European Economic Review 20 (1983) 41–57. North-Holland Publishing Company

THE EFFECTS OF AGE AND UNEMPLOYMENT PERCENTAGE ON THE DURATION OF UNEMPLOYMENT

Evidence from Aggregate Data

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Received December 1980, final version received October 1981

In this paper models for the duration of unemployment are estimated using aggregate data on incomplete unemployment spells. In particular the elasticities of the probability of leaving unemployment with respect to age and unemployment percentage are estimated. Special attention is paid to the time dependence of the re-employment probability and to the effect of omitted regressors. Because models are fitted for male and female unemployed separately, these groups can be compared. We find that their position on the labour market is different.

1. Introduction

Recently much work has been done on the specification and estimation of models for the duration of unemployment [Salant (1977), Lancaster (1979), Nickell (1979), Lancaster and Nickell (1980), Kiefer and Neumann (1979)]. With these models which are based on the theory of jobsearch [see Lippman and McCall (1976)] the effects of variables on the probability of leaving unemployment can be studied. Hitherto most estimates were obtained using a sample of unemployed individuals. In this paper models similar to those used by Lancaster (1979) are estimated from aggregate data about the numbers unemployed. In the Netherlands data on the duration of unemployment have not been extensively analysed. As far as we know the pioneering study by Siddré c.s. (1976), in which mean complete durations of unemployment are estimated for several groups of the labour force, is the only one which is, in this respect, comparable to the present paper.

The paper is organized as follows. In section 2 the specification of models for the duration of unemployment is considered. Section 3 is devoted to a description of the data. In section 4 the estimation results are presented and in section 5 some conclusions are drawn.

2. The model

2.1. *Theoretical foundations: Search theory*

Models for the duration of unemployment can be specified using results

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from the economic theory of jobsearch. In the economic theory of jobsearch an unemployed person is solving a sequential decision problem. More specifically, he is confronted with a distribution of wage rates corresponding to available job openings. From this distribution wage rates are drawn, possibly at a stochastically determined rate. Each time the unemployed person receives a job offer, he has to decide whether to accept it or to continue searching. If he decides to stay unemployed he foregoes some income (the difference between the offered wage and the unemployment benefit). Moreover searching itself is not costless.

The unemployed person balances the costs of continued unemployment and obtaining another job offer against the possible income gain from continued search. The resulting solution of the sequential decision problem is to choose a wage level such that the marginal cost of another period of search is equal to the marginal income gain in this period. This wage level is called the reservation wage of the unemployed person. The optimal strategy is then to accept a job offer if the corresponding wage rate exceeds the reservation wage and to refuse it if it falls short of the reservation wage [for an extensive discussion of jobsearch models the reader is referred to Lippman and McCall (1976)].

2.2. *Jobsearch and the probability of leaving unemployment*

It would be natural to study jobsearch behaviour of unemployed persons using data on reservation wages. Unfortunately such data are, at least for the Netherlands, not available. Therefore to make inferences about jobsearch behaviour we have to find a variable that is dependent on the reservation wage and also is directly or indirectly observable. Following Lancaster (1979) and Nickell (1979) we choose for this quantity the probability of leaving unemployment.

The relation between the reservation wage and the probability of leaving unemployment can be seen as follows. Consider a person indexed by i who is unemployed for t periods, during an infinitesimal short time interval $[t, t+\mathrm{d}t)$. The probability that during this interval he receives exactly one job offer is $\mu_i(t)\,\mathrm{d}t$, the probability of receiving more than one offer is zero. If the distribution of wage rates corresponding to possible job offers is given by the distribution function F_i, then the probability that he leaves unemployment during $[t, t+\mathrm{d}t)$, given that he has been unemployed for t periods, is

$$\mu_i(t)(1-F_i(\tilde{w}_i(t)))\,\mathrm{d}t, \tag{1}$$

where $\tilde{w}_i(t)$ is the reservation wage of i after t periods of unemployment. It is clear from (1) that, without data on reservation wages it is impossible to distinguish the effect of a variable on the probability of receiving a job offer from the effect of that variable on the probability that the wage offer exceeds the reservation wage. We therefore write for the probability of leaving unemployment

$$\lambda_i(t)\,\mathrm{d}t, \tag{2}$$

where a functional form for $\lambda_i(t)$ remains to be chosen. With reference to renewal theory we also call $\lambda_i(t)$ the hazard or failure rate of the process that governs the flow out of unemployment.

