CentER

Discussion Paper

No. 2004–104

BALANCE OF POWER

By J. Boone

October 2004

ISSN 0924-7815

Tilburg ◆ University

Balance of Power

Jan Boone*

25 October 2004

Abstract

This paper argues that the efficiency distribution of players in a game determines how aggressively these players interact. We formalize the idea of balance of power: players fight very inefficient players but play softly versus equally (or more) efficient players. This theory of conduct predicts that entry by new firms leads to a less aggressive outcome if it creates a balance of power. A balance of power is created if more players get technologies that are close to the most efficient technology. Using a related argument, we show that an increase in entry costs can lead to more aggressive outcomes.

Keywords: pricing games, Folk theorem, refinement of predicted outcomes, supergames, contestable market

J.E.L. codes: D43, C72, L41

1. Introduction

An important determinant of how aggressively agents behave is the response their behavior will elicit from other agents. If you expect an aggressive response from your opponent you tend to behave more friendly than when your opponent is so weak that he cannot retaliate if you increase your utility at his expense. This idea that a balance of power in efficiency levels causes less aggressive behavior leads to the surprising comparative static result that entry by new players into the game can lead to a less aggressive outcome instead of a more aggressive

*Department of Economics, CentER, TILEC, Tilburg University, ENCORE, IZA and CEPR. Address: Department of Economics, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. Email: j.boone@uvt.nl. Comments and suggestions by Jos Jansen and seminar participants at the 2004 WZB conference on 'Collusion and Cartels' are much appreciated. Financial support from NWO, KNAW and VSNU through a Vernieuwingsimpuls grant is gratefully acknowledged.

one. More precisely, if n_1 denotes the highest efficiency level in the game, then entry which increases the number of firms with efficiency n_1 creates a balance of power and hence leads to a less aggressive outcome. In terms of a pricing game: such entry leads to higher equilibrium prices.

This idea of balance of power is not confined to economic examples with competing firms. Casual observation suggests a number of other examples. To illustrate, most people have the experience of rather dull sport matches at the highest level. For instance, boring soccer games for the world championship. Flat stages in the Tour de France cycling competition where hardly anyone attacks and the winner is determined in the final sprint. In contrast, mountain stages in the same cycling contest are full of action and attacks. The explanation for these phenomena stressed in this paper is that aggressive actions do not pay off if the differences between players or teams are small. However, if the differences in ability are big, as in the mountain stages, aggressive behavior is more rewarding. Also, over time it appears that more and more flat stages are won in the final sprint and not by groups breaking away from the bunch of riders. Paradoxically, some commentators explain this by their observation that the number of top quality cyclists has increased. This is the crowding effect explained below. Intuitively, the more high quality riders there are which can counterattack, the less likely it becomes that a break away group can be successful. Hence we see fewer attacks in flat stages now than, say, ten years ago. Finally, the balance of power between the US and USSR created a cold war instead of a real one. At the same time, these two countries were picking fights with adversaries that appeared a lot weaker to them.

Economic evidence suggesting that simple Bertrand and Cournot models may be missing something is, for instance, the empirical finding on the relation between entry and prices (as in Knetter (1994), Geroski (1989) and Klette (1999)). This relation is not as strong as traditional Cournot and Bertrand models would predict. This evidence is only suggestive and the reason why there is no direct evidence is that -to the best of our knowledge- no paper tries to explain the observed outcome by the dispersion of efficiency levels. To illustrate, the traditional structure-conduct-performance approach (as in Ravencraft (1987) and Schmalensee (1985)) tried to predict the price level of an industry on the basis of the first moment of the cost distribution (and other factors) but not on the basis of higher moments. Hence the empirical implication of our paper is that the shape of the efficiency distribution is important, not just the average efficiency level.

Although the results below are derived for a general class of games (pay off structures), an important subclass is pricing games. For the purpose of illustration we consider a pricing game with homogenous good firms each producing with constant marginal costs. As is well known (see, for instance, Tirole (1988)) there are two extreme predictions here, neither of which is fully satisfactory. On the one hand, we have the Bertrand Nash equilibrium. It's two main disadvantages are: (i) two firms with zero fixed costs is enough to get the outcome with price equal to marginal costs; (ii) any positive fixed or entry cost leads to the monopoly price. It is hard to believe that two firms are enough to get such a competitive outcome. The price discontinuity in fixed costs also seems unlikely. Surely, the outcome of the market should not differ much between zero entry cost and an entry cost equal to, say, one dollar. On the other hand, in a repeated game setting we have the Folk theorem. If players put enough weight on future pay offs any price between the Nash equilibrium price and the monopoly price can be sustained by trigger strategies. Since 'anything goes' in this case, comparative static exercises become hard and sometimes meaningless. The main result from the traditional literature stresses the effect that entry in such games makes cooperation harder and tends to reduce the maximum sustainable price. We will derive conditions under which entry leads to a higher price.

In the supergame tradition, there is a recent literature by Rothschild (1999), Compte, Jenny and Rey (2002) and Vasconcelos (2002). These papers establish that more symmetric firms (in terms of marginal cost levels, production capacities and capital) can more easily sustain the fully collusive outcome. More precisely, as the industry becomes more symmetric (along the mentioned dimensions) there is a bigger set of discount factors for which the maximal collusive price can be sustained as a subgame perfect equilibrium. This is related to our paper, as we also find that the efficiency distribution (not just the average efficiency level) is an important determinant of the outcome.

The main difference between these results and our approach is the following. We assume that players put enough weight on the future that the most collusive outcome is always sustainable. Then we ask which of the set of prices 'predicted' by the Folk theorem are plausible. We refine the set of sustainable prices by focusing on outcomes Preferred by Efficient Players (PEP). The following example illustrates the main point. Consider a consumer who is willing to buy (at most) one product at a reserve (maximum) price equal to 1 (one). There are three firms in the market, two with marginal costs equal to 0 and one with marginal costs equal to 0.9. Assume that fixed costs are equal to zero. If this game is repeated (in some way; details follow below) and firms value the future enough, the set of prices [0,1] is sustainable as subgame perfect equilibrium prices. Consider the price equal to 1: the two (most) efficient firms make an expected profit equal to 0.33 (assuming that the consumer randomizes between three firms charging the same price). If the price is reduced to 0.9, these two firms make (expected) profits equal to 0.45. Now what stops these firms from charging the lower (limit) price $p = 0.9$? Nothing actually; the only firm that loses from charging the lower price cannot (credibly) retaliate as it is priced out of the market. Hence the folk theorem does not rule out a price equal to one, but we do because the $p = 1$ is not preferred by the efficient players. It is not a PEP outcome.

Below we define and further discuss PEP outcomes. Here we note two things. First, consider the case where three players are supposed to play $p = 1$ can they actually get to a lower price? They can, if they play the following 'clever' trigger strategies (instead of the standard ones). Suppose one of your opponents undercuts the current price $p = 1$ to p'. If at this lower price your expected discounted profits are actually higher than they were at the original price, then charge p' from then onward (and do not revert to playing Bertrand Nash). If the new price p' yields lower expected discounted profits than p then revert to playing Bertrand Nash from then onward. These trigger strategies allow the efficient firms to get to a price equal to 0.9 without triggering a punishment phase. Alternatively, one can argue that if three firms can coordinate on a price equal to $p = 1$ and on the required punishment strategies, why is it not feasible for the two efficient firms to get together and agree on the lower, more profitable, price? In many scenarios it seems realistic to assume that the efficient firms will be able to do just that and hence focussing on PEP outcomes is reasonable. Second, the point is to distinguish between two types of deviations (a distinction that the folk theorem does not make). A firm which just (slightly) undercuts the current price to gain the whole market at the expense of its opponents is different from a firm that reduces the price to one where all the remaining firms are actually better off. In the former case punishment is called for, in the latter case it is not. Put differently, in the latter case, the firm can even announce its price reduction beforehand and the price cut is still profitable; in the former case this is clearly not true. The 'clever' trigger strategies discussed above take this distinction into account.

