European Journal of Operational Research 59 (1992) 405–412
North-Holland

Theory and Methodology

Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs

M.J.G. van Eijs, R.M.J. Heuts and J.P.C. Kleijnen

Tilburg University, Postbox 90153, 5000 LE Tilburg, Netherlands

Received April 1990; revised August 1990

Abstract: Multi-item inventory systems with joint replenishment costs are studied for constant deterministic demand. Two different types of strategies are distinguished: direct grouping strategies and indirect grouping strategies. For these types of strategies different heuristics are reviewed. The performances of the strategies is measured as the percentage cost savings of a joint replenishment strategy relative to an independent strategy. These performances are quantified through simulation. The input–output behaviour of several simulation experiments is summarized by regression analysis.

Keywords: Multi-item inventory systems, heuristics, simulation, regression, experimental design

1. Introduction

Joint replenishment strategies can be used in multi-item inventory systems. A characteristic of multi-item systems is the existence of some kind of interaction among items. Joint replenishment strategies are based on the interaction of the *set-up or order costs*. These costs can be subdivided into major and minor costs. Interaction arises because the major set-up cost is independent of the number of items in the replenishment. In addition to the major set-up cost, there is a minor set-up cost, charged to each particular item included in the replenishment. Cost savings can be obtained by coordinating the replenishments of several items: the major set-up cost is shared if two or more items are jointly replenished. In many practical situations it makes sense to coordinate replenishments of individual items. If several items are purchased from the same supplier, the fixed order cost can be shared by replenishing two or more items jointly. Joint replenishments may also be attractive if a group of items use the same vehicle or the same machine.

In the case of constant demands, the strategies can be classified into two classes, which will be called 'indirect grouping strategies' and 'direct grouping strategies'. Both classes of strategies assume that the replenishment cycle, which is the time between two subsequent replenishments of an individual item, is constant. A group is defined as the set of those items that have the same replenishment cycle. Consequently, items of the same group are jointly replenished.

Under an *indirect* grouping strategy, a family replenishment is made at constant intervals. The replenishment cycle of each item (or group) is an integer multiple of this basic cycle time. The problem is to determine the basic cycle time and the replenishment frequencies of all items simul-

0377-2217/92/$05.00

taneously. A group is then (indirectly) formed by those items that have the same replenishment frequency. In the last two decades several authors have studied this sort of joint replenishment problem. For extended reviews we refer to Aksoy and Erenguc (1988) and Goyal and Satir (1989). A different type of strategy which is not mentioned in these surveys is a *direct* grouping strategy. Here, the replenishment cycles of the groups are not an integer multiple of a basic cycle, so the family replenishments are not equally spaced. In this case the problem is to form (directly) a predetermined number of groups in such a way that the total costs of the items in the family are as low as possible.

One might conjecture that indirect grouping strategies outperform direct grouping strategies for high major set-up cost, because different groups are jointly replenished when using an indirect grouping strategy. However, indirect grouping strategies are less flexible in setting replenishment cycles, since these cycles are restricted to integer multiples of the basic cycle time. One can imagine that direct grouping strategies outperform indirect grouping strategies when the savings from coordination are low (small major set-up cost). To the best of our knowledge, a comparison between the class of indirect grouping and direct grouping strategies has never been made. The purpose of our study is twofold: first, to find out whether there is a threshold value of the major set-up cost above which it makes sense to use an indirect grouping strategy; secondly, to determine the effects of some factors in the performance evaluation of joint replenishment strategies. Performance is measured as the percentage cost savings when a joint replenishment strategy is used instead of an independent strategy.

The paper is organised as follows: Section 2 gives a short review of the literature on joint replenishments, and discusses the kind of algorithms for direct grouping and indirect grouping. Section 3 describes the experimental design and simulation results. Section 4 gives conclusions.

2. Literature review

The joint replenishment problem has been investigated under a set of assumptions that are the same as those for the classical economic order quantity (EOQ) model, except for the major set-up cost. Because of these assumptions the relevant costs are the set-up costs and the carrying inventory cost. We review the literature for both grouping strategies.

