The Repeated Lake Game[1]

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Abstract

In common property ecological systems such as shallow lakes, an increasing number of users leads to additional Nash equilibria with considerably lower welfare. When the game is repeated, these bad outcomes can be turned into an advantage.

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1. Introduction

Lakes provide a variety of services such as water for irrigation, drinking and industrial uses, fishery, recreation and aesthetic enjoyment. But lakes are also used for dumping pollutants such as phosphorus from agricultural activities. These services are conflicting in the sense that the release of too many pollutants makes the lake useless in the other services. Especially shallow lakes have been studied intensively and it has been shown that due to internal loading processes, a sudden loss of ecological services may occur which is very difficult to restore (Carpenter and Cottingham, 1997; Scheffer, 1997). Lakes are usually also open-access resources. If a lake is used by a number of communities, open access implies the lake is not managed efficiently. In a Nash equilibrium the communities will release more phosphorus than they would in case of joint management of the lake, with a welfare loss as the result of that.

It has been shown that the internal loading processes in the lake also affect the number of Nash equilibria of this game (Mäler, Xepapadeas and de Zeeuw, 2000). More specifically, for a low number of communities the Nash equilibrium is unique in their model, with a modest welfare loss, but when the number of communities is increased, a second Nash equilibrium occurs with a high loss in welfare. This result is worrisome at first sight but we like to argue in this paper that it may also be turned into an advantage.

Remember that the folk theorem in repeated games teaches us that cooperation can be sustained by an equilibrium in trigger strategies if the discount factor is high enough. The basic idea is that the communities stick to efficient loadings of phosphorus under the threat that deviation triggers the repeated static Nash equilibrium with lower welfare. The higher that loss of welfare, the lower is the minimal discount factor for which cooperation can be sustained by trigger strategies. An increase in the number of communities that use the lake has many effects. Welfare per community under optimal management changes as well as welfare in the static Nash equilibrium and welfare in case of cheating but as long as the static Nash equilibrium is unique, the minimal discount factor will not change much. However, when the second static Nash equilibrium occurs, this one will be used as trigger threat point and a non-monotonic jump in the minimal discount factor will occur. A similar result was found in the effect of changing numbers of firms on cartel formation in repeated Bertrand oligopolies with capacity constraints (Brock and Scheinkman, 1985).

2. The lake model

The most important indicator for the pollution of lakes is the amount of phosphorus sequestered in algae. The essential dynamics of the pollution process (called lake eutrophication) is given by (see Carpenter, Ludwig and Brock, 1999)

(1)P dot (t) ~=~ L `-` sP(t) `+` r{P sup 2 (t)} over {P sup 2 (t) `+` m sup 2}, ~P(0) `=` P sub 0 ,

where P is the amount of phosphorus in algae, L (for loading) is the release of phosphorus on the lake, s is the rate of loss consisting of sedimentation, outflow and sequestration in other biomass, r is the maximum rate of internal loading and m is the anoxic level (for an extensive treatment of the lake model see Carpenter and Cottingham, 1997, or Scheffer, 1997). It is assumed that the lake dynamics is much faster than the changes in the agriculture activities that are responsible for the release of phosphorus. It follows that the loading L can be treated as a parameter: each level L leads to a steady-state level P for the differential equation (1).

By substituting $x = P/m$, $a = L/r$, $b = sm/r$ and by changing the time scale to rt/m, equation (1) can be rewritten as

(2)x dot (t) ~=~ a `-` bx(t) `+` {x sup 2 (t)} over {x sup 2 (t) `+` 1}, ~x(0) `=` x sub 0 .

For most lakes it is found that $½ < b < 3\ 3/8$. Figure 1 depicts for $b = 0.6$ the steady-states x of differential equation (2) as a function of the loading parameter a. If the loadings a are increased from 0, at a certain point the steady-state phosphorus level x jumps to a high level which explains the sudden loss of ecological services. It is said that the lake flips from an oligotrophic state to a eutrophic state. If the loadings a are gradually decreased again, the lake remains eutrophic for a while before it flips back to an oligotrophic state (hysteresis effect).

