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An alternative proposal for measuring occupational segregation*

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Abstract

This paper offers a general framework in which to study the occupational segregation of a target group when involving a categorization of individuals in two or more groups. For this purpose, it proposes to compare the distribution of the target group against the distribution of total employment across occupations. In doing so, this paper first presents an axiomatic set-up within which segregation measures can be evaluated and defines an alternative segregation curve. Next, a class of additive segregation indexes, related to the generalized entropy family and consistent with the above curves, is characterized. Finally, decompositions of these measures by subgroups of occupations and by subgroups of individuals are proposed.

JEL Classification: J71; J16; D63
Keywords: Occupational segregation; Segregation curves; Inequality measures; Gender

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1. Introduction

When analyzing segregation, most measures compare the distribution of a target group across units with the distribution of another group considered as the standard. Thus, when studying school segregation, the distribution of black students is usually compared with that of whites (Duncan and Duncan, 1955; James and Taeuber, 1985), while when focusing in occupational segregation the distribution of female workers is compared with that of males (Hutchens, 2001, 2004; Chakravarty and Silber, 2007).[1] By considering one of the categories of individuals as the ideal--such that segregation represents deviations from that distribution--it is assumed that the population of reference is not "segregated." Nevertheless, as documented by Anker (1998, p 285), there are occupations around the world that are strongly feminized (nursing, secretary/typist, housekeeper, bookkeeper/cashier, building caretaker/cleaner and tailor/sewer), which suggests that male workers do not distribute evenly across occupations, even though "the value of these niches to women is often of dubious value as these occupations tend to have low pay and status".

In measuring occupational segregation, it would be reasonable to compare the distribution of the target group against the distribution of total employment across occupations. This approach seems especially interesting when there are more than two categories of individuals or when there are two categories with similar population sizes, since in these cases a single group should not be considered the standard. Suppose, for example, that we are concerned with the segregation of an ethnic group by sex. If the traditional approach was followed, the distribution of male workers (including all ethnic groups) would be considered as the ideal, and therefore, the segregation measurement of the female ethnic group would be tackled in a different way than that of the male ethnic group. The reason is that the distribution of the former would be compared with an independent distribution (that of males), but the distribution of the latter would be compared with a correlated one. However, when choosing a more general distribution as the ideal, as the one represented by total employment, the criterion used for measuring the segregation of any population subgroup is the same. Moreover, this approach permits one to emphasize the (labor) demand side, since the weight of each occupation is measured in terms of total employment, i.e., the benchmark is not the

[1] Flückiger and Silber (1999) offer a revision of occupational segregation measures by following this approach.

distribution of a particular population group but the occupational structure of the economy. In addition, this framework allows quantifying not only the occupational specialization pattern of women but also that of men, while the traditional approach actually measures segregation by gender, since those indices compute the differences between the male and female distributions, but they do not analyze each of them separately.

There are some previous proposals in the literature in this respect. On one hand, this idea appears in the field of regional economics in analysis of the geographical concentration of industries (Krugman, 1991; Amiti, 1999). In this regard, the employment distribution of a particular industry across locations is compared with that of total employment. Thus, for example, the distributions of the chemistry and textile industries are compared with the distribution of manufacturing employment in order to see which one is more spatially concentrated.[2] On the other hand, in the field of segregation, there are also some previous works that take a similar approach to the one proposed here. Moir and Selby Smith (1979) offered a variation of the index of dissimilarity, proposed by Duncan and Duncan (1955), by taking the distribution of employment over industry categories as the standard in order to measure industrial segregation in the Australian labor market.[3] Furthermore, papers that use the entropy index proposed by Theil and Finizza (1971) to analyze occupational segregation also consider in some respect that the distribution of reference is that of total population since the female and non-female (i.e., male) ratios are calculated with respect to this total value (Mora and Ruiz-Castillo, 2003, 2004, among others). However, we should note that this approach differs form ours, since that index takes into account the distribution of women and its complementary, i.e., that of men, together, not separately. In other words, it allows measuring gender segregation but not exactly the segregation of female workers.[4]

The aim of this paper is to propose a general framework in which to study the occupational segregation of any population subgroup. In doing so, this paper proposes to compare the

[2] See Brülhart and Traeger (2005) for a discussion about this geographical concentration measurement.
[3] Gross (1968) used another variant of the dissimilarity index, as proposed by Gibbs (1965), to analyze the evolution of sex segregation in the USA. In doing so, first, the ratios of female and male workers with respect to total workers are obtained. Next, these ratios are adjusted in order to make all occupations of the same size. Finally, the differences between female and male figures are calculated. Therefore, the ideal distribution in this case is a mixture between total employment and male employment.
[4] Reardon and Firebaugh (2002) extended several inequality indexes, such as that of Gini, Theil and the squared coefficient of variation, to measure multigroup segregation across schools. In doing so they also take into account the distribution of total students, rather than that of white students, but their approach also differs from ours since they measure racial segregation rather than segregation of a particular racial group.

distribution of the target group across occupations with the distribution of total employment. First, it presents an axiomatic set-up within which segregation measures can be evaluated. For this purpose, some of the basic properties established in the traditional approach by James and Taueber (1985) and Hutchens (1991, 2001) are modified in order to make them suitable to the new framework. In particular, in the traditional approach the distribution of the target group (female workers) and the benchmark (male workers) are independent, while now the distribution of the former has to be compatible with the distribution of total employment. Second, an alternative segregation curve is defined. This curve does not change when the occupational structure of the economy varies so long as the target group experiences the same type of evolution. Third, segregation indexes consistent with these curves are shown. In particular, the Gini index of segregation, the index of dissimilarity, and those recently proposed by Chakravarty and Silber (2007) are modified to make them compatible with our approach. Even though our analysis is similar to that previously undertaken by Hutchens (1991) for traditional segregation curves, there are some differences. In order to link segregation and inequality measurement, we propose to construct a fictitious "income distribution" in a world of "replicants" in such a way that each occupation is represented by so many identical individuals as jobs it has. Fourth, a class of segregation indexes, related to the generalized entropy family, and satisfying some basic properties is characterized. This family includes a variation of the indexes proposed by Hutchens (2004) in the traditional approach, but also new ones. Finally, decompositions of these measures by subgroups of occupations and by subgroups of individuals are proposed.

This paper is structured as follows. Section 2 proposes an axiomatic framework to evaluate segregation measures when having at least two population subgroups. In Section 3 an alternative segregation curve is defined and differences with respect to the traditional measurement are shown. Section 4 presents the relationship between our segregation curves and segregation indexes satisfying our basic properties, while Section 5 characterizes some of these indexes and proposes two types of decompositions. This segregation measurement is then illustrated in Section 6 by using Spanish labor force data for 2007. Section 7 concludes.

2. Basic axioms for a measure of occupational segregation

Consider an economy with $J > 1$ occupations among which total employment, denoted by *T*, is distributed according to distribution $t \equiv (t_1, t_2, ..., t_J)$, where $t_j > 0$ represents the number of jobs in occupation j $(j = 1, ..., J)$ and $T = \sum_j t_j$. Let us denote by $c \equiv (c_1, c_2, ..., c_J)$ the distribution of the target group we want to analyze, where the total number of individuals in this set is $C = \sum_j c_j$. Since this set represents a subset of the total number of employees, $c_j \leq t_j$. Distribution *c* could represent, for example, the number of women employed in each occupation, but it could also represent the number of individuals of an ethnic group or any other group of citizens in which we are interested.

