Society for the Study of Economic Inequality

Working Paper Series

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ECINEQ WP 2007 – 70

ECINEQ 2007-70
June 2007

www.ecineq.org

Partial and complete equality-of-opportunity orderings*

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Abstract

This paper proposes a partial equality-of-opportunity ordering based on the inequality-of-opportunity curve, a mechanism that gives preference to those who are worse in terms of opportunity. Moreover, it provides a complete ordering that depends on a sensitivity parameter representing the degree of priority in the equality-of-opportunity policy. The Moreno-Ternero approach is obtained as a particular case. This proposal is applied to a set of 11 countries to compare their degree of equality of opportunity. Results show the relevance for economic policy of observing inequality of opportunity over tranches. Denmark dominates, in terms of posttax income, all other economies.

Key Words: equality of opportunity, partial ordering, circumstances, responsibility.
JEL Classification: D63, H00.

* This paper was written during my visit to the Department of Political Science at Yale University. I am very grateful for the hospitality and helpful comments of John Roemer. I also acknowledge useful comments and suggestions by Juan D. Moreno-Ternero. This paper has benefited from the support of the Spanish Ministry of Science and Technology [Project #SEC2003-08397] and Fundación BBVA. The usual disclaimer applies.
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1. Introduction

The modern economic theory of justice recognizes that an individual's outcome (for example, income, life expectancy or wage-earning capacity) is a function of effort (for example, investment in human capital, number of hours worked and occupational choice) and circumstances (for example, genes, socioeconomic and cultural background, race and sex). However, individuals are only responsible for their effort, because circumstances are beyond the individual's control. Society should, therefore, compensate individuals who suffer from bad circumstances but let the agents exercise their responsibility without trying to distort their outcomes. Opportunities must be equalized (*leveling the playing field*) before the competition starts, but after it begins, individuals are on their own.

The measurement of equality of opportunity is concerned with this level-playing-field principle.[1] A first difficulty in this theory is to determine the line separating effort from circumstances. If we suppose that society has reached a political agreement on the list of circumstances then a second difficulty appears: how to make outcome comparisons between individuals. To do this, people with the same set of circumstances are grouped into types, and then comparisons across types are carried out. Types are constructed according to circumstances like parents' education or intelligence quotient.

The final step is the determination of a policy that allocates some finite amount of resources across types so that it makes the degree to which an individual achieves the outcome a function only of his/her effort. Typically, a different policy arises for each individual so a compromise is required. Roemer (1993, 1998, 2002 and 2003) proposes

[1] See, among others, Roemer (1993, 1998, 2002 and 2003), Van de Gaer (1993), Peragine (2002 and 2004), Roemer *et al.* (2003), Ruiz-Castillo (2003), Betts and Roemer (2006), Lefranc *et al.* (2006a and 2006b), Moreno-Ternero (2007), Ooghe *et al.* (2007) and Fleurbaey and Maniquet (2007).

implementing the policy that maximizes the average of the outcome enjoyed by the worst-off type at each relative effort level. Moreno-Ternero (2007) proposes (from a pure equity framework) to minimize the average of outcome inequality across relative effort levels. Both proposals rely on the average. Therefore, every tranche receives the same weight independently of the degree of inequality of opportunity suffered by individuals at each tranche.

In this paper, we propose a program that is sensitive to the magnitude of inequality of opportunity at each tranche. In particular, we propose a priority axiom according to which equality-of-opportunity policy should allocate resources giving preference to those who are worse off. In doing so, an equality-of-opportunity partial ordering is provided. This partial ordering is based on the inequality-of-opportunity curve, a mechanism adapted from the poverty literature. However, this ordinal criterion is not conclusive if inequality-of-opportunity curves cross. For that case, a complete ordering is also provided. The proposed cardinal program will depend on a sensitivity parameter that represents the degree of priority in the inequality-of-opportunity policy. A particular case of this complete ordering is the Moreno-Ternero approach. This proposal resorts to a pure equity framework where we are only concerned with equalizing the outcome of all individuals, across types, who expended the same relative effort.

Both proposals, a partial ordering and a complete ordering, are applied to a set of 11 countries to compare their degree of equality of opportunity. The dataset for this empirical exercise is the data used in Roemer *et al.* (2003) and Lefranc *et al.* (2006b). Results show the relevance for economic policy of observing inequality of opportunity at each tranche because inequality of opportunity over tranches has a particular shape in each country. Moreover, posttax income experiences a lower level of inequality of opportunity than pretax income, although that particular equalization of inequality of opportunity is not homogenous among countries. By economies, Denmark and Spain are the best and worst countries, respectively, in the sample in terms of inequality of opportunity.

The paper is organized as follows. Section 2 provides a brief review of equality-of-opportunity measurement. Section 3 presents an intuitive version of the proposal. The partial ordering and the complete ordering are presented in Section 4, and the empirical exercise is summarized in Section 5. The final section includes some concluding remarks.

2. Equality-of-opportunity measurement

In this section, we summarize several approaches to equality-of-opportunity measurement but first we provide some notation and definitions.

Let the members of a population enjoy a certain kind of advantage u, for example, income, life expectancy or wage-earning capacity. This advantage is a function of the amount of effort e they expend and the amount of resources they consume x. Moreover, population is partitioned into a set of types $t = \{1, \ldots, T\}$, where all individuals in a type have the same set of circumstances. Therefore, the achieved level of advantage enjoyed by an individual of type t is $u^{t}(x, e)$.

Suppose that there exists an amount w per capita of the resource to allocate among individuals. In order to achieve equality of opportunity, society must choose a policy for allocating w among the population. Let $\varphi^{t}: \Re_{+} \rightarrow \Re_{+}$ be an allocation rule that indicates the amount of the resource that an individual of type t receives with respect to the effort she/he exerts. Then, the T-tuple $\varphi = \left(\varphi^1,...,\varphi^T\right) \in \Phi$ is the policy of the social planner, where Φ is the set of feasible policies.

Finally, assume that the distribution of effort exerted by individuals of type t is F_φ^t and that $e^t(\pi,\varphi)$ is the level of effort exerted by the individual at the π^{th} quantile of that effort distribution when facing the policy φ.[2] We may hence define the indirect advantage function as follows:

$$v^t(\pi,\varphi) = u^t(\varphi^t(e^t(\pi,\varphi)), e^t(\pi,\varphi)). \qquad (1)$$

In what follows, we assume that π is a discrete variable but the analogous result applies, *mutatis mutandis*, to an infinite set.

