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A DIFFUSION MODEL FOR NEW PRODUCT INTRODUCTION IN EXISTING MARKETS

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ABSTRACT

This paper reports on the development, estimation and validation of a diffusion model for new products introduced in an existing (competitive) market.

The proposed model structure incorporates the impact of the main company and competitive marketing actions that affect new product penetration.

The parameterization procedure allows for early estimation of the model parameters, by making optimal use of information available on the complete market. The procedure is also dynamic: as the new product penetration proceeds, some model parameters are re-evaluated on the basis of newly available data.

In this paper, the model is applied in a specific segment of the pharmaceutical market. It is indicated, however, that many features of the proposed procedure can be of interest outside of this specific market context.

INTRODUCTION

The literature on quantitative approaches in the area of new product introduction is extensive. Part of that literature deals with so-called "diffusion models", and has been inspired by the publication of the original Bass model (Bass 1969). Since then a considerable number of replications have been done and refinements and extensions developed. More recently, interesting analyses of the characteristics, merits and shortcomings of existing diffusion models have been conducted by Mahajan and Müller (1981) and Kalish and Sen (1984).

In this paper, we develop, and illustrate the use of, a diffusion-type model in a relatively innovative segment of the pharmaceutical market. As a starting point, we take the approach of Lilien, Rao and Kalish (1981) (hereafter LRK), who developed a model that successfully predicted sales of a new ethical drug. It turns out, however, that using their model for the data set available to us poses problems in terms of multicollinearity and significance of effects. This forces us to develop an approach which, although similar to the procedure proposed by LRK, involves some changes in model structure and estimation method.

The approach is essentially designed to predict the penetration of a new product in a highly competitive market segment as early as possible. The model structure and parameterization procedure are designed in such a way that not only information from the introduction phase of past product launches, but also data on more mature products in the segment, can be of use in estimating the diffusion parameters at an early stage of new product introduction. Typically, the latter source of information will be more extensive and easily accessible than the first one. The discussion is organized as follows. Section 1 contains an outline of the problem context. In section 2, we review the model of LRK, and discuss the problems involved in using it for our data set. Section 3 specifies and discusses the proposed model structure. In section 4, we examine the post-diffusion behavior of the model. In section 5, we successively estimate and validate the diffusion model in three cases: firstly, only using historical data, secondly, exclusively considering new product information and finally, combining both types of data. Section 6 contains some concluding remarks.

1. OUTLINE OF THE PROBLEM

The introduction of a new product is a strategic and uncertain decision. Especially in so-called "innovative markets" (Kalish & Coughlan, 1983), characterized by frequent introductions, the risk and uncertainty involved in new product launch are considerable. This uncertainty can, however, be reduced by systematic analyses of the information available on the new product and the target market (segment). Along the same lines, Lilien & Kalish (1983) state that

"... a key ingredient in a well-conceived program of new product development should be the use of sound, explicit models of planning and forecasting new product sales...".

The empirical analysis conducted here is situated in a well defined segment of the pharmaceutical market. In recent years several new products have been launched in this particular segment. A "diffusion model", based on available information on the market segment, could provide some insight into the future performance of such introductions, as well as some guidelines for the marketing policy to be implemented. In the market studied, "detailing" constitutes the major marketing instrument. Other elements, such as price, advertising and direct mailing, are clearly of minor importance. A large portion of sales stems from "repeat purchases". For any company in the segment, it is thus indispensable to create a certain goodwill for its products.

Furthermore, a number of opinion leaders can be identified who strongly influence the sales evolution of a product.

Total sales (e.g. total number of prescriptions) in the segment remain rather stable. As such, sales for a particular (new) product can only be realized at the expense of other products in the segment.

More detailed insight can be gained by analysing available market data. On the market we will study 12 different products are offered at present. For 9 of them we have 34 monthly observations at our disposal. The other 3 brands have been introduced within the last 3 years. For every product, information is available on sales (absolute number of prescriptions), market share, and share of detailing effort in the segment. These data will be of help in analysing the diffusion process and eventually in constructing a model to support new product introduction.

A problem context very similar to the one briefly described here, has been studied by Lilien, Rao and Kalish (1981). These authors succeeded in developing and estimating a diffusion model for an ethical drug aimed at a certain class of specialists. Given the satisfactory results obtained with their approach, it is only natural that we take the model of LRK as a starting point for analysing our data set. The next section reports on the results of such an effort.

2. ESTIMATING THE LRK-MODEL

The model of LRK is based upon a "trial and repeat" structure of product adoption, e.g. drug prescription, and designed to develop "good" detailing policies. For practical purposes, two classes of doctors - prescribing vs. not prescribing - are analysed with the following specification:

$$N(t) = N(t-1) + \lambda_1(d'(t))(N-N(t-1)) + \lambda_2(N(t-1)-N(t-2)) \cdot (N-N(t-1)) - \lambda_3(\bar{d}'(t))N(t-1) \quad (1)$$

where $N(t)$ = number of doctors prescribing at t
N = total number of specialists (potential prescribers) in the class
$d'(t)$ = level of detailing at t
$\bar{d}'(t)$ = level of competitive detailing at t

The flow between the two classes of doctors $(N(t)-N(t-1))$ is controlled by company detailing $(\lambda_1(d'(t)))$, competitive detailing effects $(\lambda_3(\bar{d}'(t)))$ and product experience or word of mouth impact $(\lambda_2(N(t-1)-N(t-2)).(N-N(t))$.

In this model, $d'(t)$ and $\bar{d}'(t)$ represent the so-called 'effective' levels of detailing. The latter are obtained by multiplying the true (observed) levels $d(t)$ and $\bar{d}(t)$ with a 'decay'-factor $f(t)$. The decay factor $f(t)$ is a non-increasing function which accounts for two phenomena: early prescribers prescribing more, and decaying of detailing effectiveness [1]. As it stands, equation (1) cannot be directly parameterized on our data set, which comprises information on prescriptions - not prescribing doctors.

Since detailers may have a tendency to visit doctors with a large practice first, expressing equation (1) in terms of prescriptions rather than prescribing doctors may in fact be desirable. Alternative forms of the LRK model were estimated using the available information on new products in the segment. Model variants were obtained by using different forms of the decay function $f(t)$, different lags for the explanatory variables, and explicit formulations of the components $\lambda_1(d'(t))$, $\lambda_2(\Delta S(t))$ and $\lambda_3(\bar{d}'(t))$. As LRK we could not use direct nonlinear estimation methods due to convergence problems. As a result, we followed the two-step estimation procedure (i.e. predetermine some parameters which are then assumed given in conditionally estimating the remaining ones) they proposed to obtain estimates of the model parameters for the alternative model forms. It turned out that none of the specifications studied yielded satisfactory outcomes. For product 10 , for example, the best results we could find were obtained with the following model:

$$\begin{aligned} S(t) = S(t-1) &+ \lambda_1(d'(t))(S-S(t-1)) \\ &+ \lambda_2(S(t-1)-S(t-2))(S-S(t-1)) \\ &- \lambda_3(\bar{d}'(t))\ S(t-1) \end{aligned} \tag{2}$$

where

$$\lambda_1(d'(t)) = a_1\ f(t)\ d(t) + a_2\ f(t)\ d^2(t)$$

$$\lambda_2(S(t-1)-S(t-2)) = a_4(S(t-1)-S(t-2))$$

$$\lambda_3(\bar{d}'(t)) = a_3\ \bar{d}(t)$$

$$f(t) = \exp\left[\alpha(t_m - t_{intro})\right]$$

with

$t_m = 6$

t_{intro} = number of periods the product has been on the market, if less than 6 months

= 6 elsewhere

$S(t)$ = number of prescriptions in month t

$d(t)$ = detailing effort for the product in t

S = potential market (maximum number of prescriptions/month)

$\bar{d}(t)$ = competitive detailing effort in t

In the two-step estimation procedure, the "intermediate" model with $\alpha = .075$ and $t_m = 6$ [2], yielded an estimate of $\hat{S} = 22{,}940$ prescriptions. The "final" model then provided the following estimation results:

coefficient	t-statistic
$\hat{a}_1 = .0067$	(.47)
$\hat{a}_2 = .0340$	(1.08)
$\hat{a}_3 = -.0138$	(-1.30)
$\hat{a}_4 = -.0046$	(-.64)

$$(MSE)^{1/2} = .0678$$ [3]

The results appear to be quite problematic. The estimate of $\hat{S}$ yielded by the intermediate model is hardly acceptable, since over the past years, total segment sales have fluctuated between 7000 and 8000 prescriptions which is only one third of $\hat{S}$. None of the coefficients significantly differs from zero. Additional tests point at the presence of multicollinearity, mainly between the explanatory variables $d(t)(S-S(t-1))$; $d^2(t)(S-S(t-1))$ and $\bar{d}(t)\,S(t-1)$ in equation (2) . The fourth variable $[S(t-1) - S(t-2)]\,.\,(S-S(t-1))$ has a small negative effect on (changes in) prescriptions. According to people in the industry, this result is counterintuitive in the sense that word-of-mouth is expected to have a strong positive effect on sales changes.

To overcome the difficulties first mentioned, we tried out several variants of the LRK model. First, instead of estimating $\hat{S}$ from the intermediate model, we simply used a forecast based on historical segment sales, as input to the final model. Second, we replaced the word-of-mouth term in equation (2) - $(S(t-1)-S(t-2)).(S-S(t-1))$ - by a representation more often encountered in the literature, i.e. $S(t-1).(S-S(t-1))$.

Unfortunately, none of these efforts led to a significant improvement of the model outcomes.

It appears then that, in order to obtain meaningful results for the data analysed here, the model structure has to be adapted such that:

i. multicollinearity between explanatory variables included is avoided
ii. word-of-mouth effects are identified appropriately.

In the next section, we comment on the development of an 'adapted' diffusion model, which maintains some basic features of the LRK approach while overcoming the problems encountered with our data set.

