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INVESTMENT IN R&D, COSTS OF ADJUSTMENT AND EXPECTATIONS

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Abstract

This paper proposes a framework which integrates convex costs of adjustment and expectations formation in the determination of investment decisions in R&D at the firm level. The model is based on cost minimization subject to the firm's expectations of the stream of output and the price of R&D, and results in equations for actual and multiple-span planned investment in R&D and for the realization error as functions of these expectations. The model accommodates alternative mechanisms of expectations formation and provides a methodology for testing these hypotheses empirically. We derive estimable equations and testable parameter restrictions for the rational, adaptive and static expectations hypotheses. The empirical results using pooled firm data strongly reject the rational and static expectations hypotheses and generally support adaptive expectations.

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This paper develops a simplified cost of adjustment model of R&D investment by private firms in which expectations play a central role. Our main objective is to provide a dynamic equilibrium framework in which alternative hypotheses of expectations formation can be tested empirically.

Most of the existing empirical work on R&D investment at the micro level is based on static equilibrium models, sometimes modified by arbitrary distributed lags, and on the assumption that firms hold static or myopic expectations on the exogenous variables in the model (e.g., Goldberg 1972; Nadiri and Bitros 1980; for a cost of adjustment model, Rasmussen 1969). It seems clear that static expectations are inadequate as an untested maintained hypothesis, and they have the additional serious drawback of making it difficult to interpret the empirically determined lag distribution in a meaningful way. It is virtually impossible to disentangle the part of the observed lag structure which is due to costs of adjustment from the lags reflecting expectational formation. Partly as an attempt to rectify this problem and to give estimated lag distributions an economically meaningful interpretation, recent work on aggregate investment in physical capital integrates rational expectations (in the sense of Muth 1961) into investment models and in some cases tests that expectations hypothesis (Abel 1979; Kennan 1979; Meese 1979). However, this approach has not been applied to R&D investment, and even more important, no attempt has been made to formulate and empirically test other less restrictive mechanisms of expectations formation. This paper represents a first attempt at these important tasks.

Our model is based on the assumption that the firm selects an R&D investment profile (i.e., a current investment decision plus a stream of future planned investment) which minimizes the present value of its costs,

given its expectations of the future price of R&D and the level of output. If there are convex adjustment costs (i.e., a rising marginal cost of R&D investment, either because of capital market imperfections or internal adjustment costs), this yields a determinate rate of current R&D and of multiple-span planned R&D. The optimal R&D profile is determined by the firm equating the marginal cost of adjustment to the shadow price of R&D expected to prevail at the time the investment is actually made. We show that the marginal cost of adjustment depends on the anticipated price of R&D, while the shadow price (which reflects the present value of savings in variable costs due to investment in R&D) depends on the anticipated demand for output. This links the optimal investment profile directly to the firm's expectations of these economic variables. The model of R&D investment also generates a realization function which relates the difference between actual and planned R&D to revisions over time in the firm's expectations of the exogenous variables. This integration of the investment profile, the firm's expectations and the realization function represents a formalization and extension of earlier work by Modigliani (1961) and Eisner (1978).

The general investment framework is designed to accommodate arbitrary expectations hypotheses, but in order to provide the model with empirical content a specific forecasting mechanism for the price of R&D and the level of output must be postulated. We consider three alternative specifications and develop a set of empirical tests for each. The first, rational expectations, is based on the idea that the firm formulates its forecasts according to the stochastic processes (presumed to be) generating the exogenous variables. Using a third order autoregressive specification for these variables, we derive a set of testable nonlinear parameter

restrictions in the actual and planned investment equations and some additional tests on the realization function. This represents an application to R&D of the methodology developed by Sargent (1978, 1979a), with some extensions to planned investment and the associated realization function. Next, the model is formulated under adaptive expectations according to which the firm adjusts its forecasts by some fraction of the previous period's forecast error. We show that this hypothesis also delivers a set of testable nonlinear restrictions on the R&D investment equations. Finally, we consider the conventional hypothesis of static expectations and show that, since it is a limiting case of adaptive forecasting, it can be tested directly by exclusion restrictions on the model under adaptive expectations.