According to the equality of opportunity from Roemer's pragmatic approach (1993, 1998, 2002 and 2003), two persons of different types have tried equally hard if and only if they are on the same rank of their respective effort distributions.[3] Then, a policy that equalizes opportunities is a policy that equalizes advantage across types, for given quantiles of effort expended. At this point, Roemer proposes to maximize the minimum level of advantage, across types, of individuals who exert the same degree of effort π^{th}:

[2] Note that the level of effort depends on the whole policy not just the allocation rule for type t (see Roemer, 2003, for this generalization).
[3] The use of rank π as an interpersonally comparable measure of effort is precisely justified in Roemer (2003).

$$\varphi_{\pi}^{R} = \arg\max_{\varphi \in \Phi} \left\{ \min_{t \in T} v^{t}(\pi;\varphi) \right\}. \tag{2}$$

Typically, we will have a different policy for each quantile. To adopt a compromise between so hypothetical a bundle of policies, Roemer proposes to give the same weight to each tranche; that is, he proposes the average policy:

$$\varphi^{R} = \arg\max_{\varphi \in \Phi} \left\{ \frac{1}{Q} \sum_{\pi=1}^{Q} \min_{t \in T} v^{t}(\pi;\varphi) \right\} \tag{3}$$

where Q is the number of quantiles; for example, $Q = 100$ for centiles.

An alternative program has been proposed in Van de Gaer (1993). This approach focuses on the set of outcomes available to the members of each type (opportunity set). The proposed policy maximizes the minimum of type-averages of the objective, over types:

$$\varphi^{V} = \arg\max_{\varphi \in \Phi} \min_{t \in T} \left\{ \frac{1}{Q} \sum_{\pi=1}^{Q} v^{t}(\pi;\varphi) \right\}. \tag{4}$$

We do not consider this approach in this paper because it does not allow the consideration of individual advantage by levels of relative effort.

Recently, different approaches have been proposed in the literature to take into account all the outcomes at each quantile, not just the minimum outcome. Peragine (2004) proposed two different methods to make ordinal welfare comparisons according to equality of opportunity. In the first method, the Generalized Lorenz Curve at each quantile for both income distributions must be compared, so Q different dominance conditions should be checked. If the number of quantiles, which must be examined to guarantee a close approximation of the responsibility exercised, is large, the application of this method may be too laborious. In the second method, the Generalized Lorenz Curve of the outcome distribution, in which each individual receives the mean of his/her type, must be compared. Dominance according to this method is a much less demanding criterion; in fact, dominance according to the first approach implies dominance according to the second method. However, this second method does not take into account dispersion in types.[4]

Moreno-Ternero (2007) provides (from a pure equity framework) some alternative mechanisms to construct equality-of-opportunity policies. A first mechanism considers that equality-of-opportunity policy must minimize the average of advantage inequality (across types) at each relative effort level. Thus, program (3) comes under this proposal:

$$\varphi^{MT1} = \arg\min_{\varphi \in \Phi} \frac{1}{Q} \sum_{\pi=1}^{Q} I(v^1(\pi;\varphi),\ldots,v^T(\pi;\varphi)) \qquad (5)$$

[4] This can cause some problems. For example, assume two different outcome distributions with two types. In the first distribution, there is equality of opportunity. In the second distribution, type means are alike but there is some inequality of opportunity. If we apply the proposed method, we will conclude that both distributions are equivalent.

where $I(\cdot)$ is an inequality index. A second proposal minimizes the maximum inequality throughout the different levels of relative effort. Finally, he proposes to minimize the inequality between the average outcome of each type of individual. In this case, program (4) is reformulated as follows:

$$\varphi^{MT2} = \arg\min_{\varphi \in \Phi} I\left(\frac{1}{Q}\sum_{\pi=1}^{Q} v^1(\pi;\varphi),\ldots,\frac{1}{Q}\sum_{\pi=1}^{Q} v^T(\pi;\varphi)\right). \qquad (6)$$

This paper generalizes the first two mechanisms. The last program is not considered because it does not allow (as in the Van de Gaer approach) the treatment of individual advantage by relative levels of effort.

Notice that Moreno-Ternero (2007) assumes that policies in Φ allocate the available resource *w* completely among individuals. In this manner, he rules out the policy that gives zero to every type, which might reach a higher (but undesirable) degree of equality. For the same reason, I make the same assumption throughout the paper.

Lefranc *et al.* (2006a) propose a mechanism to contrast equality of opportunity in a model that considers not only circumstances and effort, but also luck. They contrast *strict equality of opportunity* (outcome distributions conditional on effort are equal) and *weak equality of opportunity* (there are not unanimous preferences over the range of possible circumstances for all possible increasing utility functions that exhibit risk aversion) by using the first and second stochastic dominance criterions.[5] Unfortunately, as the authors acknowledge, this characterization of equality of opportunity does not allow us to rank situations in which equality of opportunity is rejected. To avoid this problem, they propose a new index to measure the degree of inequality of opportunity (see Lefranc *et al.*, 2006b). In what follows, we consider luck, in any form, as a circumstance that must be compensated.[6]

3. An intuitive version of the proposal

Here we provide a simple example with three different populations to show the relevance of the proposed method. Let us assume two types in each population and the same distribution of advantage within-type 1 for all of them (see Figures 1a, 1b and 1c).

Figure1a.

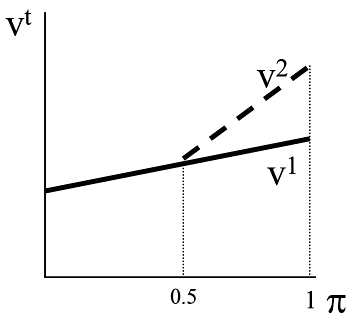


Figure 1b.

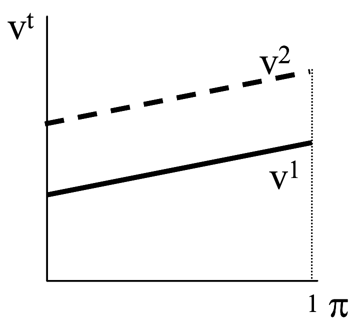
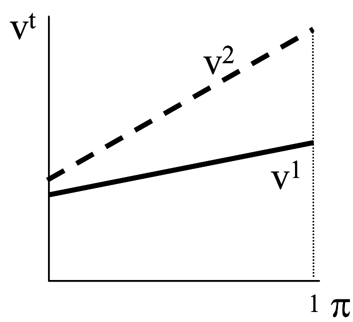


Figure 1c.