In what follows, we show a list of desirable properties for any measure of occupational segregation $\Phi : D \to \mathbb{R}$ where $D = \bigcup_{J>1} \left\{ (c;t) \in \mathbb{R}^J_+ \times \mathbb{R}^J_{++} : c_j \leq t_j \forall j \right\}$. The first axiom is *scale invariance*, which means that the segregation index does not change when the total number of jobs in the economy and/or the total number of individuals of the target group varies, so long as their respective shares in each occupation remain unaltered. This is an axiom similar to the one considered in the income distribution literature regarding relative inequality measures.

Axiom 1. *Scale Invariance*: Let α and β be two positive scalars such that when $(c;t) \in D$ vector $(\alpha c; \beta t) \in D$, then $\Phi(\alpha c; \beta t) = \Phi(c;t)$.

As opposed to Hutchens' formulation (1991; 2004), the above property requires compatibility between distributions *c* and *t*, and this is why not every pair of positive scalars is possible but only those that allow $(\alpha c; \beta t) \in D$, so that $\alpha c_j \leq \beta t_j$. In Figure 1, we can see that αc is a distribution located in the ray passing by *c*. We can also see that the only βt distributions compatible with αc are those belonging to the ray passing by *t* that are in the dash line.

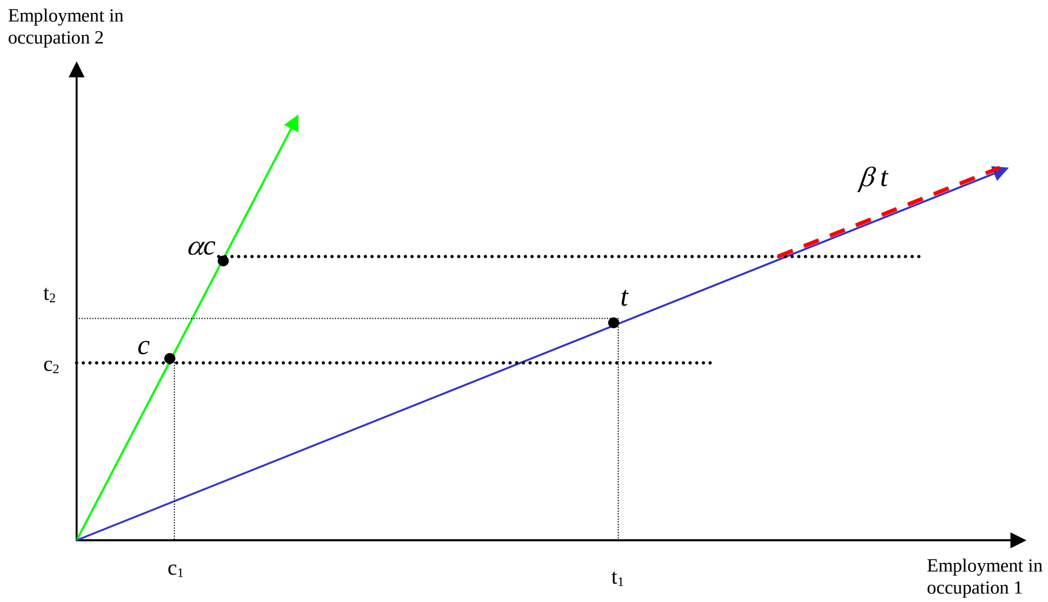

Figure 1. Relationship between αc and βt in a two-dimensional case.

Consider, for instance, that we are interested in measuring the occupational segregation of female workers. Since we do not compare the female distribution with the male one, as in the traditional approach, but with the employment distribution, some reflections are in order. If $\beta = 1$ and $\alpha = 2$ are two possible scalars, then vector $(2c;t) \in D$ represents a scenario in which total employment has not changed, the female share in each occupation (with respect to the total number of female workers) has not changed either, but the number of female employees has actually doubled. For example, suppose that there are three occupations and that the initial number of women working in each of them is, respectively, 2, 3 and 5, while the number of jobs in each occupation is, respectively, 30, 50 and 20.

$$\begin{array}{ll} & \text{Occupations} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{l} \text{Women} \\ \text{Total employment} \end{array} & \begin{pmatrix} 2 & 3 & 5 \\ 30 & 50 & 20 \end{pmatrix} \end{array} \quad \text{or equivalently} \quad (c;t) = (2,3,5;30,50,20).$$

Assume now that the employment distribution remains the same, but the number of women in each occupation doubles:

$$\begin{array}{ll} & \text{Occupations} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{l} \text{Women} \\ \text{Total employment} \end{array} & \begin{pmatrix} 4 & 6 & 10 \\ 30 & 50 & 20 \end{pmatrix} \end{array} \quad \text{or equivalently} \quad (2c;t) = (4,6,10;30,50,20).$$

Given that in this example our target group is female workers, we are only interested in the distribution of women among occupations. Since neither the proportion of women in each occupation has changed (20% of female workers are still in the first occupation, 30% in the second, and 50% in the third) nor has the employment structure, any measure of female segregation should remain unaltered, even though the distribution of other groups (in this case, men) have changed. In fact, in our example, some men have lost their previous positions since more women have entered each occupation while the number of jobs has remained the same (the distribution of men among occupations has changed from $(28,47,15)$ to $(26,44,10)$). Therefore, the gender rate (i.e., the number of female workers against the number of male workers) in each occupation has actually changed. Certainly, segregation for men could have changed, but that can only be measured by calculating the segregation index for that particular group.

When considering the case where $\alpha = \beta$, the above axiom becomes the s*ize invariance* or *replication invariance* axiom.

Axiom 2. *Size Invariance*: Let α be a positive scalar and $(c;t) \in D$, then $\Phi(c;t) = \Phi(\alpha c;\alpha t)$.

This axiom means that if we have an economy obtained by the replication of an initial one, segregation does not change.[5] The next axiom is *symmetry in groups*, which means that the "occupation's name" is irrelevant, so that if we rank occupations in a different order, the segregation measurement remains unchanged.

Axiom 3. *Symmetry in Groups* (Hutchens, 1991): If $(\Pi(1),...,\Pi(J))$ represents a permutation of occupations $(1,...,J)$ and $(c;t) \in D$, then $\Phi(c\Pi;t\Pi) = \Phi(c;t)$, where $c\Pi = (c_{\Pi(1)},...,c_{\Pi(J)})$ and $t\Pi = (t_{\Pi(1)},...,t_{\Pi(J)})$.

The next axiom is *movement between groups*, which requires that when an occupation with a lower number of target individuals than another (but with the same employment level) loses

[5] Note, however, that this axiom differs from that proposed by James and Taeuber (1985). Consider, for instance, that our target group is that of young male workers. Our formulation requires that when both the number of young male workers and jobs double, segregation remains unaltered. Their formulation requires an a priori classification of the whole population in different exclusive subgroups, so that only when all subgroups double, segregation is unaffected. Certainly, James and Taeuber's criterion can be considered as a particular case of the above axiom.

target jobs in favor of the latter, segregation must increase. This property is similar to the Pigou-Dalton principle of the income distribution literature.

