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Abstract

We propose the Heterogeneous Autoregressive (HAR) model for the estimation and prediction of realized correlations. We construct a realized correlation measure where both the volatilities and the covariances are computed from tick-by-tick data. As for the realized volatility, the presence of market microstructure can induce significant bias in standard realized covariance measure computed with artificially regularly spaced returns. Contrary to these standard approaches we analyse a simple and unbiased realized covariance estimator that does not resort to the construction of a regular grid, but directly and efficiently employs the raw tick-by-tick returns of the two series. Montecarlo simulations calibrated on realistic market microstructure conditions show that this simple tick-by-tick covariance possesses no bias and the smallest dispersion among the covariance estimators considered in the study. In an empirical analysis on S&P 500 and US bond data we find that realized correlations show significant regime changes in reaction to financial crises. Such regimes must be taken into account to get reliable estimates and forecasts.

Keywords

High frequency data; Realized Correlation; Market Microstructure; Bias correction; HAR; Regimes.

JEL Classification

C13; C22; C51; C53

1 Introduction

Asset returns cross correlation is pivotal to many prominent financial problems such as asset allocation, risk management and option pricing. Recently, for the measure of the asset volatility, the use of high frequency data has been advocated to improve the precision of the estimation: the so-called Realized Volatility (RV) approach proposed in a series of breakthrough papers by Andersen, Bollerslev, Diebold and Labys (2001a,b), Barndorff-Nielsen and Shephard (2001a,b 2002a,b, 2004)and Comte and Renault (1998). As for the realized volatility approach, the idea of employing high frequency data in the computation of covariances and correlations between two assets leads to the analogous concept of *realized covariance* (or covariation) and *realized correlation* .

The standard way to compute the realized covariance is to first choose a time interval, construct an artificially regularly spaced time series by means of some interpolation scheme and then take the contemporaneous sample covariance of those regularly spaced returns. But simulations and empirical studies indicate that such covariance measure presents a bias toward zero which rapidly increases with the reduction of the time length of the fixed interval chosen. As for the realized volatility, the presence of market microstructure can induce significant bias in the standard realized covariance measure. However, the microstructure effects responsible for this bias are different. In fact, bid-ask bouncing, which is the major source of bias for the realized volatility, will just increase the variance of the covariance estimator but it will not induce any bias. On the contrary, the so called non-synchronous trading effect (Lo and MacKinlay 1990) strongly affects the estimation of the realized covariance and correlation. In fact, since the sampling from the underlying stochastic process is different for different assets, assuming that two time series are sampled simultaneously when, indeed, the sampling is non-synchronous gives rise to the non-synchronous trading effect. As a result, covariances and correlations measured with high frequency data will posses a bias toward zero which increases as the sampling frequency increases. This effect of a dramatic drop of the absolute value of correlations among stocks when increasing the sampling frequency was first reported by Epps (1979) and hence called “the Epps effect”. Since then, the Epps effect has been confirmed on real data and simulations by many other authors, such as Dacorogna and Lundin (1999) Renó (2003) and Martens (2004), among others.

Existing empirical studies on realized covariance usually compute the sample covariance based on the 5 or 30 minutes return interval. Such frequencies are heuristically chosen to try to avoid the bias and market microstructure effects. In some cases, a number of leads and lags covariance are added to reduce the remaining bias. However, this type of correction will increase the variance of the estimator. Though the optimal choice of the frequency of the returns and the number of leads and lags would substantially lower the RMSE compared to the heuristic choices, these optimal values are unknown in empirical application (Martens 2004).

Instead, following the general statistical principles which tell us to never "throw data away", we analyse an unbiased realized covariance measure directly built on the raw tick-by-tick data series. The tick-by-tick covariance estimator presented here, has been also independently proposed and formally analysed under the assumption of no microstructure noise by Hayashi and Yoshida (2005) and already appeared in Martens (2004) as a more efficient version of the De Jong and Nijman estimator in the absence of true leads and lags cross covariances[1]. This estimator has been recently investigated also by Griffin and Oomen (2006), Palandri (2006) and Sheppard (2006).

This paper differs from the above mentioned studies and contributes to the literature on realized correlations along several aspects. First, we investigate through extensive Monte Carlo simulations the behavior of the tick-by-tick covariance estimator under market microstructure conditions analogous to that of the financial data studied in the empirical part.

Second, combining tick-by-tick realized volatility and covariance estimators we obtain a highly accurate measure of the daily correlation between S&P 500 and 30 years Treasury Bond futures. Analyzing the time-varying dynamics of the constructed daily correlations using models allowing for different regime specifications (like, for example, the regime-switching models introduced by Hamilton and Susmel, 1994, or the tree-structured threshold models introduced by Audrino and Bühlmann, 2001), we collect empirical evi-

[1]As a result of informal discussions with us, as kindly acknowledged by the author in his footnote 9.

dence that correlations show drastic regime shifts. Other studies in the recent literature on stock-bond correlations have already reported that stock-bond correlations have gone from being positive to negative after 1997; see, for example, Ilmanen (2003). The reasons advocated to explain this pattern are different. A first one is related to market uncertainty and risk, introducing the "flight-to-quality" effect that suggests the phenomenon of fleeing from stock to bond markets in times of worsening economic conditions (see, for example, Ilmanen, 2003, or Connolly et al., 2005). Another reason advocated to explain the change of sign in bond-stock correlations is related to differences in inflation expectation or in other macroeconomic announcements (see, for example, Li, 2002, or Christiansen and Ranaldo, 2006). Finally, in their empirical study Pastor and Stambaugh (2003) found that changes in stock-bond correlations are related to different levels of liquidity. In particular, they found that a kind of "flight-to-quality" effect appears in months with exceptionally low liquidity, i.e. months in which liquidity drops severely tend to be months in which stocks and fixed-income assets move in opposite directions.

Our result is different and adds another possible explanation for the changes in the stock-bond correlation behavior. In our analysis we collect empirical evidence that regime changes occur in reaction to big financial crises. In this context, the "flight-to-quality" phenomenon seems to be a consequence of the bad economic conditions implied by financial crises. To our knowledge, this is the first study that empirically shows that there is a relationship between changes in stock-bond correlations and financial crises like the Western European monetary crisis of 1992-1993 or the Asian crisis of 1997-1999. Note that this can be due to the fact that we use an highly accurate measure for correlations constructed on tick-by-tick data. This allows us to identify not only the well-investigated change in correlations that occurred at the end of 1997 (with stock-bond correlations moving from positive to negative), but also other changes, in reaction to possible significant increases in stock-bond correlations, too.

Third, we propose the Heterogeneous Autoregressive (HAR) model for the estimation and prediction of the tick-by-tick realized correlations. The empirical results show that the proposed model is able to mimic well the dynamic properties of the daily realized correlation process and to provide accurate out of sample forecasts.

