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# Tail Estimates and the EMS Target Zone*

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## Abstract

Characteristically, distributions of exchange-rate returns are fat-tailed. We use a nonparametric tail-index estimator based on extreme value theory for seven EMS currencies between April 1979 and October 1991. We find that the behavior of the Belgian franc, the Danish Krone, the French franc, and the Italian lira has become significantly less fat-tailed over time. We attribute this to the decline in the exchange-rate variance as observed in the EMS, which according to the target-zone literature should lead to a convergence of fixed exchange-rate behavior to that of floating rates. A comparison of tail estimates for the Deutsche mark and dollar exchange rates supports this notion.

## 1. Introduction

A well-established stylized fact in the empirical research on exchange-rate changes is the fat-tailedness of the statistical distribution. The consequences of this finding for the choice of distribution and for testing stability over time are less clear. Both Student-$t$ and stable distributions have been suggested as candidates for modeling exchange-rate return distributions.[1] These two distributions are not nested in the parameter space, however, and hence, results appear to depend on the maintained distributional hypothesis.

In this paper we apply extremal value analysis using the econometric techniques introduced by Koedijk, Schafgans, and de Vries (1990). Extremal analysis investigates the distribution of the maximum (minimum) in large samples, thereby determining the shape of the tails of a distribution. In fact, the limit law for the maximum is characterized by the so-called tail index, $\alpha$, which happens to be equal to the number of moments that exist. Thus $\alpha$ is a good indicator for the thickness of the tails. Both the stable and the Student-$t$ distribution are nested within the limit law for extremes. Using direct estimates of this tail index, we test for possible regime switches affecting the type of distribution. The gain of this procedure is that one can nest and test for different tail sizes. The loss consists of information about the center characteristics of the distribution. Given the predominance of outliers in exchange-rate return series, however, one may benefit from this trade-off. In this paper, we apply extremal value analysis to the case of the European Monetary System (EMS). One of the unresolved issues with respect to the EMS is whether and to what extent its functioning has changed over time. We contribute to the issue by empirically investigating the tail characteristics of the distributions of seven currencies participating in the EMS between March 1979 and October 1991.

Using the empirical estimates of the tail index, $\alpha$, first we formally test for the existence of finite second moments in EMS exchange-rate returns ($H_0$: $\alpha < 2$ and $H_1$: $\alpha \geq 2$), on which, for example, the applicability of the Central Limit Theorem depends. Second, we test for the stability of the tail index across subperiods. We find substantial instability in the EMS distributions over time. Three possible

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explanations of the observed instability are evaluated: the decline in frequency and magnitude of realignments over time, the change in intervention behavior of EMS monetary authorities over time, and the change in target-zone behavior implied by the decline in the variance of exchange-rate returns. This last factor appears to be the most important.

## 2. Extreme Value Theory and the Tail-Index Estimator

Consider $\phi_1, \phi_2, \ldots, \phi_n$ to be a stationary sequence of independently and identically distributed (iid) exchange-rate returns with distribution function $F(\cdot)$. Define $M_n$ as the maximum of this sequence of returns:

$$M_n: = \max(\phi_1, \phi_2, \ldots, \phi_n). \tag{1}$$

It may then be shown that the distribution function $F^n(x)$ of $M_n$ for large $n$ converges towards the same limiting distribution $G(x)$, independent of whether the exchange-rate returns were generated by a Student-$t$ or sum-stable distribution. As the competing distributions are thus nested within the same limit law $G(x)$, no maintained hypothesis about the correct $F(x)$ is required. The limiting distribution $G(x)$ is of the following form, with $\gamma > 0$ and the tail index $\alpha$ equal to $1/\gamma$:

$$\begin{aligned} G(x) &= 0, & x < 0, \\ &= \exp(-x)^{-1/\gamma} = \exp(-x)^{-\alpha}, & x \geqslant 0. \end{aligned} \tag{2}$$

Relevant references on this result are Mood, Graybill and Boes (1974, p. 261) and Leadbetter, Lindgren, and Rootzen (1983, ch. 1). The latter group have shown (1983, ch. 3) that the theory also holds in case the assumption of independence for the exchange-rate returns is inappropriate, provided the dependency is not too strong.[2]

For the family of symmetric stable Paretian distributions the tail index $\alpha$ in (2) may be interpreted as the characteristic exponent of the stable distribution, which ranges between 0 and 2. For the class of Student-$t$ distributions the tail index $\alpha$ in (2) equals the number of degrees of freedom of the distribution, ranging from zero to infinity. Roughly speaking, the lower the value of $\alpha$, the thicker are the tails of the distribution, all other things being equal.