3. THE ADAPTED MODEL STRUCTURE

In formulating a model specification appropriate for our data set, we combine our experience with the LRK model, with more general insights from the (theoretical) diffusion literature. As a first point, we intend to reduce the collinearity due to relatedness of company and competitive detailing, by expressing the diffusion in terms of market shares - and not absolute number of prescriptions. This will allow us to combine the detailing effects into one relevant explanatory variable expressed in relative terms (company vs. competitive effort). Also, given the strong competitive pressure in the segment studied, market share is at least as vital as number of prescriptions, in characterizing the diffusion process. From the more general diffusion literature, and keeping in mind that we deal with a repeat purchase environment, our market share diffusion model could comprise three components, formalized in equation (3):

$$m_j(t) = r_j(t)\, m_j(t-1) + e_j(t)(1 - m_j(t-1)) + i_j(t)\, m_j(t-1)(1 - m_j(t-1)) \qquad (3)$$

where $r_j(t)$, $e_j(t)$ and $i_j(t)$ represent the coefficients of repeat, innovation and imitation respectively. From empirical analyses on the data set - cfr section 2 - it appears that introduction of the third component, $i_j(t)\, m_j(t-1)(1 - m_j(t-1))$, is not the most appropriate way of modeling word-of-mouth, since it yields unexpected insignificant effects, and is bound to introduce collinearity in the data matrix. To avoid these problems, we propose to reduce equation (3) to the following expression

$$m_j(t) = r_j'(t)\, .\, m_j(t-1) + e_j'(t)(1-m_j(t-1)) \qquad (4)$$

in which $r_j'(t)$ and $e_j'(t)$ are functions that vary over time. In this equation, the word-of-mouth effect is incorporated in $r_j'(t)$. $r_j'(t)$ is non-increasing during the introduction period (ceteris paribus), to represent - among other phenomena - the decrease in the untapped market as $m_j(t-1)$ increases. Equation (4) can be re-written as follows:

$$m_j(t) = \lambda_{0j}(t)\ m_j(t-1) + \lambda_{1j}(t) \tag{5}$$

with

$$\lambda_{0j}(t) = r'_j(t) - e'_j(t)$$

$$\lambda_{1j}(t) = e'_j(t)$$

Here $\lambda_{1j}(t)$ represents the "innovation" component in equation (4), and indicates what portion of the market autonomously - without impact from other practitioners - starts prescribing the product. $\lambda_{0j}(t)$ is the net result of three effects:

- an "imitation" effect: prescribing doctors (market share gained) may stimulate others to start using the product
- a "repeat" effect: a fraction of previous users ($m_j(t-1)$) will go on prescribing the product
- a "negative" innovation effect: logically, as more doctors prescribe the product, the number of (potential) innovators decreases.

Obviously, $\lambda_{0j}(t)$ and $\lambda_{1j}(t)$ will be influenced by the detailing effort for the new product relative to competitors. This impact is specified as follows [4] :

$$\begin{aligned} \lambda_{0j}(t) &= a_1\left[f_j(t_{0j})(d_j(t) + \varepsilon)\right]^{d_1} \\ \lambda_{1j}(t) &= b_1\left[f_j(t_{0j})(d_j(t) + \varepsilon)\right]^{d_2} \end{aligned} \tag{6}$$

where a_1, b_1, d_1, d_2, ε = parameters, and $d_j(t)$ = detailing share for product j in period t, defined as

$$\frac{\text{detailing for j in t}}{\text{detailing for all products in the segment in t}}$$

As introduction proceeds, and the product becomes more mature, its level of detailing might drop to zero in some periods. As such, the parameter ε enters the components $\lambda_{0j}(t)$ and $\lambda_{1j}(t)$ to avoid "zero value" problems [5].

The effect of detailing $d_j(t)$ is further "corrected" with the function $f_j(t_{0j})$, which is similar to the decay function introduced in the LRK model. It is non-increasing in t_{0j}, the number of periods elapsed since introduction of product j. Different specifications could be used for $f_j(t_{0j})$. For our problem situation, the following forms were estimated and compared:

$$f_j(t_{0j}) = \lambda_j \quad \text{for} \quad t_{0j} \leqslant x \qquad (7.1)$$
$$= 1 \quad \text{for} \quad t_{0j} > x$$

$$f_j(t_{0j}) = [x + 1 - t_{0j}]^{\alpha_j} \quad \text{for} \quad t_{0j} \leqslant x \qquad (7.2)$$
$$= 1 \quad \text{for} \quad t_{0j} > x$$

$$f_j(t_{0j}) = \exp\ (\alpha_j(x-t_{0j})) \quad \text{for} \quad t_{0j} \leqslant x \qquad (7.3)$$
$$= 1 \quad \text{for} \quad t_{0j} > x$$

where x was set at 6,8,9 and 12 months, resp. for each specification. Unlike LRK - who used equation (7.1) with x set at 12 for their specific model structure - we found that, for our data, the highest model validity - cfr. infra - was obtained with specification (7.3), and x equal to 6. Ultimately, we thus retained the following structure for our diffusion model:

$$m_j(t) = a_1[f_j(t_{0j})(d_j(t)+\epsilon)]^{d_1}\ m_j(t-1) + b_1[f_j(t_{0j})(d_j(t)+\epsilon)]^{d_2} \qquad (8)$$

with

$$f_j(t_{0j}) = \exp\ [\alpha_j(6-t_{0j})] \quad \text{for} \quad t_{0j} \leqslant 6$$
$$= 1 \quad \text{for} \quad t_{0j} > 6$$

where a_1, b_1, d_1, d_2, ϵ and α_j are parameters.

Equation (8) possesses some desirable properties. First, the basic model structure (equation (5)) is fairly simple. Second, by using specifications (6), detailing enters the model in a nonlinear way, such that, for reasonable levels of the parameters d_1 and d_2 ($d_1 < 1$ and $d_2 < 1$) the model shows decreasing returns to detailing effort at a given point in time. Also interaction between lagged market share and detailing effectiveness is allowed for. An interesting point is that some more "naive" relationships are nested in the model presented here. Indeed, for $d_1 = d_2 = 0$, we obtain a simple autoregressive function. For $d_1 = 0$ and $d_2 = 1$ a linear form in $d_j(t)$ and $m_j(t-1)$ is obtained. If $b_1 = 0$ and $\varepsilon = 0$, equation (8) becomes a multiplicative function with lagged market share elasticity equal to 1. Finally, for $\alpha_j = 0$, we are confronted with a function invariant over time. In short, the model put forward here is quite flexible, and can easily be tested against some simpler relationships.

In equation (8) the decay function f(t) captures the dynamics of introduction. As product launch proceeds, the level of f(t) gradually declines, and after 6 months of introduction, the market share response function reaches some "equilibrium" level, characterized by the (time-invariant) parameters a_1 , d_1 , b_1 , d_2 (and ε).

4. EXISTING PRODUCTS: MODEL ESTIMATION AND VALIDATION

The formulation of the model structure put forward in the previous section implies that, after an introduction period of 6 months, new products in the segment reach some "equilibrium" response level. Indeed, after 6 periods, according to equation (8), $f_j(t_{0j})$ levels off to 1, and the product's market share function becomes

$$m_j(t) = a_1\left[d_j(t) + \varepsilon\right]^{d_1} m_j(t-1) + b_1\left[d_j(t) + \varepsilon\right]^{d_2} \qquad (9)$$

As a first step in validating the proposed diffusion model, we could analyse the validity of this "equilibrium" part using information on existing products in the segment, for which $f_j(t_{0j})$ has become 1. If equation (9) fails to describe the behavior of existing products in the market, serious doubt is shed on the appropriateness of using equation (8) to predict the behavior of future introductions.

In trying to assess the level of a_1, b_1, d_1, d_2 and ε, we use the data available on the nine (out of twelve) products in the segment for which $f_j(t_{0j}) = 1$. For each of these products, an estimation sample of 27 observations is reserved. This leaves us with a holdout sample of 7 observations for each product, which can be used for predictive model validation. Pooling the data on the 9 "existing" products, and taking into account that 9 observations are lost due to the introduction of lagged market share, parameter estimation is based on a total of 234 observations. Using a nonlinear estimation routine supplied by Harwell (1981), the following results are obtained [6]:

$$\hat{a}_1 = .977 \qquad (43.8)$$
$$\hat{b}_1 = .000815 \qquad (3.3)$$
$$\ln \hat{d}_1 = -4.65 \qquad (-.4)$$
$$\ln \hat{d}_2 = -3.99 \qquad (-.1)$$
$$c = 1.E-6 \qquad (16.8)$$

$$(MSE)^{1/2} = .0051$$

where the values between brackets indicate the t-values corresponding to the (asymptotic) standard deviations, and $(MSE)^{1/2}$ represents the square root of the mean square error of estimation. In terms of "fit" (descriptive validity) the model performs only slightly better than a linear model:

$$m_j(t) = \alpha + \beta m_j(t-1) + \delta d_j(t) \qquad (10)$$

$$\hat{\alpha} = -.00069 \qquad (-.8)$$
$$\hat{\beta} = .976 \qquad (177.0)$$
$$\hat{\delta} = .021 \qquad (2.8)$$

$$(MSE)^{1/2} = .0053$$

Some basic t-tests indicate, however, that the nonlinear model does not reduce to any of the simpler structures nested in it. Furthermore, in terms of long term prediction [7], the nonlinear model performs far better than the linear one for 6 out of 9 products. The square root of the MSE of the forecasts, pooled over 9 products, amounts to .0086 for equation (9) compared to .0108 for equation (10).

In summary, it seems that the "equilibrium part" of the proposed diffusion model appropriately represents the behavior of products in the segment after the introduction phase. The question remains, however, whether it adequately describes (predicts) the evolution of new introductions. This will be examined next.

5. ESTIMATING AND VALIDATING THE MODEL FOR NEW PRODUCTS

In the previous section, we assessed the validity of equation (8) for existing products in the segment, and obtained segment-specific estimates of the parameters a_1, b_1, d_1, d_2 and ε.

In this section, we will analyse the model's performance for new products. As indicated earlier, within the pharmaceutical segment studied, 3 products have been introduced recently - in addition to the 9 existing ones. Product 10 has been introduced in January 1980 (33 monthly observations available), for product 11, introduced 9 months later, we have 24 data points at our disposal, whereas for product 12, 7 observations are available.

First, we will be concerned with different options for model parameterization. After having chosen a suitable estimation method, we will concentrate on the model's (long term) predictive properties.

The discussion is organized as follows:

Paragraph 1 examines the consequences of simply using historical info on existing products (past introduction) to characterize new product diffusion.

Paragraph 2 analyses the other extreme, namely, parameterization of the diffusion model using data on the particular new product to be studied, as the sole source of information.

Since neither approach works well, we propose, in paragraph 3, a parameterization procedure that combines inputs from both new and existing products. In that paragraph, we analyse the model's descriptive and predictive properties in a dynamic setting.

5.1 Estimating the diffusion model using information on existing products only

In the previous section, we obtained some "mean", or segment-specific levels of the parameters a_1, b_1, d_1, d_2 and ε, on the basis of pooled info on existing products in the segment. Since we are interested in diffusion of new products a first thing we might want to investigate is, to what extent these estimates are useful in predicting the evolution of new products in the segment.

A preliminary test on the usefulness of the estimated equation (9) can be to investigate the match between real and predicted market share for new products ex post, on a period by period basis. The fit between real and predicted market share can be evaluated by graphical inspection.

