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HOW TO BEAT THE RANDOM WALK An Empirical Model of Real Exchange Rates

Kees G. KOEDIJK and Peter SCHOTMAN*

Erasmus University, 3000 DR Rotterdam, The Netherlands

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An econometric model is developed for all possible bilateral real exchange rates between the United States, the United Kingdom, Germany and Japan for the period February 1977 to June 1987. We extend the standard Dornbusch–Frankel type of models using an error correction approach with an observable macro-economic determinant of the long-run real exchange rate. For the econometric analysis we develop an efficient estimator by pooling the data for all currencies. Contrary to previous empirical tests on the long-run behaviour of real exchange rates, we find a notable and significant mean reversion component.

1. Introduction

Empirical models of exchange rate determination have been criticized on several grounds. First, in general the fit of these models during the floating exchange rate period is very poor. The parameter estimates of popular models, like for instance the Dornbusch–Frankel overshooting model, are often statistically insignificant and/or have the theoretically wrong sign. The poor fits are dramatized in a series of papers by Meese and Rogoff (1983a, b, 1988), who show that none of the popular exchange rate models performed better than a random walk model in predicting nominal as well as real exchange rates out of sample.

It is difficult to draw any definite conclusions from this poor performance, however. The low explanatory power of models that attempt to explain exchange rate changes need not be in conflict with the theory. The overshooting model of Dornbusch (1976), and the asset market approach in general, stresses that exchange rates will be highly sensitive to news, and that the variance of the error term in an exchange rate equation can be large

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compared with the variance of the explanatory variables. In econometric terms, as long as we do not identify the news variables, a low R^2 is unavoidable. Such a low signal to noise ratio, together with the small sample of only about 15 years, might explain why it is so difficult to obtain accurate parameter estimates. Efficient estimation is therefore an important issue, whatever the precise formulation of the model. One of the aims of this paper is to construct an efficient estimator for exchange rate models by pooling the data for several currencies and exploiting the properties of the measurement system of exchange rates. Exchange rates have the property that the ratio of two exchange rates vis-à-vis the same numeraire currency is again an exchange rate. Subtracting the equation for the log of the dollar/yen rate from the equation for the log dollar/Dmark should give a consistent model for the yen/Dmark cross rate.

Efficient estimation clearly cannot counter the second and more serious line of criticism, that despite their poor fits exchange rate models are severely misspecified. Since the late 1970s the monetary model, developed by Dornbusch (1976) and Frankel (1979), has been the principal tool for explaining exchange rate movements. The vast empirical literature has established a number of serious shortcomings of this class of models. The symptoms of misspecification are autocorrelated residuals, time varying parameters, structural breaks, heteroskedasticity and omitted variables.¹

The literature suggests that at least two extensions to the Dornbusch–Frankel model are important.² First, because we cannot reject the presence of a unit root in real exchange rates, purchasing power parity might not be the most appropriate long-run equilibrium concept. Instead one needs an *observable* macro-economic variable that measures the shifts in the long-run equilibrium real exchange rate. Second, it has been argued that the Dornbusch–Frankel model should allow for a more general dynamic specification than that derived from static money demand and a partial adjustment mechanism for relative prices.

The purpose of this paper is to use efficient estimation to develop and test an empirical model of real exchange rates that incorporates more flexible dynamics and a time varying long-run equilibrium real exchange rate, while preserving some of the implications of the sticky price monetary model. The plan of this paper is as follows. In section 2 we set out the theoretical framework and discuss how movements in the tradables/non-tradables price differential can be a relevant proxy for movements in the long-run equilibrium exchange rate. In section 3 we develop the pooled estimator and tests for the restrictions it imposes on the specification of the model. In section 4 we present empirical evidence for all possible bilateral exchange rates

¹See Smith and Wickens (1986), Campbell and Clarida (1987), and Boughton (1987) for econometric evidence.

²See Frankel and Meese (1987) and Dornbusch and Frankel (1987).

between the United States, the United Kingdom, Germany and Japan for the period February 1977 to June 1987. In order to assess the results, we compare the implied final-form estimates of the exchange rate with actual values in section 5. Section 6 contains our conclusions.

2. Dynamic specification

The standard Dornbusch–Frankel, sticky price, monetary model, as developed in Frankel (1979, 1983), has two fundamental elements: instantaneous asset market equilibrium and long-run goods market equilibrium. The first element is embodied in the uncovered interest parity condition. The second element describes the adjustment of the exchange rate to its long-run purchasing power parity (PPP) level. The conclusion in the empirical literature is that this second part of the model fails to account for the dynamics of long-run movements of the real exchange rate. Below we will retain the asset market view, but introduce a more flexible dynamic structure.

In its simplest form asset market equilibrium is reflected in the uncovered interest parity (UIP) condition, which relates the international linkage of interest rates and the expected change in the exchange rate:

$$E_t(\Delta e_{t+1}) = i_t - i_t^* \quad (1)$$

where e is the log of the nominal exchange rate (measured as the domestic price of foreign currency), and i is the nominal one-period short-term interest rate. An asterisk (*) denotes a foreign country. Although the strict UIP condition appears rejected by the data, we do not explicitly augment (1) with a risk premium, since it has proven difficult to obtain an econometric specification in which the risk premium shows up as an economically important factor.³

Our interest is in a model for the real exchange rate, so we subtract the expected inflation differential, $E_t(\Delta p_{t+1} - \Delta p_{t+1}^*)$, from both sides of (1). After rearrangement we have:

$$q_t = E_t(q_{t+1}) - (r_t - r_t^*), \quad (2)$$

where $q = e - (p - p^*)$ is the real exchange rate, p is the log of the price index, and $r_t = i_t - E_t(\Delta p_{t+1})$ is the ex-ante real interest rate. To eliminate the unobservable expectation $E_t(q_{t+1})$ we assume that the real exchange rate is

³See Hodrick (1987) for an extensive survey of the empirical literature on time varying risk premia. See also Frankel (1988). Note, however, that the derived regression model (10) in the text leaves room for a risk premium that is proportional to the real interest rate differential, as in Campbell and Clarida (1987).

expected to move towards its long-run equilibrium according to the error correction model (ECM):

$$E_t(q_{t+1}) = q_t - \theta(q_t - x_t) + \gamma_1 \Delta x_t + \gamma_2 \Delta q_t, \quad (3)$$

in which x_t is the long-run determinant of the real exchange rate, which will be discussed below. In the special case $\gamma_1 = \gamma_2 = 0$ and $x_t = \bar{q}$ (a constant), substitution of (3) into the UIP condition (2) yields the familiar relation between the PPP deviation, $q_t - \bar{q}$, and the real interest differential,

$$q_t = \bar{q} - \frac{1}{\theta}(r - r^*)_t, \quad (4)$$

that appears as the solution in most versions of the monetary model.⁴ Since econometric analysis has found that models based on eq. (4) are seriously misspecified, we have introduced three additional elements into the expectations equation (3). First, we allow for persistent deviations from long-run PPP, represented by some variable x_t . Its importance is stressed in, among others, Stockman (1987) and Meese and Rogoff (1988), who suggest that real shocks are responsible for a major part of the variation in real exchange rates.⁵ The other two elements are the Δx_t and Δq_t terms. Both generalize the dynamic structure of the model. The first term, Δx_t , measures real shocks impinging on the real exchange rate (assuming that x_t is close to a random walk).

