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ON A MODEL REGARDING THE PRODUCT LIFE CYCLE

Alexandru ISAIC-MANIU
Academy of Economic Studies, Bucharest
Viorel Gh. VODĂ
„Gh. Mihoc – C. Iacob" Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy

Abstract. *In this paper we analyze a modified model for the so-called PLC – Product Life Cycle, the original model being proposed by N. Al. Pop (2000). This PLC is considered here from a reliability theory viewpoint: our modification lies in the fact that the PLC - curve is transformed in a probability density function which allows the application of statistical inferential procedures.*

Key words: average sales, PLC – Product Life Cycle, PLC – curve, Pop's model, reparametrization, pdf – probability density function.

1. Introduction

According to *Enciclopedia calitatii* (2005, p. 135), by „product life cycle" we understand the deployment of successive and interrelated phases regarding the accomplishment and use of a product, starting from the acquisition of raw materials and/or materials (components) necessary to the manufacturing process – including the exploitation of some natural resources – and finishing with the phase of post-utilization.

There are two main directions in approaching this concept: the first is given by the theories and practices of marketing and the second is imposed by the aspects regarding the viability of the product.

Another important fact is that we also have the concept and in the same time the activity called „life cycle analysis".

The concept/notion derived from the idea that a product generically represents a multitude of elements both concrete and abstract that define the claims of a consumer/user (also generically!) towards a certain product – regarding the effective characteristics (physical, of use) as well as regarding the expectations referring to use and, ulterior, to the satisfaction, the degree of „contentment" achieved after using that product.

Because of the fact that the economic context where we stand is supposed –at least theoretically – to be the one of a market economy, the demand for a product, its acceptance and evolution on this market tends to register, in a considerable degree, a predictable trend, called life cycle. This cycle consists of several phases, generally four or five – for example: the introduction (on the market), the growth, the maturity, the saturation and the decline. Some authors (Drăgan and Demetrescu, 1998) also take into consideration the phase of disappearance from the market of a product due to objective causes or to deliberate interventions of certain economic „actors". All these phases (the number is not important!) refer to the actual selling process.

The purpose of this life cycle analysis is precisely that of formulating a prognosis as realistic as possible regarding the possibilities and sales evolution of the product on a given market and in the same time it aims to identify the necessities and ways of supporting the product in surpassing all phases in its specific cycle.

Consequently, the investigation refers not only to the products of a company but also to the ones of the competition- case in which a complex comparative analysis is performed, that includes quantitative indicators regarding the sales volumes, profitability as well as the results of sociologic inquests/surveys regarding the opinions of consumers about a certain product.

Enciclopedia calitatii (mentioned above) considers that the life cycle analysis was also imposed by the extending in practicing the ecological marking in the EU (and not only): the famous „Grüne Punkt" – among others. The quantification of the impact of products over the environment is indeed necessary not only during their economic life but also in the phase of post-utilization when they are submitted to a process of cassation, becoming dump goods, preferably recyclable.

However, extracting the role of environment protection in this particular context contributes neither theoretically nor practically to the development and applying the cycle ideas respectively the life cycle analysis.

In the last few years a justified interest was observed worldwide, regarding the issue of pollution, but many times this real situation alarming even on medium term is speculated „extra-scientifically" by authorities from different countries by introducing supplementary taxes under the excuse of „environmental protection" (see, for example, the famous first matriculation tax introduced in Hungary, Romania and Poland; if Poland and Hungary had to quit this nonsense under the EU pressure, the Romanian government is hesitant in taking measures in practice, since trucks and buses are not registered as pollution sources, against all evidences).

Coming back, we can say that the modern econometric literature presents the different models - generally statistical ones - regarding the life cycle. The most frequent – at least for the first three sales phases (introduction, growth, maturity- somehow equivalent to a „limit" or a constant trend for a determined period of time of the sales volume) – is the logistic model or the Verhulst model (Boşcaiu et al., 2007). This model is seen as a chronological series (time series), the independent variable being the time (t – weeks, months, years, etc) and the dependent one (Y) representing the sales volume on time unit:

$$Y = \frac{A}{B + e^{-Ct}}, \quad t \geq 0, \quad A, B, C > 0 \tag{1}$$

where $e \approx 2{,}718$ (Euler); A, B, C are characterizing parameters that must be estimated from real data. We do not mention here the well-known theoretical problems concerning this subject (Johnson et al., 1994).

