiles of high and low educated respondents; worst case bounds with bracket responses

	No Anchoring (A0)			Anchoring following Hurd et al. (A1)		
	Low education	High education	Test Statistic	Low education	High education	Test Statistic
20 th Percentile	\$8,900	\$12,750	3.53	\$9,800	\$7,700	-2.21
25 th Percentile	\$10,800	\$17,500	6.17	\$11,900	\$11,900	0
30 th Percentile	\$12,750	\$20,800	6.54	\$13,000	\$15,800	2.19
40 th Percentile	\$17,500	\$25,900	7.98	\$17,900	\$25,000	7.42
50 th Percentile	\$21,900	\$32,500	9.29	\$22,500	\$29,900	7.42
60 th Percentile	\$25,000	\$39,400	21.58	\$25,000	\$35,600	11.31
75 th Percentile	\$31,500	\$50,000	13.21	\$34,600	\$48,600	10.31
80 th Percentile	\$36,900	\$56,400	10.79	\$39,400	\$52,500	6.42
90 th Percentile	\$50,000	\$76,000	9.70	\$50,000	\$66,000	9.24

Note: See Table 10 for the test statistic.

6 Conclusions

Manski’s approach to deal with item nonresponse avoids the assumptions usually associated with parametric and semi-parametric methods. On the other hand, it identifies the unknown parameters up to an upper and a lower bound only. In this paper, these bounds are extended to take account of the information contained in follow up categorical questions for initial nonrespondents. Such questions are included in many current household surveys. Several studies have shown that responses to such questions can be subject to response errors due to anchoring

effects. Some existing studies model this response error with a parametric set up. We have extended the bounds to allow for anchoring in a nonparametric way, starting from various nonparametric anchoring assumptions inspired by the existing parametric models. These bounds simultaneously allow for any type of selective nonresponse and various forms of anchoring.

Using the variable wages and salary of the household representative taken from the 1996 wave of the Household and Retirement Survey, the empirical section shows estimates of Manski's basic worst case bounds that do not use the bracket respondents information and compares these with estimates of the new bounds. For the wages and salaries variable considered, the initial nonresponse rate is 12.4%. Most of the initial nonrespondents answer unfolding bracket questions, and the percentage of full nonresponse is 3.3%. Incorporating information provided by bracket respondents tightens the bounds. Allowing for anchoring effects reduces the gain in information but still leads to bounds that are substantially more informative than the bounds not using the bracket information. This is illustrated by using the bounds to test for equality of quantiles of high and low educated respondents. Adding the information provided by bracket respondents improves the power of the tests, and leads to rejecting the null more often. How much the power of the tests increases depends on whether and how anchoring is allowed for.

Manski's bounds are an elegant, intuitively plausible and extremely flexible way to allow for selective nonresponse. Their flexibility is at the same time their main weakness: the bounds are often so wide that they do not provide enough information for the economic issue under consideration. This paper shows that additional information on bracket responses by initial nonrespondents can be useful to make the bounds more informative. This is still true if anchoring is allowed for, though to a lesser extent.

The bounds are estimated allowing for different types of anchoring each generalizing a different parametric model in the existing literature. The paper does not analyze which model of anchoring is most appropriate: this is not a relevant question for this framework. With the current data, however, selective nonresponse and anchoring interact, and it is hard to say something about anchoring without making strong assumptions about the nature of nonresponse. For an analysis of anchoring itself, therefore, experimental data where all respondents get bids that vary randomly across the sample, such as in the experimental HRS module used by Hurd et al. (1998), is more appropriate. With more knowledge about the nature of the anchoring process, the analysis here could be refined.

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Appendix: Bounds in Case of Two Bracket Questions with Anchoring

This appendix derives the bounds for the case of two bracket questions allowing for anchoring along the lines of Hurd et al. (1998), Jacowitz and Kahneman (1995), and Herriges and Shogren (1996). The bounds are “worst case” in the sense that any type of selective nonresponse or bracket response is allowed for. This implies that the data on full respondents carry no information on the bracket respondents, complete bracket respondents provide no information on incomplete bracket respondents, etc. Bounds on incomplete bracket respondents are straightforward, using the assumptions for the one bracket question case. This appendix focuses on complete bracket respondents. Using (24) and (4), these can be used to obtain bounds for the distribution in the complete population.

