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Hubbert's Oil Peak Revisited by a Simulation Model

Pierre-Noël Giraud, Aline Sutter, Timothée Denis & Cédric Léonard

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Cerna, Centre d'économie industrielle
MINES ParisTech
60, boulevard Saint Michel
75272 Paris Cedex 06 – France
Tél. : 33 (1) 40 51 90 00

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HUBBERT'S OIL PEAK REVISITED BY A SIMULATION MODEL

Working paper. Comments welcomed.

Pierre-Noël Giraud	CERNA Mines ParisTech	+33 (0)1 40 51 92 06	pierre-noel.giraud@ensmp.fr
Aline Sutter	EDF-R&D	+33 (0)1 47 65 16 61	aline.sutter@edf.fr
Timothée Denis	EDF-R&D	+33 (0)1 47 65 40 44	timothee.denis@edf.fr
Cédric Léonard	EDF-R&D	+33 (0)1 47 65 53 25	cedric.leonard@edf.fr

Abstract

As conventional oil reserves are declining, the debate on the oil production peak has become a burning issue. An increasing number of papers refer to Hubbert's peak oil theory to forecast the date of the production peak, both at regional and world levels. However, in our views, this theory lacks microeconomic foundations. Notably, it does not assume that exploration and production decisions in the oil industry depend on market prices. In an attempt to overcome these shortcomings, we have built an adaptative model, accounting for the behavior of one agent, standing for the competitive exploration-production industry, subjected to incomplete but improving information on the remaining reserves.
Our work yields challenging results on the reasons for an Hubbert type peak oil, lying mainly "above the ground", both at regional and world levels, and on the shape of the production and marginal cost trajectories.

Keywords : oil, economics, exploration, production, peak, Hubbert, adaptative, simulation, fundamental, microeconomics, reserves

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Chapter 1

Introduction

The main difficulty about forecasting the future of the oil market is to anticipate the date when the demand for liquid fuels will go short of conventional oil. At that time, the oil market price must reach a level that renders an abundant production of substitutes profitable, since the demand for oil products, particularly transportation fuels, will keep increasing. The volume of the reserves of conventional oil still in the ground whose exploration plus extraction costs are below the production cost of an abundant substitute, is indeed limited. Therefore, the annual production of conventional oil is bound to level off and reach a peak, the so called "peak oil". It will then decrease till exhaustion of the reserves. All the debate is thus about the shape of the production profile till the end of conventional oil.

The pioneer papers on peak oil are those of Marion King Hubbert, a geophysicist ([14, 15]). In 1956, at a meeting of the American Petroleum Institute held in San Antonio, Texas, he forecast that the oil production in the USA (the "lower 48 states", i.e. outside Alaska and deep water) would peak in 1970 and that the production profile along time would be rather symmetrical. Fifteen years later, in 1971, the peak was reached. Hubbert had made an error of just one year and became famous in the oil industry.

Hubbert's more general thesis was that the production profile over time in every oil province is bound to follow a bell-shaped symmetrical curve, with the maximum level of production being reached when about half of the total reserves of the province have been produced.

Since then, this type of production profile has been noted in other oil provinces ([8]). As the reserves of conventional oil have been declining for years, and since new discoveries do not offset the depletion of producing fields, the debate about the world production peak date has become a burning issue. Many oil geologists, engineers and even economists use Hubbert's thesis to forecast this peak oil at the world level, by aggregating the production profile of the main oil provinces, including the gulf area in the Middle east, these provinces being all supposed to follow a Hubbert's type production profile ([2, 8, 9]). In most of the academic literature since Hubbert's papers and reports (see [19] for a brief survey, and [10, 13]), the most convincing reasons given for the production profile being time deterministic and symmetrical are the following: the geological knowledge of any oil province increases with the cumulative amount of oil discovered. Consequently, the probability of discovering a new barrel of oil in the province increases first with the cumulative discoveries, and the cost of discovering oil decreases. Thus, the rate of discoveries (the amount of oil discovered per period of time) first increases with time. Note that this assumption implies an hypothesis on the amount of the exploration expenses: it should at least remain constant over time. But on the other hand, the more oil has already been discovered in a given province, the less remains to be discovered, and consequently the probability of discovering a new barrel then decreases, leading to an increase in exploration cost. Therefore, at a certain

point of time, the rate of discoveries peaks and then decreases.
The supporters of Hubbert's thesis claim that the combination of these two factors results in the cumulative discoveries profile over time being a logistic curve and the rate of discoveries a symmetrical bell-shaped curve, derived from the former. The other important claim is that the production rate (i.e. the amount of oil produced per period of time) follows the same symmetrical bell-shaped curve, simply translated by the time lag (supposed constant over time) necessary to put the discovered oil into production. One can see that Hubbert's thesis relies on three different hypothesis:

1. When the exploration expenses remain constant over time, the cumulative discoveries follow a pure symmetrical logistic curve and the discovery rate (the amount of annual discoveries in a discrete model) follows a bell shaped curve, symmetrical around the line where half of reserves have been discovered. Illustrations of such cumulative discoveries and discovery rates curves are given on the two first charts of Figure 1.1.

2. The exploration expenses remain constant over time both in every oil province and at the world level, i.e. neither depends on the market price of oil, nor, in one given province, on the discovery of new oil provinces soon proving to be more attractive for exploration than the former ones.

3. The time lag between discovery and production is constant (this constant time lag 'TL' is illustrated on the third chart of Figure 1.1).

Under these assumptions, it is not surprising to conclude that the production profile, both at the province and the world level, is purely time-deterministic and driven only by geological factors, the so called "under the ground" factors.
All these assumptions can be discussed. Why should the cumulative discovery result from a simple symmetrical logistic growth, and not a more complex one, like some growth paths observed in other natural capital dynamics, as described by [20, 16] (see an example on Figure 1.2, charts 1 and 2)? We won't discuss this first assumption. Let's trust the geologists and petroleum engineers on this point. But as regards the two others, why should an economist adopt them? An economist would at least guess that the exploration level, in a specific province and all the more at the world level, is elastic to oil prices. Indeed, since the beginning of wide oil price fluctuations with the first oil shock in 1973, there is strong evidence of such a positive and high elasticity ([5, 1]). Moreover, an economist would guess that the time lag between discovery and production depends on the extraction cost of the discovered oil field and on the market price. At the world level, if the market price of oil increases, exploration expenses will increase, and reserves at higher costs may be more rapidly put into production. Everything else being equal, the peak oil will occur later and will be higher. Chart 3 of 1.2 illustrates how increasing with time exploration expenses and different time lags between discovery and production ('TL1' and 'TL2') yield production rate curves that are not symmetrical.

In fact, according to different professional sources, estimates of the date of the peak still vary widely from 2010 to 2040 or more ([3, 18]. This is not mainly due to different assumptions on the remaining reserves of oil. Different assumptions on the shape of the production profile also matter. Although increasingly adopted in academic papers on oil prices ([19, 13]), Hubbert's peak oil theory still lacks, in our view, micro-economic foundations.

This paper is an attempt to overcome these shortcomings, while acknowledging and providing an explanation of Hubbert's remarkable success in forecasting the production profile of the lower 48 states in the US. We present a model where the production and marginal cost profiles of oil

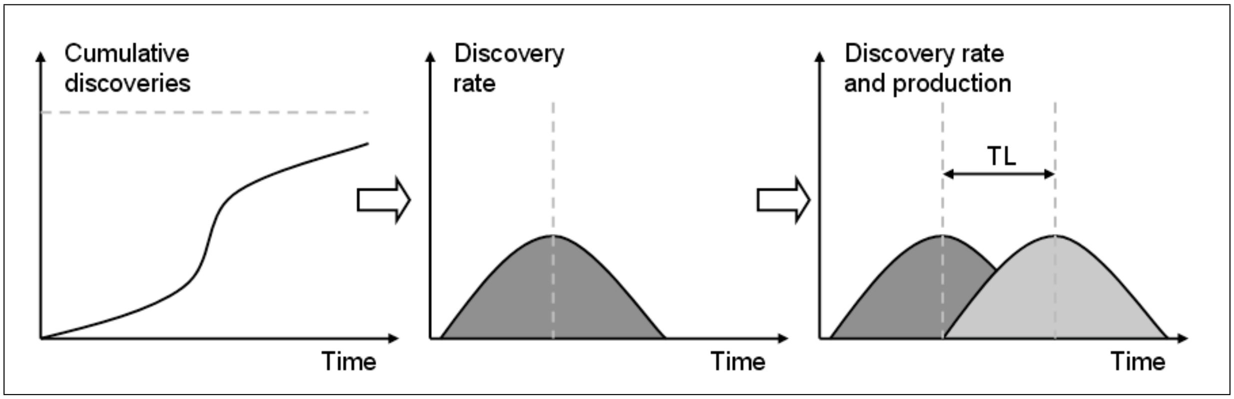

Figure 1.1: Resulting production profile with Hubbert's hypothesis

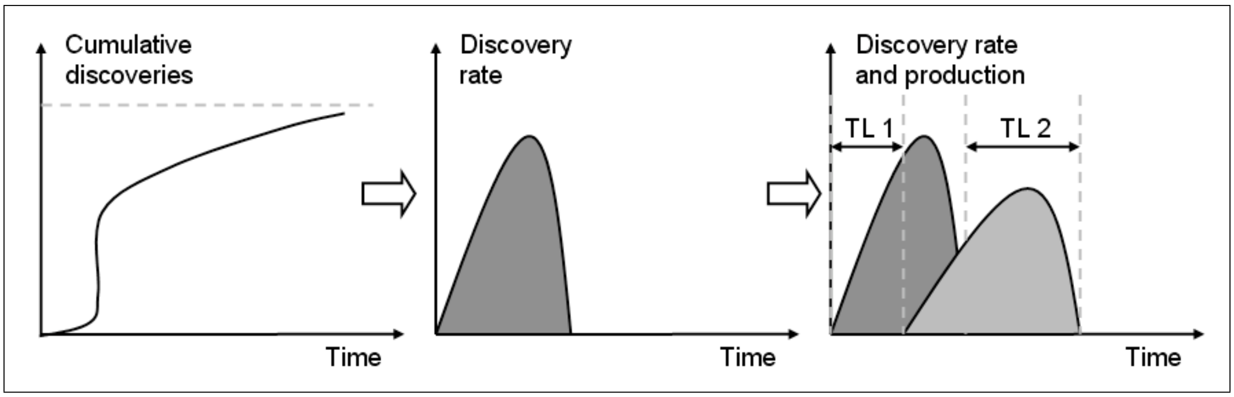

Figure 1.2: Resulting production profile with Hubbert's hypothesis relaxed

are explained by the rational microeconomic behavior of oil exploring and producing companies facing incomplete, but improving information on the remaining reserves.