The present paper will study in a detailed manner the model proposed by professor N. Al. Pop (2000, pp. 204 - 210) through the transformation of function

Y= f(t) in a probability density, which allows the application of statistical inferential techniques on the parameters involved in the function.
We will also approach in this context, the issue of life cycle from the viability point of view, an aspect less developed by the specific literature.

2. Two points of view on the product life cycle: the supplier and the beneficiary

Any product meant to be consumed or utilized can be considered a link between the producer and its effective or potential user. But the two participants to the economic process have relatively distinct perspectives and viewpoints about the product and the way of interpreting the product life cycle.

Consequently, the producer has two approaches: the first refers to marketing aspects while the second to the actual production.

From the „commercial" point of view, the product life cycle is imagined or shaped by a curve that describes/represents the units of product sold in a period of time defined from the moment of entering the market until the moment of definitive disappearance from the market. It's about the four classical phases regarding sales: very slow growth, explosive growth, relative constancy in sales, decrease (slow or sudden).

From the point of view of the actual production, the product life cycle is equivalent to the time it spends from its conception (stages of project, of theoretical model, of idea) until its withdrawal from the market (and by „withdrawal" we do not exclusively refer to the basic meaning but also to the natural disappearance of the product). From this perspective, the life cycle can be divided into two large phases: the pre-launching and the launching.

The pre-launching consists of the activities that the producers develop **before** the introduction of the product on the market. These activities are:

a) conceiving the product (project, idea, model);
b) evaluating the product (characteristics, performances, costs);
c) the factual design;
d) creating the prototype (or the „zero series") and testing it;
e) the production.

The launch of the product on the market can be made by the producer through his own commercial system or by the organizations specialized in this activity (cash and carry and retail).

The post-launch phase also includes two important elements:

i) the study of the market reaction to the newly launched product;
ii) the after-sale service supply for the buyers.

The buyer/user understands the product life cycle as the period of time from the moment of acquisition until the moment of discarding the product, whether it has arrived at the end of its period of use or it is technically and/or esthetically damaged,

or even if the user is „tired" of using it and desires its replacement with another, similar from the point of view of the services offered but better in performances.
From this perspective, the product life cycle seems to have three phases:

1) the acquisition of the product;
2) its exploitation and maintenance operations, if the case may be;
3) the discarding and eventual replacement with a "new" one, when necessary or when the beneficiary wishes to quit using the product.

It is interesting to mention the fact that the American document SAE M - 110/1993 entitled „Reliability and Maintainability Guideline for Manufacturing and Equipment" (SAE = Society for Automotive Engineers, in USA) considers that the life cycle of a product from the viability point of view having five phases:

1) the concept of the product;
2) the project and development;
3) fabrication and installation;
4) exploitation and maintenance;
5) conversion, improvement or discarding.

By „conversion" the document refers to the reengineering of the product's initial functions into new ones (if it is possible and useful for the customer) and by „improvement" it refers to enhancing the initial performances or adding new functions – through technical interventions, directly in the process of product manufacturing.

3. A few specifications to Kotler's ideas

The famous econometrician – creator of the marketing school – Philip Kotler seems to have been among the first specialists that remarked the fact that the issue of the product life cycle must be approached through marketing theory means as well as through statistical instruments, specific to the reliability theory. Kotler notices the analogy between the processes of **economic rustiness** and physical degradation of a product, process that takes place as a consequence of its utilization.

In his monograph *Marketing Management* (1998) Kotler ties the concept of life cycle of a generic product (a merchandise, a service, even an idea – he writes) directly to the concept of economic rustiness of a product. Certainly, it is quite difficult to quantify the economic rustiness of an idea (idea – in the sense given by Plato) but we can talk about the economic effects of using „rusty" strategies in different domains (the strategies can be considered as collections of ideas).
Also, he makes the difference between the service duration – term frequently used by engineers, the life duration (preferred by demographers and actuaries) and the product life cycle used in econometrics.

Kotler insists that PLC defined on a specific period of time, must have as „interval limits" the moment of introduction on the market, respectively the moment of integral disappearance from the economic-commercial processes of that market.
This „disappearance" or „elimination" of a product from a certain market can take place through various manners and because of various reasons. Thus: 1) the product

gets „rusty" from the point of view of the performances offered (a new similar product appears, but with superior abilities, that inevitably draws the attention and interest of the clients especially if the price is acceptable for the target-buyers market); 2) the product is withdrawn from the market by the producer himself, as a consequence of discovering major flaws, that couldn't be tracked down in the quality inspection process (the so-called hidden flaws, that usually out-burst in the phase of using the product); 3) unfavorable economic circumstances appear that incapacitate the product from being bought by the potential clients in the specific market; 4) the phenomenon of extended pauperization appears, leading the segment of population in discussion to re-orient only to vital needs (food, clothing etc.) while the products that used to be „normal" now become a useless luxury and lie on the shelves.