Axiom 4. *Movement between Groups*: If vector $(c';t') \in D$ is obtained from vector $(c;t) \in D$ in such a way that **a)** $c'_i = c_i - d$ and $c'_h = c_h + d$ $(0 < d \leq c_i)$, where i and h are two occupations with the same employment share, $\frac{t_i}{T} = \frac{t_h}{T}$, and $\frac{c_i/C}{t_i/T} < \frac{c_h/C}{t_h/T}$; and **b)** $c'_j = c_j$ $\forall j \neq i,h$, and $\frac{t'_j}{T'} = \frac{t_j}{T}$ $\forall j$; then $\Phi(c';t') > \Phi(c;t)$.[6]

Note that since occupations i and h have the same employment share, condition $\frac{c_i/C}{t_i/T} < \frac{c_h/C}{t_h/T}$ is equivalent to condition $c_i < c_h$. In other words, occupation i has initially the same number of jobs as occupation h but a lower number of positions for the target group. Therefore, a movement of some of these citizens from occupation i to occupation h is a disequalizing movement.

In order to highlight the possible consequences of these disequalizing movements, we will analyze three different scenarios in the case of female segregation. First, consider the case where the total number of jobs does not change and the employment distribution does not change either. Therefore, a movement of women between i and h means that female employment in occupation i has been replaced by male employment, while the opposite holds for occupation h. Second, suppose that the total number of jobs increases while shares remain the same. Now, employment in each occupation does increase, so that when occupation i loses some women, even more men than before enter this occupation. And third, consider the case where the total number of jobs decreases, even though employment shares do not change. Then, occupation i loses both women (who move to occupation h) and men (due to the employment reduction). Therefore, a disequalizing movement may involve not only

[6] Note that this axiom differs from that of Hutchens (2004) not only because we compare the female distribution with the total employment distribution, but also because our definition allows the possibility of changes in the job distribution, so long as the employment shares in each occupation remains unaltered. In Hutchens' definition, however, disequalizing movements involve only changes in the female population, while the benchmark distribution (that of males) remains necessarily unaltered. In our approach, when the female distribution changes, the benchmark distribution (total employment) could change. This is why we invoke specific assumptions about those changes. In particular, it is required that the proportion of employment in each occupation does not change, which involves changes in the distribution of men.

changes in the female distribution but also in other population subgroups. In any case, we should keep in mind that, since we compare the distribution of the target group with the distribution of jobs among occupations, when an occupation *i* with a worse position than another *h* (i.e. $\frac{c_i/C}{t_i/T} < \frac{c_h/C}{t_h/T}$), faces a decrease in the number of individuals belonging to the target group, any segregation measure of that group should increase independently of changes in the distribution of other groups of individuals. As mentioned before, segregation for those other groups could have changed, but that can only be measured by calculating segregation indexes for those particular groups.

Finally, we present the axiom of *insensitivity to proportional divisions*, which means that subdividing an occupation in several categories of equal size, both in terms of total employment and in terms of individuals of the target group, does not affect segregation measurement.

Axiom 5. *Insensitivity to Proportional Divisions* (Hutchens, 2004): If vector $(c';t') \in D$ is obtained from vector $(c;t) \in D$ in such a way that **a)** $c'_j = c_j$, $t'_j = t_j$ for any $j = 1,..., J-1$; and **b)** $c'_j = c_J/M$, $t'_j = t_J/M$ for any $j = J,..., J+M-1$, then $\Phi(c';t') = \Phi(c;t)$.

The axioms presented in this section appear as reasonable properties for a segregation measure, and that is why they will be invoked in later sections.

3 Alternative segregation curves: S^*

In the traditional approach, segregation curves are obtained by comparing the distribution of the target group with the distribution of a particular group that is considered the benchmark. In particular, the occupational segregation curve by sex represents the cumulative proportion of female workers corresponding to the cumulative share of male workers, once those occupations have been ranked by increasing gender ratios (the number of women divided by the number of men in each occupation).[7]

[7] A segregation curve is, therefore, similar to the Lorenz curve obtained when having groups of homogeneous income recipients, instead of individual data, so that the distribution of incomes between groups is compared

In order to analyze occupational segregation, this paper proposes instead to compare the distribution of the target group with the distribution of total employment. Thus, to calculate our segregation curve, denoted by S^*, we plot the cumulative proportion of employment, $\sum_{i \leq j} \frac{t_i}{T}$, on the horizontal axis and the cumulative proportion of individuals of the target group, $\sum_{i \leq j} \frac{c_i}{C}$, on the vertical axis $\left(j = 1,.., J \right)$, once those occupations are lined up in ascending order of the ratio $\frac{c_j / C}{t_j / T}$, which is equivalent to ranking according to $\frac{c_j}{t_j}$. This leads to the next definition.

Definition. Denoting by $\tau_j \equiv \sum_{i \leq j} \frac{t_i}{T}$ the proportion of cumulative employment represented by the first *j* occupations ranked according to the above criterion, the segregation curve is

$$S^*_{(c;t)}(\tau_j) = \frac{\sum_{i \leq j} c_i}{C}.$$

Therefore, the benchmark distribution we propose is a general one, total employment, so that it does not depend on which is the target group considered. This allows for an analysis not only of female segregation but also of the segregation of other groups of citizens.

Definition. As with Lorenz curves, we say that $\left(c;t \right) \in D$ *dominates in occupational segregation* $\left(c';t' \right) \in D$ if the segregation curve of the former lies at no point below the latter and at some point above, which can be denoted as $S^*_{(c;t)} > S^*_{(c';t')}$.

In what follows we show several examples in order to compare our approach with the traditional one. In the first example, we consider an economy with 100 female workers and 300 jobs distributed among occupations according to vector $\left(c;t \right) = \left(10, 40, 50; 90, 60, 150 \right)$. In Figure 2, we plot the segregation curve $S^*_{(c;t)}$ obtained from comparing the female distribution *c* with the employment distribution *t*. From these distributions we can also obtain the number

with that of population. In this case, groups would be first ranked by their average income, and later the cumulative proportion of population would be plot on the horizontal axis, and the cumulative proportion of income on the vertical axis.

of male workers in each occupation and compare that distribution to the female distribution, so that we can obtain the traditional segregation curve S. This curve is also plotted in Figure 2, even though in this case the horizontal axis represents the cumulative proportion of male workers instead of total employment. [8] We observe that S^* is closer to the equity line, which is reasonable since it compares the female distribution to the employment distribution, which includes female workers, while S compares the former with the male distribution. Therefore, our measure gives rise to a lower segregation level.

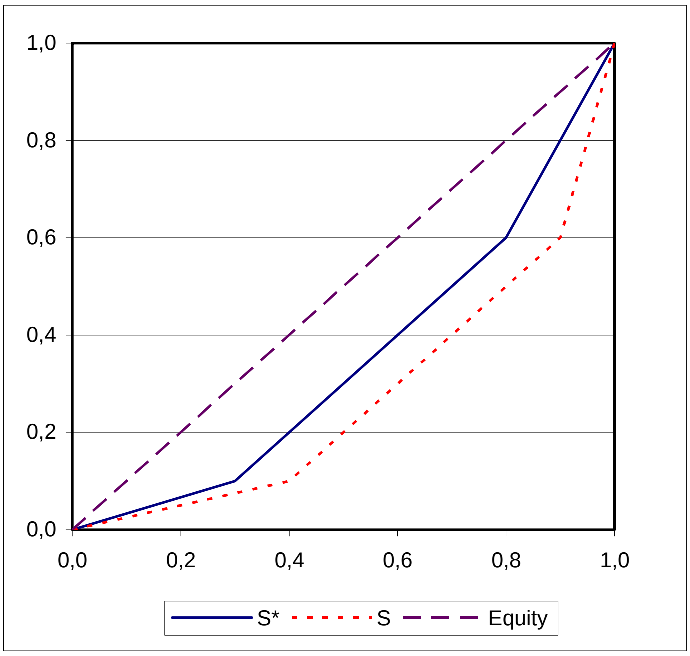

Figure 2. Segregation curves S^* and S in example 1.