The comparative statics exercise that we stress here is that a balance of power can be created through entry. This is a comparative static exercise on the set of PEP outcomes. It can be illustrated as follows. Suppose a price reduction $\Delta p = -10$ reduces the number of opponents by 5. Then this may well be profitable for the efficient firms in the industry if there are 10 firms to start with. However, if the reduction in the number of firms is from 105 to 100, this is less likely to be profitable for the efficient firms. Hence in the former case the low price is a

PEP outcome, but not in the latter case. This we call a balance of power through crowding.

Finally, we would like to mention two strands of literature that are related to our work. First, the literature on dynamic games which are not supergames. Maskin and Tirole (1988) consider a game where firms commit to a price for a certain period. This leads to reaction lags which can sustain prices above the Bertrand equilibrium price. Their main interest is price dynamics (e.g. possibilities of cycles) and not comparative statics with respect to industry structure (they focus on the case of symmetric duopoly). Farm and Weibull (1987) consider a rapid response game, which can be seen as a special case of the model introduced below. First, in their model all firms are symmetric while we allow for asymmetries between firms. Second, in their model firms can react with probability 1 (one) to price changes of opponents while in our model this probability can be less than one as well.

Second, there is a literature that considers situations where entry by a new firm raises the equilibrium price. In Stiglitz (1989) the entry of a new firm makes the market less transparent, which raises the equilibrium price. Amir and Lambson (2000) derive conditions on demand and cost structures such that entry in a symmetric Cournot equilibrium leads to higher prices. Finally, Bulow and Klemperer (2002) consider an (almost) common value auction where entry by a bidder leads to lower expected revenue. The intuition is that with more bidders the winner's curse gets worse and hence all bidders shade their bids more strongly after entry. Interestingly, in their set up the entrant is feared to have a low valuation (bad news about the common value) while we find the balance of power result in case of a high valuation (high efficiency entrant).

This paper is organized as follows. The next section introduces the framework used in this paper, in particular the pay off functions, the dynamic structure of the game and the definition of a PEP outcome. Section 3 considers the PEP outcome for a simple pricing game and illustrates the main results in the paper. Section 4 derives conditions under which entry leads to a less aggressive outcome. Section 5 discusses the results and section 6 concludes. All proofs are in the appendix.

2. Model and PEP outcome

Consider a static game with N players. We order players in terms of their efficiency, with firm 1 being the most efficient: $n_1 \geq n_2 \geq \ldots \geq n_N$. Each player chooses an action a_i from a bounded subset of $\mathbb{R}^m$ (to ease notation, we work with $m = 1$), where higher values of a_i are interpreted as more aggressive actions. We assume that there exists a continuous real valued function $G(a_1, \ldots, a_N)$ which summarizes the effect of all players actions on a player's pay off where $\frac{\partial G(a_1,\ldots,a_N)}{\partial a_i} \geq 0$ for each i. In particular, we assume that player i's pay off can be written as $\pi(a_i, A, \nu, n_i)$ where $A = G(a_1, \ldots, a_N)$ and $\nu \leq N$ is the number of active players (see below). The value of A is referred to as the aggregate aggression level.

This implies that it does not matter for player i how the different actions are distributed over the different players, the only pay off relevant information is the aggregate aggression outcome A. In the pricing example introduced in the introduction, the aggregate aggression outcome is the lowest price charged by any firm in the market. The number of active firms ν is then the number of firms charging this lowest price.

As another example, consider a patent race where firms invest a_i in R&D to be the first to find an innovation. The value of the patent to firm i is denoted by n_i and the costs of doing R&D are given by $c(a_i)$. Then the pay off structure could take the form $\frac{a_i}{\sum_{j=1}^{N} a_j} n_i - c(a_i)$. In our terminology pay offs would be written as $\frac{a_i}{\nu A} n_i - c(a_i)$ where ν is the number of firms with positive investments ($a_j > 0$) and $A = \frac{1}{\nu} \sum_j a_j$ is the average investment level of the firms that do invest in R&D.

We make the following assumptions on the pay off structure.

Assumption 1 *(i) The function* $\pi(a_i, A, \nu, n_i)$ *is continuous in* a_i *and* A.

If $\pi(a_i, A, \nu, n_i) > 0$ *then*

(ii) $\pi(a_i, A, \nu, n_i)$ *is decreasing in* A*;*

(iii) $\pi(a_i, A, \nu, n_i)$ *is decreasing in* ν*;*

(iv) $\pi(a_i, A, \nu, n_i)$ *is increasing in* n_i*.*

Further, $\pi(a_i, A, \nu + x, n_i) - \pi(a_i', A', \nu + x, n_i) \neq 0$ *implies*

(v) $\lim_{x\to+\infty} \frac{\pi(a_i,A,\nu+x,n_i)-\pi(a_i,A,\nu'+x,n_i)}{\pi(a_i,A,\nu+x,n_i)-\pi(a_i',A',\nu+x,n_i)} = 0$*;*

Assumption 1 *(ii)* states that more aggressive outcomes (for given action a_i and given number of active players) reduce i's pay offs. Assumption *(iii)* says that it is better to have a smaller number of opponents to share the pay offs with. Assumption *(iv)* defines efficiency: higher n_i leads to higher pay offs. The next assumption introduces the idea of 'sharing games'. Keeping the action a_i and the aggregate aggression level A constant, assumption *(iii)* implies that lower ν yields higher pay offs. However, the difference in pay off between sharing with $\nu + x$ and $\nu' + x$ firms goes to zero as x goes to infinity faster than a pay off difference with the same number of firms $(\nu + x)$. As one can easily verify, this is, for instance, the case if pay offs are of the form $\frac{S(a_i,A,n_i)}{\nu} - c(a_i, A, n_i)$ where $S(.)$ is a sales or revenue function and $c(.)$ a cost function. Hence the pricing examples used in this paper have this property. The reason is that $\frac{\partial\pi}{\partial\nu} = -\frac{S(a_i,A,n_i)}{\nu^2}$ goes to zero faster as ν increases than $\frac{\partial S(a_i,A,n_i)}{\partial a_i}\frac{1}{\nu}$ and $\frac{\partial S(a_i,A,n_i)}{\partial A}\frac{1}{\nu}$ do.

This static game with pay offs $\pi(a_i, A, \nu, n_i)$ is embedded in the following dynamic structure. Each period players choose actions a_i. With a probability q these pay offs are realized in this period and the game ends. With probability $1 - q$ the pay offs are not realized, the game continues and players can react to the actions observed from their opponents. To illustrate with a pricing game, each period firms choose simultaneously and independently prices. Then there is a probability q that the consumer actually arrives this period and buys from one of the firms with the lowest price. In which case the game ends. With probability $(1 - q)$ the consumer does not arrive and firms can adjust their prices in response to the prices observed in the current period.[1] In a patent race model, the idea is that firms invest in R&D every period. With a probability q there is a scientific break through and the more a firm has invested, the more likely it is to win the patent. The patent race then ends. With probability $(1 - q)$ there is no breakthrough, the game continues and firms choose how much to invest in R&D for the next period. In terms of a simple cycling model: if you attack and sprint away from the other riders at some point in the race, there is a probability that you are not caught and you win the race. However, with some probability the other riders come back (say, because you had a flat tire) and you do not necessarily win the race.

To simplify notation we assume there is no discounting $(r = 0)$.[2] We are interested in outcomes that deviate from the Nash equilibrium. To analyze this we need to define the set of sustainable or incentive compatible (IC) actions. In determining the set of IC actions, we follow the literature in assuming that players use trigger strategies. If someone deviates, players revert to playing the most aggressive (highest A) Nash equilibrium from then onwards.[3] Actually, for our purposes it does not matter much what the punishment phase looks like exactly. Alternatively, firms could have punishment periods of finite duration and return to a collusive outcome afterwards. The details of the punishment strategies determine the set of IC strategies, our interest is in which outcome in this set is actually preferred by efficient firms. The comparative statics focus on this preferred outcome, not on the set of sustainable outcomes.