The model under each expectations mechanism is estimated using a set of pooled firm data which contains both actual and (one year ahead) planned R&D. The empirical results indicate strong rejection of the parameter restrictions implied by rational expectations, and general support for the adaptive expectations hypothesis. The hypothesis most favored by the evidence appears to be a mixed one, with adaptive forecasting on the level of output and static expectations on the price of R&D. We provide some discussion of the possible reconciliation of rational expectations with this mixed forecasting hypothesis.

Section 1 develops the general model of R&D investment. The specifications of the model under rational, adaptive and static expectations are provided in Section 2. Section 3 provides a brief description of the data, and presents the empirical results and their interpretation. Brief concluding remarks follow.

1. Investment Model for R&D

Consider a firm with a production function which exhibits constant returns to scale in traditional inputs (labor and capital) and which faces fixed factor prices for those inputs. The firm's decision problem is to select an R&D investment profile which minimizes the discounted value of costs, given its expected factor prices and levels of output. This "certainty equivalence" separation of the optimization problem and the formation of expectations is justified by the separable adjustment costs specified below. Formally, the decision problem is:

$$(1) \qquad \underset{<\bar{R}_{t,s}>}{\text{Min}} \sum_{s=0}^{\infty} \alpha^s \left[C(K_{t,s}, Q_{t,s}, w_{t,s}) + \bar{R}_{t,s} \; h(\bar{R}_{t,s}) \right]$$

$$\text{s.t.} \quad K_{t,s+\theta} - K_{t,s+\theta-1} = \bar{R}_{t,s} - \delta K_{t,s+\theta-1}$$

where $\bar{R}_{t,s}$ is R&D investment in real terms planned in period t for t+s (we refer to t as the base period, t+s as the target period, and s as the anticipations span), $C(\cdot)$ is the restricted cost function defined over the stock of knowledge K and the vector of prices for variable inputs w, $\alpha = 1/(1+r)$ and δ are the (constant) discount factor and the rate of depreciation of the stock of knowledge, θ is the mean gestation lag between the outlay of R&D and the production of new knowledge, and $h(\cdot)$ describes the unit cost of R&D investment.

Specific functional forms are assumed for $h(\cdot)$ and $C(\cdot)$. First, we assume that the unit cost of R&D rises linearly with the level of R&D:

$$(2) \qquad h(\bar{R}_{t,s}) = P_{t,s}(1+A\bar{R}_{t,s}) \qquad A>0$$

where $P_{t,s}$ is the anticipated price of R&D. This formulation implies that total costs of R&D, $\bar{R}h(\bar{R})$, are a quadratic function of the level of R&D. Second, the assumption of constant returns to scale implies that $C(K,Q,w) = QF(K,w)$. We also assume that $F(\cdot)$ is separable and can be written $F(K,w) = f(w) - vK$ where $v > 0$, whence $C(K,Q,w) = Q(f(w) - vK)$.[1]

Two limitations of the basic model should be noted. First, the model treats the stock of knowledge as the only (quasi-fixed) capital asset and implicitly views traditional capital as variable in the short run. A more general model would treat both capital and R&D as quasi-fixed assets with associated costs of adjustment, but such a model would be considerably more complicated. The advantage of the present formulation is that it obviates the need to introduce the capital constraint in the determination of the level of the firm's output. The second limitation is the assumption that the parameter "v" is known and is the same for all firms. This parameter is one determinant of the savings in variable costs due to a marginal investment in the stock of knowledge ($\partial C/\partial K = vQ$). One might expect differences across firms or uncertainty about the "productivity" of R&D (for example, technological opportunity) to be reflected in the parameter "v". This important aspect of the problem is not treated in the present model.