Recently, the following simple and efficient estimator of the tail index has been proposed:

$$\hat{\gamma} = 1/\hat{\alpha} = \frac{1}{m} \sum_{i=1}^{m} [\log \phi_{(n+1-i)} - \log \phi_{(n-m)}]. \tag{3}$$

Here, $n$ represents the total number of return observations and $m$ the number of tail observations used to estimate $\alpha$. The statistic $\hat{\gamma}$ first appears in Hill (1975). Mason (1982) proves that under some regularity conditions $\hat{\gamma}$ is a consistent estimator for $\gamma$. Goldie and Smith (1987) show that $(\hat{\gamma} - \gamma)m^{1/2}$ is asymptotically normal with mean zero and variance $\gamma^2$. Consequently, $\hat{\alpha}$ is also asymptotically normal with mean $\alpha$ and variance $\alpha^2/m$, and asymptotic confidence intervals may be constructed to test specific hypotheses. An empirical application of this estimator may be found in Koedijk, Schafgans and De Vries (1990).

As is clear from (3), the above estimator only uses the positive (right) tail of exchange-rate returns to estimate $\alpha$ and neglects the information content of the large negative observations in the other tail. Conditional on the right and left tails having the same tail index, for which we test explicitly, we combine the information in the right and left tails by taking absolute values of the exchange-rate returns before ordering them and applying (3). This way, the precision of our tail index estimates may be significantly improved. The number of tail observations $m$ to be used is determined through Monte Carlo simulations.[3]

## 3. Data Description

Our data consist of weekly Wednesday quotations of the Dutch guilder, Belgian franc,[4] Italian lira, Irish punt, Danish krone, British pound, and French franc quoted against the German mark. The full sample covers the period starting 4 April 1979 and ending 31 October 1991.

In many studies of the EMS at least two subperiods are distinguished. The first subperiod is usually taken to start with the founding of the EMS in March 1979 and to end in March 1983. This period is characterized by strong deflationary policies in all EMS countries, though with different timing and at different speeds, and by frequent realignments of the central EMS exchange rates. The second subperiod is often taken from March 1983 to the present. Since March 1983 realignments have become less frequent and smaller in magnitude. Only five realignments have taken place between 1983 and October 1991, four of which occurred between 1983 and 1987.

Here, we use September 1987 as a second watershed, following, for example, Dominguez and Kenen (1992). At that time, the countries participating in the EMS reached the so-called Basle-Nyborg Agreement, to deter speculation and avoid intramarginal intervention.

As a consequence, we divided the full sample into three subperiods of approximately equal length. Our first subperiod covers 4 April 1979 to 22 June 1983 (220 observations); our second subperiod covers 29 June 1983 to 9 September 1987 (220 observations), and our third subperiod covers 16 September 1987 to 31 October 1991 (216 observations). The periods encompass seven, four, and one realignment respectively.

In applied work with exchange rates, returns generally are preferred to levels. Where the logarithm of the spot rate is nonstationary and often strongly skewed, returns are stationary and exhibit little or no serial correlation. In general, sample moments of the empirical return distributions indicate absence of skewness for freely floating currencies, but may be skewed for other exchange-rate regimes. The kurtosis usually points to fatness in the tails. The EMS data reported in Tables 1A to 1C, when scrutinized by the usual tests, accord well with these stylized facts. The weaker currencies such as the Italian lira, Irish punt, and French franc tend to be skewed, as can be seen in Tables 1A and 1B. This is probably due to recurrent devaluations. Note that skewness has disappeared, except for the Italian lira, in the third subperiod. All rates are significantly leptokurtic as reported by the $J$ test on overall normality.[5] All calculated $J$ values considerably exceed the critical value of the $\chi^2(2)$ statistic at the 1% significance level of 9.21.

## 4. Empirical Results

### *Tail Estimates*

In Table 2 we summarize the results with respect to the tail-index estimates for the seven EMS currencies for the three subperiods. For each period, the first column

*Table 1A. Empirical Distribution Characteristics of* EMS *Currencies, April 1979 to June 1983 (N = 220)*

| *Currency* | *Mean* | *Variance* | *Skewness* | *Kurtosis* | *S Test*[a] | *J Test*[b] |
|---|---|---|---|---|---|---|
| Dutch guilder | 0.017 | 0.11 | 0.21 | 5.48 | 0.13 | 58.17 |
| Belgian franc | 0.100 | 1.45 | 0.05 | 5.21 | 1.08 | 45.02 |
| Italian lira | 0.128 | 0.59 | 1.80 | 15.02 | 2.16 | 1443.07 |
| Irish punt | 0.090 | 0.28 | 1.99 | 14.52 | 2.29 | 1361.75 |
| Danish krone | 0.120 | 0.49 | 1.05 | 17.30 | 2.70 | 1915.75 |
| British pound | 0.004 | 1.63 | 0.52 | 5.14 | 1.62 | 52.15 |
| French franc | 0.122 | 0.46 | 5.14 | 41.10 | 3.91 | 14274.41 |

[a] *S* Test: 2 (# obs. below mean − $n/2$) $\sqrt{n}$ ~ $N(0, 1)$ for $n \rightarrow \infty$.
[b] *J* Test $n$[Skewness$^2$/6 + (kurtosis-3)$^2$/24] ~ $\chi(2)$.