Table 1
Glossary of most important symbols

A:	major set-up cost
a_i:	minor set-up cost for item i
D_i:	demand per period for item i
h_i:	inventory carrying cost per unit of item i per period
k_i:	the number of basic cycles between two successive replenishments of item i
M:	number of groups to be formed
N:	number of items in the family
S:	set of all items in the family
S_j:	set of items in group j
T:	basic cycle time, the time between two successive family replenishments
T_j:	replenishment cycle of group j, the time between two successive replenishments of group j
TRC:	total carrying plus set-up cost of the family per period (subscripts refer to the strategy used)
y_s:	percentage cost savings of joint replenishment strategy s relative to an independent strategy

2.1. Indirect grouping

The decision variables in the indirect grouping model are T, the basic cycle time (the time between two successive family replenishments), and k_i, the number of basic cycles between two successive replenishments of item i, with $i = 1,\dots, N$ where N is the number of items in the family. The objective is to find combinations (T, k_i ($i = 1,\dots, N$)) such that TRC, the total relevant cost of the family, is as low as possible. Let A denote the major set-up cost, a_i the minor set-up cost of item i, D_i the demand per period for item i, and h_i the inventory carrying cost per unit of item i per period (the symbols are also shown in the glossary of Table 1), then

$$\mathrm{TRC} = \frac{1}{T}\left(A + \sum_{i=1}^{N} \frac{a_i}{k_i}\right) + \tfrac{1}{2}T\sum_{i=1}^{N} k_i D_i h_i \qquad (1)$$

s.t. $k_i \in \{1, 2, 3,\dots\}$.

By taking the first derivative of TRC with respect to T and k_i (k_i is then treated as a continuous variable), we can derive the optimal T^* and the optimal k_i^*. However, T^* can not be determined without knowing k_i^*, and vice versa. Several authors have encountered this problem: Brown

(1967), Goyal (1973a, 1973b, 1974a, 1974b, 1988), Goyal and Belton (1979), Silver (1979), Kaspi and Rosenblatt (1983, 1985), etc. Only one of them (Goyal, 1974a) presented an (enumerative) algorithm that gives the global optimum under the assumption that the actual family replenishments are equally spaced. In this case at least one k_i must be equal to one. So Goyal's approach results in an optimal solution but it may be computationally prohibitive. Therefore, heuristic algorithms were developed. These heuristics may be classified into two classes: iterative algorithms and noniterative algorithms. It is not our intention to give a detailed review of the literature. We refer back to the extensive surveys of Aksoy and Erenguc (1988) and Goyal and Satir (1989).

2.2. Direct grouping strategies

The main difference between indirect grouping and direct grouping strategies is that the replenishment cycles of the groups formed by indirect grouping are multiple integers of some basic cycle time, whereas this is not the case for groups formed by direct grouping. Note that the number of groups is an output variable in indirect grouping, whereas the number of groups (denoted by M) is predetermined in direct grouping. Let S_j denote the set of items in group j, then the direct grouping problem is to divide S, the set of all items in the family, into M disjunct sets S_j, with $j = 1, \dots, M$. If the time between two successive replenishments of all items in group j is denoted by T_j, then

$$\mathrm{TRC} = \sum_{j=1}^{M} \left(\frac{A + \sum_{i \in S_j} a_i}{T_j} + \tfrac{1}{2} T_j \sum_{i \in S_j} D_i h_i \right). \qquad (2)$$

The problem of dividing N items into M groups is hard, because there may be numerous combinations. Fortunately, Chakravarty (1981) and later Bastian (1986) proved a theorem that they called the 'consecutiveness property'. This property means that when the items are arranged in increasing order with respect to the ratio $D_i h_i / a_i$, then the optimal groups can be created from this sequential list. For example, consider a set of items {1, 2, 3, 4}, which is arranged in increasing order of the ratio $D_i h_i / a_i$ (so, item 1 is the item with the smallest ratio). In this case, the groups $S_1 = \{1, 2\}$ and $S_2 = \{3, 4\}$ can be optimal, but $S_1 = \{1, 3\}$ and $S_2 = \{2, 4\}$ cannot.