Mäler, Xepapadeas and de Zeeuw (2000) consider the situation that n communities have open access to the lake and introduce welfare indicators $\ln a_i - cx^2$, $i = 1,...,n$, $c > 0$. These indicators reflect the benefits of loading and the damage of accumulated pollution. Optimal management of the lake requires us to solve

(3)maximize ~sum from {i=1} to n `ln `a sub i `-` ncx sup 2, ~s.t. ~ a `-` bx `+` {x sup 2 } over {x sup 2 `+` 1} ~=~ 0, ~a ~=~ sum from {i=1} to n `ln `a sub i .

It is easy to show that the stationary points are given by

(4)a ~=~ bx `-` {x sup 2 } over {x sup 2 `+` 1} ~=~ 1 over c `(b over {2x} `-` 1 over {(x sup 2 `+` 1) sup 2 }),

which is independent of *n*. For *b* = *0.6* and *c* = *2*, figure 1 depicts the result: optimal management leads to one oligotrophic steady-state level of phosphorus.

Mäler, Xepapadeas and de Zeeuw (2000) also show that the Nash equilibria, resulting from

(5)maximize ~ln `a sub i `-` cx sup 2, ~i `=` 1,...,n, ~s.t. ~ a `-` bx `+` {x sup 2 } over {x sup 2 `+` 1} ~=~ 0,

are given by

(6)a ~=~ bx `-` {x sup 2 } over {x sup 2 `+` 1} ~=~ n over c `(b over {2x} `-` 1 over {(x sup 2 `+` 1) sup 2 }).

More precisely, if equation (6) has one solution, one oligotrophic Nash equilibrium results, and if it has three solutions, two Nash equilibria result, one oligotrophic and one eutrophic. It is easy to see graphically, in figure 1, what happens. Compare the right-hand sides of equations (4) and (6): increasing *n* means that the dashed curve is stretched out while the intersection points with the *x*-axis remain in place. For *n* large enough, the curves have two more intersection points, of which the right one is a Nash equilibrium in the eutrophic area. Note that for *n* , the series of oligotrophic Nash equilibria approaches the flip point.

[Insert Figure 1 here]

3. Trigger strategy equilibria

Suppose now that the game in section 2 is infinitely repeated. In each period the communities choose phosphorus loadings, the lake adjusts and welfare levels result. Future welfare levels are discounted with discount factor per period. Trigger strategies prescribe that the communities cooperate but switch to non-cooperative behavior when one of the communities deviates. The switch to the repeated static Nash equilibrium can be seen as a punishment that is credible since it is an equilibrium of the repeated game: trigger strategies are subgame perfect. If it does not pay to deviate unilaterally, trigger strategies are in equilibrium and support optimal management of the lake. This holds if (e.g. Friedman, 1989)

(7)W sub d `+` over {1 `-` } `W sub c ~<~ 1 over {1 `-` } `W sub o ~ ~ ~>~ {W sub d `-` W sub o } over {W sub d `-` W sub c },

where W_d denotes the highest welfare for a community if it deviates and the other communities stick to cooperative behavior, W_c denotes the welfare per community in the Nash equilibrium for the stage game and W_o denotes the welfare per community per period under optimal management. It is, of course, possible to use equilibria in more sophisticated strategies and achieve stronger properties like renegotiation proofness, but we prefer to use the simplest framework to make our point. Welfare levels W_o and W_c are given by $ln\ a_0$ - $ln\ n$ - $2x_0^2$ and $ln\ a_c$ - $ln\ n$ - $2x_c^2$, where (a_o, x_o) is the solution of equation (4) (for $b = 0.6$ and $c = 2$) and (a_c, x_c) is a solution of equation (6). If the number of communities n increases, both welfare levels W_o and W_c decrease by $ln\ n$ but W_c decreases more because (a_c, x_c) moves away from the optimal point (a_o, x_o). More importantly, however, for n big enough ($n > 3$) two Nash equilibria occur, one oligotrophic and one eutrophic. Because the Nash equilibrium is used as a threat, the eutrophic one is chosen, so that moving from $n = 3$ to $n = 4$ shows a significant drop in W_c. The numbers are given in table 1. It remains to derive W_d. Suppose that community i decides to deviate, the best it can do is described by

