CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Research Papers in Economics

MPRA
Munich Personal RePEc Archive

The Inequality Process vs. The Saved Wealth Model. Two Particle Systems of Income Distribution; Which Does Better Empirically?

Angle, John
The Inequality Process Institute

10. February 2010

Online at http://mpra.ub.uni-muenchen.de/20835/
MPRA Paper No. 20835, posted 20. February 2010 / 07:40

The Inequality Process vs. The Saved Wealth Model. Two Particle Systems of Income Distribution; Which Does Better Empirically?

John Angle
The Inequality Process Institute, Post Office Box 215, Lafayette Hill, Pennsylvania 19444, USA
[http://www.inequalityprocess.org]

Email: angle@inequalityprocess.org

Version February 10, 2010

This paper is a revision of "A test of two similar particle systems of labor income distribution conditioned on education" presented to the Joint Statistical Meetings in Washington, DC, USA in August 2009 [1].

Abstract: The Inequality Process (IP) is a stochastic particle system in which particles are randomly paired for wealth exchange. A coin toss determines which particle loses wealth to the other in a randomly paired encounter. The loser gives up a fixed share of its wealth, a positive quantity. That share is its parameter, ω_ψ, in the ψ^{th} equivalence class of particles. The IP was derived from verbal social science theory that designates the empirical referent of $(1-\omega_\psi)$ as worker productivity, operationalized as worker education. Consequently, the stationary distribution of wealth of the IP in which particles can have different values of ω (like workers with different educations) is obliged to fit the distribution of labor income conditioned on education. The hypothesis is that when a) the stationary distribution of wealth in the ψ^{th} equivalence class of particles is fitted to the distribution of labor income of workers at the ψ^{th} level of education, and b) the fraction of particles in the ψ^{th} equivalence class equals the fraction of workers at the ψ^{th} level of education, then c) the model's stationary distributions fit the corresponding empirical distributions, and d) estimated $(1-\omega_\psi)$ increases with level of education. The Saved Wealth Model (SW) was proposed as a modification of the particle system model of the Kinetic Theory of Gases (KTG). The SW is isomorphic to the IP up to the stochastic driver of wealth exchange between particles. The present paper shows that 1) the stationary distributions of both particle systems pass test c): they fit the distribution of U.S. annual wage and salary income conditioned on education over four decades, 2) the parameter estimates of the fits differ by particle system, 3) both particle systems pass test d), but 4) the IP's overall fits are better than the SW's because 5) the IP's stationary distribution conditioned on larger $(1-\omega_\psi)$ has a heavier tail than the SW's fitting the distribution of wage income of the more educated better, and 6) since the level of education in the U.S. labor force rose, the IP's fit advantage increased over time.

Keywords: labor income distribution; goodness of fit; Inequality Process; particle system model; Saved Wealth Model

Classification: Physics PACS Subject Code: 89.65.Gh, 89.65.-s
Mathematics AMS Subject Code MSC: 60K35, 82C22
Economics JEL Subject Code: D31, C63, B59

---

1. Introduction

The Inequality Process (IP) [1-14] and the Saved Wealth Model (SW) [15-17], previously compared in [18-22], are two stochastic binary interacting particle systems whose stationary distributions have been fitted to income distributions. The two particle systems are isomorphic to each other up to their stochastic drivers of exchange between particles and a consequence of that difference. The IP, derived from an old verbal theory in economic anthropology, was published in 1983. The IP has been used to discover stable empirical patterns in income and wealth. The IP has been shown to quantitatively explain:

1. The universal pairing (all times, all places, all cultures, all races) of the appearance of social inequality and the concentration of wealth with the appearance of a storeable food surplus among hunter/gatherers. [2,3]
2. Why the gamma family of probability density functions (pdfs) is a useful approximation to wage income distributions conditioned on education and why the unconditional distribution of wage income has a right tail whose heaviness approximates that of a Pareto pdf; [2,3,6,8,9,11,12]
3. How the unconditional distribution of personal income appears to be gamma distributed at the national level and in successively smaller regions although the gamma distribution is not closed under mixture, i.e., aggregation by area; [6]
4. The shapes of the distribution of wage incomes of workers by level of education, why this sequence of shapes changes little over decades, and why it is similar to the sequence of shapes of the unconditional distribution of wage income over the course of techno-cultural evolution; [2,3,8,9,11,12]
5. The dynamics of the distribution of wage income conditioned on education as a function of the unconditional mean of wage income and the distribution of education in the labor force; [9,12]
6. Why the distribution of wage income is different from the distribution of income from tangible assets; [7]
7. Why the IP's parameters estimated from certain statistics of the wage incomes of individual workers in longitudinal data on annual wage incomes are ordered as predicted by the IP's meta-theory and approximate estimates of the same parameters from the fit of the IP's stationary distribution to the distribution of wage income conditioned on education; [8]
8. The Kuznets pattern in the Gini concentration ratio of earned income during the industrialization of an agrarian economy; [14]
9. In an elaboration of the basic IP: if a particle in a coalition of particles has a probability different from 50% of winning a competitive encounter with a particle not in the coalition, this modified IP can reproduce features of the joint distribution of income to African-Americans and other Americans: a) the % minority effect on discrimination (the larger the minority, the more severe discrimination on a per capita basis); b) the relationships among: i) % of a U.S. state's population that is non-white; ii) median non-African-American male earnings in a U.S. state; iii) the Gini concentration of non-African-American male earnings in a U.S. state; and iv) the ratio of African-American male to non-African-American male median earnings in a U.S. state. [5]

The SW, published in 2000, is a generalization of the Kinetic Theory of Gases (KTG) stochastic particle system model [23]. It has not been as extensively tested against data, prompting the question of which particle system fits data better. The IP has a social science meta-theory that assigns empirical referents to its parameters and sets consistency tests. The SW does not, prompting the question of whether the SW passes the same tests that the IP must pass and has passed. Verbal description of the difference between the IP and the SW may seem deceptively insignificant. This paper addresses the question whether the IP and SW are, if not mathematically, equivalent for all practical purposes in empirical work. Data collected by the U.S. Bureau of the Census over four decades are used in this comparison.

1.1 The SW, an Elaboration of the Kinetic Theory of Gases (KTG)

The stochastic particle system model of the Kinetic Theory of Gases (KTG) randomly pairs particles for random exchanges of a positive quantity modeling the exchange of kinetic energy between the molecules of a dilute gas in collision. The equations of the exchange are [23]:

$$\begin{aligned} x_{it} &= \varepsilon_t \left(x_{i(t-1)} + x_{j(t-1)} \right) \\ x_{jt} &= (1-\varepsilon_t)\left(x_{i(t-1)} + x_{j(t-1)} \right) \end{aligned} \qquad (1a,b)$$

where,

$$\begin{aligned} x_{i(t-1)} &= \text{particle } i\text{'s kinetic energy at time} - \text{step } (t-1) \\ x_{jt} &= \text{particle } j\text{'s kinetic energy at time} - \text{step } t \\ \varepsilon_t &= \text{an i.i.d.} [0,1] \text{ uniform continuous r.v. at time} - \text{step } t \end{aligned}$$

In the KTG the sum of kinetic energy of particles i and j after a collision equals the sum before. Given that the population of particles is isolated in a reflecting container, the sum of kinetic energy over all particles does not change. The stationary distribution of particle kinetic energy in the KTG is a negative exponential distribution. Dragulescu and Yakovenko [24] re-label the KTG. Re-labeled, its particles represent people instead of gas molecules, the positive quantity exchanged by particles becomes wealth rather than kinetic energy. Dragulescu and Yakovenko [24] have to argue that the stationary distribution of kinetic energy in the KTG, the negative exponential, is also that of the distribution of income. They perceive a fit between the negative exponential distribution and the distribution of adjusted gross income reported by the U.S. Internal Revenue Service. It is not a close fit and in later work they propose a model that sutures a heavier than exponential right tail (Dragulescu and Yakovenko, [25]) to the negative exponential left tail. The present paper shows that a negative exponential distribution is not a good model of the distribution of labor income of workers with post-secondary educations.

1.2 The Saved Wealth Model (SW)

Chakraborti and Chakrabarti [15] re-label the KTG as Dragulescu and Yakovenko [24] do, but Chakraborti and Chakrabarti also modify the mathematics of the KTG. They introduce a parameter λ, the proportion of a particle’s wealth not at risk of loss in any one transaction with another particle. When λ = 0, the Chakraborti and Chakrabarti model is equivalent to the KTG. Chakraborti and Chakrabarti call λ “savings”. The justification given for thinking the model relevant to income distribution is an image of market transactions between agents in which λ represents an agent’s “savings”. The label is not apt. λ is not the share of a gain that is saved, but rather the fraction of a particle’s wealth not at risk of loss in any one encounter with another particle. λ is like a tax exemption rather than a partially saved profit. In fact, asset income is distributed differently from labor income [7]. The development of the SW is chronicled in Yakovenko [22]. The equations of the exchange of wealth between two particles in the Chakraborti and Chakrabarti model, the Saved Wealth Model (SW), are:

$$
\begin{aligned}
x_{it} &= \lambda x_{i(t-1)} + \varepsilon_t (1-\lambda)(x_{j(t-1)} + x_{i(t-1)}) \\
x_{jt} &= \lambda x_{j(t-1)} + (1-\varepsilon_t)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})
\end{aligned}
\qquad (2a,b)
$$

where:

$$
\begin{aligned}
x_{i(t-1)} &= \textit{particle } i\textit{'s wealth at time step } (t-1) \\
x_{jt} &= \textit{particle } j\textit{'s wealth at time step } t \\
\varepsilon_t &= \textit{an i.i.d. } [0,1] \textit{ uniform continuous r.v. at time-step } t \\
0 \;\leq\; &\lambda \;< 1 \\
&\textit{All particles have equal } \lambda.
\end{aligned}
$$

Other features of the KTG particle system remain in place, such as random binary matching of particles for exchanges, the sum of the wealth of two paired particles before an encounter equaling the sum after, and the isolation of the population of particles and their immortality. Patriarca, Chakraborti, and Kaski [17] report that the stationary distribution of the SW is a gamma probability density function (pdf):

$$
f(x) \;\equiv\; \frac{1}{\beta^{\alpha}\,\Gamma(\alpha)}\, x^{\alpha-1}\, e^{-x/\beta} \qquad (3)
$$

where, x > 0; x is interpreted as income; α is the shape parameter and β is the scale parameter. (3) is denoted GAM(α,β). Patriarca et al. [17] find the shape parameter, α, of the gamma pdf of the stationary distribution of (2a,b) to be:

$$
\alpha \;=\; \frac{1+2\lambda}{1-\lambda} \qquad (4)
$$

In their model all particles have an equal value of λ. Since mean particle wealth is pre-determined in the model, and the expression for the mean of GAM(α,β) is α/β, (3)'s parameters can be expressed in terms of λ and mean particle wealth.