Using this ranking scheme, several authors proposed algorithms for direct grouping: Page and Paul (1976), Chakravarty (1981, 1985), and Bastian (1986). We note that in the original papers of Page and Paul, and Chakravarty, the major set-up cost is not incorporated explicitly. The algorithms can be adjusted easily for the major set-up cost, except for that of Chakravarty (1985). Chakravarty (1981) uses dynamic programming to create groups. This algorithm identifies the global minimum of (2). However, computer time increases exponentially with the size of the problem. After analyzing the heuristics of Bastian (1986), Page and Paul (1976) and Chakravarty (1985), we found that Bastian's algorithm was the best heuristic with respect to both costs and complexity. This simple heuristic starts with N consecutive groups (an individual item forms a group). Each iteration combines two neighbouring groups such that the increase (decrease) of the objective function is minimal (maximal). The procedure terminates as soon as M groups are formed. Bastian proved that this grouping heuristic is optimal when the major set-up cost is zero.

2.3. Choice of algorithms for comparison

In this section we select an indirect grouping and a direct grouping algorithm for our comparison of the two different types of strategies. The selection is based on complexity, deviation from the optimal solution and computer time needed.

2.3.1. Indirect grouping

Because the optimal solution method of Goyal (1974a) is complex and computationally prohibitive for large-size problems, we use a heuristic method. In a simulation study, Kaspi and Rosenblatt (1985) compared iterative algorithms due to Brown (1967) and Goyal (1974b), and noniterative algorithms due to Silver (1976), Goyal and Belton (1979), and Kaspi and Rosenblatt (1983). They also suggested a combined approach that uses the noniterative heuristic of Silver (1976) with the modification of Goyal and Belton (1979) as starting point in the iterative algorithm of Goyal (1974b). The heuristic with the smallest

Table 2
Average computer time of 500 runs (expressed in milli-seconds)

N	Indirect grouping		Direct grouping	
	Iterative [a]	Noniterative [b]	Optimal [c]	Heuristic [d]
10	2.2	1.4	3.0	2.4
20	4.9	2.8	12.1	5.9
40	11.0	5.5	45.9	13.3
80	25.0	11.2	187.0	34.1

[a] Combined approach based on Goyal (1974b).
[b] Algorithm of Kaspi and Rosenblatt (1983).
[c] Algorithm of Chakravarty (1981).
[d] Algorithm of Bastian (1986).

average deviation from the optimal solution turned out to be the combined approach, followed by that of Goyal, Brown, Kaspi and Rosenblatt, Goyal and Belton, and finally that of Silver. We compare the computer time of the best iterative algorithm (the combined approach) with the best noniterative algorithm (Kaspi and Rosenblatt). For each item $D_i h_i$ and a_i are randomly generated where $D_i h_i$ and a_i are uniformly distributed respectively over the range [200; 1800] and [7.5; 12.5]. Different values of A and N are considered. The computer times on a VAX-8700-computer for $A = 20$ are tabulated in Table 2. Although the computer time for the iterative heuristic is twice as much as for the single-iteration heuristic, we conclude that this difference is not important in an absolute sense. Consequently, we use the iterative algorithm.

2.3.2. Direct grouping

Test examples show that the differences between Bastian's heuristic solution and Chakravarty's optimal solution are very small, even with a high major set-up cost. We also consider the computer time needed for both algorithms using the same problem settings as for the indirect grouping algorithms. To avoid the effect of the number of groups (M), we fix M at the value five. The computer times for $A = 20$ are also given in Table 2. As expected, the differences in computer time are important. For these two reasons we use the heuristic of Bastian.

After simulating various inventory situations we conclude that direct grouping algorithms take more computer time than indirect grouping algorithms. However, the difference between the computer times required by the iterative indirect grouping algorithm and the heuristic direct grouping algorithm is small.

3. Experimental design and simulation results

Several inventory situations with constant demands are simulated to compare the performances of direct grouping and indirect grouping strategies. We analyse the differences between these two ways of grouping, and compare the performances of the strategies with the performance of an independent single-item strategy. We use regression analysis to summarise the output of several simulation runs.