(8)maximize ~ln `a sub i `-` cx sup 2 ~s.t. ~ a sub i `+` {n `-` 1} over n `a sub o `-` bx `+` {x sup 2 } over {x sup 2 `+` 1} ~=~ 0,

(9)a sub i ~=~ bx `-` {x sup 2 } over {x sup 2 `+` 1} `-` {n `-` 1} over n `a sub o ~=~ 1 over c `(b over {2x} `-` 1 over {(x sup 2 `+` 1) sup 2 }).

which leads to

It follows that W_d is simply given by $ln\ a_{id}$ - $2x_d^2$, where (a_{id}, x_d) is the solution of equation (9) (for $b = 0.6$ and $c = 2$). The numbers are presented in table 1. The welfare level W_d decreases but not as much as the welfare level W_o under optimal management. The reason is that a large number of cooperating communities is relatively beneficial for the one that deviates.

[Insert Table 1 here]

We are now ready to make our point. As can be seen from table 1, when n is increasing from 2 to 3 and from 4 onwards, the minimal discount factor for which optimal management of the lake can be sustained by trigger strategies, increases gradually. However, when n increases from 3 to 4, this value drops dramatically. The occurrence of a second Nash equilibrium of the static game, with much lower welfare, threatens a much stronger punishment in case of deviation so that also for low discount factors, deviation is deterred.

In order to keep our result transparent, we have chosen to present it in a simple framework with fixed parameter values. It is not so easy to derive the result analytically, because the dependence of the minimal discount factor $_m$, given by the right-hand side of inequality (7), on the number of communities n is rather complicated. It is shown in figure 2 how the minimal discount factor $_m$ depends on n if the parameter b is disturbed to $b = 0.58$ and $b =$

0.62. The qualitative pattern remains the same. Numerical calculations for other parameter values show the same picture.

[Insert Figure 2 here]

4. Conclusion

The folk theorem in repeated games teaches us that optimal management of a shallow lake, seen as repeated game, can be sustained by trigger strategy equilibria for high enough discount factors. How high the discount factor should be depends on several factors such as how serious the threat is in case of deviation. Previous papers have shown that when the number of communities that use the lake is increased, additional Nash equilibria occur with low welfare. This paper employs that result to show that at some point a further increase in the number of communities leads to a large decrease in the minimal discount rate for which optimal management can be sustained. What may be seen as a negative result in the static context, can be turned into a positive one when the game is repeated.

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<u>Table 1</u>

n	W_o	W_c	W_d	>
1	-2.507			
2	-3.200	-3.214	-3.179	0.606
3	-3.606	-3.635	-3.546	0.669
4	-3.893	-7.514	-3.792	0.027
5	-4.117	-8.656	-3.972	0.031

Figure 1 The lake model under optimal management

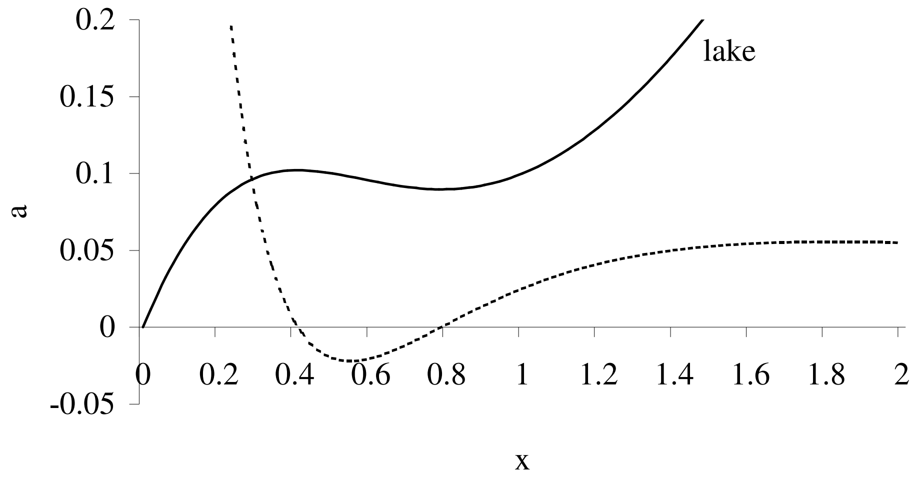

Figure 2 The minimal discount factor as a function of the number of communities