2.3. Complete and incomplete spells of unemployment

The probability of leaving unemployment determines completely the distribution of the length of a complete spell of unemployment, $\underline{t}_i$ (random variables are underlined). When G denotes the distribution function of $\underline{t}$ (for simplicity the index i is omitted), then

$$G(t)=1-\exp\left\{-\int_0^t \lambda(s)\,\mathrm{d}s\right\}, \tag{3}$$

[see Lancaster (1979)],

$$g(t)=\lambda(t)\exp\left\{-\int_0^t \lambda(s)\,\mathrm{d}s\right\}. \tag{4}$$

Given data on complete spells of unemployment one could use this density function to study the probability of leaving unemployment. However, lacking data on complete spells in this paper data on incomplete spells of unemployment are analysed. Therefore we have to derive the distribution of incomplete spells using the distribution of complete spells given in (3).

Consider an individual who is unemployed at time 0. The probability that at time 0 the elapsed duration of unemployment is equal to t is

$$\begin{aligned}&\Pr(\text{into U at } -t \text{ and } \underline{t}\geqq t \mid \text{U at time } 0)\\ &=\frac{\Pr(\text{into U at } -t \text{ and } \underline{t}\geqq t)}{\int_0^\infty \Pr(\text{into U at } -s \text{ and } \underline{t}\geqq s)\,\mathrm{d}s},\end{aligned} \tag{5}$$

[see Nickell (1979)], where 'U' stands for unemployment. Eq. (5) can be written as

$$\frac{\Pr(\underline{t}\geqq t \mid \text{into U at } -t)\Pr(\text{into U at } -t)}{\int_0^\infty \Pr(\underline{t}\geqq s \mid \text{into U at } -s)\Pr(\text{into U at } -s)\,\mathrm{d}s}. \tag{6}$$

In section 2.4 we make an assumption about the distribution of $\underline{t}$ given the time at which the spell of unemployment began. More specifically we assume that the effect of the time at which one becomes unemployed can be captured by including the unemployment percentage among the regressors. Because complete spells of unemployment are rather short — the mean duration is about 6 months [Siddré c.s. (1976)] — and because we analyse data for the month of May in which seasonal unemployment is typically low, we assume that the unemployment percentage at the end of May can be considered as a proxy for the effect of the economic environment (e.g. the business cycle) on the distribution of the duration of unemployment. Therefore in the following conditioning of the time at which unemployment started is suppressed.

With respect to (6) we also assume that for every individual the probability of becoming unemployed is constant over time. This seems a rather strong assumption. However, consider the case in which for one reason or another there has been a change in this probability at time $-t_0$. If we assume that the probability of becoming unemployed is q_0 before $-t_0$ and q_1 after $-t_0$, then (6) can be rewritten as

$$\frac{\Pr(\underline{t} \geqq t) q_1}{q_0 \int_{t_0}^{\infty} \Pr(\underline{t} \geqq s)\,\mathrm{d}s + q_1 \int_{0}^{t_0} \Pr(\underline{t} \geqq s)\,\mathrm{d}s}. \tag{7}$$

Because complete spells of unemployment are short (see above) the approximation implied by our assumption will generally be good. Again analysing data for the month of May implies that many unemployed with non-stationary probabilities of unemployment are excluded from the sample. However, it should be noted that if unemployment probabilities are non-stationary our estimates can be subject to a selection bias. Testing of the assumptions made above would require individual data on durations of employment and unemployment and transitions between those states.

If we make these assumptions eq. (5) reduces to

$$\frac{\Pr(\underline{t} \geqq t)}{\int_{0}^{\infty} \Pr(\underline{t} \geqq s)\,\mathrm{d}s} = \frac{1 - G(t)}{E(\underline{t})}, \tag{8}$$

which is a well-known result from renewal theory. When $\tilde{\underline{t}}$ denotes the elapsed duration of unemployment as observed at a point in time (the incomplete spell of unemployment), then the density of $\tilde{\underline{t}}$ is given by (8),

$$\tilde{h}(t) = \frac{1 - G(t)}{E(\underline{t})}, \qquad t \geqq 0. \tag{9}$$

Given data on incomplete spells of unemployment as observed at a point in time this density function can be used to make inferences about the probability of leaving unemployment.