*Table 1B. Empirical Distribution Characteristics of* EMS *Currencies, June 1983 to September 1987 (N = 220)*

| *Currency* | *Mean* | *Variance* | *Skewness* | *Kurtosis* | *S Test*[a] | *J Test*[b] |
|---|---|---|---|---|---|---|
| Dutch guilder | 0.003 | 0.05 | 0.05 | 7.87 | 0.27 | 215.48 |
| Belgian franc | 0.015 | 0.34 | 0.17 | 5.77 | 0.13 | 71.20 |
| Italian lira | 0.092 | 0.25 | 1.58 | 11.40 | 2.02 | 738.717 |
| Irish punt | 0.078 | 0.35 | 5.34 | 58.00 | 2.15 | 28784.38 |
| Danish krone | 0.035 | 0.10 | 0.47 | 4.90 | 0.81 | 40.92 |
| British pound | 0.120 | 1.36 | 0.28 | 4.31 | 1.35 | 18.52 |
| French franc | 0.050 | 0.13 | 4.17 | 44.59 | 2.16 | 16492.19 |

[a] *S* Test: 2 (# obs. below mean − $n/2$) $\sqrt{n}$ ~ $N(0, 1)$ for $n \rightarrow \infty$.
[b] *J* Test $n$[Skewness$^2$/6 + (kurtosis-3)$^2$/24] ~ $\chi(2)$.

*Table 1C. Empirical Distribution Characteristics of* EMS *Currencies, September 1987 to October 1991 (N = 216)*

| *Currency* | *Mean* | *Variance* | *Skewness* | *Kurtosis* | *S Test*[a] | *J Test*[b] |
|---|---|---|---|---|---|---|
| Dutch guilder | 0.001 | 0.009 | 0.04 | 4.99 | 0.27 | 35.52 |
| Belgian franc | −0.003 | 0.110 | 0.50 | 4.81 | 0.14 | 38.55 |
| Italian lira | 0.015 | 0.110 | 1.51 | 8.77 | 2.18 | 381.64 |
| Irish punt | 0.0003 | 0.055 | −0.41 | 5.26 | 0.13 | 52.07 |
| Danish krone | 0.002 | 0.087 | 0.14 | 3.75 | 0.14 | 5.76 |
| British pound | 0.009 | 0.712 | 4.53 | 4.30 | 1.22 | 15.24 |
| French franc | 0.010 | 0.050 | 1.20 | 10.22 | 0.27 | 521.15 |

[a] *S* Test: 2 (# obs. below mean − $n/2$) $\sqrt{n}$ ~ $N(0, 1)$ for $n \rightarrow \infty$.
[b] *J* Test $n$[Skewness$^2$/6 + (kurtosis-3)$^2$/24] ~ $\chi(2)$.

*Table 2. Tail Index Estimates*

| | 1979–83 | | | 1983–87 | | | 1987–91 | | |
|---|---|---|---|---|---|---|---|---|---|
| Currency | $\hat{\alpha}$ | $\hat{\alpha}^+ = \hat{\alpha}^-$ | $\hat{\alpha} = 2$ | $\hat{\alpha}$ | $\hat{\alpha}^+ = \hat{\alpha}^-$ | $\hat{\alpha} = 2$ | $\hat{\alpha}$ | $\hat{\alpha}^+ = \hat{\alpha}^-$ | $\hat{\alpha} = 2$ |
| Dutch guilder | 2.55 (0.46) | 0.29 | 1.20 | 2.08 (0.37) | 0.28 | 0.22 | 3.01 (0.54) | 0.88 | 1.87 |
| Belgian franc | 2.13 (0.48) | 1.54 | 0.27 | 2.50 (0.45) | 1.67 | 0.75 | 3.17 (0.57) | 0.00 | 2.05 |
| Italian lira | 1.47 (0.26) | 0.69 | −2.04 | 2.52 (0.45) | 1.76 | 1.16 | 2.69 (0.48) | 1.04 | 1.44 |
| Irish punt | 2.46 (0.44) | 0.11 | 1.05 | 1.94 (0.35) | 0.43 | −0.17 | 2.49 (0.45) | 1.20 | 0.74 |
| Danish krone | 1.51 (0.27) | 0.35 | −1.81 | 3.66 (0.66) | 1.03 | 2.52 | 3.67 (0.66) | 0.03 | 2.53 |
| British pound | 2.52 (0.45) | 0.08 | 1.16 | 2.77 (0.49) | 1.12 | 1.54 | 2.73 (0.49) | 1.25 | 1.49 |
| French franc | 1.57 (0.28) | 1.93 | −1.53 | 2.10 (0.38) | 0.31 | 0.26 | 3.42 (0.61) | 0.45 | 2.33 |

Notes: Asymptotic standard errors are below the tail-index estimates in parentheses; $\alpha^+$ and $\alpha^-$ are the right and left tail-index estimates respectively.

contains the tail-index estimate using both tails ($\hat{\alpha}$), with asymptotic standard errors given in parentheses below the estimates. In the next column we test for equality of the tail indices of the right and left tails of the return distributions, using the asymptotic normality of the tail-index estimates. In the third column we test for $\hat{\alpha}$ smaller or larger than 2.