The remainder of the paper is organized as follows: Section 2 defines the tick-by-tick realized covariance estimator, Section 3 shows some simulation results and comparisons with some existing methodologies and Section 4 presents an empirical application to a bivariate series of S&P 500 and 30 years US Treasury Bond futures tick-by-tick data. Section 5 proposes the HAR process as a simple and parsimonious model for the estimates and forecasts of realized correlations. We summarize results and conclude in Section 6.

2 Realized Covariance tick-by-tick

Contrary to the existing standard approaches the simple realized covariance estimator presented here does not resort on the construction of a regular grid, being based on the whole tick-by-tick raw data series. This approach has the twofold advantage of exploiting all the information available in the data and be able to avoid the bias toward zero of the realized covariance. In fact, the claim is that the non-synchronous trading effect induces a bias in the usual covariance measure as a consequence of the synchronization of the two series i.e. as a consequence of the construction of a regular grid in physical time.

The bias of the covariance estimator based on fixed interval returns can be intuitively seen as arising from two distinct effects. First, the absence of trading on one asset in a certain interval produces a zero return for that interval and then artificially imposes a zero value to the cross product of returns inducing a bias toward zero in the realized covariance (which, in its standard version, is simply the sum of those cross products). Secondly, the construction of a regular grid, depending on the frequency of tick arrivals, affects the computation of the realized covariance. For the more liquid assets with higher average arrival rates, the last tick falling in a certain grid interval is typically much closer to the end point of the grid compared to that of a less liquid asset. Any difference in the time stamps between these last ticks in grid for the two assets, will correspond to a portion of the cross product returns which will not be accounted for in the computation of the covariance. This is because for the more liquid asset, the (unobserved) returns corresponding to this time difference will be imputed to the current grid interval while for the less liquid asset such portion of returns will be ascribed to the next grid interval, so that the two will be no longer matched and their contribution to the cross products sum will be lost. This lost portion of covariance in each interval also induces a downward bias in the realized covariance computed with a regular grid; a bias which will also increase with the number of intervals and hence with the frequency.

Under the assumption of no true leads and lags cross-covariance, an unbiased covariance estimator can be computed by simply summing all the cross products of returns which have a non zero overlapping of their respective time span. In other words, a given tick-by-tick return on one asset is multiplied with any other tick-by-tick return of the other asset which has a non zero overlap in time, i.e. which share (even for a very small fraction) the same time interval.

Analytically, this tick-by-tick Realized Covariance estimator can be defined as

$$RC_t = \sum_{s=1}^{M_{i,t}} \sum_{q=1}^{M_{j,t}} r_{i,s}\, r_{j,q}\, I(\tau_{q,s} > 0) \tag{1}$$

with $I(\cdot)$ the indicator function and

$$\tau_{q,s} = \max(0, \min(n_{s+1}, n_{q+1} - \max(n_s, n_q)) \tag{2}$$

being the overlap in time between any two tick-by-tick returns $r_{i,s}$ and $r_{j,q}$.

The simplest way to intuitively show the unbiasedness of this estimator, is by assuming an underling discrete time process, with arbitrary clock time interval δ. In this setting, the expectation of the cross product of two overlapping tick-by-tick returns $r_{i,s},\ r_{j,q}$ can be expressed as a linear combination of the cross-covariances $\gamma(h) = \mathrm{Cov}(r_{i,s},\ r_{j,s-h\cdot\delta})$. But, being all the cross-covariances with $h \neq 0$ equal to zero, it reduces to $\mathbb{E}\left[r_{i,s}\ r_{j,q}\right] = \tau_{q,s}\gamma(0)$. Therefore, the expectation of the tick-by-tick covariance estimator RC_t, is

$$\mathbb{E}\left[RC_t\right] = \left[\sum_{s=1}^{M_{i,t}} \sum_{q=1}^{M_{j,t}} r_{i,s}\, r_{j,q}\, I(\tau_{q,s} > 0)\right] = \sum_{s=1}^{M_{i,t}} \sum_{q=1}^{M_{j,t}} \tau_{q,s}\gamma(0). \tag{3}$$

and given that the sum of all the overlapping intervals $\tau_{q,s}$ in a day is the whole trading day itself, we can conclude that RC_t is an unbiased estimator of the daily covariance.

Hence, loosely speaking, this estimator is unbiased because no portion of covariance will be lost while the portion of cross product which does not overlap will have zero mean. Moreover, avoiding the noise and the discarding of price observations caused by the regular grid interpolation, will considerably reduce its variance. However, in the presence of a fix amount of market microstructure noise, the estimator in this form will not be consistent because, although unbiased, his variance will diverge as the number of observations tends to infinity. For a nice and simple adjustment, based on sub sampling and averaging, that makes this estimator consistent see Palandri (2006).

3 Simulations

In this section we evaluate the performance of different covariance estimators in a simulation environment based on the Lo and MacKinlay's (1990) non-synchronous trading model. In this model the true return of any asset i is given by a single factor model. Considering only two assets, the two return series are then given by

$$r_{i,t} = \mu_i + \beta_i f_t + \epsilon_{i,t} \qquad i = 1, 2 \tag{4}$$

where β_i is the factor loading of asset i, $\epsilon_{i,t}$ represents the idiosyncratic noise of asset i and f_t is the zero mean common factor.

Under the assumptions that the idiosyncratic noises $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are mutually uncorrelated and both uncorrelated with the common factor f_t, the true covariance between the two assets is simply

$$\sigma_{1,2} = \beta_1 \beta_2 \sigma^2_{f,t} \tag{5}$$

where $\sigma^2_{f,t}$ is the variance of the common factor f_t.

In the Lo and MacKinlay's model the common factor f is assumed to be a simple homoskedastic process and, hence, the variance of f is a constant σ^2_f. As a consequence, also the true covariance remains constant. In the version adopted here, however, in order to give more dynamics and realism to the DGP, the common factor f is assumed to follow the stochastic volatility model of Heston (1993) so that, also the true covariance will dynamically change through time.

Therefore, the dynamics of the common factor is given by the following continuous time process

$$df_t \;=\; \left(\mu - \frac{v_t}{2}\right) dt + \sigma_{f,t}\, dB_t \tag{6}$$

$$dv_t = k(\alpha - v_t)dt + \gamma v_t^{1/2} dW_t \tag{7}$$

where $v = \sigma_f^2$ and the initial value $v(0)$ is drawn from the unconditional Gamma distribution of v.

The values of the parameters are chosen so to have a process with zero mean, expected annualized volatility of 15% and satisfying the Feller's condition $2k\alpha = \gamma^2$. Thus, the following annualized values for the parameter are chosen: $\mu = 0$, $k = 8$, $\alpha = 0.0225$, $\gamma = 0.5$ and a correlation coefficient between the two Brownian motions of $\rho = -0.5$. Those parameter values, will remain constant throughout the simulations. The Heston model for the factor and the true return process of the two assets will be simulated at the usual Euler clock of one second.