[5] Recall that second-order stochastic dominance is equivalent to generalized Lorenz dominance (Shorrocks, 1983).

[6] Dworkin (1981a and 1981b) claims that *option luck* (luck that is avoidable) should not be compensated. On the contrary, Fleurbaey (1995), and other authors, have provided arguments for full compensation for this type of luck. Moreover, libertarians, according to their strong form of self-ownership, consider that *genetic luck* must not be compensated (see, for example, Nozick, 1977).

It is easy to observe that the same equality-of-opportunity policy arises from these three populations, although they have a completely different distribution of advantage within-type 2. This happens whichever program in (3) or (4) is applied. This result is surprising. In the first population, there is equality of opportunity in the range [0, 0.5] as individuals in both types achieve the same amount of advantage in that range (see Figure 1a). However, they are taken into account in the policy program. This might provoke an inefficient allocation of resources because the recommended policy could give resources to people who do not suffer from inequality of opportunity. Worse still, resources transferred from the planner to people of the first type who are in the range [0, 0.5] would introduce inequality of opportunity where it did not exist before. Then, policy should be applied to tranches where inequality of opportunity exists, that is, to tranches where circumstances are not irrelevant. This intuitive result calls for a *focus axiom*: only the tranches where there is inequality of opportunity matter. This axiom is broadly used in poverty analysis where incomes of the nonpoor are irrelevant for the calculation of poverty (see, for example, the survey in Zheng, 1997). In the same way, we propose the use of this axiom in the equality-of-opportunity framework. Notice that Moreno-Ternero's (2007) approaches satisfy this axiom by construction. Thus, for instance, the program in (5) considers the inequality of outcomes across quantiles; therefore, tranches where individuals with different circumstances obtain the same outcome do not contribute to the total inequality. Notice, however, that the number of quantiles where there are not differences in outcomes is relevant to inequality-of-opportunity measurement.[7]

[7] In the poverty context, Seidl (1988) provides a version of the focus axiom, which says a poverty measure

Now, let us concentrate on a different problem. The programs in (3) and (5) give the same weight to every quantile; therefore, they just consider the average. A consequence of this is that the same aggregate statistic may be consistent with very different distributions of outcomes. For example, the outcome distributions in Figures 1b and 1c produce the same policy, although inequality of opportunity is distributed in a very different manner. Of course, if inequality of opportunity is homogenous over quantiles, the average is the appropriate statistic; however, this is not the generally observed case (see, for example, Figure 3 in Section 5).

Because equality-of-opportunity policies should allocate resources among all the tranches where there is inequality of opportunity, it is reasonable to adopt a policy based on the complete distribution. Of course, an approach like this will depend on the aggregation method that is applied. Furthermore, the aggregation method will imply a range of different judgments about how differentials are aggregated. Note, however, that the average procedure also implies a judgment of value: all tranches, regardless of the size of their inequality of opportunity, receive the same weight by the policy maker.

Let us order the tranches according to their inequality of opportunity, measured by an inequality index, in an inverse way, that is, from the largest to the lowest observed inequality of opportunity. In principle, three options are possible: a concave, a linear or a convex aggregation. In the concave case, the larger the inequality of opportunity in the tranche, the higher the weight it will receive. The linear case is the one established in the literature, where all tranches receive the same weight. In the convex case, the larger the inequality of opportunity in the tranche, the smaller the weight it will receive. In this

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is independent of the number of nonpoor.

paper, we propose the concave aggregation of tranches. In that case, the equality-of-opportunity policy would allocate resources giving preference to those who are worse off. This aggregation method does not depend on the effort distribution like, for example, the Fleurbaey (2002) proposal; therefore, the value judgments that the social planner would adopt are not about charity (see Roemer, 2003) but priority in the allocation of resources.

A new axiom, therefore, may be imposed: the *priority axiom*.[8] Once this axiom is assumed, the equality-of-opportunity policy is sensitive to the distribution of inequality of opportunity, across tranches. As a result, outcome distributions in Figures 1b and 1c are not considered alike anymore.

Another advantage of considering these two axioms is that an ordinal method for comparing distributions is applicable. We see in the next section that an inequality-of-opportunity curve can be constructed for making ordinal comparisons. This dominance criteria is an adaptation of the TIP's dominance criteria applied in the poverty literature (see Spencer and Fisher, 1992; Jenkins and Lambert, 1997 and 1998; Shorrocks, 1998; and Zheng, 2000).[9] Furthermore, in the case that inequality-of-opportunity curves cross, a complete ordering can be achieved by applying poverty indexes adapted to the equality-of-opportunity framework.

[8] Parfit (1997) defines the term *prioritarism* as the view that worse off individuals should be given priority over the better off with respect to resource allocation, but the former need not necessarily receive the extreme priority that characterizes maximin.

[9] The TIP curve is also called *cumulative poverty gap* curve or *poverty gap profile*.

4. The model

Consider the vector of outcome inequality between types across quantiles, $y = (I_1,..., I_Q)$ where Q is the number of quantiles, I (·) is an inequality index and $I_j = I(v^1(\pi_j;\varphi),...,v^T(\pi_j;\varphi))$ for all j =1,…, Q. Let $p(y)$ be the level of inequality of opportunity in distribution y for a given measure p (·). We now impose the focus axiom and the priority axiom[10] over the measure p (·).

<u>*Focus axiom*</u>: If $I_z = 0$ for any $z \in \{0,…, Q\}$, then $p(y) = p(y_{-z})$ where y_{-z} is the vector y without the element I_z.

This axiom states that quantiles with no inequality between types are not taken into account when measuring aggregated inequality of opportunity.

<u>*Priority axiom*</u>:[11] If $I_i > I_z$ for any i and $z \in \{1,…, Q\}$, then a transfer of an outcome that reduces inequality of opportunity in quantile i and increases inequality of opportunity in quantile z by the same amount, that is, $\nabla I_i = -\Delta I_z$, without any reranking between I_i and I_z, reduces $p(y)$.

The distribution of inequality of opportunity across quantiles is relevant. In particular, the larger the inequality of that distribution, the higher the inequality-of-opportunity index $p(\cdot)$.

[10] See Moreno-Ternero and Roemer (2006) for a different formal definition of the priority axiom.
[11] This formulation is an adaptation of the *weak transfer axiom* in the income inequality literature. It originally states that any income transfer between two persons, from the richer to the poorer without any reranking, reduces income inequality.