Figures 1.1, 1.2 and 1.3 show the real market share, and one-period-ahead forecasts, for product 10, 11 and 12 respectively, using equation (9). From these figures, the following conclusions emerge:

* Market share of the new products in early periods of introduction (6 months) is systematically underestimated. A possible explanation - to be verified in later sections - is that in those periods, new products behave differently than later on. Equation (9) - which assumes α to be zero - does not allow for this difference.

* Even after early periods of introduction, the performance of individual products cannot be very accurately described by segment-specific parameters. The predictions of model (9) always seem to be one step behind reality. Clearly, instead of using only "mean" parameter levels, allowance must be made for individual product differences.

For purposes of comparison, we have also depicted one period-ahead forecasts for the linear model - equation (10) - in figures 2.1, 2.2 and 2.3 . These figures support the points just made. In short, parameters obtained from information on existing products in the segment only, do not provide an adequate description of new product performance.

5.2 Estimating the diffusion model using information on new products only

Since historical info on existing products does not suffice to describe the penetration of a new product, a logical alternative would be to parameterize the diffusion model using data on the particular new product under study. Since, in practice, estimates must be obtained as early as possible in the diffusion process, we analyse the results of estimating equation (8) - the nonlinear diffusion model - on the basis of 6 monthly observations on the new product studied.

Figures 3.1 and 3.2 show the resulting market share predictions (one period-ahead forecasts) for products 10 and 11 [8]. For product 10, graphical inspection would suggest that the model performs well, not only on the estimation sample of observations (periods 1 to 6), but also on the analysis sample (periods 7 to 33). The estimated parameters, however, lead to quite different conclusions. Indeed, the results obtained for product 10 are [9]:

$$\hat{a}_1 = .7236$$

$$\hat{d}_1 = .0000$$

$$\hat{b}_1 = .0505$$

$$\hat{d}_2 = .0302$$

$$\hat{\alpha} = -2.832$$

These results - and more in particular the estimate of α - lack face validity and seem to indicate that the quality of the predictions is merely a matter of luck. This view is supported by figure 3.2 , showing the results for product 11. For this product, the one-period-ahead predictions considerably deviate from the real levels of market share during the validation period (observations 7 to 27).

In summary, estimating the diffusion model on the basis of new product data from the first six months of introduction, leads to a lack of face validity and/or predictive validity of the results. With 6 observations, and 5 parameters to be estimated, this is probably due to a lack of degrees of freedom. For illustrative purposes, we also depicted the results obtained with the linear model (equation (10)), estimated using 6 new product observations, in figures 4.1 and 4.2 . These results further support the conclusion that early estimation of all the model parameters, based on a few observations on the new product's performance only, is not likely to yield an accurate prediction of the diffusion process.

5.3 Estimating the diffusion model combining existing and new product information

In paragraphs 5.1 and 5.2 , we examined two "extreme" alternatives for parameterization of the diffusion process. Since neither approach worked well, we propose, in this paragraph, an estimation procedure that combines the advantages of the previous alternatives while overcoming their major problems.

In a first part (5.3.1), the proposed procedure is outlined, together with some checks on ex post (descriptive and predictive) validity. In a concrete situation of new product launch, however, we are not primarily interested in ex post considerations, but rather in estimating diffusion parameters as early as possible, and providing long term predictions of the diffusion process. These aspects are dealt with in a second part of this paragraph.

5.3.1 Combining historical and new product data: parameterization and ex post model validation

From the previous paragraphs, we conclude that neither information on existi products in the segment, nor specific data on the new product studied, taker separately, do suffice as a basis for parameterization of the diffusion m: Consequently, we suggest that a more workable estimation procedure migh' one that uses inputs from both data sources. Let us examine this issue more detail.

As a starting point, we let a_1, b_1, d_1, d_2 and ϵ assume their segment-levels and estimate α_j for the subsequent new products using the data availab after their introduction.

Given the specification chosen for $f_j(t_{0j})$ (equation (8)) the "optimal" leve of α_j for each of the products can be determined using 6 historical data points on their market and detailing share. The following results are obtained:

$\hat{\alpha}_{10} = 28.58$	(193.1)	$(MSE_{10})^{1/2} = .0173$
$\hat{\alpha}_{11} = 27.43$	(171.4)	$(MSE_{11})^{1/2} = .0183$
$\hat{\alpha}_{12} = 23.26$	(750.3)	$(MSE_{12})^{1/2} = .0030$

The 'order of magnitude' of α is clearly the same for all three products. From the first step in the parameterization procedure, and after some rearrangement, we obtain the following diffusion models:

$$m_{10}(t) = a_1 m_{10}(t-1)\ \exp(.1429(6-t_{0_{10}}))\ .\ (d_{10}(t) + \epsilon)^{d_1}$$
$$+\ b_1\ \exp(.486(6-t_{0_{10}}))(d_{10}(t) + \epsilon)^{d_2} \qquad (11.1)$$

$$m_{11}(t) = a_1 m_{11}(t-1)\ \exp(.1372(6-t_{0_{11}}))\ .\ (d_{11}(t)+\varepsilon)^{d_1} + b_1\ \exp(.494(6-t_{0_{11}}))(d_{11}(t)+\varepsilon)^{d_2} \quad (11.2)$$

$$m_{12}(t) = a_1 m_{12}(t-1)\ \exp(.1163(6-t_{0_{12}}))\ .\ (d_{12}(t)+\varepsilon)^{d_1} + b_1\ \exp(.419(6-t_{0_{12}}))(d_{12}(t)+\varepsilon)^{d_2} \quad (11.3)$$

with $a_1 = .977$ $b_1 = .000835$ $\varepsilon = 1.E-6$
$d_1 = .005$ $d_2 = .018$

Unless the relationships are able to provide a satisfactory description of the new products' sales future **after the facts**, we can hardly expect them to predict this future at a stage where only little information is available.

The validity of equations (11.1), (11.2) and (11.3) is illustrated in figures 5.1, 5.2 and 5.3. These figures indicate that, in early periods of introduction, the models capture - describe - new product behavior rather well, thanks to the introduction of the α_j-parameter. Later on, as the effect of α_j has died out, the model with segment-specific parameters lags behind reality. This was already indicated in paragraph 5.1 . On the other hand, paragraph 5.2 clearly indicates that re-assessment of all the parameters for the specific new product is not an advisable strategy. As a result, we opt for an intermediate solution, in which

- only some segment parameters are made product-specific, or "free"
- these parameters are re-estimated not at the outset of new product introduction, but as more data on the diffusion become available.

For our particular application, we suggest to let only a_1 vary from product to product, while keeping b_1 to ε at their segment level. This solution is logically appealing in the sense that intrinsic product characteristics are most likely to affect the intensity of repeat and word-of-mouth, which can be captured by a_1. Empirically, the descriptive validity of model (8) clearly increases by making a_1 product-specific. For products 10 and 11, "optimal" levels of a_1 and α (ex post) are given in table 1, together with the $(MSE)^{1/2}$ of the resulting models. Figures 6.1 and 6.2 further illustrate the models' descriptive properties. Ex post, they clearly perform quite well. The question remains, however, how the parameters a_1 and α can be assessed in an early stage of new product analysis, and what the resulting model predictions look like. This will be explored in the next section.

5.3.2 Combining historical and new product data: dynamic parameterization and ex ante validation

In evaluating the diffusion model in terms of practical use, we must concentrate on **ex ante** model validity. In particular, we are interested in the model's long term predictive properties at an early stage of introduction. These issues will now be analysed for the three products recently introduced in the market segment. In order to obtain proper estimates for the diffusion parameters on the basis of a limited number of data on the new product, we suggested to take the segment parameters in equation (9) as a starting point, and let the shape of the diffusion curve be further determined by an estimate of α_j obtained from this limited new product information. Over time, as more data on the product launch become available, they can be used not only to "update" the parameter α_j , but also to obtain a product-specific level for some segment parameter(s). For our model and data set re-estimation of a_1 seemed to be advisable (cfr. supra). The question remains at what point this parameter should be made product specific, that is, when adaptation of a_1 is both required - to maintain model validity in future periods - and possible - sufficient information being available for re-estimation.
In the previous section, we found that the diffusion model's descriptive validity with segment-specific a_1 was high during the first six months of introduction, and declined from then on. This observation is related to the fact that both parameters a_1 and α are somewhat interwoven. The level of α is positively related to the speed of diffusion in the first six months of introduction, whereas the level of a_1 refers to the degree of brand loyalty and positive word-of-mouth over a longer period. It can be expected that objective data on market and detailing share in the first six months of a product's life do not allow for a sharp distinction between introductory (α) and more permanent (a_1) effects, even if both enter the model in a different way. Also, with 6 data points (or fewer) and 2 parameters to be estimated, the number of degrees of freedom left would be very small, resulting in low reliability of the estimates. Table 2 provides estimates of a_1 and α obtained for products 10 and 11 during the first 6 months of in-

troduction, and compares them to "optimal" product- and segment-specific levels. Clearly, before six months have passed, using the segment level of a_1 is better practice. After the first half year, the new product reaches some equilibrium response level: the effect of α disappears, and a_1 can be "updated".

From the foregoing discussion, the following approach is proposed for the first six months of product launch:

- ... parameters a_1 to ε at the level given in equation (9)
- ...imate and update $α_j$ on the basis of historical data available in the ...urse of introduction [10].

... and 7.3 illustrate some results of this procedure for the ...different products. Each picture indicates the real evolution of market share over the first year of introduction (indicated by R), as well as

i. For the estimation period (observations used to estimate α): market share computed by the nonlinear diffusion model.
... o... he remainder of the first year: long term predictions by this model.

...e estimated diffusion models seem to approach reality quite well. On the other hand, the figures suggest that parameter estimation based on only a few historical observations remains rather uncertain: one additional data point for estimation can produce rather different parameters, and even worsen model results. Yet, on the whole, we find that the procedure provides satisfactory predictions for the first year even at a very early stage of introduction.

As already indicated in section 4, the diffusion model based on segment parameters alone does not accurately describe (let alone predict) the evolution of a new product after early introduction. We therefore suggest to introduce a product specific value for a_1 **after six months** of introduction. Table 3 shows estimated levels of a_1 for T = 7, 12 and 20, for products 10 and 11 . Clearly, as more information becomes available, the estimates approach their "optimal" level, and lead to improved predictions. This is illustrated in figures 8.1 and 8.2 , which provide long term model predictions corresponding to the different estimation periods, as compared to real market share evolution. Over time, predictions of the new products' diffusion clearly become more accurate, as expected.