In the Lo and MacKinlay's model the prices are assumed to be observed with a certain probability $1 - \pi_i$, where π_i is the so called non-trading probability. We found more convenient to express the frequency of the price observations in terms of the corresponding average intertrade duration between ticks[2] τ_i.

Each time a price is observed we simulate market microstructure noises by randomly adding or subtracting half of the spread to the true price. The size of the spread is chosen so to obtain an average level of the noise to signal ratio of the observed returns process equal to one.

In addition to the proposed tick-by-tick estimator, the other covariance measures included for comparison in the simulation are:

- The standard realized covariance computed with an interpolated regular grid of 1 minute returns.
- The standard realized covariance computed with a fix return time interval of 5 minutes.
- The Scholes and William (1977) covariance estimator, which add to the contemporaneous sample covariance of fix interval returns, one lead and lag cross covariance.

[2] For example, a non-trading probability of 90% corresponds to an exponential distribution of the intertrade duration with a mean value of 10 seconds.

To improve the performance of this estimator we chose the frequency of the fix interval returns which seems to provide the best results given the observation frequency of the two assets.

- The estimator proposed by Cohen et al. (1983), which is a simple generalization of the Scholes and Williams estimator, where more than one lead and lag are considered. Here, as in Bollerslev and Zhang (2003) we compute the Cohen et al. estimators with 12 leads and lags and at the frequency which seems to be the best performing given the corresponding simulation set up.

- The Lo and MacKinlay's estimator given by

$$\hat{\sigma}_{1,2} = \frac{1-\hat{\pi}_1\hat{\pi}_2}{(1-\hat{\pi}_1)(1-\hat{\pi}_2)}\text{Cov}\left[r^s_{1,t}, r^s_{2,t}\right] \tag{8}$$

  where $\text{Cov}\left[r^s_{1,t}, r^s_{2,t}\right]$ is the covariance between the observed 1 second returns $r^s_{i,t}$. Contrary to the highly noisy non-trading probability estimation proposed by Lo and MacKinlay's where $\hat{\pi}_1 = \text{Cov}\left[r^s_{1,t}, r^s_{2,t+1}\right]/\text{Cov}\left[r^s_{1,t}, r^s_{2,t}\right]$ and analogously for $\hat{\pi}_2$, we estimate those probabilities by simply counting the observed number of ticks in each day and dividing it by the total number of seconds in the day.

We first simulate 25,000 paths at a moderate observation frequency of $\tau_1 = 30$ seconds and $\tau_2 = 1$ minute. With the factor loadings $\beta_1 = 0.8$ and $\beta_2 = 1.25$, and an average value for the volatility of the common factor of 15% per annum, the true covariance is, on average, 2.25% per annum, which together with the volatilities of the idiosyncratic noises $\sigma_{\epsilon,1} = 0.16$ and $\sigma_{\epsilon,2} = 0.16535$ implied an average correlation of 45%. Given the relatively low frequency of the two series we compute the Scholes and Williams estimator with 3 minutes returns and the Cohen et al., which is able to correct for higher level of bias, at the 20 seconds interval (i.e. at the average frequency of the more liquid asset). The results are reported in Figure 1 and in the left panel of Table 1.

Insert Figure 1 about here

Insert Table 1 about here

With those observation frequencies the 1 minute realized covariance is highly biased: on average it would correspond to a 20% correlation against the true value of 45%. Under these conditions, the 5 minutes realized covariance gives better results both in terms of dispersion and in terms of bias (though a significant bias still exists being the implied correlation equal to 37.5%). Despite the direct estimation of the non-trading probabilities the Lo and MacKinlay's estimator (though unbiased) is extremely inaccurate, probably because of the significant presence of market microstructure noise. With the carefully chosen frequency both the Scholes and Williams and the Cohen et al. estimators are almost unbiased and reasonably accurate. However, the best estimator is clearly the proposed one (termed "All-Ticks" in figures and tables) with no bias and the smallest dispersion.

We repeat the simulation with a higher observation frequency for the two assets, choosing $\tau_1 = 5$ seconds and $\tau_2 = 10$ seconds. Now, the return frequency for the Scholes and Williams estimator is chosen at 30 seconds and that of Cohen et al. at 5 seconds. Figure 2 and the right panel of Table 1 report the results.

The 1 minute realized covariance, though less disperse now, is still significantly biased with an implied average correlation of about 39%. The 5 minutes realized covariance, instead, is unbiased but with a large variance. The higher number of price observations seems to be of little help for the performance of the Lo and MacKinlay's estimator in the presence of market microstructure noise. As before, at the chosen frequencies both the Scholes and Williams and the Cohen et al. estimators are almost unbiased, with the second one being slightly more precise. But, again, the tick-by-tick covariance estimator remains unbiased and the most precise among the estimators considered.

Summarizing the results of this simulation study, the simple tick-by-tick estimator proposed results to be the best performing for both choices of trading frequencies of the two assets. Surprisingly, it also performs favorably compared to the Scholes and Williams and the Cohen et al. estimators even if their return frequency has been chosen according to the simulation settings to give the best results. The proposed tick-by-tick estimator, however, does not require any choice of return frequency or interpolation scheme since it can be directly applied to the raw tick-returns series of any two assets, always providing unbiased results.

4 Empirical application

We apply the proposed tick-by-tick covariance estimator to the bivariate series of S&P 500 and 30 years US Treasury Bond futures. The data are from Price-data.com and we consider the period from January 1990 to October 2003 for a total of 3,391 daily data points. The tick frequency of the S&P 500 data is about two times that of the Bond, the intertrade duration of the former being less than 10 seconds while that of the latter almost 20 seconds.

Unfortunately, since the time stamps of the data in our disposal, are rounded at the 1 minute level, the proposed estimator can not be directly implemented in such a simple way but it requires a slightly different scheme. In fact, the rounding of the seconds to the minute, precludes the knowledge of the correct time ordering among the ticks of the two series inside the 1 minute interval, which is necessary for the application of the tick-by-tick estimator. To overcome this problem, we construct the tick-by-tick estimator by simply considering only the first and last ticks of each 1 minute interval. This version will hence be termed "First-Last" tick-by-tick estimator.

Using a subsample of the total number of ticks employed by the "All-Ticks" estimator, we expect the "First-Last" to be less efficient. In order to evaluate this loss of efficiency of the modified tick-by-tick estimator on this type of data, we perform a simulation study which tries to reproduce as much as possible the econometric properties of the two empirical series, i.e. the parameters of the simulation will be chosen to match as closely as possible the empirical observation frequencies, level of volatilities, noise structure and intensities and so on.