And the list of motivations can continue. We make the suggestion of bringing some specifications to Kotler's considerations regarding PLC as following:

a) the life cycle DOES NOT refer to a generic product – for example the „no frost" refrigerator or the armor plate tank thicker than 50 centimeters – but to a certain type of refrigerator (the brand X manufactured by producer Y) and to a certain type of tank (brand W, produced by Z).
b) the life cycle DOES NOT refer to the existence of a producing company: it can disappear at a certain point, from various causes, or it can change its activity, while the product may still be on the market for a long period of time;
c) the life cycle DOES NOT regularly refer to the so-called "collector's items" that theoretically have ended their life cycles although they can continue to function, and some actually do: telephonic devices dating since Graham Bell (1874-1922), radio devices fabricated in the '20s (the 20^{th} century), mechanical sawing machines (with pedal) produced by I. Merrit SINGER (1811- 1875), lanterns „with dynamo", typing machines etc.

In his doctoral dissertation (2001), Emil Petrescu suggests a partition of the product life cycle phases, which he calls „mega phases", which refer to a product so-called „auto-carrying capacitated". By „auto-carrying capacity" we mean the ability of the object/system to assure, independently, a certain service, to the direct user. Thus, a vehicle, a truck, a hydraulic excavator with cup and so on, are „auto-carrying capacitated" products, or how Petrescu defines them – Integral finite products (IFP) – while a gearbox for example is merely a finite product, because it can not independently assure a service: normally, the gearbox can not be separated from the complex technical system through which and where it develops the functions that it was created for.

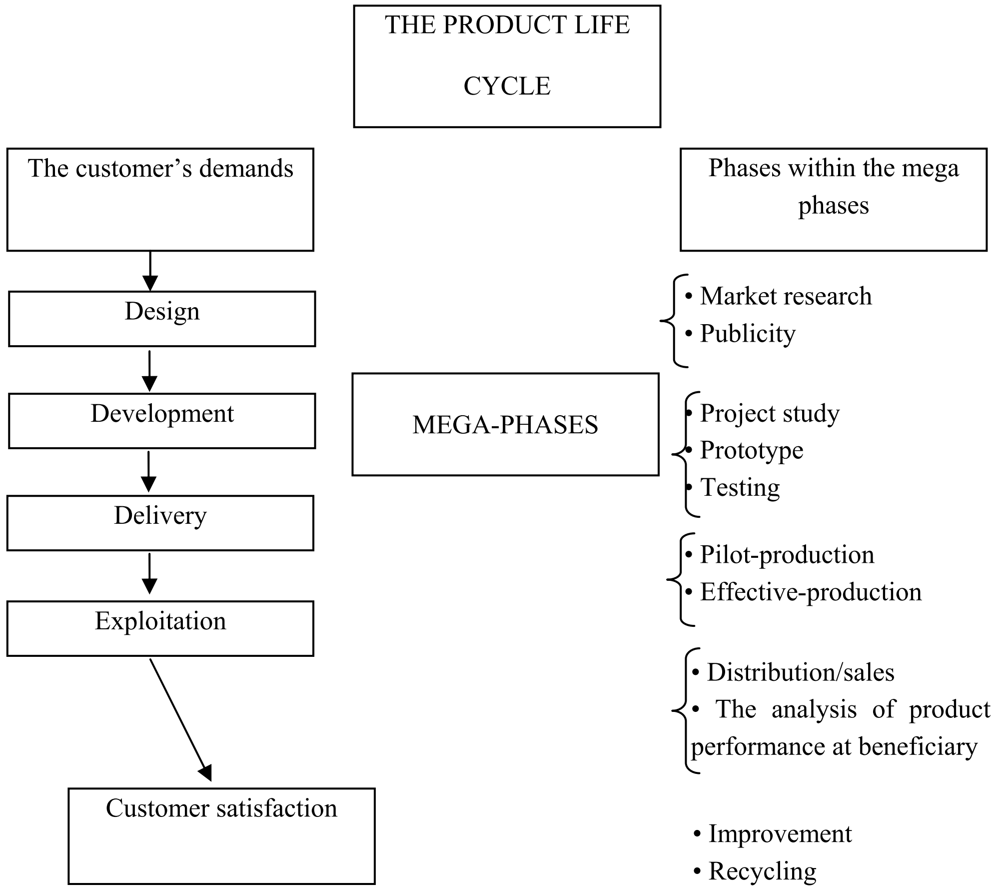

Figure 1. The main phases during the product life cycle (according to Emil Petrescu)