The second section describes the logic of the model (the underlying equations being detailed in appendix 1). Section 3 briefly presents the data used for our numerical applications (detailed in appendix 2). The first subsection of section 4 deals with a competitive oil industry exploring one global area. Firstly, we analyse the results obtained without any constraint on the exploration expenses. In a second stage, we introduce such a constraint, showing, through our simulation outcomes, that a Hubbert oil production peak can happen at a world level only when a strong exogenous constraint curbs the investments in exploration. In the second subsection of section 4, we represent two different oil provinces, one of them appearing progressively more promising for exploration than the other, and we set a constraint on exploration in the most favorable area. We therefore simulate a market power of the states having the cheapest and most abundant reserves of oil, like the Middle East, and study its impact on the production paths, both at world and regional levels. Finally, we summarize the conclusions and open paths for future research based on this type of model in Section 5.

Chapter 2

Model description

The economic hypothesis of our adaptative simulation model are presented hereafter. Appendix 1 gives the underlying equations and the algorithms of the minimization problems solved at each time step.

2.1 Basic structure of the model

Two types of agents are represented in the model : explorer-producers and reserve owners. One single explorer-producer agent represents a set of competing oil exploration-production firms, sharing a common knowledge of the remaining reserves. The reserve owners are the states controlling the rights to explore and produce oil on their territory. In this paper, the market for reserves is not modeled. Therefore, we will focus on the evolution of the marginal production cost along time, but not on the market price itself, which would be, in a competitive market, the sum of:

- the marginal production cost,
- the scarcity rent asked by the reserve owner of the marginal field.

For simplicity, the demand is assumed to be inelastic to price. Actually, the price elasticity of oil demand is very low in the short-run, and even relatively low in the long-run ([6, 1]). Moreover, we want to explore a situation where conventional oil production has to be complemented by a substitute, since the demand for transportation fuels is steadily growing ([11]).

The general structure of the model is the following (Figure 2.1). At each time step, namely each year, the explorer-producer computes the exploration program whose expected outcome minimizes the cost of meeting the demand, according to his current evaluation of the remaining reserves. He then performs this exploration program, optimal in expectation. The exploration process returns a random result. The explorer-producer then adds the newly discovered oil to its reserves portfolio and puts oil into production in the ascending extraction cost order. The exploration outcome of year t improves the quality of the evaluation of the remaining reserves for the next year. The uncertainty on the remaining reserves, and thus on the substitution date is therefore decreasing with time.

After a certain number of years, the explorer-producer needs to exploit the abundant substitute to meet the demand. The "substitution date" is then reached and the marginal cost is the production cost of the substitute.

Thus, the represented agent explores to minimize the cost of meeting demand only for the following time steps. Our work is therefore a simulation model, as there is no inter-temporal

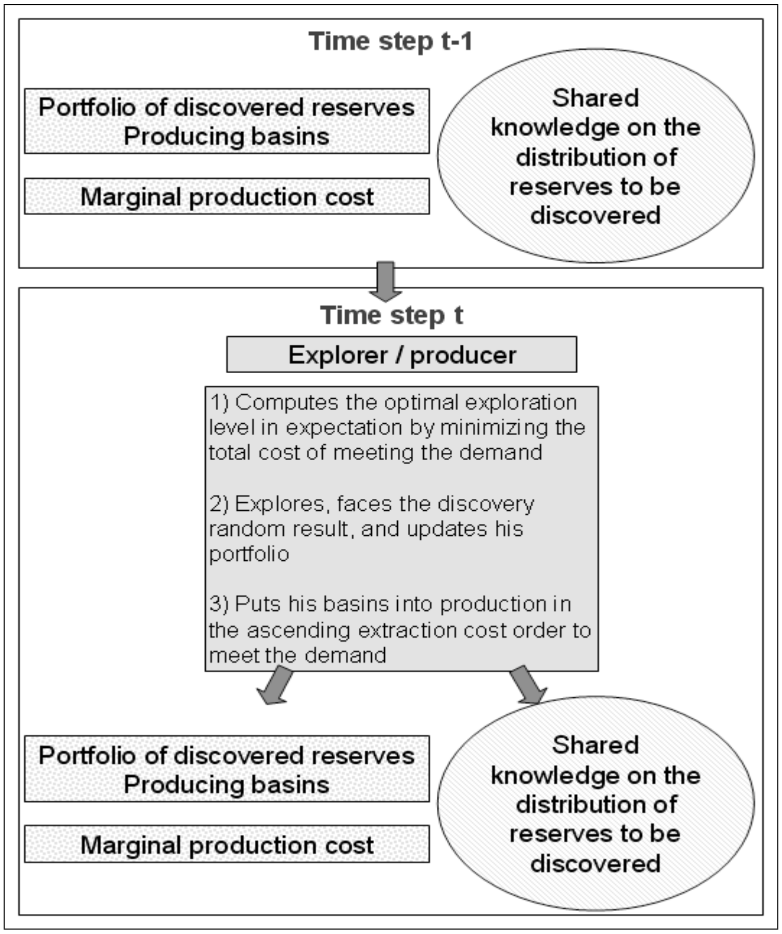

Figure 2.1: Model structure

optimization, even though a minimization problem is solved at each time step, to determine the optimal level of investment in exploration. Let us describe more precisely the different modules of the model.

2.2 Learning process on remaining reserves

At the beginning, the agent only knows, through general geological data, the total number of sedimentary basins where oil can be discovered. We assume for simplicity that each basin contains one and only one oil field. But the agent ignores the size and the extraction cost of the reserves to be discovered in a sedimentary basin. Each year, he uses the outcome of his exploration campaigns to progressively update his knowledge. He simply assumes that the actual distribution, by sizes and extraction costs, of the totality of the basins existing on earth, is homothetic to the sample he has already discovered: each year, on the basis of what he has already discovered, he updates the probabilities of discovering a basin of a given size and extraction cost during the next year.

2.3 Exploration-production decisions

The oil industry being assumed to be competitive, the representative agent has a cost minimizing behaviour. Each year, he seeks to hold a portfolio of discovered oil that will allow him to minimize the cost of meeting demand, not strictly for the next year, but for a given number of years. Indeed, knowing that exploration might yield poor results in terms of quantity of oil discovered, he wants to hold a portfolio able to meet the demand even in the case of a sequence of disappointing exploration campaigns. The optimum size of the portfolio could be computed. But, in order to simplify and recalling that the ratio reserves/annual production of the big private oil companies has proved to be remarkably constant over time, we choose to keep this number of years constant.
Exploring a sedimentary basin is assumed to cost a constant (temporally and geographically) bulk sum in geological, geophysicals and exploration drilling works. It yields an evaluation of the extraction cost and volume of the oil contained in the province. All the subsequent costs - the precise evaluation of the reservoirs, development costs and operating expenses - differ according to the province and are summed up in the "extraction cost" of the province.
The optimal (i.e. cost minimizing) exploration program is then carried out, new reserves are discovered and the model returns randomly their sizes and extraction costs. The explorer-producer then adds these reserves to its portfolio and puts basins into production to meet the demand, in the ascending extraction cost order. This gives the marginal production cost for year t, defined as the extraction cost of the most expensive producing sedimentary basin.

Therefore, in the model, both the exploration expenses and the time lag between discovery and production are sensitive to the price of oil, more precisely to the marginal extraction cost. An increase of the marginal production cost gives an incentive to explore more, since it increases the probability of discovering fields with lower extraction costs than the marginal ones. It also renders profitable to put into production already discovered, but expensive, reserves, thus shortening the time between discovery and production for these reserves. The model is thus endowed with two main microeconomic characteristics of the oil production and price dynamics that, in our view, the traditional Hubbert's type peak oil theories lack.

In the model however, we do take into account a simplified technical constraint lying "under the ground", like Hubbert and his followers do. We assume that each basin can only be put into production according to a specific profile: a plateau of constant production lasting τ years, the basin natural depletion time. A basin containing a volume of reserves V_i yields a constant rate of production of $\frac{V_i}{\tau}$ during τ years.

This production profile mimics the technico-economic observed production profile in three phases (build-up, plateau and decline).

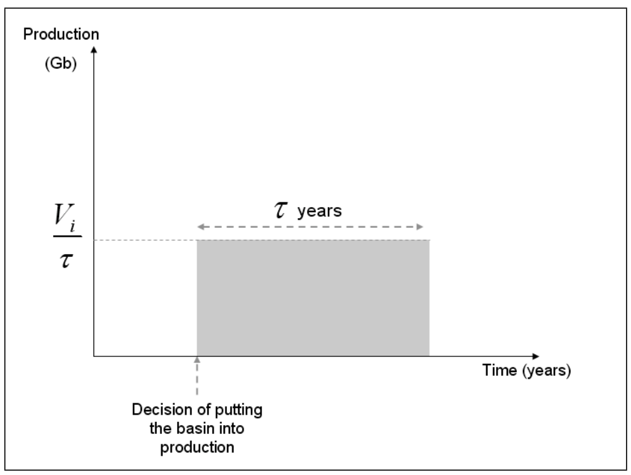

Figure 2.2: Sedimentary basins simplified production profile

Chapter 3

Numerical illustration

Let's briefly introduce the data chosen for the simulations of the next section. The exogenous and inelastic demand starts from 0 Gb and grows by 0.5 Gb/year, which is close to the demand evolution rate observed during the past decades.
The reserves of oil represent a total amount of 2000 Gb. The chosen figure allows for about 100 years of consumption before oil exhaustion, which is the time horizon of the simulation.
These reserves are distributed among N=330 sedimentary basins and divided into three size classes:

- small (2 Gb),
- medium (12 Gb),
- giant (58 Gb).

