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Liquidity, Assets and Business Cycles

By Shouyong Shi

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Shouyong Shi*
University of Toronto
(shouyong@chass.utoronto.ca)

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Abstract

Equity price is cyclical and often leads the business cycle by one or two quarters. These observations lead to the hypothesis that shocks to equity market liquidity are an independent source of the business cycle. In this paper I construct a model to evaluate this hypothesis. The model is easy for aggregation and for the construction of the recursive competitive equilibrium. After calibrating the model to the US data, I find that a negative liquidity shock in the equity market can generate large drops in investment and output but, contrary to what one may conjecture, the shock generates an equity price boom. This response of equity price occurs as long as a negative liquidity shock tightens firms' financing constraints on investment. Thus, liquidity shocks to the equity market cannot be the primary driving force of the business cycle. For equity price to fall as it typically does in a recession, a negative liquidity shock must be accompanied or caused by other shocks that reduce the need for investment sufficiently and relax firms' financing constraints on investment. I illustrate that a strong negative productivity shock is a good candidate of such concurrent shocks.

JEL classifications: E32; E5; G1
Keywords: Liquidity; Asset prices; Business cycle.

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1. Introduction

Asset prices move in the business cycle with other macro variables such as investment, output, and employment. In recent episodes, asset prices lead the business cycle. Figures 1.1 - 1.3 depict the time series of a broad stock price index and some macro variables in the US from 1999 to 2011, all of which are percentage deviations of the quarterly data from the trend.[1] It is clear from these graphs that stock prices fluctuate in a much larger scale than other variables. In fact, to bring the variations in the variables to roughly the same scale, I have multiplied the deviation of investment from its trend by 2 in Figure 1.1, the deviation of GDP from its trend by 10 in Figure 1.2, and the deviation of the bond price from its trend by 50 in Figure 1.3. It is also clear that investment and output move very closely with stock prices while the bond price moves in the opposite direction to stock prices. Moreover, stock prices lead the business cycle in the two recessions that occurred in the period. In the recent recession, stock prices peaked in the third quarter of 2007 before falling to the trough in the first quarter of 2009. Investment and output did not peak until the second quarter of 2008 and did not reach the trough until the second quarter of 2009. Similarly, in the 2000 recession, stock prices reached the peak and the trough about two quarters ahead of investment and output.

Figures 1.1 - 1.3 here.

These patterns of the business cycle can be consistent with the standard view that asset prices reflect expectations about the future health of the aggregate economy. It is possible that immediately after stock prices peaked in the first quarter of 2000 and the third quarter of 2007, asset market participants received news that the conditions of the aggregate economy would soon deteriorate rapidly. Such expectations could drive asset prices down, and the ensuing fall in investment and output could be just fulfilling the expectations. Although possible, this view is difficult to be checked with reliable evidence because prior to both recessions, there was wide dispersion in the forecast on the future health of the economy. It is difficult to believe that most market participants in the first quarter of 2000 and the third quarter of 2007 had forecast that the overall stock price would fall by nearly 57% in a short period of time.

[1] The stock price index is the Wilshire 5000 price full cap index (Wilshire Associates Incorporated, also available at the Federal Reserve Data Center). This is an index of the market value of all stocks actively traded in the US, weighted by market capitalization. The designation "full cap" signifies a float adjusted market capitalization that includes shares of stocks not considered available to ordinary investors. The data is available on the daily basis, but the series used here is the price of the last trading day in each quarter. Investment is real private nonresidential fixed investment and GDP is the real gross domestic product, both of which are available at the US Department of Commerce: Bureau of Economic Analysis. The bond price index is the price of the three-month Treasury bills at the secondary market rate, available from the Board of Governors of the Federal Reserve System. All variables depicted here are quarterly and filtered through the Hodrick-Prescott filter with a parameter 1600.

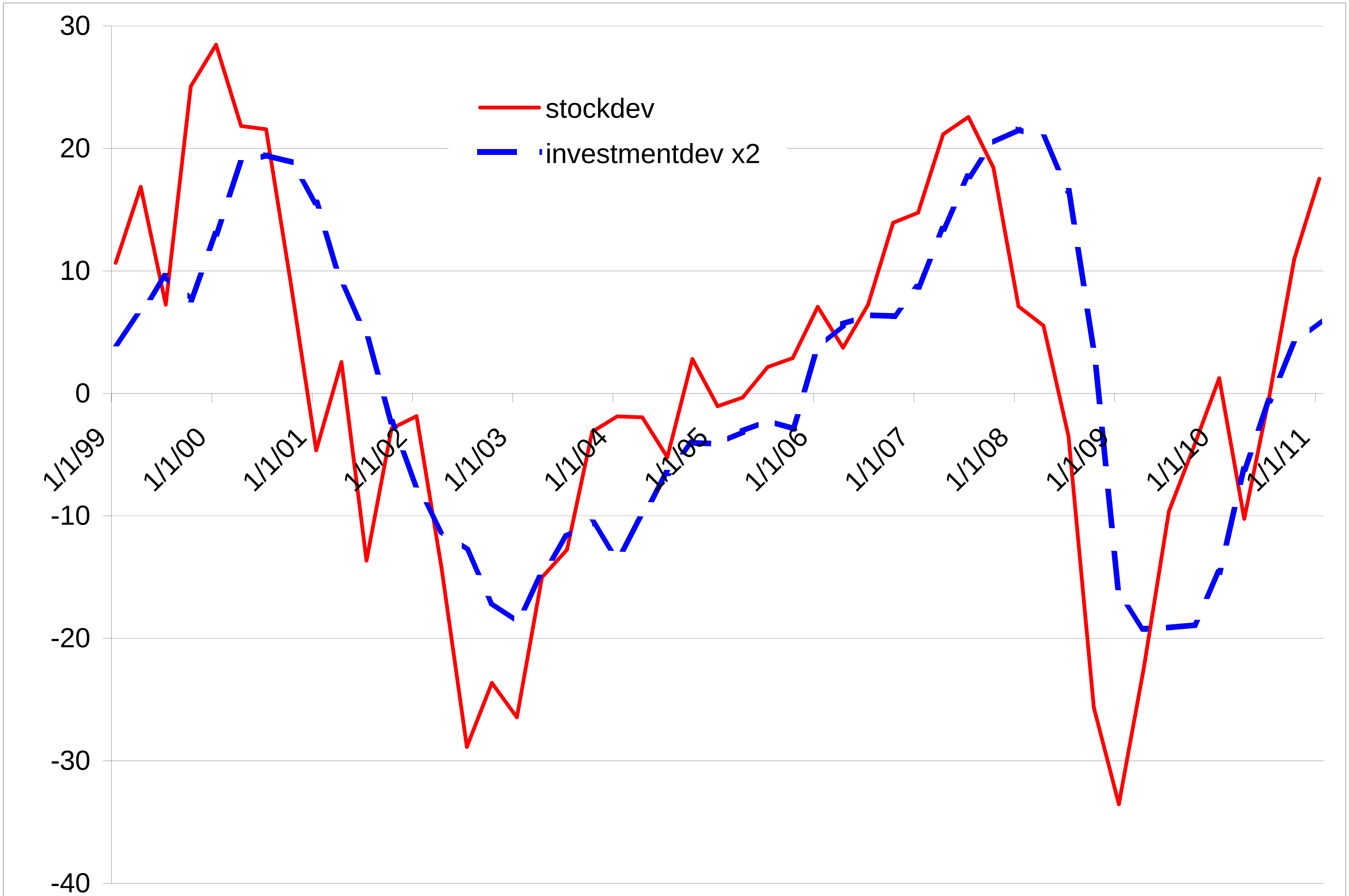

Figure 1.1. Deviation of stock price and investment from trend (%)

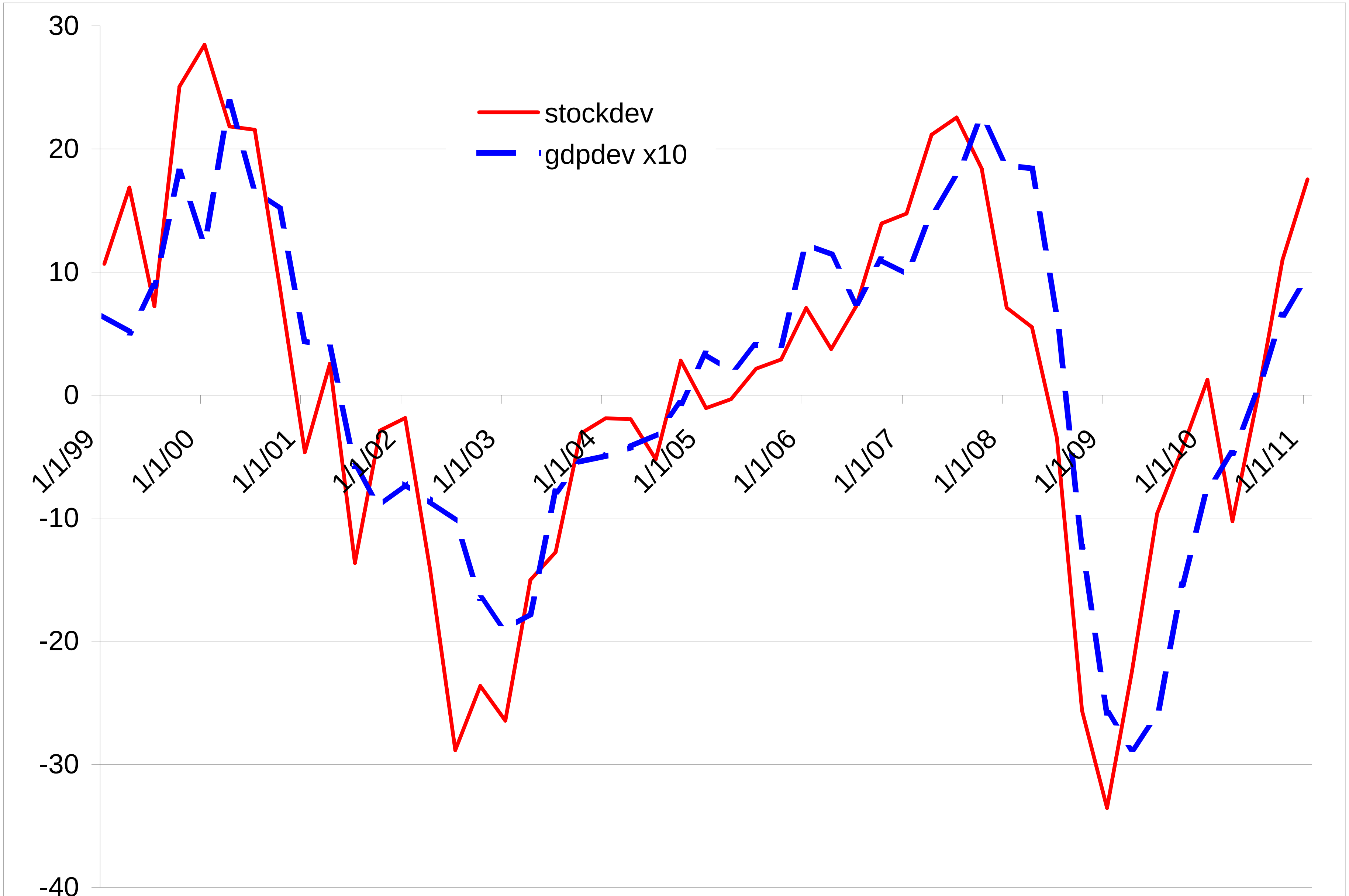

Figure 1.2. Deviation of stock price and GDP from trend (%)