Of course, someone might say that a car battery can also be used independently, as electric energy source for lightning or in other purposes, but that is not the intended purpose of the object: its destination is precise: embodiment in the structure of a vehicle.

The history of technology (Stoichițoiu – Vodă, 2002) shows us that the idea of using components (finite products) destined to a certain purpose in a different structure and implicitly with a different mission, is not new, but it constitutes at least at the beginning, a curiosity. We also mention here, the case of the famous French inventor Jacques de VAUCANSON (1709-1782) who built a series of „automatic" toys with a watch mechanism (without „dwarfs inside" – as once thought!): these true masterpieces of the mechanism excellently imitated the real models (for example the famous „Vaucanson's duck" that quacked, pecked seeds, and processed them inside, and so on…).

Nowadays, the mechanical and electro-electronic toys industry has developed its own sub-providing base, so the „loans" of components initially destined to other purposes are no longer necessary.

4. N. Al. Pop's model: modifications

It is well known the fact that Verhulst's logistic model efficiently describes the first three phases (of the usual four) of PLC. Because of the fact that the curve is superior limited by a horizontal asymptote (Y=A/B when $t \to \infty$ - see formula (1)), it can no longer illustrate the decline phase, that involves a decreasing branch – eventually asymptotic towards the horizontal axis of the time. In order to oust this lack of the logistic curve, we have several possibilities, among which:

1) Considering a curve of type (1) truncated at the right, at a $t= t_0$ moment, situated at a convenient distance from the abscise of the inflection point of the Verhulst curve and then coupling the truncated curve to a branch decreasing towards the time axis – asymptotic, or even tangent in a t finite value; so, we would have a form of the following type: $Y = f(t;t_0) + g(t_0;t)$ with $g(t_0;t) \to 0$ when $t \to \infty$ or $g(t_0;t) = 0$ for $t = t_1 < \infty$ (here $f(t;t_0)$ is the logistic truncated at the $t= t_0$ abscise).

The procedure is difficult from the analytical point of view, because of the complicated problems that appear when estimating the parameters in the two components. We also mention the fact that the logistic can be linearized, in order to apply the ordinary least squares method, only in particular forms, for example $Y = A/(B + e^{-t})$ with $\frac{1}{Y} = \frac{A}{B} + \frac{1}{A}e^{-t}$, replacing $\frac{1}{Y} = U$, $\frac{B}{A} = a$, $e^{-t} = V$, $\frac{1}{A} = b$, obtaining the U = a + bV straight line, easy to apply on the minimization of the sum $S = \sum_{1}^{n} [U_i - (a + bV_i)]^2$ using the Gauss-Legendre-Markov method (ordinary least squares).

2) Choosing from the very beginning a curve able to approximately describe all four phases of PLC.

From the multitude of curves than can illustrate PLC, Professor Nicolae Al. Pop suggests for the study of the product life evolution – from the launching to disappearance – a function:

$$f(t) = K \cdot t^a \exp(-bt), \quad t \geq 0, \quad k, a, b > 0 \tag{2}$$

that depends on three parameters and can efficiently characterize the evolution. Unlike the logistic model, function (2) has a maximum, obtained from the equation

$$f'(t) = Kt^{a-1} \cdot e^{-bt} \cdot (a - bt) = 0 \quad \rightarrow \quad \boldsymbol{x}_{max} = a / b \tag{3}$$

that is, it has the maximum value given by the coordinates (a/b, f (a/b)), considered by econometricians **„the moment of saturation"**, when the sales volume is maximum.

The second derivative:

$$f''(t) = K \cdot \boldsymbol{x}^{a-2} \cdot e^{-bt} \cdot \left(b^2 t - 2abt + a^2 - a\right) \tag{4}$$

gives us through the equation $f''(t) = 0$ two inflection points symmetric to the saturation point, their abscissas being:

$$t_{1,2}^{(i)} = \left(a \pm \sqrt{a}\right) / b \tag{5}$$

These two points (moments) show us how the sales curve modifies its rhythm of evolution. From $t_1^{(i)}$ to t_{max} we have a maturity period (the growth takes place from 0 to $t_1^{(i)}$; from t_{max} to $t_2^{(i)}$ we have a decreasing phase, followed by the "agony period" when sales tend to zero, if te producer does not adequately intervene.