2.4. The probability of leaving unemployment: Time dependence and sample heterogeneity

The next problem we turn to is the specification of the probability of leaving unemployment $\lambda_i(t)\,\mathrm{d}t$. The most general specification that is considered in this paper is

$$\lambda_i(t)\,\mathrm{d}t = \exp\{x_i'\beta\}\psi(t)\cdot \underline{v}_i\,\mathrm{d}t. \tag{10}$$

In this expression x_i is a vector of regressors, β a vector of parameters, $\psi(t)$ expresses the time dependence of the probability of leaving unemployment and $\underline{v}_i$ a random disturbance representing the effect of omitted variables. This specification, which is known in renewal theory as the proportional hazard model, is the same as that used by Lancaster (1979).

With respect to the disturbances we assume that $\underline{v}_i$ has a distribution with mean 1, e.g. a gamma-distribution

$$f(v) = ((\sigma^2)^{-\sigma^{-2}}/\Gamma(\sigma^{-2}))v^{\sigma^{-2}-1}\mathrm{e}^{-v\sigma^{-2}}, \qquad 0 < v < \infty. \tag{11}$$

In the following we also consider restricted versions of (10). In particular we consider the case of time independence of the probability of leaving unemployment, $\psi(t) \equiv 1$, and the case of no uncontrolled heterogeneity in the sample, $v_i \equiv 1$ for all i.

This specification of the probability of leaving unemployment implies the following distribution function for a single complete spell of unemployment, $\underline{t}$,

$$G(t) = E_v(G(t \mid \underline{v})) = 1 - E_v(\exp\{-\mathrm{e}^{x'\beta}\underline{v}z(t)\}), \tag{12}$$

where E_v indicates that the expectation is taken with respect to the distribution of $\underline{v}$ and $z(t) = \int_0^t \psi(s)\,\mathrm{d}s$.

As noted by Lancaster and Nickell (1980) a major problem with the distribution function given in (12) is that it is difficult to distinguish between time dependence and uncontrolled sample heterogeneity. Their argument is as follows. Write $\phi(x,\beta) = \mathrm{e}^{x'\beta}$, then

$$1 - G(t) = L_v(\phi(x,\beta)z(t)), \tag{13}$$

where

$$L_v(s) = \int_{-\infty}^{\infty} \mathrm{e}^{-sv} f(v)\,\mathrm{d}v. \tag{14}$$

Eq. (13) expresses that the tail of the distribution of $\underline{t}$ is given by the Laplace transform of $\underline{v}$ in $\phi(x,\beta)z(t)$. If there are no regressors in the model, i.e., $x=0$ or $\phi(x,\beta)=1$, then (13) reduces to

$$1-G(t)=L_v(z(t)). \tag{15}$$

Writing $L_v^{-1}(s)$ for the inverse of $L_v(s)$, (15) is equivalent to

$$z(t)=L_v^{-1}(1-G(t)). \tag{16}$$

Now suppose that from a random sample of durations we determine $1-G(t)$. Then it follows from (16) that for an arbitrary distribution of a positive random variable $\underline{v}$, it is possible to choose $z(t)$ such that the combination of this distribution with the chosen function $z(t)$ yields $1-G(t)$.

Conversely, an arbitrary choice of $z(t)$ is not allowed. To see this, note that $z(t)$ is a monotone increasing function of t on $[0,\infty)$. Thus (15) can be rewritten as

$$L_v(t)=1-G(z^{-1}(t)), \qquad 0\leqq t<\infty. \tag{17}$$

A necessary and sufficient condition for $L_v(t)$ to be a Laplace transform of some probability distribution is that

$$(-1)^k L_v^{(k)}(t)>0, \qquad 0\leqq t<\infty. \tag{18}$$

where $L_v^{(k)}$ denotes the kth derivative of L_v. Given $1-G(t)$, this condition restricts the choice of $z(t)$. The condition in (18) implies that $L_v^{(2)}(t)>0$, and this is equivalent to

$$\psi'(t)/\psi(t)>g'(t)/g(t), \qquad 0\leqq t<\infty. \tag{19}$$

A consequence of the inequality in (19) is, that on intervals where the density of $\underline{t}$ increases $\psi'(t)$ must be positive.