To understand better the differences between the traditional segregation curve and ours, example 2 posits that the number and distribution of jobs, in addition to the distribution of female workers, are the same as in example 1, but now there are 120 women. Thus, $(c';t')=(12,48,60;90,60,150)$. In this scenario the distribution of total employment among occupations and that of female workers have not changed; therefore, S^* does not vary (see Figure 3). In other words, female segregation remains the same because there have been

[8] In the case of segregation by sex, total employment is the result of adding female and male workers, so that curve S can be obtained by calculating S^* for distribution $(c;t-c)$. In example 1 vector $(c,t-c)=(10,40,50;80,20,100)$. However, if we were interested in other types of occupational segregation involving more than 2 groups of individuals--for instance female segregation by age, or race segregation, etc.--both approaches would substantially differ.

changes neither in their distribution nor in the employment structure. However, S has varied, since there has been a change in the distribution of male workers among occupations, which moved from representing 40% in the first occupation, 10% in the second and 50% in the third, to 43%, 7% and 50%, respectively. We cannot deny that the economy has experienced a change when moving from example 1 to 2, but we find it interesting to distinguish between changes that affect the target group from those that do not. Female segregation should not vary so long as the employment and female labor force structures remain unaltered. If we are interested in other target groups that may also suffer from occupational segregation, it is possible to measure their segregation, within this framework.

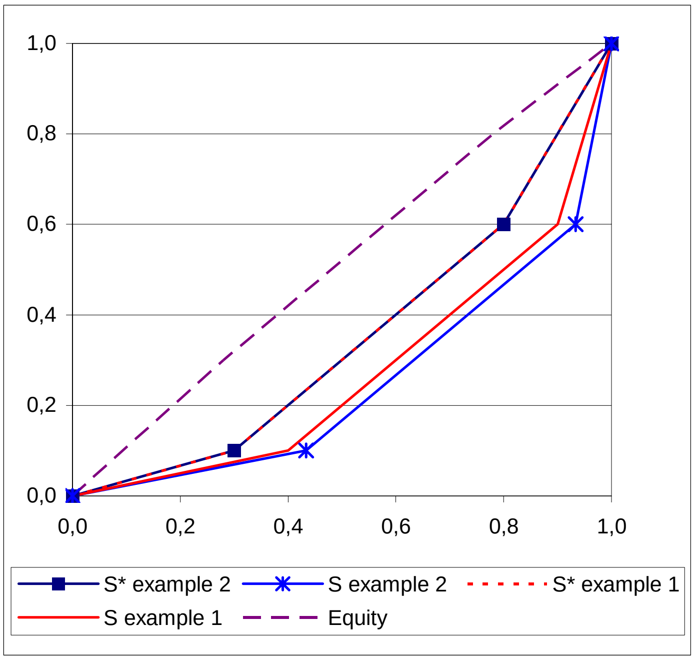

Figure 3. Segregation curves S* and S in examples 1 and 2.

In what follows we show a scenario in which changes in the distributions lead to changes in the segregation level when using S^*, but not when using S. Imagine now that the number of jobs in the economy remains constant, but that occupation one loses 6 jobs in favor of occupation two. This means that the employment share decreases in occupation one, which represented 30% of jobs in example 1 and 28% now, and increases in occupation two (20% against 22%). Assume also that there are 120 female workers, like in example 2, with a distribution among occupations that keeps the same female shares as before, so that the first occupation still represents 10% of female jobs, the second represents 40%, and the third, 50%. Thus, $(c";t") = (12,48,60;84,66,150)$. If we calculate curve S^* for examples 1 and 3, we observe that they are different (see Figure 4). In particular, $S^*_{(c";t")} > S^*_{(c;t)}$, which implies that female segregation is higher in the first example. How can we explain this fact? When comparing $(c;t)$ with $(c'';t'')$, we note that there has been a job reduction in occupation 1--where female workers had a low presence--and a growth in occupation 2--where women had a higher presence. Thus, the segregation level decreases, since distribution *c''* is closer to distribution *t''* than *c* to *t*. It follows, then, that this segregation measurement does not care about situations where an occupation has a high female employment share while another has a low female share so long as they are consistent with the overall job distribution.

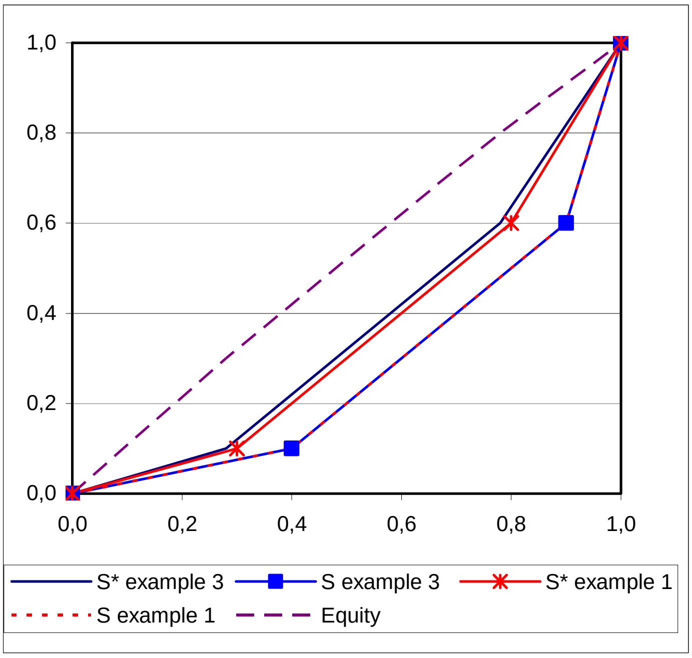

Figure 4. Segregation curves S^* and S in examples 1 and 3.

Definition. According to curve S^*, *complete integration* for the target group occurs when $\frac{c_j}{C} = \frac{t_j}{T}\ \forall j$ (i.e. when $\frac{c_j}{t_j} = \frac{C}{T}$). This distribution is called the equalitarian distribution.

In measuring segregation by sex, complete integration happens when the female and male distributions among occupations coincide. Therefore, in a 2-group framework, when there is complete integration according to our approach, there is also complete integration in the traditional approach, and vice-versa. However, both approaches differ about what complete segregation is.

Definition. For a given employment structure, *complete segregation* according to curve S^* occurs when the target group works in a single occupation.

According to curve S, however, complete segregation by gender occurs not only in the above case but also when women work in several occupations with no men, i.e., when there is perfect complementary between the two sexes' occupations. Notice that this implies the existence of complete segregation of both female and male workers.

4. Relationship between segregation indexes and curves S^*

In this section, we show the relationship between our segregation curve and segregation indexes satisfying the basic properties proposed in a previous section.