We assume that there are more reserves in the medium size class than in the small and the giant classes (see appendix 2). The extraction costs are distributed between 5 categories: 15, 25, 35, 45 and 55 \$/b. The detailed distribution by sizes and extraction costs is presented in the appendix 2.
The production cost of the abundant substitute is 100 \$/b.
The depletion time of a sedimentary basin, τ, is 20 years. The cost of an exploration campaign, assumed to be uniform geographically and temporally, is 60 MM\$.
The explorer-producer computes the level of investment in exploration in order to be able to meet the demand at expected minimum cost for the next 3 years.
He builds his initial reserve portfolio during the first ten years of each simulation. During this initial 10 years period, the number of exploration campaigns is limited to 10 per year, to avoid excessive and unrealistic variations of global exploration expenses. After this initialization phase, one important feature of the model is that it is possible to introduce an exogenous cap on the number of exploration campaigns launched by the explorer-producer at each time step, in order to model political decisions made by states to limit exploration, thus production, in their territory.

Chapter 4

Results

4.1 One exploration area

In this section, we consider a cost minimizing agent exploring a single global area, the whole world.

4.1.1 Exploration uncapped

In this first case, the exploration level is totally free after the 10 year initialization period, where it is limited to 10 campaigns per time step, as said above.
In this subsection, we will first analyze the results obtained with one particular scenario of discovery, and then we focus on the mean results over 100 scenarios.

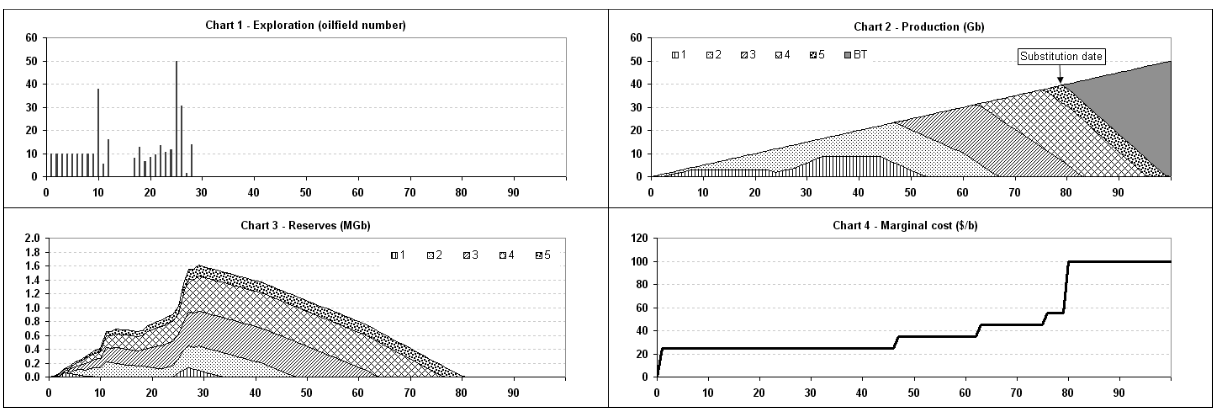

Figure 4.1: 1 area - A given scenario simulation outcome, exploration uncapped

Figure 4.1 shows the result of one simulation with free exploration worldwide. It shows that the exploration expenses, after the 10 year initialization period, vary widely along time (Figure 4.1 - Chart 1), depending among others on the results of previous exploration campaigns and the resulting total extraction cost. After a period of about 30 years, all sedimentary basins in the world are explored, about 50 years before the substitution date, which is also the date of the production peak of conventional oil. The substitution date and production peak, both occurring at time step 80, are represented on Figure 4.1 - Chart 2. On this chart, the productions (in Gb) of each extraction cost category reserve, satisfying the linearly increasing demand, are showed, from cheaper (cost category 1 at 15 $/b) to more expensive basins (cost category 5 at 55 $/b).

The substitute, or backstop technology, is also represented (area named "BT" on Figure 4.1 - Chart 2). Figure 4.1 - Chart 3 gives an insight into the evolution of the agent's reserves portfolio (already discovered, but not yet producing basins). Figure 4.1 - Chart 2 and 3 show that even if high cost oil is discovered early (for example, some oil of cost category 5 oil is discovered during the first 10 time steps), it is put into production only at the end of the period (cost category 5 basins are put into production from time step 76, as Figure 4.1 - Chart 2 shows). The time lag between discovery and production thus varies with the extraction cost of the discovered oil. The production profile (Figure 4.1 - Chart 2) is obviously not a Hubbert's type symmetrical profile. The declining slope after the peak reflects only the technical production constraints of the sedimentary basins successively put into production. The linear decline lasting exactly 20 years is the result of the successive exhaustion, after 20 years of production, of the last sedimentary basins put into production. We will call that type of peak oil a "purely technical peak oil".
In that particular simulation, the marginal cost of production, defined as the extraction cost of the more expensive producing oil category, is increasing with time (Figure 4.1 - Chart 4) since in this scenario, relatively cheap oil has been discovered during the very first time steps (cost category 2 at 25 $/b, as it can be seen on Figure 4.1 - Charts 2 and 4). Then, the explorer-producer needs to put more and more expensive oil fields into production, in order to satisfy the steadily increasing demand. This leads to an increasing marginal cost along time. At time step 80, the substitute, with a production cost of 100 $/b, is needed to meet the demand.

One particular simulation outcome may not be representative of a large set of discovery scenarios. We need to check if the results we have obtained still hold on average with a higher number of scenarios. We therefore present the mean model outputs of 100 scenarios.

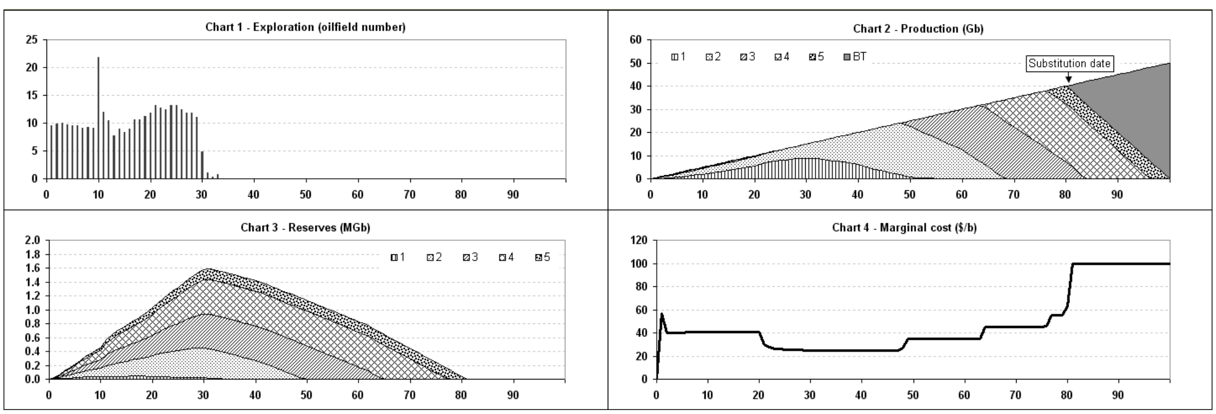

Figure 4.2: 1 area - 100 scenarios simulation outcome, exploration uncapped

As regards the exploration period and the production profile, the mean results of 100 scenarios are very close to the previously analyzed single scenario. The exploration expenses vary along time, and the exploration period lasts around 30 years (in all of the 100 scenarios, the exploration is over before time step 33). After this period, all sedimentary basins are explored. The substitution date and production peak occur at time step 80 on average (Figure 4.2 - Chart 2). Again, the time lag between discovery and production varies with the extraction cost of the reserves. Our first conclusion about the production profile is still valid. The production profile (Figure 4.2 - Chart 2) is not a Hubbert's type symmetrical one and the peak oil is "purely technical". We can therefore conclude that, if exploration is uncapped, all sedimentary basins containing oil whose extraction cost is lower than the production cost of the substitute are dis-

covered and put into production before the substitute is introduced. This simply results from the cost minimizing behavior of the explorer-producer. Concerning the marginal cost, however, we see that the particular simulation scenario presented on Figure 4.1 is not representative of the mean of many simulations. Indeed, on average over 100 scenarios, we observe a marginal cost profile which is first declining, then increasing up to the substitute cost, thus following a "U-shaped" profile (Figure 4.2 - Chart 4). This result is similar to those of several papers ([12, 17, 4] and [7] for a survey), where the conditions for such an "U-shaped" profile of oil marginal cost of production are discussed. Our simulations show that the marginal cost profile is an "U-shape curve" since there is a low probability of discovering first the large and low cost sedimentary basins. This phenomenon has been observed in the real world: oil provinces initially explored have been chosen close to the refineries and consumption areas (in the USA) and haven't proved later to be hosting the largest and cheapest fields.
The sensitivity of the production and marginal cost profiles to the randomness of the exploration process is illustrated by Figure 4.3.

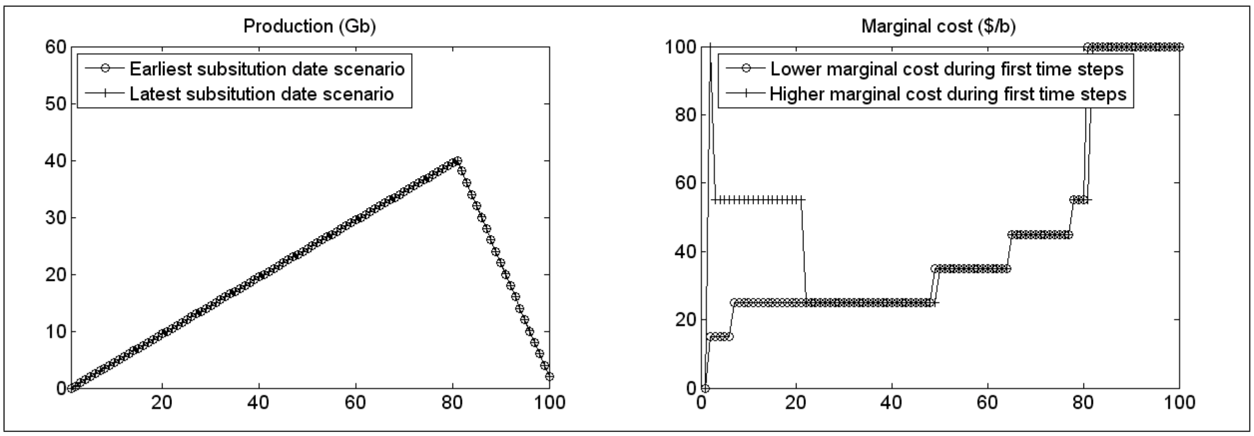

Figure 4.3: 1 area - Results sensitivity to discovery outcome, exploration uncapped

Figure 4.3 shows the production profiles that lead to the earliest and to the latest substitution date out of 100 scenarios. It also presents the two extreme marginal cost profiles: the one with the highest and the one with the lowest levels during the initial 10 year period. We see that the randomness of the discovery process has no impact on the global production profile, but a significant influence on the marginal cost profiles during the beginning of the simulation period. The difference between the high and low profile is still above 20 $/bl in year 20, 10 years before the end of exploration which occurs around year 30.