A further problem with the argument of Lancaster and Nickell is that they fail to consider the identifying restrictions imposed by the proportional hazard specification. To see this consider (13). This expression is equivalent to

$$z(t)=\frac{L_v^{-1}(1-G(t,x))}{\phi(x,\beta)}, \qquad 0\leqq t<\infty, \tag{20}$$

where we write $G(t,x)$ to make explicit the dependence of the distribution function of $\underline{t}$ on the vector of regressors x. The proportional hazard specification implies that the right-hand side of eq. (20) is not a function of x. As has been shown by Elbers and Ridder (1982), this implies that given the distribution function $G(t,x)$ it is possible to distinguish between sample heterogeneity and time dependence.

A consequence of this result is that it makes sense to consider different specifications of $G(t,x)$. In particular, it is in principle possible to test for time dependence of the probability of leaving unemployment or for the presence of sample heterogeneity. This is done in section 4. However, it must be noted that this result does not mean that parameter estimates are robust against arbitrary assumptions for the time dependence and the sample heterogeneity, i.e., assumptions not derived from $G(t,x)$ [see Heckman (1980) for some Monte Carlo evidence on this point].

3. The data

The data consist of numbers of unemployed registered at the Labour Exchange, cross-classified by sex, age, province and duration of registration. The duration of registration at a particular moment is defined as the period that has elapsed since an individual was registered as unemployed at the Labour Exchange. We consider this classification of the unemployed at the end of May 1979. We choose May, because we expect that in that month the composition of the stock of unemployed is not affected by seasonal factors. The data were obtained from the Netherlands Central Bureau of Statistics. Somewhat less detailed data are published monthly in the Monthly Bulletin of Social Statistics.

The form in which the data are given determines the choice of the regressors. These regressors are:

Age. Because the data are classified by age class, we choose the midvalue of each class.

Unemployment percentage. Because the data are classified by province, we use the unemployment percentage in the province concerned at the end of May 1979.

Actually the logs of these variables are used. The corresponding coefficients represent elasticities of the hazard with respect to age and unemployment percentage. Sex is not used as a regressor, but separate distributions are fitted for male and female unemployed.

The grouped nature of the data implies that no data are available of individual registration durations. The individual durations are grouped into five duration classes. The implications of this classification of individual durations are considered in the following section. More details on the data can be found in appendix 1.

4. Estimation methods and results

4.1. The likelihood function

The models for the duration of unemployment as specified in the preceding sections are estimated with the maximum likelihood method. Therefore we have to derive the likelihood function of the data. Let i denote a group of unemployed that is homogeneous with respect to age, province (i.e., unemployment percentage) and sex, and let the corresponding probability density function of an incomplete spell of unemployment be given by $\tilde{h}_i(t)$, $i=1,\ldots,I$. Then the proportion of unemployed of group i with an incomplete spell of unemployment in the duration class 0–<1 month is given by

$$p_{1i}=\int_0^1 \tilde{h}_i(t)\,\mathrm{d}t. \tag{21}$$

In the same way we can derive the proportions of unemployed of group i with duration in the other four duration classes. Denote these proportions by respectively p_{2i}, p_{3i}, p_{4i} and p_{5i}. If the number of unemployed of group i with registration duration in duration class k is given by n_{ki}, then the likelihood function of the data is given by

$$L(\theta; n_{1i},\ldots,n_{5i},\ldots,n_{1I},\ldots,n_{5I}) \alpha \prod_{i=1}^{I}\prod_{k=1}^{5} p_{ki}^{n_{ki}}(\theta), \tag{22}$$

where θ denotes the parameters of the model.

Next we turn to the specification of $\tilde{h}_i(t)$. As shown in eq. (9), $\tilde{h}_i(t)$ is given by

$$\tilde{h}_i(t)=(1-G_i(t))\bigg/\left(\int_0^\infty (1-G_i(s))\,\mathrm{d}s\right), \tag{23}$$

with $G_i(t)$ being the distribution function of a complete spell of unemployment in group i. The tail $1-G_i(t)$ then is given by eq. (12),

$$1-G_i(t)=E_v\left(\exp\left\{-\mathrm{e}^{x_i'\beta}\underline{v}\int_0^t \psi(s)\,\mathrm{d}s\right\}\right). \tag{24}$$

In the following four special cases of (24) are considered:

Case I. No sample heterogeneity, $\underline{v}\equiv 1$, and the probability of leaving unemployment constant over time, $\psi(s)\equiv 1$.