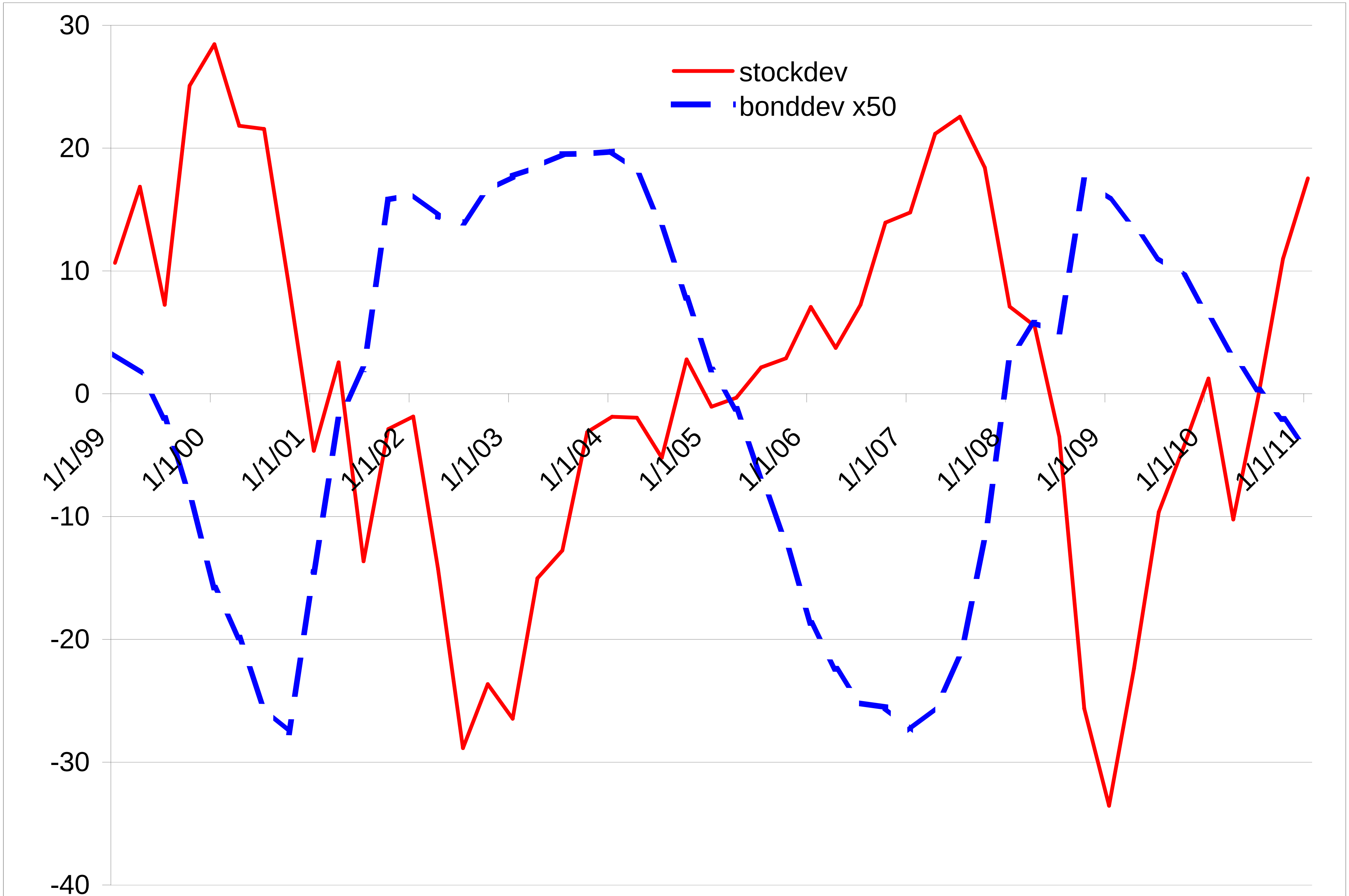

Figure 1.3. Deviation of stock price and bond price from trend (%)

A more intuitive explanation for the patterns depicted in Figures 1.1 - 1.3 is that the asset market is an important cause of, instead of a mere response to, the business cycle. Because the price of liquid assets such as Treasury bills increases in a recession and decreases in a boom, a natural hypothesis is as follows: There are shocks to the liquidity of the equity market that serve as an independent cause of the business cycle. In particular, sudden drops in the liquidity of the equity market cause equity price to fall. In a world where firms face financing constraints on investment, this reduction in equity price reduces firms' ability to finance investment through the equity market. Investment falls, output falls and a recession starts.

Kiyotaki and Moore (2008) have formulated this hypothesis (the *Kiyotaki-Moore hypothesis*, henceforth) with a model that places two equity-market frictions at the center. One is the difficulty to issue new equity: a firm can issue new equity to finance at most a fraction $\theta \in (0, 1)$ of investment. Another friction is the lack of resaleability of the existing equity; that is, only a fraction $\phi \in (0, 1)$ of the existing equity can be resold in any given period. Although both frictions are necessary for making the financing constraint on investment binding for firms in the equilibrium, Kiyotaki and Moore (2008) focus on the resaleability of the existing equity. This focus is justified by the fact that new equity is very small relative to the value of the existing equity. Modeling a negative shock to equity liquidity as an unexpected exogenous drop in equity resaleability, they argue that the shock has large and persistent effects on aggregate variables.

The Kiyotaki-Moore hypothesis has received wide attention because of its immediate policy implication. If unexpected fluctuations in equity liquidity are a cause of the business cycle, then a government should and can attenuate the business cycle by making the supply of liquid assets counter-cyclical. Specifically, at the onset of a recession, a government should use liquid assets to buy up some of the illiquid equity in order to prevent equity price from falling precipitously. The increase in the supply of liquid assets relaxes firms' financing constraints, and the stabilization of equity price improves firms' ability to use the equity market to finance investment. Both effects can reduce the magnitude of a recession. This policy implication seems to provide a justification for the large and repeated injections of liquidity by the US Federal Reserve System and some other central banks in the recent recession.

Given the intuitive appeal and the immediate policy implication of the Kiyotaki-Moore hypothesis, it is important to investigate formally and clearly how a liquidity shock works in a macro model. In their paper, Kiyotaki and Moore (2008) formulate the equilibrium as a sequence problem which makes it difficult to illustrate how the liquidity shock works in a stochastic and dynamic environment. Another issue is aggregation. For the financing constraint on investment to have force, the model must distinguish entrepreneurs, who undertake investment, from other individuals such as workers. To aggregate the decisions of different types of individuals, the model requires a special utility function (logarithmic) and, even in that case, aggregation is quite involved. A few authors have extended the model of Kiyotaki and Moore (2008) by adding the elements that are popular in a dynamic stochastic general equilibrium model, such as wage/price rigidity, adjustment costs in investment and habit persistence in consumption. After calibrating such models, Ajello (2010) find that liquidity shocks account for a large part of the US business cycle and Del Negro, et al. (2010) find that the US Fed policy might have prevented a greater recession in the last few years. The additional elements in these models are intended to be realistic for analyzing monetary policy, but they make it difficult to dissect the models to evaluate the Kiyotaki-Moore hypothesis.

In this paper, I try to accomplish two tasks. The first is to construct a theoretical model to simplify the formulation of the Kiyotaki-Moore hypothesis. This model provides straightforward aggregation and leads to a natural definition of the recursive competitive equilibrium in subsection 2.3. This tractable formulation may be useful broadly for studying the role of the asset market in macro. The second task is to calibrate the model to evaluate the Kiyotaki-Moore hypothesis quantitatively. By computing the dynamic response of the equilibrium to shocks, I reach some surprising results regarding the role of equity market liquidity in the business cycle, some of which are opposite to what one may conjecture. The transparent formulation in my paper allows me to explain these results in section 3 and suggest directions of future research in section 4.

Financial frictions have been the focus of business cycle research for quite some time. The literature is too large to be surveyed here (see Bernanke, et al., 1999, for a partial survey). A majority of this literature emphasizes the role of financial intermediation and the implied financial multiplier/accelerator of the business cycle. One particularly fruitful approach is the so-called costly state verification (e.g., Townsend, 1979). In this approach, financial intermediaries arise to economize on the cost of lending to and monitoring entrepreneurs in a world where entrepreneurs have private information on their projects' outcome. Williamson (1987) is one of the first to integrate this approach into a dynamic macro model to study the business cycle. The approach has gained momentum through influential papers such as Bernanke and Gertler (1989, 1990). A more recent example is Liu, et al. (2010). The main character in the play in this literature is net worth of entrepreneurs and/or financial intermediaries, whose pro-cyclical fluctuations generate the financial multiplier. This approach provides important insights into the role of financial intermediaries in the business cycle, but it does not necessarily require the equity market to play a central role. In contrast, the equity market is the central character of the Kiyotaki-Moore hypothesis. To evaluate this hypothesis, I abstract from financial intermediaries and, in fact, from debt-financing entirely as Kiyotaki and Moore (2008) do.[2]

2. A Macro Model with Asset Market Frictions

2.1. The model environment

Consider an infinite-horizon economy with discrete time. The economy is populated by a continuum of households, with measure one. Each household has a unit measure of members. At the beginning of each period, all members of a household are identical and share the household's assets. During the period, the members are separated from each other and each member receives a shock that determines the role of the member in the period. With probability $\pi \in (0,1)$, a member will be an entrepreneur and, with probability $1-\pi$, the member will be a worker in the period. These shocks are *iid* across the members and time. An entrepreneur has an investment project and no labor endowment, while a worker has one unit of labor endowment and no investment project. The members' preferences are aggregated and represented by the following utility function of the household:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi \; u\left(c_t^e\right) + (1-\pi)\left[U(c_t^w) - h(\ell_t)\right] \right\}.$$

Here, the expectation is taken over aggregate shocks (A, ϕ) which will be described below. The parameter $\beta \in (0,1)$ is the discount factor, c_t^e an entrepreneur's consumption, c_t^w a worker's consumption, and ℓ_t a worker's labor supply. The functions u, U and h are assumed to have standard properties. To maximize the above utility function, the household chooses the actions for all members, and the members simply implement the household's choices. In the presence of ex post heterogeneity among the individuals, this household structure allows me to easily aggregate individuals' quantities to construct macro aggregates.[3]

Let me describe the technologies in the economy together with the timing of events in an arbitrary period t. A period is divided into three stages: households' decisions, investment/production, and consumption. In the stage of households' decisions, all members of a household are together and they pool their assets. Aggregate shocks (A_t, ϕ_t) are realized.[4] The household holds (physical) capital k_t, equity claims s_t, and liquid assets b_t. Capital resides in the household and will be rented later to firms that produce consumption goods in the second stage. Equity consists of all of the household's claims on the household's own capital and other

[2]The focus on the equity market may bring up the question whether monetary policy should take asset price fluctuations into account. For the debate on this issue, see Gilchrist and Leahy (2002).

[3]A similar household structure has been used in monetary theory (e.g., Lucas, 1990, and Shi, 1997).