6. CONCLUSION

In this paper, we developed and discussed a diffusion model for predicting the behavior of new products introduced in existing market segments. A procedure for sequential parameterization of the diffusion model, based on available information from both existing products in the market segment, and the product to be launched, was proposed. The approach was illustrated using data from a particular pharmaceutical market segment. Though the model is able to predict new product evolutions rather well, at an early stage of introduction, it is also clear that the model results must be extensively validated, since they are based on a few historical new product observations only. In practice, such validation may be based on subjective insight and experience from managers knowledgeable about the product and the market segment. For the data set analysed here, we started with 1 product-specific and 5 segment-specific diffusion parameters, and "freed" one particular segment coefficient in the course of analysis.

In a different context, several segment parameters may have to be adapted. It is clear that final acceptance of the model and/or sequential estimation procedure developed here will necessitate further analysis and validation on different data sets.

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NOTES

1 We should observe that the two phenomena are related. Pharmaceutical firms have good knowledge of the identity of the innovative doctors, and the size of the practice of the doctors they visit regularly. Systematically calling upon the more promising doctors first will of course lead to decreasing detailing effectiveness over time.

2 Alternative specifications for $f(t)$ and different levels of t_m and α were considered. It was found that $f(t) = \exp(\alpha(t_m - t_{intro}))$, with $\alpha = .075$ and $t_m = 6$, yielded the highest descriptive validity for model (2). Also, $t_m = 6$ was consistent with what people in the industry thought to be the "length of the introduction period". A more elaborate discussion on the specification of the decay function is provided in the next section.

3 We use the square root of the mean square error as a measure of descriptive validity instead of the R^2, since for a model without constant term, the latter is of little significance. Also, using $(MSE)^{1/2}$ enhances comparability with the outputs of the nonlinear estimation routine used later on.

4 We also analysed the results obtained when different f_j functions ($f_{j1}(t_{0j})$ and $f_{j2}(t_{0j})$) were allowed for in the coefficients $\lambda_{0j}(t)$ and $\lambda_{1j}(t)$. In that case, we found the model performed well in terms of descriptive validity, but not in terms of predictive validity, especially if only a limited number of observations was used for estimation. This already points at the danger of "overparameterization". A detailed discussion of model estimation and results will be provided in the following sections.

5 For a more elaborate discussion of this problem, see Naert and Weverbergh (1980).

6 $\ln d_1$ and $\ln d_2$ were estimated rather than d_1 and d_2 because of convergence problems in the nonlinear estimation. The estimated values of d_1 and d_2 are .005 and .018 respectively.

7 Long term prediction - sometimes referred to as "full" prediction - takes the real market share as a starting point, and uses the model thereafter. The alternative is "period-by-period" or "periodic" prediction, in which forecasts in each period are obtained by using the real level of lagged market share - instead of the previous model prediction - as an input.

8 For product 12, only 7 data points are available, such that model validity after the first half year cannot be examined.

9 For new products, d_{jt} is different from zero, and the ϵ-parameter can be omitted from the model.

10 Of course, we need at least one observation on the new product to estimate α_j . In the absence of this, i.e. before the actual launch, one could either use an α estimate from another recent launch in the product category, or a subjective assessment.

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Table 1

Estimates of a_1 and α , and SSR, based on the products' market share history (a_1 = .977 is the "segment level" of this parameter).

product n° (j)	a_1	α	$(MSE)^{1/2}$
10	.977	28.58	.0115
	1.0251	24.44	.0088
11	.977	27.43	.0132
	.943	32.01	.0113

Table 2

Estimates of a_1 and α based on T observations

	T	product n° 10	product n° 11
a_1	all	1.0251	.943
α	all	24.44	32.01
a_1	historical	.977	.977
α	6	28.58	27.43
a_1	4	.777	.8085
α	4	38.415	37.99
a_1	6	.9916	.969
α	6	27.32	30.185

Table 3

Estimates of a_1 and α after the first half year of introduction (based on T observations)

	T=	7	12	20	all
10	a_1	1.0142	1.019	1.0226	1.0251
	α	25.39	24.95	24.66	24.44
11	a_1	.9826	.962	.9605	.9431
	α	29.17	30.70	30.81	32.00

Figure 1.1 : real vs. predicted (one-period-ahead) market share (product 10, equation 9, parameters for products 1 to 9)

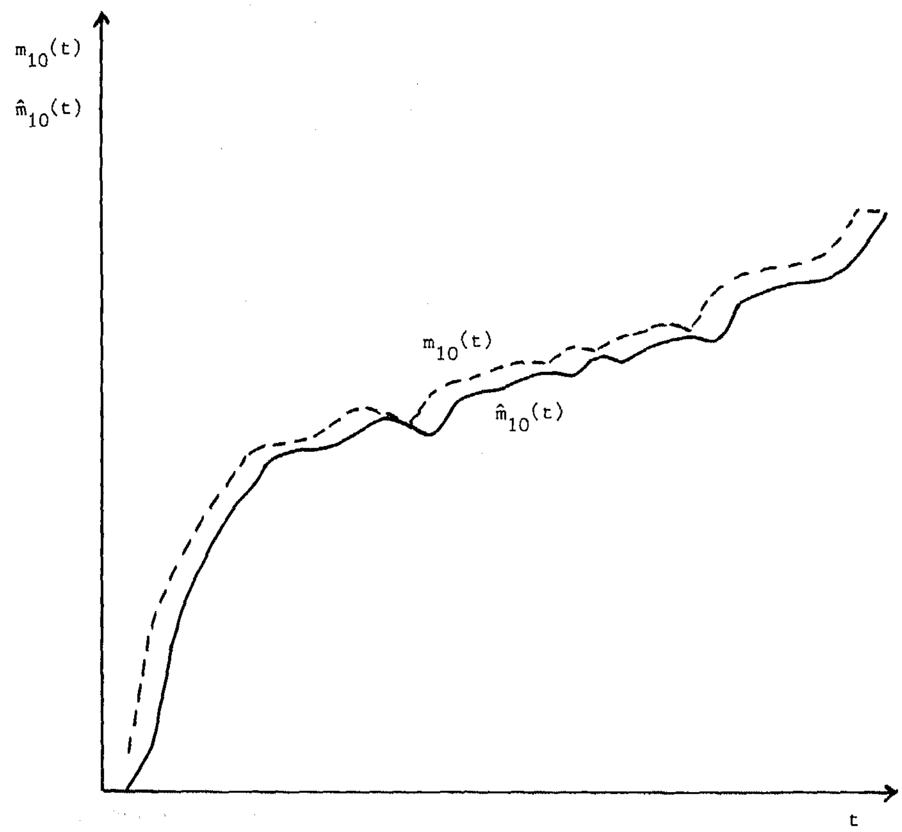

Figure 1.2 : real vs. predicted (one-period-ahead) market share
(product 11, equation 9, parameters for products 1 to 9)

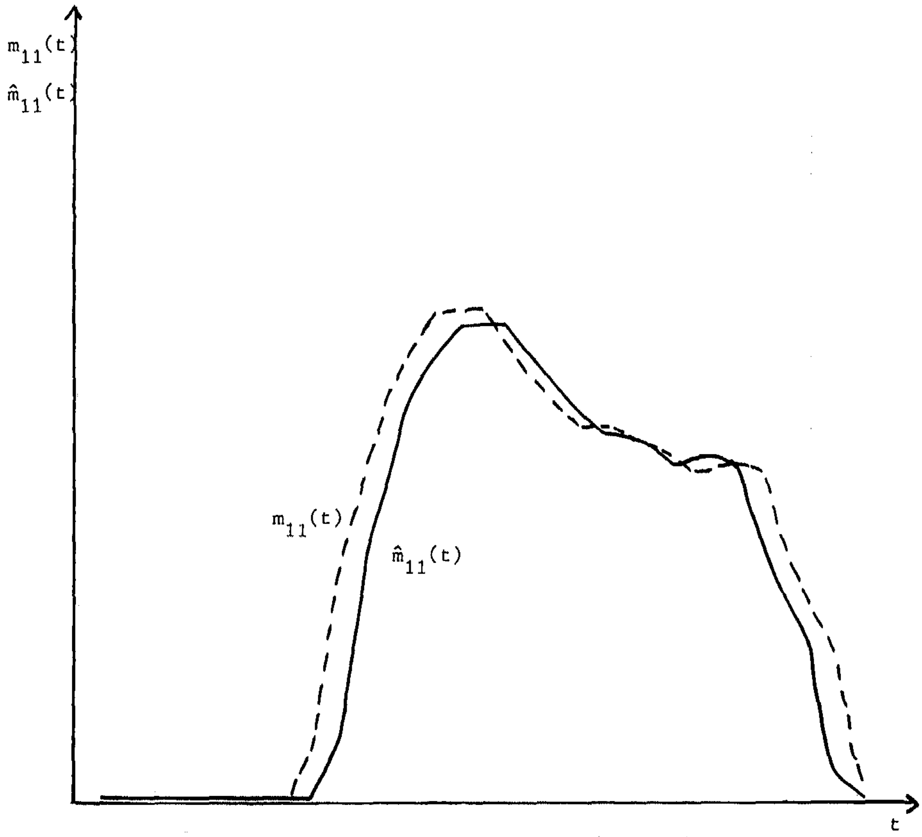

Figure 1.3 : real vs. predicted (one-period-ahead) market share
(product 12, equation 9, parameters for products 1 to 9)

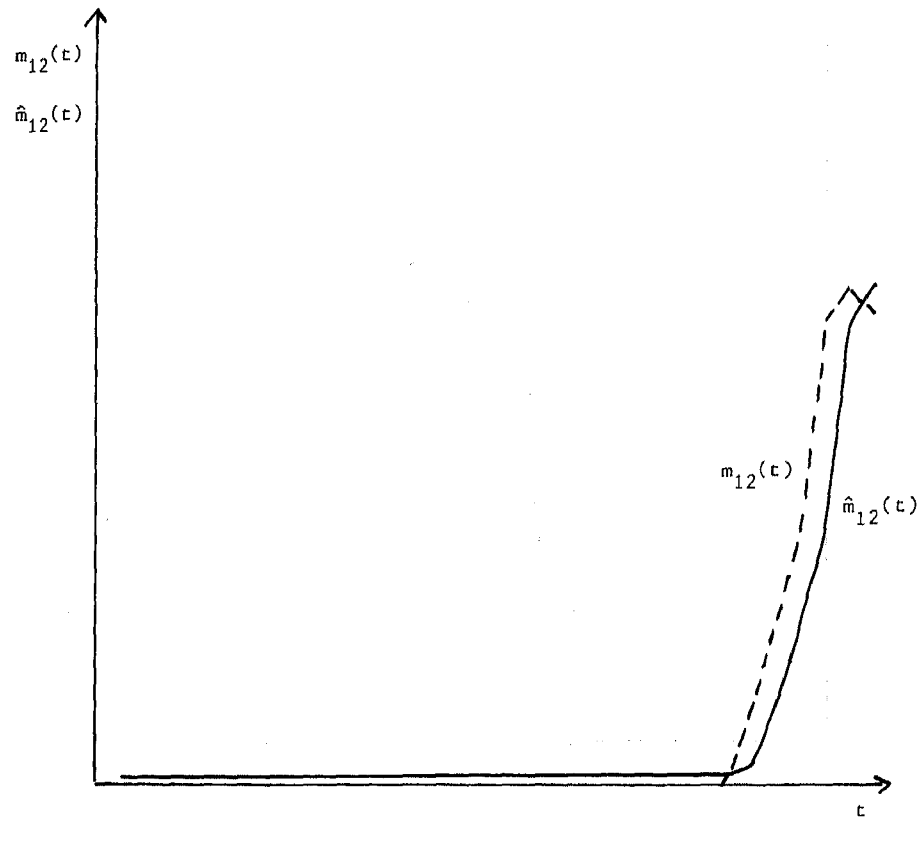

Figure 2.1 : real vs. predicted (one-period-ahead) market share
(product 10, equation 10, pareameters for products 1 to 9)