Kleijnen (1987) gives the following hierarchical modelling approach: (a) determine the response or criterion variable of the study; (b) determine the independent variables or factors; (c) construct a regression metamodel (a cause–effect relation between the response variable and the independent variables of the simulation); (d) determine the experimental design (the situations that will be simulated); and (e) estimate the regression parameters and validate the metamodel. When the model is not valid, Step (b) or (c) is repeated; otherwise conclusions can be drawn.

Several authors have used simulation to study joint replenishment models. Goyal and Satir (1989, p. 11) list some simulation studies. A popular response variable is the average cost savings of a joint replenishment strategy expressed as a percentage of the total cost of an independent strategy. This is a dimensionless variable, which we denote by y_s. So if $\mathrm{TRC}_{\mathrm{eoq}}$ denotes the total cost of the family of items under an independent EOQ-strategy, and TRC_s is the total cost of joint replenishment strategy s, then

$$y_s = 100 \cdot \frac{\mathrm{TRC}_{\mathrm{eoq}} - \mathrm{TRC}_s}{\mathrm{TRC}_{\mathrm{eoq}}}. \tag{3}$$

Potential cost factors in the joint replenishment problem are: the major set-up cost (A), the minor set-up cost (a_i), and the inventory carrying cost of stocking the periodic demand of item i for one period ($D_i h_i$). This factor consists of the demand for item i per period D_i, and the inventory carrying cost per unit per period h_i, which in turn is a constant percentage h of the unit cost v_i; so $h_i = h v_i$. Other potential factors are: the number of items in the family (N), the number of groups to be formed (M), and the joint replenishment strategy s which is used.

Instead of blindly incorporating all these factors in a full-fledged simulation experiment, these factors are first examined in pilot experiments. After an extensive analysis, it turns out that only two factors must be included in the metamodel:

(i) Instead of the individual $D_i h_i$ and a_i, we can use the means $\overline{Dh}$ and $\bar{a}$ in the analysis of the simulation. In the remainder of this study the bar over a and Dh will be deleted to simplify the notation.

(ii) Instead of the major set-up cost (A) and minor set-up cost (a) separately, we use their ratio (A/a). It can easily be shown that a different combination of the major set-up cost (A) and the minor set-up cost (a) with an equal set-up ratio (A/a) indeed yields the same value of the response variable y_s; see Van Eijs et al. (1990, p. 18).

(iii) It is easy to prove that an increase of Dh does not affect the response variable y_s, all other things being equal. Therefore, the factor Dh is not a separate factor in the simulation.

(iv) The number of groups is an input variable in direct grouping, whereas it is an output variable in indirect grouping. Therefore, we change Bastian's direct grouping algorithm a little: the algorithm does not terminate when M groups are formed; instead it terminates when the objective function starts to increase when combining two neighbouring groups. In this way the number of groups is not a factor any more.

(v) After performing several pilot experiments we concluded that the set-up cost ratio (A/a) and the number of items (N) are the only important factors. We also incorporated other factors such as $(A+a)/Dh$, the variance of Dh, and the variance of the minor set-up cost a, but these factors were not important. In the remainder of this study we concentrate on these two factors: the set-up cost ratio (A/a) and the number of items (N).

A graphical analysis of the pilot experiments showed that an increase of the set-up cost ratio yields decreasing returns to scale and so does the number of items. Therefore we specify a regression metamodel with decreasing marginal procentual cost savings for the variables A/a and N. Possible metamodels with decreasing marginal procentual cost savings are quadratic models, square root models, logarithmic models, and reciprocal models. All these models are linear in the parameters, so we can apply linear regression analysis to estimate the parameter vector β of these regression models.

Table 3
Factors and values

Factor	Levels					
A/a	1	2	4	8	12	16
N	10	20	30	60		

By definition, an experimental design determines which combinations of factor values are simulated. The choice of the experimental design is affected by the metamodel. Since in our case there are only two factors, a full factorial design can be used. The factor A/a is varied over six values; the factor N over four values; see Table 3.

So, there are 24 different combinations. Every combination is simulated for both joint replenishment strategies, which gives $24 * 2$ responses (percentage cost savings of both strategies relative to an EOQ-strategy).