[4]This timing of aggregate shocks simplifies the analysis. If A_t and ϕ_t are realized in the second stage, instead, there may be precautionary holdings of assets.

households' capital.[5] Liquid assets include government bonds. Because all members of the household are identical in this stage, the household evenly divides the assets among the members. The household also gives each member the instructions on the choices in the period contingent on whether the member will be an entrepreneur or a worker in the second stage. For an entrepreneur, the household instructs him to consume an amount c_t^e, invest i_t, and hold a portfolio of equity and liquid assets (s_{t+1}^e, b_{t+1}^e) at the end of the period. For a worker, the household instructs him to consume an amount c_t^w, supply labor ℓ_t, and hold a portfolio of equity and liquid assets (s_{t+1}^w, b_{t+1}^w) at the end of the period. After receiving these instructions, the members go to the market and will remain separated from each other until the beginning of the next period.

At the beginning of the investment/production stage, each member receives the shock whose realization determines whether the individual is an entrepreneur or a worker. Competitive firms rent capital from the households and hire labor to produce consumption goods according to:

$$y_t = A_t F(k_t^d, \ell_t^d),$$

where the superscript d indicates the demand for productive factors and the function F has constant returns to scale. Total factor productivity A follows a Markov process. A worker supplies the amount ℓ_t of labor to the firms. An entrepreneur receives an investment project which uses consumption goods as the input to produce capital goods. To simplify, let me assume that each unit of the input produces one unit of capital good, and so an amount of investment i_t will increase the household's capital stock next period by i_t. In this stage, the asset market and the goods market are open. Individuals trade assets to finance new investments and to achieve the portfolios of equity and liquid assets instructed earlier by their households. Wage income and the rental income of capital are available to the individuals in this stage.[6]

In the consumption stage, a worker consumes the amount c_t^w and an entrepreneur the amount c_t^e. A fraction $(1-\sigma)$ of existing capital depreciates, where $\sigma \in (0,1)$, and the newly produced capital goods add to the capital stock. Then, individuals return to their households, arriving at the beginning of the next period.

There are two liquidity frictions that constrain an entrepreneur's ability to finance new investment, as modeled by Kiyotaki and Moore (2008). The first is a constraint on new equity: an entrepreneur can issue equity on only a fraction $\theta \in (0,1)$ of new investment. The second is a constraint on re-selling equity: an individual can re-sell only a fraction $\phi_t \in (0,1)$ of the existing

[5] As in Kiyotaki and Moore (2008), I assume that claims on the household's own capital and other households' capital have the same liquidity. This assumption simplifies the analysis because it implies that the two subsets of claims have the same price.

[6] I will discuss this assumption on timing at the end of subsection 3.3.

equity in his portfolio. One may be able to explicitly specify the impediments in the asset market that generate these bounds, θ and ϕ.[7] As a first pass, however, I take θ and ϕ as exogenous elements of the model as do Kiyotaki and Moore (2008). Let me refer to θ as the *equity-issuing bound* and ϕ as *equity resaleability.* Also following Kiyotaki and Moore (2008), I focus on equity resaleability by assuming that θ is constant and ϕ follows a Markov process. Shocks to ϕ are interpreted as shocks to equity liquidity.

The asset market frictions amount to putting a lower bound on the amount of equity that an entrepreneur must hold at the end of the period. Because of the equity-issuing bound, an entrepreneur must hold onto an amount $(1-\theta)i_t$ of the new capital to be formed by investment i_t. On the existing equity, s_t, the entrepreneur must hold onto at least the amount $(1-\phi_t)s_t$, and only a fraction σ of such equity survives depreciation. These new and old equities will both become the existing equity at the beginning of the next period. Thus, the amount of equity that the entrepreneur holds at the end of the period, s^e_{t+1}, must satisfy the following constraint:

$$s^e_{t+1} \geq (1-\theta)i_t + (1-\phi_t)\sigma s_t. \tag{2.1}$$

This is the liquidity constraint on equity.

For the liquidity constraint (2.1) to be effective, an entrepreneur must face a tight borrowing limit. I set this limit to zero. Note that the borrowing constraint is made explicitly by the timing of events in a period and, particularly, by temporary separation of the members from each other in a period. This separation ensures that a household cannot shift funds among the members in the investment/production stage. If a household could freely channel funds among the members in this stage, there would be a need to take some resource from workers and give it to entrepreneurs to finance investment. In this case, (2.1) would not be binding.

2.2. A household's decisions

I formulate a household's decisions with dynamic programming and the equilibrium as a recursive competitive equilibrium. To do so, let me specify the aggregate state of the economy and price functions. Consider an arbitrary period, suppress the time subscript t, and add the subscripts $+1$ to the variables in the next period. The aggregate state of the economy at the beginning of the period is (K, Z), where K is the stock of capital per household and $Z = (A, \phi)$ is the

[7] For example, if new investment differs in quality which is the entrepreneur's private information, then the entrepreneur may not be able to finance the investment entirely with equity. Also, if investment requires an enterpreneur's (non-contractible) labor input as well as the input of goods, then moral hazard on labor input may put an upper bound on θ (see Hart and Moore, 1994). The upper bound ϕ may be caused by the common characteristics of all equity that are unknown to the market.

realizations of exogenous shocks to total factor productivity and equity resaleability. I omit the amount of equity per household from the list of aggregate state variables because it is equal to the aggregate capital stock. Also, I fix the supply of liquid assets at $B \geq 0$ and omit it from the list of aggregate variables. Let equity price be $q(K, Z)$, the price of liquid assets $p_b(K, Z)$, the rental rate of capital $r(K, Z)$, and the wage rate $w(K, Z)$. These prices of assets and productive factors are expressed in terms of the consumption good, which is the numeraire in the model.[8]

A household's state variables consist of equity claims, s, and liquid assets, b, in addition to the aggregate state. Denote the household's value function as $v(s, b; K, Z)$. Recall that a household's choices are as follows. For an entrepreneur, the household instructs him to invest i, consume c^e, and hold a portfolio (s^e_{+1}, b^e_{+1}) at the end of the period. The subscripts $+1$ are used here because the member will hold onto the portfolio until the beginning of the next period. For a worker, the household instructs him to supply labor ℓ, consume c^w, and hold a portfolio (s^w_{+1}, b^w_{+1}) at the end of the period. The household's choices $(i, c^e, s^e_{+1}, b^e_{+1}, \ell, c^w, s^w_{+1}, b^w_{+1})$ solve:

$$v(s, b; K, Z) = \max \ \{\pi \ u(c^e) + (1-\pi)\left[U(c^w) - h(\ell)\right] + \beta \ \mathbb{E}v(s_{+1}, b_{+1}; K_{+1}, Z_{+1})\} \tag{2.2}$$

subject to (2.1) and the following constraints:

$$rs + q(i + \sigma s - s^e_{+1}) + (b - p_b b^e_{+1}) - \tau \geq i + c^e, \tag{2.3}$$

$$rs + w\ell + q(\sigma s - s^w_{+1}) + (b - p_b b^w_{+1}) - \tau \geq c^w, \tag{2.4}$$

$$i \geq 0, \ c^e \geq 0, \ s^e_{+1} \geq 0, \ b^e_{+1} \geq 0, \tag{2.5}$$

$$c^w \geq 0, \ s^w_{+1} \geq 0, \ b^w_{+1} \geq 0, \tag{2.6}$$

$$s_{+1} = \pi s^e_{+1} + (1-\pi)s^w_{+1}, \quad b_{+1} = \pi b^e_{+1} + (1-\pi)b^w_{+1}. \tag{2.7}$$

The expectation in the objective function is taken over next period's aggregate state (K_{+1}, Z_{+1}). I have suppressed the arguments of price functions r, w, q and p_b in the constraints above.

Constraint (2.3) is an entrepreneur's resource constraint. The entrepreneur needs to finance consumption, c^e, investment, i, and the tax liability, τ. The funds include income from equity, rs, the receipt from selling the existing equity and issuing new equity, $q(i + \sigma s - s^e_{+1})$, and the receipt from adjusting the holdings of liquid assets, $(b - p_b b^e_{+1})$. Note that the liquidity constraint (2.1) ensures that the receipt from equity is strictly positive, which prevents the entrepreneur from going short on equity. Also note that I model liquid assets in a way similar to bonds; that is, each unit of matured liquid asset can be sold for one unit of consumption good while new

[8] As is standard, the price of equity is the so-called post-dividend price; i.e., it is measured after the rental income of capital is distributed to shareholders.

liquid assets are sold at price p_b. Thus, liquid assets have a positive net rate of return if and only if $p_b < 1$. Constraint (2.4) is an analogous constraint on a worker's resource, except that a worker receives labor income and does not have an investment project.

The non-negativity constraints in (2.5) ensure that an entrepreneur cannot borrow, and the constraints in (2.6) are similar constraints on a worker. The conditions in (2.7) are adding-up constraints on the members' holdings of equity and liquid assets at the end of the period.

To emphasize the choices for the entrepreneurs, I reformulate the household's decisions as the choices of $(i, c^e, s^e_{+1}, b^e_{+1})$ and $(\ell, c, s_{+1}, b_{+1})$, where c is the average consumption level per member in the household defined as

$$c = \pi c^e + (1 - \pi) c^w. \tag{2.8}$$

Using (2.7) and (2.8), I can replace the choices for a worker, $(c^w, s^w_{+1}, b^w_{+1})$, with functions of $(c^e, s^e_{+1}, b^e_{+1})$ and (c, s_{+1}, b_{+1}). In particular, I can replace the resource constraint on a worker, (2.4), with a resource constraint on the household. Multiplying (2.3) by π and (2.4) by $1 - \pi$, and adding up, I obtain the household's resource constraint as

$$rs + (1 - \pi) w\ell + (q - 1)\pi i + q(\sigma s - s_{+1}) + (b - p_b b_{+1}) - \tau \geq c. \tag{2.9}$$

Similarly, the non-negativity constraints in (2.6) can be expressed in terms of $(c^e, s^e_{+1}, b^e_{+1})$ and (c, s_{+1}, b_{+1}). This new formulation of the household's decision problem has already incorporated the constraints in (2.7). So, the constraints are (2.1), (2.3), (2.5), (2.6) and (2.9).

It is instructive to combine the liquidity constraint, (2.1), and an entrepreneur's resource constraint, (2.3), to eliminate s^e_{+1}. Doing so yields

$$(r + \phi\sigma q)s + (b - p_b b^e_{+1}) - \tau \geq c^e + (1 - \theta q) i. \tag{2.10}$$

This constraint reveals two features of an entrepreneur's financing constraint. First, the resaleability of equity increases an entrepreneur's ability to finance investment. Second, the cost of each unit of investment to an entrepreneur is $1 - \theta q$, which can be interpreted as the "downpayment" on the investment. This interpretation is valid because the entrepreneur can issue θ shares of new equity on each unit of investment to raise the amount of fund θq. The remainder of the required fund, $1 - \theta q$, has to come from the rental income and selling assets.

Note that an entrepreneur's resource constraint (2.3) holds with equality, because the marginal utility of consumption is strictly positive. Thus, an entrepreneur's liquidity constraint (2.3) is binding if and only if the combined constraint (2.10) is binding. For this reason, I refer to (2.10) as the *equity liquidity constraint*. Let $\lambda^e \pi U'(c^w)$ be the Lagrangian multiplier of the entrepreneur's constraint, (2.10), where the rescaling by $\pi U'(c^w)$ simplifies various expressions below. The multiplier λ^e is a measure of liquidity services provided by the resource to an entrepreneur.