1.3 The Inequality Process (IP)

The Inequality Process was abstracted from the Surplus Theory of Social Stratification, economic anthropology’s explanation of an invariant: the pairing of the earliest evidence of extreme economic inequality in the same archeological strata as the earliest evidence of abundant storeable food [2,3]. This verbal theory asserts that:

a) people compete for surplus, storeable food, a form of wealth,

b) competition distributes wealth and concentrates it,

and,

c) when wealth in the form of storeable food appears among people, usually via the acquisition of agriculture, its concentration overwhelms the apparent egalitarianism of subsisting without much stored food.

The society that emerges out of a hunter/gatherer population when it acquires a storeable food surplus is called a "chiefdom" by anthropologists, who view it as the most inegalitarian societal form.

The Surplus Theory has a prominent flaw: no answer to the question of why inequality of wealth decreases over the course of techno-cultural evolution beyond the chiefdom when more wealth is produced per capita than in the chiefdom. Gerhard Lenski [26] addressed this flaw with a speculative explanation for why inequality of wealth, defined as concentration of wealth, decreased over the course of techno-cultural evolution. The speculation is that the production of more wealth per capita requires workers who are more skilled and that a more skilled worker retains a larger fraction of the wealth that worker produces. The Inequality Process (IP) operationalizes and tests this hypothesis. Wherever verbal theory offers no help in specifying a mathematical model, the principle of parsimony was used in the specification of the IP. [4] notes in 1990 that the IP is a particle system similar to the KTG.

1.3.1 The One Parameter Inequality Process (OPIP)

A two parameter version of the IP appeared in Angle [2,3]. This model was later simplified where one parameter is adequate to explain income and wealth phenomena. The one parameter Inequality Process (OPIP) is isomorphic to the SW as defined in (2a,b) up to the stochastic driver of wealth exchange and a consequent difference in the intervals on which the parameters of the two particle systems are defined.

The IP's meta-theory makes (1 – ω), where ω is the fraction of wealth lost by a particle in an encounter with another particle, a measure of worker skill, a semi-permanent trait. A particle's ω is only apparent when it loses an encounter. A coin toss determines which of two particles randomly paired for competition loses. The share of wealth a loser transfers to a winner, its ω, is pre-determined and to some degree permanent (like a worker's skills), its parameter. Winning and losing are asymmetric. The equations for the exchange of wealth between two particles in the OPIP are:

$$\begin{aligned} x_{it} &= x_{i(t-1)} + d_t \omega x_{j(t-1)} - (1-d_t)\omega x_{i(t-1)} \\ x_{jt} &= x_{j(t-1)} - d_t \omega x_{j(t-1)} + (1-d_t)\omega x_{i(t-1)} \end{aligned} \qquad (5a,b)$$

where:

$$d_t = \begin{pmatrix} 1\ with\ probability\ .5 \\ 0\ otherwise \end{pmatrix}$$

$$\omega = (1-\lambda)\ where\ \lambda\ is\ the\ SW\ parameter$$
$$except\ that\ 0 < \omega < 1$$
$$while\ 0 \leq \lambda < 1$$

All particles are randomly paired at each time-step. There are no particle deaths, births, or migration in or out of the population. The number of particles is an even number. (5a,b) can be re-expressed to resemble (2a,b):

$$\begin{aligned} x_{it} &= (1-\omega)x_{i(t-1)} + d_t\omega(x_{j(t-1)} + x_{i(t-1)}) \\ x_{jt} &= (1-\omega)x_{j(t-1)} + (1-d_t)\omega(x_{j(t-1)} + x_{i(t-1)}) \end{aligned} \qquad \text{(5c,d)}$$

The OPIP maps into the SW, (2a,b), if (1-λ) is substituted for ω and a continuous [0,1] uniform random variate, ε_t, is substituted for the discrete (0,1) uniform random variable, d_t. The latter substitution may seem subtle and unimportant, but, as this paper shows, this substitution gives the OPIP, defined by (5a,b), properties substantially different from those of (2a,b) although the SW is isomorphic to the IP up to that substitution. The substitution requires a different interval on which the model's parameter is defined. The SW's particle parameter, λ, can be mapped into the complement of the parameter of the IP, (1-ω), one-to-one, except for λ = 0 (the KTG). The IP is not ergodic at ω = 1.0 .

1.3.1.1 Implications of the Different Stochastic Drivers of Wealth Exchange in the OPIP and SW

The difference between the OPIP of (5c,d) and the one parameter SW of (2a,b) is clear in (5e,f), the OPIP expressed in terms of SW notation:

$$\begin{aligned} x_{it} = \ & \lambda x_{i(t-1)} + \varepsilon_t(1-\lambda)(x_{j(t-1)} + x_{i(t-1)}) \\ & + (1-\varepsilon_t)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})(\varepsilon_t > .5) \\ & - \varepsilon_t(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})(1-((\varepsilon_t > .5)) \\ x_{jt} = \ & \lambda x_{j(t-1)} + (1-\varepsilon_t)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)}) \\ & - (1-\varepsilon_t)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})(\varepsilon_t > .5) \\ & + \varepsilon_t(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})(1-((\varepsilon_t > .5)) \end{aligned}$$

(5e,f)

where (ε_t > .5) equals 1.0 if the condition is true, 0.0 otherwise. (5e,f) shows that the one parameter Inequality Process (OPIP), expressed in SW notation, is the SW particle system (2a,b) with gains and losses maximally exaggerated. The OPIP is the SW with ε_t rounded up to 1.0 or down to 0.0, which can be expressed by the logical term (ε_t > .5). The other difference between the OPIP and the SW, the difference at the endpoint of the intervals on which the particle parameters are defined, follows from the difference in the stochastic driver of wealth: the OPIP is not ergodic at ω = (1-λ) = 1.0 whereas the SW is ergodic where λ = 0.0 .

The OPIP of (5e,f) reduces to:

$$x_{it} = \lambda x_{i(t-1)} + (\varepsilon_t > .5)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})$$
$$x_{jt} = \lambda x_{j(t-1)} + (1-((\varepsilon_t > .5)(1-\lambda)(x_{j(t-1)} + x_{i(t-1)})$$

(5g,h)

In SW notation and (5e,f) and (5g,h), if (ε_t > .5) is not true (equals 0.0), particle i of the OPIP always loses a (1-λ) fraction of its wealth. However, in the SW (2a,b), ε_t > .5 merely means a greater than even probability of gain for particle i ceteris paribus. Particle i actually gains wealth in the SW (2a,b) when ε_t > .5 if:

$$\begin{aligned} \varepsilon_t (1-\lambda)(x_{j(t-1)} + x_{i(t-1)}) &> (1-\lambda) x_{i(t-1)} \\ \varepsilon_t (x_{j(t-1)} + x_{i(t-1)}) &> x_{i(t-1)} \\ \varepsilon_t x_{j(t-1)} &> x_{i(t-1)} - \varepsilon_t x_{i(t-1)} \\ \varepsilon_t x_{j(t-1)} &> (1-\varepsilon_t) x_{i(t-1)} \end{aligned}$$

(6)

i.e., the question of whether there is a gain or loss experienced by particle i, a general particle, in the SW (2a,b) depends on three variables: ε_t, , $x_{j(t-1)}$, and , $x_{i(t-1)}$, its own wealth. The magnitude of particle i's gain in the SW (2a,b), if particle i has a gain, depends on three variables and the parameter:

$$\begin{aligned} \lambda x_{i(t-1)} + \varepsilon_t (1-\lambda)(x_{i(t-1)} + x_{j(t-1)}) - x_{i(t-1)} &> 0 \\ -(1-\lambda)(1-\varepsilon_t) x_{i(t-1)} + \varepsilon_t (1-\lambda) x_{j(t-1)} &> 0 \\ (1-\lambda)\left[\varepsilon_t x_{j(t-1)} - (1-\varepsilon_t) x_{i(t-1)}\right] &> 0 \end{aligned} \qquad (7)$$

In the SW (2a,b), particle i loses wealth if the inequalities of (6) are reversed. The magnitude of particle i's loss in the SW (2a,b), if particle i has a loss, is (7) with the inequalities reversed, i.e., also dependent on three variables and the parameter.

In the OPIP, (5e,f) and (5g,h) in SW notation, particle i experiences a gain if ε_t > .5, a loss otherwise, i.e., gain or loss depends on a single variable, ε_t. If particle i gains wealth that amount is, in SW notation, (1-λ) $x_{j(t-1)}$; the gain depends on the parameter and one variable. From particle i's point of view the wealth of its competitors, i.e., $x_{j(t-1)}$, is a random variable. If particle i loses wealth that amount is (1-λ) $x_{i(t-1)}$, i.e., dependent on the parameter and one variable. From particle i's point of view, however, the loss is just dependent on the parameter since, if it is like a person, it knows its own wealth. Fewer variables in the OPIP determine whether a particle wins or loses and the amount won or lost than in the SW. From the losing particle's point of view, the magnitude of a loss, in the OPIP is entirely determined by the parameter. Not only is gain and loss dependent on fewer variables in the OPIP, there is an asymmetry between gain and loss from the point of view of the general particle, say particle i. Gains are random from the point of view of particle i whereas losses from the point of view of particle i are determined by the parameter and its wealth going into the encounter with another particle: (1-λ) $x_{i(t-1)}$.