The Δq_t term is motivated by the empirical analysis of survey data by Frankel and Froot (1987). Frankel and Froot refer to the Δq_t term in the expectations equation as representing the so-called bandwagon expectations if $\gamma_2 > 0$; with $\gamma_2 < 0$, expectations are called inelastic. The term $\theta(q_t - x_t)$ contains the regressive part of expectations, implying that the real exchange rate is expected to return to its long-run equilibrium eventually. Using survey data, Frankel and Froot (1987) find evidence that both terms are relevant in investors' expectations. In particular they find that $\gamma_2 < 0$, and that a regressive term is important for longer-term expectations.

Substituting (3) into (2) and solving for q_t gives an error correction model for the real exchange rate:

$$\Delta q_t = \frac{1}{\theta - \gamma_2} [-(r - r^*)_t - \theta(q_{t-1} - x_{t-1}) + (\theta + \gamma_1)\Delta x_t]. \quad (5)$$

⁴See Frankel (1979, p. 619, eq. (A.4)). In general, the parameter θ is a function of the structural parameters underlying money demand, aggregate demand, and a price or trade balance adjustment mechanism.

⁵For example, the Hooper-Morton (1982) model fits into this framework.

Like the standard version of the model, eq. (5) implies a negative relation between the real interest rate differential and the current spot exchange rate if $\theta - \gamma_2 > 0$, which, if $\theta > 0$, we must assume anyway as part of the stability condition $\gamma_2 < \frac{1}{2}\theta$ for (5). The model also incorporates the overshooting property if we make the auxiliary assumption that the real interest rate differential is expected to converge to zero. In that case a monetary contraction in the domestic country leads (given price stickiness) to a positive real interest differential and thus to a temporary appreciation of the currency below the equilibrium determinant x_t . Overshooting also occurs with respect to x_t . With $(\theta + \gamma_1)/(\theta - \gamma_2) > 1$ a shock in x_t raises the real exchange rate initially by more than the unit response in the long run.

So far we have not been explicit about the meaning of x_t , which can be any empirically relevant observable proxy (or proxies) of the long-run equilibrium real exchange rate. Our approach here is to link shifts in the long-run equilibrium real exchange rate to changes in the tradable/non-tradable price differential.⁶ The important point in the distinction between the prices of tradable and non-tradable goods is that PPP is supposed to hold for internationally traded goods only:

$$e - p_T + p_T^* = 0, \quad (6)$$

where p_T is the log of an index for traded goods. Now assume, like in Hsieh (1982), that the aggregate price index is a weighted average of the prices of traded and non-traded goods:

$$p = (1 - \alpha)p_T + \alpha p_{NT}, \quad (7)$$

where p_{NT} is the log of the price of non-traded goods, and α is the share of non-traded goods in the economy. A similar relation is assumed to hold for the foreign country. The real exchange rate will then depend on the relative prices between tradable and non-tradable goods as well as on the size of the tradable goods sectors in the economies:

$$q = e - p + p^* = \alpha(p_T - p_{NT}) - \alpha^*(p_T^* - p_{NT}^*). \quad (8)$$

⁶Another approach to explain movements in the equilibrium real exchange rate is to incorporate a balance of payments constraint, see Hooper and Morton (1982) and Meese and Rogoff (1988).

The distinction between tradables and non-tradables becomes important if there are persistent deviations between the prices of tradable and non-tradable goods in one of the two economies.

In empirical work one often takes the wholesale price index, P_w , as a proxy for the price of tradable goods, P_T .⁷ Mecagni and Pauly (1987) performed unit root tests with nominal exchange rates deflated by P_w for a number of currencies. They concluded that the ratio of wholesale prices to consumer prices does pick up some of the low frequency characteristics of exchange rates. In the empirical analysis we will use the wholesale price index as our proxy for tradable goods prices and the consumer price index, P_c , as the aggregate price index p .⁸ The determinant of the long-run real exchange rate then becomes:

$$x = \alpha(p_T - p_{NT}) - \alpha^*(p_T^* - p_{NT}^*) = (p_w - p_c) - (p_w^* - p_c^*). \quad (9)$$

With x defined as in (9) we are ready to implement the exchange rate model (5) empirically. After reparameterizing, this model can be written as:

$$\Delta q_t = \beta_1(q_{t-1} - x_{t-1}) + \beta_2(r_t - r_t^*) + \beta_3 \Delta x_t, \quad (10)$$

which is linear in the parameters. The exchange rate specification (10) does not necessarily imply that the time series of the real exchange rate has a unit root. But if $\{q_t\}$ has a unit root, co-integration of q_t and x_t is a necessary condition.⁹ The crucial parameter for the long-run implications is β_1 . If it is significantly negative there is a significant error correction mechanism in (10).

Like much of the literature we view the regression model (10) as a semi-reduced form. Because Δx_t and $(r - r^*)_t$ are stationary, series simultaneity issues can be important. Boughton (1987) and Frankel and Meese (1987), however, note that there is generally not much difference between OLS results and instrumental variables (IV) procedures that are meant to overcome the simultaneity bias. Since OLS minimizes the residual variance of a regression, an IV estimator will necessarily provide a worse fit. If OLS does not provide any significant results, application of an IV estimator will not improve the statistical fit. If OLS cannot beat a random walk, then neither

⁷See, for example, Clements and Frenkel (1980) and Wolff (1987).

⁸Since the wholesale price index is still an imperfect proxy for the price of tradable goods, one has attempted to construct other proxies. Balassa (1964) stressed different sectoral productivity trends as the principal cause for movements in the ratio of tradable to non-tradable goods' prices. See also Marston (1986), Edison and Klovland (1987), and Kravis and Lipsey (1988). In this paper we will not attempt the indirect approach using proxies for productivity differentials.

⁹The existence of a unit root in real exchange rates can also be a consequence of persistence in the real interest differential. Campbell and Clarida (1987) and Meese and Rogoff (1988) failed to find evidence of this possibility, however.

can IV. Estimation by IV techniques can, however, produce different parameter estimates. We will therefore estimate all models both by least squares as well as by IV and report the latter results in footnotes in the empirical section.

3. Pooled estimation

The theoretical model described in section 2 was not developed for one specific exchange rate or currency, but rather served as a general framework. In the empirical analysis we will test the model for all six possible bilateral real exchange rates between the United States, the United Kingdom, Germany and Japan. Only three of these exchange rates can be independent. If we have modelled the pound/dollar rate and the Dmark/dollar rate we have implicitly modelled the pound/Dmark exchange rate. To obtain the same type of specification for this cross rate some cross equation parameter restrictions must be imposed. If these restrictions are valid, pooling the data for the four currencies will improve the efficiency of the parameter estimates.