Therefore, with asset 1 mimicking the S&P and asset 2 the US bond, the following configuration of the parameters are chosen: $\tau_1 = 8$ seconds, $\tau_2 = 18$ seconds, an average annualized volatility of about 20% for asset 1 and 10% for asset 2 and with a correlation of 30% . From the empirical study of the tick-by-tick series of those assets, we found significant departure from the standard i.i.d. assumption on the structure of the market microstructure noise. In fact, studying the autocorrelation of the tick-by-tick returns of those series, more complex structures than those of a simple MA(1) expected under the standard i.i.d assumption, were found. We suggest that such autocorrelation patterns of the tick-by-tick returns could be explained by assuming more complex ARMA structure for the microstructure noise. In particular, the noise structure is closely reproduced by introducing an MA(2) for the asset 1 mimicking the S&P 500 with $\theta_1 = 0.85$, $\theta_2 = 0.25$ and a noise to signal of 0.45, and a strong oscillatory AR(1) with $\phi_1 = -0.6$ and noise to signal of 0.6 for the asset 2 corresponding to the US bond.

Insert Figure 3 about here

Insert Table 2 about here

Figure 3 and Table 2 reports the results of the 25,000 simulations. As expected, the First-Last estimator results to be less precise than the All-Ticks. However, this loss of efficiency due to the lower number of ticks employed, is contained and the First-Last remains the best performing measure compared to the other covariance estimators.

Insert Figure 4 about here

Applying to the S&P 500 and US bond series the First-Last estimator we obtain the realized covariance time series shown in Figure 4. To better appreciate the remarkable difference between the tick-by-tick realized covariance and the standard cross product of daily returns (the usual proxy for daily covariance in standard multivariate volatility models) both measures are plotted together on the same scale.

Combining the First-Last covariance measure together with a tick-by-tick realized volatility estimator we are now able to obtain a realized correlation measure where both the volatilities and the covariance are computed from tick-by-tick data. Here, we employ the Multi-Scales DST realized volatility estimator proposed by Curci and Corsi (2006)[3]. Figure 5 shows the time series of 3,391 daily tick-by-tick correlations from 1990 to 2003[4].

[3] We also experiment with the Two Scales estimator of Zhang, Mykland and Aït-Sahalia (2004) obtaining very similar results.

[4] In two days (out of 3,391) the estimated realized correlation resulted to be slightly outside the [-1, 1] correlation boundary, in those two cases we arbitrary set the correlation absolute value to 0.9999.

Insert Figure 5 about here

Simply looking at the correlation dynamics, there seems to be two important changes of regime around the end of 1993 and the end of October 1997. In fact, the correlation between the two series oscillates around a positive stable value of about 20% until '94 and around 40% from '94 to '97, while after the end of '97 the correlation starts to exhibit a stronger dynamics and becomes predominantly negative. Note that in the first case we report a positive regime-shift, whereas in the second case a negative regime-shift.

Insert Figure 6 about here

These structural changes in the dynamics of the correlation between S&P and US bond is also apparent from the different behavior of the autocorrelation function computed on the three periods January 1990 to the end of 1993 (from now on called '90-'94 period), from the beginning of 1994 to the end of 1997 ('94-'98 period) and from the beginning of 1998 to the end of the sample ('98-'03 period) (Figure 6). In the first '90-'94 period the level of the autocorrelation is very low and quickly decay. While in the second period '94-'97 the autocorrelation level and its persistence significantly increase. Finally, after the end of '97 the memory of the process, in particular the short and medium one, rises further. Hence, there seems to be a consistent increase in the memory persistence of the stock-bond correlation in the most recent years. It should be noticed that this new stylized fact of the stock-bond correlation wouldn't be so easily identifiable without the employment of high frequency data and a precise realized correlation measure.

Another interesting effect revealed by Figure 6 is how the structural change affects the global autocorrelation function computed on the full sample inducing an artificially high level in the autocorrelation coefficients. Nonetheless, even without this structural break effect, the autocorrelation function of the realized correlation remains highly persistent as shown by the separated sub sample autocorrelation functions.

To end this section, we provide some more statistical evidence and an economic intuition to support the visual impression of regime changes in stock-bond correlations. To this purpose, we estimate the time-varying dynamics of realized stock-bond correlations with two different models allowing for regime changes: the regime-switching specification of

Gray (1996), in the framework of regime-switches governed by an unobservable Markovian state variable firstly introduced by Hamilton and Susmel (1994), and the tree-structured model introduced by Audrino and Bühlmann (2001), in the framework of regimes defined using thresholds for some relevant predictor variables.

Insert Table 3 about here

As it is shown in Table 3, using both models we find strong empirical evidence that realized correlations are subjected to regime changes: both Akaike and Schwarz Bayesian information criterium (AIC and BIC, respectively) clearly favor models allowing for different regime specifications. In particular, using the tree-structured approach, where regimes are better identifiable as particular cells of the predictor space, we identify approximately the same three different regimes that were apparent by the visual inspection of the realized correlation series. In fact, the most important predictor in the analysis of the realized correlation series is time, and searching for breaks in time we find exactly the regimes '90-'94, '94-'98 and '98-'03. Moreover, there seems to be some important predictive information included also in the original return series of S&P500 returns and 30 years US Treasury Bond returns.[5]

The two significant regime changes we identified for realized stock-bond correlations can be naturally associated with two big financial crises: the Western European monetary crisis of 1992-1993, mere outcome of a self-fulling foreign financial panic against slowing economies with rising unemployment, and the Asian crisis of 1997-1999. Stock-bond correlation dynamics drastically changed in reaction to the different behavior of investors during or after such bad economic conditions ("flight-to-quality" phenomenon, different allocation between corporate and treasury bonds, different liquidity levels, ...). As a consequence, in the first case we have a significant increase in stock-bond correlations, whereas in the second case correlations between stocks and bonds drop severely, changing sign from positive to negative.

[5]Detailed results of this analysis can be obtained from the authors upon request.

5 Modeling realized correlations

Corsi (2004) and Corsi et al.(2006) recently proposed a class of time series models called Heterogeneous Autoregressive (HAR) that seems to successfully achieve the purpose of modelling the long memory behavior of financial variables in a very simple and parsimonious way.

The basic idea stems from the so called "Heterogeneous Market Hypothesis" (Müller et al.1993), which explains the long memory observed in the volatility as the superimposition of only few processes operating on different time scales. Hence, Corsi(2004) proposed a stochastic additive cascade of three different realized volatility components corresponding to the three main different time horizons operating in the market (daily, weekly and monthly). This stochastic volatility cascade leads to a simple AR-type model in the realized volatility with the feature of considering realized volatilities defined over different time horizons (the HAR-RV model). Although the HAR model doesn't formally belong to the class of long-memory models, it is able to reproduce a memory decay which is almost indistinguishable from that observed in the empirical data.