It is straightforward to use the first-order condition of c to verify that the shadow price of the household's resource constraint, (2.9), is equal to $U'(c^w)$.[9] Also, if an entrepreneur faces a binding liquidity constraint, then it is optimal for him to spend the resource to finance new investment rather than acquire new liquid assets. This implies $\lambda^e b^e_{+1} = 0$. Moreover, the the optimal choices of (ℓ, c^e, i) yield the following conditions:

$$\frac{h'(\ell)}{U'(c^w)} = w, \tag{2.11}$$

$$u'(c^e) = U'(c^w)\left(1 + \lambda^e\right), \tag{2.12}$$

$$q - 1 \leq (1 - \theta q)\lambda^e \text{ and } i \geq 0, \tag{2.13}$$

where the two inequalities in (2.13) hold with complementary slackness. Condition (2.11) is the standard condition for optimal labor supply. Condition (2.12) captures the fact that a marginal unit of the resource is more valuable to an entrepreneur than to a worker if an entrepreneur's liquidity constraint is binding, in which case the additional value to an entrepreneur is captured by $\lambda^e U'(c^w)$. The conditions in (2.13) characterize the optimal choice of investment. As explained above, the downpayment on each unit of investment in terms of goods is $1 - \theta q$, the cost of which in terms of utility is $(1 - \theta q)\lambda^e U'(c^w)$. For the household, a unit of investment increases the resource by $(q - 1)$, the benefit of which in terms of utility is $(q - 1)U'(c^w)$. Investment is zero if the cost exceeds the benefit, and positive if the cost is equal to the benefit. The liquidity constraint is binding, i.e., $\lambda^e > 0$, if and only if $1 < q < 1/\theta$.

Finally, the optimality conditions on asset holdings at the end of the period and the envelope conditions on asset holdings together give rise to the asset-pricing equations below:

$$q = \beta\, \mathbb{E}\left\{\frac{U'(c^w_{+1})}{U'(c^w)}\left[r_{+1} + \sigma q_{+1} + \pi\lambda^e_{+1}\left(r_{+1} + \phi_{+1}\sigma q_{+1}\right)\right]\right\}, \tag{2.14}$$

$$p_b = \beta\, \mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1 + \pi\lambda^e_{+1}\right)\right]. \tag{2.15}$$

The shadow price λ^e_{+1} enters the right-hand sides of both pricing equations because the existing equity and liquid assets can both be sold to raise funds for new investment, thereby relaxing the liquidity constraint on an entrepreneur. However, only a fraction ϕ_{+1} of the existing equity can be sold next period while all liquid assets can be sold. Thus, ϕ_{+1} appears in the pricing equation for equity but not in that for liquid assets.

[9] The non-negativity constraints $c^e \geq 0$, $c^w \geq 0$, $b_{+1} \geq 0$ and $s_{+1} \geq 0$ do not bind. The constraint $s^e_{+1} \geq 0$ is not binding under (2.1), because the latter constraint imposes a strictly positive lower bound on s^e_{+1}.

2.3. Definition of a recursive equilibrium

Before defining an equilibrium, let me specify government policies. As said earlier, the supply of liquid assets per household is fixed as $B \geq 0$. Let me also assume that government spending per household is a constant g. The government's budget constraint is

$$g = \tau + (p_b - 1)B. \tag{2.16}$$

Let $\mathcal{K} \subset \mathbb{R}_+$ be a compact set which contains all possible values of K and $\mathcal{Z} \subset \mathbb{R}_+ \times [0,1]$ a compact set which contains all possible values of Z. Let $\mathcal{C}_1$ be the set containing all continuous functions that map $\mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$, $\mathcal{C}_2$ the set containing all continuous functions that map $\mathcal{K} \times [0, B] \times \mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$ and $\mathcal{C}_3$ the set containing all continuous functions that map $\mathcal{K} \times [0, B] \times \mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}$.

A *recursive competitive equilibrium* consists of asset and factor price functions (q, p_b, r, w) belonging in $\mathcal{C}_1$, a household's policy functions $(i, c^e, s^e_{+1}, b^e_{+1}, \ell, c, s_{+1}, b_{+1})$ belonging in $\mathcal{C}_2$, the value function $v \in \mathcal{C}_3$, the demand for factors by final-goods producers, (k^d, ℓ^d), and the law of motion of the aggregate capital stock such that the following requirements are met:

(i) Given price functions and the aggregate state, a household's value and policy functions solve a household's optimization problem in (2.2);

(ii) Given price functions and the aggregate state, factor demands satisfy the optimality conditions, $r = AF'_1(k^d, \ell^d)$ and $w = AF'_2(k^d, \ell^d)$, where the subscript 1 indicates that the derivative is taken with respect to the first argument and the subscript 2 the second argument;

(iii) Given the law of motion of the aggregate state, prices clear the markets:

$$\text{goods:} \quad c(s, d; K, Z) + \pi i(s, d; K, Z) + g = AF(k^d, \ell^d), \tag{2.17}$$

$$\text{labor:} \quad \ell^d = (1 - \pi)\ell(s, d; K, Z), \tag{2.18}$$

$$\text{capital:} \quad k^d = K = s, \tag{2.19}$$

$$\text{liquid assets:} \quad b_{+1}(s, b; K, Z) = b \equiv B, \tag{2.20}$$

$$\text{equity:} \quad s_{+1}(s, b; K, Z) = \sigma s + \pi \ i(s, b; K, Z); \tag{2.21}$$

(iv) The law of motion of the aggregate capital stock is consistent with the aggregation of individual households' choices:

$$K_{+1} = \sigma K + \pi \ i(K, B; K, Z). \tag{2.22}$$

Requirements (i) and (ii) are self-explanatory. In requirement (iii), the market clearing conditions for goods, labor, capital and liquid assets are easy to understand after taking into account that only the entrepreneurs invest and only workers supply labor. The equality $K = s$ in the capital market clearing condition states the fact that the households in the economy own all capital in the form of equity. In the equity market clearing condition, the amount of new investment that adds to the existing stock of equity includes not only new equity sold to other households but also new equity that a household retains. This is so because I define s to include the part of equity that a household holds on its own capital. Condition (iv) is also self-explanatory. I impose this requirement explicitly here because it is used by the households to compute the expectations in (2.2). However, because $K = s$, the law of motion of the capital stock duplicates the equity market clearing condition – a reflection of the Walras' law.

Determining an equilibrium amounts to solving for asset price functions $q(K, Z)$ and $p_b(K, Z)$. Once these functions are determined, other equilibrium functions can be recovered from a household's first-order conditions, the Bellman equation in (2.2), the market clearing conditions and factor demand conditions. To solve for asset price functions, I can use the right-hand sides of asset pricing equations, (2.14) and (2.15), to construct a mapping T that maps a pair of functions in $\mathcal{C}_1$ back into $\mathcal{C}_1$ (see Appendix A). The pair of functions (q, p_b) in an equilibrium is a fixed point of T. I will implement this procedure numerically in subsection 3.1.

2.4. Value of liquidity and the equity premium

Liquidity has a positive value in this model only if an entrepreneur's equity liquidity constraint, (2.10), is binding. Precisely, the *value of liquidity* in the aggregate state (K, Z) can be measured as $\pi\lambda^e(K, Z)$ units of consumption, where $\lambda^e \pi U'(c^w)$ is the shadow price of (2.10). Note that the price of liquid assets in a standard economy without the liquidity constraint is $\beta \mathbb{E}\frac{U'(c^w_{+1})}{U'(c^w)}$. According to (2.15), liquid assets in the current model have a higher price than the standard one if and only if liquidity is expected to have a positive value in the next period.

While a liquid asset provides full liquidity, equity provides only partial liquidity. By assumption, the rental income generated by the existing equity, i.e., the one associated with r, can be used to finance new investment. So can the fraction ϕ of the equity itself. Thus, an intuitive measure of the value of liquidity provided by a unit of the existing equity is $(\pi\lambda^e)\frac{r+\phi\sigma q}{r+\sigma q}$. The value of the additional liquidity generated by liquid assets relative to the existing equity is:

$$\Delta \equiv \pi\lambda^e \left[1 - \frac{r + \phi\sigma q}{r + \sigma q}\right] = \pi\lambda^e \frac{(1-\phi)\sigma q}{r + \sigma q}. \tag{2.23}$$

Intuitively, this additional value is strictly positive if and only if liquidity has a positive value (i.e., if $\lambda^e > 0$) and if the existing equity is not fully liquid (i.e., if $\phi < 1$).

The measure Δ is in terms of consumption units. An alternative way to measure the lower liquidity of equity is the *equity premium*, i.e., the additional rate of return that equity must provide relative to liquid assets in order to compensate for its lower liquidity. The gross rate of return to equity, not counting its role in relaxing the liquidity constraint, will be $(r_{+1} + \sigma q_{+1})/q$, and the gross rate of return to liquid assets will be $1/p_b$. The realized equity premium in the next period will be $(\frac{r_{+1}+\sigma q_{+1}}{q} - \frac{1}{p_b})$. The equity premium is closely related to the measure Δ. The asset-pricing equations (2.14) and (2.15) imply that the equity premium is strictly positive on average if and only if the value of the additional liquidity provided by liquid assets relative to the existing equity is positive on average. Precisely, the following inequality holds

$$\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1+\pi\lambda^e_{+1}\right)\left(\frac{r_{+1}+\sigma q_{+1}}{q} - \frac{1}{p_b}\right)\right] > 0$$

if and only if

$$\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(\frac{r_{+1}+\sigma q_{+1}}{q}\right)\Delta_{+1}\right] > 0.$$

2.5. Non-stochastic steady state

In the non-stochastic steady state, the exogenous state is constant at $Z = Z^*$, where $Z^* = (A^*, \phi^*)$, and all endogenous variables are constant over time. In the steady state, investment is strictly positive because it is equal to $i^* = (1-\sigma)K^*/\pi > 0$. By (2.13), the shadow price of the equity liquidity constraint in the steady state is

$$\lambda^{e*} = \frac{q^* - 1}{1 - \theta q^*}. \tag{2.24}$$

The asset-pricing equations, (2.14) and (2.15), yield the following steady-state relations:

$$\lambda^{e*} = \frac{(\beta^{-1} - \sigma)q^* - r^*}{\pi(r^* + \phi^*\sigma q^*)}, \tag{2.25}$$

$$p_b^* = \beta(1 + \pi\lambda^{e*}). \tag{2.26}$$

The two equations, (2.24) and (2.25), determine (λ^{e*}, q^*). Substituting these solutions into (2.26) yields p_b^*. Other steady-state conditions are:

$$u'(c^{e*}) = U'(c^{w*})\frac{(1-\theta)q^*}{1-\theta q^*}, \tag{2.27}$$

$$c^* = AF(K^*, (1-\pi)\ell^*) - (1-\sigma)K^* - g, \tag{2.28}$$

$$p_b^* B = g + c^{e*} - \left[r^* + \phi^*\sigma q^* - \frac{1-\sigma}{\pi}(1-\theta q^*)\right]K^*. \tag{2.29}$$

Equation (2.27) is the steady-state version of the first-order condition of c^e, (2.12), with λ^{e*} being substituted. Equation (2.28) is the steady-state version of the goods-market clearing condition, and (2.29) is the steady-state version of the liquidity constraint, (2.10). Together with $r^* = AF_1$ and $h'(\ell^*) = U'(c^{w*})AF_2$, (2.25) - (2.29) solve for $(q^*, p_b^*, c^{e*}, c^*, K^*, r^*, \ell^*)$.