The explanatory variables in the theoretical model are all relative variables like the real interest rate differential and the tradable/non-tradable price differential. Relative variables have the same measurement property as exchange rates. For example, for the real interest differential we have that $(r^{JP} - r^{WG}) = (r^{JP} - r^{US}) - (r^{WG} - r^{US})$. A general linear specification with relative variables reads:

$$q_j^{(i)} = \beta_j^{(i)} x_j^{(i)} + u_j^{(i)}, \quad (11)$$

where

$q_j^{(i)}$ = logarithm of (real) exchange rate of currency j ($j=0, \dots, m; j \neq i$) in units of the currency i . The superscript in parentheses denotes the numeraire, the subscript is the running index.

$x_j^{(i)}$ = $(n \times 1)$ vector of explanatory variables for exchange rate $q_j^{(i)}$; the variables are all in relative form, i.e. $x_j^{(k)} = x_j^{(i)} - x_k^{(i)}$.

$\beta_j^{(i)}$ = $(1 \times n)$ parameter vector.

$u_j^{(i)}$ = error term in equation for exchange rate $q_j^{(i)}$.

In eq. (11) we have not specified one particular currency. The same general specification therefore applies to the exchange rates in units of currency k :

$$q_j^{(k)} = \beta_j^{(k)} x_j^{(k)} + u_j^{(k)}. \quad (12)$$

Since $q_j^{(k)} = q_j^{(i)} - q_k^{(i)}$ ($j \neq k, i$), and $q_i^{(k)} = -q_k^{(i)}$ the model for $q_j^{(k)}$ is also implicit in eqs. (11):

$$\begin{aligned}
 q_j^{(k)} &= \beta_j^{(i)} x_j^{(i)} + u_j^{(i)} - \beta_k^{(i)} x_k^{(i)} - u_k^{(i)} \\
 &= \beta_j^{(i)} (x_j^{(i)} - x_k^{(i)}) + (\beta_j^{(i)} - \beta_k^{(i)}) x_k^{(i)} + u_j^{(i)} - u_k^{(i)} \\
 &= \beta_j^{(i)} x_j^{(k)} + (\beta_j^{(i)} - \beta_k^{(i)}) x_k^{(i)} + u_j^{(i)} - u_k^{(i)}.
 \end{aligned}
 \tag{13}$$

Expressions (12) and (13) must be mutually consistent, which implies that the parameters should be equal in all equations, i.e. $\beta_j^{(i)} = \beta_k^{(i)} = \beta_j^{(k)} = \beta$. We will call these parameter restrictions the ‘consistency’ conditions.¹⁰

The ‘consistency’ conditions seem overly strong. The equality restrictions arise solely because the set of explanatory variables for $q_j^{(i)}$ is limited to variables relating directly to countries i and j . An unrestricted specification will include all explanatory variables in all equations:

$$q_j^{(i)} = B_j^{(i)} X^{(i)} + u_j^{(i)},
 \tag{14}$$

where $B_j^{(i)}$ is a $(1 \times mn)$ parameter vector, and $X^{(i)}$ an $(mn \times 1)$ vector of explanatory variables obtained by stacking all vectors of explanatory variables $x_j^{(i)}$ ($j=0, \dots, m; j \neq i$). In this model the ‘consistency’ property is automatically fulfilled, since the implicit equation for the cross rate, $q_j^{(k)}$, is also of an unrestricted form containing all explanatory variables, $x_j^{(k)}$ ($j \neq k$), that are linear combinations of the original $x_j^{(i)}$ ($j \neq i$). The parameters $B_j^{(k)}$ are simple linear combinations of the parameters $B_j^{(i)}$. The pure bilateral model obtains if, in the equation for $q_j^{(i)}$, all parameters on the ‘indirect effects’, $x_h^{(i)}$ ($h \neq i, j$), are zero. But as soon as we are back in the purely bilateral model, ‘consistency’ requires that the parameters on the direct effects are equal and do not depend on the numeraire.

The relative variables in the equations for a common numeraire currency (say currency i) contain all data information. It is thus sufficient to consider the system of m equations for a single common numeraire, and estimate this system using SUR. The full system of exchange rate equations reads:

$$q^{(i)} = B^{(i)} X^{(i)} + u^{(i)},
 \tag{15}$$

where

$$\begin{aligned}
 q^{(i)} &= (q_0^{(i)} \dots q_{i-1}^{(i)} q_{i+1}^{(i)} \dots q_m^{(i)})' \text{ is the } m \times 1 \text{ vector of exchange rates expressed} \\
 &\text{in numeraire currency } i, \\
 B^{(i)} &= (B_0^{(i)'} \dots B_{i-1}^{(i)'} B_{i+1}^{(i)'} \dots B_m^{(i)'})', \text{ an } (m \times mn) \text{ matrix,} \\
 u^{(i)} &= (u_0^{(i)} \dots u_{i-1}^{(i)} u_{i+1}^{(i)} \dots u_m^{(i)})',
 \end{aligned}$$

¹⁰The constant terms in the equations are left out. They are unrestricted, since the difference between two constants is again a constant, and thus satisfies the ‘consistency’ conditions.

The pooling restrictions for the full system can thus be represented as:

$$B^{(i)} = \begin{bmatrix} \beta & 0 & & & 0 \\ 0 & \beta & & & \vdots \\ \vdots & 0 & & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & & & \beta \end{bmatrix}, \quad (16)$$

which entails $(m^2 - 1)n$ testable cross equation restrictions. The restrictions in (16) will be tested in section 4. Rejection of the 'consistency' restrictions implies that the theoretical models have unjustly focused on a two-country world, failing to recognize third-country effects. Since the unrestricted system has all explanatory variables in all equations, the estimator is independent of the structure of the covariance matrix: SUR reduces to OLS. The 'consistency' conditions (16) can then easily be tested by Wald or likelihood ratio tests.

The SUR estimator is in general not numerically invariant with respect to the choice of the numeraire. Appendix A motivates the restrictions on the contemporaneous covariance matrix of the errors $u_j^{(i)}$ that we impose. These restrictions simplify the SUR estimator, since the covariance matrix is specified up to a scalar variance.

4. Empirical results

In the regression analysis we use monthly data for the United States, the United Kingdom, Germany and Japan from February 1977 to July 1987. The data are described in appendix B. We start by looking at the pooled estimates for the basic model of eq. (6), which entails that the parameters in all six bilateral equations are equal. The first column of table 1 reports estimation and test results of the pooled model. This model reduces to a random walk if the parameters are jointly zero, which hypothesis is rejected at any conventional significance level. If β_1 equals zero, the real exchange rate and the tradable/non-tradable price differential do not co-integrate and the error correction term, $q_{t-1} - x_{t-1}$, has a unit root. To test for significance of β_1 one must therefore refer to the Dickey-Fuller critical values for unit root tests. The t -value of 3.1 is above the 5 percent critical value of 2.89, thus supporting the long-run equilibrium part of the model. Hence, real exchange

Table 1

The basic model (1977:2-1987:6): $\Delta q_t = \beta_0 + \beta_1(q_{t-1} - x_{t-1}) + \beta_2(r_t - r_t^*) + \beta_3 \Delta x_t + u_t$.