With these qualifications in mind, we proceed with the derivation of the optimal R&D profile. Using the specific forms for $h(\cdot)$ and $C(\cdot)$ and the constraint in (1), the decision problem can be expressed

$$(3)\qquad \underset{<K_{t,s}>^t}{\text{Min}}\ V_t = \sum_{s=0}^{\infty} \alpha^s \left[Q_{t,s}(f(w_{t,s}) - vK_{t,s}) + P_{t,s}(K_{t,s+\theta} - (1-\delta)K_{t,s+\theta-1}) \right.$$

$$\left. + AP_{t,s}(K_{t,s+\theta} - (1-\delta)K_{t,s+\theta-1})^2 \right]$$

where we note that the decision variable here is the stock of knowledge. The first order (Euler) conditions are:

$$(4)\qquad \frac{\partial V_t}{\partial K_{t,j}} = -v\alpha^j Q_{t,j} + \alpha^{j-\theta} P_{t,j-\theta} + 2A\alpha^{j-\theta} P_{t,j-\theta}(K_{t,j} - (1-\delta)K_{t,j-1})$$

$$- (1-\delta)\alpha^{j+1-\theta} P_{t,j+1-\theta} - 2A\alpha^{j+1-\theta} P_{t,j-\theta}(K_{t,j+1} - (1-\delta)K_{t,j})$$

$$= 0$$

for $j \geq \theta$. Noting that $K_{t,j} - (1-\delta)K_{t,j-1} = \bar{R}_{t,j-\theta}$ and defining $R_{t,j} = P_{t,j}\bar{R}_{t,j}$ for $j \geq 0$, the Euler conditions can be written

$$(5)\qquad \left(1 - \frac{1}{\beta} L\right) R_{t,j+1-\theta} = -aP_{t,j+1-\theta} + \frac{a}{\beta} P_{t,j-\theta} - \frac{av\alpha^{\theta}}{\beta} Q_{t,j}$$

where $a = 1/2A$, $\beta = (1-\delta)/(1+r)$ and L denotes the lag operator.

Since $\beta < 1$ we can obtain the forward solution to the difference equation in (5) (see Sargent 1979b, Chapter 9). Letting $s = j+1-\theta$ for simplicity, this yields after some manipulation

$$(6)\qquad R_{t,s} = -aP_{t,s} + \left[\alpha^{\theta} \sum_{j=s+\theta}^{\infty} \beta^{j-s-\theta} vQ_{t,j} \right]$$

Planned R&D depends on the expected price of R&D investment goods and the stream of future expected levels of output. To gain more insight into the solution, note that the term $vQ_{t,j}$ $(j \geq s+\theta)$ represents the expected savings in variable costs in period t+j due to a unit increase in the stock of knowledge in t+j. This in turn reflects the marginal dollar of R&D planned for $t+j-\theta$. Hence the bracketted expression in (6) is the discounted value (in terms of period $t+j-\theta$) of cost savings due to planned R&D and may be interpreted as the expected shadow price of R&D, $q_{t,s}$. Then (6) expresses the optimal planned expenditures on R&D as a linear function of the anticipated price of R&D investment goods and the implicit shadow price of R&D.

Figure 1

The model is illustrated in Figure 1. The marginal cost of R&D schedule rises linearly with the level of R&D, and is shifted by anticipated changes in the price of R&D investment goods. The shadow price relevant to investment planned for year t + s in year t, $q_{t,s}$, depends on the expected future stream of output (which determines the cost savings due to R&D investment), but it is independent of the level of R&D. The optimal amount of planned R&D, $\overline{R}^*_{t,s}$, is fixed by the intersection of the shadow price and marginal cost schedules. Both the supply and demand schedules of R&D are driven by the firm's expectations. Any shift in expected output or the anticipated price of R&D will alter the optimal level of planned R&D.