Moreover, if three common axioms like continuity, anonymity and monotonicity are also considered we will be able to develop a dominance criterion to rank distributions according to their inequality-of-opportunity level.[12] Next, we will propose a correspondence between these rankings and those obtained by using complete inequality of opportunity indexes that also satisfy these axioms.

Continuity axiom: $p(\cdot)$ must be a continuous function for any vector y of outcome inequality between types, across quantiles.

This axiom guarantees that small changes in the inequality within quantiles produce a small change in the level of inequality of opportunity.

Between-tranche anonymity axiom: $p(y) = p(\mathrm{A}\cdot y)$ where A is a permutation matrix.

This axiom guarantees that the inequality-of-opportunity index does not favor any particular quantile.

Weak monotonicity axiom: Let y_0 and y_1 be two vectors of outcome inequality between types across quantiles that only differ in their i element, $I_i^{y_0} > I_i^{y_1}$. Then, $p(y_0) > p(y_1)$.

An increase in the outcome inequality within a particular quantile implies a worsening of inequality of opportunity.

[12] In other frameworks, for example, poverty analysis (see Chakravarty, 1983, and Thon, 1983) and wage discrimination (see Jenkins, 1994, and Del Rio *et al.*, 2006), a *replication axiom* is also imposed. However, this axiom is not necessary in our case, because we consider the same number of quantiles for every distribution.

Now the inequality-of-opportunity curve will be defined so as to satisfy the above axioms.

Let y' be the vector y where its elements have been ranked from a higher to a lower inequality level, $I_1^{'} \geq I_2^{'} \geq ... \geq I_Q^{'}$. The inequality-of-opportunity curve represents, for each $0 \leq q \leq 1$, the sum of the first 100·q percent of $I_i^{'}$ values divided by the total number of quantiles, Q. In formal terms:

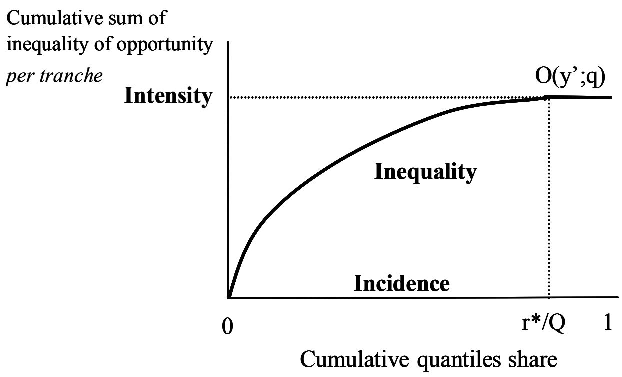
$$O(y';q) = \frac{1}{Q}\sum_{i=1}^{r} I_i^{'} \tag{7}$$

where $q = \frac{r}{Q}$ and r is any integer such that $r \leq Q$.[13] Actually, this curve is the Inverse Generalized Lorenz Curve (see Jenkins and Lambert, 1997 and 1998; and Shorrocks, 1998) defined for the values of outcome inequality between types across quantiles.

The inequality-of-opportunity curve in (7) is an increasing and concave function that takes the value zero at the origin and a constant value of $\frac{r^*}{Q}$, where r^* is the last tranche with positive outcome inequality between types (see Figure 2).

[13] If we assume π is a continuous variable and consider H to be the distribution of inequality between types across quantiles, we have: $O(H;q) = \int_{1-q}^{1} H^{-1}(q)dq, \quad q \in [0,1]$. See Shorrocks (1998) for a derivation of this expression in the deprivation literature.

Figure 2. Inequality-of-opportunity curve



This curve gives information about the incidence, intensity and dispersion of inequality of opportunity. The length of the nonhorizontal portion of the curve reveals the proportion of quantiles with positive inequality of opportunity, that is, the incidence of inequality of opportunity. The height of the curve informs us about the intensity of inequality of opportunity: the higher the curve, the larger the total inequality of opportunity. The curve is the horizontal axis if there is no inequality of opportunity in society. Finally, the degree of concavity of the curve shows the dispersion in the distribution of inequality of opportunity. In particular, the curve is the 45° line if each quantile experiences the same inequality of opportunity. Notice that the inequality-of-opportunity curve defined in (7) satisfies all the axioms above.

The dominance of one equality-of-opportunity curve over another is the criterion to rank distributions according to their inequality-of-opportunity level.

Assume two different distributions of outcome inequality between types across quantiles, $y_1^{'}$ and $y_2^{'}$. We say that distribution $y_1^{'}$ dominates distribution $y_2^{'}$ in inequality-of-opportunity terms, if $O(y_1^{'};q) \le O(y_2^{'};q) \quad \forall\, q \in [0,1]$. Let P be the set of inequality-of-opportunity indexes that satisfy the axioms above, then a useful result follows (see Marshall and Olkin, 1979; Jenkins and Lambert, 1993; and Shorrocks, 1993 and 1998).[14]

Result: Given outcome inequality (between types, across quantiles) distributions $y_1^{'}$ and $y_2^{'}$, dominance of $y_1^{'}$ over $y_2^{'}$ is necessary and sufficient to ensure $p(y_1^{'}) \le p(y_2^{'})$ for any $p(\cdot) \in P$.

That is, population A is better off than population B, according to all aggregate indexes of inequality of opportunity that satisfy the focus, priority, continuity, between-tranche anonymity and weak monotonicity axioms if and only if its inequality-of-opportunity curve goes below the inequality-of-opportunity curve for population B in the entire domain.

However, this ordinal criterion is not conclusive about whether inequality-of-opportunity curves cross. In this case, a complete ordering can be achieved by applying inequality-of-opportunity indexes that satisfy the axioms above (elements of *P*). This set of indexes is actually the set of poverty indexes that satisfy the axioms above, adapted to our inequality-of-opportunity framework, where we have vectors of outcome inequality.