3. Equilibrium Response to Shocks

In this section I calibrate the model and examine the response of the equilibrium to shocks to total factor productivity and the resaleability of equity.

3.1. Calibration and computation

For the utility and production functions, I choose the following standard forms:

$$U(c^w) = \frac{(c^w)^{1-\rho} - 1}{1-\rho}, \quad u(c^e) = u_0 U(c^e),$$

$$h(\ell) = h_0 \ell^\eta, \quad F(K, (1-\pi)\ell) = K^\alpha \left[(1-\pi)\ell\right]^{1-\alpha}.$$

For the exogenous state of the economy (A, ϕ), I assume that $\log A$ and $-\log(\frac{1}{\phi} - 1)$ obey:

$$\log A_{t+1} = (1-\delta_A)\log A^* + \delta_A \log A_t + \varepsilon_{A,t+1}, \tag{3.1}$$

$$-\log(\frac{1}{\phi_{t+1}} - 1) = -(1-\delta_\phi)\log\left(\frac{1}{\phi^*} - 1\right) - \delta_\phi \log\left(\frac{1}{\phi_t} - 1\right) + \varepsilon_{\phi,t+1}. \tag{3.2}$$

These processes are convenient because they ensure $A \geq 0$ and $\phi \in [0,1]$. The quantitative analysis below will often take ε_A and ε_ϕ as one-time shocks.

I use the non-stochastic steady state to calibrate the model and choose the length of a period in the model to be one quarter. The procedure is described is Appendix B. The following targets and parameter values are standard in macro calibration. The value of the discount factor β and the relative risk aversion are common. The elasticity of labor supply is equal to two and aggregate hours of work in the steady state are 0.25. The share of labor income in output is $1 - \alpha = 0.64$, the ratio of annual investment to capital in the steady state is $4(1-\sigma) = 0.076$, and the ratio of capital to annual output is 3.32. The steady state value of productivity is normalized to $A^* = 1$ and the degree of persistence of productivity shocks is $\delta_A = 0.95$. The level of government spending g is set to be 18% of the steady state level of output. Note that the parameter u_0 is identified by the ratio of capital to output because u_0 affects entrepreneur's consumption which in turn affects the rental rate of capital and the capital stock.

Let me discuss the remaining identification restrictions. First, the parameter π can be interpreted as the fraction of firms that adjust their capital in a period. The estimate of this fraction on the annual basis ranges from 0.20 (Doms and Dunne, 1998) to 0.40 (Cooper, et al., 1999). I choose a value 0.24 in this range, which leads to $\pi = 0.06$. Second, I set θ to be equal to ϕ^* as a benchmark. If it is more difficult to issue new equity than resell equity, then $\theta < \phi^*$ and the liquidity constraint is more binding than in the benchmark case. Third, for the shocks to the resaleability of equity to generate persistent effects, the shocks must be persistent. Thus, I set $\delta_\phi = 0.9$ in the baseline calibration. Fourth, the rate of return to liquid assets and the fraction of liquid assets in the total value of assets come from the evidence in Del Negro, et al. (2010). These authors report that the annualized net rate of return to the US government liabilities is 1.72% for one-year maturities and 2.57% for ten-year maturities. I choose the value 0.02, which lies between these two rates of return.[10] Finally, Del Negro, et al. (2010) use the US Flow of Funds between 1952 and 2008 to compute the share of liquid assets in asset holdings. Their measure of liquid assets consists of all liabilities of the Federal Government, that is, Treasury securities net of holdings by the monetary authority and the budget agency plus reserves, vault cash and currency net of remittances to the Federal Government. The sample average of the share of liquidity assets is close to 0.12, which I target in the calibration.

Table 1. Parameters and calibration targets

parameter	value	calibration target
β: discount factor	0.992	exogenously chosen
ρ: relative risk aversion	2	exogenously chosen
π: fraction of entrepreneurs	0.06	annual fraction of investing firms $= 0.24$
u_0: constant in entrep. utility	44.801	capital stock/annual output $= 3.32$
h_0: constant in labor disutility	17.005	hours of work $= 0.25$
η: curvature in labor disutility	1.5	labor supply elasticity $1/(\eta - 1) = 2$
α: capital share	0.36	labor income share $(1 - \alpha) = 0.64$
σ: survival rate of capital	0.981	annual investment/capital $= 0.076$
A^*: steady-state TFP	1	normalization
δ_A: persistence in TFP	0.95	persistence in TFP $= 0.95$
B: stock of liquid assets	2.0204	fraction of liquid assets in portfolio $= 0.12$
ϕ^*: steady-state resaleability	0.276	annual return to liquid assets $= 0.02$
δ_ϕ: persistence in resaleability	0.9	exogenously chosen
θ: fraction of new equity	0.276	set to equal to ϕ^*
g: government spending	0.1928	government spending/GDP $= 0.18$

Let me briefly mention some of the identified values. First, equity resaleability in the steady state is $\phi^* = 0.276$. Because this is significantly less than one, the resale market for equity is far from being liquid. Notice that ϕ^* is identified by the target that the annual yield on liquid assets

[10]Note that the pricing equation for liquid assets in the steady state imposes the constraint $p_b^* \geq \beta$. Thus, given the value of β, the upper bound on the annual rate of return to liquid assets is $\beta^{-4} - 1 = 0.0327$. Thus, the value chosen for the rate of return to liquid assets is close to the middle point in the feasible region.

is 0.02. If assets were liquid, then the yield on liquid assets would be $\beta^{-1/4}-1=0.0327$. So, another way to phrase the result on ϕ^* is that equity resaleability needs to fall from one to 0.276 in order for the steady state to generate a differential of 127 basis points between the annual discount rate and the yield on liquid assets. Second, in the steady state, the rental rate of capital is $r^*=0.0271$ and the price of equity is $q^*=1.0367$. It is difficult to map this price of equity to a specific price index of stocks because equity in this model represents a broad collection of assets other than the particular group of liquid assets mentioned above. Third, the annualized equity premium in the steady state is $4(r^*/q^*+\sigma-1/p_b^*)=0.0087$. This premium is significant, considering that it is the equity premium in a steady state where no risk is present.

Suppose that the error terms in the processes of A and ϕ are zero. That is, the paths of A and ϕ are all realized at the beginning of $t=1$, which is the case with one-time shocks. I follow the procedure in Appendix A to compute equilibrium asset price functions $(q,p_b)(K,Z)$, where $Z=(A,\phi)$. Then, I recover an individual household's policy functions $x(s,b;K,Z)$, where x is any element in the list $(c,i,c^e,s_{+1}^e,\ell,c^w,s_{+1},b_{+1})$. Since the equilibrium has $s=K$ and $b=B$, where B is a constant, I shorten the notation $x(K,B;K,Z)$ as $x(K,Z)$.

Most of the policy functions have predictable properties. For example, consumption, investment and output are increasing functions of the capital stock, K. An exception might be the dependence of asset prices on the capital stock. For most values of the capital stock, equity price and the price of liquid assets are decreasing functions of the capital stock. On equity price, a plausible explanation is that as the capital stock increases, the rental rate of capital falls which reduces equity price. On the price of liquid assets, a plausible explanation is that as the capital stock increases, the need for further investment falls, which reduces the demand for liquid assets.

3.2. Equilibrium response to an asset liquidity shock

Suppose the economy is in the non-stochastic steady state at time $t=0$. Then, at the beginning of $t=1$, there is an unanticipated drop in the resaleability of equity from ϕ^* to $\phi_1<\phi^*$. After this shock, equity resaleability follows the process in (3.2), with $\varepsilon_{\phi,t}=0$ for all $t\geq 2$. To focus on the effect of this shock to equity resaleability, let me assume for the moment that the fraction of new investment that can be financed by issuing equity, θ, does not change. Similarly, total factor productivity, A, is assumed to remain at the steady state level. I compute the dynamics of the equilibrium using the procedure described in Appendix B. Note that I use the policy functions and asset pricing functions to compute the dynamics instead of linearizing the equilibrium system. This approach has the advantage of being able to deal with large shocks. To dramatize the effect of the liquidity shock, I assume that the shock reduces equity resaleability from 0.276 to 0.05, a

82% drop that nearly shuts down the resale market for equity temporarily.