An alternative form of the investment equation can be obtained in which the infinite series of expected output does not appear. Leading the target period in (6), multiplying by β and subtracting the result from (6) we obtain

$$(7) \qquad R_{t,s} = -aP_{t,s} + a\beta P_{t,s+1} + bQ_{t\ s+\theta} + \beta R_{t,s+1} \cdot$$

where $b = av\alpha^{\theta}$. We refer to equations (6) and (7) as the structural and reduced form investment equations, respectively. One advantage of the reduced form in (7) is that it contains a testable implication of the cost of adjustment formulation independent of the particular specification of expectations. Specifically, the coefficient on the leading R&D anticipation, $R_{t,s+1}$, should be approximately equal to the gross discount factor $\beta = (1-\delta)/(1+r)$.

The realization function relates the difference between actual and planned investment in R&D for a given target period (the realization error) to its determinants. Using (6) the general form for the realization function is

$$(8) \quad D_{t,s} \equiv R_{t,o} - R_{t-s,s} = -a(P_{t,o} - P_{t-s,s} + b \sum_{j=0}^{\infty} \beta^{j}(Q_{t,j+\theta} - Q_{t-s,j+\theta+s})$$

Note that the realization error depends on the error in predicting the price of R&D and the discounted value of the revisions in expected output (i.e., the revision in the shadow price of R&D). Hence, the realization function reflects the use of new information regarding the exogenous variables in the investment model which becomes available between the formation and implementation of the investment plans. However, the precise form of the realization function (and of the underlying investment function) depends critically on how the new information is used, that is, on the manner in which expectations are formed.

One general point of interest is that the realization errors will have zero mean under a variety of expectational mechanisms. It follows

from (8) that $E_t D_{t,s} = 0$ if two conditions hold, i) $E_t P_{t,0} = E_t P_{t-s,s}$ and ii) $E_t Q_{t,j+\theta} = E_t Q_{t-s,j+\theta+s}$, where E_t is the expectation operator over t. A sufficient condition for (i) and (ii) to hold is that the firm forms unbiased predictors of the price of R&D and the level of output.

2. Model Under Specific Expectations Hypotheses

In this section we derive estimable forms of the investment and realization functions under three alternative expectations hypotheses. It should be noted that the available data set (described in section 3) contains actual and one-span planned R&D expenditures; no multiple-span anticipations are provided. Though the model applies to multi-span investment decisions, we are limited in the empirical work to the actual and one-span structural investment equation, the reduced form equation for actual R&D, and the one-span realization function (refer to (6) - (8) above).

2.1. Rational Expectations

The test of the rational expectations hypothesis is based on the assumption that the firm forms expectations of the price of R&D and the level of output according to the stochastic processes (presumed to be) generating these exogenous variables. We assume that each variable evolves according to an autoregressive process:

$$P_t = b_1 P_{t-1} + \ldots + b_m P_{t-m} + \varepsilon_t \tag{9}$$

$$Q_t = c_1 Q_{t-1} + \ldots + c_n Q_{t-n} + u_t \tag{10}$$

where ε_t and u_t are mutually uncorrelated white noise disturbances.[2] Define

$$x_t = \begin{bmatrix} P_t \\ P_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ P_{t-m+1} \end{bmatrix}, \quad z_t = \begin{bmatrix} Q_t \\ Q_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ Q_{t-n+1} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 & \cdots & & b_m \\ 1 & 0 & & 0 \\ 0 & 1 & 0 \cdots & \vdots \\ \cdot & & & \cdot \\ \cdot & & & \\ \cdot & & & \\ 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} c_1 & & & c_n \\ 1 & 0 & & 0 \\ 0 & 1 & 0 \cdots & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \\ \cdot & & & \\ 0 & \cdots & 1 & 0 \end{bmatrix} \qquad \varepsilon_t^* = \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad u_t^* = \begin{bmatrix} u_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}.$$

and the $1 \times m$ and $1 \times n$ vectors $d = (1\ 0\ \ldots\ 0)$ and $e = (1\ 0\ \ldots\ 0)$. If the eigenvectors of B and C are distinct, we can write $B = M\Lambda M^{-1}$ and $C = N\Omega N^{-1}$, where Λ and Ω are diagonal matrices of eigenvalues and M and N are matrices of associated eigenvectors. Then one can show that under the rational expectations hypothesis the following set of equations results:[3]