Lemma 1. *A segregation index satisfying scale invariance, symmetry in groups, movement between groups, and insensitivity to proportional divisions can be interpreted as a relative inequality index satisfying symmetry, the Pigou-Dalton transfer principle, and population invariance.*

Proof:

For simplicity, let us assume that vector $(c;t) \in D$ is ordered according to shares $\frac{c_j}{t_j}$ from low to high values. From the above vector we can build a hypothetical "income distribution" so that we have t_1 "replicants" with an individual "income" of $\frac{c_1}{t_1}$, t_2 "replicants" with an individual "income" of $\frac{c_2}{t_2}$, and so on. Therefore, we have a fictitious (ordered) income distribution $(\underbrace{\frac{c_1}{t_1},...,\frac{c_1}{t_1}}_{t_1 \text{ replicants}},...,\underbrace{\frac{c_J}{t_J},...,\frac{c_J}{t_J}}_{t_J \text{ replicants}})$ in a world of $T = \sum_j t_j$ replicants where total income is $C = \sum_j t_j \frac{c_j}{t_j}$. An inequality index evaluated at this distribution can be defined as the segregation index evaluated at the original vector $(c;t)$, i.e., $I(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}) := \Phi(c;t)$. Since Φ satisfies the axiom of insensitivity to proportional divisions, the above inequality index is well defined. Certainly, a given distribution of replicants can be obtained from different vectors $(c;t)$, having the same number of jobs (T) and individuals belonging to the target group (C), depending on the way the occupations had been grouped. Note, however, that all these vectors have the same segregation level, since they can be obtained from each other through proportional divisions.

If segregation index Φ satisfies axioms 1, 3, 4 and 5, then index *I* satisfies the basic properties of a relative inequality index:

a) *I* is scale invariant since $I(\alpha\frac{c_1}{t_1},...,\alpha\frac{c_1}{t_1},...,\alpha\frac{c_J}{t_J},...,\alpha\frac{c_J}{t_J}) = \Phi(\alpha c;t)$, which is equal to $\Phi(c;t)$ because Φ is a scale invariant segregation index.
b) *I* satisfies the replication invariance axiom since a k-replication of the fictitious distribution leads to a k-replication of vector $(c;t)$ and Φ satisfies the corresponding axiom, as a particular case of axiom 1.
c) *I* is symmetric since any permutation of the replicants distribution leads to the same ordered vector $(c;t)$ or to another ordered vector that is segregation-equivalent.
d) *I* satisfies the Pigou-Dalton transfer principle. Any possible regressive transfer in this fictitious economy of replicants corresponds to a situation where an occupation *i* transfer individuals of the target group to another occupation *k* where $t_i = t_h$ and $c_i < c_h$. Since Φ satisfies the movement between groups axiom, this second situation leads to a higher segregation index, and therefore, to a higher value of *I*. □

Theorem 1. *Let us consider two vectors* $(c;t),(c';t') \in D$. $S^*_{(c;t)} > S^*_{(c';t')}$ *if and only if* $\Phi(c;t) < \Phi(c';t')$ *for any segregation index* Φ *satisfying axioms 1, 3, 4 and 5.*

Proof:

On one hand, from lemma 1, any segregation index Φ satisfying axioms 1, 3, 4 and 5 leads to a relative inequality index satisfying symmetry, the Pigou-Dalton transfer principle and replication invariance. On the other hand, note that the segregation curve for vector $(c;t)$ is like the Lorenz curve for the fictitious distribution $(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J})$ obtained as in lemma 1's proof. Given the relationship between Lorenz curves and relative inequality measures established by Foster (1985), the Lorenz curve of a distribution dominates another if and only if any relative inequality measure satisfying the above three basic properties takes a lower value at the former distribution. Therefore, $\Phi(c;t) > \Phi(c';t') \Leftrightarrow S^*_{(c;t)} < S^*_{(c';t')}$, which completes the proof. □

The Gini index is an inequality measure satisfying scale invariance, replication invariance and the Pigou-Dalton transfer principle, and it is therefore consistent with the Lorenz criterion (Foster, 1985). Given the similarity between segregation curves and Lorenz curves, an adequate version of the classic Gini index works as a relative segregation measure consistent with non-intersecting S^* curves:[9]

$$G^* = \frac{\sum_{i,j} \frac{t_i}{T}\frac{t_j}{T}\left|\frac{c_i}{t_i} - \frac{c_j}{t_j}\right|}{2\frac{C}{T}}.$$

If there is complete integration (i.e. if $\frac{c_j}{C} = \frac{t_j}{T}\ \ \forall j$), then the Gini index is equal to zero, while if there is complete segregation so that all target citizens work in a single occupation, for example in occupation one, the Gini index is equal to $\frac{T-t_1}{T}$.

The index of dissimilarity proposed by Duncan and Duncan (1955), the most popular segregation measure, is also related to the traditional segregation curve since it equals the maximum vertical distance between the traditional curve S and the 45º line. It can be interpreted as "the proportion of male workers plus the proportion of female workers who would need to change occupations in order to have the same proportion of women in every occupation" (Anker, 1998, p 90). This measure can also be conveniently adapted to make it consistent with our segregation curve S^* so that:[10]

$$D^* = \frac{1}{2}\sum_j \left|\frac{c_j}{C} - \frac{t_j}{T}\right|.$$

It is easy to see that the value of this index coincides with that of Gini's when there is either complete segregation or complete integration.

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[9] Another version of the Gini index is consistent with the traditional segregation curve, as discussed by Flückinger and Silber (1999).

[10] This index was initially proposed by Moir and Selby Smith (1979).

By following an axiomatic approach, Chakravarty and Silber (2007) have recently proposed relative segregation indexes bounded between zero and one that are consistent with the ordering produced by traditional segregation curves. These measures can also be conveniently modified to make them consistent with our segregation curves as follows:

$$\bar{K}_{\alpha}^{*} = 1 - \left[\frac{1}{J} \sum_{j} \left(\frac{c_j}{C} \right)^{\alpha} \left(\frac{t_j}{T} \right)^{\alpha} \right]^{\frac{1}{2\alpha}}, \ \bar{K}^{*} = 1 - \left[\prod_{j} \left(\frac{c_j}{C} \right)^{0.5} \left(\frac{t_j}{T} \right)^{0.5} \right]^{\frac{1}{J}} \quad ,$$

where parameter α is such that the lower its value, the larger the increase of the index due to disequalizing movements between occupations. As noticed by the authors, one limitation of these measures is that they are not suitable when either the target group or the population of reference has a zero value in at least one occupation. Certainly, the use of these indexes in a framework as the one proposed here, where the distribution of reference is total employment instead of a particular population group, reduces that problem.[11]

5. Aggregative segregation indexes consistent with *S**

In the literature of income distribution, scholars usually invoke another axiom, *aggregation*, in order to characterize the class of relative inequality indexes satisfying some basic axioms. This axiom can also be invoked here as follows.

Axiom 6. *Aggregation* (Hutchens, 2004): Let us assume that occupations can be partitioned in two mutually exclusive classes such that $(c;t) = (c^1, c^2; t^1, t^2)$, where the number of jobs in class 1 (2) is denoted by T^1 (T^2), while C^1 (C^2) represents the number of individuals of the target group who work in those occupations. Φ is aggregative if there exists a continuous aggregator function *A* such that $\Phi(c,t) = A\left(\Phi(c^1;t^1), \frac{C^1}{T^1}, T^1, \Phi(c^2;t^2), \frac{C^2}{T^2}, T^2 \right)$, where *A* is strictly increasing in the first and fourth argument.