Case II. No sample heterogeneity, $\underline{v}\equiv 1$, and $\psi(s)=\alpha s^{\alpha-1}$, $\alpha>0$ (time dependence of the probability of leaving unemployment).

Case III. $\underset{\sim}{v}$ has a gamma distribution as given in (11), with $\mathrm{var}(\underset{\sim}{v})=\sigma^2$ (sample heterogeneity) and no time dependence of the probability of leaving unemployment, $\psi(s)\equiv 1$.

Case IV. $\underset{\sim}{v}$ has a gamma distribution as given in (11) (sample heterogeneity) and $\psi(s)=\alpha s^{\alpha-1}$, $\alpha>0$ (time dependence).

It is important to realize that these four cases can be distinguished on the basis of the information contained in the data (see section 2.4).

If we define

$$\phi_i(\beta)=\exp\{x_i'\beta\}, \tag{25}$$

the four cases lead to the following specifications of $\tilde{h}_i(t)$:

Case I

$$\tilde{h}_i(t)=\phi_i(\beta)\exp\{-\phi_i(\beta)t\}, \tag{26}$$

Case II

$$\tilde{h}_i(t)=\frac{\phi_i(\beta)^{1/\alpha}\cdot\alpha}{\Gamma(1/\alpha)}\exp\{-\phi_i(\beta)t^{\alpha}\}, \tag{27}$$

Case III

$$\tilde{h}_i(t)=\phi_i(\beta)(1-\sigma^2)(1+\sigma^2\phi_i(\beta)t)^{-1/\sigma^2}, \tag{28}$$

Case IV

$$\tilde{h}_i(t)=\frac{\alpha(\sigma^2\phi_i(\beta))^{1/\alpha}}{B(1/\alpha,1/\sigma^2-1/\alpha)}(1+\sigma^2\phi_i(\beta)t^{\alpha})^{-1/\sigma^2}, \tag{29}$$

with

$$B(1/\alpha,1/\sigma^2-1/\alpha)=\frac{\Gamma(1/\alpha)\cdot\Gamma(1/\sigma^2-1/\alpha)}{\Gamma(1/\sigma^2)}, \tag{30}$$

(for the derivations, see appendix 2).

To evaluate the likelihood function (22) it is necessary to integrate the given probability density functions. For Cases I and III, this can be done analytically. For the integration of the densities corresponding to Cases II and IV, numerical integration techniques must be employed. The technique used is Gaussian quadrature (Gauss–Legendre for finite intervals and Gauss–Laguerre for semi-infinite intervals) as provided by computer routines from the NAG Library. The resulting likelihood function is maximized using a quasi-Newton algorithm which requires no (analytical) derivatives (NAG Library). The (asymptotic) variance matrix of the maximum likelihood estimators is estimated by the inverse of the (numerically calculated) Hessian of the min-loglikelihood function,

$$\operatorname{var}(\hat{\theta}) = \left(\frac{\partial^2 \ln L(\theta)}{\partial\theta\, \partial\theta'} \right)^{-1}_{\theta=\hat{\theta}}, \tag{31}$$

with $\hat{\theta}$ the maximum likelihood estimate of θ.

4.2. Estimations results

The results of the maximum likelihood estimation are summarized in table 1 (estimates of the corresponding asymptotic correlation matrices can be found in appendix 3). The estimates show that as could be expected there is an inverse relation between the probability of leaving unemployment and the unemployment percentage. There is also an inverse relation between this probability and the age of the unemployed. One can think of a number of explanations for this result. Among these are less willingness to move or to work in another occupation, higher wages due to job-specific human capital,

Table 1

Maximum likelihood estimates and asymptotic standard errors.

Case	Const.	ln (age)	ln (unempl.)	α	σ^2	$-2\ln L$[a]
			Male unemployed			
I	2.59	−1.21	−0.47	—	—	361624.8
	(0.039)	(0.0099)	(0.012)			
II	2.58	−1.03	−0.40	0.80	—	361046.0
	(0.012)	(0.0047)	(0.010)	(0.0067)		
III	3.28	−1.30	−0.51	—	0.17	361012.4
	(0.05)	(0.013)	(0.013)		(0.0046)	
IV	3.12	−1.24	−0.49	0.95	0.14	361012.2
	(0.11)	(0.038)	(0.020)	(0.027)	(0.022)	
			Female unemployed			
I	2.74	−1.14	−0.54	—	—	211296.8
	(0.054)	(0.014)	(0.013)			
II	2.71	−0.99	−0.47	0.82	—	210995.4
	(0.060)	(0.014)	(0.013)	(0.0085)		
III	3.20	−1.20	−0.57	—	0.12	211109.4
	(0.064)	(0.013)	(0.015)		(0.0081)	
IV[b]	2.72	−1.00	−0.47	0.83	10^{-8}	

[a]Up to a constant.