1.3.1.2 Information on the Parameter in the OPIP

A vector containing a particle's wealth at each time step in both the SW and the OPIP, (2a,b) and (5a,b), contains information about the parameter of the process. This information is clearer in the OPIP than the SW because of the exaggeration of gain and loss in the OPIP, (5a,b). In the OPIP a particle's parameter is so clear that it can be calculated without error from the first instance of a decrease in any particle's wealth. The clarity of the OPIP is such that the direction of time itself, were it unknown, can be inferred from a vector of a single particle's wealth amounts: time flows in the direction of the first two equal proportional decreases from an adjacent wealth amount in the sequence. It takes two such decreases, given one of the two hypotheses about which way the vector of particle wealth is oriented in time, because the first such decrease might be an increase if the hypothesis is wrong. Such an inference from the SW (2a,b) is not similarly deterministic. It requires many vectors of particle wealth histories with the number of such vectors needed for an estimate of a given precision dependent on λ.

1.3.1.3 An Approximation to the Stationary Distribution of the OPIP

Angle [8, 11] uses the run-like (generalized runs) character of the solution of the OPIP to specify, via the relationship of the gamma pdf to the negative binomial pf, a shape parameter of a gamma pdf , α, (3), approximating the OPIP's stationary distribution. It is:

$$\alpha \approx \frac{1-\omega}{\omega} \qquad (8a)$$

(4) is the expression for the shape parameter of the gamma pdf approximating the stationary distribution of the SW [17]. [17] gives (4) in terms of OPIP notation. Thus translated [17]'s expression for shape parameter of the gamma pdf approximating the stationary distribution of the SW is on the RHS of (8b). The OPIP analogue, (8a), is on the LHS of (8b):

$$\frac{1-\omega}{\omega} \neq \frac{3-2\omega}{\omega} \qquad (8b)$$

The LHS and RHS of (8b) are not equal. (8b) suggests that equivalent parameters in the SW and OPIP yield different stationary distributions.

Angle [8,11] makes no claim that the OPIP has an exactly gamma stationary distribution. He gives a proof that no conservative particle system scattering a positive quantity via binary particle interactions, a class that includes the OPIP and the SW, has an exactly gamma stationary distribution. Patriarca, Chakraborti, and Kaski's [17] finding that the SW's stationary distribution is a gamma pdf is a numerical finding, unable to distinguish among an exactly gamma stationary distribution, an asymptotically gamma stationary distribution, or a gammoidal distribution.

1.3.1.4 A Mis-statement of the OPIP

The OPIP (or any other published version of the IP) is mis-stated in Patriarca, Heinsalu, and Chakraborti [20] as:

$$x_{it} = x_{i(t-1)} + d_t \varepsilon_t \, \omega x_{j(t-1)} - (1-d_t)\, \varepsilon_t \omega x_{i(t-1)}$$
$$x_{jt} = x_{j(t-1)} - d_t \varepsilon_t \, \omega x_{j(t-1)} + (1-d_t)\, \varepsilon_t \omega x_{i(t-1)} \qquad (9a,b)$$

where ε_t is an i.i.d. [0,1] continuous uniform random variable and d_t is 1 with probability p if $x_i > x_j$, , 0 otherwise, and $0 < \omega < 1$.

1.4 Models of the Distribution of Earned Income Conditioned on Education: The Inequality Process With Distributed Omega (IPDO) and Its SW Analogue, An SWDO

The Inequality Process (IP) must explain the distribution of wage income conditioned on education since it is derived from verbal theory that asserts that more skilled workers lose less in the competition for wealth, identifying (1-ω) as a measure of worker skill. Education is a measure of worker skill available in large surveys and is often used as a proxy for worker skill level in economics. Such a fit requires identifying the stationary distribution of the wealth of particles in the ω_ψ equivalence class with the distribution of wage income at the ψ^{th} level of education. The fraction that the ω_ψ equivalence class of particles forms of the whole population of particles is set equal to the fraction that workers at the ψ^{th} level of education are in the labor force. The Inequality Process with distributed omega (IPDO) [8,11] has the following equations for the exchange of wealth between particles i and j:

$$x_{i\psi t} = x_{i\psi(t-1)} + d_t \omega_\theta \, x_{j\theta(t-1)} - (1-d_t)\omega_\psi x_{i\psi(t-1)}$$
$$x_{j\theta t} = x_{j\theta(t-1)} - d_t \omega_\theta \, x_{j\theta(t-1)} + (1-d_t)\omega_\psi \, x_{i\psi(t-1)} \qquad (10a,b)$$

The IPDO is isomorphic to (5a,b) except that particle i is in the ψ^{th} ω equivalence class (all particles whose parameter is ω_ψ), while particle j is in the θ^{th} ω equivalence class. Particles i and j are distinct although they may be drawn from the same equivalence class, i.e. it is possible that $\omega_\psi = \omega_\theta$. The stationary distribution of wealth in each IPDO ω_ψ equivalence class is not in general equal to that of the OPIP with equal ω_ψ unless the ω_ψ equivalence class includes the entire particle population, in which case the IPDO is identical to the OPIP.

1.4.1 The Saved Wealth Model Analogue of the IPDO, the SWDO

No publication has appeared in the literature with the Saved Wealth Model (SW) analogue of the IPDO, because there is no SW meta-theory that would lead one to assert that λ or in IP notation (1-ω), is a semi-permanent particle trait and hence a particle parameter. To compare the IPDO to its SW analogue, a Saved Wealth Model with distributed omega (SWDO) has to be specified. Since the difference between the IP and SW is well defined, it is easy to specify the SW analogue of the IPDO. The equations for the exchange of wealth between particles of the SW analogue of the IPDO, the Saved Wealth Model with Distributed Omega (SWDO), expressed in the IPDO's notation, are:

$$x_{i\psi t} = x_{i\psi(t-1)} + \varepsilon_t \omega_\theta \, x_{j\theta(t-1)} - (1-\varepsilon_t)\omega_\psi \, x_{i\psi(t-1)}$$
$$x_{j\theta t} = x_{j\theta(t-1)} - \varepsilon_t \omega_\theta \, x_{j\theta(t-1)} + (1-\varepsilon_t)\omega_\psi \, x_{i\psi(t-1)} \qquad (11a,b)$$

where ε_t is an i.i.d. [0,1] continuous uniform random variate and $0 < \omega \le 1$.

The SWDO should not be confused with the model in the Saved Wealth (SW) literature by Chatterjee, Chakrabarti, and Manna [16] whose equations for the exchange of wealth between two particles are:

$$\begin{aligned} x_{i\psi t} &= x_{i\psi(t-1)} + \varepsilon_t \omega_{\theta t}\, x_{j\theta(t-1)} - (1-\varepsilon_t)\omega_{\psi t}\, x_{i\psi(t-1)} \\ x_{j\theta t} &= x_{j\theta(t-1)} - \varepsilon_t \omega_{\theta t}\, x_{j\theta(t-1)} + (1-\varepsilon_t)\omega_{\psi t}\, x_{i\psi(t-1)} \end{aligned} \qquad (12a,b)$$

where ε_t is an i.i.d. [0,1] continuous uniform random variate, and $\omega_{\psi t}$ is an i.i.d. (0,1] continuous uniform random variate, as is $\omega_{\theta t}$. This SW model violates the Inequality Process' (IP's) meta-theory that asserts that a particle's ω is semi-permanent in the same way that a person's education or a worker's skill level is semi-permanent, i.e., in (10a,b) the particle's parameter, ω_ψ, is not a random variate at each time step. As with all SW models (12a,b) is the result of numerical tinkering. It is intended to yield a stationary distribution that has a gammoidal left tail and a heavier than exponential right tail. Angle [6,9] shows that the unconditional distribution of wealth in the IPDO, with ω_ψ's estimated from the distribution of earned income conditioned on education and ω_ψ equivalence classes forming the same fraction of the population of IPDO particles as groups of workers with the corresponding level of education, has a heavier than exponential right tail, one heavy enough to account for aggregate labor income in the U.S.' National Income and Product Accounts.

2. Does the Saved Wealth Model with Distributed Omega (SWDO) Pass The Tests Set by The IP's Meta-Theory?

The social science meta-theory of the Inequality Process (IP) requires the stationary distribution of particle wealth in the Inequality Process with Distributed Omega (IPDO) to have certain properties. This meta-theory designates the empirical referent of ($1-\omega_\psi$) as worker productivity, operationalized as worker education. Consequently, the stationary distribution of wealth of the IPDO, the IP in which particles can have different values of ω (like workers with different educations) is obliged to fit the distribution of labor income conditioned on education. The hypothesis is that when a) the stationary distribution of wealth in the ψ^{th} equivalence class of particles is fitted to the distribution of labor income of workers at the ψ^{th} level of education, and b) the fraction of particles in the ψ^{th} equivalence class equals the fraction of workers at the ψ^{th} level of education, then c) the model's stationary distributions fit the corresponding empirical distributions, d) estimated ($1-\omega_\psi$) increases with level of education, and, subsidarily, e) mean IPDO wealth, μ_ψ, of particles in each ω_ψ equivalence class scales from low to high with ($1-\omega_\psi$).

It is unlikely that the permutation of ω_ψ's predicted by the IP's meta-theory would occur if all permutations were equally likely, since the predicted permutation is one out of 6! = 720 in the fit of the IPDO to the distribution of annual wage and salary income conditioned on education in a single year. With 43 years of data and 36 independent fits per year, the permutation of ω_ψ's in all 43 years predicted by the IP's meta-theory is one out of 43 x 36 x 6! = 1,114,560 permutations.