4.1.2 Exploration capped

We now introduce a cap on the exploration expenses. In the framework of this model, only a coalition of the reserve owners can constrain the access to their reserves and cap exploration. After the 10 year initialization period, exploration may be capped by the reserve owners. The cap is measured by the number of sedimentary basins that are allowed to be explored each year. While in the uncapped case, the number of exploration campaigns varied, except for some years, from 7 to 13 campaigns per year in average (see Figure 4.2 - Chart 1), we will now study the consequences of a cap at 3 and of a very stringent cap at 1 campaign per year. Figure 4.4 shows the mean of 100 simulations with exploration capped to 3:

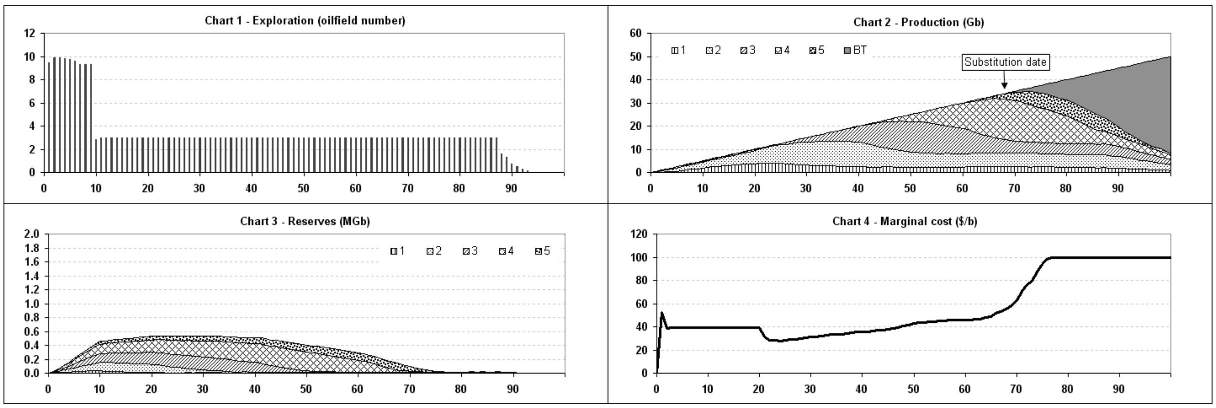

Figure 4.4: 1 area - 100 scenarios simulation outcome, exploration capped to 3 campaigns

The main difference with the uncapped case is that the exploration period lasts much longer (about 90 years), and that the production profile does not exhibit a purely technical peak oil anymore. The slope of the conventional oil production rate (the annual production in our discrete model) after the peak is less steep and lasts more than 20 years.
With exploration still capped at 3 campaigns, let's analyze now the sensitivity of the results to the randomness of exploration outcomes (Figure 4.5):
The various exploration random processes yield not only various price paths, but also various

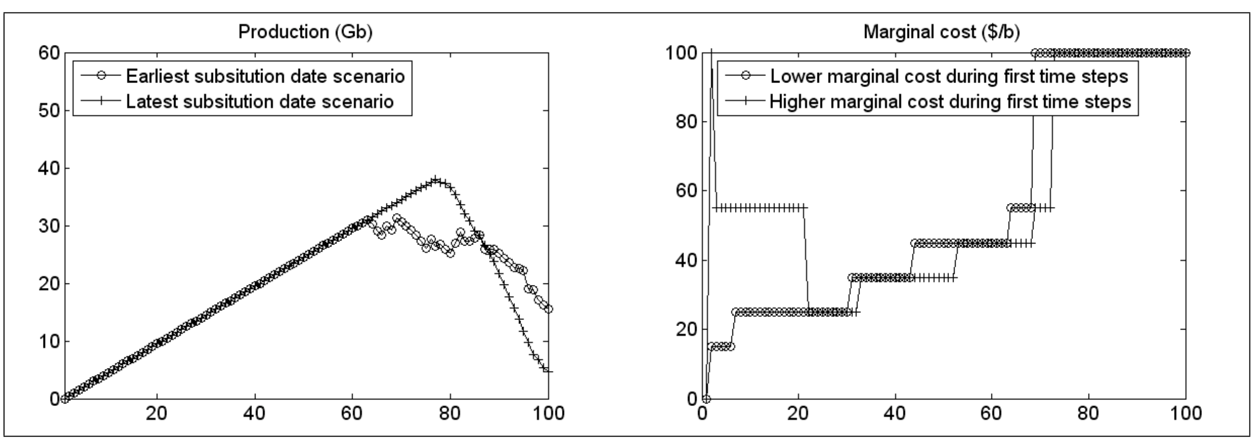

Figure 4.5: 1 area - Results sensitivity to discovery outcome, exploration capped to 3 campaigns

peak oil dates, ranging from year 62 to 78, a large bracket of 16 years. The sooner the peak, the more reserves remain unknown at the peak and the closer to a Hubbert's type is the production

profile. Such a peak results from two causes: the cap on exploration, and "bad luck" in the initial exploration process, which has kept the large and cheap deposits for the end. In the real world, according to geologists, the best sedimentary basins have already been discovered. The only reason for a Hubbert's type peak oil at the world level would then be a cap on exploration expenses.
The cap also increases the differences between the low and high marginal cost profiles, as compared to the uncapped case (Figure 4.5).

Let's now examine how the cap intensity impacts the production and price profiles. We compare the production and the marginal cost profiles for a mean of 100 simulations, with exploration uncapped, capped to 3 as shown and commented above and with a very strong cap of 1 (Figures 4.6 and 4.7):

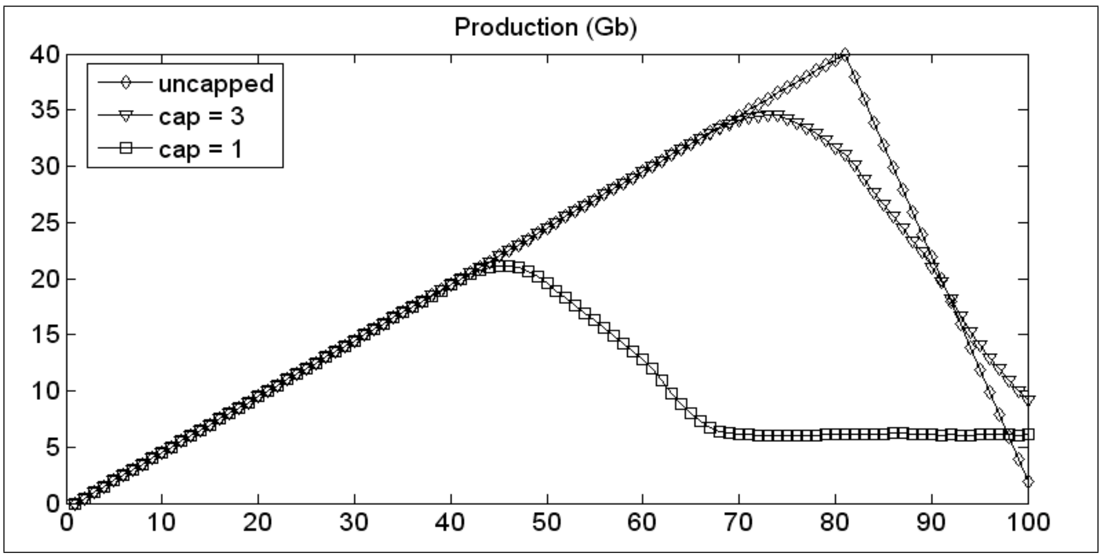

Figure 4.6: 1 area - Mean production path for varying cap levels

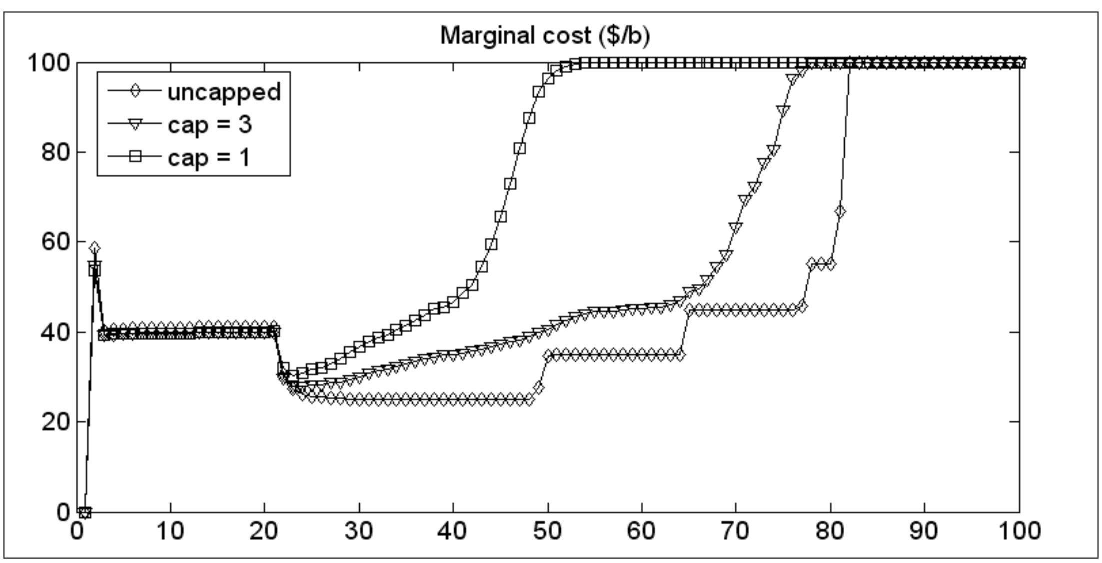

Figure 4.7: 1 area - Mean marginal cost path for varying cap levels

If the cap is very stringent (cap at 1 campaign per year) exploration goes on and conventional oil is still produced all over the simulation period. The substitution date varies from year 80 in the uncapped case to year 70 when the cap is 3 and 45 when the cap is 1. The more stringent the cap is, the flatter, tending to be symmetrical and closer to a Hubbert's type, is the production profile. The more stringent the cap, the higher the marginal cost profile. Ricardian rents are thus higher in the capped cases, reflecting the fact that, by constraining the access to their reserves, the reserve owners exert a market power which yields higher differential rents for explorer-producers. At that stage, we can summarize the results as follows. At the world level, there is no reason for a Hubbert's type peak oil, unless the exploration is capped. The reasons for a Hubbert's type peak oil therefore lie entirely "above the ground". In the real world, they lie in the political will of the governments of the Middle east oil producing countries, where more than 50% of the lower cost reserves are located, to constrain the access to their reserves. These results challenge what most of geological papers on the subject claim, which locate the main reasons for a Hubbert's type peak oil "under the ground".