[b]In this case the numerical optimization algorithm failed to converge at an interior point of the parameter space. The variance σ^2 reached its lower bound.

etc. However, it is impossible to choose among these explanations on the basis of the results of this paper.

The estimates in table 1 indicate that the effect of age and unemployment percentage on re-employment probabilities is different for male and female unemployed. For both groups age is more important than unemployment percentage. However, age seems to be less important for female unemployed than for male unemployed, while the reverse is true for local labour market conditions represented by the unemployment percentage. This is also true for the elasticities of the mean duration of a complete spell of unemployment with respect to age and unemployment percentage (see table 2). This result could be due to differences in the occupational structure of male and female (un)employment, but a full explanation of this phenomenon would require additional (and preferably individual) data.

Table 2

Elasticities of mean unemployment duration with respect to age and unemployment percentage.

Case	Age	Unemployment percentage
	Male unemployed	
I	1.21	0.47
II	1.28	0.50
III	1.30	0.51
IV	1.37	0.54
	Female unemployed	
I	1.14	0.54
II	1.20	0.57
III	1.20	0.57
IV	1.20	0.56

The role played by the time dependence of the re-employment probability and the sample heterogeneity is also different for male and female unemployed. The values of $-2\ln L$ and the attendant likelihood ratios of table 1 indicate that there is heterogeneity but no time dependence for male unemployed, while there is time dependence but no sample heterogeneity for female unemployed. Probably this difference is a result of a poor specification of the time dependence and the sample heterogeneity.

The estimates of table 1 can be compared with those obtained by Lancaster (1979) (see table 3). Lancaster's estimates are based on a sample of 479 unemployed, mainly men. Note that the elasticities of the re-employment probability with respect to age are lower (in absolute value) than those in table 1. A second difference is that the standard errors in table 3 are larger.

Table 3

Lancaster's estimates.

Case	ln(age)	ln(unempl.)	α	σ^2	$-2\ln L$
I	−0.87	−0.44	—	—	436.72
	(0.23)	(0.17)			
II	−0.66	−0.34	0.77		429.68
	(0.25)	(0.18)	(0.09)		
III	−0.84	−0.43	—	0.18	429.42
	(0.30)	(0.24)		(0.08)	
IV	−0.77	−0.34	0.90	0.11	429.28
	(0.32)	(0.24)	(0.22)	(0.17)	

This is due to the different sample sizes. The ratios of the standard errors in tables 1 and 3 are of the same order of magnitude as the square root of the ratio of the sizes of the samples.

5. Conclusions

Two kinds of conclusions can be drawn from the results presented in this paper. First, conclusions concerning the econometric methods used, and secondly, implications of the estimates obtained by these methods.

A first conclusion is that the methods developed by Lancaster and Nickell for the estimation of the probability of leaving unemployment from data on incomplete unemployment spells can also be used for the analysis of the aggregate data considered in this paper. However, a major problem with these models is the specification of the time dependence of the re-employment probability and the heterogeneity of the sample. Given that it is possible to distinguish between time dependence and sample heterogeneity, the large difference between male and female unemployed in respect of either time dependence or heterogeneity indicates that other specifications should be used. This conclusion is strengthened by the small decrease in the maximized likelihood due to the introduction of the particular specifications for time dependence and sample heterogeneity used in this paper. However, further research on more flexible specifications will require data on individual durations of unemployment.

Although only two explanatory variables appear in the estimated models, some conclusions can be drawn from the estimates especially from the estimated elasticities of the re-employment probability with respect to the explanatory variables (age and regional unemployment percentage). For both groups that are considered in this paper, male and female unemployed, the age elasticity is greater (in absolute value) than the unemployment elasticity. However, the effect of individual characteristics (age elasticity) is smaller (in absolute value) for female unemployed, while the effect of labour market conditions (unemployment elasticity) is larger (in absolute value) for this group. Subject to the qualifications concerning the methods used, these results seem to stress the different positions on the labour market of male and female unemployed.