Before we define IC outcomes, let's introduce some more notation. Let $\pi^{NE}(n_i)$ denote

[1] Another interpretation is that in every period a fraction of total demand is realized. Both interpretations are equally valid.

[2] This is without loss of generality since q takes on the role of discount factor.

[3] We do not yet worry here about deviations that make all surviving players better off. We come back to this below.

player i's pay off in the (most aggressive) Nash equilibrium outcome. Further, the (static) best response of player i to aggregate aggression level A is denoted by $R_i(A)$. In principle, the best reply may also depend on the number of active players, but this is suppressed to ease notation.

Definition 1 *The set of IC actions is defined as*

$$\mathbb{A} = \left\{ a \;\middle|\; \begin{array}{c} \pi(a_i, G(a), \nu(G(a)), n_i) \geq q\pi(R_i(G(a)), G(a), \nu(G(a)), n_i) + \\ (1-q)\pi^{NE}(n_i) \quad \forall i = 1, 2, \ldots, \nu(G(a)) \end{array} \right\}$$

where the number of active firms $\nu(G(a))$ is determined by

$$\nu(G(a)) = \max_{i=\{1,2,\ldots,N\}} \{i | q\pi(R_i(G(a)), G(a), i, n_i) + (1-q)\pi^{NE}(n_i) > f\}$$

Hence an action profile a is an element of $\mathbb{A}$ if for all active firms it is optimal to stick to a instead of playing a best reply to $G(a)$ and playing the Nash equilibrium from then onwards. If a player deviates, there is a probability q that pay offs are realized and the deviator gets $\pi(R_i(G(a)), G(a), i, n_i)$. With probability $1-q$ pay offs are not realized and the deviator gets the Nash equilibrium pay off $\pi^{NE}(n_i)$.[4] The number of active firms $\nu(G(a))$ is determined as follows. Firm i cannot be kept out of the market if it is profitable for i to deviate and enter. If i is supposed to stay out and yet enters, there is a probability q that pay offs are realized and he gets $\pi(R(G(a)), G(a), i, n_i)$. With probability $1-q$ pay offs are not realized in that period and i gets the Nash equilibrium pay off from then onward. If these expected profits exceed the entry cost f, player i cannot be kept out of the market. Note the similarity between this and the hit-and-run entrant in contestable market theory (Baumol (1982) and Baumol and Willig (1981)). We come back to this below. If the expected profits do not exceed f then firm i cannot profitably enter.

[4]Note that with a discount rate $r > 0$ the weights on the profits would be $\frac{r+q}{1+r}$ and $\frac{1-q}{1+r}$ resp. As these weights add up to one as well, it is without loss of generality that we focus on the case with $r = 0$.

Definition 2 *We say that an aggregate aggression level A is sustainable if there exists $a \in \mathbb{A}$ with $G(a) = A$.*

With this definition we can make the final assumption on the pay off structure.

Assumption 2 *Take two sustainable A and $A' < A$ with $\nu(A) < \nu(A')$. Then for each $a \in \mathbb{A}$ with $G(a) = A$ there exists $a' \in \mathbb{A}$ with $G(a') = A'$ such that*

$$\pi(a_i', A', \nu(A), n_i) > \pi(a_i, A, \nu(A), n_i)$$

for each $i = 1, \ldots, \nu(A)$.

This assumption generalizes the idea in Assumption 1 *(ii)* that a lower aggregate aggression level A is more profitable. In particular, in Assumption 1 we consider the partial effect of A for given a. However, if all a_i are kept constant, we cannot have a different value of A. Hence, assumption 2 states that if A is reduced while the number of firms is kept constant at $\nu(A)$ then we can always find a' such that the most efficient firms are better off at the reduced level $A' = G(a')$ than they are at the more aggressive level A. In the pricing example of the introduction where a consumer buys at most one product, this can be illustrated as follows. A firm with cost level $c_i < p$ has profits equal to $\frac{p-c_i}{\nu}$ given that it shares the market with $\nu - 1$ competitors. The assumption says that at a higher price $p' > p$ (less aggressive outcome) we can find an action profile such that firm i is better off if the number of firms is unchanged. In this example, this assumption holds (trivially) since $\frac{p'-c_i}{\nu} > \frac{p-c_i}{\nu}$ for each firm i with $c_i < p$.

Now we introduce the equilibrium concept used in this paper. We say that an action profile a is a Preferred by Efficient Players (PEP) outcome if there is no other action profile a' such that the firms that are active under both outcomes prefer a' over a. More formally, this can be stated as follows.

Definition 3 *An action profile* $a \in \mathbb{A}$ *with* $G(a) = A$ *is a PEP outcome if there does not exist* $a' \in \mathbb{A}$ *with* $G(a') = A'$ *such that*

$$\pi(a_i', A', \nu(A'), n_i) \geq \pi(a_i, A, \nu(A), n_i)$$

for each $i = 1, \ldots, \min\{\nu(A), \nu(A')\}$ *and with strict inequality for at least one* i.

In other words, a PEP outcome has a Pareto optimality property, but not with all players taken into account. When we compare action profile a' with a more aggressive action profile a in the sense that $G(a) > G(a')$, then a' is not preferred by efficient players if all surviving players $1, \ldots, \nu(G(a))$ are better off under a. Since players $\nu(G(a)) + 1, \ldots, \nu(G(a'))$ see their pay offs reduced to zero, it is not the case that a Pareto dominates a', but it is the case that a is better for all the firms that could potentially retaliate.

The motivation for focussing on PEP outcomes is the following. Suppose that, following the folk theorem, we imagine that firms $1, \ldots, \nu(A')$ can coordinate on an action profile a' and corresponding punishment strategies, then in many scenarios it is hard to understand what stops a coalition of efficient players to coordinate on an action profile that makes all of these efficient players better off. If the action profile is determined by some bargaining procedure, one would expect that it is easier to come to an agreement with a smaller number of players. Hence, any outcome that can be supported as a collusive outcome in the folk theorem but that is not a PEP outcome is discarded in this paper.

Our concept of a PEP outcome is related to the core in cooperative game theory. A PEP outcome is one where no coalition can break away and guarantee itself a higher pay off. We do not pursue this relation here further to avoid technicalities.[5]

[5] In particular, the action space for a player here (who chooses an action every period) is not finite dimensional and the relation between PEP outcomes and the core only holds in the dynamic version of the game. Also we have more structure here than in the general framework used in cooperative game theory. This allows us to give a rather straightforward proof of existence of PEP outcomes (where we only use Weierstrass' theorem).

Before turning to some examples of PEP outcomes, we briefly discuss existence and uniqueness issues.

Lemma 1 *Assume that actions a_i are chosen from a bounded set, and that the functions $\pi(a_i, A, \nu, n_i), G(a)$ and $R_i(A)$ are continuous in a and A. Then if there exists a Nash equilibrium for the static game, there exists a PEP outcome.*

The assumption that actions are taken from a bounded set is not restrictive. In most economic examples it is the case that there exists a_i such that for all a_{-i} it is the case that $\pi(a_1, G(a_1, a_{-1}), \nu(G(a_1, a_{-1}), n_1), n_1) < \pi^{NE}(n_1)$ which cannot be optimal. In a pricing example, if firm i charges a price equal to the lowest marginal cost level in the industry, we already have this effect. In an R&D race model where the aggregate aggression level is related to the sum of the R&D investments chosen, we find the same effect as $\lim_{a_i \to \infty} G(a_i, a_{-i}) = +\infty$ which cannot be optimal for a convex cost of R&D function.

The idea of the proof is to use the Weierstrass theorem on the existence of global solutions to a particular set of optimization problems. The continuity assumptions are needed to apply the theorem. Further, if a Nash equilibrium exists, we know that the set of IC outcomes $\mathbb{A}$ is nonempty, as the Nash equilibrium itself is IC.