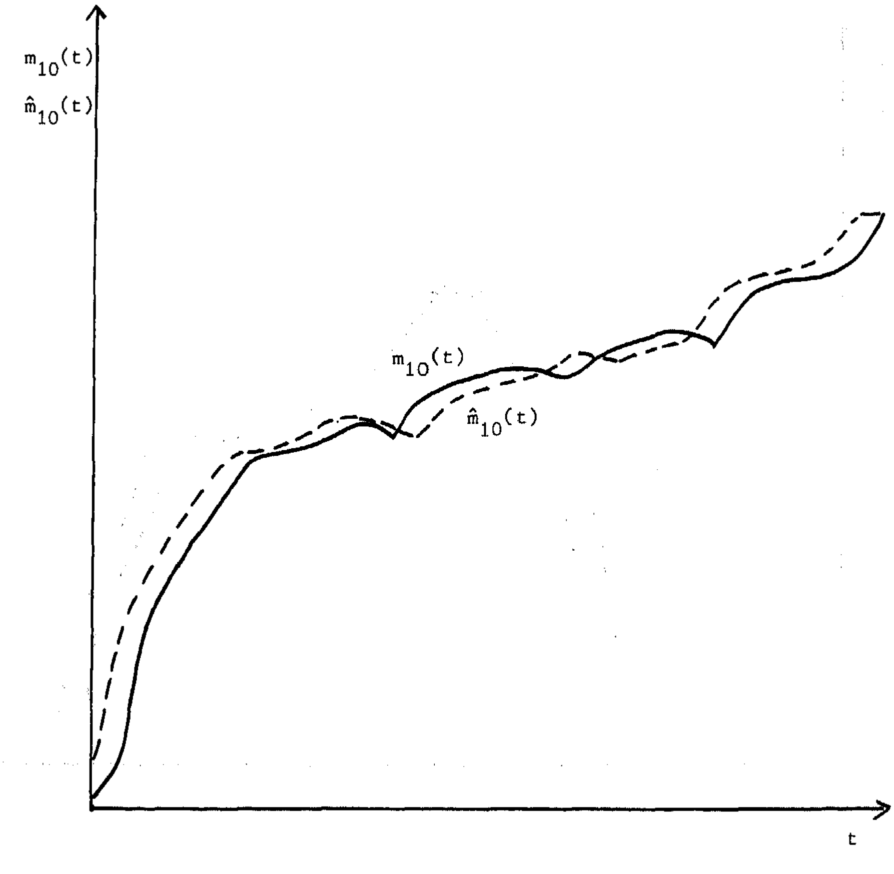

Figure 2.2. real vs. predicted (one-period-ahead) market share (product 11, equation 10, parameters for products 1 to 9)

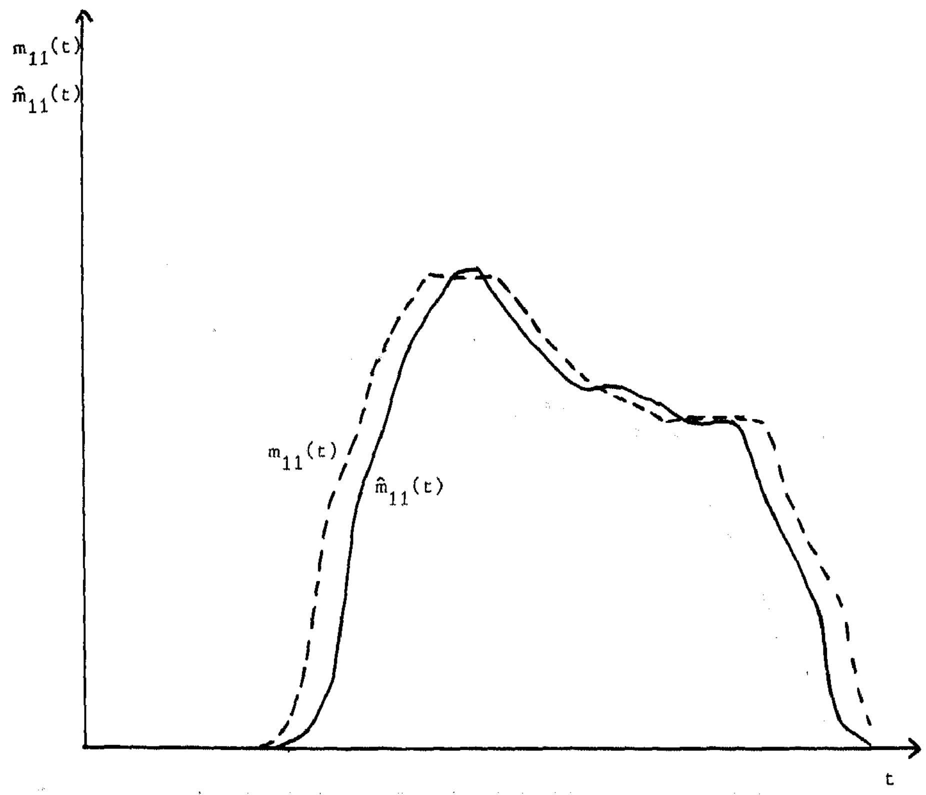

Figure 2.3. : real vs. predicted (one-period-ahead) market share (product 12, equation 10, parameters for products 1 to 9)

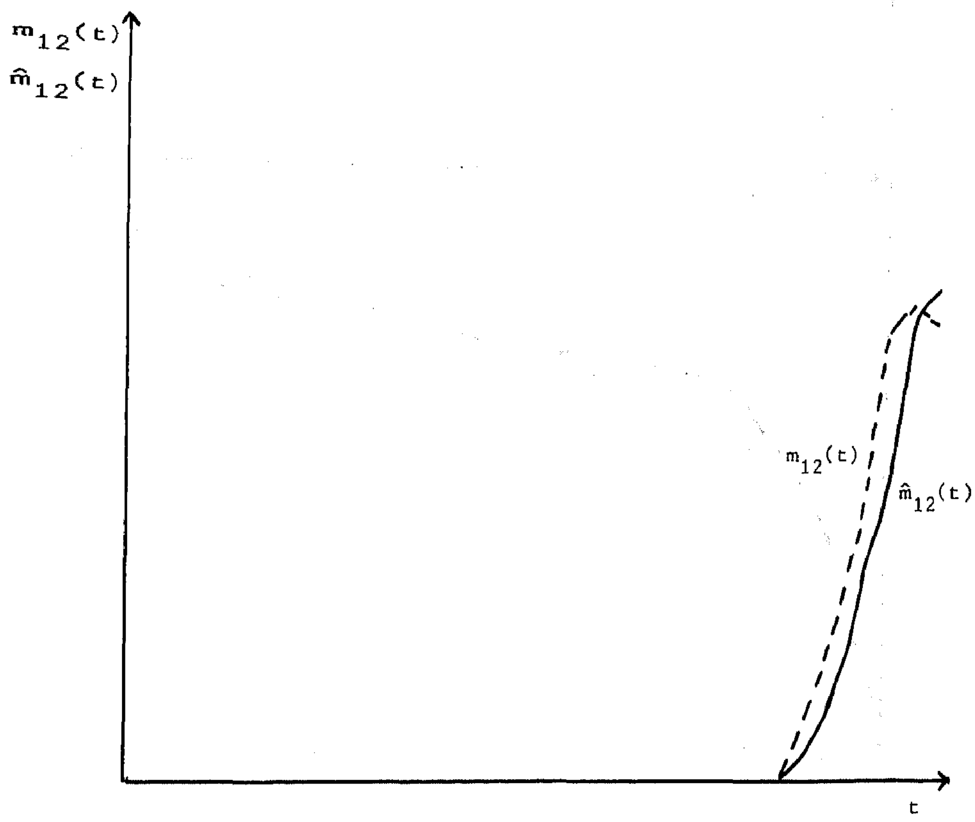

Figure 3.1 : real vs predicted (one-period-ahead) market share (product 10, equation 8)

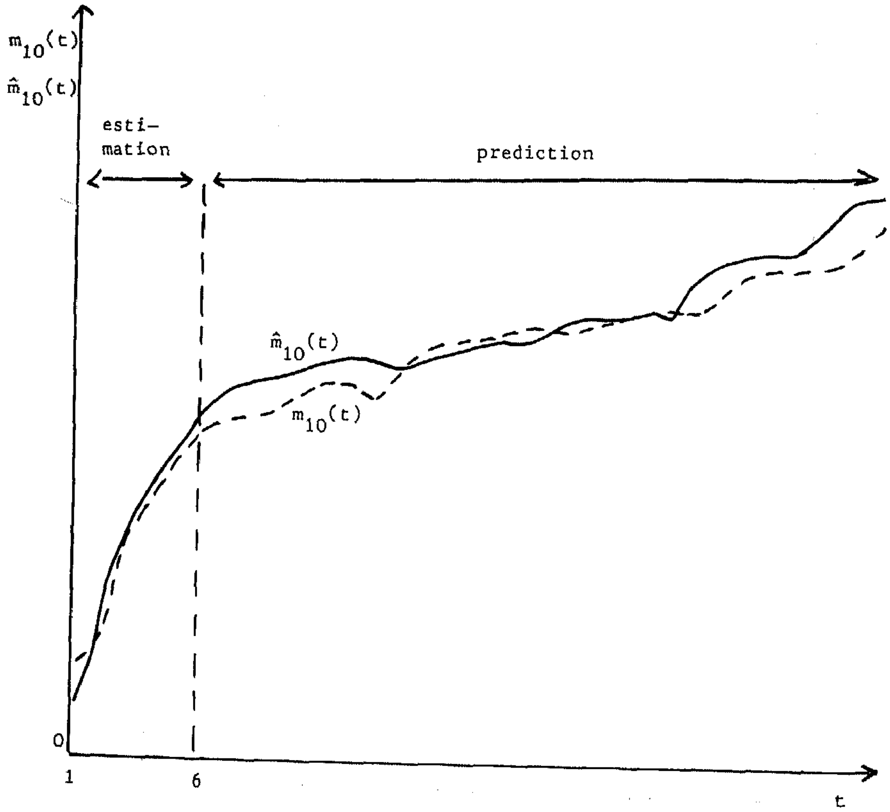

Figure 3.2 : real vs predicted (one-period-ahead) market share (product 11, equation 8)