From Table 2 it appears that all point estimates of $\alpha$ hover around 2. Equality of the left and right tail indices is never rejected, although the French franc in the first period and the Italian lira in the second period are quite close to rejection. In all cases, the test statistic is below the 5% critical value of 1.96. This warrants simultaneous use of both tails in the further analysis. From now on, we will focus, therefore, on $\hat{\alpha}$.

The null hypothesis of $H_0$: $\alpha < 2$ against the alternative $H_1$: $\alpha \geqslant 2$, is rejected at the 5% level if the test statistic exceeds 1.64. For the alternative test of $H_0$: $\alpha \geqslant 2$ versus $H_1$: $\alpha < 2$, $H_0$ is rejected at the 5% level if the test statistic is below −1.64. Summarizing the results in the last column of Table 2, we conclude that $H_0$: $\alpha \geqslant 2$ is rejected for the Italian lira and Danish krone before 1983, while $H_0$: $\alpha < 2$ is rejected for the Danish krone after 1983 and for the Belgian franc, the French franc, and the Dutch guilder after 1987. This suggests the existence of finite second moments for at least four of the seven EMS currencies.

Note that for those currencies where the null hypothesis of $\alpha < 2$ is rejected, the sum-stable distribution is rejected as a potential generating distribution of exchange-rate returns. Similarly, Student-*t* distributions with less than two degrees of freedom are rejected for these currencies.

Comparing the distributional properties of exchange-rate returns is hazardous, as results have generally been obtained through the use of methods requiring a maintained hypothesis about the underlying distribution. Nevertheless, our tail-index estimates for the first period of the EMS are close to the level reported by Westerfield (1977) for the Bretton Woods era. More recent exchange-rate behavior in the EMS yields tail estimates closer to those reported by Boothe and Glassman (1987) for floating rates. An intriguing aspect from Table 2 is the apparent rise in the tail index estimates over time: the average tail index is 2.03 for the period April 1979 to April 1983 and 3.03 for the period September 1987 to October 1991. For some countries, the evidence even tentatively suggests that the EMS exchange-rate distributions have experienced a regime switch from sum-stable to Student-*t*. We therefore turn to the issue of regime changes and their effect on EMS exchange-rate distributions.

### *Regime Changes*

Next, we test whether the exchange-rate behavior of a specific country has significantly changed over time. To do so we formally test for the equality of the $\alpha$ estimates from Table 2 over time. We use the asymptotic normality of the tail-index estimates and, thus, of the difference between them. The results are shown in Table 3.

Table 3 shows a tendency for the tail-index estimates to rise over time. The economic interpretation of this rise in tail estimates over time is that extreme exchange-rate changes in the EMS have become less frequent over time. For the traditionally weak currencies, in particular, we find significant increases in the tail index. For the Italian lira and the Danish krone the tail index is significantly higher after 1983, while the tail-index estimate for the French franc is significantly higher after 1987 at the 10% level. The results for the French franc corroborate the results in Dominguez and Kenen (1992) that the French franc experienced a regime change in 1987. They attribute this result to the fact that the French franc made fuller use of the band width after September 1987. For Belgium, a gradual rise in the tail index is observed. The hypothesis that the Belgian franc tail index is higher in the third period than in the first cannot be rejected at the 10% level.

We now turn to possible explanations of the observed phenomenon of systematically rising tail-index estimates over the lifetime of the EMS. First, we investigate to what extent our results are influenced by the realignments in the EMS. To this end, we estimate tail indices over subperiods for each country with the realignment

*Table 3. Stability of Tail Indices Across Subperiods*

| | *Test Statistics* | | |
|---|---|---|---|
| *Currency* | $\hat{\alpha}_1 = \hat{\alpha}_2$ | $\hat{\alpha}_2 = \hat{\alpha}_3$ | $\hat{\alpha}_1 = \hat{\alpha}_3$ |
| Dutch guilder | 0.80 | 1.42 | 0.65 |
| Belgian franc | 0.63 | 0.92 | 1.52 |
| Italian lira | 2.00 | 0.25 | 2.27 |
| Irish punt | 0.92 | 0.97 | 0.05 |
| Danish krone | 3.03 | 0.01 | 3.03 |
| British pound | 0.37 | 0.06 | 0.31 |
| French franc | 1.13 | 1.83 | 2.74 |