2.1 Fitting the IPDO to the Distribution of U.S. Annual Wage and Salary Income Conditioned on Education in 1986

The present paper fits the IPDO's stationary distribution of wealth conditioned on ω_ψ to the distribution of annual wage and salary income of workers at each level of education. To illustrate the procedure, fits to a single year's data are discussed before fits to the whole pooled cross-sectional time-series of survey data on annual wage and salary income in 43 years. 1986 data on annual wage and income by level of education are from the 1987 March Current Population Survey (CPS) of the U.S. civilian population conducted by the U.S. Bureau of the Census [27]. After data from1986 are examined, the whole data set is examined. The CPS asks for wage and salary income in the previous calendar year, for age and education at the time of the survey.

2.1.1 Ordering Categories of Education

Testing the IPDO requires an ordered set of education categories. The order of a set of education categories is clear if education level is coarsely categorized. The U.S. Bureau of the Census changed its education categories in 1990. A single set of education categories for the period 1962-2004 has to be sufficiently coarse to be insensitive to the change of Census Bureau detailed categories in 1990. Another consideration is the amount of information in a set of ordered categories. The amount of information is at a maximum if, subject to the coarseness requirement to assure order and insensitivity to the change in Census Bureau categories in 1990, the number of categories is as large as possible and the distribution of observations falling into the categories uniform. It is conventional in U.S. labor economics to restrict the age range of workers to 25 to 65 when examining the relationship of education to earned income. A minimum age of 25 allows workers to complete advanced educations. In an ordered set of categories, the high and low categories are "open end" and vulnerable to "definition creep" in the period 1962-2004 during which the level of education of the U.S. labor force rose. Table 1 has a categorization of U.S. workers 1962-2004 by level of education that takes these constraints into consideration.

Table 1. Ordered Set of Education Categories

eight years or fewer years of primary education (including illiteracy); open end category
some high school education
high school graduate (completion of four years of secondary education
some college (some post-secondary education)
college graduate (completion of four years of post-secondary education)
at least some post-graduate education (including academic and professional degree programs); open end category

2.1.2 Estimating the Vector of IPDO ω_ψ's that Minimizes the Sum of Weighted Squared Errors

The IPDO's stationary distribution is fitted to the distribution of wage income in 1986 conditioned on education (the six categories of table 1) by a) the current optimum vector of six ω_ψ's, b) simulating the IPDO each time the current optimal parameter vector is perturbed until its distribution of wealth converges to its stationary distribution, c) fitting the IPDO's stationary distribution of wealth conditioned on ω_ψ to the distribution of wage income of workers at the ψ^{th} level of education, d) calculating the closeness of the fit, and then e) updating the current optimum parameter vector, if the sum of squared errors weighted by the fraction of workers at each level of education is less the current optimum parameter vector.

There are constraints on this search. A perturbed value of ω_ψ smaller than .001 is replaced by .001. Similarly, a perturbed value of ω_ψ greater than .999 is replaced by .999. The start vector of ω_ψ's is six .5's, the midpoint of the interval on which the ω_ψ's are defined. The number of particles in each ω_ψ equivalence class is round($w_{\psi t}$ x 1,000) where $w_{\psi t}$ is the relative frequency of workers at the ψ^{th} level of education in year t. Each IPDO simulation is run for 300 iterations before sampling. Then at the 301st the wealth of each particle is recorded. Each particle's membership in the ω_ψ equivalence class is known. The simulation runs for another twenty-five iterations. At the 326th simulation, the wealth of each particle is recorded again, and so on to the 401st iteration, at which point there are 5 observations on the wealth of each of 1,000 particles for 5,000 observations altogether. Particle wealth in each IPDO ω_ψ equivalence class is adjusted via (13) so that mean wealth in each ω_ψ equivalence class equals mean annual wage and salary income of workers with a given level of education. Particle wealth is then aggregated into ten relative frequency bins, i.e., $1 to $10,000, $10,001 to $20,000, etc. up to $90,001 to $100,000, the same bins empirical incomes are aggregated into. The relative frequencies of the IPDO's stationary distribution in the ω_ψ equivalence class are fitted to the empirical relative frequencies of workers at the ψ^{th} level of education. The differences between the fitted relative frequencies (those of the stationary distribution in the ω_ψ equivalence class) and the empirical relative frequencies are squared. These squared errors are weighted by the fraction of the sample at each level of education and summed over the six levels of education. Thus ends the first iteration of the fitting algorithm to one year's data.

The fits are done year by year. The estimates of the ω_ψ in each year are denoted $\hat{\omega}_{\psi t}$. If the perturbed vector fits better than the optimum parameter vector, the mean of the two is taken, and it becomes the current optimum parameter vector. The current optimum parameter vector is successively and independently perturbed seven more times, each an independent simulation of the IPDO, whose stationary distribution is then fitted to the data. The current optimum parameter vector may be replaced at any time. In each of the seven iterations the vector of perturbation factors is multiplied by .5 raised to successively higher powers with each iteration, damping the perturbations to the current optimal estimates of the parameters. The damping factor is re-set to 1.0 after the eight iterations have been performed. There are 100 iterations of the 8 successively damped perturbations of the parameter vector regardless of how closely the fitted relative frequencies approximate the empirical relative frequencies. Thus ends the first fitting of the IPDO's stationary distribution in each ω_ψ equivalence class to each partial distribution of earned income at the ψ^{th} level of education in a particular year. This fitting of the IPDO is independently performed 36 times for each year's data. The fitting procedure for the SWDO is identical.

Figure 1 here.

2.1.3 IPDO Fits in 1986

The fitting procedure described in section 2.1.2 fits the IPDO's stationary distribution conditioned on the particle parameter, ω_ψ, to the distribution of U.S. annual wage and salary income conditioned on education. The fits achieved in a particular year, 1986, are shown in figure 1.
The IPDO relative frequencies fitted in figure 1 are the mean of the 36 sets of estimated relative frequencies, each set independently fitted to the 1986 data. The 36 $\hat{\omega}_{\psi t}$ vectors estimated in the fit to 1986 data are all ordered as predicted by the IP's meta-theory. All 36 1986 IPDO estimated parameter vectors are ordered as predicted. Out of the 43 x 36 = 1,548 fits of the IPDO to the distribution of annual wage and salary income conditioned on education, 1961-2003, with 36 independent fits per year, in only 31 instances (2%) did the $\hat{\omega}_{\psi t}$ vector fail to be ordered exactly as the IP's meta-theory predicts. There are 36 weighted sums of squared errors produced by the 36 fits of the IPDO to the 1986 data. The weights are the fraction of the sample in each education category. The mean of the 36 sums of weighted squared errors for 1986 is 0.0040189. Their standard deviation is 0.0001495 . The IPDO $\hat{\omega}_{\psi t}$'s and the $\hat{\mu}_{\psi t}$'s shown in table 2 are the mean of the estimates of the 36 IPDO fits.

Table 2. Estimated Particle Parameters $\hat{\omega}_{\psi t}$ and the Estimated Mean of Wealth, $\hat{\mu}_{\psi t}$, in Each IPDO ω_ψ Equivalence Class Based on 31 Fits of the IPDO to the Distribution of Wage Income Conditioned on Education in U.S. in 1986

education	estimated $\hat{\omega}_{\psi t}$	mean standard error of estimate of $\hat{\omega}_{\psi t}$ in 36 replications of fit to 1986 data	estimated $\hat{\mu}_{\psi t}$ where μ = 1.0 (mean of 36 independent replications of fit to 1986 data)	mean standard error of estimate of $\hat{\mu}_{\psi t}$ in 36 replications of fit to 1986 data
eight years or less	.4733	.0200	0.6571	.0280
some high school	.4261	.0173	0.7826	.0273
high school graduate	.3674	.0096	0.8602	.0130
some college	.3162	.0104	1.0046	.0234
college graduate	.2528	.0090	1.2568	.0353
some post-graduate education or more	.1940	.0078	1.6152	.0402

Nothing in the fitting and estimation procedure forces the $\hat{\omega}_{\psi t}$'s to scale inversely with level of education or the $\hat{\mu}_{\psi t}$'s to scale with education, the orders predicted by the IP's meta-theory.
For numerical reasons, all IPDO simulations have an unconditional mean of particle wealth of 1.0. The $\hat{\mu}_{\psi t}$'s in table 2 are estimated as the mean of the 36 IPDO estimates. Their standard errors of estimate are in table 2, column 4. The small standard errors of the $\hat{\omega}_{\psi t}$'s and the $\hat{\mu}_{\psi t}$'s show the unlikelihood of a different ordering of each. The estimate of mean 1986 annual wage and salary income at each level of education, its $\hat{\hat{x}}_{\psi t}$ (in constant 2003 dollars), is estimated as:

$$\hat{\hat{x}}_{\psi t} = \left(\frac{\hat{x}_{(50)\psi t}}{x_{(50)\psi}} \right) \hat{\mu}_{\psi t} \qquad (13)$$

where,

$\hat{x}_{(50)\psi t}$ = median annual wage and salary income of workers at ψ^{th} level of education, in constant 2003 dollars, estimated from data;

$x_{(50)\psi}$ = median wealth of particles in the IPDO's $\omega_{\psi t}$ equivalence class;

$\hat{\mu}_{\psi t}$ = mean wealth of particles in the IPDO's $\omega_{\psi t}$ equivalence class.

With the $\hat{\bar{x}}_{\psi t}$'s in hand, the unconditional mean of annual wage and salary income in 1986 of people aged 25 to 65, $\hat{\bar{x}}_t$, is estimated as:

$$\hat{\bar{x}}_t = \sum_{\psi} w_{\psi t} \hat{\bar{x}}_{\psi t}$$

where,

$w_{\psi t}$ = fraction of sample at the ψ^{th} level of education .