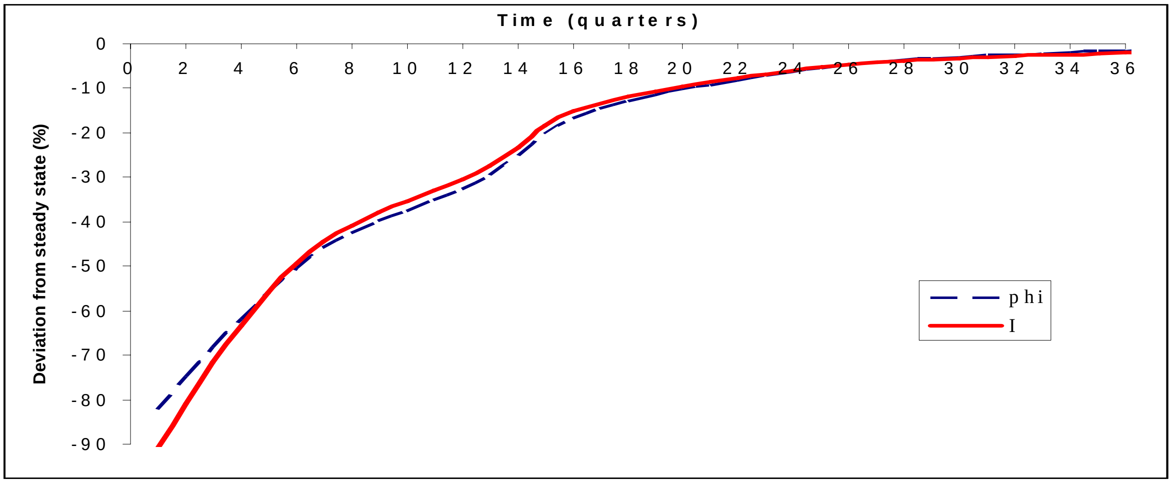

Figure 2.1. Investment and equity resaleability after a negative liquidity shock

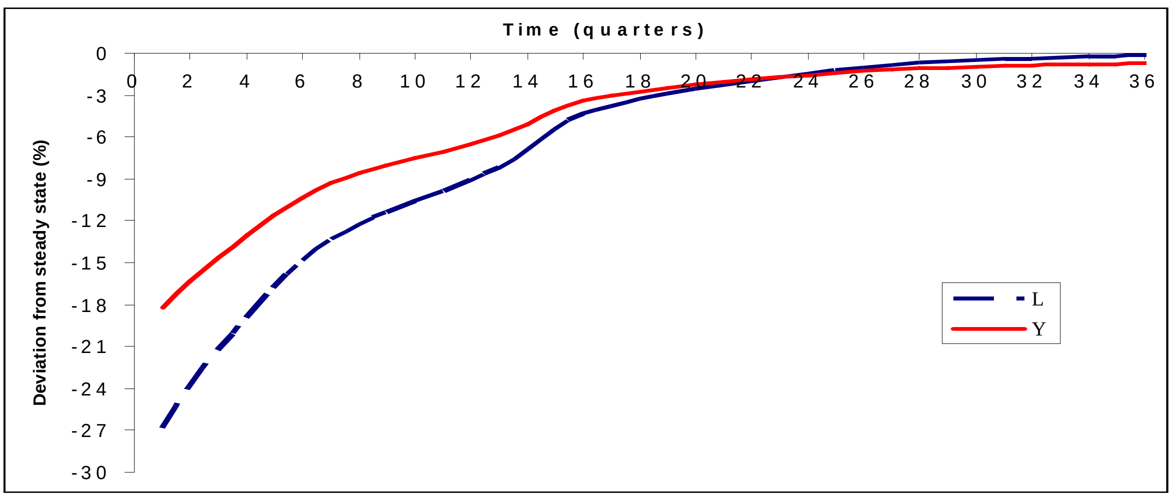

Figure 2.2. Employment and output after a negative liquidity shock

Figure 2.1 graphs aggregate investment ($I = \pi i$) and equity resaleability, where the vertical axis is percentage deviations of the variables from their steady-state levels and the horizontal axis is the number of quarters after the shock. In period 1, the negative shock to equity resaleability reduces investment by 90%. Although the negative shock to the resaleability is large by construction, this magnitude of the reduction in investment may still be surprising in the following sense. Because θ is not reduced with ϕ in this experiment, entrepreneurs can still issue new equity to finance new investment. The large fall in investment indicates that a majority of new investment is financed by selling existing equity rather than issuing new equity. Figure 2.1 also shows that investment closely follows the dynamics of equity resaleability. Because equity resaleability is assumed to be persistent, with a persistence parameter 0.9, the shock has persistent effects on investment. Four years after the shock, investment is still about 16% below the steady state.

Figure 2.2 exhibits aggregate employment ($L = (1 - \pi)\ell$) and output (Y). Both variables fall by a large amount when the negative shock to equity resaleability hits. Employment falls by 27% and output by 18% in period 1. Such magnitudes are comparable to the ones in the Great Depression. Note that the reduction in output in period 1 comes entirely from the reduction in employment, because the capital stock in period 1 is predetermined and total factor productivity is fixed. After period 1, however, the capital stock also falls below the steady state due to lower investment, which contributes to the lower level of output. The effects of the negative shock to equity resaleability are persistent. Four years after the shock, employment is still 5% below the steady state and output 4% below the steady state. These responses of aggregate investment, employment and output seem to suggest that a liquidity shock to the resale market of equity can be a potent cause of aggregate fluctuations.

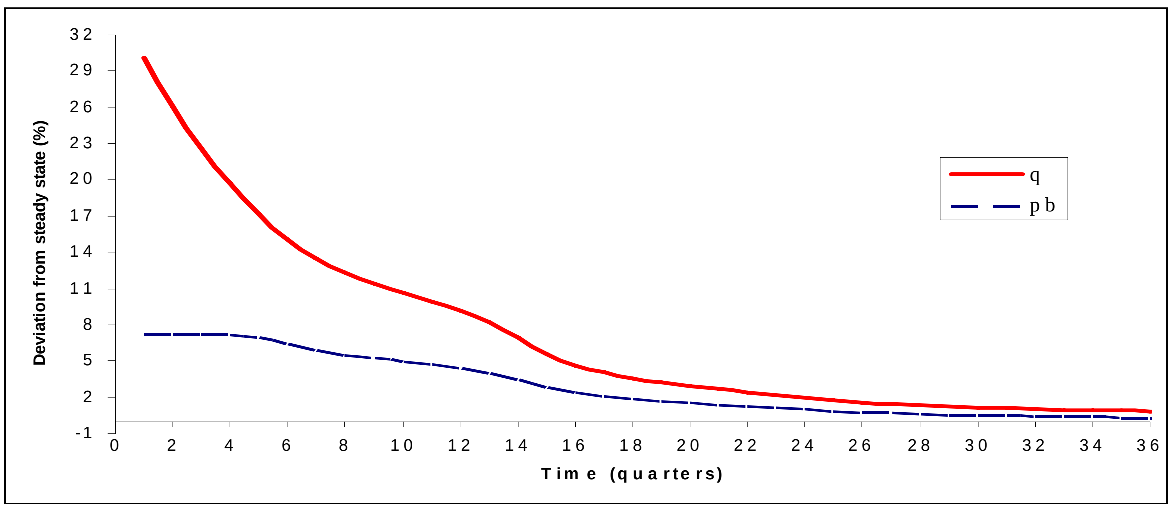

Figure 2.3. Asset prices after a negative liquidity shock

One can question whether shocks to equity resaleability are as persistent as I assumed. Instead of getting into this debate, let me check how asset prices respond to the liquidity shock. As Figure 2.3 shows, a negative liquidity shock to the equity market generates an asset price boom! Immediately after the shock, equity price increases by 30% and the price of liquid assets increases by 7%. Asset prices stay above the steady state for quite a long time. Four years after the shock, equity price is still 4.5% above the steady state and the price of liquid assets is 2.3% above the steady state. Thus, the negative liquidity shock to the asset market can generate large and persistent fluctuations in equity price, but the problem is that the direction of these fluctuations is opposite to what is typically observed in the business cycle (e.g., Figure 1.1).[11]

3.3. What is the source of this problem?

One suspicion is that the liquidity shock is so large that non-linearity of the equilibrium system generates erratic responses in asset prices. To eliminate this suspicion, I have computed smaller shocks to equity liquidity and still found that a negative shock to equity resaleability increases asset prices. The second suspicion is that the exercise above has fixed θ. Whatever that makes it more difficult to re-sell equity, it may also increase the difficulty to issue new equity. That is, θ is likely to fall in an episode when ϕ falls. This joint reduction in θ and ϕ can certainly be investigated and I will take up the issue later.

The other suspicion is that the model lacks some realistic ingredients. As a result, some other variables respond to the liquidity shock in the incorrect magnitude or direction and, to make up for these unrealistic responses in other variables, equity price ends up responding in the wrong direction. Indeed, one can have a long list of the ingredients in a typical dynamic stochastic general equilibrium model that I have omitted. The following is a partial list:

(i) Wage rigidity: the absence of it in my model may imply that output does not fall enough after the negative liquidity shock;

(ii) Adjustment costs of investment: the absence of these costs may imply that investment may fall by too much immediately after the negative liquidity shock;

(iii) Habit persistence in consumption: the absence of this ingredient may imply that consumption responds to the liquidity shock by too much or in the wrong direction.

These items are related. In particular, if the fall in output is sufficiently deficient and the fall in investment is sufficiently large, then consumption must increase after the negative liquidity shock in order to satisfy the resource constraint. To check this, I have computed the dynamics of consumption (not graphed here). After the negative liquidity shock described above, an entrepreneur's consumption falls by about 6% and returns to the steady state slowly from below, and a worker's consumption increases by about 14.4% and falls toward the steady state from above. Because workers are 94% of the population in the model, aggregate consumption per capita increases by 8.4% on the impact of the shock and falls toward the steady state from above.

[11] Other authors (e.g., Ajello, 2010) also find this paradoxical response of equity price to an equity liquidity shock. However, their models have many additional elements that make this result less transparent. Instead of focusing on this result, they focus on other predictions of the model, such as the role of liquidity shocks in explaining output fluctuations and the effects of monetary policy.

Thus, indeed, the absence of items (i)-(iii) above makes aggregate consumption respond to the negative liquidity shock in a direction opposite to what one observes in business cycles. Given this apparent defect of the model, one may start putting items (i)-(iii) above into the model.

Such an effort will be futile in correcting the response of equity price to the liquidity shock. So will the effort of allowing θ to fall together with the negative liquidity shock. I make these statements basing on a simple calculation of the marginal benefit and cost of investment. Specifically, the optimal amount of investment that a household should instruct an entrepreneur to undertake must satisfy the complementary slackness conditions in (2.13). For the sake of argument, let me focus on the case where investment is positive. In this case, (2.13) becomes:

$$q - 1 = (1 - \theta q)\lambda^e. \tag{3.3}$$

Because this equation is central to the argument, let me repeat the meanings of the terms in it. For each share of equity issued on investment, the price is q, and so the benefit from issuing equity to finance a unit of investment is $(q-1)$. If there were no frictions on issuing new equity, this term would be the net benefit of issuing equity, in which case investment could be positive and finite in the equilibrium if and only if $q = 1$. When there are frictions in issuing new equity, as modeled by $\theta < 1$, the fraction of investment financed with new equity is θq. The entrepreneur must use other (liquid) resources to finance the remainder of investment, $1 - \theta q$, which is referred to as the downpayment on a unit of investment. The cost of this downpayment depends on how tight the equity liquidity constraint (2.10) is, as measured by the shadow price λ^e of the constraint. Thus, $(1 - \theta q)\lambda^e$ is the marginal cost of a unit of investment. Condition (3.3) requires the net marginal benefit of investment to be zero.

The marginal benefit of investment is a strictly increasing function of q and the downpayment on investment is a strictly decreasing function of q. Thus, for any given λ^e, the net marginal benefit of investment is a strictly increasing function of q. When there is a negative shock to liquidity, the implicit cost of raising funds to finance the downpayment of investment increases. That is, the equity liquidity constraint (2.10) becomes tighter and its shadow price λ^e increases. The higher λ^e reduces the net marginal benefit of investment for any given equity price. To restore the balance between the marginal benefit and cost of investment, equity price must increase. As liquid resources become more scarce, the price of liquid assets, p_b, also increases.

This argument is quite general, because it only requires the negative liquidity shock to tighten the liquidity constraint, which is what the shock is supposed to do. At the risk of over-simplifying the problem, let me phrase it in terms of the demand for and the supply of equity. A reduction in equity resaleability reduces the supply of equity. Because the demand for equity is not affected so much by the reduction, the price of equity must increase to clear the market. Mathematically, the equation on which I have relied to illustrate the problem, (3.3), involves only two variables, (q, λ^e), and one parameter θ. All other details, such as the resaleability of equity and the price of liquid assets, do not direct enter this condition. Instead, whatever effect they can exert on equity price must go through λ^e. With this generality, the argument can survive a wide range of extensions/modifications of the model and the liquidity shock.

Consider first the possibility that the same force that makes existing equity less liquid also makes it more difficult to issue new equity. In this case, θ falls when there is a negative shock to equity resaleability ϕ. This concurrent fall in θ with ϕ is likely to make the equity price boom even larger. To see this, note that, for any given equity price, a fall in θ increases the downpayment on each unit of investment. This increases the marginal cost of investment. To restore the balance between the marginal benefit and cost of investment, equity price must rise even further after a negative liquidity shock.

Now let me make inferences on what will happen if the ingredients (i)-(iii) above are introduced into the model. It is clear that wage rigidity and habit persistence in consumption do not directly affect the marginal benefit and cost of investment. Their indirect effects may tighten the equity liquidity constraint, (2.10), even further and exacerbate the problem of the response of equity price to a liquidity shock. To see this, consider wage rigidity first. When there is a fall in equity liquidity, output is likely to fall by more with rigid wages than with flexible wages. Similarly, the rental income of capital will fall by more when wages are rigid. Because this rental income is part of the resource which an entrepreneur uses to finance investment, the equity liquidity constraint (2.10) is likely to become tighter, and so equity price is likely to rise by more with rigid wages than with flexible wages. Next consider habit persistence in consumption. When an entrepreneur's consumption cannot respond quickly to a negative liquidity shock, the entrepreneur needs resource not only to finance investment but also to support persistently high consumption. Again, the equity liquidity constraint (2.10) is likely to be tighter, which requires equity price to increase by more than if there is no habit persistence.