A1: The Hurd et al. (1998) model

It is easy to show that, apart from (18), Assumption 2 also implies the following monotonicity condition:

$$P(QI=1|Y<t,x,BR) \leq P(QI=1|x,BR) \quad \text{for each } t \quad (\text{MON})$$

Together with (11), (19), and (20), this property will be used to determine what the three probabilities $P(QI=1/x, BR)$, $P(Q20=1/QI=0,x, BR)$, and $P(Q21=1/QI=1,x, BR)$ that can be identified from the data, imply for the conditional distribution of Y given $X=x$ among bracket respondents. First, bounds are derived on the values of the distribution function at the bids $B20$, $B1$, and $B21$. The bounds on the value of the conditional distribution function at an arbitrary value y of Y then follow straightforwardly, as in the no anchoring case. For notational convenience, we abbreviate conditioning on $X=x$ and BR , using P_c where $P_c(\dots)=P(\dots/x, BR)$ and $P_c(\dots/\dots)=P(\dots/\dots,x, BR)$.

Upper bound on $P_c(Y<B20)$:

$$\begin{aligned} P_c(Y<B20) &= P_c(Y<B20|QI=0)P_c(QI=0) + P_c(Y<B20|QI=1)P_c(QI=1) \\ &\leq \min[1, 2P_c(Q20=0|QI=0)]P_c(QI=0) + P_c(Y<B20)\min[0.5, P_c(QI=1)] \end{aligned} \quad (\text{A.1})$$

Here (19) is used to obtain an upper bound on the first term; the second term uses $P_c(Y<B20|QI=1)P_c(QI=1)=P_c(Y<B20)P_c(QI=1/Y<B20)$ together with (18) and (MON). Thus,

$$Pc(Y < B20) \leq \min[1, 2Pc(Q20=0|QI=0)]Pc(QI=0)/(1 - \min[0.5, Pc(QI=1)]) \quad (\text{A.2})$$

Considering the two cases $Pc(QI=1) > 0.5$ and $Pc(QI=1) \leq 0.5$ separately, it is easy to see that (A.2) can also be written as

$$Pc(Y < B20) \leq \min[1, 2Pc(Q20=0|QI=0)]\min[1, 2Pc(QI=0)] \quad (\text{A.3})$$

Upper bound on $Pc(Y < B1)$:

Inequality (11) directly gives $Pc(Y < B1) \leq 2Pc(QI=0)$. The second question gives the following additional information.

$$\begin{aligned} Pc(Y < B1) &= Pc(Y < B1|QI=0)Pc(QI=0) + Pc(Y < B1|QI=1)Pc(QI=1) \\ &\leq Pc(QI=0) + Pc(Y < B21|QI=1)Pc(QI=1) \\ &\leq Pc(QI=0) + \min[1, 2Pc(Q21=0|QI=1)]Pc(QI=1) \end{aligned} \quad (\text{A.4})$$

Taken together, the first and second question lead to the bound

$$Pc(Y < B1) \leq \min[1, 2Pc(QI=0), Pc(QI=0) + 2Pc(Q21=0|QI=1)Pc(QI=1)] \quad (\text{A.5})$$

If $Pc(QI=0) \geq 0.5$ and $Pc(Q21=0|QI=0) \geq 0.5$, the upper bound in (A.5) is 1. If at least one of the two probabilities is less than 0.5, the upper bound is smaller than one.