	Pooled	UK/US	WG/US	JP/US	UK/WG	UK/JP	WG/JP
β_1	-0.043 (3.1)	-0.036 (1.5)	-0.026 (1.0)	-0.057 (2.1)	-0.055 (2.2)	-0.071 (2.0)	-0.063 (2.3)
β_2	-1.36 (1.8)	-2.25 (1.8)	-1.34 (0.7)	-4.99 (2.8)	0.13 (0.2)	0.07 (0.0)	-1.80 (1.0)
β_3	1.25 (5.9)	1.03 (2.2)	0.74 (1.1)	1.21 (3.8)	1.63 (3.8)	1.34 (4.6)	1.36 (4.2)
σ	3.28	3.43	3.55	3.45	2.86	3.46	3.02
RW	40.69*	9.42*	1.79	22.76*	19.75*	23.24*	21.24*
CHOW	4.98	10.68*	2.63	4.33	1.73	3.47	1.94
AUTO	0.01	0.05	0.16	0.02	0.81	0.00	0.08
ARCH	0.31	0.02	5.30*	0.01	3.59	0.81	0.35

Notes: US=United States; UK=United Kingdom; WG=Germany; JP=Japan. *t*-values are reported in parentheses. An asterisk (*) denotes significance at the 5 percent level. The standard error of the equation (σ) is given as a percentage of the level q . RW is a χ^2 likelihood ratio test for joint significance of all parameters (degrees of freedom (df) are 6 for the pooled system and 4 for the single equation models); CHOW is a χ^2 likelihood test for constant parameters, with the sample split after 81:12 (df=6 for pooled system, df=4 for single equation estimates); AUTO is a χ^2 LM-test for first-order autocorrelation of the residuals (df=3 pooled, df=1 single); ARCH is a χ^2 LM-test for ARCH type heteroskedasticity (df=3 pooled, df=1 single).

The unrestricted error covariance (correlation) matrix in the pooled model:

$$\sum^{US} = \text{Var} \begin{pmatrix} u_{UK}^{(US)} \\ u_{WG}^{(US)} \\ u_{JP}^{(US)} \end{pmatrix} = \begin{pmatrix} 0.00114 & 0.65 & 0.50 \\ 0.00077 & 0.00123 & 0.64 \\ 0.00058 & 0.00077 & 0.00119 \end{pmatrix}.$$

rates show significant mean reversion when wholesale prices are used as deflator.^{11,12} The estimates of the parameters β_2 on the real interest rate differential and β_3 on the change in the fundamentals, Δx_t , are both consistent with an overshooting reaction of the exchange rate ($\beta_2 < 0$, $\beta_3 > 1$). Furthermore, the implied estimate of $\gamma_2 = (\beta_1 + 1)/\beta_2 = -0.70$ corresponds to the finding of Frankel and Froot (1987) that expectations have an inelastic component; an observed appreciation generates expectations of a future depreciation. The coefficient on the interest rate effect is insignificant, however. An alternative specification for the real interest rate effect will be discussed further below.¹³

¹¹We have also experimented with a model that assumes that PPP already holds for consumer price indices. The relevant *t*-statistic on the error correction term was only 2.5, which is not significant at the 5 percent level under the null hypothesis of a unit root in $\{q_t\}$.

¹²Some evidence for mean reversion in real exchange rates has also been reported in Huizinga (1987) and Frankel and Meese (1987). One should note, however, that Huizinga only finds mean reversion in the case of the U.S. dollar/Canadian dollar real exchange rate.

¹³When we estimated the system by IV using the lagged real interest differentials as instruments, the interest rate parameter, β_2 , retained its negative sign, but was still not significant. The mean reversion effect remained significant. A table with all IV results is available from the authors upon request.

The rest of the first column of table 1 is devoted to misspecification tests of this basic model. The CHOW statistic tests for a structural break after 1981, when the dollar started its long continuous upswing. The insignificant value indicates that parameters can be regarded as constant over the full sample period. Table 1 also reports standard diagnostic tests for autocorrelation and ARCH-type heteroskedasticity. The residuals do not show signs of either.¹⁴

At the bottom of the table we report the unrestricted covariance/correlation matrix of the errors taking the dollar as the numeraire. The assumption underlying the pooled estimator discussed in appendix A implies that all equations have equal residual variance, and that the correlation between residuals is a half. The unrestricted covariance matrix closely matches this assumed pattern.

For comparison the other columns of table 1 contain estimates for the six bilateral real exchange rates individually. The single equation estimates do not look impressive. Contrary to the pooled estimates, the error correction is never significant at Dickey-Fuller levels. This implies that one would not have detected the co-integration of the real exchange rate and the tradable/non-tradable price differential for any of the individual exchange rate models. The RW statistic indicates that the Dmark/dollar equation does not even fit significantly better than the random walk. The model that performs best is the yen/dollar equation, which is the only equation with a strong and significant interest rate effect.

The parameters in all six equations must be equal to satisfy the cross equation restrictions employed in the pooled estimates. If, for example, the equation for the Dmark/dollar and the equation for the pound/dollar have different parameters, the implied equation for the Dmark/pound does not solely depend on variables relating to Germany and the United Kingdom, but also on U.S. variables. So, as discussed in section 3, the test for equal parameters in the separate equations is in fact a test for omitted third-country effects in some of the equations. The pooled model is a special case of the unrestricted system (15). All three equations in the unrestricted system have the same explanatory variables. Under the null hypothesis the parameter matrix $B^{(i)}$ satisfies the restrictions in (16). With three types of explanatory variables (real interest differentials, deviations from long-run equilibrium, and real shocks), and a system of three equations, there are 27

¹⁴Further evidence on the constancy of the parameters can be provided by the predictive failure test. The model is re-estimated omitting one or more observations. The predicted failure test indicates whether the omitted observations can be explained by the model estimated for the rest of the sample. We computed the predictive failure test for all single observations (PF1) and all possible half year periods (PF6). Only three of the PF1 statistics are significant at the 1 percent level (78:11, 82:11, and 85:3), and only one of the PF6 statistics is significant at the 1 percent level (85:3). These numbers are not improbable for 125 observations. Our conclusion is that parameters are fairly constant over the full sample.

Table 2
Specification tests of pooled exchange rate system.

(A) All cross equation restrictions versus fully unrestricted:	
LR(24) = 36.9**	(covariance matrix restrictions maintained)
LR(24) = 37.5**	(unrestricted Σ under null and alternative)
(B) Cross equation restrictions for individual explanatory variables:	
$\rightarrow (q-x)_{i-1}$:	LR(8) = 13.4*
$\rightarrow (r-r^*)_i$:	LR(8) = 10.9
$\rightarrow \Delta x_i$:	LR(8) = 7.3
(C) Separate interest rate parameters:	
pooled versus general unrestricted:	LR(24) = 22.2
separate interest parameters versus basic model:	LR(3) = 12.5***

Notes: Likelihood ratio (LR) tests are computed as:

$$LR(df) = (T-n) \ln(\det(\Sigma_0)/\det(\Sigma_1)),$$

where df is the number of restrictions, T the length of the sample, n the number of explanatory variables per equation, Σ_0 the estimated covariance matrix of residuals under the null hypothesis, and Σ_1 the estimated covariance matrix under the alternative. Asterisks denote significance at the 10 percent (*), 5 percent (**), and 1 percent (***), respectively.

parameters in the fully unrestricted system, but only three parameters in the pooled basic model.¹⁵

Table 2 summarizes the tests of the exchange rate system. The basic model of table 1 is rejected in favour of the fully unrestricted model at the 5 percent level. The rejections do not depend on the restrictions on the error covariance matrix, Σ . In order to examine whether one particular explanatory variable is responsible for the overall rejection, we tested the pooling restrictions for each variable individually. As panel (B) of table 2 shows, none of the test statistics exceeds the 5 percent critical value, so it is impossible to assign the overall rejection to any particular variable in the model.