As we show with a simple pricing example below a PEP outcome is generally not unique. However, this does not matter for our purposes. The comparative statics result that we want to show is that entry can lead to a less aggressive outcome. This we will show for all PEP outcomes. In particular, we start with a situation where all PEP outcomes feature $A > A_0$ for some $A_0 > 0$. Then we prove that if enough firms enter all PEP outcomes feature $A < A_0$.

3. Pricing game examples

In this section we use the simple pricing game in the introduction to illustrate the following results: entry by a new firm can lead to a higher PEP outcome, the PEP outcome is not necessarily unique but still we find that entry leads to a lower price. Further, we show that an increase in entry costs can lead to a lower price and that the PEP outcome can be achieved in a non cooperative game when players use 'clever' trigger strategies.

Consider the following simple pricing game between firms producing a homogenous good. There is one consumer who buys maximally one unit of the good and has a reservation price equal to $v > 0$. Firm $i = 1, \ldots, N$ has cost c_i and we define firm i's efficiency level as $n_i = v - c_i$. Firms set prices, but we define actions in such a way that more aggressive actions are associated with higher values. Firm i's action is defined as $a_i = v - p_i$. In words, firm i's action is the surplus granted to the consumer. The pay off to firm i can now be written as

$$\pi(a_i, A, \nu(A), n_i) = \begin{cases} 0 & if \quad a_i < A \\ \frac{n_i - A}{\nu(A)} & if \quad a_i = A \end{cases}$$

where $A = \max\{a_1, \ldots, a_N\}$ and $\nu(A)$ is the number of firms choosing action A. That is, the most aggressive firm captures the whole market. If firms are equally aggressive, they share the market equally.

We embed this static game into the following dynamic structure. In each period, firms choose their actions a_i. With probability q the consumer arrives and buys from the firm that grants him the highest surplus (randomizes, if necessary). After the consumer has bought the good, the game ends. With probability $(1-q)$ the consumer does not arrive and firms can react to each other's actions. In the next period there is again a probability q that the consumer arrives etc. This seems a reasonable description of, say, a high street firm selling TV sets. Each morning the firm's owner decides what price tag to put on a certain TV model. If a consumer arrives that day, she buys the TV from one of the sellers charging the lowest price. If the consumer does not arrive, the firm's owner can change the price the next morning taking into account the prices charged by his competitors down high street. If the probability q is determined by the probability that the customer's old TV breaks down, it is exogenous and does not depend on the price charged by the firms. Below we also consider an example with a downward sloping demand curve $X(p)$. Expected sales are then $qX(p)$ which can (under some conditions) also be interpreted as saying that lower prices make it more likely that the consumer arrives today.

The set of incentive compatible action profiles, takes a simple form in this context:

$$\mathbb{A} = \{A | \frac{n_1 - A}{\nu(A)} \geq q(n_1 - A) + (1-q)(n_1 - n_2)\}$$

where the number of active players is determined by

$$\nu(A) = \max_{i \in \{1,\ldots,N\}} \{i | q(n_i - A) > f\}$$

Since $A = \max\{a_1, \ldots, a_N\}$ all active players have $a_i = A$ otherwise they will never sell. Next, if an action A is IC for firm 1 it is IC for all active firms. This is because firm 1 has the biggest incentive to deviate: the most efficient firm has the highest incentive to price low and the most efficient firm still (potentially) makes a positive profit in the Nash equilibrium played after a deviation. If firm 1 deviates he chooses an action slightly above A (slightly undercuts the price) and captures the whole market. If the consumer does not arrive, firm 2 plays $a_2 = n_2$ from then onward and we get the Nash outcome. Finally, only firms $i > 1$ can enter who can finance the entry cost f out of expected profits if the consumer arrives in the period of entry.

From the definition of $\nu(A)$, we see that the limit action (price) that keeps firm $i > 2$ out of the market equals $A = n_i - f/q$ ($p = c_i + f/q$). In other words, f/q measures how contestable a market is. However, this does not imply that it is optimal to keep the next efficient firm 2 out.

If firm 2 is efficient enough, the PEP outcome will exceed $n_2 + f/q$. In other words, although the logic of contestable markets applies in this context, it does not follow that the outcome is necessarily a low price. This depends on the efficiency distribution in the industry.

In this sense, the concept of a PEP outcome bridges the gap between predatory pricing keeping firm 2 out of the market and full collusion where firms charge the monopoly price. The efficiency distribution determines which limit price is chosen, or put differently, the degree to which predatory pricing is used. Alternatively, starting from the monopoly price, the efficiency distribution determines the degree to which the firms collude. By focussing on PEP outcomes, predatory pricing-collusion is no longer a zero-one decision.

Note that an action A that keeps a firm j out with $A > n_j + f/q$ cannot be a PEP outcome: there exists another action ($A = n_j + f/q$) that makes all surviving firms $(1, \ldots, j-1)$ better off. Hence below we only consider limit prices and the monopoly price ($A = 0$) when determining PEP outcomes.

For all examples here we take $v = 1$ and $n_1 = 1$. Other parameters are given in each example.

Example 1 *Assume that* $f = 0, n_2 = 0.4$. *Hence there are only two potential PEP outcomes* $A = 0$ *and* $A = 0.4$. *Assuming that* q *is small enough that both are IC, we find that the profits for firm 1 equal* $\pi_1 = 0.5$ *at* $A = 0$ *and* $\pi_1 = 0.6$ *at* $A = 0.4$. *Consequently, firm 1 prefers to keep 2 out of the market and be the only producer in the market. The unique PEP outcome is* $A = 0.4$. *Now consider the entry of a new firm into the market with* $n_3 = n_1 = 1$. *The monopoly price,* $A = 0$, *now yields expected profits* $\pi_1 = \pi_3 = 0.33$ *while keeping 2 out of the market yields* $\pi_1 = \pi_3 = 0.3$. *Hence, after entry by firm 3, the unique PEP outcome is less aggressive than before* ($A = 0 < 0.4$)*, assuming that* q *is small enough that* $A = 0$ *is IC.*

There are two things to note about this example. First, using the folk theorem it can also happen that entry by a very efficient firm leads to a less aggressive outcome. This happens because the punishment phase can be harsher due to entry and therefore more collusive (less aggressive) outcomes can be sustained that could not be sustained before. However, this is not the intuition of the effect above. By assumption $A = 0$ was also IC before firm 3 entered the market. The intuition here is a balance of power through crowding. Increasing aggression from $A = 0$ to 0.4 first reduced the number of firms from 2 to 1. After entry the same increase in aggression leads to a reduction in the number of firms from 3 to 2 which is less profitable.

Second, although this paper focusses on the effect of entry, similar results can be derived for efficiency gains. To see this, assume that instead of firm 3 entering, we consider a gain in efficiency for firm 2 from $n_2 = 0.4$ to $n_2 = 0.9$. Such an efficiency gain would lead to lower prices in both a Cournot and a Bertrand model. Here, however, the PEP outcome will become less aggressive after the efficiency gain: $A = 0 < 0.4$. The intuition is that it is profitable for firm 1 to fight a rather inefficient firm ($n_2 = 0.4$) but it is not profitable to fight an efficient firm with $n_2 = 0.9$. Hence the efficiency gain creates a balance of power that did not exist before.[6]

The next example shows that a PEP outcome is not necessarily unique, yet the main comparative static result (entry can lead to less aggressive outcomes) still holds.

Example 2 *Consider an industry with* $n_2 = 0.7, n_3 = 0.3, f = 0$. *Then it is routine to verify that* $A = 0$ *and* $A = 0.3$ *are both PEP outcomes (1 prefers* $A = 0.3$ *over* $A = 0$ *while for 2 it is the other way around). If a new firm enters with* $n_4 = n_1 = 1$ *then* $A = 0$ *is the unique PEP outcome. Hence entry, if it changes anything, leads to a less aggressive outcome.*

Next we consider the effect of a rise in entry cost. Here we see the 'mirror image' of the crowding effect. A rise in entry cost allows each limit price to rise and still keep the marginal entrant out. This has more impact on profits if the number of active players is small. Hence, a rise in entry costs, tends to make more aggressive outcomes relatively more attractive.