Then, given the set of poverty indexes that satisfy the axioms above (see Zheng, 1997), we propose to adapt the Foster–Greer–Thorbecke family of poverty measures (see Foster *et al.*, 1984). This choice is based on its simplicity (see expression 8 below), its *subgroup*

[14] This result was used for wage discrimination measurement in Jenkins (1994) and Del Rio *et al.* (2006).

consistency (this family is consistent with a change in the inequality-of-opportunity level of a subgroup of quantiles) [15] and the fact that the first two mechanisms developed by Moreno-Ternero are obtained as particular cases. Thus, the proposed program is as follows:

$$\varphi_\gamma = \arg\min_{\varphi \in \Phi} \frac{1}{Q} \sum_{j=1}^{Q} (I_j)^\gamma \quad , \gamma \geq 0 \tag{8}$$

where $I_j = I\left(v^1\left(\pi_j;\varphi\right),...,v^T\left(\pi_j;\varphi\right)\right)$ and γ is a sensitivity parameter that represents the degree of priority in the inequality-of-opportunity policy: the higher the parameter value, the larger the weight that quantiles with larger outcome inequality will receive. This family of inequality-of-opportunity measures is the proportion of quantiles with positive inequality of opportunity, that is, the incidence of inequality of opportunity, when $\gamma = 0$. The linear case ($\gamma = 1$) is the first Moreno-Ternero mechanism. Finally, the quantiles with larger outcome inequality receive higher weights (priority axiom) when $\gamma > 1$. A consequence of the latter is that the minimization of maximum inequality throughout the different levels of relative effort (the second Moreno-Ternero mechanism) is applied when γ converges to infinite.

In the next section, we apply these proposals to a sample of countries.

[15] As a matter of fact, this family of measures verifies a stronger condition: additive decomposability (see Foster and Shorrocks, 1991).

5. Partial and complete orderings among countries

The dataset for this empirical exercise (see Table 1) is the data used in Roemer *et al.* (2003), which contain information on 11 countries: Belgium (1992), Denmark (1993), France (1994), Great Britain (1991), Italy (1993), The Netherlands (1995), Norway (1995), Spain (1991), Sweden (1991), the United States (1991) and West Germany (1994).[16,17] Samples refer to individuals who are male heads of household 25 to 40 years old (30 to 40 years old for Denmark, 25 to 50 years old for West Germany). Pretax income is calculated as the individual's labor income plus the household capital income divided by the number of adults in the household (calculations do not include self-employed income, nor capital income for Belgium). Posttax income is calculated as pretax income plus cash transfers from the government, minus income tax payments and social security contributions. Tax payments were simulated for all countries other than the Nordic economies. Moreover, the samples are partitioned into types based on individual parents' level of education for all countries except France and Great Britain where the occupational typology is used. Finally, the population of these countries is split into centiles, that is, $Q = 100$ (see Roemer, 1998).

First, we compute the mean income of each type at each quantile. We need to reduce income dispersion within types at each quantile into its mean income to keep separate inequality because of circumstances and inequality because of effort (see Ruiz-Castillo, 2003). Then, the Gini coefficient is applied to measure income inequality between types

[16] I am grateful to John Roemer and Nicolas Pistolesi for providing this dataset. The authors of this dataset were: I. Marx (Belgian data), T. Tranaes (Danish data), A. Lefranc and A. Trannoy (French data), S. Jenkins (British data), U. Colombino (Italian data), E. Pommer (Dutch data), R. Aaberge (Norwegian data), J. Ruiz-Castillo, M. J. San Segundo and I. Zubiri (Spanish data), J. Fritzell (Swedish data), M. Page and J. Roemer (US data) and G. G. Wagner (German data).

[17] East Germany was also included in Roemer *et al.* (2003). However, we disregard this data (following Lefranc *et al.*, 2006b) because West Germany and East Germany were very different economies in 1994.

at each quantile. Figure 3 shows inequality of opportunity for pretax and posttax incomes by tranches. The relevance of observing inequality of opportunity at each tranche becomes apparent. Inequality of opportunity in every country has a particular shape that calls for different policy responses. For example, inequality of opportunity in the USA and Spain is completely different. Inequality of opportunity in the USA is concentrated at the lower levels of relative effort while inequality of opportunity in Spain is concentrated at the last three deciles of relative effort. Thus, there is a significant problem regarding incentives in Spain that does not appear in the USA: incomes at high levels of relative effort depend on circumstances much more than incomes at low and middle levels of relative effort. As a consequence, the Spanish might be avoiding very high levels of relative effort because incomes there depend too much on circumstances.

Table 1. Country databases[18]

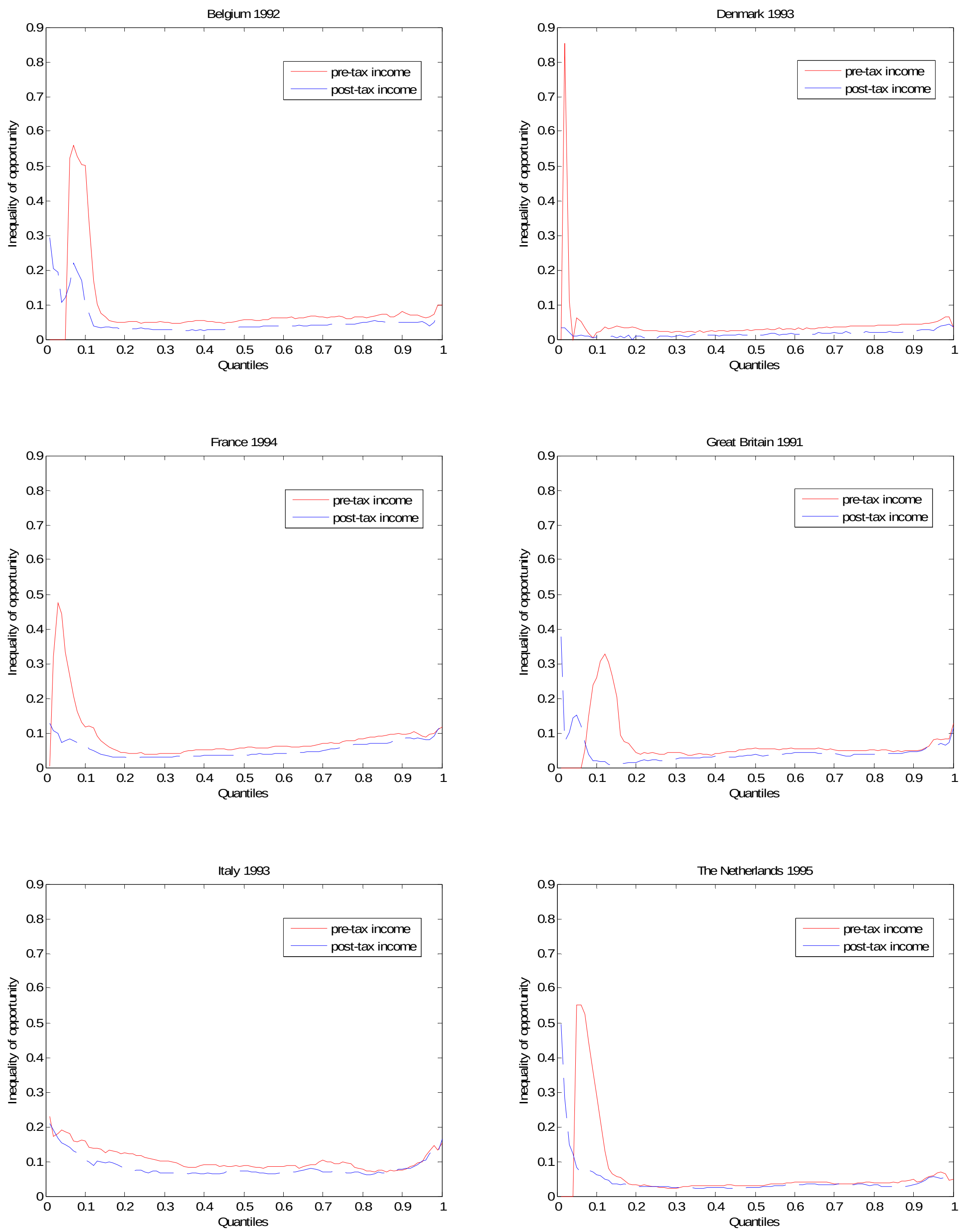
Country	Year	Database	Observations
Belgium	1992	Panel survey of Belgian households (PSBH)	933
Denmark	1993	Danish integrated database for labor market research with income registers (IDA-ISR)	32070
France	1994	French household survey (BdF)	2763
Great Britain	1991	British household panel survey (BHPS)	991
Italy	1993	Italian survey of household income and wealth (SHIW)	1435
The Netherlands	1995	Dutch facilities-use survey (AVO)	1758
Norway	1995	Norwegian survey of level of living (SLL)	576
Spain	1991	Spanish survey on class structure, social biography and class consciousness (ECBC)	1986
Sweden	1991	Swedish level of living survey (LNU)	823
USA	1991	Panel study of income dynamics (PSID)	1140
West Germany	1994	German socioeconomic panel (GSOEP)	1113