This intervention refers to a „resuscitation" of the product that is an activity of re-design, of improving the performances – in one word, re-engineering the product. The discussion upon the nature and role of the three parameters was made by N. Al. Pop (2000, p. 209) leads us to the idea of connecting the multiplying factor (k) to the other two parameters (a and b), as they all contribute to the shaping in the sales evolution of the product.

In a reparametrization of Pop's function was suggested in such a manner that it can become a probability density:

$$T : f(t; a, b) = \left(\frac{a}{b}\right)^a \cdot \frac{t^{a-1}}{\Gamma(a)} \cdot \exp(-at / b), \quad t \geq 0, \quad a, b > 0 \tag{6}$$

where

$$\Gamma(a) = \int_0^\infty u^{a-1} e^{-u} du \tag{7}$$

is the famous Gamma function of Leonhard EULER (1707-1783).

Indeed, the (b) form immediately verifies the conditions of a probability density:

$$f(t; a, b) \geq 0, \quad t \geq 0, \quad a, b > 0 \text{ and } \int_0^\infty f(t; a, b) = 1 \tag{8}$$

Professor Pop quoted the operation in a probabilistic context because its shape (2) can be linear after applying the logarithm:

$$\ln f = \ln k + a \ln t - bt, \quad t \geq 0, \quad a, b > 0 \tag{9}$$

and replacing Y= ln f, ln k = A, a = B, ln t = U, b = c and t = V we have the two-dimensional "regression" linear in U and V

$$Y = A + BU + CV \tag{10}$$

The estimation of the A, B, C parameters can also be easily done with the ordinary least squares method, a method also suggested by the author.

We observe that the form (6) can also easily be linear:

$$\ln f = a \ln a - a \ln b + (a-1)\ln t - \ln \Gamma(a) - at/b \tag{11}$$

but the appearance of the natural logarithm in the Gamma function somehow "complicates" the estimation process. However, considering the fact that f in this case has 0 density, we can apply the methods specific to the statistic inference – such as the moment estimation method, proposed by the British scientist Karl PEARSON (1857-1936).

It consists of the equalization of the theoretical moments (average and variance in the case of two parameters) with the empirical ones, calculated with variables $\{t_1, t_2, \ldots, t_n\}$, the average and variance of sample being given by the classical expressions:

$$\bar{t} = \frac{1}{n}\sum_{1}^{n} t_i, \quad s^2 = \frac{1}{n}\sum_{1}^{n}(t_i - \bar{t})^2 \tag{12}$$

(in practice, if n is sufficiently large, the s^2 indicator will be considered with the n-1 denominator; a discussion upon this motivation is made by Isaic-Maniu et al., 2003). In the case of (6), the average and variance of the T variable are:

$$E(T) = \int_0^\infty t \cdot f(t;a,b)dt = b \text{ and } Var(T) = \int_0^\infty [t - E(T)]^2 f(t;a,b) = b^2/a \tag{12 bis}$$

which allows the immediate estimation of b with $\bar{b} = \bar{t}$ and of a with $\hat{a} = \bar{t}^2/s^2$.

From the econometric point of view, the advantage in form (9) is given by the fact that **the average theoretical sales volume is equal to parameter b of the frequency curve f.**

We also mention the following observations:

(i) if a=1, as $\Gamma(1) = 1$ (see Dorin et al., 1994 [10] page 239), the form (9) becomes the exponential law $f(t;1,b) = (1/b)\exp(-t/b), \quad t \geq 0, \quad b > 0$; this case (a=1) would shape in our case only the decline period in the product life cycle. As the density f (t; 1, b) is initialized for t=0 at 1/b value, followed by a continuous decrease, asymptotically towards zero, it means that the product had just been launched and the sales already begin to decrease!

(ii) The $\bar{t}/s$ indicator that appears in expressing the parameter (a) is nothing else but the inverse of the variation coefficient known under the name of **signal-to-noise ratio**; this indicator is used in the measurement theory as a barometer of the closeness between an observed value of a random variable and its average.

In conclusion, the PLC shaping with the help of the statistical distribution theory can represent a fertile foundation in the econometric research.

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