[6]Note that entry can also be viewed as an efficiency gain. Firm 3 in the example above may have started out with efficiency level $n_3 = 0$ and experience an efficiency gain to $n_3 = 1$.

Example 3 *Consider an industry with* $n_2 = 0.65, n_3 = 0.1, f/q = 0.1$. *Hence the relevant limit actions are* $A = 0$ *and keep firm 3 out and* $A = 0.55$ *and keep firms 2 and 3 out of the market. Clearly,* $A = 0$ *is the unique PEP outcome here. Now the entry cost* f *goes up such that* $f/q = 0.2$. *Now it becomes optimal for player 1 to keep 2 out of the market by limit action* $A = 0.45$. *Hence the rise in entry cost leads to a more aggressive outcome.*

Two brief remarks on this example. First, note that the PEP outcome for $f = 0$ equals $A = 0.1$ to keep firm 3 out of the market. For small increases in f the outcome is simply the limit action $A = 0.1 - f/q$ which is continuous in f. This is in sharp contrast to the Bertrand outcome. Bertrand competition in this context predicts $A = 0.65$ for $f = 0$ while $A = 0$ for any $f > 0$ no matter how small. This discontinuity is seen as a serious drawback of the Bertrand model. It disappears when considering PEP outcomes.[7]

Our second remark concerns the different interpretations of the entry cost f. If f is a sunk entry cost, then the comparative static in the example should be interpreted as the comparison between two identical industries where one has a higher entry cost. In particular, it cannot be interpreted as a change over time in the same industry. If firm 2 has already sunk its investment f then only a limit action $A = n_2 = 0.65$ can remove it from the industry. However, if f is interpreted as a recurring per period fixed cost (*e.g.* cost of keeping your shop open) then an increase in f (say, the rental rate of the shop goes up) can induce firm 1 to push 2 out of the market. In that case, the comparative static can be interpreted as happening over time within the same industry.

We conclude this section with a discussion of the way firms can get to a PEP outcome in a noncooperative way. Suppose that firms currently charge a sustainable price which is not a PEP outcome in the sense that there is a lower price that makes all surviving firms better

[7]Other ways in which this problem disappears is to consider heterogenous goods instead of homogenous ones, or to assume increasing marginal costs instead of constant marginal costs in a model with a downward sloping demand function. See Tirole (1988) for details.

off. Consider the case where firms use the following 'clever' trigger strategies. Suppose you are in the cooperative stage (*i.e.* not in a punishment phase) and you are supposed to charge a price p. Now one of your opponents in the previous period charged $p' < p$. If p' is below your marginal cost level, charge your marginal cost level from then onward. If p' is above your marginal cost level and your expected discounted profits are lower at p' than they were at p then play the punishment strategy (*e.g.* play the Nash equilibrium strategy from then onward). If, on the other hand, your expected discounted profits are higher at p' than they were at p, charge p' from then onward. In words, the 'clever' trigger strategy differs from the usual trigger strategy in that it distinguishes between deviations that are profitable for all surviving firms and deviations that go at the expense of some of the surviving firms. Firms that are forced to exit due to the price reduction are worse off, but they cannot retaliate.

If the deviating firm can announce beforehand that it will reduce its price from p to p', then there is not even the risk that pay offs will be realized (with probability q) before the others can follow the price reduction. Hence with such announcements there is no risk that the other firms lose out because of the price reduction. As is well known, such price announcements happen frequently in reality. Examples go back to Stigler (1947) which includes examples of pre announcements in the cigarette and anthracite industries. Adams and Brock (2001) describe the price announcements in the US car and airline industries. Recent European antitrust cases where, for instance, trade associations made 'price recommendations' include the Dutch concrete industry, German fire insurers and Belgian manufacturers and importers of tobacco.[8]

Hence it seems that it is relatively straightforward for firms to get to a PEP outcome in a noncooperative way. Admittedly, this is simplest in a pricing game, where a firm either charges the lowest price in the market or exits the market. In a more general game, firms need to coordinate both on A and on the vector of actions a that leads to $A = G(a)$, which is harder.

[8]The cases are Cementhandelaren v. Commission [1972] ECR977, Verband der Sachversicherer v. Commission [1987] ECR405 and Van Landewyck v. Commission [1980] ECR3125.

4. The effect of entry on the PEP outcome

In this section we derive sufficient conditions under which entry by efficient firms leads to a less aggressive PEP outcome. Next we derive for a pricing game with a more general demand function than the one used in the examples above that an increase in the entry cost f tends to lead to more aggressive outcomes.

The next proposition derives that if enough efficient firms enter the market, the outcome becomes less aggressive. Although we derive this as a limit result, the examples above have shown that it is easy to find cases where one firm entering can be enough to get this result. Note that we (simply) assume that q is small enough that the less aggressive outcomes are IC. Hence, the relevant effect here is not that entry allows for harsher punishments and therefore makes more collusive outcomes possible. By assumption the more collusive outcomes are (always) possible, they are just not preferred by efficient firms.

Proposition 1 *Consider a game with N players where $n_1 \geq \ldots \geq n_N$ and $f/q \geq 0$. Assume that there exists $A_0 > 0$ such that initially any $A < A_0$ is not a PEP outcome. Then there exists $\overline{x} \geq 0$ such that if $x > \overline{x}$ firms enter with efficiency level n_1 then any $A > A_0$ cannot be a PEP outcome (assuming q is small enough that at least one $A < A_0$ is sustainable).*

This shows that entry by efficient firms leads to a less aggressive outcome. Initially, all PEP outcomes have an aggregate aggression level above $A_0 > 0$.[9] After entry, all PEP outcomes lie below A_0 (assuming that lemma 1 applies and a PEP outcome exists).

The intuition for this result is balance of power through crowding. Suppose that an increase in aggregate aggression $\Delta A > 0$ causes five firms to leave the market. This may be profitable for efficient (surviving) firms if there are ten firms to start with. To illustrate, if the sharing

[9] This is without loss of generality. If $A = 0$ is a PEP outcome at the start, entry cannot lead to a (strictly) less aggressive outcome.

takes the simple form of $1/\nu$ then a share $\frac{1}{10-5}$ is substantially bigger than $\frac{1}{10}$. However, for x big enough the fall in profits due to $\Delta A > 0$ is not compensated by a gain in share from $\frac{1}{10+x}$ to $\frac{1}{10+x-5}$ since $\lim_{x\to+\infty} \frac{1}{10+x-5} - \frac{1}{10+x} = 0$.

Why is it important that the x new firms enter with the highest efficiency level n_1? Suppose they would all enter with some efficiency level $n_x \in [A_0, n_1\rangle$ then for x big enough, it becomes optimal for firm 1 (and other firms with $n_i > n_x$) to keep these x firms out of the market by choosing $A = n_x$. Hence, entry by inefficient firms can lead to a more aggressive outcome in this case. To exclude this, we consider the case where entry happens by firms with efficiency level n_1. Note that if the firms enter with efficiency levels below A_0, there is going to be no effect on the PEP outcome. Since it was not optimal before to lower A below A_0, it is certainly not optimal now because even more firms would become active if A is reduced to such low levels. Finally, if the new firms enter with efficiency levels above n_1 they may be so much more efficient than 1 that they want to push 1 from the market as well. That would imply that entry makes the outcome more aggressive because it does not create a balance of power. In fact, in that case entry upsets the balance.

Summarizing, we get that entry leads to a less aggressive outcome if it creates a balance of power. This happens if the entrants' efficiency levels are close (or equal) to that of the leaders in the industry.