4.2 Two exploration areas

We now attempt to explain why a Hubbert's type peak oil can be noted in one particular area, as anticipated by Hubbert, and actually verified in the USA (outside Alaska), but not necessarily at the world level. In this case, the 330 sedimentary basins in the world are divided between two areas (165 basins in each area), but with more large and low cost basins in area 1 than in area 2 (the distribution actually used in the simulations is presented in appendix 2). The explorer-producer guesses that there may be differences between the two areas, but it does not initially know which area is the more attractive to explore. Therefore, at the beginning, it divides equally its exploration expenses between the two areas. According to its exploration results, it starts to progressively favor the most likely promising one. If the trend is confirmed, it ends up by abandoning the less endowed area, but is bound to come back to it when the better-off area runs into depletion (The algorithm of choice between the two areas is given in appendix 1).

4.2.1 Exploration uncapped

Figure 4.8 shows the results for the mean of 100 scenarios.

As Figure 4.8 - chart 1 shows, the area 2, which rapidly proves to be less favorable, is abandoned after year 8 and exploration then takes place in area 1 only, until area 1 is completely explored. Then exploration in area 2 resumes. Exploration expenses, in both areas, vary along time. Reserves in area 2 are put in production only when there are no cheaper reserves left in area 1 anymore. Their rate of production is thus slower than in the one single area case. This gives birth to a flatter production profile. Production profiles are close to Hubbert's type, both in area 1 and 2, although the declining slopes remain stiff. But this is not the case at the world level, where the peak remains of a purely technical type.

4.2.2 Exploration capped

Now, let exploration be capped in the more favorable area, as shown in Figure 4.9. In that case, after the initialization period, exploration is capped at 1 campaign per time step in area 1, while it remains free in area 2. Note two interesting results. First, when exploration resumes around year 12 in area 2, after the cap set in area 1, the exploration expenses are not constant (Figure 4.9 - chart 1), contrary to the second hypothesis of Hubbert's theory (see section 1). For any

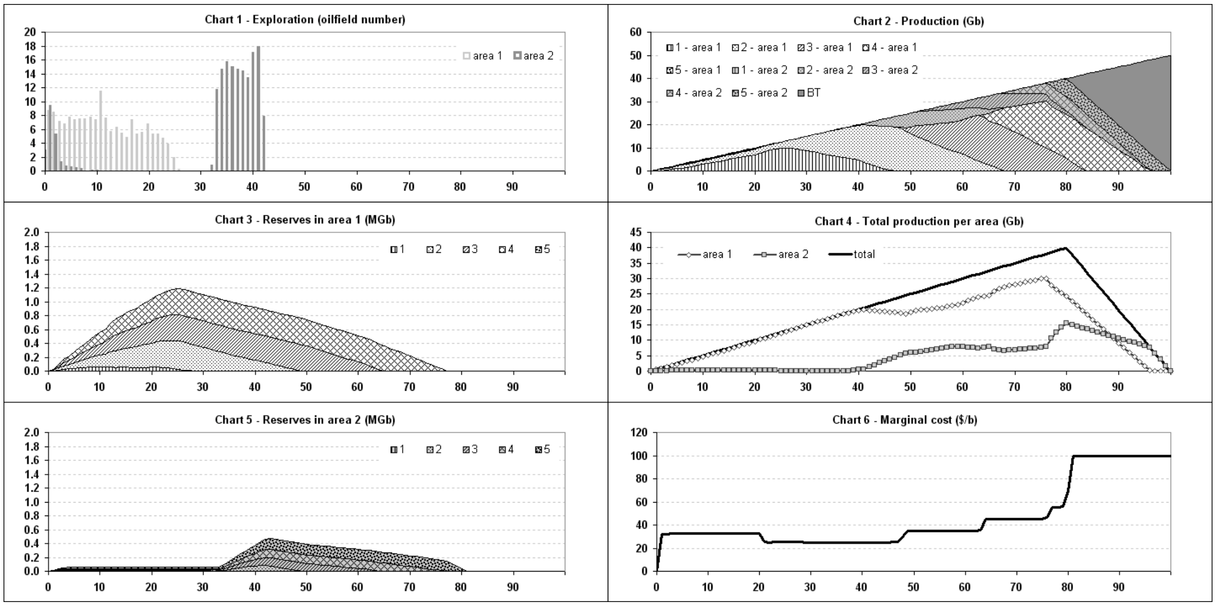

Figure 4.8: 2 areas - 100 scenarios simulation outcome, exploration uncapped

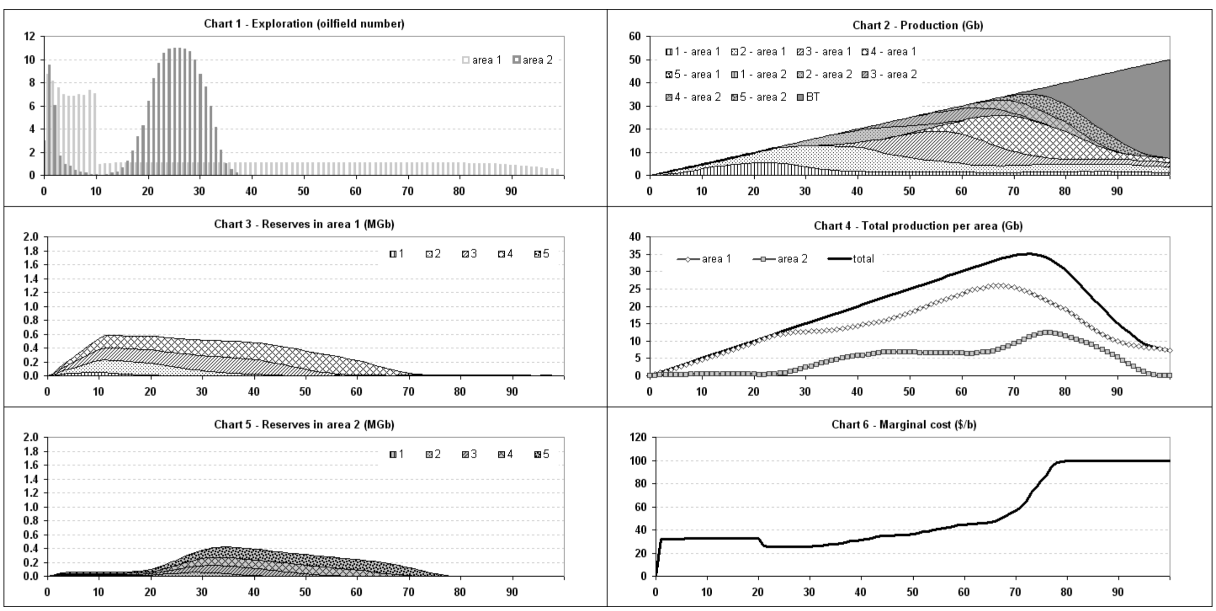

Figure 4.9: 2 areas - 100 scenarios simulation outcome, exploration capped to 1 campaign

specific simulation, it actually depends on the comparative outcome of previous exploration in both areas. For example, if many low cost large fields are discovered in area 1, exploration then slows down in area 2, and conversely. Second, the reserve portfolio built in area 1 before the cap is set, and maintained thereafter by the continuous, although moderate, exploration expenses, is made of cheaper oil than in area 2 (see and compare charts 3 and 5 of Figure 4.9). Because of the cap, the reserves of the two zones "compete" for a longer time than in the uncapped case (compare charts 3 and 5 of Figure 4.8 with those of Figure 4.9). Therefore, the time lag between discovery and production in area 2 (which is the average life time of each cost category of reserves shown in charts 5) is longer in average in the capped case and it increases with the

cost of production of the reserves. As a consequence of these two results, the production profiles, both in area 2 and 1 can be flatter and closer to symmetrical, as it is illustrated in Figure 4.9, chart 4. Note however that the peak at the world level is closer to a Hubbert type symmetrical peak than the global peak observed in the uncapped case (Chart 2 of Figure 4.8).
The main result is then: there are micro economic foundations for a Hubbert's peak oil in a specific region, if:

1. one region appears progressively less favorable than the other,
2. the exploration expenses are constrained in the most favorable one.

We have thus found reasons for the success of Hubbert's forecast on the production profile in a specific province, the USA, that are compatible with a rational economic behavior of the explorers-producers, i.e. exploration expenses and production decisions being sensitive to prices.