[11] Note that the indices proposed by Chakravarty and Silber (2007) in the traditional approach may not take a zero value when there is no segregation. In fact, if $\alpha = 0.5$, and if the distribution of females across occupations coincides with that of males, then $\bar{K}_{\alpha} = 1 - \left[\frac{1}{J} \sum_{j} \left(\frac{c_j}{C} \right)^{\alpha} \left(\frac{t_j - c_j}{T - C} \right)^{\alpha} \right]^{\frac{1}{2\alpha}} = 1 - \frac{1}{J}$. The same problem has the modified version $\bar{K}_{\alpha}^{*}$ when the distribution of female workers coincides with the occupational structure of the economy.

Theorem 2. *Let* Φ *be a continuous segregation index that takes a zero value at the equalitarian distribution (i.e., when* $\frac{c_j}{t_j}=\frac{C}{T}$ $\forall j$*). Then,* Φ *is an aggregative segregation measure satisfying scale invariance, symmetry in groups, movement between groups, and insensitivity to proportional divisions if and only if there exists a strictly increasing function* $F:[0,\infty)\to\mathbb{R}$*, with* $F(0)=0$*, such that* $F(\Phi)=\Phi_a$ *for some parameter a, where*

$$\Phi_a(c;t)=\begin{cases}\frac{1}{a(a-1)}\sum_j \frac{t_j}{T}\left[\left(\frac{c_j/C}{t_j/T}\right)^a-1\right] & \text{if } a\neq 0,1\\ \frac{1}{\mathrm{T}}\sum_j t_j\left[\frac{c_j/C}{t_j/T}\ln\left(\frac{c_j/C}{t_j/T}\right)\right] & \text{if } a=1\end{cases}$$

Proof: See Appendix.

Remark 1. Note that these indexes are related to the generalized entropy family used in the income distribution literature.[12] One of their advantages is that they are additively decomposable. They can be decomposed by subgroups of occupations and by subgroups of individuals, which corresponds to the decompositions of inequality by subpopulations and by factor components, respectively (Shorrocks, 1980, 1982):

i) *Decomposition by subgroups of occupations.* Given a partition of occupations in K categories, $(c;t)=(c^1,...,c^K;t^1,...,t^K)$, our indexes can be decomposed as follows:[13]

$$\Phi_a(c^1,...,c^K;t^1,...,t^K)=\sum_k\left(\frac{C^k}{C}\right)^a\left(\frac{T^k}{T}\right)^{1-a}\Phi_a(c^k;t^k)+\Phi_a(C^1,...,C^K;T^1,...,T^K)\quad \text{if } a\neq 0$$

where the first addend of the above formula represents the *within* component, i.e. the weighted sum of segregation inside each occupation class, while the second addend reflects the *between* component.

ii) *Decomposition by subgroups of individuals.* In order to analyze segregation differences between individuals of the target group, let us classify them into several mutual exclusive subgroups. Without loss of generality, consider that there are only two subgroups A and B

[12] The case $a\neq 0,1$ is similar to the one obtained by Hutchens (2004) when comparing the female distribution with male distribution.
[13] These decompositions follow from the aggregator function defined in the Appendix.

such that $(c;t)=(c^A+c^B;t)$. Then the contribution of subgroup A to the segregation level of the whole target group according to index Φ_2 is

$$IC_A=\rho_A\left(\frac{C^A}{C}\right)\sqrt{\frac{\Phi_2(c^A;t)}{\Phi_2(c;t)}},$$

where ρ_A is the correlation between $(\underbrace{\frac{c_1}{t_1},...,\frac{c_1}{t_1}}_{t_1\text{ replicants}},...,\underbrace{\frac{c_J}{t_J},...,\frac{c_J}{t_J}}_{t_J\text{ replicants}})$ and $(\underbrace{\frac{c_1{}^A}{t_1},...,\frac{c_1{}^A}{t_1}}_{t_1\text{ replicants}},...,\underbrace{\frac{c_J{}^A}{t_J},...,\frac{c_J{}^A}{t_J}}_{t_J\text{ replicants}})$, which represent two fictitious income distributions in the world of replicants.[14]

The above theorem proposes a family of relative segregation indexes that are consistent with our segregation curves S^* since they satisfy the basic properties. However, these measures are not necessarily bounded between zero and one, which could be helpful in some empirical analyses. In the next corollary we propose aggregative segregation measures that are bounded within this interval.

Corollary. $\tilde{\Phi}_a(c;t)=1-\sum_j\left(\frac{c_j}{C}\right)^a\left(\frac{t_j}{T}\right)^{1-a}$, with $a\in(0,1)$, is a family of relative and aggregative segregation indexes consistent with segregation curves S^* and is bounded between zero and one.[15]

Proof:

Note that $\tilde{\Phi}_a(c;t)=F^{-1}[\Phi_a(c;t)]$ with $F(y)=\frac{1}{a(1-a)}y$. Therefore, by theorem 2, $\tilde{\Phi}_a$ is a relative segregation index satisfying the basic axioms and, by theorem 1, it is then consistent with our segregation curves. Trivially, $\tilde{\Phi}_a$ is up bounded by 1. To show that it is low bounded by 0, note that if $\sum_j\left(\frac{c_j}{C}\right)^a\left(\frac{t_j}{T}\right)^{1-a}>1$ and using that $a\in(0,1)$, then $\sum_j\frac{t_j}{T}>1$, which is impossible. Therefore, $\sum_j\left(\frac{c_j}{C}\right)^a\left(\frac{t_j}{T}\right)^{1-a}\leq 1$, which completes the proof. □

[14] This decomposition follows from the relationship between segregation and inequality measurement shown in a previous section.

[15] The case where $a=0.5$ is analogous to the square root index proposed by Hutchens (2001) when comparing the female distribution with that of males.

Remark 2. $\tilde{\Phi}_a$ never reaches the upper bound, since $\tilde{\Phi}_a(c;t)=1$ if and only if $c_j=0\ \forall j$, which is impossible. In Section 3, we saw that segregation is maximal when all individuals of the target group work in the same occupation. Without loss of generality, assume that this happens in occupation one, i.e., $(c;t)=(c_1,0,...,0;t_1,t_2...,t_J)$. Then $\tilde{\Phi}_a(c;t)=1-\left(\frac{t_1}{T}\right)^{1-a}$. Therefore, the lower the weight of that occupation in terms of employment, the higher the segregation level.

6. An empirical illustration

To illustrate the above ideas, our segregation indexes and curves are calculated by using labor force data from the *Encuesta de Población Activa* (EPA) conducted by the Spanish Institute of Statistics (INE) by following EUROSTAT's guidelines. This survey offers labor market information of a representative sample of Spanish households and is commonly used for international comparisons. Our data corresponds to the second semester of 2007 and occupations are considered at a two-digit level, so that the list includes 65 occupations.[16] Our target group is workers between the ages of 16 and 40, which group has been partitioned into two groups in order to determine whether the distribution of young females and young males differs across occupations. Occupations have been divided into three categories of similar sizes, according to their average wage. Since the EPA does not gather any salary data, this information comes from the earning survey (*Encuesta de Estructura Salarial*) conducted by the INE in 2002, which is the most recent available year.