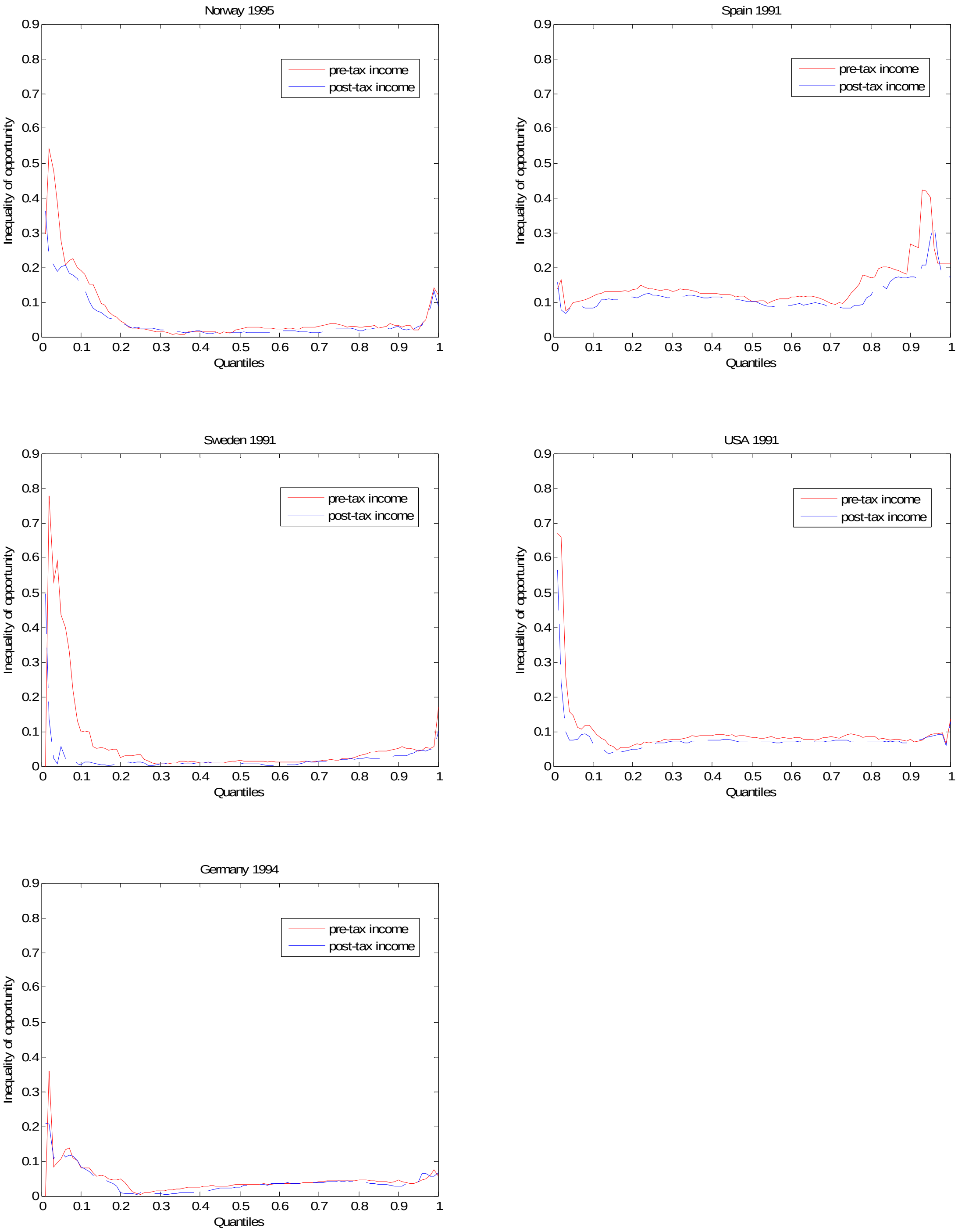
[18] We have not considered observations whose type is unknown.