Appendix 1: The data

The data on the number of male and female unemployed registered at the Labour Exchange at the end of May 1979 cross-classified by age and duration of registration and by province and duration of registration can be found in the Monthly Bulletin of Social Statistics of July 1979. This Bulletin is published by the Netherlands Central Bureau of Statistics.

The provincial unemployment percentages at the end of May 1979 can also be found in the Monthly Bulletin of Social Statistics of July 1979.

The values of the variable *age* are given by:

Age class	1	2	3	4	5	6	7	8
Age	17.0	21.0	24.0	32.5	45.0	52.5	57.5	62.5

Appendix 2: Derivations

The tail of the distribution of $\underline{t}$ is given by

$$1-G_i(t)=E_v\left(\exp\left\{-\phi_i(\beta)\underline{v}\int_0^t \psi(s)\,\mathrm{d}s\right\}\right), \qquad t\geqq 0. \tag{A.1}$$

The density of an incomplete spell of unemployment is given by

$$\tilde{h}_i(t)=(1-G_i(t))\bigg/\left(\int_0^\infty (1-G_i(s))\,\mathrm{d}s\right), \qquad t\geqq 0. \tag{A.2}$$

Case I. $\underline{v}\equiv 1, \quad \psi(s)\equiv 1$

Then

$$1-G_i(t)=\exp\{-\phi_i(\beta)t\}, \qquad t\geqq 0 \tag{A.3}$$

and

$$\int_0^\infty (1-G_i(s))\,\mathrm{d}s=1/\phi_i(\beta). \tag{A.4}$$

Thus

$$\tilde{h}_i(t)=\phi_i(\beta)\exp\{-\phi_i(\beta)t\}. \tag{A.5}$$

Case II. $\underline{v}\equiv 1,\quad \psi(s)=\alpha s^{\alpha-1},\quad \alpha>0$

Then

$$1-G_i(t)=\exp\{-\phi_i(\beta)t^{\alpha}\},\qquad t\geqq 0, \tag{A.6}$$

and

$$\int_0^{\infty}(1-G_i(s))\,\mathrm{d}s=\int_0^{\infty}\exp(-\phi_i(\beta)s^{\alpha}\}\,\mathrm{d}s. \tag{A.7}$$

By a change of variable [setting $u=\phi_i(\beta)s^{\alpha}$],

$$\int_0^{\infty}(1-G_i(s))\,\mathrm{d}s=(1/\alpha)\phi_i(\beta)^{-1/\alpha}\Gamma(1/\alpha). \tag{A.8}$$

Thus

$$\tilde{h}_i(t)=(\phi_i(\beta)^{1/\alpha}/\Gamma(1/\alpha))\exp\{-\phi_i(\beta)t^{\alpha}\},\qquad t\geqq 0. \tag{A.9}$$

Case III. $\underline{v}$ *gamma-distributed,* $\psi(s)\equiv 1$

Then

$$1-G_i(t\,|\,v)=\exp\{-\phi_i(\beta)vt\},\qquad t\geqq 0. \tag{A.10}$$

Since $\underline{v}$ is gamma-distributed with mean 1 and variance σ^2, the tail of the distribution of $\underline{t}$ is given by the Laplace-transform of $\underline{v}$ in the point $\phi_i(\beta)t$. Thus,

$$1-G_i(t)=(1+\sigma^2\phi_i(\beta)t)^{-1/\sigma^2},\qquad t\geqq 0. \tag{A.11}$$

Then

$$\int_0^{\infty}(1-G_i(s))\,\mathrm{d}s=\int_0^{\infty}(1/\{1+\sigma^2\phi_i(\beta)s\})^{1/\sigma^2}\,\mathrm{d}s. \tag{A.12}$$

By a change of variable, $u=(1/\{1+\sigma^2\phi_i(\beta)s\})$, we find

$$\int_0^{\infty}(1-G_i(s)\,\mathrm{d}s=1/\{\phi_i(\beta)(1-\sigma^2)\}. \tag{A.13}$$

Thus,

$$\tilde{h}_i(t)=\phi_i(\beta)(1-\sigma^2)(1+\sigma^2\phi_i(\beta)t)^{-1/\sigma^2}, \qquad t\geqq 0. \tag{A.14}$$