In Spain, workers in this range of age, whom we shall call young workers for the sake of simplicity, represent 56.7% of total employment, from which 24.16% corresponds to women and the remaining 32.54% to men. Within this group, the education level of women is higher than that of men. In particular, the proportion of young females with a secondary school education or a university degree is 26.71% and 43.73%, respectively, while those of males are 25.23% and 30.75%.

[16] Armed forces have been excluded from the analysis.

As shown in Table 1, the occupational segregation level of young females is much higher than that of males. In fact, according to index Φ_α with $\alpha = 0.1$ and 0.5, the segregation of females doubles the segregation of males.[17] In fact, as shown in Figure 5, the segregation curve S^* for young males dominates that of young females, which means that any segregation index satisfying scale invariance, symmetry in groups, movement between groups and insensitivity to proportional divisions, would take a higher value for the female group. On the other hand, we also observe that the segregation of young workers is much lower than that of the two population subgroups, which suggests that the occupational distribution pattern of young males and females must be complementary since they balance in the aggregate.

Table 1. Occupational segregation indexes for Spanish young workers.

	$\Phi_{0.1}$	$\Phi_{0.5}$	Φ_1	Φ_2	D^*	G^*	$\bar{K}^*_{0.5}$
Young workers	0.0207	0.0203	0.0198	0.0191	0.0812	0.1111	0.9847
Young female workers	0.5580	0.4171	0.3321	0.2792	0.3337	0.4268	0.9862
Young male workers	0.2438	0.2154	0.1949	0.1825	0.2583	0.3446	0.9854

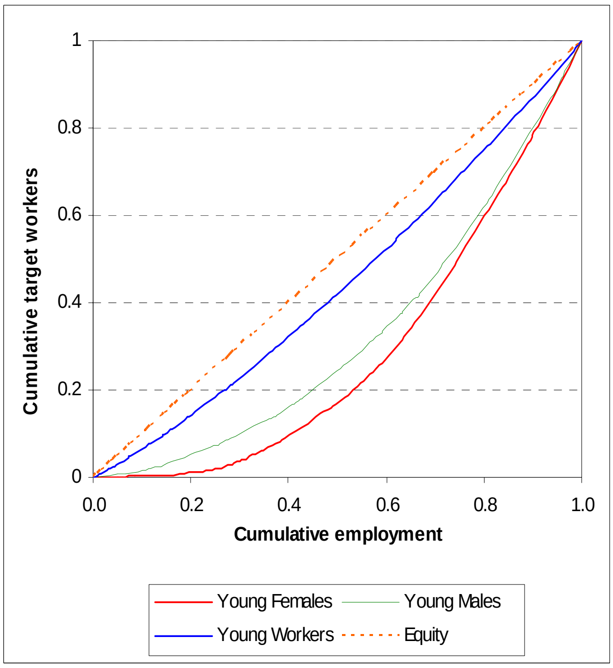

Figure 5. Segregation curves S^* for Spanish young workers

In Table 2, we can see that young workers represent between 50% and 59% of total employment in the three occupational categories considered, those with low, intermediate and

[17] The index proposed by Chakravarty and Silber (2007) seems to show extraordinary high values in both cases (Table 1, column 7). The behavior of this index was also shown by these authors in their empirical analysis based on USA data.

high wages (see fourth column).[18] We also observe that the proportion of young female workers working in the high- and intermediate-wage category is higher than that of males (see fifth column). When comparing the value of index Φ_1 in each occupational category, we observe that the values for the low- and intermediate-wage categories are much higher for young females, while in the high-wage category differences between sexes are lower (see first column). In fact, the contribution of these two categories, jointly considered, to explain female segregation is higher than their contribution to male segregation (see second column).

Table 2. Distribution of young workers in Spain and decompositions of segregation index Φ_1.

	Φ_1	**Contribution of each occupational category to the within component Φ_1**	**Within-Between decomposition Φ_1**	**Target group weight in each occupational category**	**Distribution of the target group between categories**
YOUNG WORKERS	0.0198		88.38% - 11.62%		
Low-wage occupations	0.0172	41.41%		59.00%	41.94%
Intermediate-wage occupations	0.0091	18.08%		58.92%	34.79%
High-wage occupations	0.0304	40.51%		50.34%	23.27%
YOUNG FEMALE WORKERS	0.3321		99.32% - 0.68%		
Low-wage occupations	0.4202	50.35%		23.70%	39.53%
Intermediate-wage occupations	0.3796	41.92%		26.29%	36.43%
High-wage occupations	0.1061	7.73%		22.17%	24.05%
YOUNG MALE WORKERS	0.1949		98.00% - 2.00%		
Low-wage occupations	0.2339	53.56%		35.30%	43.73%
Intermediate-wage occupations	0.1805	31.72%		32.63%	33.58%
High-wage occupations	0.1239	14.72%		28.17%	22.69%

The decomposition of index Φ_1 in the within-between components shows that the classification of occupations by wage explains around 11.62% of the segregation of young workers across occupations (Table 2, third column). However, the value of the between component reduces to 2% in the case of young males and even to 0.68% for female workers. This suggests again that the distributions of young males and females across occupations must

[18] If considering the range 16 to 35 years old, the proportion of young workers in the high-wage category would decrease to 35% and in the others to approximately 45%.

substantially differ since the earning variable appears as relevant for young workers but not for each population subgroup.

The decomposition of index Φ_2 by population subgroups, which is not included in the above tables, shows that women contribute to 25.4% of the segregation of young workers, while the contribution of males is 74.6%. The reason for this disparity is that, even though the segregation level of the former is much higher, young male workers represent a higher proportion of youth employment, and its correlation with youth employment is also higher.

7. Conclusions

When focusing on female segregation, the indexes commonly used are defined as a function of the female and male ratios in each occupation. Some of these ratios are calculated with respect to the population size of the corresponding group, while others are obtained by dividing the number of men and women in each occupation by its total employment. In any case, these indexes actually measure gender segregation because they quantify the differences between the occupational distributions of men and women.

This paper has proposed a general framework in which to study the occupational segregation of any population subgroup by comparing its distribution with respect to the occupational structure of the economy, rather than doing it with respect to the distribution of a particular group considered as the standard or ideal. Within this new set-up, the basic axioms for a segregation measure have been redefined. Also, alternative segregation curves have been proposed and new indexes consistent with them have been characterized. This proposal has been illustrated with Spanish data for 2007. Several measures have been used to analyze whether the generations of young female workers (who have a higher human capital level than men) have a segregation level similar to that of young males. We found that, even though young workers have a certain level of segregation among occupations, perhaps due to their life cycle, segregation is much higher for women.

Appendix

Proof of Theorem 2.

First step: *Any segregation index* Φ *satisfying axioms 1, 3, 4, 5 and 6 can be written as a strictly increasing monotonic transformation of* Φ_a.

In order to prove this implication, we make use of the relationship between segregation and inequality, and also of Shorrocks' (1984) theorem, which characterizes aggregative relative measures as reinterpreted by Foster (1985).