The HAR model for the daily realized volatility is defined by

$$\mathbb{E}_t \left[RV_{t+1}^{(d)} \right] = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} \tag{9}$$

where $RV_t^{(d)}$, $RV_t^{(w)}$, and $RV_t^{(m)}$ are respectively the daily, weekly and monthly annualized realized volatilities obtained as simple averages of the daily realized volatility. The notation $\mathbb{E}_t [\cdot]$ indicates as usual the conditional expectation given the information up to time t.

The empirical evidence on the high degree of persistence of correlations, suggests that the parsimonious HAR model could also be successfully applied to model the time series of realized correlations

$$\mathbb{E}_t \left[RC_{t+1}^{(d)} \right] = a + b^{(d)} RC_t^{(d)} + b^{(w)} RC_t^{(w)} + b^{(m)} RC_t^{(m)} \tag{10}$$

where $RC_t^{(d)}$, $RC_t^{(w)}$, and $RC_t^{(m)}$ are respectively the daily, weekly and monthly realized correlations. Many extensions of this simple HAR model for realized correlations can be envisaged: include realized volatilities at different frequencies as explanatory variables for correlations or add matrices of cross-product returns measured over the three different horizons with possibly different coefficients for the positive and negative values to account for asymmetric effects.

Insert Table 4 about here

Insert Table 5 about here

To analyse the dynamic evolution of the realized correlation process, we first estimate the HAR(3) model, together with a simple benchmark AR(1) model, on the full sample and on the three different sub periods (regimes) '90-'94, '94-'98 and '98-'03 to investigate possible differences in (local) regime-dependent behavior. Table 4 reports the results of the 1 day ahead in sample prediction in the different samples. A first thing to be noticed is the significantly higher value of the R^2 for the full sample compared to that of any of the sub samples. This is true for both models and it is akin to the artificially high level of the autocorrelation function induced by the structural breaks (as showed in Figure 6). Hence, the exceptionally high values of the R^2 (75% for the AR(1) and 81% for the HAR(3)) should be considered with extreme caution.

It is also interesting to note that while the R^2 in the three sub periods tends to steadily increase also the RMSE and MAE tends to increase over time. This apparently contradictory result could be due to an increase of the total variance of the process which is however accompanied by an increase in the persistency of the process and hence in a better ability of the models to explain and predict it (hence producing a larger R^2). In other words, although the total variance of the process increases, the noise to signal ratio decreases.

Table 5 reports the values of the estimated coefficients of the HAR(3) process on the full sample and sub periods, showing the high significance level of the three heterogeneous components (daily, weekly and monthly). Consistently with the increase of the memory persistence depicted in Figure 6, the weights (i.e. the loadings of the different time horizons) in the HAR(3) model change accordingly in the sub periods indicating how this simple model is able to easily fit and reproduce very different degrees of memory persistence.

Insert Figure 7 about here

In order to study better the time evolution of the weights of the three different market components, we estimate the HAR(3) model on a rolling window of 1,000 days. Note, that this is a very simple strategy that allows us to take implicitly regime changes in realized correlations into account. In the bottom panel of Figure 7, we show how already from the beginning of the sample (which is now 1994 because the first 4 years are used in the burn in of the 1,000 days rolling window) the weight of the daily component steadily increases until 2002 going from about 10% (the smallest one of the three) to about 40% (the highest one). Such an increment is only partially compensated by a decline in the weekly component (from 40% to 30%), while the monthly component remains substantially the same in the sub samples. The growth of the daily component weight could be responsible for the increase in the short period memory observed in the autocorrelation function of the last part of the sample. This analysis shows how the identification of the different market components and the study of their dynamics, which is made possible by the HAR model, can help explain (and maybe also predict) interesting properties and dynamics of realized correlations.

Insert Table 6 about here

A comparison of the in sample and out of sample forecasting performance of the HAR(3) model with respect to different standard models is shown in Table 6. The benchmark models are the random walk model (RW), the AR(1) and AR(3) processes and the ARMA(1,1) model. Once gain, note that we implicitly take into account the presence of structural breaks by estimating all the models on a rolling windows of 1,000 daily observations, making 1 day ahead predictions at each step. The results of the table clearly show the better forecasting performance of the HAR(3) model which, in terms of the out of sample RMSE and MAE, outperform the AR(1) by more than 20%, the AR(3) by more than 12% and the ARMA(1,1) by more than 8%.

The last row of Table 6 reports a slight extension of the HAR(3) model where, in the rolling window estimation, more weights are given to the more recent observations with respect to the more distant ones. The idea is to have the estimated parameters of the HAR(3) to adapt faster to the structural changes of the process. This idea is simply implemented by performing a rolling window WLS regression with exponentially decaying weights in place of the equally weighted OLS. Moreover, a dynamical variance targeting for the constant is also implemented to have the intercept to quickly adapt to the shifts in the mean. We refer to this model as Weighted HAR (WHAR). The results of the last row of Table 6 show that WHAR(3) outperforms HAR(3), although the performance improvement is relatively small (around 1%).

6 Conclusions

In this paper we extend the approach of directly using all the available tick-by-tick data to the realized covariance and realized correlation estimation. As for the realized volatility, the presence of market microstructure can induce significant bias in standard realized covariance measures computed with artificially regularly spaced returns. Contrary to these standard approaches we adopt a very simple and unbiased realized covariance estimator which does not resort to the construction of a regular grid, but directly and efficiently employs the raw tick-by-tick returns of the two series. Montecarlo simulations calibrated on realistic conditions show that this simple tick-by-tick covariance estimator possesses no bias and the smallest dispersion, resulting to be the best performing measure among the covariance estimators considered in the study. Combining this realized covariance measure together with the tick-by-tick volatility estimator we obtain a realized correlation measure where both the volatilities and the covariances are computed from tick-by-tick data.

In the empirical analysis performed on S&P 500 and US bond data, we investigate the time-varying dynamics of the time series of realized correlations constructed using tick-by-tick data finding empirical evidence that such dynamics are subjected to regime changes in reaction to big financial crises. In a second step, we apply the Heterogeneous Autoregressive HAR model to the tick-by-tick realized correlation series obtaining highly significant coefficients for all the three heterogeneous components and remarkably good out of sample forecasting performance.

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COVARIANCE ESTIMATION SIMULATION RESULTS

	$\tau_1 = 30$ sec, $\tau_2 = 1$ min			$\tau_1 = 5$ sec, $\tau_2 = 10$ sec		
	bias	std	RMSE	bias	std	RMSE
1 min no correction	-1.2438	1.1592	1.7002	-0.2921	0.4352	0.5241
5 min no correction	-0.3786	0.8783	0.9564	-0.0599	0.7233	0.7258
All-Ticks	0.0095	0.6930	0.6931	0.0057	0.2988	0.2988
Scholes-Williams	-0.0685	1.0148	1.0171	-0.0168	0.4229	0.4233
Cohen 12 leads-lags	-0.0795	0.9925	0.9957	-0.0121	0.4919	0.4920
LoMac-Kinlay	-0.0074	7.8983	7.8983	0.0143	1.4013	1.4014

Table 1: The table reports the mean, standard deviation and RMSE of the estimation errors on the annualized covariance (on average 2.25%) obtained at different observation frequencies for the two assets and a noise to signal ratio equal to one.