Given a certain combination of A/a and N, the simulation program generates particular inventory situations: the number of items (N), the major set-up cost (A), and the individual values of a_i and $D_i h_i$. Individual values of a_i and $D_i v_i$ are randomly generated from a uniform distribution on the intervals [1; 5] and [1000; 9000], respectively. $D_i h_i$ is obtained by multiplying $D_i v_i$ by the given carrying charge ($h = 0.2$); the major set-up cost is selected such that A/a is equal to the given value (thus $A = 3 \cdot A/a$). So we use sampling to generate a situation, but once a situation has been created, the inventory problem is deterministic. Both direct grouping and indirect grouping are applied to the same inventory situation. Consequently, the responses (y_s) of different joint replenishment strategies s are based on the same random numbers. Each factor combination is replicated 500 times (a_i and $D_i v_i$ differ, whereas N, A and h are fixed). The performances of the strategies for the given factor combination is then measured by the cost savings (in %) averaged over 500 replications.

The simulation output of the 24 factor combinations is summarized by regression analysis. Since common random numbers have been used, the linear metamodels are estimated with Estimated Generalised Least Squares; also see Kleijnen (1987). We validate the models with Rao's lack of fit test (1959), Kleijnen's cross validation test (1988), and interpolation. We find that a logarithmic model fits and predicts the simulation data well within the range over which the two factors are varied. This yields eqs. (4) and (5), where standard errors are shown in parentheses; $\hat{y}_{dg}$ denotes the cost savings (in %) of Bastian's direct grouping algorithm, and $\hat{y}_{ig}$ denotes the cost savings (in %) of the combined indirect grouping algorithm:

$$\hat{y}_{dg} = 6.6588 + 15.9710 \cdot \ln(A/a) + 5.6209 \cdot \ln N \qquad (4)$$
$$\qquad\qquad (1.E\text{-}05) \qquad\qquad (2.3E\text{-}04)$$

$$\hat{y}_{ig} = 6.3064 + 15.7797 \cdot \ln(A/a) + 5.9964 \cdot \ln N. \qquad (5)$$
$$\qquad\qquad (1.6E\text{-}05) \qquad\qquad (2.2E\text{-}04)$$

The interaction between the variables is not significant. We use Rao's F-test (1959) for linear hypotheses to see if the effects of the independent variables are equal for both strategies. All coefficients differ significantly, because the standard errors were virtually zero.

Figures 1 and 2 show the predicted responses $\hat{y}_{ig}$ and $\hat{y}_{dg}$ as a function of the cost set-up ratio and the number of items, respectively. Over the observed factor ranges of Table 3, the indirect grouping strategy always performs better than the direct grouping strategy does, but the difference is small. So the coefficients in (4) and (5) differ significantly but not importantly. The estimate of β shows that all coefficients in (4) are higher than in (5) except for the coefficient of N. So, the better performance of the indirect grouping strategy depends on the effect of the number of items in the family.

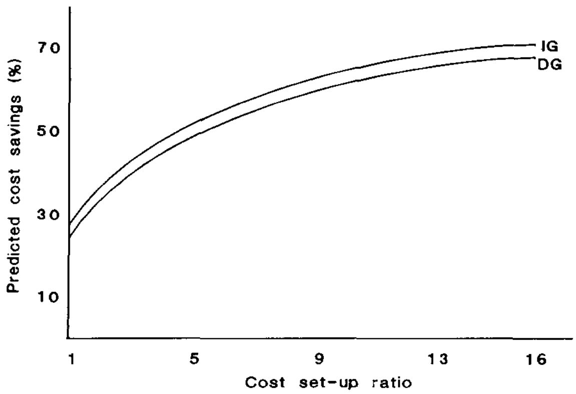

Figure 1. Predicted costs savings $\hat{y}_{dg}$ and $\hat{y}_{ig}$ as a function of the set-up cost ratio, given $N = 20$. DG = direct grouping, IG = indirect grouping.