The 1986 IPDO estimate of the unconditional mean of annual wage and salary income, $\hat{\bar{x}}_{1986}$, is calculated as the mean of its estimates in the 36 independent fits of the IPDO to the empirical distribution: \$31,043 in 2003 constant dollars. The standard error of this estimate of $\hat{\bar{x}}_{1986}$ is the mean of the 36 estimates, \$234.

2.3 SWDO Fits in 1986

The computer program fitting the SWDO to the distribution of annual wage and salary income conditioned on education in the U.S. in 1986 differs from that of the IPDO in only one line of code, that generating the 0,1 discrete uniform random variate that drives the exchange of wealth between each paired particle in (10a,b). Instead, the SWDO program generates a [0,1] continuous uniform random variate, as in (11a,b), the equations of wealth exchange in the SWDO. Figure 2 displays the fit of the mean of the 36 SWDO expected relative frequencies to the empirical 1986 relative frequencies. SWDO fits are similar to those of the IPDO. Both the SWDO and the IPDO fit the annual wage income distribution of the least well educated groups well.

Figure 2 here.

The SWDO relative frequencies fit in figure 2 are the mean of the 36 sets of relative frequencies estimated in the 36 fits of the SWDO's stationary distribution to 1986 data. 34 out of the 36 SWDO $\hat{\omega}_{\psi t}$ vectors estimated in this fit to 1986 data are ordered as predicted by the IP's meta-theory. Out of the 43 x 36 = 1,548 fits of the SWDO to the 43 years of data on the distribution of annual wage and salary income conditioned on education with 36 independent fits per year, there are 87 instances of the ω_{ψ} vector failing to be ordered exactly as the IP's meta-theory predicts (about 5.6%). The IPDO had 31 such failures out of its 1,548 fits (2%).

Each SWDO fit has a weighted sum of squared errors. The mean of the 36 SWDO sums is 0.0057261, 42% larger than that of the IPDO. The standard deviation of the SWDO sums of squared errors is much greater than that of the IPDO. The SWDO's standard deviation, .00159842, is over ten times that of the IPDO. The SWDO fits the six empirical partial distributions but not as well as the IPDO nor as reliably.

Table 3. Estimated Particle Parameter Vector, $\hat{\omega}_{\psi t}$, and the Estimated Mean of Wealth, $\hat{\mu}_{\psi t}$, in Each SWDO ω_{ψ} Equivalence Class Based on 36 Fits of SWDO to Distribution of Wage Income Conditioned on Education in U.S. in 1986

education	estimated $\hat{\omega}_{\psi t}$	standard error of $\hat{\omega}_{\psi t}$ (estimated from 36 replications of fit to 1986 data)	$\hat{\mu}_{\psi t}$ estimated where $\mu_t = 1.0$ (mean of 36 independent replications of fit to 1986 data)	standard error of $\hat{\mu}_{\psi t}$ (estimated from 36 replications of fit to 1986 data)
eight years or less	.9697	.0221	0.6811	0.0269
some high school	.9055	.0350	0.7426	0.0248
high school graduate	.8006	.0214	0.8517	0.0121
some college	.6928	.0298	0.9837	0.0328
college graduate	.5346	.0252	1.2786	0.0406
some post-graduate education or more	.4138	.0191	1.6425	0.0406

However, table 3 shows that the SWDO passes the tests set by IPDO meta-theory, that is, its estimated vector of $\omega_{\psi t}$'s, its $\hat{\omega}_{\psi t}$, (where $\hat{\omega}_{\psi t} = 1 - \hat{\lambda}_{\psi t}$) varies inversely with level of education and its estimated vector of $\mu_{\psi t}$'s, its $\hat{\mu}_{\psi t}$. Estimates of the parameters of the fitted stationary distributions of the SWDO and IPDO do not approximate each other: compare the SWDO's $\hat{\omega}_{\psi t}$'s (data column 1 of table 3) to those of the IPDO (data column 1 of table 2). The SWDO's $\hat{\omega}_{\psi t}$'s are over twice those of the IPDO. The standard error of estimate of the SWDO's $\hat{\omega}_{\psi t}$ are larger than the IPDO's particularly for the more educated. The SWDO's estimate of the unconditional mean of annual wage and salary income approximates the IPDO's. The SWDO's estimate is $30,921 in 2003 dollars whereas the IPDO's is $31,043. The s.e.e. of the IPDO's estimate of the unconditional mean of annual wage and salary income in 1986 is $234 while the SWDO's is $286.

Figure 3 here.

Figure 3 displays the IPDO (solid curves) and SWDO (dashed curves) partial stationary distributions fitted to the partial distribution of 1986 annual wage and salary income of people with an elementary education or less (red curves), to the partial distribution of people with some college (green curves), and to the partial distribution of people with a post-graduate educations (purple curves). The fitted IPDO partial stationary distributions transition from more leptokurtic than the SWDO's in the case of the least educated to more platykurtic than the SWDO's in the case of the most educated The IPDO partial stationary fitted to the annual wage and salary income distributions of people with at least some college have heavier tails than the fitted SWDO partial stationary distributions. The comparisons of fitted partial stationary distributions not shown in figure 3 are intermediate between the comparisons that are.

Figure 4 here.

3. Comparing IPDO and SWDO Fits

The findings of in the previous section of the paper about IPDO and SWDO fits to 1986 data generalize to March CPS data from 1961 through 2003. In each year the IPDO and the SWDO are fitted in the same way as in 1986. In each year of the 43 years of data, each model is independently fitted 36 times. In not a single year is the mean IPDO fit (the mean of 36 independent IPDO fits) poorer than the mean SWDO fit. See figure 4 which graphs the weighted sum of squared errors of the six partial distributions fitted 36 times independently in each year. Both models fit the empirical distributions well, but the IPDO fits better on average in every year. Figure 4 shows at a glance that it is redundant to do 43 two sample difference of means tests. The two samples of independent fits in each year hardly overlap and the difference between their means is as large or larger than the range of the 36 IPDO and SWDO fits. The standard deviation of the 36 IPDO fits in most years is smaller than that of the 36 SWDO fits. The IPDO grand total of the 36 sums of weighted squared errors in each of 43 years, 1,548 fits altogether, is 6.4559; the SWDO's is 8.9676. The sum of IPDO weighted squared errors is 72% of that of the SWDO. The IPDO's advantage in fit over the SWDO increases over the decades.

Figure 5 here.

3.1 Why the IPDO Fits the Data Better Than the SWDO

Figure 5 shows the ratio of the IPDO weighted sum of squared errors to the SWDO weighted sum of squared errors at each level of worker education in each year. The IPDO is often a better fit than the SWDO to the annual wage and salary income distributions of the two least educated groups, but its superiority is not uniform over time. However, among workers with at least some college education, the IPDO provides a distinctly superior fit to annual wage and salary income distribution, a superiority that grows over the decades. The education level of the U.S. labor force steadily rose, so the IPDO's advantage grew overtime. See figure 6.

Figure 6 here.

4. Conclusions

The Inequality Process (IP) and the Saved Wealth Model (SW) are stochastic binary interacting systems. Both models randomly pair particles for interaction. In both models the population of particles is isolated and the positive quantity exchanged between particles when they are paired and interact is neither created nor destroyed. Since the populations of particles in both models are isolated, the sum of the positive quantity, called “wealth”, does not vary over time. The IP was abstracted in the early 1980's from an old theory of economic anthropology about the origin of substantial economic inequality in competition for stored food, as speculatively extended by a sociologist to account for decreasing inequality, in the sense of concentration, over the course of techno-cultural evolution. The extension is that more skilled workers retain a larger share of the wealth they create. This verbal meta-theory assigns an empirical referent to the IP's parameter, a characteristic of particles. This parameter is denoted ω, $0.0 < \omega < 1.0$, and is the share of its wealth a particle loses when it loses to another particle. This meta-theory designates the empirical referent of $(1-\omega_\psi)$ as worker productivity, operationalized as the ψ^{th} level of worker education.

Consequently, the stationary distribution of wealth of the Inequality Process with Distributed Omega (IPDO), the IP in which particles can have different values of ω (as workers can have different educations), is obliged to fit the distribution of labor income conditioned on education. This obligation is tested by showing that when a) the stationary distribution of wealth in the ψ^{th} equivalence class of particles is fitted to the distribution of labor income of workers at the ψ^{th} level of education, and b) the fraction of particles in the ψ^{th} equivalence class equals the fraction of workers at the ψ^{th} level of education, then c) the model's stationary distributions fit the corresponding empirical distributions, d) estimated $(1-\omega_\psi)$ increases with level of education, and, subsidarily, e) mean IPDO wealth, μ_ψ, of particles in each ω_ψ equivalence class scales from low to high with $(1-\omega_\psi)$. The IPDO passed these tests [1-14] repeated in this paper for comparison to the SWDO.

4.1 Does The Saved Wealth Model (SW) Pass the Tests the IPDO Passed?

The Saved Wealth Model (SW) results from tinkering with the Kinetic Theory of Gases (KTG) particle system. The SW has not been as extensively tested empirically as the IP because a) it appeared 17 years later, and b) it was not abstracted from a social science meta-theory assigning an interpretation to its parameter and setting tests that it must pass. Despite the difference in provenance, the SW is isomorphic to the IP up to the stochastic driver of wealth exchange between particles and the end point of the interval on which its parameter is defined. The IP's driver of wealth exchange is a 0,1 discrete random variate; the SW's is a continuous [0,1] uniform random variate. The IP's particle parameter, ω, is the fraction of wealth a particle loses to another particle; the SW's parameter, denoted λ, is the complement of ω, $\lambda = 1 - \omega$, with an exception at the end point of the interval on which the parameters are defined. The SW's particle parameter, λ, can be mapped into the complement (1-ω) of the IP's parameter, ω, one to one, except for $\lambda = 0$ (the special case of the KTG) because the IP with $\omega = 1.0$ is not ergodic.