CALIBRATED S&P-US BOND SIMULATION RESULTS

	bias	std	RMSE
1 min no correction	-0.1746	0.2729	0.3240
5 min no correction	-0.0382	0.3385	0.3406
Scholes and Williams Cov	-0.0076	0.2625	0.2626
10 sec Cohen 12 leads-lags	-0.0061	0.2751	0.2752
First-Last Tick	-0.0009	0.2018	0.2018
All Ticks	0.0019	0.1580	0.1581

Table 2: The table reports the mean, standard deviation and RMSE of the estimation errors on the annualized covariance for a simulation set up which reproduces the statistical properties of the S&P 500 and US bond future data.

REGIME CHANGES IN STOCK-BOND CORRELATION

Model	N° of Param	AIC	BIC
ARMA(1,1)-GARCH(1,1)	6	-3054.994	-3018.221
Regime-Switching with 2 regimes	16	-3164.220	-3066.158
Tree-structured GARCH(1,1) with 3 regimes	18	-3184.952	-3074.632

Table 3: Goodness-of-fit statistic results: number of parameters, Akaike information criterium (AIC) and Schwarz Bayesian information criterium (BIC) for searching for regime changes in the daily realized stock-bond correlation time series between 1990 and 2003. The models that we consider are a Markovian regime-switching specification with two regimes, a tree-structured threshold regime specification with (an endogenously estimated number of) three regimes, corresponding to the different time intervals '90-'94, '94-'98 and '98-'03, and a global benchmark ARMA(1,1)-GARCH(1,1) model with no regimes. Note that the local dynamics of the realized correlations in the different regimes also follows an ARMA(1,1)-GARCH(1,1) process.

S&P - US BOND CORRELATION IN SAMPLE RESULTS

	AR(1)			
	full sample	'90 - '94	'94 - '98	'98 - '03
Mincer-Zarnowitz R^2	0.7511	0.0573	0.2129	0.5879
RMSE	18.0820	14.7759	16.3873	17.1867
MAE	13.9322	11.0718	12.5828	13.6263
	HAR(3)			
	full sample	'90 - '94	'94 - '98	'98 - '03
Mincer-Zarnowitz R^2	0.8143	0.0953	0.2678	0.6622
RMSE	15.6271	14.5023	15.8050	15.5685
MAE	11.9408	10.8208	12.0230	12.2302

Table 4: 1 day ahead in sample forecast results of S&P500 and 30-years US Treasury bond realized correlations in the period between 1990 and 2003. Results are reported for the full sample and the three subperiods (regimes) identified in the in sample analysis.

HAR(3) ESTIMATED COEFFICIENTS

	full sample	'90 - '94	'94 - '98	'98 - '03
const	0.0002 (0.9438)	0.0708 (0.0000)	0.0862 (0.0006)	-0.0149 (0.0238)
$RC^{(d)}$	0.2667 (0.0000)	0.1198 (0.0338)	0.2600 (0.0000)	0.3245 (0.0000)
$RC^{(w)}$	0.3646 (0.0000)	0.2394 (0.0019)	0.3163 (0.0000)	0.3317 (0.0000)
$RC^{(m)}$	0.3489 (0.0000)	0.2818 (0.0039)	0.2028 (0.0345)	0.2919 (0.0000)
$\sum RC^{(\cdot)}$	0.9803	0.6411	0.7791	0.9483

Table 5: In sample estimation of the Newey-West adjusted least-squares regression of HAR(3) model for the S&P and 30-years US Treasury bond realized correlation (p-values are given in parentheses).

COMPARATIVE FORECASTING RESULTS

	In sample			Out of sample		
	RMSE	MAE	MedianAE	RMSE	MAE	MedianAE
RW	1.1981	1.2024	1.2201	1.4152	1.4046	1.3897
AR(1)	1.1571	1.1667	1.2010	1.2378	1.2591	1.2816
AR(3)	1.0470	1.0480	1.0756	1.1263	1.1317	1.1415
ARMA(1,1)	1.0102	1.0090	1.0250	1.0842	1.0868	1.0761
HAR(3)	1	1	1	1	1	1
	(15.627)	(11.941)	(9.536)	(16.151)	(12.401)	(9.996)
WHAR(3)				0.9909	0.9929	0.9868

Table 6: Comparative results of 1 day ahead forecasts of S&P - US Bond realized correlations expressed as ratios to the benchmark heterogeneous autoregressive HAR(3) values (reported in parentheses). The models considered in the analysis are a simple random walk RW, standard autoregressive AR models of order 1 and 3, an autoregressive moving average ARMA(1,1) model and an extension of the HAR model where more weights are given to more recent observations in the rolling window WHAR(3). The forecasting time period is between 1994 to 2003. Out of sample forecasts are computed using a rolling window of 1,000 days to implicitly take into account regime changes.

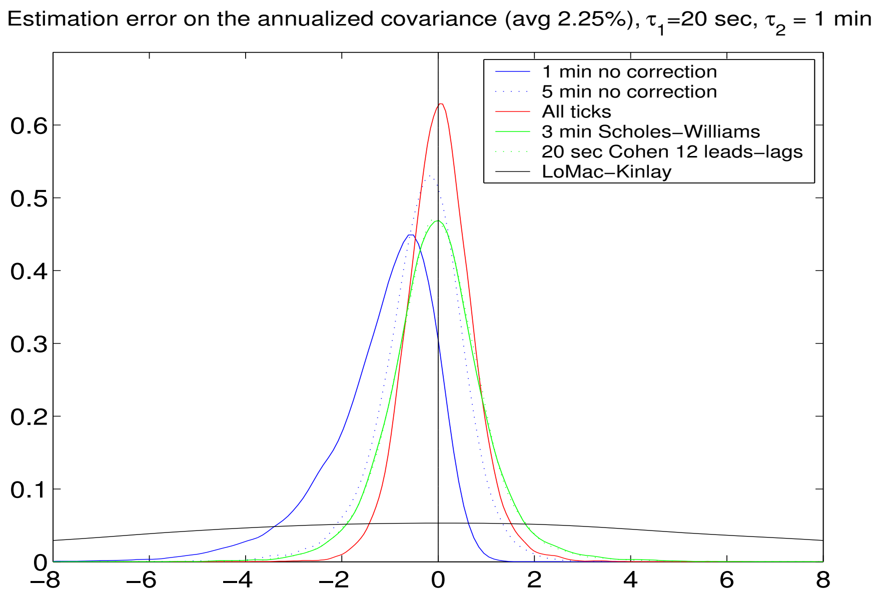

Figure 1: Comparison of the pdf of the covariance estimation errors with noise to signal one and average observation frequencies $\tau_1 = 30$ seconds and $\tau_2 = 1$ minute.