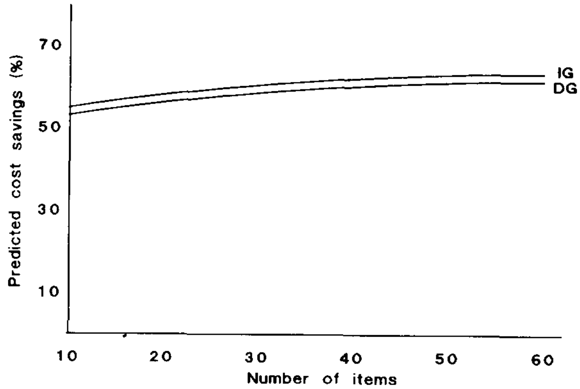

Figure 2. Predicted cost savings $\hat{y}_{dg}$ and $\hat{y}_{ig}$ as a function of the number of items in the family given $A/a = 8$. DG = direct grouping, IG = indirect grouping.

It is not possible to extrapolate the logarithmic model to the left of the observed range, since for values of A/a smaller than one, the variable $\ln(A/a)$ will be negative. Extrapolation to the right of the observed range may result in responses $\hat{y}_i$ larger than 100, which is impossible; see (3). So the metamodel is only valid for situations within the observed ranges.

Next, various situations are simulated with a set-up cost ratio larger than sixteen, the upper limit of the range in Table 3. Part a of Table 4 shows that the responses grow very slowly with an increasing set-up cost ratio when the ratio is higher than twenty-five. When the ratio is larger than seventy-five, the direct grouping and indirect grouping strategy become identical, because only one group is created.

We already mentioned that indirect grouping strategies perform slightly better than direct grouping strategies within the observed range of Table 3. Table 4 (part b) shows that for very small values of the set-up cost ratio, direct grouping strategies perform better than indirect grouping strategies. With a set-up cost ratio of 0.01, the

Table 4
Simulations with $A/a > 25$ and $A/a < 1$ [a]

Part a: Simulations with $A/a > 16$ ($N = 20$)			Part b: Simulations with $A/a < 1$ ($N = 20$)			Part c: Threshold value of A/a	
Set-up cost ratio (A/a)	Cost savings (%)		Set-up cost ratio (A/a)	Cost savings (%)		Number of items (N)	Threshold value A/a
	Direct [1] grouping	Indirect [2] grouping		Direct [1] grouping	Indirect [2] grouping		
25.00	69.34	69.44	0.01	0.28	−0.56	10	0.08
50.00	72.73	72.74	0.05	1.78	1.33	20	0.14
75.00	73.94	73.94	0.10	3.66	3.50	30	0.20
100.00	74.59	74.59	0.25	8.87	9.24	40	0.30
500.00	76.25	76.25	0.50	15.76	16.56	50	0.42
1000.00	76.49	76.49	0.75	21.24	22.26	60	0.56

[a] 1 = Heuristic algorithm of Bastian (1986).
2 = Combined approach based on Goyal (1974b).

indirect grouping strategy performs even worse than the independent strategy, because the replenishment cycles of the groups are restricted to an integer multiple of the basic cycle time. In this case the extra carrying cost is greater than the major set-up cost saved. In these situations, however, a joint replenishment strategy does not make much sense.

One of the purposes of our study was to find a threshold value of the major set-up cost (relative to the average minor set-up cost) above which indirect grouping outperforms direct grouping. From Table 4 (part b) it follows that the threshold value of the set-up cost ratio is between 0.10 and 0.25 for $N = 20$. The threshold value of A/a for different values of n is tabulated in part c of Table 4.

4. Conclusions

In this paper we investigated two types of joint replenishment inventory strategies, namely indirect and direct grouping strategies, assuming constant demands. We reviewed optimal and heuristic solution procedures for both types of strategies. For reasons of complexity and computer time we selected heuristic methods to represent direct and indirect grouping strategies. Direct grouping algorithms turned out to consume more computer time than indirect grouping algorithms.

We presented a simulation design to study the effect of some factors that were expected to be important. The performances of the strategies were measured as the percentage cost savings of a joint replenishment strategy relative to an independent strategy. After some pilot experiments we concluded that only two factors are important, namely: (i) the ratio of the major set-up cost (A) to the mean minor set-up cost (a), and (ii) the number of items in the family (N). Regression analysis was used to model the input–output behaviour of the simulation experiments with these two factors. A logarithmic model fitted and predicted the experimental data well within the range over which the two factors were varied. We also performed some extra simulation experiments outside the observed range.