The consequences of introducing (ii), an adjustment cost in investment, are more complicated than introducing (i) and (iii), because the adjustment cost directly affects the condition for optimal investment. For concreteness, let $i^* \Psi(i/i^*)$ be the additional resource that an entrepreneur must incur in order to invest an amount i, where i^* is the steady-state level of investment by an individual entrepreneur.[12] Impose the usual assumptions: $\Psi(1) = 0$, $\Psi'(1) = 0$, and $\Psi'' > 0$. With the adjustment cost, a unit of investment at the margin needs $(1 + \Psi')$ units of resources.

[12] Del Negro et al. (2010) use this specification of the adjustment cost.

Thus, the optimality condition for investment is modified as

$$q - (1 + \Psi') = (1 + \Psi' - \theta q)\lambda^e. \tag{3.4}$$

For any given (q, λ^e), the marginal benefit of investment is a decreasing function of i and the marginal cost an increasing function of i. Thus, the net marginal benefit of investment is a decreasing function of i. If investment falls after a negative shock to equity liquidity, then the marginal adjustment cost falls. Such savings of the adjustment cost increase the net marginal benefit of investment for any given (q, λ^e) and mitigates the increase in equity price caused by the increase in λ^e. For this effect to be strong, the marginal cost of adjusting investment, Ψ', must be sufficiently steep and the reduction in investment must be sufficiently small, which may be inconsistent with the large observed reduction in investment at the beginning of a recession (see Figure 1.1). Moreover, when the adjustment cost prevents investment from falling sharply after a negative shock to equity liquidity, an entrepreneur must raise enough resource to maintain such investment. As a result, the equity liquidity constraint (2.10) is likely to be tighter with the adjustment cost than without. That is, λ^e is likely to increase by more with the adjustment cost than without. This effect of the adjustment cost on λ^e is likely to dominate the savings in the adjustment cost, which will require equity price to rise by more in order to satisfy the optimality condition for investment, (3.4).

Let me now examine whether other specific assumptions of this model play a role in making equity price increase after a negative liquidity shock. One is the structure of large households in which the two groups of individuals, entrepreneurs and workers, pool their asset holdings at the beginning of each period. This structure has simplified aggregation because heterogeneity between the two groups of individuals does not persist through their asset holdings. In contrast, Kiyotaki and Moore (2008) allow this heterogeneity to persist and so their model admits aggregation only when entrepreneurs' utility function is logarithmic. However, the household structure cannot be the reason why equity price increases after a fall in equity resaleability. Relative to Kiyotaki and Moore (2008), the household structure allows entrepreneurs in the next period to use some of workers' assets, which reduces the persistence of the tightening effect of the negative liquidity shock on the liquidity constraint. Thus, equity price should increase by even more in response to a fall in equity resaleability if the household structure is not available.

Another assumption in this model and in Kiyotaki and Moore (2008) is that the return to existing shares in the period is immediately available to an entrepreneur. That is, the income rs is available for financing current investment, as can be seen from the entrepreneur's constraint (2.3). One may consider the alternative timing according to which the income rs is available only for financing consumption at the end of the period but not for financing investment in the current period. In this case, a fall in equity resaleability will tighten the liquidity constraint to a greater extent than it does in the current model and, hence, will increase equity price by even more.

4. Some Solutions to the Problem

Let me summarize the finding in the section above. A negative shock to equity resaleability can have large negative effects on investment, employment and output. But the same shock increases the implicit cost of financing new investment, which requires equity price to rise rather than fall in order to restore the balance between the marginal benefit and cost of investment.

For equity price to fall after a negative liquidity shock, the equity liquidity constraint must become less tight. To generate this paradoxical outcome, there must be other concurrent shocks that sufficiently reduce the need for investment in the economy. Various modifications of the model examined in the previous section failed to accomplish this task because they had either no direct effect or very weak effect on the need for investment. In this section I discuss some candidates that may sufficiently reduce entrepreneurs' desire to invest. These concurrent shocks do not have to be unrelated to the liquidity shock; instead, they may even be the cause of the change in equity liquidity.

One obvious candidate is a negative shock to total factor productivity, A. An unexpected drop in productivity reduces investment by reducing the marginal productivity of capital. If the shock to productivity is sufficiently persistent, then a household will scale down all members' consumption, including entrepreneurs' consumption. These reductions in investment and consumption reduce an entrepreneur's expenditure, given by the right-hand side of the equity liquidity constraint (2.10). However, the negative shock to productivity may also reduce the rental income of capital, rs, which appears on the left-hand side of (2.10). If the reductions in investment and consumption dominate the reduction in the rental income, then the equity liquidity constraint becomes less tight and equity price falls.

To illustrate this possibility, I compute the response of the equilibrium to simultaneous shocks to ϕ and A. Suppose that the economy is in the steady state before $t = 1$ and, at the beginning of $t = 1$, there are unanticipated reductions in ϕ about 8% and in A about 5%. After these shocks, A and ϕ follow the deterministic dynamics of the processes in (3.1) and (3.2). Figure 3.1 depicts percentage deviations of A and ϕ from the steady state, while Figures 3.2 - 3.3 depict percentage deviations of some endogenous variables from the steady state. By construction, total factor productivity A and equity resaleability ϕ are both persistent, but total factor productivity is relatively more persistent, as shown by Figure 3.1.

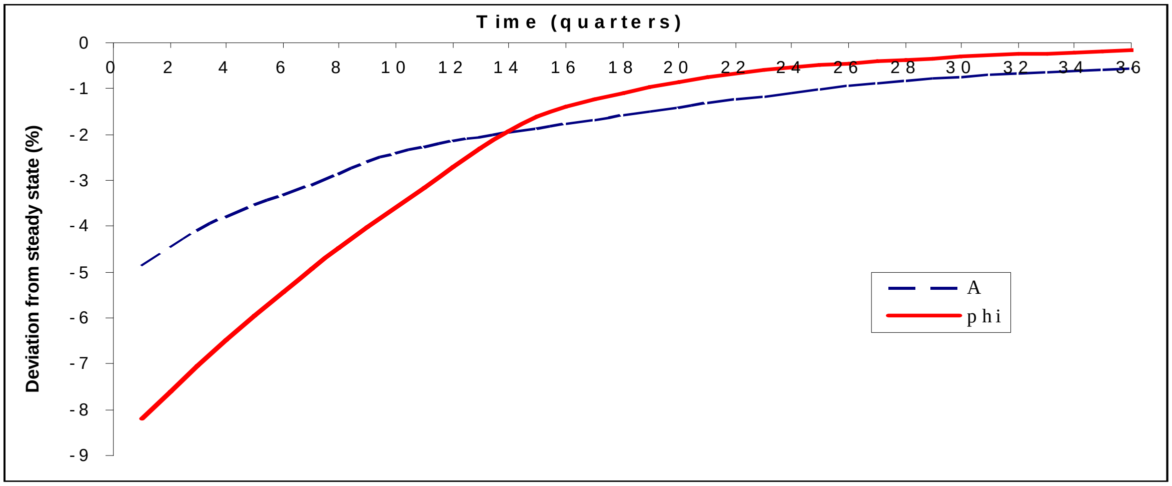

Figure 3.1. Negative shocks to A and ϕ

Figure 3.2 confirms the analysis that the two shocks reduce investment, consumption and output. As explained earlier, the negative shock to liquidity alone increases consumption in the current model. Thus, the reduction in consumption in Figure 3.2 reflects the fact that the negative productivity shock dominates the liquidity shock. Figure 3.3 shows that, after the two shocks, equity price falls and approaches the steady state from below. This response must mean that the negative productivity shock reduces the tightness of the equity liquidity constraint. Moreover, the bond price increases after the two shocks and stays above the steady state for eight quarters before falling below the steady state. Thus, the liquidity shock does play a visible role in these graphs. If the negative shock to productivity were the only shock, the bond price would fall.

The patterns in Figures 3.2 - 3.3 are broadly consistent with the patterns in the two recessions depicted in Figures 1.1 - 1.3. In terms of the magnitude, the response of equity price in Figure 3.3 is about one-tenth of that observed in the trough of the two recessions. For other variables, the responses in the two sets of graphs are not far apart in the magnitude. For example, in the trough of the recent recession, investment is about 10% below the trend, GDP is 3% below the trend, and the bond price is 0.35% above the trend. In Figures 3.2 - 3.3, the two shocks reduce investment by 15% in investment, reduce output by 5%, and increase the bond price by 0.25%. The broad consistency between the two sets of graphs suggests that productivity shocks are important for explaining the cyclical behavior of asset prices with other macro variables.