Chapter 5

Conclusions

The paper has inferred the profiles of oil production and marginal cost from the micro economic behavior of a cost minimizing explorer-producer who is facing an incomplete but improving information on the remaining reserves, with reserves owners able to constrain the access to their reserves. The model yields challenging results on the production and marginal cost profiles both at a global and at a regional level. A Hubbert's type peak oil at the world level appears only for reasons that lies "above the ground": a constrained level of exploration. However, a Hubbert's type peak oil can appear in specific oil provinces, if these provinces progressively appear to be less favorable to exploration than others. This could explain the success of Hubbert's forecast of the production profile of the lower 48 states in the USA. However we are not allowed to extend this result at the world level, as some experts tend to do.
Developments of the model would be:

- Model the reserves market, and therefore the market price, with a scarcity rent computed by the reserves owners in a framework of incomplete but improving information on the expected substitution date.
- Model the behavior of the low cost reserves owners, not only by their capacity to cap the exploration expenses, but through a Stackelberg oligopoly in a framework of shared, but incomplete and improving information.

Chapter 6

Appendix 1: Model underlying equations

6.1 Learning process on remaining reserves

It is assumed that the actor has no prior idea of the sedimentary basins distribution by size and extraction costs. However, he is given access to N, the total number of sedimentary basins containing oil, assumed to be shared by all market participants ("shared knowledge").

After the exploration outcome, the exporer-producer updates its estimate for cumulated number of sedimentary basins of size class i and cost j category in the world, in the following manner:

$\tilde{n}_{ij}(t) = N\frac{\tilde{n}_{ijsample}(t)}{\tilde{N}_{sample}(t)}$

with:

$\tilde{n}_{ij}(t)$	cumulated number of sedimentary basins of size i and cost category j in the world, as estimated by the actor at time step t
N	total number of sedimentary basins in the ground
$\tilde{n}_{ijsample}(t)$	cumulated number of sedimentary basins of size class i and cost category j in the actor's sample at time step t
$\tilde{N}_{sample}(t)$	cumulated number of exploration campaigns conducted at time step t

In other words, the actor makes a dilatation with a factor of $\frac{\tilde{n}_{ijsample}(t)}{\tilde{N}_{sample}(t)}$.

Therefore, the probabilities of discovery used by the actor for his heuristic exploration are:

$$\tilde{\rho}_i(t) = \frac{\sum_{j=1}^{N_C} \tilde{n}_{ij}(t)}{N} \tag{6.1}$$

$$\tilde{\rho}_j(t) = \frac{\sum_{i=1}^{N_S} \tilde{n}_{ij}(t)}{N} \tag{6.2}$$

with:

$\tilde{\rho}_i(t)$	probability of discovering a i size class sedimentary basin, as estimated by the actor, after the $\tilde{N}_{sample}(t)$ exploration
$\tilde{\rho}_j(t)$	probability of discovering a j cost category sedimentary basin, as estimated by the actor, after the $\tilde{N}_{sample}(t)$ exploration
N_C	number of different sedimentary basins extraction costs on earth
N_S	number of different sedimentary basins sizes on earth

6.2 Model constraints

The equality constraints placed upon the model are the following, for each $t \in [1, ..., t_e]$:

1. **Meeting demand**

$$D(t) = \sum_{i=1}^{N_C} P_j(t) + P_S(t) \tag{6.3}$$

with:

t_e	last time step of the simulation period, in years
$D(t)$	demand at time step t, in Gb
N_C	number of different sedimentary basins extraction costs on earth
$P_j(t)$	production of cost category j oil at time step t, in Gb
$P_S(t)$	production of backstop technology at time step t, in Gb

Note that we assume that the backstop technology is abundant, immediately available, and not submitted to production constraints.

The marginal cost is then defined as the extraction cost of the more expensive producing oil cost category (or the substitute production cost C_S if $P_S(t) > 0$).

2. **Production of cost category j oil**
In our modelling, it is assumed that oil is produced according to a simplified profile, which stands for a representative oil well production curve. The profile we use is based on an oil field containing a reserve volume V_i, yielding a constant rate of production during τ years. V_i stands for the size of class i sedimentary basin, i varying from 1 to N_S, the total number of different sizes (see Figure 2.2).

Thus, if the actor decides to put $r_{P_j}(t-\tau+1)$ Gb of cost category j oil into production at time step $t-\tau+1$, the corresponding cost category j sedimentary basins will still be producing at time step t, as a fatal production, because the actor has no possibility of stopping it, even if he could put cheaper oil (due to discoveries made between time steps $t-\tau+2$ and t) into production. This is illustrated on Figure 6.1, case a. If we assume that he puts into production another amount of cost category j oil at time step t' (with $t' \in [t-\tau+2, ..., t-1]$), then these sedimentary basins will although still be producing at time step t (Figure 6.1, case b). In this simplified example, the total production of cost category j oil at time step t will be $\frac{r_{P_j}(t)+r_{P_j}(t')}{\tau}$, as shown on Figure 6.1, case c:

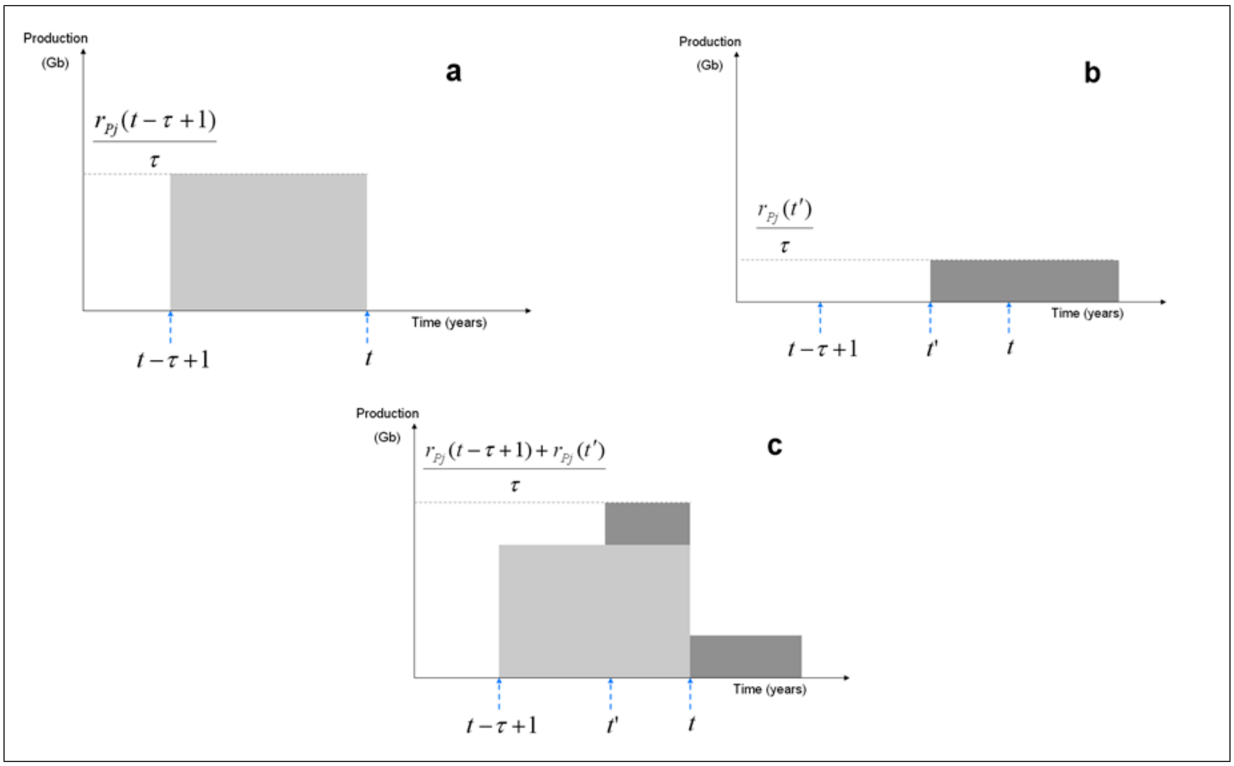

Figure 6.1: Illustration of the calculus of cost category j oil production

Finally, if we want to know the total production $P_j(t)$ of cost category j oil at time step t, we have to take into account all the quantities of cost category j oil put into production during the $\tau - 1$ previous time steps:

$$P_j(t) = \sum_{t'=t-\tau+1}^{t} \frac{r_{P_j}(t')}{\tau} \tag{6.4}$$

3. **Evolution of cost category j discovered but not producing sedimentary basins**

$$R_{N_j}(t+1) = R_{N_j}(t) + r_{D_j}(t) - r_{P_j}(t)$$

with:

$R_{N_j}(t)$	cumulated cost category j discovered (during previous time steps), but not producing basins, in Gb
$r_{D_j}(t)$	cost category j reserves discovered at time step t, in Gb
$r_{P_j}(t)$	cost category j reserves put into production at time step t, in Gb

For a given cost category j, at time step $t+1$, the amount of discovered but not producing sedimentary basins equals the amount of discovered but not producing sedimentary basins at time step t plus the new discoveries made at time step t, $r_{D_j}(t)$, minus the reserves put into production at time step t, $r_{P_j}(t)$.

4. **Evolution of the number of discovered sedimentary basins**

$$\tilde{N}_{sample}(t+1) = \tilde{N}_{sample}(t) + \hat{n}_{dmax}(t)$$

with:

$\tilde{N}_{sample}(t)$	cumulated number of exploration campaigns conducted at time step t
$\hat{n}_{dmax}(t)$	optimal number of explored sedimentary basins, $\hat{n}_d(t)$, at time step t, capped by the exogenous limitation $n_{dmax}(t)$

At time step $t+1$, the cumulative number of already discovered sedimentary basins, $\tilde{N}_{sample}(t+1)$, equals the cumulative number of already discovered sedimentary basins at time step t, $\tilde{N}_{sample}(t)$ (the results of all exploration campaigns launched from time step 1 to time step $t-1$) plus $\hat{n}_{dmax}(t)$, the optimal number of explored sedimentary basins at time step t, capped by the exogenous limitation $n_{dmax}(t)$.

The exploration level $n_d(t)$ is a decision variable of the model. The way the actor optimally chooses it at each time step is described in part 6.3.