Lemma 1 shows that any segregation index Φ satisfying axioms 1, 3, 4 and 5 gives rise to an inequality index *I* satisfying scale invariance, symmetry, the Pigou-Dalton transfer principle and replication invariance, where $I(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}) := \Phi(c;t)$. Also, it is easy to see that if Φ is a continuous function, so too is *I*. In what follows, we show that *I* is an aggregative inequality index. For the sake of simplicity, assume that class 1 includes occupations $j = 1,...,i$, while class 2 is the complementary. By definition

$$I\left(\underbrace{\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i}{t_i},...,\frac{c_i}{t_i}}_{\text{class 1}}\underbrace{\frac{c_{i+1}}{t_{i+1}},...,\frac{c_{i+1}}{t_{i+1}},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}}_{\text{class 2}}\right) = \Phi(c;t) \cdot$$

On the other hand, since Φ is an aggregative segregation index:

$$\Phi(c;t) = \Phi(c^1, c^2; t^1, t^2) = A\left(\Phi(c^1;t^1), \frac{C^1}{T^1}, T^1, \Phi(c^2;t^2), \frac{C^2}{T^2}, T^2\right).$$

Note that $\Phi(c^1;t^1) = I(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i}{t_i},...,\frac{c_i}{t_i})$, and $\Phi(c^2;t^2) = I(\frac{c_{i+1}}{t_{i+1}},...,\frac{c_{i+1}}{t_{i+1}},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J})$. Therefore,

$$I\left(\underbrace{\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i}{t_i},...,\frac{c_i}{t_i}}_{\text{class 1}}\underbrace{\frac{c_{i+1}}{t_{i+1}},...,\frac{c_{i+1}}{t_{i+1}},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}}_{\text{class 2}}\right) = A\left(I(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i}{t_i},...,\frac{c_i}{t_i}), \frac{C^1}{T^1}, T^1, I(\frac{c_{i+1}}{t_{i+1}},...,\frac{c_{i+1}}{t_{i+1}},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}), \frac{C^2}{T^2}, T^2\right),$$

where $\frac{C^1}{T^1}$ (respectively, $\frac{C^2}{T^2}$) represents the average income of replicants in class 1 (respectively, 2), while T^1 (respectively, T^2) is the number of replicants in that class. Therefore, the inequality index *I* is aggregative.[19]

[19] An inequality index $I(x)$ is defined as aggregative if there exists a continuous function *A*, which is strictly increasing in the first and fourth argument, so that $I(x) = A(I(x^1), \mu(x^1), n(x^1), I(x^2), \mu(x^2), n(x^2))$, where

Finally, note that I is equal to zero when all replicants have the same income or, put another way, when all occupations have the same shares of the target group (i.e., when $\frac{c_j}{t_j} = \frac{C}{T} \quad \forall j$).

Following Shorrocks (1984) and Foster (1985), any continuous inequality measure I taking a zero value at the egalitarian distribution and satisfying scale invariance, replication invariance, the Pigou-Dalton transfer principle, symmetry and aggregation can be written as $I(x) = F^{-1}(I_a(x))$ for some parameter a, where F is a strictly increasing function such that $F : [0,\infty) \to \mathbb{R}$, with $F(0) = 0$ and

$$I_a(x) = \begin{cases} \dfrac{1}{na(a-1)} \sum_i \left[\left(\dfrac{x_i}{\frac{1}{n}\sum_k x_k} \right)^a - 1 \right] & \text{if } a \neq 0,1 \\ \dfrac{1}{\mathrm{n}} \sum_i \left[\dfrac{x_i}{\frac{1}{n}\sum_k x_k} \ln \left(\dfrac{x_i}{\frac{1}{n}\sum_k x_k} \right) \right] & \text{if } a = 1 \\ \dfrac{1}{\mathrm{n}} \sum_i \ln \left(\dfrac{\frac{1}{n}\sum_k x_k}{x_i} \right) & \text{if } a = 0 \end{cases}$$

The above inequality indexes are the well-known generalized entropy family. In our case, our "income" distribution is $x = (\underbrace{\frac{c_1}{t_1},\dots,\frac{c_1}{t_1}}_{t_1 \text{ replicants}},\dots,\underbrace{\frac{c_J}{t_J},\dots,\frac{c_J}{t_J}}_{t_J \text{ replicants}})$, and the average of that distribution is equal to $\frac{C}{T}$. Therefore, $\Phi(c;t) = I(x) = F^{-1}(I_a(x)) = F^{-1}(\Phi_a(c;t))$ for $a \neq 0,1$ or $a = 1$. The case where $a = 0$ is discarded because when an occupation j has no employees belonging to the target group (i.e., when $c_j = 0$), the index value would be infinite.[20]

$\mu(.)$ represents the average of the corresponding distribution, $n(.)$ is the number of individuals and x^i represents class i. This definition can be seen in Shorrocks (1984).

[20] The case where $a = 1$ does not have the same problem since $\lim_{c_j \to 0} \frac{c_j/C}{t_j/T} \ln\left(\frac{c_j/C}{t_j/T}\right) = 0$

Second step: $F^{-1}(\Phi_a)$ *is a segregation index satisfying scale invariance, symmetry in groups, movement between groups, insensitivity to proportional divisions, and aggregation.*

In order to prove this, it suffices to show that Φ_a satisfies the above properties, which is done in what follows. It is easy to see that Φ_a verifies scale invariance, symmetry, and insensitivity to proportional divisions.

To demonstrate that any disequalizing movement from occupation i to h , where $t_i = t_h$ and $c_i < c_h$, leads to a higher value of Φ_a, note that this movement from $(c;t)$ to $(c';t')$ implies moving from distribution $x = \left(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i}{t_i},...,\frac{c_i}{t_i},...\frac{c_h}{t_h},...,\frac{c_h}{t_h},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}\right)$ in the world of replicants to distribution $x' = \left(\frac{c_1}{t_1},...,\frac{c_1}{t_1},...,\frac{c_i-d}{t_i},...,\frac{c_i-d}{t_i},...,\frac{c_h+d}{t_h},...,\frac{c_h+d}{t_h},...,\frac{c_J}{t_J},...,\frac{c_J}{t_J}\right)$. Since $\Phi_a(c;t) = I_a(x)$, $\Phi_a(c';t') = I_a(x')$ and I_a is an inequality measure satisfying the Pigou-Dalton transfer principle, it follows that $\Phi_a(c';t') > \Phi_a(c;t)$ (*x'* can be obtained from *x* by a finite sequence of regressive transfers).

To prove that Φ_a is aggregative, note that it can be written as $\Phi_a(c^1,c^2;t^1,t^2) = A\left(\Phi_a\left(c^1;t^1\right),\frac{C^1}{T^1},T^1,\Phi_a\left(c^2;t^2\right),\frac{C^2}{T^2},T^2\right)$ since

$$\Phi_a(c^1,c^2;t^1,t^2) = \begin{cases} -\frac{1}{a(a-1)} + \left[\begin{array}{l} \left(\frac{T^1}{T}\right)^{1-a}\left(\frac{C^1}{C}\right)^{a}\left(\Phi_a\left(c^1;t^1\right)+\frac{1}{a(a-1)}\right)+ \\ +\left(\frac{T^2}{T}\right)^{1-a}\left(\frac{C^2}{C}\right)^{a}\left(\Phi_a\left(c^2;t^2\right)+\frac{1}{a(a-1)}\right) \end{array}\right] & \text{for } a \neq 0,1 \\ \frac{C^1}{C}\left[\Phi_a(c^1;t^1)+\ln\left(\frac{C^1}{T^1}\frac{T}{C}\right)\right]+\frac{C^2}{C}\left[\Phi_a(c^2;t^2)+\ln\left(\frac{C^2}{T^2}\frac{T}{C}\right)\right] & \text{for } a=1 \end{cases}$$

and $T = T^1 + T^2$ and $C = C^1 + C^2$, which completes the proof. □

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