While there is no published SW analogue of the IPDO, an SWDO, it is clear how to define an SW version that is isomorphic to the IPDO (the Inequality Process with Distributed Omega) up to the difference in definition between the OPIP and the SW. The SWDO passes tests posed by the IP's social science meta-theory that the IPDO passed although with different parameters and stationary distributions. Specifically, 1) although the two particle systems have different stationary distributions, both models' stationary distributions provide a good fit to the distribution of U.S. annual wage and salary income conditioned on education over four decades, 2) the parameter estimates of the fits differ by particle system, although 3) both particle systems have estimated $(1-\omega_\psi)$'s that increase with the mean wealth, μ_ψ, of particles in each ω_ψ equivalence class and scale from low to high with worker education. However, 4) the IPDO's fits are better than the SWDO's because 5) the IPDO's stationary distribution has a heavier tail than the SWDO, fitting the distribution of wage income of the more educated better, and 6) since the level of education in the U.S. labor force rose, the IPDO's fit advantage increased over time.

5. References

1. Angle, John. 2009. "Two similar particle systems of labor income distribution conditioned on education". In **JSM Proceedings, Business and Economics Statistics Sections**. Pp. 1003-1017. CD-ROM. Alexandria, VA: American Statistical Association.

2. _____. 1983. "The surplus theory of social stratification and the size distribution of personal wealth." **1983 Proceedings of the American Statistical Association, Social Statistics Section.** Pp. 395-400. Alexandria, VA: American Statistical Association.

3. _____. 1986. "The surplus theory of social stratification and the size distribution of Personal Wealth." **Social Forces** 65:293-326.

4. _____. 1990. "A stochastic interacting particle system model of the size distribution of wealth and income." **1990 Proceedings of the American Statistical Association, Social Statistics Section**. Pp. 279-284. Alexandria, VA: American Statistical Association.

5. _____, 1992. "The Inequality Process and the distribution of income to blacks and whites". ***Journal of Mathematical Sociology*** 17:77-98.

6. _____. 1996. "How the gamma law of income distribution appears invariant under aggregation". **Journal of Mathematical Sociology**. 21:325-358.

7._____. 1997. "A theory of income distribution". **1997 Proceedings of the American Statistical Association, Social Statistics Section**. Pp. 388-393. Alexandria, VA: American Statistical Association.

8. _____. 2002. "The statistical signature of pervasive competition on wages and salaries". **Journal of Mathematical Sociology**. 26:217-270.

9. _____. 2003a. "Imitating the salamander: a model of the right tail of the wage distribution truncated by topcoding". November, 2003 ***Conference of the Federal Committee on Statistical Methodology***, [http://www.fcsm.gov/events/papers2003.html].

10. _____. 2003. "Inequality Process, The". An entry in T. Liao, et al., (eds.), **The Encyclopedia of Social Science Research Methods**. Volume 2: 488-490. Thousand Oaks, CA: Sage.

11. _____. 2006. "The Inequality Process as a wealth maximizing process". **Physica A** 367: 388-414.

12. _____. 2007a. "The Macro Model of the Inequality Process and The Surging Relative Frequency of Large Wage Incomes". Pp. 171-196 in A. Chatterjee and B.K. Chakrabarti, (eds.), **The Econophysics of Markets and Networks (Proceedings of the Econophys Kolkata III Conference, March 2007**). Milan: Springer.

13. _____. 2007b. "The Inequality Process is an evolutionary process". ***The Constructal Theory of Social Dynamics***. Adrian Bejan and Gilbert Merkx, eds. (Proceedings of the Conference on the Constructal Theory of Social Dynamics, Duke University, April 2006). New York: Springer.

14._____, François Nielsen, and Enrico Scalas. 2009. "The Kuznets Curve and the Inequality Process". In Banasri Basu, Bikas K. Chakrabarti, Satya R. Chakravarty, Kausik Gangopadhyay, editors, ***Econophysics and Economics of Games, Social Choices and Quantitative Techniques***. Milan: Springer.

15. Chakraborti, A., B.K. Chakrabarti. 2000. "Statistical mechanics of money: How saving propensity affects its distribution". **European Physics Journal B**: 17: 167- 170.

16. Chatterjee, A., B.K. Chakrabarti, and S. Manna. 2004. "Pareto law in a kinetic model of market with random saving propensity". **Physica A** 335: 155-163.

17. Patriarca, M., A. Chakraborti, and K. Kaski. 2004. "A statistical model with a standard gamma distribution". **Physical Review E** 70: article # 016104.

18. Lux, Thomas. 2005. "Emergent statistical wealth distributions in simple monetary exchange models: a critical review".Pp. 51-60 in A. Chatterjee, S. Yarlagadda, and B.K. Chakrabarti, (eds.), **Econophysics of Wealth Distributions**, (the proceedings volume of the International Workshop on the Econophysics of Wealth Distributions, March, 2005, Kolkata, India). Milan, Italy: Springer.

19. Lux, Thomas. 2008. "Applications of Statistical Physics in Economics and Finance". In J. Barkley Rosser Jr., (ed.). **Handbook of Research on Complexity**. London: Edward Elgar.

20. Patriarca, Marco, Els Heinsalu, and Anirban Chakraborti. 2006. "The ABCD's of statistical many-agent economy models". [on-line at http://arxiv.org/abs/physics/0611245/].

21. Scalas, Enrico, Mauro Gallegati, Eric Guerci, David Mas, and Allessandra Tedeschi. 2006. "Growth and Allocation of Resources in Economics: The Agent-based Approach".

22. Yakovenko, Victor. forthcoming. "Econophysics, Statistical Mechanical Approaches to". **Encyclopedia of Complexity and System Science**. [on-line at http://arxiv.org/abs/0709.3662].

23. Whitney, Charles. 1990. **Random Processes in Physical Systems**. New York: Wiley, page 220.

24. Dragulescu, A. and V.Yakovenko. 2000. "Statistical mechanics of money". **European Physics Journal B** 17: 723-729.

25. Dragulescu, A.. and V. Yakovenko. 2001. "Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States". **Physica A** 299: 213-221.

26. Lenski, G. 1966. **Power and Privilege**. New York: McGraw-Hill.

27. Current Population Surveys, March 1962-2004. [machine readable data files]/ conducted by the Bureau of the Census for the Bureau of Labor Statistics. Washington, DC: U.S. Bureau of the Census [producer and distributor], 1962-2004. Santa Monica, CA: Unicon Research Corporation [producer and distributor of CPS Utilities], 2005.

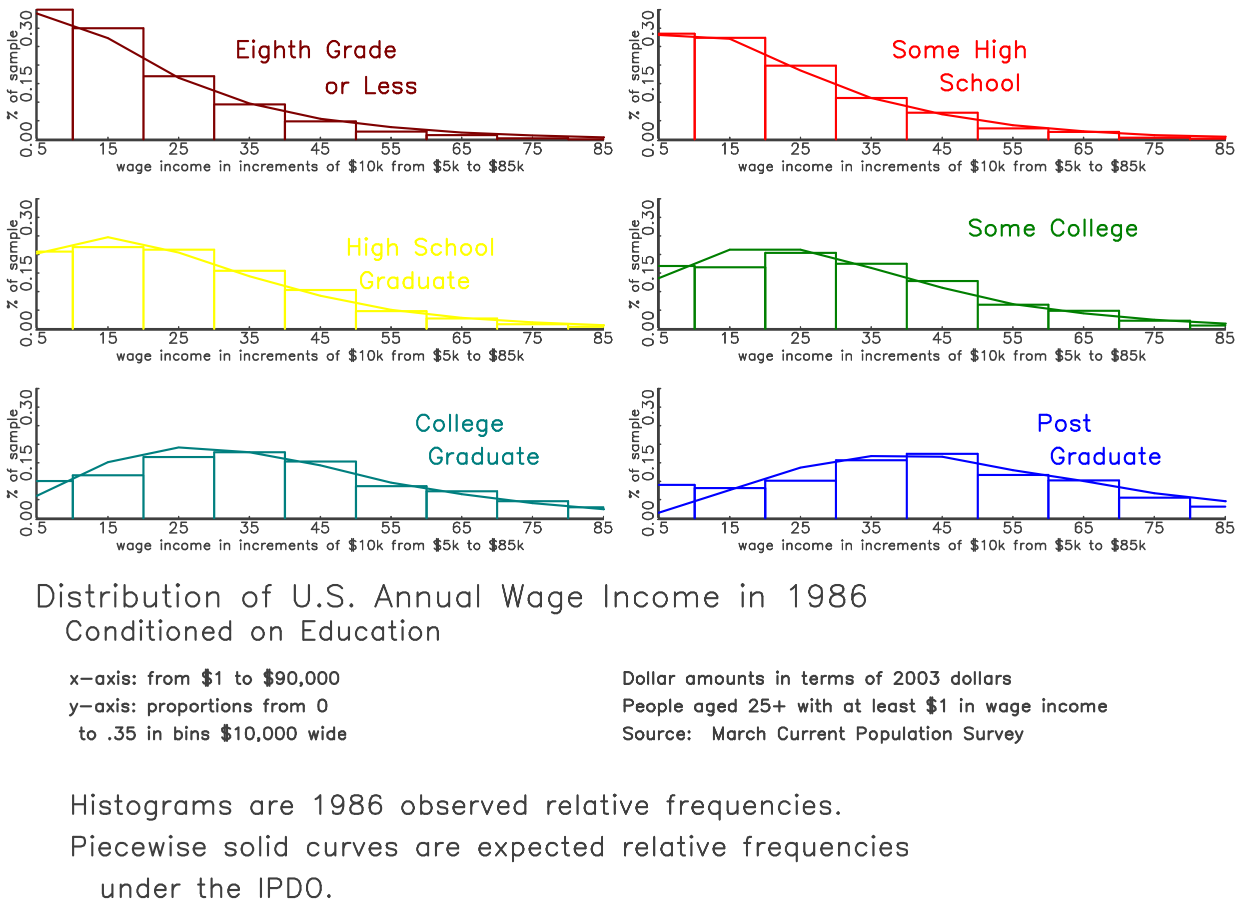
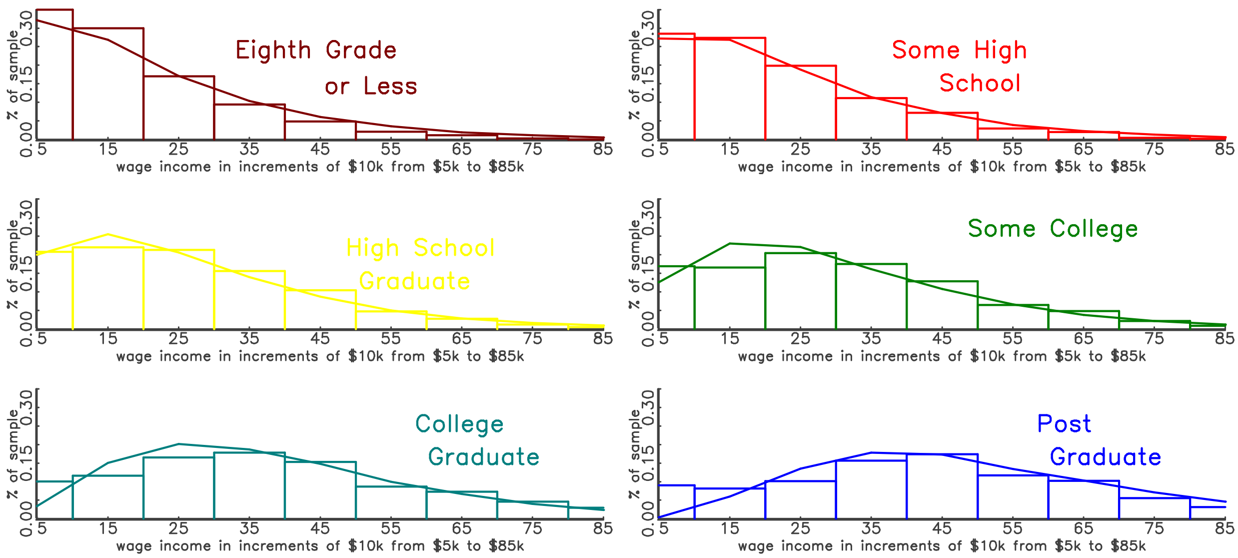