A few inequality constraints although exist, regarding positivity of reserves, sedimentary basins numbers and productions: $\forall t \in \{1, ..., t_e\}$, $\forall j \in \{1, ..., N_C\}$:

$R_{N_j}(t) > 0$
$n_d(t) \quad > 0$[1]
$r_{D_j}(t) > 0$
$r_{P_j}(t) > 0$
$P_j(t) \quad > 0$
$P_S(t) \quad > 0$

Finally, there are two inequality constraints on the total number of sedimentary basins on earth and on exploration capacity:

$\forall t \in \{1, ..., t_e\}$:

$\tilde{N}_{sample}(t) \leq N$

$n_d(t) \leq n_{dmax}(t)$

where N stands for the total number of sedimentary basins in the world and $n_{dmax}(t)$ the exogenous restriction on exploration capacity.

6.3 Exploration-production decisions

The actor meets residual demand by bringing its reserves into production in the ascending cost order. At each time step, the optimal amount of extraction cost j oil to be put into production is calculated thanks to a recurrent relation (j varying from 1 to N_C). In other words, he fixes firstly the amount of oil from cost category 1 (the cheapest) to be produced, and then, if necessary, he uses more expensive oil categories, in the ascending extraction cost order:

$$r_{P_j}(t) = \min\left(\tau\left(D(t) - \sum_{k=1}^{j-1} \frac{r_{P_k}(t)}{\tau} - P_F(t)\right), R_{N_j}(t) + r_{D_j}(t)\right) \tag{6.5}$$

Where $P_f(t) = \sum_{k=1}^{N_C}\left(\sum_{t'=t-\tau+1}^{t-1} \frac{r_{P_k(t')}}{\tau}\right)$

$P_f(t)$ stands for the total fatal production, due to the explorer-producer's previous production decisions (see the constraint linked to the simplified sedimentary basins production profile described in part 2.3).
$D(t) - P_f(t)$ is therefore the residual demand the actor has to face by putting new sedimendary basins into production at time step t. These decisions of putting new basins into production are taken by solving equation 6.5, with j varying successively from 1 to N_C, at each time step, in order to calculate the $r_{P_j}(t)$.
The first term of equation 6.5 stands for the quantity of cost category j oil needed to satisfy the residual demand when all available lower extraction cost reserves are producing. The second one represents the available amount of cost category j oil after exploration.
Once the decision variable $r_{P_j}(t)$ is calculated, each extraction cost category production is then calculated thanks to equation 6.4 and the backstop technology production is finally deduced from equation 6.3.
It should be noticed that the decisions $r_{P_j}(t)$ are depending on new discoveries made at time step t, $r_{D_j}(t)$, which are functions of:

1. **The number of explored basins at time step t**
   The number of explored basins is the major decision variable in the model. This part explains how it is optimally defined by the explorer-producer. This optimal number, $\hat{n}_d(t)$, can then be exogenously capped by $n_{dmax}(t)$. Finally, the number of explored basins at time step t is defined as $\hat{n}_{dmax}(t) = \max\left(\hat{n}_d(t), n_{dmax}(t)\right)$.

2. **The random outcome of exploration**
   When the actor decides to explore one sedimentary basin, he can randomly find a basin of cost category j and size class i (j varying from 1 to N_C, the total number of existing basins extraction costs, and i varying from 1 to N_S, the total number of existing basins sizes).

At each time step t, the agent calculates the optimal exploration investment $\hat{n}_d(t)$. In other words, he decides how many sedimentary basins he wants to explore (we assume that the agent knows the total number of sedimentary basins in the world and their geographical localisation). Therefore, each exploration campaign will lead to the discovery of one sedimentary basin. in order to meet the demand $D(t)$ at minimum expected total cost. The decision variable is therefore $n_d(t)$, and the total cost in expectation $\tilde{C}^T(t, n_d(t))$ is minimized to find $\hat{n}_d(t)$:

$$\hat{n}_d(t) = \min_{n_d(t)} \left(\tilde{C}^T\left(t, n_d(t)\right)\right) \tag{6.6}$$

The total cost in expectation, $\tilde{C}^T\left(t, n_d(t)\right)$ (in \$) is defined as:

$$\tilde{C}^T\left(t, n_d(t)\right) = \tilde{C}_P\left(t, n_d(t)\right) + C_e n_d(t) \tag{6.7}$$

with:

C_e	exploration cost for one sedimentary basin, in \$ (the temporally and geographically constant bulk sum in geological, geophysicals and exploration drilling works)
$C_e n_d(t)$	total exploration cost, in \$
$\tilde{C}^P\left(t, n_d(t)\right)$	total extraction cost, in \$

The total extraction cost is calculated as follows:

$$\tilde{C}_P\left(t, n_d(t)\right) = \sum_{i=1}^{N_C} C_j \tilde{P}_j(t, n_d(t)) + C_S \tilde{P}_S(t, n_d(t)) \tag{6.8}$$

with:

$\tilde{P}_j(t, n_d(t))$	expected production of cost category j basins as a function of $n_d(t)$
$\tilde{P}_S(t, n_d(t))$	expected production of the backstop technology as a function of $n_d(t)$
C_j	extraction cost of category j sedimentary basin, in \$/b
C_S	backstop technology production cost, in \$/b

Several stages are necessary to compute these expected productions:

1. The explorer-producer uses his estimated probabilities $\tilde{\rho}_i(t)$ and $\tilde{\rho}_j(t)$ (see equations 6.1 and 6.2 in part 6.1). $\tilde{\rho}_i(t)$ is the estimated probability of discovering a i size class sedimentary basin and $\tilde{\rho}_j(t)$ is the estimated probability of discovering a j cost category sedimentary basin. On the basis of these probabilities, he calculates the expected discovered volume if $n_d(t)$ exploration campaigns are launched, for each extraction cost j:

   $\tilde{r}_{D_j}(t, n_d(t)) = n_d(t)\tilde{V}_{expj}(t)$

   with: $\tilde{V}_{expj}(t) = \tilde{\rho}_j(t) \sum_{i=1}^{N_S} \tilde{\rho}_i(t) V_i$

2. Then, he solves equation 6.5 by induction, by replacing the $r_{D_j}(t)$ by $\tilde{r}_{D_j}(t, n_d(t))$ corresponding to his vision in expectation, j varying successively from 1 to N_C, in order to calculate $\tilde{r}_{P_j}(t, n_d(t))$, the expected amount of cost category j oil that will be put into production, if $n_d(t)$ exploration campaigns are launched.

3. Finally, he computes the expected productions $\tilde{P}_j(t, n_d(t))$, for j varying from 1 to N_C, and $\tilde{P}_S(t, n_d(t))$ thanks to equations 6.4 and 6.3.

For one considered time step t; it can be shown by induction that $\tilde{C}_P\,(t, n_d(t))$ is a piecewise linear function of $n_d(t)$ from $\Re^+$ to $\Re^+$. Moreover, there is $\bar{n}_1(t)$ such as, for $n_d(t) \in [0, \bar{n}_1(t)]$, $\tilde{C}_P\,(t, n_d(t))$ has a number of removable discontinuities less than or equal to N_C, and for $n_d(t) \in [\bar{n}_1(t), +\infty[$, $\tilde{C}_P\,(t, n_d(t))$ is constant (see the case $N_C = 5$ on Figure 6.2).
After this consideration, we deduce from equation 6.7 that $\tilde{C}^T\,(t, n_d(t))$ is although a piecewise linear function of $n_d(t)$ from $\Re^+$ to $\Re^+$ with N_C removable discontinuities. For $n_d(t) \in [\bar{n}_1(t), +\infty[$, the function $\tilde{C}^T\,(t, n_d(t))$ is strictly increasing with a slope of C_e, which means that the explorer-producer only needs to calculate $\tilde{C}^T\,(t, n_d(t))$ for a finite number of values of $n_d(t)$ (corresponding to the removable discontinuities of the piecewise linear function) to be able to find the global minimum of it, $\hat{n}_d(t)$, as illustrated hereafter:

If we consider a given time step t, Figure 6.2 highlights the following key elements:

1. The sum of the 2 functions $\tilde{C}^P\,(t, n_d(t))$ and $C_e n_d(t)$ admits a minimum due to the fact that reducing the expected total cost of production becomes less and less interesting with the number of sedimentary basins explored (the exploration fee C_e remains constant).

2. In area 1 and 2 of figure 6.2, we observe that exploration diminishes the expected total cost of the actor. The exploration earning or gain (in \$) is defined as the money saved in expectation by exploration compared to the situation without exploring: $\tilde{G}(t) = \tilde{C}^T(t, 0) - \tilde{C}^T(t, n_d(t))$. In area 3 of figure 6.2, exploration is not profitable, the gain becomes negative.

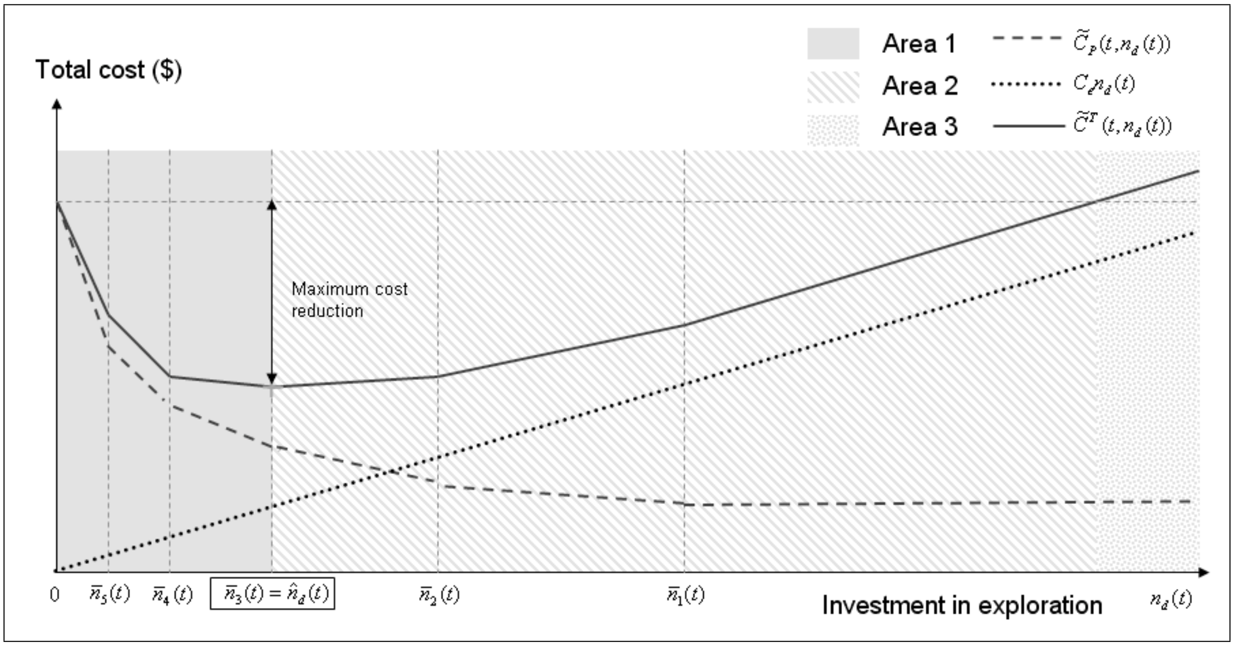

Figure 6.2: The piecewise linear total cost as a function of exploration

3. The optimal level of investment $\hat{n}_d(t)$ decided by the actor corresponds to the number of investment campaigns that maximizes the expected gain of exploration: $\hat{n}_d(t) = \max_{n_d(t)}\left(\tilde{G}(t, n_d(t))\right)$.