Summarizing, the empirical evidence indicates two shortcomings of the basic regression model: the rejection of the 'consistency' conditions, and the insignificant interest rate effect. We will therefore consider a somewhat more general specification for the real interest rate effect. The problem with the interest rate specification might originate from the restriction that real interest rates enter in differential form ($r-r^*$) and not with separate coefficients. An early reference to this point is the critique of Haynes and Stone (1981) on the original Frankel (1979) model. Allowing for separate interest rate parameters for all four countries introduces three more parameters into the general model (four individual interest rates instead of three differentials). The specification of an equation in the purely bilateral system then becomes:

¹⁵There are three additional parameters due to the constant terms. These are not subject to cross equation restrictions.

$$\Delta q_{j,t}^{(i)} = \beta_1(q_{j,t-1}^{(i)} - x_{j,t-1}^{(i)}) + \beta_2^j r_t^j - \beta_2^i r_t^i + \beta_3 \Delta x_{j,t}^{(i)} + u_{j,t}^{(i)}. \quad (17)$$

This specification entails that the domestic and foreign real interest rates have separate parameters. It is more general than the basic model, since the interest rate effects are not restricted to a single parameter. The basic model obtains if

$$\beta_2^{\text{US}} = \beta_2^{\text{UK}} = \beta_2^{\text{WG}} = \beta_2^{\text{JP}} = \beta_2. \quad (18)$$

This bilateral specification with country specific explanatory variables obviously satisfies the 'consistency' conditions, since the parameter β_2^i cancels after subtracting any two of the equations with numeraire i . In order to test the pooling restrictions (18) we need a general model that has all $(m+1)$ interest rates of all $(m+1)$ ($j=0, \dots, m$) countries as explanatory variables. With four countries in the system the number of restrictions are 3 (pooled basic model versus separate interest rate parameters model) and 24 (general versus pooled system with separate interest rate parameters).

The likelihood ratio test of the 'consistency' restrictions within the general model does not reject. But the further simplification from separate interest rate parameters to the interest differentials model is rejected, which is consistent with our earlier results. Allowing interest rates to enter the equation with separate coefficients preserves a purely bilateral specification for the real exchange rate and also satisfies the 'consistency' conditions. A full set of estimates for the pooled specification is given in table 3.

As in table 1, the random walk is strongly rejected. The mean reversion effect is now even stronger than in the basic model. The error correction parameter β_1 is larger in size and obtains a t -value of 4.5, implying that the real exchange rate the tradable/non-tradable price differential are likely to be co-integrated. All interest rate parameters have the theoretical correct sign; for the United States and Japan they are now significant and much larger than the parameters for the United Kingdom and Germany. Especially, the U.S. real interest rate has a very strong effect on exchange rates.¹⁶

The other columns of table 3 report the corresponding single equation regression results for all six bilateral exchange rates. The U.S. and Japanese real interest rates always have large coefficients, just as in the pooled model. The German real interest rate is virtually zero in all equations where it is included. The U.K. real interest rate is only important for the pound/dollar exchange rate.

¹⁶JV results are similar. We used lagged real interest rates as instruments. The coefficients for the United States and Japan still appear to be the only two important ones. Detailed results are available upon request.

Table 3

Augmented model with individual interest rate effects (77:2-87:6):

$$\Delta q_t = \beta_0 + \beta_1(q_{t-1} - x_{t-1}) + \beta_2 r_t - \beta_2^* r_t^* + \beta_3 \Delta x_t + u_t.$$

	Pooled	UK/US	WG/US	JP/US	UK/WG	UK/JP	WG/JP
β_1	-0.071 (4.5)	-0.096 (3.7)	-0.046 (1.7)	-0.055 (2.0)	-0.049 (1.5)	-0.051 (1.4)	-0.093 (2.6)
β_2^{US}	-5.00 (4.0)	-8.88 (4.6)	-2.88 (1.4)	-4.95 (2.7)	-	-	-
β_2^{UK}	-0.68 (0.9)	-2.67 (2.3)	-	-	0.14 (0.2)	-2.21 (1.2)	-
β_2^{WG}	-0.19 (0.1)	-	1.73 (0.7)	-	-0.56 (0.2)	-	0.22 (0.1)
β_2^{JP}	-4.08 (2.4)	-	-	-5.32 (2.1)	-	-7.08 (1.9)	-3.42 (0.1)
β_3	1.27 (6.2)	0.89 (2.0)	0.68 (1.0)	1.21 (3.7)	1.64 (3.8)	1.24 (4.2)	1.47 (4.4)
σ	3.21	3.20	3.52	3.47	2.88	3.42	3.01
RW	53.27*	29.12*	4.59	21.88*	19.05*	27.47*	22.41*
CHOW	11.95	3.54	12.94	8.71	4.19	6.78	2.18
AUTO	0.01	0.10	0.85	0.04	0.79	0.12	0.03
ARCH	0.31	0.56	4.36*	0.01	3.18	0.81	0.77

See table 1 for explanatory notes. Degrees of freedom for RW and CHOW test are now $df=9$ for the pooled model and $df=5$ for the single equation models.

The unrestricted error covariance (correlation) matrix in the pooled model:

$$\Sigma^{(US)} = \text{Var} \begin{pmatrix} u_t^{(US)} \\ u_t^{(UK)} \\ u_t^{(WG)} \\ u_t^{(JP)} \end{pmatrix} = \begin{pmatrix} 0.00102 & 0.64 & 0.48 \\ 0.00071 & 0.00121 & 0.63 \\ 0.00053 & 0.00075 & 0.00116 \end{pmatrix}.$$

5. Evaluation of the model

The diagnostic tests lead to the conclusion that the model is able to describe a significant part of the movements of real exchange rates. We will now investigate what part of the recent behaviour of exchange rates can be ascribed to fundamentals and the implied dynamics of the model. For this purpose we use the final form of the model for the nominal exchange rate. The test results lead us to prefer the model with separate coefficients for interest rates, which for each individual exchange rate has the general specification:

$$\Delta q_t = \beta_0 + \beta_1(q_{t-1} - x_{t-1}) + \beta_2 r_t - \beta_2^* r_t^* + \beta_3 \Delta x_t + u_t. \quad (19)$$