Upper bound on $Pc(Y < B21)$:

$$\begin{aligned} Pc(Y < B21) &= Pc(Y < B21|QI=0)Pc(QI=0) + Pc(Y < B21|QI=1)Pc(QI=1) \\ &\leq Pc(QI=0) + \min[1, 2Pc(Q21=0|QI=1)]Pc(QI=1) \end{aligned} \quad (\text{A.6})$$

The lower bounds follow by symmetry from (A.3), (A.5), and (A.6):

Lower bound on $Pc(Y < B20)$:

$Pc(Y < B20) = 1 - Pc(Y \geq B20)$; an upper bound on $Pc(Y \geq B20)$ is obtained in the same way as the upper bound on $Pc(Y < B21)$ given in (A.6). This gives:

$$Pc(Y \geq B20) \leq Pc(QI=1) + \min[1, 2Pc(Q20=1|QI=0)]Pc(QI=0) \quad (\text{A.7})$$

And, thus,

$$\begin{aligned} Pc(Y < B20) &\geq 1 - Pc(QI=1) - \min[1, 2Pc(Q20=1|QI=0)]Pc(QI=0) \\ &= \max[0, \{1 - 2Pc(Q20=1|QI=0)\}Pc(QI=0)] \end{aligned} \quad (\text{A.8})$$

Lower bound on $Pc(Y < B1)$:

$Pc(Y < B1) = 1 - Pc(Y \geq B1)$; an upper bound on $Pc(Y \geq B1)$ is obtained in the same way as the upper bound on $Pc(Y < B1)$ given in (A.5)

$$Pc(Y \geq B1) \leq \min[2Pc(QI=1), Pc(QI=1) + 2Pc(Q20=1|QI=0)Pc(QI=0)] \quad (\text{A.9})$$

and (A.9) implies

$$\begin{aligned} Pc(Y < B1) &\geq 1 - \min[2Pc(QI=1), Pc(QI=1) + 2Pc(Q20=1|QI=0)Pc(QI=0)] \\ &= \max[1 - 2Pc(QI=1), Pc(QI=0)(1 - 2Pc(Q20=1|QI=0))] \end{aligned} \quad (\text{A.10})$$

Lower bound on $Pc(Y < B21)$:

$Pc(Y < B21) = 1 - Pc(Y \geq B21)$; an upper bound on $Pc(Y \geq B21)$ is obtained in the same way as the upper bound on $Pc(Y < B20)$ given in (A.3),

$$Pc(Y \geq B21) \leq \min[1, 2Pc(Q21=1|QI=1)] \min[1, 2Pc(QI=1)] \quad (\text{A.11})$$

and thus a lower bound is obtained as

$$Pc(Y < B21) \geq 1 - \min[1, 2Pc(Q21=1|QI=1)] \min[1, 2Pc(QI=1)] \quad (\text{A.12})$$

A2: The Jacowitz and Kahneman (1995) Assumption

In this case, expressions (15), (22) and (MON) are the basis for deriving the bounds. The sample analogues of $P_c(QI=1)$ and $P_c(Q2I=1|QI=1)$ in our case are smaller than 0.5, while that of $P_c(Q20=1|QI=0)$ is larger than 0.5. Thus $B1$ and $B2I$ are “large” and $B20$ is “small.” Ignoring sampling error, this means that (15) and (22) imply

$$\begin{aligned}
P_c(Y \geq B1) &\leq P_c(QI=1) \\
P_c(Y \geq B2I|QI=1) &\leq P_c(Q2I=1|QI=1) \\
P_c(Y \geq B20|QI=0) &\geq P_c(Q20=1|QI=0)
\end{aligned} \tag{JK}$$

Upper bound on $P_c(Y < B20)$:

$$\begin{aligned}
P_c(Y < B20) &= P_c(Y < B20|QI=0)P_c(QI=0) + P_c(Y < B20|QI=1)P_c(QI=1) \\
&\leq P_c(Q20=0|QI=0)P_c(QI=0) + P_c(Y < B20)P_c(QI=1|Y < B20) \\
&\leq P_c(Q20=0|QI=0)P_c(QI=0) + P_c(Y < B20)P_c(QI=1)
\end{aligned} \tag{A.13}$$

where (MON) was used in the last step. Rewriting (A.13) and dividing by $P_c(QI=0)$ yields

$$P_c(Y < B20) \leq P_c(Q20=0|QI=0) \tag{A.14}$$

Upper bound on $P_c(Y < B1)$:

None of the three assumptions in (JK) help to find a nontrivial upper bound, either directly or using the same decomposition used above. Thus, all that can be said is

$$P_c(Y < B1) \leq 1 \tag{A.15}$$

Upper bound on $P_c(Y < B2I)$:

This immediately follows from (A.15):

$$P_c(Y < B2I) \leq 1 \tag{A.16}$$

Lower bound on $P_c(Y < B20)$:

None of the three assumptions in **(JK)** help to find a nontrivial lower bound, so that all can be said is

$$Pc(Y < B20) \geq 0 \quad (\text{A.17})$$

Lower bound on $Pc(Y < B1)$:

The first assumption in **(JK)** immediately gives:

$$Pc(Y < B1) \geq Pc(Q1 = 0) \quad (\text{A.18})$$

The other two assumptions do not add anything here.

Lower bound on $Pc(Y < B21)$:

$$\begin{aligned} Pc(Y < B21) &= Pc(Y < B21 | Q1 = 0)Pc(Q1 = 0) + Pc(Y < B21 | Q1 = 1)Pc(Q1 = 1) \\ &\geq Pc(Y < B21)Pc(Q1 = 0 | Y < B21) + Pc(Q21 = 0 | Q1 = 1)Pc(Q1 = 1) \\ &\geq Pc(Y < B21)Pc(Q1 = 0) + Pc(Q21 = 0 | Q1 = 0)Pc(Q1 = 1) \end{aligned} \quad (\text{A.19})$$

where **(MON)** is used in the last step. Rewriting (A.19) and dividing it by $Pc(Q1 = 1)$ gives:

$$Pc(Y < B21) \geq Pc(Q21 = 0 | Q1 = 0) \quad (\text{A.20})$$

Moreover, the first inequality in **(JK)** directly implies

$$Pc(Y < B21) \geq Pc(Y < B1) \geq Pc(Q1 = 0) \quad (\text{A.21})$$

Combining (A.20) and (A.21) yields the lower bound

$$Pc(Y < B21) \geq \max[Pc(Q1 = 0), Pc(Q21 = 0 | Q1 = 0)] \quad (\text{A.22})$$

A3: The Herriges and Shogren (1996) Model

The assumptions about anchoring in this model can be summarized as

$$\begin{aligned}
P_c(Y < B1) &= P_c(Q1 = 0) \\
P_c(Y < B21 | Q1 = 1) &\leq P_c(Q21 = 0 | Q1 = 1) \\
P_c(Y < B20 | Q1 = 0) &\geq P_c(Q20 = 0 | Q1 = 0)
\end{aligned}
\tag{HS}$$

The derivations are much easier than in the previous two cases.

Upper bound on $P_c(Y < B20)$:

$$P_c(Y < B20) \leq P_c(Y < B1) = P_c(Q1 = 0) \tag{A.23}$$

Upper and lower bound on $P_c(Y < B1)$:

$$P_c(Y < B1) = P_c(Q1 = 0) \tag{A.24}$$

Upper bound on $P_c(Y < B21)$:

$$\begin{aligned}
P_c(Y < B21) &= P_c(Y < B21 | Q1 = 0)P_c(Q1 = 0) + P_c(Y < B21 | Q1 = 1)P_c(Q1 = 1) \\
&\leq P_c(Q1 = 0) + P_c(Q21 = 0 | Q1 = 1)P_c(Q1 = 1)
\end{aligned}
\tag{A.25}$$

Lower bound on $P_c(Y < B20)$:

$$\begin{aligned}
P_c(Y < B20) &= P_c(Y < B20 | Q1 = 0)P_c(Q1 = 0) + P_c(Y < B20 | Q1 = 1)P_c(Q1 = 1) \\
&\geq P_c(Q20 = 0 | Q1 = 0)P_c(Q1 = 0)
\end{aligned}
\tag{A.26}$$

The lower bounds on B1 and B21 are also given by (A.24). Thus, the set of nontrivial upper bounds is given by (A.23) to (A.25), whereas the nontrivial lower bounds are given by (A.26) and (A.24). Only (HS) is used, (MON) is not needed.