Another relevant fact from Figure 3 is the different role of taxes. Here we do not estimate the extent to which each country's fiscal system equalizes opportunity for income as in Roemer *et al.* (2003); however, we can appreciate the difference between the inequality-of-opportunity distributions before and after taxes. We examine this issue empirically after the presentation of cardinal orderings among countries (see below). In general terms, posttax income experiences lower inequality of opportunity than pretax income. However, the equalization of inequality of opportunity is not homogenous among countries. We can distinguish between three groups of economies: first, Denmark and Sweden, where inequality of opportunity in terms of posttax income is much lower than in terms of pretax income. In this sense, Roemer *et al.* (2003) reported that these two economies had taxed more than equality of opportunity would have required; second, countries like West Germany, Spain, Italy and the USA, where differences in inequality of opportunity between pretax income and posttax income are small. Roemer *et al.* (2003) reported that Italy and the USA clearly had low effective tax systems. However, they considered West Germany as one of the economies that had been overtaxing; third, Belgium, France, The Netherlands, Great Britain and Norway, where differences in inequality of opportunity before and after taxes are intermediated. Two countries, The Netherlands and Norway, are classified as overtaxing economies in Roemer *et al.* (2003); however, The Netherlands is no longer overtaxing when IQ test results are used as a circumstance.[19]

[19] Roemer *et al.* (2003) used data on IQ tests taken during youth for four countries: Denmark, Sweden, The Netherlands and the USA.

Figure 3. Inequality of opportunity by tranches.





Partial orderings among countries are represented in Table 2 (pretax income) and Table 3 (posttax income). These partial orderings are based on the dominance relationship between inequality-of-opportunity curves (see Section 4). To contrast the dominance property we have applied the bootstrap method. Cowell and Flachaire (2007) find that bootstrap tests usually improve numerical performance. Moreover, with small sample sizes it could be better to use a bootstrap approach that guarantees a better level of approximation to the nominal confidence intervals (Davison and Hinkley, 2005). The procedure we have adopted is the following:

1. Compute confidence intervals for every inequality-of-opportunity curve according to the percentile method at the 0.05 level.[20] The number of replications (R) and quantiles are 1000 and 100, respectively.

2. If confidence intervals for two inequality-of-opportunity curves do not cross, the curve that is below at every tranche dominates the curve above. Otherwise (see Figure 4), significance tests are applied to assess the dominance hypotheses (step 3).

3. Given any two economies, A and B, the null hypothesis to be contrasted at every centile q is: $H_0 : O(q)^A - O(q)^B \geq 0$. This hypothesis is a composite hypothesis, which means that some aspects of the distribution remain unknown, for example, in our case the variance is unspecified. For this hypothesis, we can use the statistic $\hat{T} = \hat{O}(q)^A - \hat{O}(q)^B$ (see Efron and Tibshirani, 1993). However, the distribution of this statistic is unknown; therefore, we use the bootstrap distribution of $\hat{T}$ at every centile q. A clean solution for this problem is to choose Student's *t*-test for a normal difference of means with unknown variances (see Kendall and Stuart, 1968). Notice that the normality assumption is derived from the

[20] The advantages of the percentile method in comparison with other approaches are discussed by Gray *et al.* (2003) and Moran (2005).

normality of the asymptotic distribution for the estimate of the Gini index (see, among others, Cowell, 1989 and Bishop *et al.*, 1998) and equation (7).

Table 2. Partial ordering (pretax income)

	Dn	**Fr**	**GB**	**It**	**Nd**	**Nw**	**Sp**	**Sw**	**USA**	**WG**
Be	↑	X	↑	X	↑	↑	X	X	X	↑
Dn		X	X	X	X	X	X	X	X	X
Fr			↑	X	X	X	X	X	X	↑
GB				X	X	X	X	X	↓	X
It					X	X	↓	X	X	X
Nd						X	X	X	X	↑
Nw							X	X	X	↑
Sp								X	X	↑
Sw									X	↑
USA										↑

X: Inequality-of-opportunity curves cross; ↑: The column dominates the row; ↓: The row dominates the column.

Table 3. Partial ordering (posttax income)

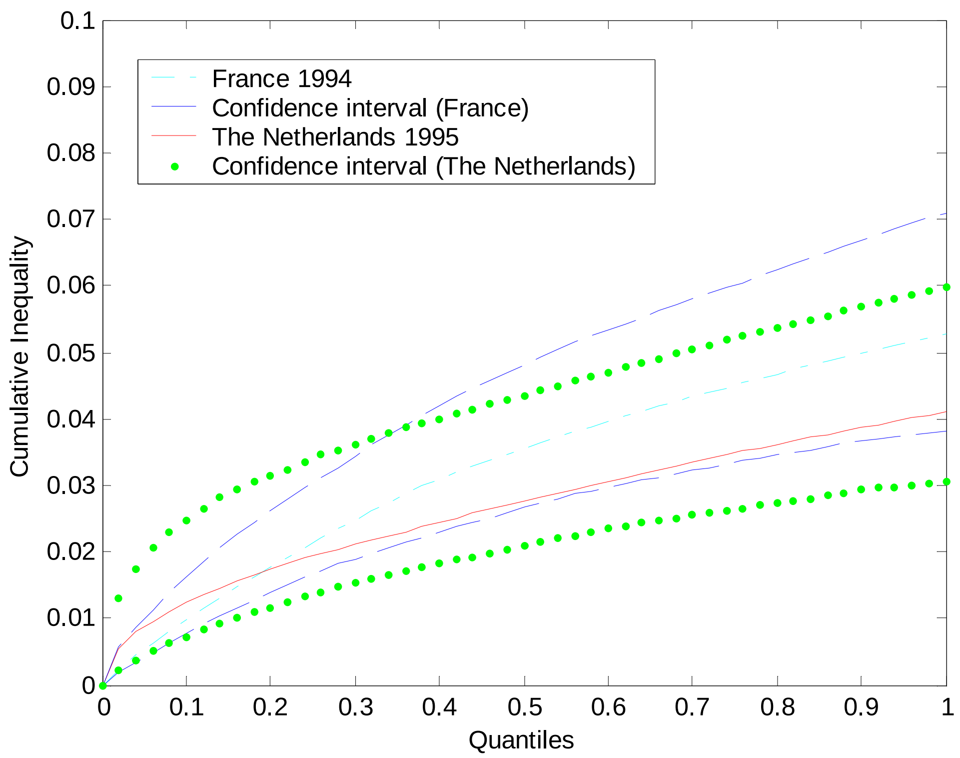
	Dn	**Fr**	**GB**	**It**	**Nd**	**Nw**	**Sp**	**Sw**	**USA**	**WG**
Be	↑	X	↑	X	X	X	↓	X	X	↑
Dn		↓	↓	↓	↓	↓	↓	↓	↓	↓
Fr			X	↓	X	X	↓	X	↓	X
GB				X	X	↓	↓	X	↓	↑
It					X	X	↓	X	X	↑
Nd						X	X	↑	↓	X
Nw							↓	X	X	↑
Sp								↑	X	↑
Sw									↓	X
USA										↑

X: Inequality-of-opportunity curves cross; ↑: The column dominates the row; ↓: The row dominates the column.