Solving eq. (19) for the nominal exchange rate defined as $e = q + p_c - p_c^*$ yields:

$$e_t = \frac{1}{1 - (1 + \beta_1)L} \times [\beta_0 + (1 - \beta_3)\Delta(p_c - p_c^*) + (\beta_3\Delta - \beta_1L)(p_w - p_w^*) + \beta_2 r - \beta_2^* r^* + u],$$

$$= z_t + \frac{1}{1 - (1 + \beta_1)L} u_t, \quad (20)$$

where L is the lag operator, and $\Delta = 1 - L$. The part of e_t that can be explained by fundamentals is given by z_t , which represents the t -period horizon 'forecast' of the nominal exchange rate using the realizations of prices and interest rates. Formally, $z_t = E[e_t | e_0, P_w(t), P_c(t), r(t), r^*(t)]$, where $P_w(t) \equiv (p_{w1}, \dots, p_{wt})$ contains all realizations of the wholesale price index from March 1973 ($t^0 = 1973:3$) up to time t . The variables $P_c(t)$, $r(t)$, and $r^*(t)$ are defined analogously. The only variable that is not in the information set is the exchange rate, e , except from the initial condition, e_0 , which is the actual nominal exchange rate in March 1973. The computed series, z_t , represents our best guess of the nominal exchange rate, if we were to only observe the actual course of prices and interest rates, and not the historical path of the exchange rate itself. The series $\{z_t\}$ is computed recursively as:

$$z_t = (1 + \beta_1)z_{t-1} + [\beta_0 + (1 - \beta_3)\Delta(p_c - p_c^*) + (\beta_3\Delta - \beta_1L)(p_w - p_w^*) + \beta_2r - \beta_2^*r^*], \quad (21)$$

with $z_0 = e_0$. One can also interpret the series z_t as the result of a dynamic simulation with the model. The final form depends heavily of course on the exogeneity of prices and interest rates that we assume here.

Figs. 1 to 6 show the final form fitted values for all six exchange rates. The series in the figures are $\exp(z_t)$. They are computed using the pooled parameter estimates in table 3. For comparison we plotted the actual course of the six exchange rates. The first thing to notice from these figures is the apparent difference between dollar and non-dollar exchange rates. With respect to the non-dollar exchange rates the model seems to track the major trends throughout the full sample period; for the dollar exchange rates the model is able to account for the major part of the fluctuations until early 1984. The model does not provide an explanation for the last part of the appreciation of the dollar against the pound, Dmark and yen between March 1984 and February 1985, nor does it explain the prolonged fall since January 1986. One likely explanation for the perceived behaviour of the dollar between March 1984 and February 1985 could be that the currency was on a speculative bubble path during this period, although we cannot rule out the possibility of omitted variables in our model.

6. Conclusions

- (1) Contrary to previous empirical tests on the long-run behaviour of real

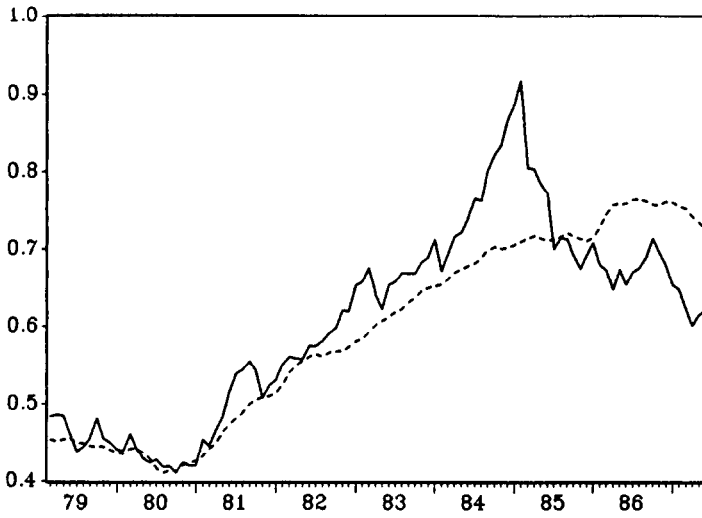


Fig. 1. Pound/dollar actual (—) and simulated (---) nominal rate.

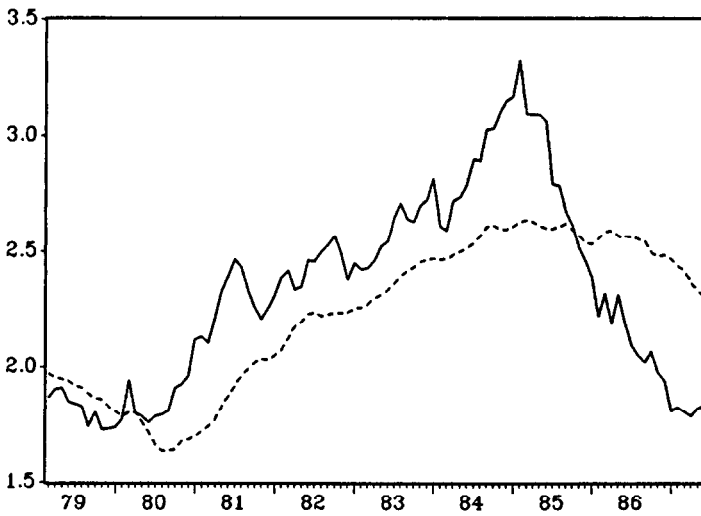


Fig. 2. Dmark/dollar actual (—) and simulated (---) nominal rate.

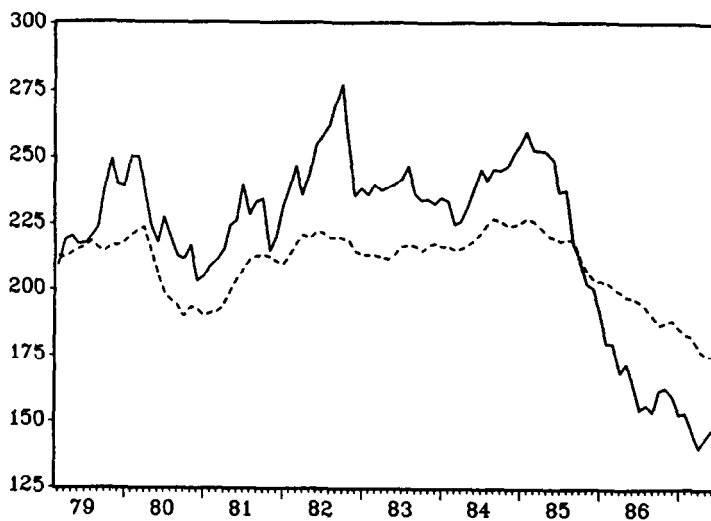


Fig. 3. Yen/dollar actual (—) and simulated (---) nominal rate.

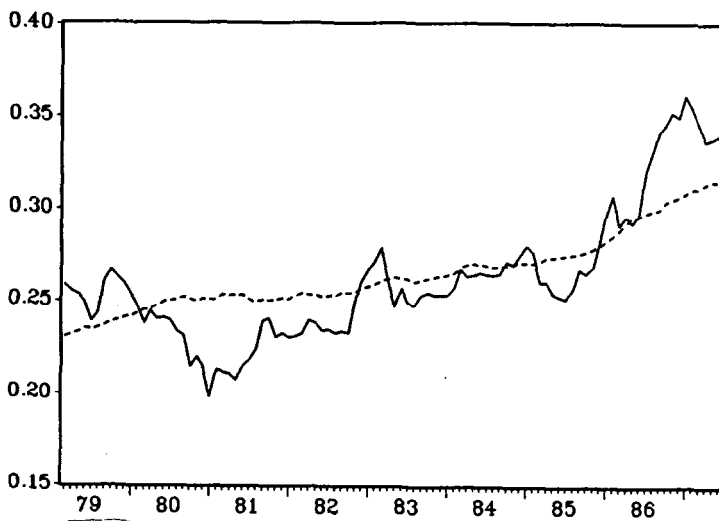


Fig. 4. Pound/Dmark actual (—) and simulated (---) nominal rate.