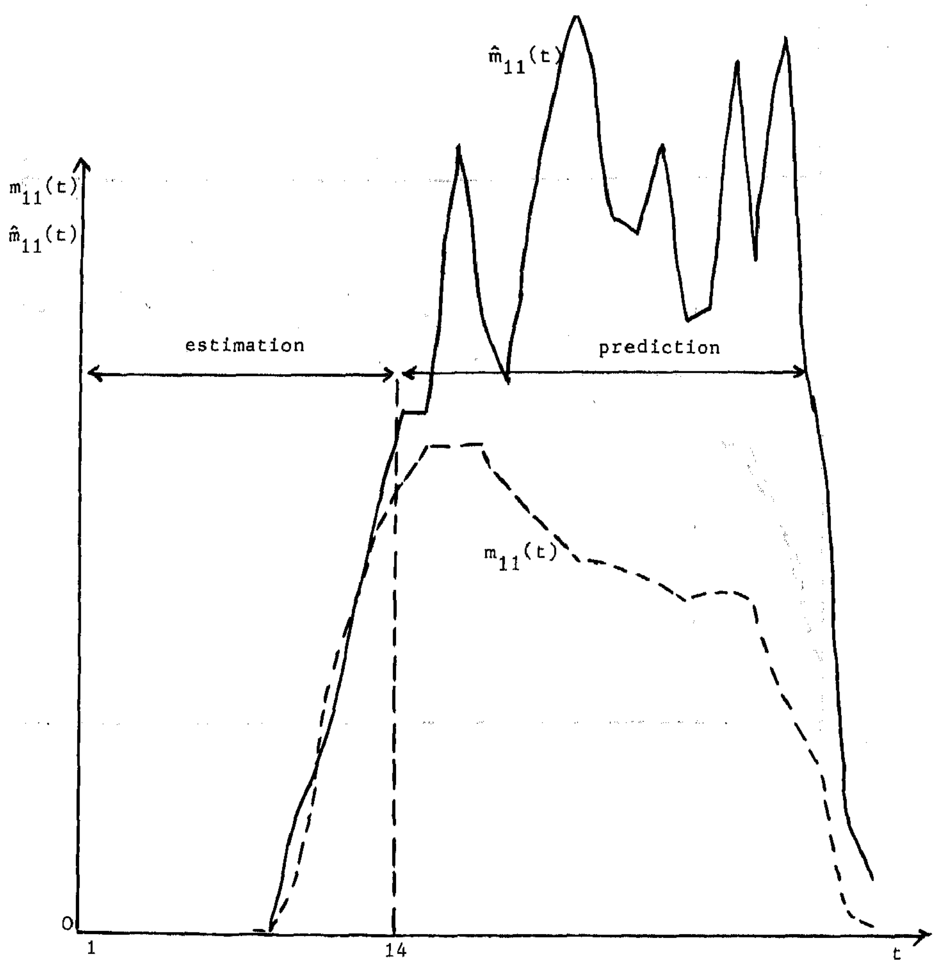

Figure 4.1 : real vs predicted (one-period-ahead) market share (product 10, equation 10)

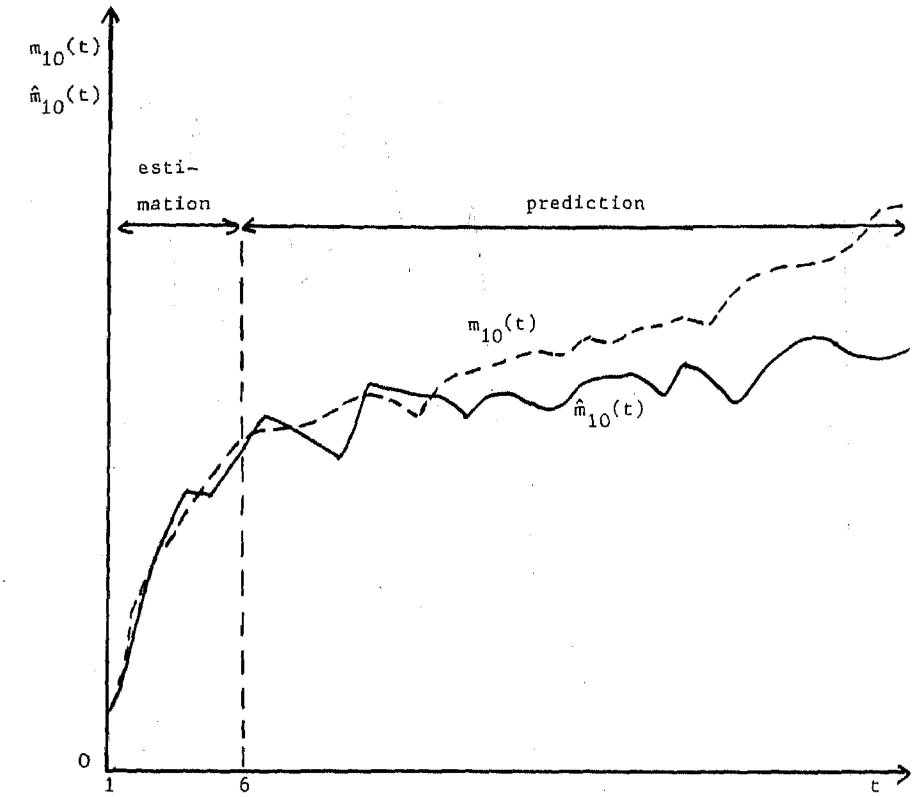

Figure 4.2 : real vs predicted (one-period-ahead) market share (product 11, equation 10)

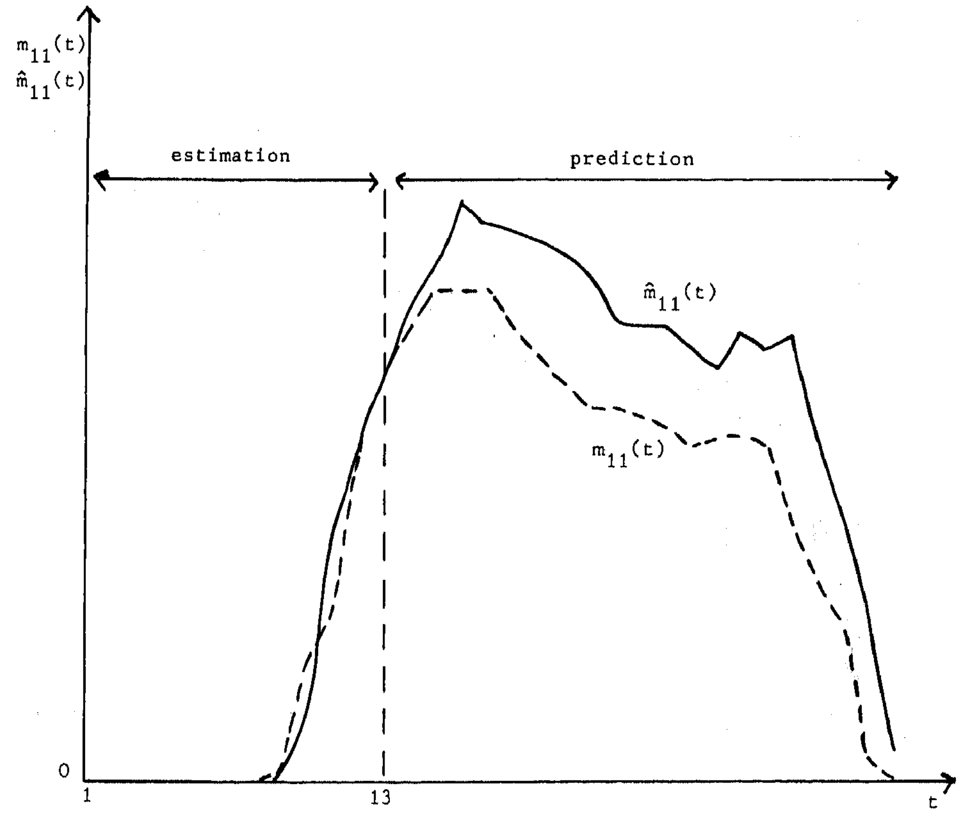

Figure 5.1 : real vs predicted (one-period-ahead) market share (product 10, equation 11.1)

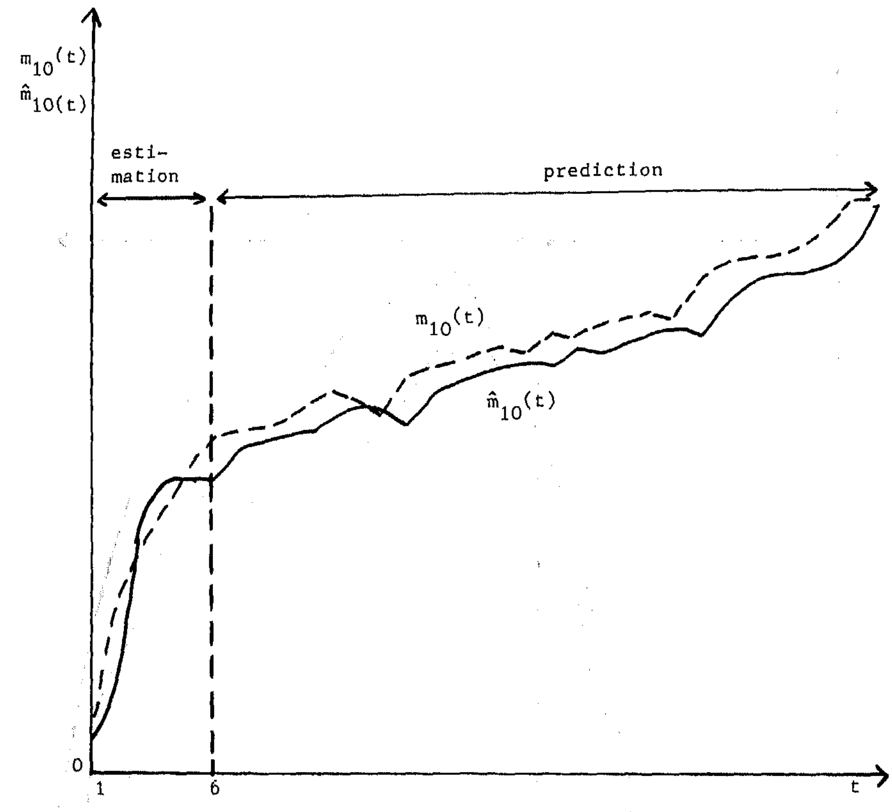

Figure 5.2 : real vs predicted (one-period-ahead) market share (product 11, equation 11.2)