Case IV. $\underline{v}$ *gamma-distributed,* $\psi(s)=\alpha s^{\alpha-1}$, $\alpha>0$

Then

$$1-G_i(t\,|\,v)=\exp\{-\phi_i(\beta)vt^\alpha\}, \qquad t\geqq 0. \tag{A.15}$$

By the same argument as in Case III, we find that

$$1-G_i(t)=(1+\sigma^2\phi_i(\beta)t^\alpha)^{-1/\sigma^2}, \qquad t\geqq 0, \tag{A.16}$$

and by a change of variable, $u=1/\{1+\sigma^2\phi_i(\beta)t^\alpha\}$,

$$\int_0^\infty (1-G_i(s))\,\mathrm{d}s=B(1/\alpha,1/\sigma^2-1/\alpha)/\alpha(\sigma^2\phi_i(\beta))^{1/\alpha}. \tag{A.17}$$

Thus,

$$\tilde{h}_i(t)=\{\alpha(\sigma^2\phi_i(\beta))^{1/\alpha}/B(1/\alpha,1/\sigma^2-1/\alpha)\}(1+\sigma^2\phi_i(\beta)t^\alpha)^{-1/\sigma^2}, \quad t\geqq 0. \tag{A.18}$$

Appendix 3: Asymptotic correlation matrices

$\beta_0\equiv$ constant,
$\beta_1\equiv$ age elasticity,
$\beta_2\equiv$ unemployment elasticity,
$\alpha\ \equiv$ time dependence parameter,
$\sigma^2\equiv$ variance of sample heterogeneity.

Case I

Male unemployed

	β_0	β_1	β_2
β_0	1.		
β_1	−0.91	1.	
β_2	−0.47	0.06	1.

Female unemployed

	β_0	β_1	β_2
β_0	1.		
β_1	−0.87	1.	
β_2	−0.60	0.13	1.

Case II

Male unemployed

	β_0	β_1	β_2	α
β_0	1.			
β_1	−0.83	1.		
β_2	−0.50	−0.42	1.	
α	−0.77	−0.85	−0.04	1.

Female unemployed

	β_0	β_1	β_2	α
β_0	1.			
β_1	−0.78	1.		
β_2	−0.60	0.28	1.	
α	−0.36	−0.13	−0.06	1.

Case III

Male unemployed

	β_0	β_1	β_2	σ
β_0	1.			
β_1	−0.91	1.		
β_2	−0.42	0.04	1.	
σ^2	0.24	−0.02	−0.10	1

Female unemployed

	β	β	β	σ^2
β_0	1.			
β_1	−0.85	1.		
β_2	−0.66	0.25	1.	
σ^2	0.48	−0.25	−0.15	1.

Case IV

Male unemployed

	β_0	β_1	β_2	α	σ^2
β_0	1.				
β_1	−0.23	1.			
β_2	−0.05	−0.32	1.		
α	0.07	0.46	0.10	1.	
σ^2	0.05	0.27	0.06	−0.09	1.

References

Elbers, C. and G. Ridder, 1982, True and spurious duration dependence: On the identifiability of the proportional hazard model. Review of Economic Studies 49, 403–409.

Heckman, J., 1980, The structural analysis of longitudinal data, Paper presented at the World Congress of the Economic Society, Aix-en-Provence.

Kiefer, N.M. and G.R. Neumann, 1979, An empirical job search model, with a test of the constant reservation-wage hypothesis, Journal of Political Economy 87, 89–107.

Lancaster, T., 1979, Econometric methods for the duration of unemployment, Econometrica 47, 939–956.

Lancaster, T. and S. Nickell, 1980, The analysis of re-employment probabilities for the unemployed, Journal of the Royal Statistical Society A 143, 141–165.

Lippman, S.A. and J.J. McCall, 1976, The economics of job search: A survey I, II, Economic Inquiry 14, 155–189, 347–367.
Numerical Algorithms Group, 1978, NAG-library manual mark 7.
Nickell, S., 1979, Estimating the probability of leaving unemployment, Econometrica 47, 1249–1264.
Salant, S.W., 1977, Search theory and duration data: A theory of sorts, Quarterly Journal of Economics, 39–58.
Siddré, W. (with A. van Ravesteijn, W. Vijverberg and J. de Vries), 1976, De duur van de werkloosheid, Discussion paper (Institute for Economic Research, Erasmus University, Rotterdam).