Notes: Test statistics have an asymptotic $N(0, 1)$ distribution. Subscripts of $\hat{\alpha}$ refer to the different subperiods 1, 2, and 3.

weeks excluded. Second, we investigate to what extent different intervention policies over time by the EMS countries can explain the observed rise in tail-index estimates. Since actual intervention data are kept secret and are not published, we perform a Monte Carlo experiment to simulate the effect of interventions on EMS tail estimates. Third, we analyze the consequences of the observed decline in the variance of EMS exchange-rate returns for the tail-index estimates in a target-zone framework. For this purpose, we compare the tail estimates for the EMS exchange rates relative to the deutsche mark to tail estimates for the corresponding US dollar exchange rates.

### *Tail Estimates Adjusted for Realignments*

To investigate whether our results are sensitive to the nonuniform occurrence of realignments over the full sample period, we estimate the tail indices with the realignment weeks excluded from the sample. The results are shown in Table 4.

Exclusion of the 7 realignments between 1979 and 1983 increases the tail-index estimates for all countries participating in the exchange-rate mechanism (ERM) except for the Netherlands. Exclusion of the 4 realignment weeks in the period 1983–87 leads to somewhat higher tail-index estimates for France, Italy, and Ireland.

Overall, the exclusion of realignment weeks thus yields higher tail-index estimates for the weak EMS currencies. That is, part of the relatively strong fat-tailedness of these currencies in the first and second subperiods has been caused by the realignments. This does not change our initial results, however, indicating that there has been a general rise in tail indices over time and that for Denmark, France, and Italy tail-index estimates have risen significantly. As can be inferred from Table 4, the average $\alpha$ increases over time, from 2.24 during 1979–83 to 3.03 during 1987–91. The fourth column of Table 4 tests the equality of tail indices in subperiods 1 and 3 and shows that the rises in the tail indices for France and Denmark remain significant at the 5% level, while the result for Italy is still significant at the 10% level. Only for Belgium is the rise in the tail index insignificant now. Hence, our conclusion that the EMS experienced a significant change in behavior still stands, even when realignments are taken into consideration. We now turn to the effect of different intervention policies on the tail estimates.

*Table 4. Tail Indices with Realignments Excluded*

| | | | | Test Statistics |
|---|---|---|---|---|
| *Currency* | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $\hat{\alpha}_3$ | $\hat{\alpha}_1 = \hat{\alpha}_3$ |
| Dutch guilder | 2.48 | 2.08 | 3.01 | 0.76 |
| Belgian franc | 2.51 | 2.50 | 3.17 | 0.91 |
| Italian lira | 1.68 | 2.93 | 2.69 | 1.78 |
| Irish punt | 2.79 | 2.45 | 2.49 | 0.45 |
| Danish krone | 1.72 | 3.68 | 3.67 | 2.68 |
| British pound | 2.52 | 2.77 | 2.73 | 0.31 |
| French franc | 2.01 | 2.45 | 3.42 | 1.99 |

Notes: Test statistics have an asymptotic $N(0, 1)$ distribution. Subscripts refer to the different subperiods 1, 2, and 3.

*The Effect of Intervention on Tail Estimates: A Simulation Study*

At the start of the EMS a clear distinction was made between marginal and intramarginal intervention.[6] While countries were obliged to support their own currency at the margin of the fluctuation band, intramarginal intervention was to be executed on a voluntary basis and, moreover, was subject to a number of restrictions. Most important is the fact that the Very Short Term Financing facility provided unlimited means to countries for marginal interventions, whereas no financial support was given to intramarginal interventions. Nevertheless, Ungerer et al. (1990) state that intramarginal intervention became the rule rather than the exception rather soon.

For ease of exposition, three intervention regimes can be distinguished over the lifetime of the EMS coinciding with the three subperiods used earlier. In the period 1979–83 both intramarginal and marginal interventions were important. In the period 1983–87, most countries undertook to coordinate their policies, aiming at convergence to the anchor-country, Germany. As a consequence, the role of marginal interventions significantly decreased, while intramarginal interventions increased. In the Basle-Nyborg Agreement in 1987, however, the decision was made to make fuller use of the fluctuation margin and to use the interest rate more frequently in supporting exchange rates, to avoid the use of intramarginal intervention. After 1987, therefore, intramarginal intervention decreased.