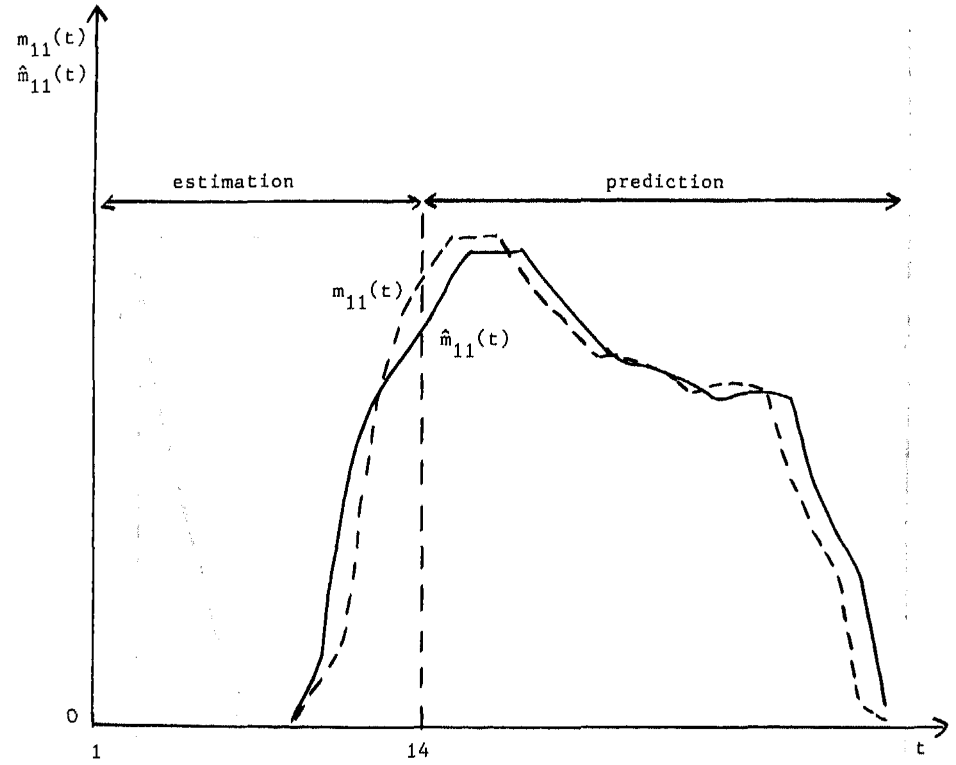

Figure 5.3 : real vs predicted (one-period-ahead) market share (product 12, equation 11.3)

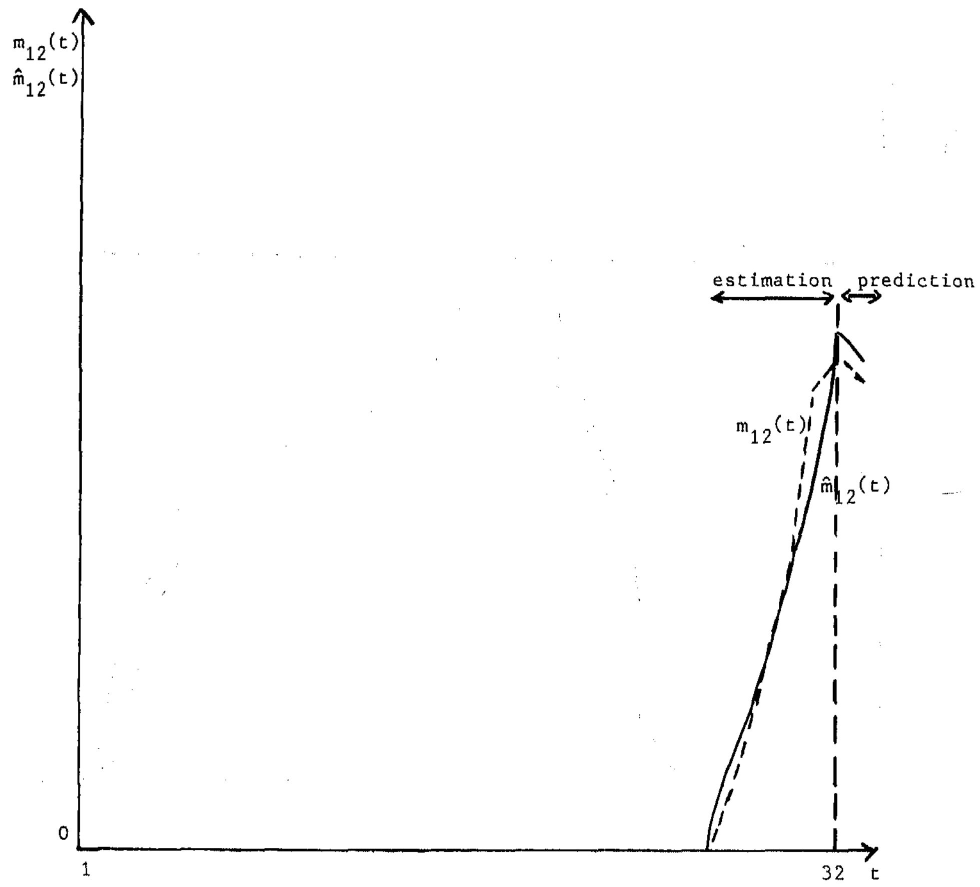

Figure 6.1 : real vs estimated market share
(product 10, equation 8)
(a_1 and α estimated on alle observations)

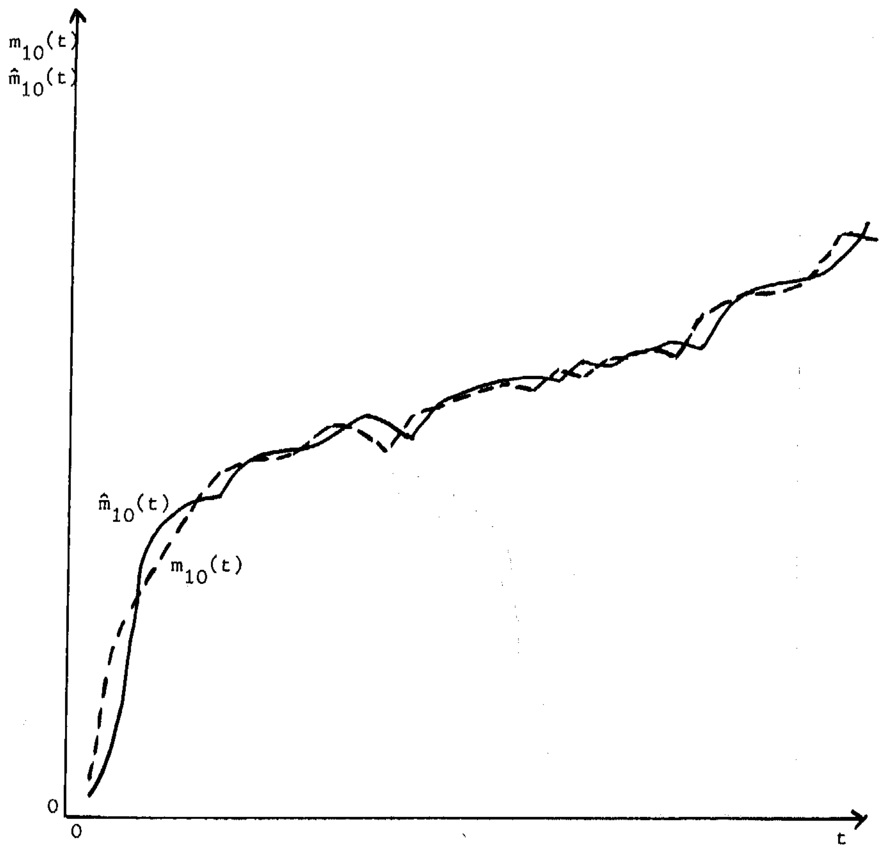

Figure 6.2 : real vs estimated market share
(product 11, equation 8)
(a_1 and α estimated on all observations)

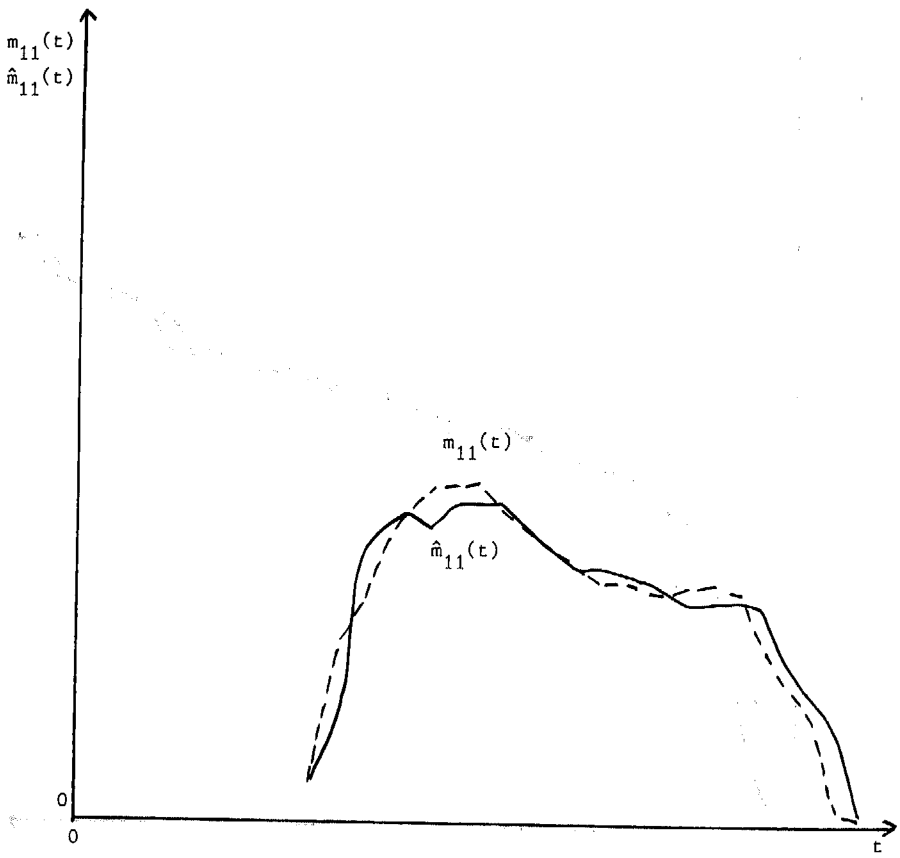

Figure 7.1 : long term market share predictions (first year) for product 10 based on 3, 4 and 6 observations