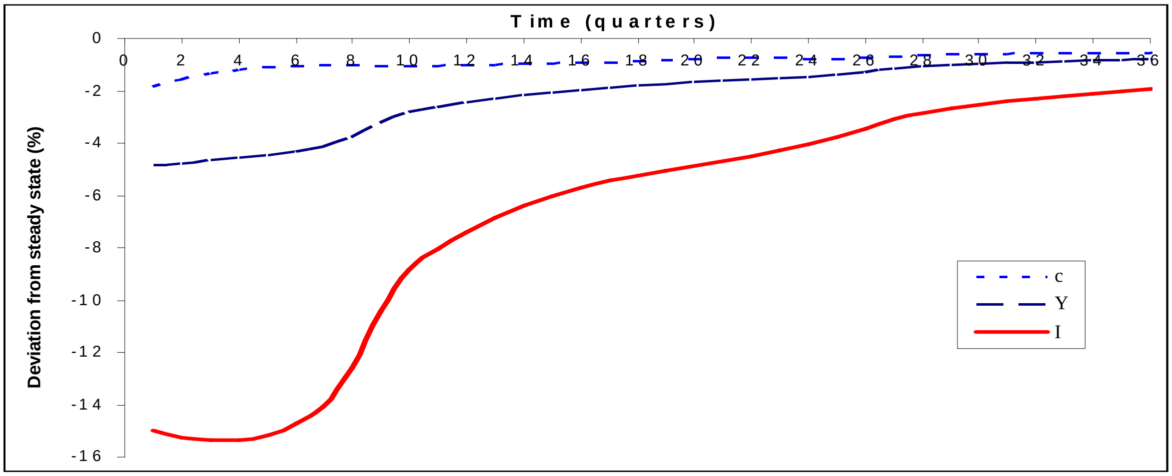

Figure 3.2. (c, Y, I) following negative shocks to A and ϕ

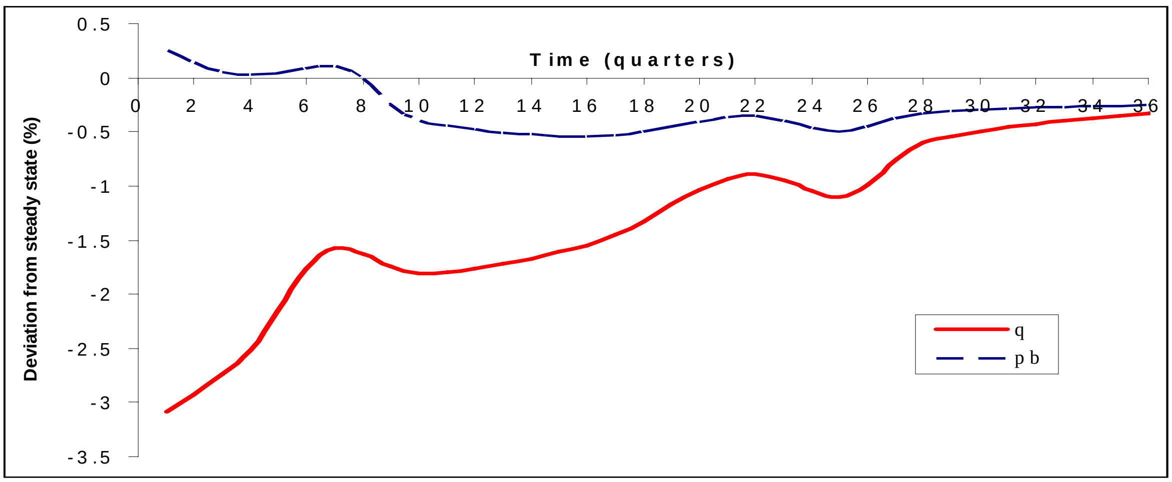

Figure 3.3. Asset prices following negative shocks to A and ϕ

Another force that can significantly reduce the need for investment is a fall in the quality of capital. If market participants perceive the quality of capital to deteriorate quickly at the onset of a recession, they will move resources from equity to relative safe and liquid assets. This will depress equity price and drive up the price of liquid assets. To model this process, let me assume that the effective capital stock, instead of the raw capital stock, appears in the production function. Let the effective capital stock be κK, where κ is the quality of capital. In this case, total factor productivity is $A\kappa^{\alpha}$. Thus, a negative shock to the quality of capital is similar to a negative shock to A examined above. This modeling approach does not allow for a non-degenerate distribution of the quality of capital among the firms. For a model where this quality difference is important, see Ajello (2010).[13]

[13] One may introduce the quality of investment rather than capital. For example, one may model the investment

Finally, a drop in investment opportunities can also reduce the need for investment. This shock can be modeled as an unexpected temporary fall in the fraction of entrepreneurs in the population, π. Because a household's utility is a weighted sum of entrepreneurs' and workers', a shock to π is also a shock to a household's preferences. Another way to model a drop in investment opportunities is to assume that π is fixed but only a fraction of entrepreneurs have investment opportunities in a period. This extension of the model is feasible, although it introduces heterogeneity among entrepreneurs.

5. Conclusion

In this paper, I have constructed a model to evaluate the hypothesis that shocks to equity market liquidity are an independent source of the business cycle. The model is easy for aggregation and for the construction of the recursive competitive equilibrium. After calibrating the model to the US data, I have computed the dynamic response of the equilibrium to shocks. A negative liquidity shock in the equity market can generate large drops in investment and output but, contrary to what one may conjecture, the shock generates an equity price boom. This response of equity price occurs as long as a negative liquidity shock tightens firms' financing constraints on investment. For equity price to fall as it typically does in a recession, the negative liquidity shock must be accompanied or caused by other shocks that reduce the need for investment sufficiently and relax firms' financing constraints on investment. I have discussed some candidates of these concurrent shocks. In particular, I have illustrated that if the negative liquidity shock is accompanied by a strong negative productivity shock, then the dynamics of major macro variables and asset prices in the model are similar to the ones observed in a recession.

The main message that should be taken from this analysis is not that shocks to equity market liquidity are not important for the business cycle but, rather, that such shocks are not the primary driving force of the business cycle. In this paper, equity resaleability is taken exogenously as is in Kiyotaki and Moore (2008). It is plausible that equity resaleability is affected by other components of the economy. For example, negative shocks to productivity or the quality of capital may increase the difficulty to re-sell equity. If this is the case, then fluctuations in equity resaleability can amplify or propagate the business cycle. Even if productivity shocks are the primary driving force of the business cycle, pro-cyclical fluctuations in equity resaleability seem

technology as such that one unit of capital needs $1/\kappa$ units of investment. A higher κ represents a higher quality of investment. However, a fall in κ may not necessarily reduce equity price. To see this, note that the introduction of κ changes the optimality condition for investment to: $q - \frac{1}{\kappa} = (\frac{1}{\kappa} - \theta q)\lambda^e$. For any given λ^e, a fall in κ reduces the marginal benefit, and increases the marginal cost, of issuing equity to finance investment. This effect exacerbates the problem, unless λ^e falls sufficiently.

necessary for generating the observed negative correlation between equity price and the price of liquid assets. Moreover, the difficulty in re-selling equity seems a potent friction that generates the equity premium. Nevertheless, this paper does illustrate in a concrete way that various policies which have been implemented to increase asset market liquidity during a recession might have missed the primary source of the recession, although they might have helped reducing the extent of the recession.

This analysis suggests that it is important to explicitly model why asset market liquidity fluctuates. Also, as discussed in the introduction, I have abstracted from financial intermediation and debt-financing in order to focus on the evaluation of the Kiyotaki-Moore hypothesis. It is useful to build an integrated model to incorporate frictions in both the equity market and financial intermediation. In particular, if firms can use the existing equity as collateral in borrowing from financial intermediaries, then a negative shock to equity resaleability reduces a firms' net worth, which can lead to lower investment and the amplification of the business cycle. The model in this paper is promising for this integration because its structure facilitates aggregation.

Appendix

A. The Mapping T on Asset Price Functions

To construct the mapping T on asset price functions, start with arbitrary functions $q, p_b \in \mathcal{C}_1$. The following procedure constructs the updated asset price functions Tq and Tp_b:
(i) Substitute the factor market clearing conditions (2.18) and (2.19) into requirement (ii) of the equilibrium definition. This step generates r and w as functions of (ℓ, K, Z).
(ii) Substitute the functions for (r, w) in step (i) and the market clearing conditions (2.18) - (2.21) into the household's optimality conditions (2.10) - (2.13). Then, solve $(i, c^e, c^w, c, \lambda^e)$ as functions of (ℓ, K, Z).
(iii) Substitute the resulting functions in steps (i) and (ii) above into the goods market clearing condition, (2.17), and solve ℓ as a function of (K, Z). With this function $\ell(K, Z)$, the functions solved in steps (i) and (ii) express (r, w) and $(i, c^e, c^w, c, \lambda^e)$ as functions of (K, Z). Similarly, (r_{+1}, w_{+1}) and $(\ell, i, c^e, c^w, c, \lambda^e)_{+1}$ can be expressed as functions of (K_{+1}, Z_{+1}).
(iv) Substitute the functions obtained in step (iii) and the law of motion of aggregate capital in requirement (iv) of the equilibrium definition into the right-hand sides of the asset pricing equations (2.14) and (2.15). The result is a pair of functions of (K, Z), provided that Z follows a Markov process. These are the updated asset price functions, denoted as $T(q, p_b)(K, Z)$.

It can be verified that for any $q, p_b \in \mathcal{C}_1$, Tq and Tp_b are continuous functions of (K, Z) and their values lie in $\mathbb{R}_+$. That is, T maps a pair of elements in $\mathcal{C}_1$ back into $\mathcal{C}_1$, where $\mathcal{C}_1$ is the set containing all continuous functions mapping $\mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$. A fixed point of T is a pair of asset price functions in the equilibrium. After obtaining the fixed point of asset pricing functions, I can retrieve the policy functions of individuals' decisions and other variables.

B. Identifying Parameters and Computing Dynamics

Let me first describe how to identify the parameters. The values of β, ρ, A^*, δ_A, δ_ϕ and π are exogenously set and the explanations for these values are given in the text. The parameter η is calculated from the elasticity of labor supply, $\frac{1}{\eta-1} = 1$, and the capital share in output is $\alpha = 0.36$. Since aggregate investment in the steady state is $\pi i^* = (1-\sigma)K^*$, the ratio of annual investment to capital is $4\pi i^*/K^* = 4(1-\sigma)$. Equating this to the target, 0.076, yields σ.

Setting total hours of work to the target yields $(1-\pi)\ell^* = 0.25$. Given π, this solves ℓ^*. Setting the ratio of capital to annual output in the steady state to the target yields $K^*/(4A^*F^*) = 3.32$. With the value of α identified above, this condition solves r^*. Since $r^* = A^*F_1'$, I can solve K^*

and recover $i^* = (1-\sigma)K^*/\pi$. Also, $w^* = A^*F_2'$. These values of $(\ell^*, K^*, i^*, r^*, w^*)$ will be used to identify some parameters below. Since the ratio of government spending to output is $g/(A^*F^*) = 0.18$, I can solve g.

Setting the annualized net rate of return to liquid assets, I have $1/(p_b^*)^4 - 1 = 0.02$. This solves for p_b^*. Substituting this value of p_b^* into the asset pricing equations in the steady state, (2.25) and (2.26), using the value of r^* identified above, and using $\theta = \phi^*$, I can solve ϕ^*, θ and q^*. Using the target on the share of liquid assets, I have $p_b^*B/(p_b^*B + q^*K^*) = 0.12$. Because p_b^*, q^* and K^* are all solved by now, this condition solves B.

There are two parameters still to be solved, (u_0, h_0). To identify them, I substitute the values of $(r^*, q^*, \phi^*, K^*, i^*, B, g)$ into the steady-state version of (2.10) to solve c^{e*}. The goods-market clearing condition yields $c^* = A^*F^* - \pi i^* - g$. Then, the definition of c yields $c^{w*} = (c^* - \pi c^{e*})/(1-\pi)$. Substituting the values of (c^{w*}, ℓ^*, w^*) into the steady-state version of the first-order condition of labor supply, (2.11), I obtain the value of h_0. Note that $i^* > 0$. Combining the steady-state versions of (2.12) and (2.13) to eliminate λ^{e*}, I obtain an equation in which u_0 is the only item still to be solved. Solve this equation for u_0.

Now let me describe how to compute the dynamics of the equilibrium after some shocks. Suppose that there are shocks to A or ϕ or both. Let me focus on various cases where the new paths of A and ϕ are completely known after the shocks are realized at the beginning of time $t = 1$, because only such cases are examined in the text. An example is a one-time shock to A or ϕ or both that occurs at $t = 1$, after which A and ϕ follow (3.1) and (3.2) with the error terms being zero for all $t \geq 2$. Another example is a change to the path of A or ϕ or both that becomes known at $t = 1$. With such shocks, the entire path of $Z = (A, \phi)$ becomes known immediately after the beginning of $t = 1$. Given this path of Z, I can use the equilibrium asset price functions and the policy functions to compute the paths of equilibrium variables. Specifically, at $t = 1$, asset prices are $q_1 = q(K_1, Z_1)$ and $p_{b,1} = p_b(K_1, Z_1)$, where K_1 is predetermined and Z_1 is known. An individual household's optimal decisions in period 1 are given by $x_1 = x(K_1, B; K_1, Z_1)$, where x is the policy function for any variable in the list $(c, i, c^e, s^e_{+1}, \ell, c^w, s_{+1}, b_{+1})$. Similarly, I can compute factor prices (w_1, r_1) and aggregate variables in period 1. Using (2.22), I can obtain K_2. With (K_2, Z_2), I repeat the process to obtain all equilibrium variables in period 2. Continuing this process yields the dynamic path of the equilibrium after the shocks.

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