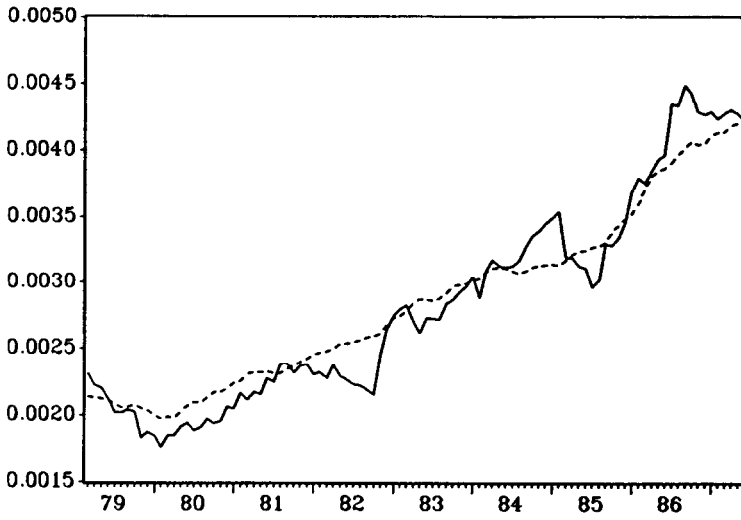


Fig. 5. Pound/yen actual (—) and simulated (---) nominal rate.

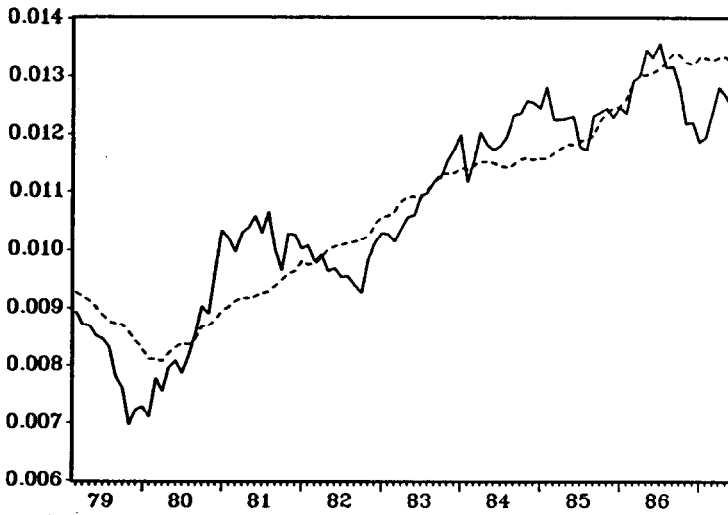


Fig. 6. Dmark/yen actual (—) and simulated (---) nominal rate.

exchange rates, we find a notable and significant mean reversion component by pooling data for the four major currencies and by introducing a dynamic model that is more general than the standard Dornbusch–Frankel model. We find that real exchange rates co-integrate with the tradables/non-tradables price differential, proxied by the relative ratio of wholesale to consumer prices.

(2) Interest rates are important, but not as hypothesized in the standard Dornbusch–Frankel model. Within the system of exchange rates between the United States, the United Kingdom, Germany and Japan the effect of real interest rates in the United States and Japan appears to be more important than real interest rates in Germany and the United Kingdom. In the monetary model only the differential between foreign and domestic interest rates enter the exchange rate equation. The results of our pooled model clearly suggest that this interest rate specification is too restrictive. Both third-country effects as well as separate parameters (instead of a differential) provide a better fit of the exchange rate equation.¹⁷

(3) Fundamentals can readily explain the major trends of the non-dollar exchange rates. The course of the dollar was roughly in line with fundamentals until March 1984. We are unable, however, to explain the strong appreciation of the dollar between March 1984 and February 1985 and its subsequent fall with an appeal to fundamentals.

Appendix A: Estimation of the system

As shown in eq. (13) a change in the numeraire from i to k implies the following transformation of the error terms:

$$\begin{aligned} u_j^{(k)} &= u_j^{(i)} - u_k^{(i)}, \quad \text{for } j \neq k, j = i, \\ u_i^{(k)} &= -u_k^{(i)}. \end{aligned} \tag{A.1}$$

Let $\Sigma^{(i)}$ be the covariance matrix of $u^{(i)}$, the vector of error terms defined in (16). Then the covariance matrix of $u^{(k)}$ is $\Sigma^{(k)} = P\Sigma^{(i)}P'$. The transformation matrix P is a permutation of the matrix that describes the transformation from numeraire $i=1$ to the new numeraire $k=2$, which is given by the $(m \times m)$ matrix:

¹⁷Improvement on the interest rate specification requires a richer model of the international linkages of interest rates. In a further analysis of the interest rate specification one would like to estimate the exchange rate system and the model of the international linkage of interest rates simultaneously. This would solve any potential endogeneity problems. But such an augmented model implies a large number of highly non-linear parameter restrictions between the interest rate and the exchange rate model.

$$P = \begin{pmatrix} -1 & \mathbf{0} \\ -\iota_{m-1} & I_{m-1} \end{pmatrix}, \tag{A.2}$$

where ι_n is a $(n \times 1)$ vector of ones. Although it is consistent, the standard two-step SUR estimator for β will in general not produce estimates that are numerically invariant with respect to the specific numeraire against which all variables happen to be expressed. There exists, however, a simple and interpretable specification of $\Sigma^{(i)}$ that ensures identical β 's, whatever the numeraire of the system. To derive it we assume that $u_j^{(i)} = u_j - u_i$, with u_j and u_i being mutually uncorrelated, and having equal variances $(\frac{1}{2}\sqrt{2})\sigma^2$. The assumption implies that the error term consists of two independent country specific components that are equally important. The full covariance matrix $\Sigma^{(i)}$ of $u^{(i)}$ now becomes:

$$\Sigma = \frac{1}{2}\sigma^2(I_m + \iota_m \iota_m'), \tag{A.3}$$

which is independent of the numeraire i . This proposition can be verified by calculating (co-)variances of all $u_j^{(i)}$ and $u_k^{(i)}$ using the transformation (A.2) and the proposed covariance matrix Σ in (A.3).¹⁸