To shed some more light on the effects of interventions on the tail behavior of exchange rates, the following simulation experiment was set up. A hypothetical exchange rate versus the German mark is assumed to have a central parity of 3.35386 (equal to the actual French franc parity since 12 January 1987). Weekly percentage changes in this exchange rate have a variance of 0.21903 (equal to the overall variance of French franc exchange-rate innovations over the period 1979–91). The actual exchange rate can deviate at most 2.25% from its parity at any moment. Intervention behavior by monetary authorities is modeled in the following way. The authorities may set an intervention limit at the 2.25% margin (mandatory within the EMS) or lower, at 75%, 50%, or 25% of the total margin. This latter attitude reflects increasing unwillingness to have the exchange rate deviate from the central parity. On the effectiveness of intramarginal or marginal intervention we assume that as soon as the actual exchange rate crosses the intervention limit (25%, 50%, 75%, or 100% of the overall margin), the effect of the intervention is to drive the exchange rate back from the intervention limit towards the central parity. This intervention effect is simulated by drawing from a uniform distribution. The resulting exchange rate equals the intervention limit plus or minus the realization of the draw from the uniform distribution. Different assumptions are made about the characteristics of the uniform distribution to reflect differences in effectiveness. For a relatively ineffective intervention, a uniform distribution is specified that only allows an exchange-rate more towards the parity of at most 25% of the difference between the intervention limit and the parity. For more effective interventions the effect may be larger (50%, 75%, or even 100%).

For each combination of assumed intervention effectiveness and assumed intervention behavior of the monetary authorities (different intervention limits), a Monte Carlo experiment consisting of 500 replications is executed. In each replication, a series of 220 (equal to the number of observations in our subperiods) exchange-rate innovations is randomly drawn from a Student-*t* distribution with 3 degrees of freedom. These innovations are added to the initial level of the exchange rate (assumed to be the parity) and compounded with random draws from the current

*Table 5. Tail Indices Including Simulated Interventions*

| Intervention Limit | Intervention Effectiveness 25% | 50% | 75% | 100% |
|---|---|---|---|---|
| 25% | 4.73 | | | |
| 50% | 3.27 | 4.07 | | |
| 75% | 3.05 | 3.43 | 3.18 | |
| 100% | 2.97 | 3.20 | 3.05 | 2.77 |

uniform distribution as soon as the intervention limit is crossed. Returns from the resulting series are used to estimate the series' tail index. Table 5 contains the average tail-index estimate across replications for each experiment.

In Table 5 two points stand out. First and most important, an increasing intervention limit—given the level of intervention effectiveness—results in lower tail-index estimates. Intuitively, this could have been expected. For a given variance of exchange-rate innovations, a wider effective fluctuation margin allows for larger changes in exchange rates without correction through interventions. This will lead to fatter tails and lower tail-index estimates. Note, though, that according to this, the EMS regime change in 1987, after which wider fluctuation margins were observed, is unable to explain the rise in tail-index estimates.

Second, increasing intervention effectiveness, for a given intervention limit, first leads to higher tail-index estimates but then to lower ones. Intuitively, this finding may be explained by pointing to two opposite effects on the exchange rate: on the one hand, a relatively large shock which makes the exchange rate cross the intervention limit is reduced in size by the compensating intervention, leading to less large shocks and thinner tails; however, if the intervention becomes so effective as to dominate the original move in the exchange rate, tails may become fatter. This latter phenomenon, however, may be more of a theoretical artifact than a real-world issue.

Overall, our results do not suggest that changes in intervention behavior over time have been the causal factor behind the general rise in tail-index estimates. We therefore turn to a discussion of the implications of the decline in exchange-rate variances within a target-zone framework.

### *An Explanation From the Target-Zone Approach*

During the past few years the exchange-rate behavior within a target zone has been formalized.[7] This literature starts from the observation that due to the forward-looking nature of rational expectations models, the presence of a band exerts an influence on the current movements of exchange rates.

Although empirical evidence in favor of the target-zone literature is scarce, the analytical framework gives an attractive theoretical explanation for the kind of circumstances under which the behavior of exchange rates within a band will differ from the behavior of fully floating exchange rates, and under which circumstances they will coincide. One of the implications of the target-zone literature is that for a given exchange-rate band, a decline in the variance of exchange-rate returns makes exchange-rate behavior in the target zone converge to the free float.

Table 1 shows that for all currencies participating in the exchange-rate mechanism of the EMS except Ireland, there has been a monotonic decline in the variance of exchange-rate returns over time. This has been most pronounced for the weaker EMS currencies. Possible explanations for this decrease in variance are convergence of fundamentals, an increase in the credibility of the system, an increased use of interest-rate instruments to protect EMS parities,[8] or a combination of these clearly interdependent factors.

The target-zone literature implies that as a consequence of this variance reduction, tail-index estimates for the various EMS exchange rates against the Deutsche mark should converge to corresponding tail-index estimates relative to floating currencies like, for example, the US dollar.

First, therefore, we estimate the tail estimates of the EMS currencies relative to the US dollar.[9] The results are reported in Table 6.