Eighth Grade
or Less
Some High
School
High School
Graduate
Some College
College
Graduate
Post
Graduate
% of sample
0.00 0.15 0.30
5 15 25 35 45 55 65 75 85
wage income in increments of $10k from $5k to $85k
Distribution of U.S. Annual Wage Income in 1986
Conditioned on Education
x–axis: from $1 to $90,000
y–axis: proportions from 0
to .35 in bins $10,000 wide
Dollar amounts in terms of 2003 dollars
People aged 25+ with at least $1 in wage income
Source: March Current Population Survey
Histograms are 1986 observed relative frequencies.
Piecewise solid curves are expected relative frequencies
under the IPDO.

Figure 1



Distribution of U.S. Annual Wage Income in 1986
Conditioned on Education

x–axis: from $1 to $90,000
y–axis: proportions from 0
to .35 in bins $10,000 wide

Dollar amounts in terms of 2003 dollars
People aged 25+ with at least $1 in wage income
Source: March Current Population Survey

Histograms are 1986 observed relative frequencies.
Piecewise solid curves are expected relative frequencies
under the SWDO.

Figure 2

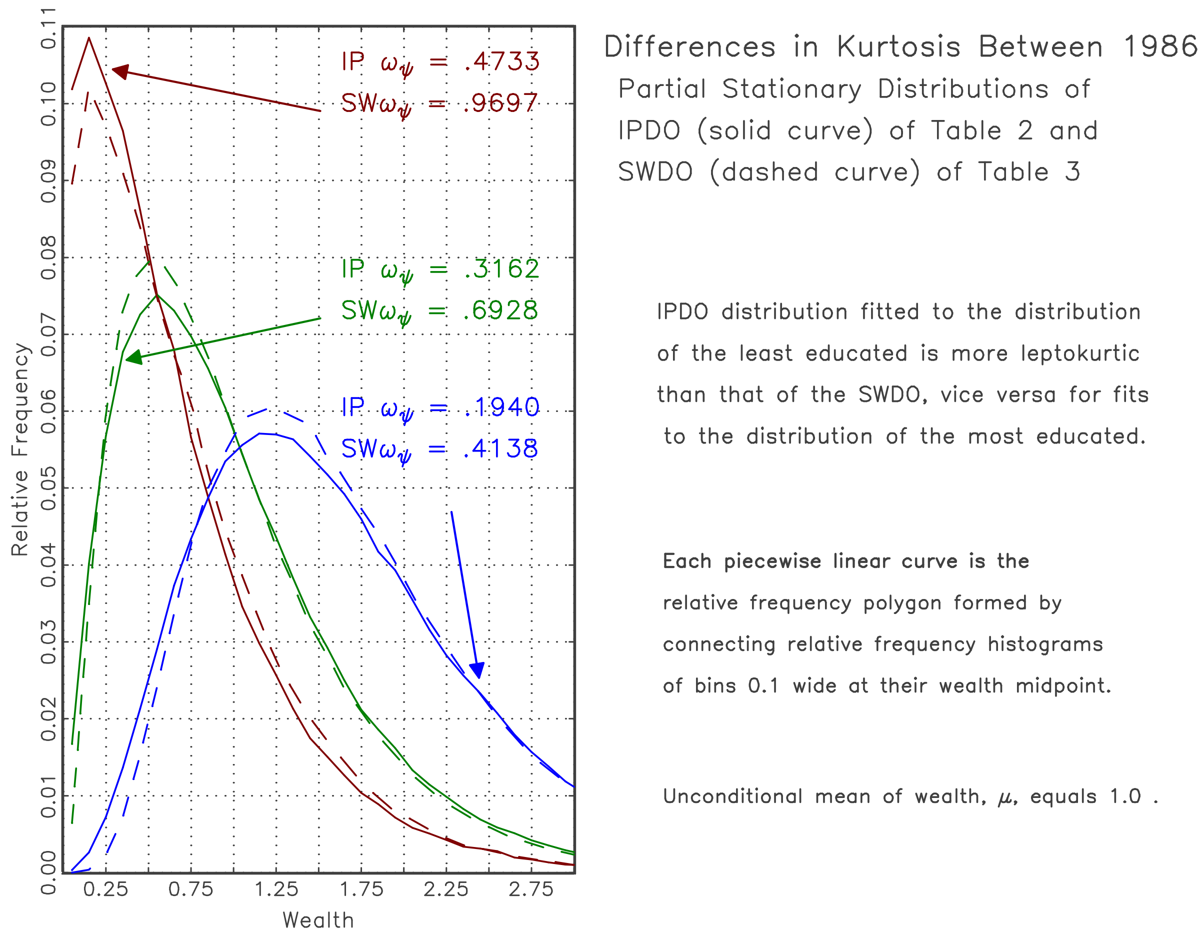

Differences in Kurtosis Between 1986
Partial Stationary Distributions of
IPDO (solid curve) of Table 2 and
SWDO (dashed curve) of Table 3
IP ω_ψ = .4733
SWω_ψ = .9697
IP ω_ψ = .3162
SWω_ψ = .6928
IP ω_ψ = .1940
SWω_ψ = .4138
IPDO distribution fitted to the distribution of the least educated is more leptokurtic than that of the SWDO, vice versa for fits to the distribution of the most educated.
Each piecewise linear curve is the relative frequency polygon formed by connecting relative frequency histograms of bins 0.1 wide at their wealth midpoint.
Unconditional mean of wealth, μ, equals 1.0 .
Relative Frequency
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.25
0.75
1.25
1.75
2.25
2.75
Wealth