The logarithmic metamodel showed that over the observed range of the experiments the indirect grouping strategy always outperforms the direct grouping strategy. The differences between the responses are, however, very small. The better performance of the indirect grouping strategy depends on the effect of the number of items in the family. The cost savings increase only slightly when the set-up cost ratio becomes larger than fifty. If the ratio is larger than seventy-five; only one group is created, and the direct grouping and indirect grouping strategy become identical. The threshold value of the set-up cost ratio under which direct grouping strategies outperform indirect grouping strategies is very small. If the set-up cost ratio is smaller than this threshold, then a joint replenishment strategy does not make much sense.

So when it makes sense to replenish items jointly, we recommend an indirect grouping strategy, since: (i) the indirect grouping strategies outperform the direct grouping strategies slightly, and (ii) the indirect grouping algorithms need less computer time than the direct grouping algorithms.

References

Aksoy, Y., and Erenguc, S. (1988), "Multi-item inventory models with coordinated replenishments: A survey", *Inventory Journal of Production Management* 8, 63–73.

Bastian, M. (1986), "Joint replenishment in multi-item inventory systems", *Journal of the Operational Research Society* 37, 1113–1120.

Brown, R.G. (1967), *Decision rules for inventory management*, Holt, Reinhart & Winston, New York.

Chakravarty, A.K. (1981), "Multi-item inventory aggregation into groups", *Journal of the Operational Research Society* 32, 19–26.

Chakravarty, A.K. (1985), "An optimal heuristic for coordinated multi-item inventory replenishments", *Journal of the Operational Research Society* 36, 1027–1039.

Goyal, S.K. (1973a), "A method for improving joint replenishment systems with a known frequency of replenishment orders", *International Journal of Production Research* 11, 195–200.

Goyal, S.K. (1973b), "Determination of the economic packaging frequency of items jointly replenished", *Management Science* 20, 232–235.

Goyal, S.K. (1974a), "Determination of optimum packaging frequency of items jointly replenished", *Management Science* 21, 436–443.

Goyal, S.K. (1974b), "Optimum ordering policy for a multi-item single supplier system", *Operational Research Quarterly* 25, 293–298.

Goyal, S.K. (1988), "Economic ordering policy for jointly replenished items", *International Journal of Production Research* 26, 1237–1240.

Goyal, S.K. and Belton, A.S. (1979), "On a simple method of determining order quantities joint replenishments under deterministic demand", *Management Science* 25, 604.

Goyal, S.K., and Satir, A.T. (1989), "Joint replenishment inventory control: Deterministic and stochastic models", *European Journal of Operational Research* 38, 2–13.

Kaspi, M., and Rosenblatt, M.J. (1983), "An improvement of Silver's algorithm for the joint replenishment problem", *IIE Transactions* 15, 264–267.

Kaspi, M., and Rosenblatt, M.J. (1985), "The effectiveness of heuristic algorithms for multi-item inventory systems with joint replenishments costs", *International Journal of Production Research* 23, 109–116.

Kleijnen, J.P.C. (1987), *Statistical tools for simulation practitioners*, Marcel Dekker, New York.

Kleijnen, J.P.C. (1988), "Analyzing simulation experiments with common random numbers", *Management Science* 34, 65–74.

Page, E., and Paul, R.J. (1976), "Multi-product inventory situations with one restriction", *Operational Research Quarterly* 27, 815–834.

Rao, C.R. (1959), "Some problems involving linear hypotheses in multivariate analysis", *Biometrica* 46, 49–58.

Silver, E.A. (1976), "A simple method of determining order quantities in joint replenishments under deterministic demand", *Management Science* 22, 1351–1361.

Van Eijs, M.J.G., Heuts, R.M.J., and Kleijnen, J.P.C. (1990), "Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs", Report No. FEW 436, Research Memorandum, Department of Economics, Tilburg University, Netherlands.