Finally, the actor will explore $\hat{n}_{dmax}(t)$ sedimentary basins at time step t, with $\hat{n}_{dmax}(t) = \max\left(\hat{n}_d(t), n_{dmax}(t)\right)$ ($n_{dmax}(t)$ being the cap on exploration).

Two major characteristics of this exploration heuristic should be underlined:

- At each time step, the explorer-producer makes the optimal decision in expectation. Once the optimal level of investment fixed, he has to face the exploration random results in terms of sedimentary basins sizes and costs. His total cost $C^T(t)$ can then be higher or lower than the expected value $\tilde{C}^T(t)$.
- The actor's decision is depending on his vision of the sedimentary basins distribution, based on $\tilde{\rho}_j(t)$ and $\tilde{\rho}_i(t)$, ie on the exploration random outputs of the former time steps.

In the reality, explorer-producers tend to ensure that they will be able to satisfy demand at minimum cost during a longer period than just the next year. For this reason, in the model, it is possible to take x years into consideration to determine the optimal level of investment. The exploration strategy remains the same, with the demand $D(t)$ replaced by $\sum_{i=0}^{x-1} D(t+i)$ in the equations above (the actor is supposed to anticipate perfectly the evolution of the demand).

6.4 Case with two areas

6.4.1 Step 1: Calculation of the optimal investment level in each area

In a first stage, the explorer-producer calculates his exploration level (optimal in expectation) in each area. For this purpose, he firstly assumes that he will have to satisfy the total residual demand $D(t) - P_F(t)$ (with $P_F(t)$ standing for the fatal production resulting from production decisions of previous time steps) with his exploration outcome from each area.

Let k be the index characterizing the area ($k \in \{1, 2\}$).

Thus, for $k \in \{1, 2\}$, he computes:

$$\hat{n}_{dk}(t) = \min_{n_{dk}(t)} \left(\tilde{C}_k^T(t, n_{dk}(t)) \right) \quad (6.9)$$

6.4.2 Step 2: Investment splitting heuristics

After the step 1, the explorer-producer has calculated $\hat{n}_{dk}(t)$ for $k \in \{1, 2\}$, but these exploration levels would be optimal if the actor had to satisfy the total residual demand $D(t) - P_F(t)$ with his exploration outcome from each area. In other words, if he explored $\hat{n}_{d1}(t)$ sedimentary basins in area 1 and $\hat{n}_{d2}(t)$ sedimentary basins in area 2, he would be able to yield twice the residual demand overall, which is not his purpose. He needs therefore a rule to split his exploration level between the two areas. Let us see the heuristics that enables him to choose which region to explore, based on his knowledge:

$$\bar{n}_{d1}(t) = \hat{n}_{d1}(t) \frac{\tilde{G}_1(t, \hat{n}_{d1}(t))}{\tilde{G}_1(t, \hat{n}_{d1}(t)) + \tilde{G}_2(t, \hat{n}_{d2}(t))} \quad (6.10)$$

$$\bar{n}_{d2}(t) = \hat{n}_{d2}(t) \frac{\tilde{G}_2(t, \hat{n}_{d2}(t))}{\tilde{G}_1(t, \hat{n}_{d1}(t)) + \tilde{G}_2(t, \hat{n}_{d2}(t))} \quad (6.11)$$

with:

$\bar{n}_{dk}(t)$	Exploration level chosen by the actor in area k
$\hat{n}_{dk}(t)$	Exploration level optimal in expectation to satisfy the total residual demand with sedimentary basins from area k
$\tilde{G}_k(t, \hat{n}_{dk}(t))$	Optimal expected exploration gain in area k

The optimal expected exploration gain is computed by the explorer-producer on the basis of his estimated probabilities $\tilde{\rho}_{ik}(t)$ and $\tilde{\rho}_{jk}(t)$, $\tilde{\rho}_{ik}(t)$ being the estimated probability of discovering a size class i sedimentary basin in area k and $\tilde{\rho}_{jk}(t)$ standing for the estimated probability of discovering a j cost category sedimentary basin in area k.

As a conclusion, the explorer-producer allocates the expected optimal levels of exploration computed between the two areas in proportion to the maximum expected earnings per area, which depends on the actor's vision of the sedimentary basin characteristics distribution in each area.

6.5 Conclusion

This first appendix presents in a detailed way the underlying equations of the oil simulation model. Firstly, the way the explorer-producer updates his knowledge on sedimentary basins size and extraction cost distribution is explained. Then, the equality constraints placed upon the model are introduced. They show how the production of cost category j oil is calculated, with a focus on the integration of oil production constraints. Afterwards, the production management decision of the agent is presented: the actor meets residual demand by bringing its reserves into production in the ascending extraction cost order by solving a recurrent relation.

Once this decision variable (ie the amount of each cost category oil put into production) is calculated, each extraction cost category production is then calculated and the backstop technology production is finally deduced.

It is underlined that these decisions depend on the optimal number of explored basins (which is the second major decision variable in the model) and on the uncertain outcome of exploration. Indeed, at each time step, the agent has to calculate its optimal exploration investment. In other words, he decides how many sedimentary basins he will have to explore in order to meet the demand at minimum expected total cost. It can be proved by induction that the explorer-producer only needs to calculate this expected total cost for a finite number of values of the exploration level to be able to find the global minimum of it.

Two important features of the model should be noticed:

- At each time step, the explorer-producer makes his optimal investment decision in expectation. Once the optimal level of investment is fixed, he has to face the exploration random results in terms of sedimentary basins sizes and extraction costs.
- The actor's decision is depending on his vision of the sedimentary basins distribution, based on the exploration random outputs of the former time steps.

Once the actor has made his exploration decision, he faces exploration random results and fixes the amount of each cost category oil to be put into production. The resulting marginal production cost is finally deduced.

Chapter 7

Appendix 2: Detailled data

7.1 Case with one exploration area

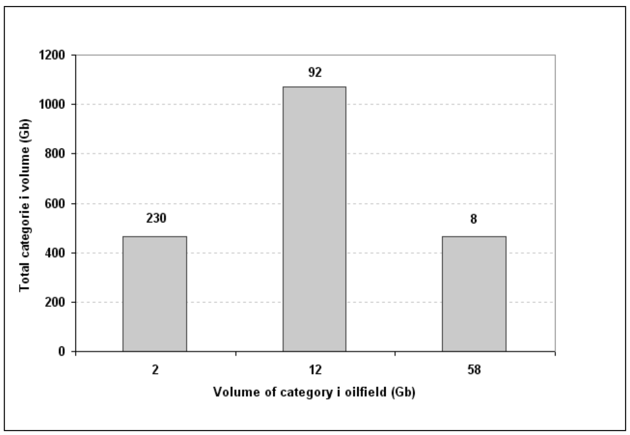

Figure 7.1: 1 area - Distribution of sedimentary basins by size class

We assume a correlation between oil fields extraction costs and sizes:

- small oil fields are divided equally between 35, 45 and 55 $/b cost categories,
- medium-sized oil fields are divided between 25, 35 and 45 $/b cost categories,
- big oil fields are divided between 15 and 25 $/b.

Finally, we end up with 8 different oil field types:

Extraction cost ($/b)/ Volume (Gb)	**15**	**25**	**35**	**45**	**55**
2	0	0	77	77	76
12	0	32	30	30	0
58	4	4	0	0	0

Table 7.1: 1 area - distribution of sedimentary basins by sizes and extraction costs

7.2 Case with two exploration areas

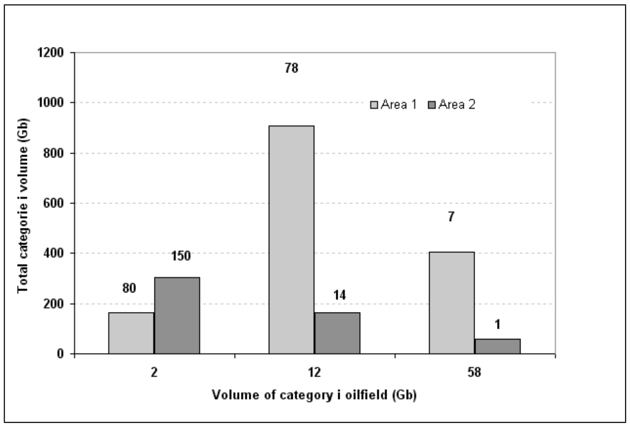

Figure 7.2: 2 areas - Distribution of sedimentary basins by size class

In this case, the 330 oil fields are divided between area 1 and area 2 (165 oil fields in each area), according to following extraction cost and size distributions:

Extraction cost ($/b)/ Volume (Gb)	**15**	**25**	**35**	**45**	**55**
2	0	0	64	46	0
12	0	26	22	30	0
58	4	3	0	0	0

Table 7.2: 2 areas - distribution of sedimentary basins by sizes and extraction costs in area 1

Extraction cost ($/b)/ Volume (Gb)	**15**	**25**	**35**	**45**	**55**
2	0	0	13	61	76
12	0	6	8	0	0
58	0	1	0	0	0

Table 7.3: 2 areas - distribution of sedimentary basins by sizes and extraction costs in area 2

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