Figure 3

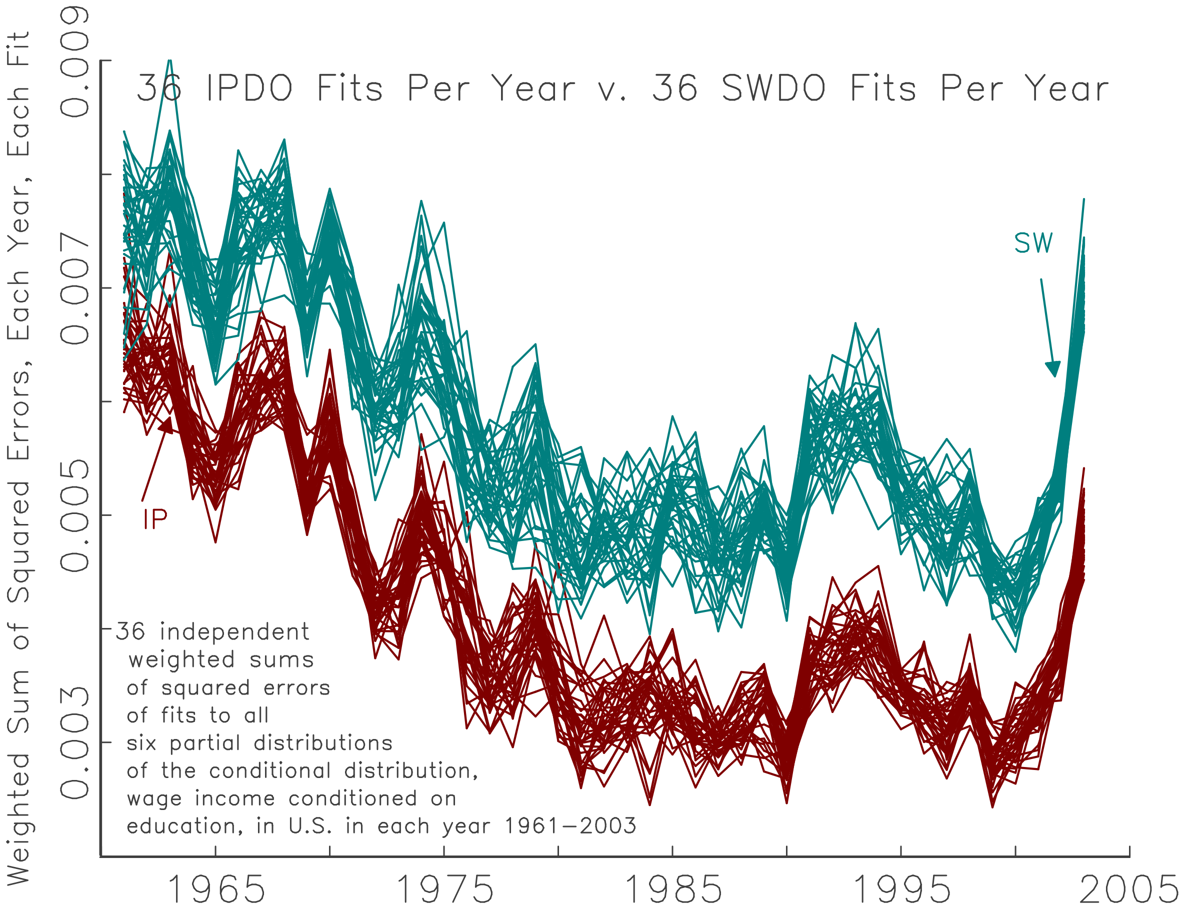

36 IPDO Fits Per Year v. 36 SWDO Fits Per Year
Weighted Sum of Squared Errors, Each Year, Each Fit
0.009
0.007
0.005
0.003
SW
IP
36 independent
weighted sums
of squared errors
of fits to all
six partial distributions
of the conditional distribution,
wage income conditioned on
education, in U.S. in each year 1961–2003
1965
1975
1985
1995
2005

Figure 4

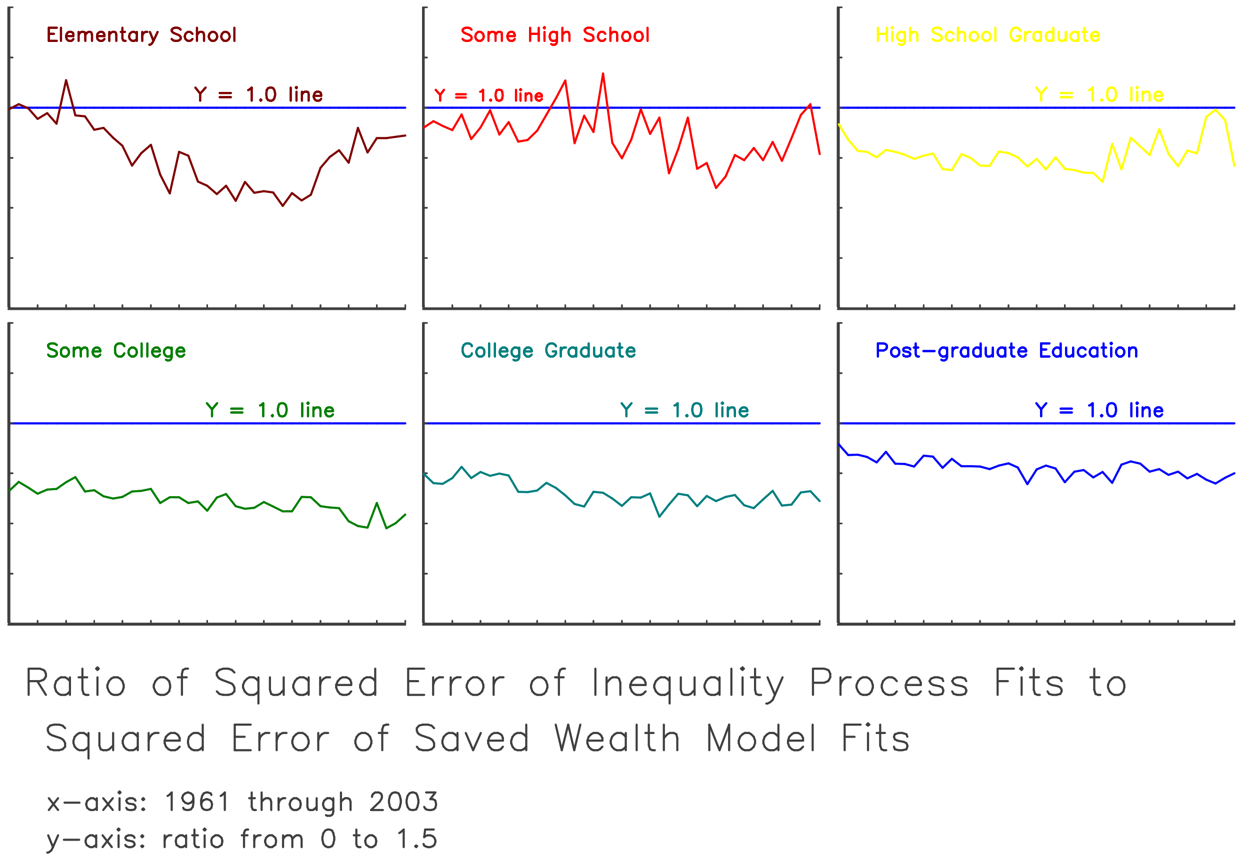
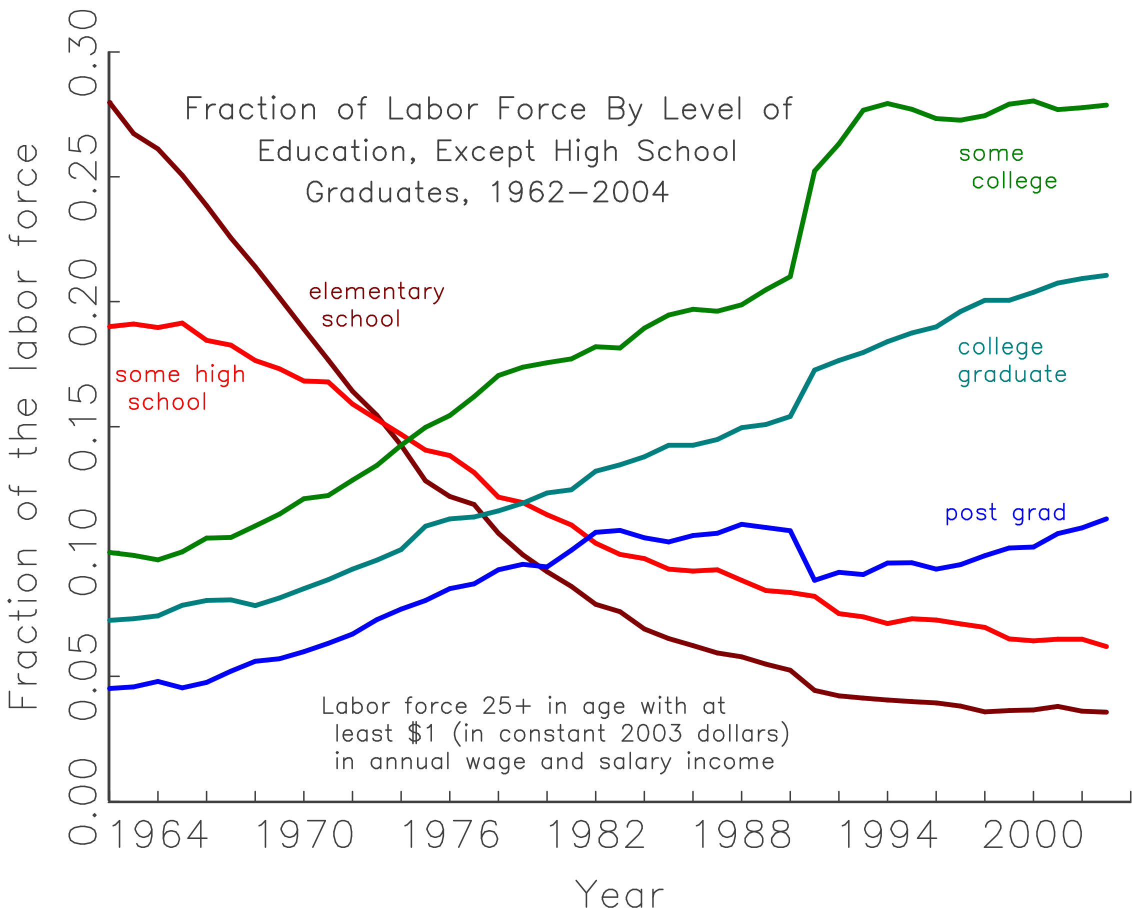

Elementary School
Y = 1.0 line
Some High School
Y = 1.0 line
High School Graduate
Y = 1.0 line
Some College
Y = 1.0 line
College Graduate
Y = 1.0 line
Post–graduate Education
Y = 1.0 line
Ratio of Squared Error of Inequality Process Fits to
Squared Error of Saved Wealth Model Fits
x–axis: 1961 through 2003
y–axis: ratio from 0 to 1.5

Figure 5



Fraction of Labor Force By Level of Education, Except High School Graduates, 1962–2004
some college
elementary school
some high school
college graduate
post grad
Labor force 25+ in age with at least $1 (in constant 2003 dollars) in annual wage and salary income
Fraction of the labor force
0.00
0.05
0.10
0.15
0.20
0.25
0.30
1964
1970
1976
1982
1988
1994
2000
Year

Figure 6