A comparison of Tables 6 and 3 shows that tail estimates relative to the US dollar tend to be higher[10] than those relative to the Deutsche mark, reflecting the lower probability mass in the tails for floating exchange rates. Moreover, there is no evidence of a change in behavior of the US dollar tail estimates over time, suggesting that the reported change in behavior is specific to the EMS. To further study the difference between Deutsche mark rates and dollar rates we formally test for the equality of the respective tail estimates.

As is apparent from Table 6, in the early 1980s the tail estimates of at least four EMS currencies against the Deutsche mark (Belgian franc, Italian lira, Danish krone, and French franc) are significantly different from the corresponding dollar tail estimates. After 1987 this difference disappears. Hence, our results show a convergence of tail-index estimates for fixed and floating exchange rates for EMS countries over time coinciding with the reduction in exchange-rate variance within the EMS. This is consistent with our null hypothesis based on the target-zone framework for fixed exchange rates.[11] It should be noted however that the recent turmoil in EMS currency markets since September 1992 may have reversed this trend.

We conclude that the observed instability in tail-index estimates within the EMS is largely due to the decrease in exchange-rate variance over time, as it renders the restrictions imposed by the fluctuation bands less binding. The decline in frequency and magnitude of realignments may have played an additional role in raising the tail estimates.

*Table 6. Tail Estimates Relative to the Dollar*

| | | | | Test Statistics | | |
|---|---|---|---|---|---|---|
| Currency | $\hat{\alpha}_1^{us}$ | $\hat{\alpha}_2^{us}$ | $\hat{\alpha}_3^{us}$ | $\hat{\alpha}_1 = \hat{\alpha}_1^{us}$ | $\hat{\alpha}_2 = \hat{\alpha}_2^{us}$ | $\hat{\alpha}_3 = \hat{\alpha}_3^{us}$ |
| Dutch guilder | 3.21 | 3.62 | 3.76 | 0.90 | 2.05 | 0.87 |
| Belgian franc | 3.99 | 3.48 | 3.72 | 2.29 | 1.27 | 0.63 |
| Italian lira | 3.34 | 3.87 | 4.00 | 2.85 | 1.63 | 1.51 |
| Irish punt | 3.58 | 3.93 | 3.67 | 1.44 | 2.52 | 1.48 |
| Danish krone | 3.18 | 4.22 | 4.01 | 2.64 | 0.56 | 0.36 |
| British pound | 3.39 | 2.37 | 3.10 | 1.15 | 0.61 | 0.50 |
| French franc | 3.62 | 3.44 | 4.06 | 2.89 | 1.85 | 0.67 |
| German mark | 3.62 | 3.83 | 3.64 | — | — | — |

Notes: Test statistics have an asymptotic $N(0, 1)$ distribution. Subscripts of $\hat{\alpha}$ refer to the different subperiods 1, 2, and 3.

Our tail-index analysis also sheds some light on the controversy in the literature about the relative "riskiness" of fixed versus floating exchange-rate regimes.[12] It is often argued that floating exchange-rate regimes tend to smooth movements on foreign-exchange markets compared with fixed exchange-rate regimes and that fixed exchange-rate regimes tend to be characterized by larger probabilities of extreme changes.[13] Under the assumption that movements in exchange rates are predominantly caused by divergences in the underlying fundamentals across countries,[14] this generally results in both a higher variance and fatter tails of exchange-rate returns in a fixed regime.

In practice, however, it has often been observed that the variance of both nominal and real exchange-rate returns in a floating-rate regime significantly exceeds the variance in a fixed-rate regime (see, for example, Mussa [1986]), maybe due to erratic and uncoordinated exchange-rate policies, speculation unrelated to fundamentals, or both. At the same time, the literature suggests fatter tails under fixed than under floating rates. Judged by the variance level of exchange-rate returns, floating may result in more riskiness. Judged by the fat-tailedness, however, fixed rates are more risky. This appears to present a trade-off: how much average exchange-rate variability in a floating exchange-rate regime is to be preferred to a particular degree of fat-tailedness (representing discrete and unpredictable large exchange-rate changes) in a fixed exchange-rate regime.

Our results suggest that it may be possible to have a fixed exchange-rate regime with absolute lower variance of exchange-rate returns than under a floating-rate regime, while enjoying comparable fat-tailedness. In general, the difference in fat-tailedness, as measured by the tail-index estimates, is suggested to depend on the degree to which the fixed exchange-rate regime imposes binding restrictions on exchange-rate behavior. This, in turn, depends on the width of the allowed fluctuation margin compared to the degree of convergence and credibility of the system. More precisely, the adherence to a fixed exchange-rate regime may reduce the overall variance of exchange-rate returns by reducing the variance of the underlying fundamentals through coordinated policies. As long as the regime is credible, it may, moreover, reduce exchange-rate variance due to speculation. When, in addition, the fluctuation margins are chosen sufficiently wide for the currently obtained degree of convergence, the need for large realignments will be small, and the fat-tailedness may approach that of a floating exchange-rate regime with a comparable degree of coordination and in the absence of speculation unrelated to fundamentals.