It is clear from Tables 2 and 3 that tax systems may be effective tools to reduce inequality of opportunity. Thus, Denmark dominates, in terms of posttax income, all other economies although it only dominates one economy (Belgium) according to pretax income. Meanwhile, West Germany does not change, or even marginally worsens its position among countries.

Both countries are the best positioned economies in the sample in terms of inequality of opportunity: West Germany in terms of pretax income and Denmark in terms of posttax income. On the contrary, Spain and the USA are the worst economies in the sample in terms of posttax income.

Figure 4. Inequality-of-opportunity curves (posttax income).



In Section 4 we said that a partial ordering is not conclusive if inequality-of-opportunity curves cross. This means that unanimity among inequality-of-opportunity indexes for all parameters of priority γ is not possible. For example, inequality of opportunity of posttax income in The Netherlands is lower than inequality of opportunity in France across the entire domain apart from the first centiles of effort. Their inequality-of-opportunity curves (and confidence intervals) cross at the beginning (see Figure 4). Therefore, if the degree of priority γ is large enough, The Netherlands will show higher inequality of opportunity than France because the former possess the highest inequality-of-opportunity level.

In Table 4 (pretax income) and Table 5 (posttax income) we present the values of the index in (8) for $\gamma = 1$, 2 and 3. Moreover, we provide the standard error estimates calculated by bootstrapping according to the formula: $\hat{\sigma}(\hat{I}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(I^* - \bar{I}^*\right)^2}$ (see, for example, Davison and Hinkley, 2005). In general terms, cardinal results reproduce the main conclusions of the ordinal measure. Thus, Denmark and Spain are the best and worst countries in the sample respectively in terms of inequality of opportunity. Moreover, the three groups of countries that we intuitively found in Figure 3 are better appreciated now: in Denmark and Sweden the decrease in inequality of opportunity (for $\gamma = 1$) is larger than 60 percent. Inequality of opportunity is reduced by 20 percent or less (for $\gamma = 1$) after tax in West Germany, Spain, Italy and the USA. Meanwhile, the change in inequality of opportunity is around 30 or 40 percent for $\gamma = 1$ in Belgium, France, The Netherlands, Great Britain and Norway. We see that as Roemer *et al.* (2003) conclude, fiscal policy may compete with other instruments for equalizing opportunities like education.

Table 4. Inequality of opportunity (pretax income)[a]

γ	Be	Dn	Fr	GB	I	Nd	Nw	Sp	Sw	USA	WG
1	84.0	40.9	84.6	65.4	105.2	64.4	62.8	145.9	60.6	96.4	45.0
	(14.1)	(12.7)	(12.5)	(9.7)	(11.3)	(10.9)	(9.0)	(13.5)	(7.8)	(11.8)	(7.2)
2	18.4	8.5	12.5	8.0	12.0	15.0	12.8	25.1	18.9	16.5	3.6
	(7.6)	(6.2)	(4.6)	(5.2)	(2.8)	(4.9)	(4.1)	(5.2)	(4.3)	(4.0)	(5.2)
3	7.8	6.3	3.3	1.7	1.5	6.6	4.4	5.4	10.3	6.7	0.6
	(5.4)	(4.7)	(2.3)	(3.5)	(0.7)	(2.8)	(2.4)	(2.0)	(3.0)	(2.3)	(4.2)

[a] Results are multiplied by 10^3. Standard deviations in parentheses.

Table 5. Inequality of opportunity (posttax income)[a]

γ	Be	Dn	Fr	GB	I	Nd	Nw	Sp	Sw	USA	WG
1	52.3	15.6	52.8	43.6	83.6	43.4	45.5	118.5	21.4	76.6	40.2
	(8.0)	(5.5)	(8.1)	(4.2)	(10.0)	(8.2)	(5.9)	(11.1)	(4.0)	(10.4)	(4.5)
2	4.9	0.3	3.3	3.6	7.8	4.9	5.8	15.8	3.1	8.7	2.9
	(2.2)	(1.0)	(1.2)	(1.2)	(2.1)	(2.6)	(1.6)	(3.2)	(2.1)	(2.7)	(0.9)
3	0.8	0	0.2	0.7	0.8	1.5	1.2	2.4	1.3	2.3	0.3
	(1.0)	(0.5)	(0.3)	(0.4)	(0.5)	(1.6)	(0.5)	(0.9)	(1.5)	(1.5)	(0.4)

[a] Results are multiplied by 10^3. Standard deviations in parentheses.

6. Concluding remarks and discussion

This paper suggests that equality-of-opportunity policies should allocate resources according to the degree of inequality of opportunity at each level of relative effort. The social planner would allocate resources giving preference to those who are worse off. If this proposal is adopted as a simple tool, the inequality-of-opportunity curve will allow us to make ordinal comparisons in terms of equality of opportunity. Nevertheless, those ordinal comparisons may not be conclusive. In that case, a cardinal program that depends on the degree of priority in the inequality-of-opportunity policy is presented. The Moreno-Ternero (2007) approach is obtained as a particular case.

In the empirical exercise, we found that West Germany is the best positioned economy in the sample in terms of pretax income. Meanwhile, Denmark dominates in terms of posttax income all other economies. On the contrary, Spain and the USA are the worst economies in the sample in terms of posttax income.

Thus far, we have not discussed the framework of this proposal. Roemer's methodology resorts to a welfare objective; however, the mechanism presented here focuses on the design of equal opportunity policies in a pure equity framework. This means that we are only concerned with equalizing the outcome of all individuals, across types, who expended a given degree of relative effort. Efficiency, therefore, is secondary. As Moreno-Ternero (2007) has pointed out this allow us to address relative deprivation that may make more sense under several scenarios (see, for example, Brown *et al.*, 2006). Furthermore, even if we are only concerned with equal opportunity policies that are efficient, the proposal in this paper is still useful because it allows us to make ordinal and complete comparisons between economies according to equality of opportunity.

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