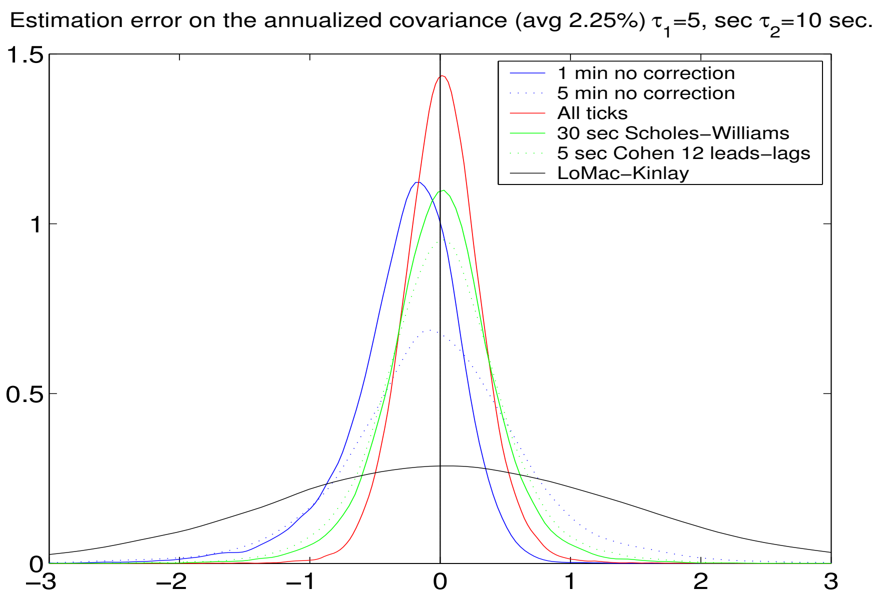

Figure 2: Comparison of the pdf of the covariance estimation errors with noise to signal one and average observation frequencies $\tau_1 = 5$ seconds and $\tau_2 = 10$ seconds.

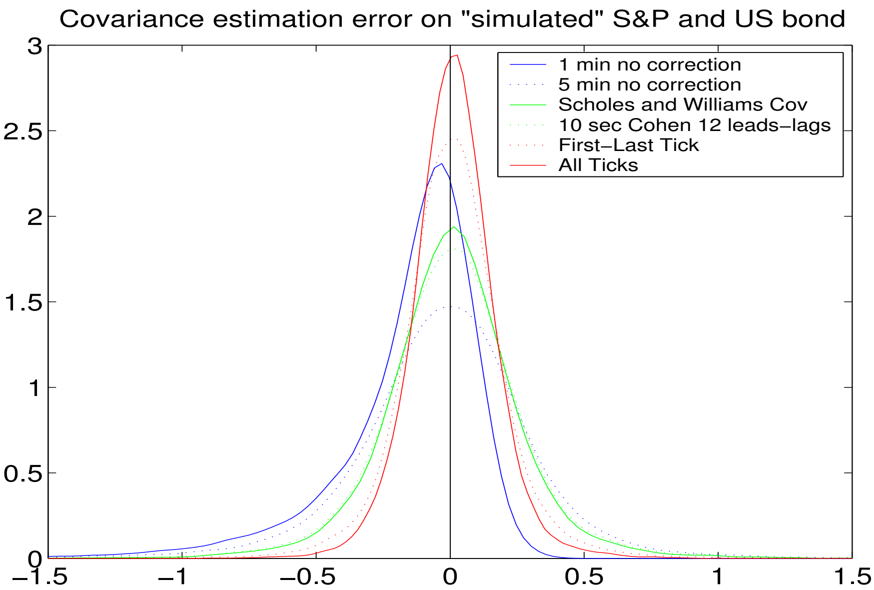

Figure 3: Comparison of the pdf of the covariance estimation errors for a simulation set up which reproduces the statistical properties of the S&P 500 and US bond future data.

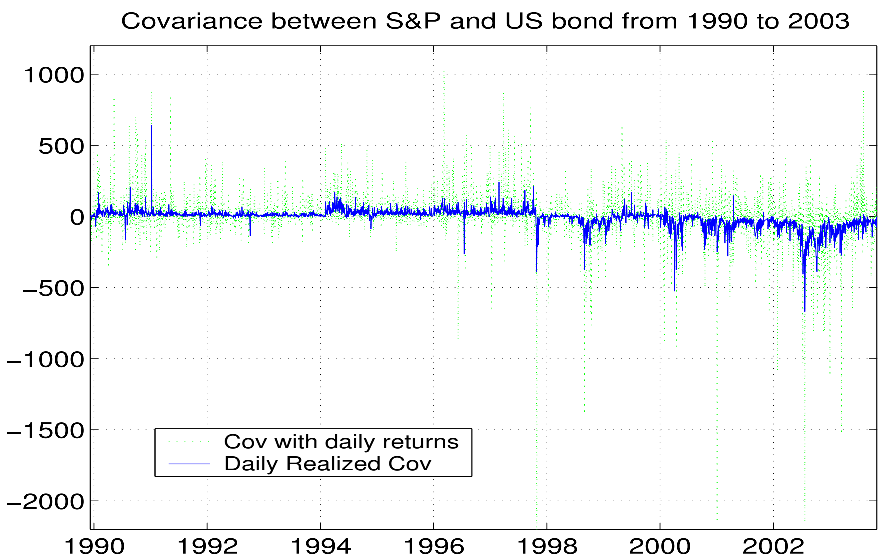

Figure 4: Time series of daily realized covariances constructed using tick-by-tick data and the daily cross product returns of S&P 500 and 30-years US Treasury bond from 1990 to 2003.

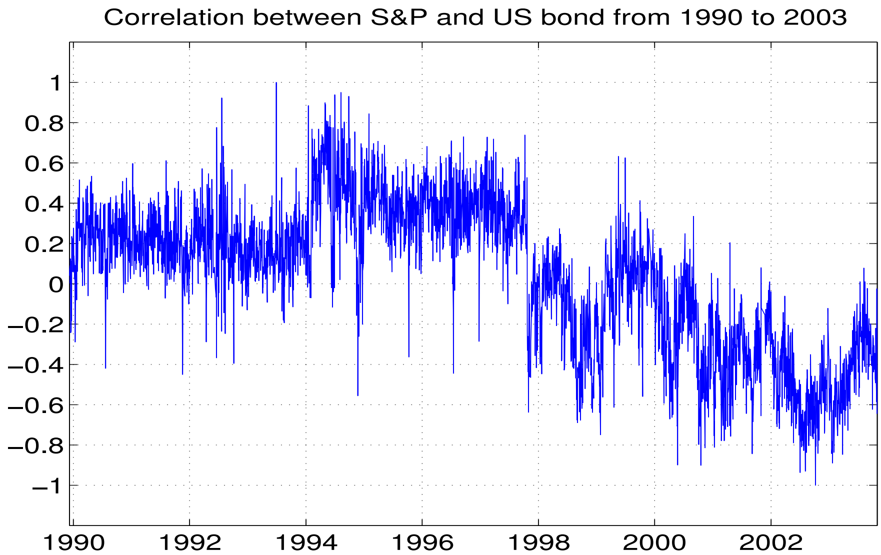

Figure 5: Time series of daily S&P 500 and 30-years US Treasury bond realized correlations constructed using tick-by-tick data. The time period under investigation goes from 1990 to 2003.