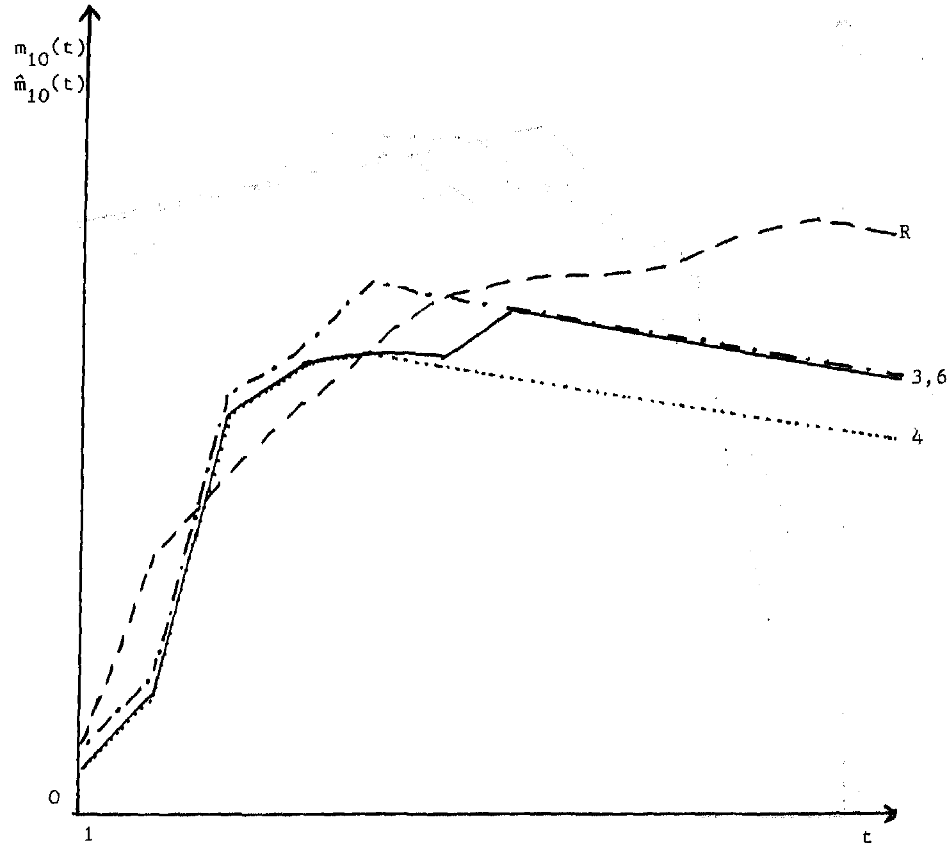

Figure 7.2 : long term market share predictions (first year) for product 11 based on 3, 4 and 6 observations

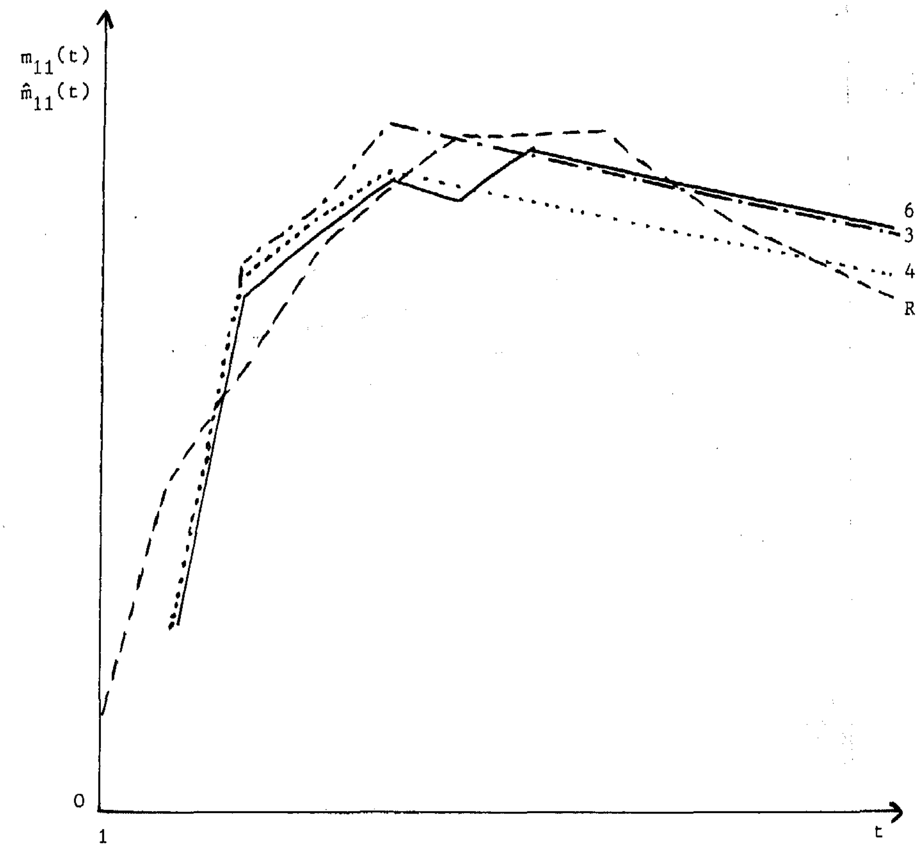

Figure 7.3 : long term market share predictions (first year) for product 12 based on 3, 4 and 6 observations

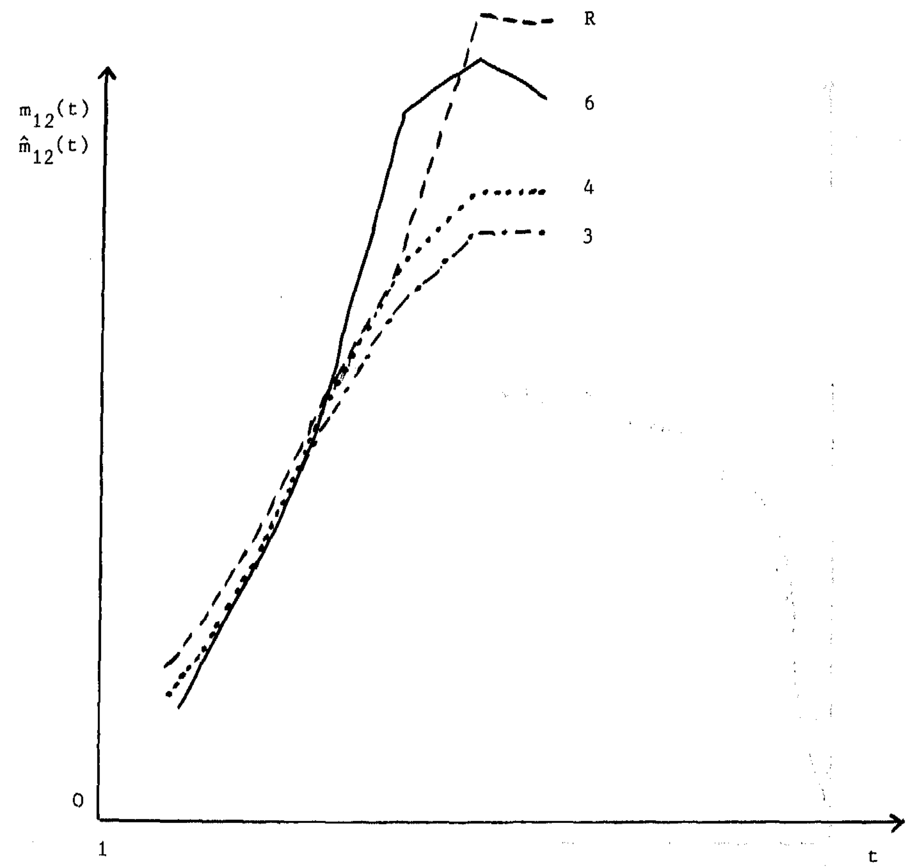

Figure 8.1 : long term market share predictions (33 periods) for product 10 based on 7, 12 and 20 observations

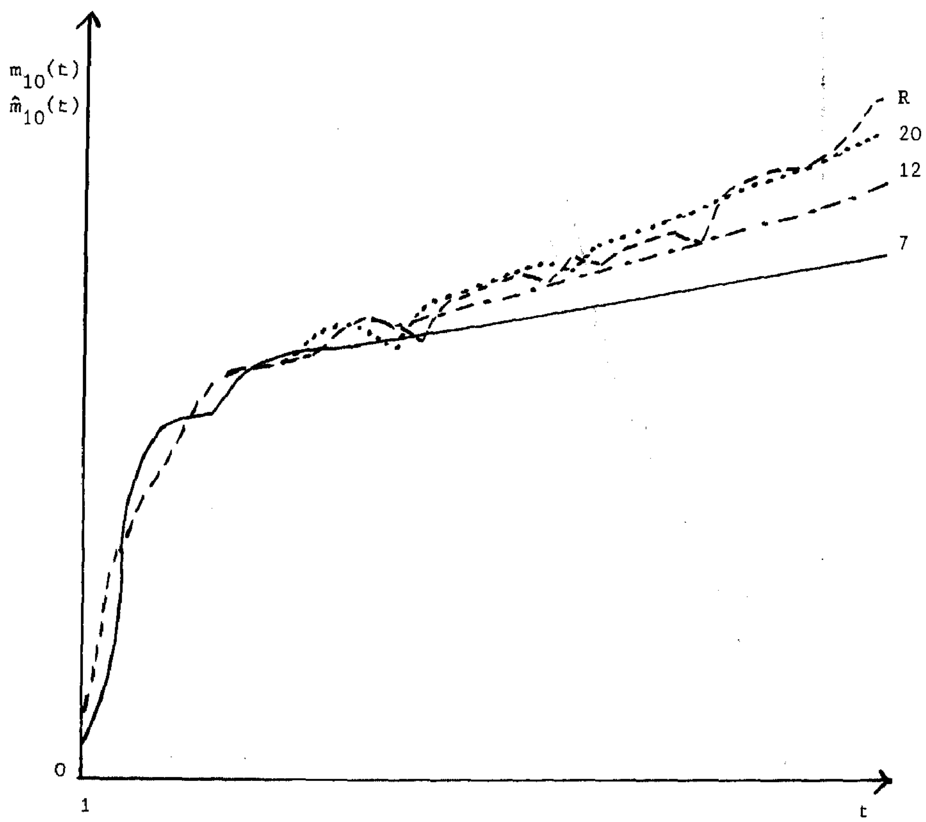

Figure 8.2 : long term market share predictions (27 periods) for product 11 based on 7, 12 and 20 observations