## 6. Conclusions

In this paper we provide new insights in the empirical distribution of seven EMS exchange-rate returns between April 1979 and October 1991 by focusing on the information in the tails of the distribution. We investigate the amount of fat-tailedness and the stability of the distribution's tail behavior over time using a tail-index estimator, $\alpha$, that is based on extreme value theory.

We find considerable kurtosis of exchange-rate returns for all periods and currencies. Point estimates of the tail index $\alpha$ are mostly above 2. The tail estimates of especially the weak EMS currencies increase substantially over time. Stability tests show that the behavior of the Italian lira and the Danish krone changed significantly over time after 1983, while the behavior of the French franc changed after 1987. This latter result corroborates the work by Dominguez and Kenen (1992). For the

Belgian franc a significant difference between tail estimates for the first and the third subperiod is found.

We investigate three possible explanations of this phenomenon: the effect of realignments, the effect of a change in intervention policies over time, and the effect of a decline in variance, as implied by the target-zone literature. While the decrease in frequency and magnitude of realignments over time has indeed contributed somewhat to the observed rise in tail estimates, it cannot be the complete story. A simulation experiment on different types of intervention policy provides no clear-cut evidence supportive of a rise in EMS tail estimates.

Finally, the target-zone literature suggests that exchange-rate behavior in a fixed exchange-rate regime should converge to exchange-rate behavior in a free float when a decline in variance occurs. Comparison of the EMS fixed exchange-rate tail estimates with tail estimates for the same currencies relative to the US dollar indeed reveals a convergence of fixed and floating tail estimates over time. While the US dollar tail estimates remain stable in the range 3–4 across subperiods, the fixed-rate tail estimates gradually rise towards this level over the lifetime of the EMS. This presents strong support for the impact of the decline in variance on our tail estimates, in line with the target-zone framework for fixed exchange rates.

Our results also shed some light on the discussion about the relative "riskiness" of fixed versus flexible exchange-rate regimes. The evidence here suggests that fixed or semi-fixed exchange-rate regimes are not necessarily more risky than floating exchange-rate regimes. According to our results, the difference in tail-index estimates of fixed and floating exchange rates appears to depend on the degree to which the fixed exchange-rate regime imposes binding restrictions on exchange-rate behavior. This, in turn, depends on the width of the allowed fluctuation margin compared to the degree of obtained convergence and credibility of the system.

## Notes

1. An excellent review of the literature on exchange-rate distributions is contained in Boothe and Glassman (1987).
2. For some EMS returns there is weak serial correlation. Full estimation results are available from the authors upon request.
3. Each Monte Carlo experiment consisted of 500 replications of n draws from different Student-t distributions with degrees of freedom ($= \alpha$) equal to 1, 2, 3 and 4. Optimal m-levels are selected by minimizing the mean squared error across replications. Full simulation results are available upon request from the authors. For one-sided tail indices the optimal $m = 20$, while for two-sided tail indices the optimal $m = 31$.
4. Over the major part of the period, a dual exchange-rate system was in use in Belgium. The official Belgian franc exchange rate is used here as it is the one the Belgian authorities have to support in the exchange-rate mechanism (ERM) of the EMS.
5. If the third and fourth moments of the underlying distributions are infinite, as suggested by estimates of $\alpha$ presented later on, one may question the usefulness of the skewness and kurtosis statistics presented here. We nevertheless provide their values as preliminary evidence of non-normality of exchange-rate returns.
6. For detailed information on the functioning of intervention within the EMS and its changes over time, we refer to Mastropasqua, Micossi, and Rinaldi (1988) and Ungerer et al. (1990).
7. For an overview of this literature, see Krugman (1991) and Krugman and Miller (1992).
8. Recent work by Gros (1992) suggests that this is the case for the post-1987 period.
9. Unpublished results for the EMS exchange rates against the Japanese yen were quite similar to those for the US dollar and are available from the authors on request.

10. Table 6 shows that our estimates for the dollar exchange rates are quite close to the results reported in Boothe and Glassman (1987) using a different methodology. The estimates reported in Table 6 suggest that a Student-t distribution with 3 or 4 degrees of freedom provides an accurate description of dollar exchange-rate behavior.
11. For the UK, no systematic rise in tail estimates for its exchange rate relative to the Deutsche mark is found, nor can equality between dollar and Deutsche mark tail estimates be rejected. This is consistent with the floating exchange rate of the pound against both dollar and Deutsche mark over most of the period.
12. See for instance Friedman (1953) and Westerfield (1977).
13. See Westerfield (1977) for evidence on the fixed Bretton Woods era and Boothe and Glassman (1987) for evidence on floating exchange rates between 1973 and 1986.
14. An additional assumption is the long-run constancy of the real exchange rate.

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