Finally, we consider the case where an increase in entry cost f leads to a more aggressive outcome. This result we derive for a pricing game for two reasons. First, as f changes, the limit action to keep the marginal entrant out changes. The question then is: how should the vector of individual actions a change to get to the new (lower) limit level $A = G(a)$? The structure introduced above is insufficient to answer this question and considerably more structure is needed to analyze this. However, in a pricing game (with homogenous goods and constant marginal costs, as above) the relation between the aggregate aggression level to keep the marginal entrant out and individual actions is very simple: all active players charge the limit price. Second, the intuition for the effect of f on the aggregate outcome is naturally described in terms of a mark up. Higher f allows for a higher mark up while keeping the marginal entrant out. This higher mark up leads to a bigger increase in profits if the market is shared with fewer firms. Moreover, pricing games are, in fact, an important class of games in economics.

We assume that demand is of the form $D(p)$ with $D'(p) < 0$ and firm i has constant marginal cost c_i which is weakly increasing in i. Then the limit price $\overline{p}_i$ to keep firm i out is implicitly defined by

$$qD(\overline{p}_i)(\overline{p}_i - c_i) = f$$

since the Nash profit for any firm $i > 1$ equals 0.

Proposition 2 *If $D''(p) \leq 0$ then an increase in f makes lower prices relatively more attractive. In particular, the increase in firm i's profits due to the rise in f is higher for lower values of p (assuming $c_i < p$).*

This proposition gives conditions under which a rise in fixed costs makes lower prices relatively more attractive. Hence it points to a tendency under which higher fixed costs leads to lower prices, which is contrary conventional wisdom. The idea is that a rise in fixed costs allows for a higher mark up of prices over the marginal costs of the marginal entrant. Such an increase in mark up has a bigger positive effect on firm i's profits if i shares the market with fewer opponents. This is the case at lower prices. We need to exclude the case where at lower prices an increase in the price (due to the increase in f) leads to a bigger fall in demand than at higher prices. In that case, the increase in f would have a less beneficial effect at lower prices than at higher prices. This case is excluded by assuming that demand $D(p)$ is weakly concave in p.

5. Discussion

Above we have illustrated the results using pricing games. Here we show that R&D race models have similar properties. This illustrates that the assumptions made above are actually quite general. We relate the results found here to the outcomes of the static Nash equilibrium and of supergames. Finally, we relate our results to the literature on price leadership.

Consider the following static version of a patent race model. Firms invest an amount a_i in R&D. The probability that firm i wins the patent is given by $\frac{a_i}{\sum_{j=1}^{N} a_j+\lambda}$ for some $\lambda \geq 0$. We assume pay offs are of the form

$$\frac{a_i}{\sum_{j=1}^{N} a_j + \lambda} n_i - a_i$$

where n_i is the value of the patent for firm i. There is an entry cost $f \geq 0$. If we define A as the average investment of the ν active firms (*i.e.* firms with $a_i > 0$), we can write i's pay offs as

$$\pi(a_i, A, \nu, n_i) = \frac{a_i}{\nu A + \lambda} n_i - a_i$$

It is straightforward to check that assumption 1 is satisfied: π is continuous in a_i and A, decreasing in A and ν and increasing in efficiency n_i. This pay off function also has the sharing property *(v)* since

$$\lim_{x\to+\infty} \frac{\frac{a_i}{(\nu+x)A+\lambda} n_i - a_i - \left(\frac{a_i}{(\nu'+x)A+\lambda} n_i - a_i\right)}{\frac{a_i}{(\nu+x)A+\lambda} n_i - a_i - \left(\frac{a'_i}{(\nu+x)A'+\lambda} n_i - a'_i\right)} =$$

$$\lim_{x\to+\infty} \frac{n_i a_i \left(\frac{\nu+x}{(\nu+x)A+\lambda} - \frac{\nu+x}{(\nu'+x)A+\lambda}\right)}{\frac{a_i(\nu+x)}{(\nu+x)A+\lambda} n_i - \frac{a'_i(\nu+x)}{(\nu+x)A'+\lambda} n_i - (\nu+x)(a_i - a'_i)} = 0$$

for $\nu' \neq \nu$ and $(a'_i, A') \neq (a_i, A)$. Finally, we check whether assumption 2 is satisfied. Take an arbitrary sustainable $(a_1, \ldots, a_n)$ such that $A = \frac{1}{\nu} \sum_{j=1}^{\nu} a_j$ Then consider a sustainable $A' = \zeta A$ with $\zeta \in \langle 0, 1\rangle$ and $a'_i = \zeta a_i$. Assumption 2 is satisfied if $\frac{\zeta a_i}{\nu\zeta A+\lambda} n_i - \zeta a_i$ is decreasing in ζ. To characterize this, we need to define the monopoly action a_i^m of firm i. This is defined as follows

$$a_i^m = \arg\max_a \frac{a}{a+\lambda} n_i - a$$

It is the action taken by i if it is the only firm in the market. Using this, one can show that

$$\frac{\partial \left(\frac{\zeta a_i}{\nu \zeta A + \lambda} n_i - \zeta a_i \right)}{\partial \zeta} < 0$$

as long as $\nu \zeta A > a_i^m$. That is, the current total investments exceed the monopoly investment of firm i for each active firm i.

There are a number of ways in which the static game can be repeated. There may be a number of innovations that have to be made and the firms have a race every period for the next innovation. Or there is one innovation and there is a probability q that there is a scientific breakthrough this period such that the innovation is found and pay offs are realized. The expected duration of the patent race is then $1/q$. Both set ups create a dynamic structure such that the results derived above apply: entry by efficient firms can lead to lower R&D investments.

This patent race model can also be used to describe certain sport games. If we denote the value of winning by V and the athlete's effort during the race by a_i then pay offs can be written as $\frac{a_i}{\sum_{j=1}^{N} a_j} V - \frac{a_i}{n_i}$ where n_i is the quality of the athlete. By increasing your effort during the race, you increase the probability of winning but it is not necessarily the case that the player with the highest effort wins with probability one. Of course, in sport events an important determinant is the shape of the day for the athletes, *e.g.* having 'good legs' in cycling. This suggests that the values n_i are known to player i but the opponents only know the distribution from which n_i is drawn. Analyzing the signalling game that follows from this is beyond the scope of this paper. Here we just want to point out that there is a formalization of sport tournaments that satisfies the assumptions made above.

Second, how do the results derived above relate to the results of the static Nash equilibrium and of supergame outcomes. The games analyzed here include games with upward sloping reaction functions. In these games, the Nash equilibrium has the property that more players lead to more aggressive outcomes. This prediction goes against the crowding effect described here which leads to less aggressive outcomes as the result of entry. In the supergame version of the game analyzed here, the main result is that more firms in the market makes collusion harder to sustain. Hence the outcome becomes 'less collusive' (more aggressive) as the result of entry. The only caveat is that entry may lead to more aggressive punishment phases and thereby may make it possible to sustain more collusive outcomes. This is a comparative static exercise on the set of sustainable prices. In contrast, we have considered the effect on the attractiveness of different sustainable outcomes.

Finally, in pricing games there is a literature on price leadership. From the set of sustainable prices, the price leader chooses the one that maximizes its profits taking the reaction of the other firms into account. This is another way to select an equilibrium outcome than the PEP outcome. We argue that the main result derived above applies here as well. In this literature, the price leader is determined in an industry by the following factors. First, in Rotemberg and Saloner (1990) the better informed firm (about demand conditions in the industry) becomes price leader. Second, in Kirkman and Schueller (1990) a firm is price leader in his home country (in an international context where a country has at most one home producer of the product). Third, in Deneckere and Kovenock (1992) the firm with the biggest capacity is price leader and in Deneckere, Kovenock and Lee (1992) the firm with the largest segment of loyal consumers is price leader. In the set up above, we would argue that the leading firm in the industry chooses an action profile that is sustainable and that maximizes its own profits taking entry behavior by other firms into account. In other words, the leader i chooses a to solve

$$\max_{a \in \mathbb{A}} \pi(a_i, G(a), \nu(G(a)), n_i)$$

It is routine to check that the proof of proposition 1 applies to this selection of the market outcome as well. Hence the same balance of power intuition applies in markets where a price leader (or group of price leaders) selects the industry outcome. Also in these markets there is a tendency for entry by efficient firms to lead to less aggressive outcomes.