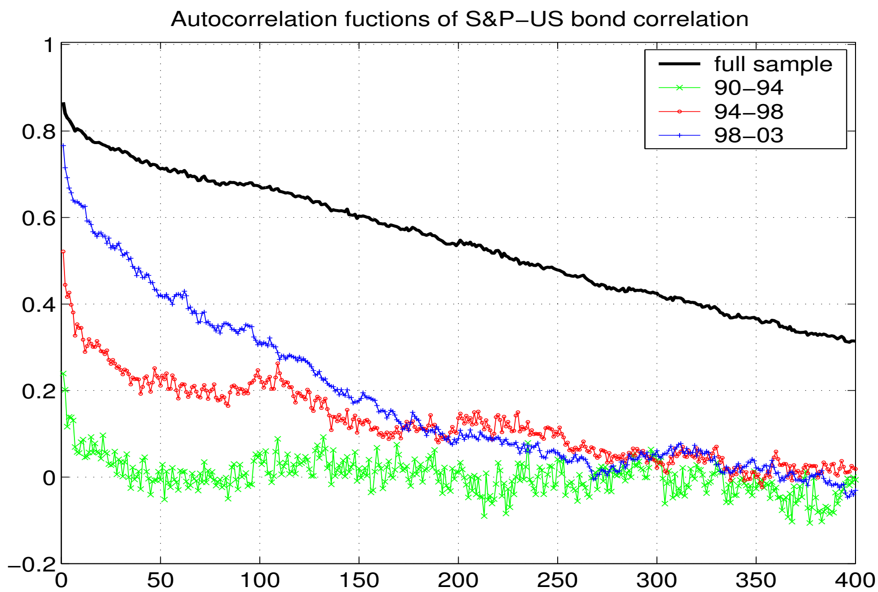

Figure 6: Autocorrelation functions of the S&P 500 and 30-years US Treasury bond realized correlation for the full sample 1990-2003 and the three sub samples periods (regimes) '90-'94, '94-'98 and '98-'03.

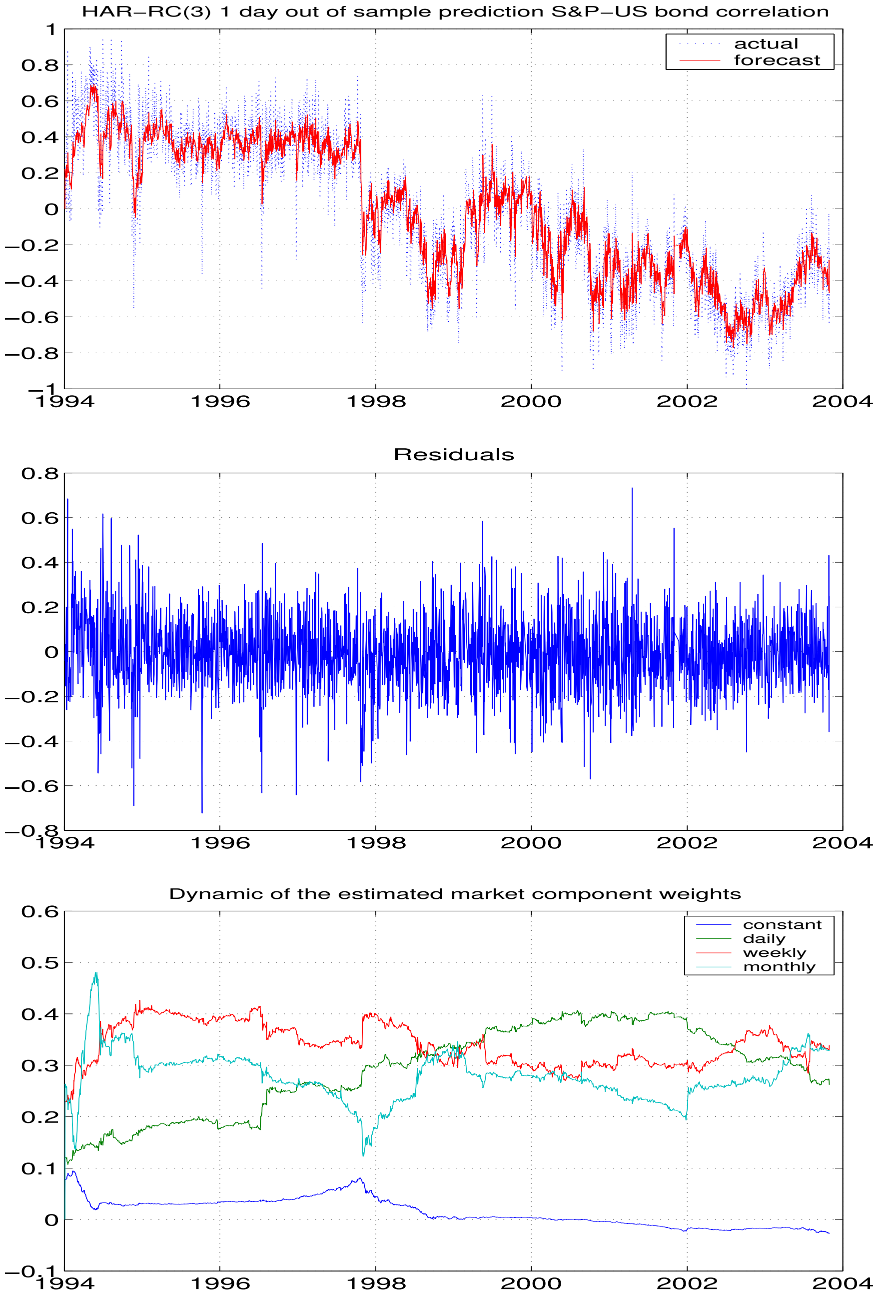

Figure 7: Top: comparison of actual (dotted) and out of sample prediction (solid) of the HAR(3) model for daily realized correlations. Middle: realized correlation residuals obtained from the HAR(3) model fit. Bottom: time evolution of the regression coefficients (loadings) of the three different time frequencies in the HAR(3) model.