Since the covariance matrix Σ is completely specified up to a scalar multiple, we are in the unusual position that we can directly apply GLS to the stacked system:

$$Q^{(i)} = \begin{bmatrix} Q_0^{(i)} \\ \vdots \\ Q_{i-1}^{(i)} \\ Q_{i+1}^{(i)} \\ \vdots \\ Q_m^{(i)} \end{bmatrix} = \begin{bmatrix} X_0^{(i)} \\ \vdots \\ X_{i-1}^{(i)} \\ X_{i+1}^{(i)} \\ \vdots \\ X_m^{(i)} \end{bmatrix} \beta + \begin{bmatrix} U_0^{(i)} \\ \vdots \\ U_{i-1}^{(i)} \\ U_{i+1}^{(i)} \\ \vdots \\ U_m^{(i)} \end{bmatrix} = X^{(i)}\beta + U^{(i)}, \tag{A.4}$$

where $Q_j^{(i)} = [q_{j,1}^{(i)}, \dots, q_{j,T}^{(i)}]'$ is a vector of T observations on $q_j^{(i)}$ ($j \neq i$), $X_j^{(i)}$ is a $(T \times n)$ matrix of observations on the n explanatory variables $x_j^{(i)}$, $U_j^{(i)}$ is a vector of disturbances of length T , so that $U^{(i)}$ is a stacked error term of length mT with zero mean and covariance matrix $\Omega = \Sigma \otimes I_T$. Applying GLS to this system yields the estimator:

$$\hat{\beta} = (X^{(i)'} \Omega^{-1} X^{(i)})^{-1} X^{(i)'} \Omega^{-1} Q^{(i)}. \tag{A.5}$$

¹⁸The decomposition of an error term into two independent country specific components always implies that the residuals of exchange rate equations with a common numeraire will be positively correlated. The correlation will be a half only in the case where the variances of the two components are equal, which we assume here.

The estimator $\hat{\beta}$ is consistent for all the single equation $\beta_j^{(i)}$, even if the covariance matrix restrictions (A.3) are false; these restrictions only serve to obtain efficiency.

Appendix B: Data sources and construction

Exchange rates (e): Nominal exchange rates are taken from the International Financial Statistics (IFS) databank, line ae.

Consumer price index (p_c): Line 64 from IFS for all countries.

Wholesale price index (p_w): Line 63 from IFS for all countries.

Interest rates (i): For the United States, the one-month Treasury bill rate is obtained from the Federal Reserve Bank of St. Louis. For Germany we use the one-month deposit rate published in the *Frankfurter Allgemeine Zeitung*. For the United Kingdom, the one-month interbank deposit rate is taken from the *Financial Times*. For Japan, we use the one-month Gensaki rate provided by the Bank of Japan. All interest rates are end-of-month.

All data are seasonally unadjusted. All series, except interest rates, are converted to logarithms. Nominal interest rates are transformed to $\ln(1 + i/100)/12$. The series used in the regressions are constructed as:

$$\begin{aligned} q_j^{(US)} &= e_j^{(US)} - (p_c^j - p_c^{US}), & j &= \text{UK, WG, JP} \\ x_j^{(US)} &= (p_w^j - p_c^j) - (p_w^{US} - p_c^{US}), & j &= \text{UK, WG, JP}, \\ r_t^j &= i_t^j - (p_{c,t}^j - p_{c,t-12}^j)/12, & j &= \text{US, UK, WG, JP}. \end{aligned}$$

Except for the Japanese interest rate, all series run from January 1972 to June 1987. The one-month interest rate, i^{JP} , is only available from February 1977.

References

- Balassa, B., 1964, The purchasing power parity doctrine: A reappraisal, *Journal of Political Economy* 72, 584-596.
- Boughton, J.R., 1987, Tests of the performance of reduced-form exchange rate models, *Journal of International Economics* 23, 41-56.
- Campbell, J.Y. and R.H. Clarida, 1987, The dollar and real interest rates, *Carnegie-Rochester Conference Series on Public Policy* 27, 103-140.
- Clements, K.W. and J.A. Frenkel, 1980, Exchange rates, money and relative prices: The dollar-pound in the 1920s, *Journal of International Economics* 10, 249-263.
- Dornbusch, R., 1976, Expectations and exchange rate dynamics, *Journal of Political Economy* 84, 1161-1176.
- Dornbusch, R. and J.A. Frankel, 1987, The flexible exchange rate system: Experience and alternatives, NBER working paper no. 2464.
- Edison, H.J. and J.T. Klovland, 1987, A quantitative reassessment of the purchasing power parity hypothesis: Evidence from Norway and the United Kingdom, *Journal of Applied Econometrics* 2, 309-333.

- Frankel, J.A., 1979, On the mark: A theory of floating exchange rates based on real interest differentials, *American Economic Review* 69, 610-622.
- Frankel, J.A., 1983, Monetary and portfolio-balance models of exchange rate determination, in: J.S. Bhandari and B.H. Putnam, eds., *Economic interdependence and flexible exchange rates* (MIT Press, Cambridge).
- Frankel, J.A., 1988, Recent estimates of time-variation in the conditional variance and in the exchange risk premium, *Journal of International Money and Finance* 7, 115-125.
- Frankel, J.A. and K.A. Froot, 1987, Using survey data to test standard propositions regarding exchange rate expectations, *American Economic Review* 77, 133-153.
- Frankel, J.A. and R. Meese, 1987, Are exchange rates excessively variable?, NBER working paper series no. 2249.
- Haynes, S.E. and J.A. Stone, 1981, On the mark: Comment, *American Economic Review* 71, 1060-1067.
- Hodrick, R.J., 1987, *The empirical evidence on the efficiency for forward and futures foreign exchange markets* (Harwood Academic Publishers, Chur).
- Hooper, P. and J. Morton, 1982, Fluctuations in the dollar: A model of nominal and real exchange rate determination, *Journal of International Money and Finance* 1, 39-56.
- Hsieh, D.A., 1982, The determination of the real exchange rate: The productivity approach, *Journal of International Economics* 12, 355-362.
- Huizinga, J., 1987, An empirical investigation of the long-run behavior of real exchange rates, *Carnegie-Rochester Series on Public Policy* 27, 149-214.
- Kravis, I.B. and R.E. Lipsey, 1988, National price levels and the prices of tradables and non tradables, *American Economic Review* 78, 474-479.
- Marston, R.C., 1986, Real exchange rates and productivity growth in the United States and Japan, NBER working paper no. 1922.
- Mecagni, M. and P. Pauly, 1987, Relative prices and exchange rates in the long-run, Paper for presentation at NBER conference on the dynamics of PPP deviations, Boston, August 1987.
- Meese, R.A. and K. Rogoff, 1983a, Empirical exchange rate models for the seventies: Do they fit out of sample?, *Journal of International Economics* 14, 3-24.
- Meese, R.A. and K. Rogoff, 1983b, The out-of-sample failure of empirical exchange rate models: Sampling error or misspecification?, in: J.A. Frenkel, ed., *Exchange rates and international macroeconomics* (University of Chicago Press, Chicago).
- Meese, R.A. and K. Rogoff, 1988, Was it real? The exchange rate - interest differential relation over the modern floating rate period, *Journal of Finance* 43, 933-948.
- Smith, P.N. and M.R. Wickens, 1986, An empirical investigation into the causes of failure of the monetary model of the exchange rate, *Journal of Applied Econometrics* 1, 143-162.
- Stockman, A.C., 1987, Equilibrium approach towards exchange rate determination, *Economic Review, Federal Reserve Bank of Richmond* 73, 12-31.
- Wolff, C.C.P., 1987, Time-varying parameters and the out-of-sample forecasting performance of structural exchange rate models, *Journal of Business and Economic Statistics* 5, 87-97.