6. Conclusion

In this paper we have formalized the balance of power intuition. The outcome of a game becomes less aggressive as players cluster around the highest efficiency level in the group of players. This can happen because of entry by such efficient players. Similar arguments can be made for efficiency gains by existing players.

To derive the balance of power result, we make three basic assumptions on the static game. First, players' pay offs are decreasing in the aggregate aggression level of the outcome. Second, more aggression can eliminate an opponent from the market and a higher aggression level is needed to eliminate more efficient players. Third, agents are in a 'sharing' game.

Our claim is that many games in real life have this structure. Consider, for instance, the cold war between the US and the USSR. The battle was over spheres of influence. In a direct open conflict, the country with the biggest military strength would probably have won. But the pay off would have been related to the differences in military strengths (efficiency). The US and USSR did pick fights with countries that they considered a lot weaker than they were. But there was (fortunately) never an open conflict between the US and USSR. One explanation is the balance of power between them. This paper argues that one should then expect a relatively peaceful outcome if there is a balance of power.

Another illustration is that Lance Armstrong does not attack in flat stages in the Tour de France because ability differences are small between him and his opponents. In the mountains the differences in ability are bigger and hence he attacks there. One should expect more aggressive strategies and outcomes when the difference between efficiency (ability) levels goes up.

Going back to economics, in competition policy there is also the idea that more symmetric firms cause prices to be higher. This is referred to as coordinated effects or joint dominance. As mentioned in the introduction, joint dominance has been formalized by showing that asymmetries between firms make it harder to sustain a collusive outcome. We have shown that aggressive play becomes less profitable as firms cluster around the highest efficiency level in the industry.

The main implication for empirical research is that not only the average efficiency level matters in determining how aggressive the outcome is. Higher moments of the efficiency distribution are important as well. It is these higher moments that reveal how big the gaps are between firms' efficiency levels and whether there is a balance of power.

Finally, the results point to the following policy implications. In a joint dominance or coordinated effects case where a merger between two firms brings the merged entity towards the technological frontier in the industry, there is indeed reason to consider the effects. Normally such efficiency gains are seen as an argument in favor of the merger (the so called efficiency defence). But if such a merger creates a balance of power by increasing the number of firms at the technological frontier there is a chance that the outcome becomes less aggressive and prices become higher. As argued by Motta (2004) it is hard, if not impossible, to punish tacit collusion. Hence a competition authority may chose to prevent it from happening in the first place. This can involve forbidding a merger if it creates a balance of power in the industry. This only applies to industries where tacit collusion is likely. Posner (2001) and Motta (2004) describe lists of characteristics that make industries susceptible to collusion. Moreover, stimulating entry or reducing entry barriers in such industries may not have the desired effects of disciplining incumbents. Both policy measures can in fact have the perverse effect of raising prices. Thus, if it is hard or even impossible for a competition authority to fight and prevent tacit collusion, mergers creating a balance of power should be prevented.

Another implication is that in order to make the Tour de France more lively and attractive again, one should consider reducing the number of cyclists that are allowed to start. Having fewer high quality cyclists reduces the crowding effect and hence the outcome may become more aggressive.

7. References

Adams, W. and J. Brock, (2001), *The structure of American industry*, Prentice-Hall, New Jersey.

Amir, R. and V. Lambson, (2000), On the effects of entry in Cournot markets, *Review of Economic Studies*, Vol. 67 (2): 235-254.

Baumol, W., (1982), Contestable markets: an uprising in the theory of industry structure, *American Economic Review*, Vol. 72 (1), 1-15.

Baumol, W. and R. Willig, (1981), Fixed costs, sunk costs, entry barriers, and sustainability of monopoly, *Quarterly Journal of Economics*, Vol. 96 (3), 405-431.

Brinkhuis, J. and V. Tikhomirov, 2004, Optimization: insights and applications, forthcoming Princeton University Press.

Bulow, J. and P. Klemperer, (2002), Prices and the winner's curse, *RAND journal of Economics*, Vol. 33 (1), 1-21.

Compte, O., F. Jenny and P. Rey, (2002), Capacity constraints, mergers and collusion, *European Economic Review*, Vol. 46, 1-29.

Deneckere, R. and D. Kovenock, (1992), Price leadership, *Review of Economic Studies*, Vol. 59, 143-162.

Deneckere, R., D. Kovenock and R. Lee, (1992), A model of price leadership based on consumer loyalty, *Journal of Industrial Economics*, Vol. XL (2), 147-156.

Farm, A. and J. Weibull, (1987), Perfectly flexible pricing in a homogenous market, *Scandinavian Journal of Economics*, 487-495.

Geroski, P. A., (1989), The effect of entry on profit margins in the short and long run, *Annales d'Économie et de Statistique*, Vol. 15/16, 333-353.

Kirman, A. and N. Schueller, (1990), Price leadership and discrimination in the European car market, *Journal of Industrial Economics*, Vol. XXXIX (1), 69-91.

Maskin, E. and J. Tirole, (1988), 'A theory of dynamic oligopoly, II: price competition, kinked demand curves, and Edgeworth cycles, *Econometrica*, Vol. 56 (3): 571-599.

Motta, M. , (2004), *Competition Policy: Theory and Practice*, Cambridge University Press.

Posner, R.A., (2001), *Antitrust Law*, 2nd edition, University of Chicago Press.

Ravenscraft, D., (1983), Structure-profit relationships at the line of business and industry level, *Review of Economics and Statistics*, Vol. 65, 22-31.

Rotemberg, J. and G. Saloner, (1990), Collusive price leadership, *Journal of Industrial Economics*, Vol. XXXIX (1), 93-111.

Rothschild, R., (1999), Cartel stability when costs are heterogeneous, *International Journal of Industrial Organization*, Vol. 17, 717-734.

Schmalensee, R., (1985), Do markets differ much?, *American Economic Review*, Vol. 75 (3), 341-351.

Stigler, G.J, (1947), 'The Kinky Oligopoly Demand Curve and Rigid Prices', *Journal of Political Economy*, Vol. 55 (5), 432-449.

Stiglitz, J., (1989), Imperfect information in the product market, In. R. Schmalensee and R. Willig, ed, *Handbook of Industrial Organization*, Volume I, Elsevier Science Publishers.

Tirole, J., (1988), The theory of industrial organization, MIT Press.

Vasconcelos, H., (2002), Tacit collusion, cost asymmetries and mergers, mimeo European University Institute.

Appendix A. Proofs of results

Proof of Lemma 1 Choose an arbitrary vector $\alpha \in \mathbb{R}^N_{++}$ and consider the following family of optimization problems

$$(\mathbb{P}_\nu) \quad \max_{a_1,\ldots,a_\nu,A \in \mathbb{C}_\nu} \sum_{i=1}^{\nu} \alpha_i \pi(a_i, A, \nu, n_i)$$

for $\nu = 1, \ldots, N$ with

$$\mathbb{C}_\nu = \left\{ (a, A) \in \mathbb{R}^{\nu+1}_+ \;\middle|\; \begin{array}{c} \pi(a_i, A, \nu, n_i) \geq q\pi(R_i(A), A, \nu, n_i) + (1-q)\pi^{NE}(n_i) \\ i = 1, \ldots, \nu \\ q\pi(R_{\nu+1}(A), A, \nu+1, n_{\nu+1}) + (1-q)\pi^{NE}(n_{\nu+1}) \leq f \\ G(a) = A \end{array} \right\}$$

Since (by assumption) there exists a Nash equilibrium of the static game, we know that the set $\mathbb{A}$ is non empty and hence for at least one value of ν the set $\mathbb{C}_\nu$ is nonempty. Note that by the continuity assumptions on the functions $\pi(.), G(.)$ and $R(.)$ the sets $\mathbb{C}_\nu$ are closed. Further, the only relevant values for A lie in the interval $[0, n_1]$ and we assumed that actions a_i are chosen from a bounded set. Hence, the sets $\mathbb{C}_\nu$ are, in fact, compact.

It follows that a problem $(\mathbb{P}_\nu)$ maximizes a continuous function on a compact set. Hence if $(\mathbb{C}_\nu)$ is non empty, it follows from the Weierstrass theorem that $(\mathbb{P}_\nu)$ has a global solution (maximum). Therefore at least one of the problems $(\mathbb{P}_\nu)$ has a solution, but possibly more.

Let $\mathbb{S}_\nu$ denote the set of solutions to problem $\mathbb{P}_\nu$. That is,

$$a \in \mathbb{S}_\nu \text{ if } (a, A) \in \text{argmax}_{(a',A') \in \mathbb{C}_\nu} \sum_{i=1}^{\nu} \alpha_i \pi(a'_i, A', \nu, n_i)$$

We define $\mathbb{S}$ as the union of the sets $\mathbb{S}_\nu$, *i.e.* $\mathbb{S} = \bigcup_{\nu=1}^{N} \mathbb{S}_\nu$.

We use the following procedure to find a candidate PEP outcome. From all the solutions in $\mathbb{S}$ choose the one(s) with the highest value for $\pi(a_1, A, \nu, n_1)$. This set is denoted by $\mathbb{S}^{\{1\}}$.If there is only one such solution, this is our candidate PEP outcome and we denote it by $\tilde{a}$. If there are more solutions yielding the same maximum pay off for player 1, then choose among these solutions in $\mathbb{S}^{\{1\}}$, the solution that maximizes the pay off of player 2. This subset is denoted $\mathbb{S}^{\{1,2\}}$. If this is a singleton, it is our candidate $\tilde{a}$, if not move on to player 3's pay off to determine the set $\mathbb{S}^{\{1,2,3\}}$ etc. until only one candidate solution is left (if one has moved on to player N and still is there a number of solutions left in $\mathbb{S}^{\{1,\ldots,N\}}$, pick an arbitrary one in this set).

Now we will argue that $\tilde{a}$ and $\tilde{A} = G(\tilde{a})$ form indeed a PEP outcome. Suppose not, that is suppose there exists a' and $A' = G(a')$ such that

$$\pi(a'_i, A', \nu(A'), n_i) \geq \pi(\tilde{a}_i, \tilde{A}, \nu(\tilde{A}), n_i)$$

for all $i = 1, \ldots, \min\{\nu(A'), \nu(\tilde{A})\}$ with a strict inequality for firm j. If $j \leq \nu(\tilde{A})$ then $\tilde{a}$ cannot have been an element of $\mathbb{S}^{\{1,\ldots,j\}}$. This contradicts the way in which $\tilde{a}$ was obtained. If $\nu(\tilde{A}) < j \leq \nu(A')$ then both $\tilde{a}$ and a' were in the set $\mathbb{S}^{\{1,\ldots,j-1\}}$. However, in the next step $\tilde{a}$ should have been deleted. In either case we get a contradiction with the way we found $\tilde{a}$. Hence $\tilde{a}$ and $\tilde{A} = G(\tilde{a})$ form indeed a PEP outcome. *Q.E.D.*

Proof of proposition 1 Take an arbitrary $a \in \mathbb{A}$ with $G(a) = A > A_0$ and take a sustainable $A' < A_0$ which exists by assumption. Then assumption 2 tells us that there exists $a' \in \mathbb{A}$ with $G(a') = A'$ such that $\pi(a'_i, A', \nu(A), n_i) > \pi(a_i, A, \nu(A), n_i)$. In fact, for any $x > 0$ we can find a'_x and a_x with $G(a'_x) = A' < G(a_x) = A$ such that

$$\pi(a'_{xi}, A', \nu(A) + x, n_i) > \pi(a_{xi}, A, \nu(A) + x, n_i) \tag{A.1}$$

We need to show that for x big enough we have

$$\pi(a'_{xi}, A', \nu(A') + x, n_i) > \pi(a_{xi}, A, \nu(A) + x, n_i)$$

This can be written as

$$\pi(a'_{xi}, A', \nu(A) + x, n_i) - \pi(a_{xi}, A, \nu(A) + x, n_i) > \tag{A.2}$$

$$\pi(a'_{xi}, A', \nu(A) + x, n_i) - \pi(a'_{xi}, A', \nu(A') + x, n_i) \tag{A.3}$$

Using equation (A.1) we can write this as

$$1 > \frac{\pi(a'_{xi}, A', \nu(A) + x, n_i) - \pi(a'_{xi}, A', \nu(A') + x, n_i)}{\pi(a'_{xi}, A', \nu(A) + x, n_i) - \pi(a_{xi}, A, \nu(A) + x, n_i)}$$

This inequality holds for x big enough since the right hand side converges to zero by assumption 1 *(v)*. *Q.E.D.*

Proof of proposition 2 Firm ν is the least efficient firm that can still enter if $qD(\bar{p}_{\nu+1})(\bar{p}_{\nu+1} - c_{\nu+1}) = f$. This (implicitly) defines the limit price $\bar{p}_{\nu+1}$ to keep $\nu + 1$ out of the market. Using the implicit function theorem, we find

$$(D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1} - c_{\nu+1}) + D(\bar{p}_{\nu+1}))\frac{d\bar{p}_{\nu+1}}{df} = \frac{1}{q}$$

Hence, for each firm $i \leq \nu$ we find that

$$\frac{d\pi(\bar{p}_{\nu+1}, \nu, c_i)}{df} = \frac{1}{\nu}(D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1} - c_i) + D(\bar{p}_{\nu+1}))\frac{d\bar{p}_{\nu+1}}{df} \tag{A.4}$$

$$= \frac{1}{\nu q}\frac{D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1} - c_i) + D(\bar{p}_{\nu+1})}{D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1} - c_{\nu+1}) + D(\bar{p}_{\nu+1})} \tag{A.5}$$

where $\pi(p, \nu, c_i) = D(p)(p - c_i)/\nu$. Now we compare the expression for $\frac{d\pi_i}{df}$ at $\bar{p}_{\nu+1}$ and at a lower value $\bar{p}_{j+1}$ for $j < \nu$. Then we see the following three effects working in the direction of

$$\frac{d(\pi(\bar{p}_{j+1}, j, c_i) - \pi(\bar{p}_{\nu+1}, \nu, c_i))}{df} > 0$$

or equivalently

$$\frac{1}{jq}\frac{D'(\bar{p}_{j+1})(\bar{p}_{j+1}-c_i)+D(\bar{p}_{j+1})}{D'(\bar{p}_{j+1})(\bar{p}_{j+1}-c_{j+1})+D(\bar{p}_{j+1})}-\frac{1}{\nu q}\frac{D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1}-c_i)+D(\bar{p}_{\nu+1})}{D'(\bar{p}_{\nu+1})(\bar{p}_{\nu+1}-c_{\nu+1})+D(\bar{p}_{\nu+1})}>0$$

First, $j<\nu$ implies that $1/j>1/\nu$. Second, $c_{j+1}<c_{\nu+1}$ implies that the denominator of the first term is smaller than the denominator of the second term. Finally, consider the effect of p for given ν and $c_{\nu+1}$:

$$\frac{\partial\left(\frac{D(p)+D'(p)(p-c_i)}{D(p)+D'(p)(p-c_{\nu+1})}\right)}{\partial p}=\frac{2(D'(p))^2(c_i-c_{\nu+1})+D''(p)D(p)(c_{\nu+1}-c_i)}{(D(p)+D'(p)(p-c_{\nu+1}))^2}$$

which is negative if $D''(p)\leq 0$ since $c_{\nu+1}>c_i$. This also implies that a lower value of p leads to a higher value of $\frac{\pi_i}{df}$ for each $i=1,\ldots,j$. *Q.E.D.*