$$(11a) \qquad R_{t,s} = \left[-daB^s\right] x_t + \left[ebN\Omega^{s+\theta}JN^{-1}\right] z_t \qquad s = 0, 1$$

$$(11b) \qquad R_{t,0} = \left[da(\beta B - I)\right] x_t + \left[ebC^{\theta}\right] z_t + \beta R_{t,1}$$

$$(11c) \qquad D_{t,1} = -da\varepsilon_t^* + \left[ebN\Omega^{\theta}JN^{-1}\right] u_t^*$$

where H is a diagonal matrix with elements $(1 - \beta\lambda_i)^{-1}$ and λ_i are the eigenvalues in Λ, J is a diagonal matrix with elements $(1 - \beta\omega_i)^{-1}$ and ω_i are the eigenvalues of Ω, and the bracketed terms represent the vector of coefficients under rational expectations.

The structural equation for planned R&D s-periods ahead in (11a) is simply a distributed lag against m past prices of R&D and n past levels of

output, where m and n are the orders of the autoregressions in (9) and (10). The reduced form equation in (11b) includes these determinants plus the leading R&D anticipation (i.e., planned R&D for one period ahead). Equation (11c) relates the one-span realization error to the unanticipated components (or "surprises") in the price of R&D and the level of output which are realized between the formulation and the implementation of the planned R&D investment. Since under the rational expectations hypothesis the firm exploits the available information on the exogenous variables fully (i.e., according to their true stochastic structures), the realization error should be determined solely by these surprises.[4]

The unconstrained version of (11a) - (11c) is overidentified. The rational expectations hypothesis delivers a set of nonlinear parameter restrictions both within and across equations (given by the bracketed terms in (11a) - (11c)) which serve to identify the parameters a, b and β. These restrictions are related to the parameters in the underlying stochastic representations of the exogenous variables in the model. It should be noted, however, that since the realization function in (11c) is definitionally related to the investment equation (11a), the parameter restrictions in (11c) contain no independent information. Therefore, the basic system of equations which we estimate consists of the autoregressions in (9) and (10), and (11a) and (11b). First the unconstrained system is estimated and then the parameter restrictions are imposed and tested. In addition to these parameter restrictions, the rational expectations hypothesis implies two testable propositions on the realization function. First, only the contemporaneous surprises in the price of R&D and the level of output should matter, since earlier surprises are known when the R&D plans are formed and should already be reflected in those plans. Hence, lagged

surprises should be statistically insignificant when added to (11c). Second, since the unanticipated components ε_t and u_t have zero means by construction, the mean of the realization errors must be zero under rational expectations. This simply reflects the unbiasedness property of rational forecasts and the linearity of the model in the stochastic exogenous variables.

2.2. Adaptive Expectations

Suppose that the firm forms its forecasts of exogenous variables according to an adaptive expectations mechanism, revising its single-span forecast by some fraction of the previous period's forecast error:

$$(12a) \qquad P_{t,1} - P_{t-1,1} = \gamma(P_t - P_{t-1,1}) \qquad 0 < \gamma < 1$$

$$(12b) \qquad Q_{t,1} - Q_{t-1,1} = \lambda(Q_t - Q_{t-1,1}) \qquad 0 < \lambda < 1$$

It is well known that this procedure implies forecasts which are geometrically weighted averages of all past realizations:

$$(13a) \qquad P_{t,1} = \gamma \sum_{i=0}^{\infty} (1 - \gamma)^i P_{t-i}$$

$$(13b) \qquad Q_{t,1} = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i Q_{t-i}$$

We also note that if (and only if) P_t and Q_t are (mean) stationary processes, the adaptive forecasts in (13a) and (13b) are unbiased predictors.[5]

For present purposes we also need multiple-span forecasts since they appear in the expression for the shadow price of R&D. However, the adaptive expectations hypothesis is silent on how agents form multiple-span forecasts. Muth (1960) has shown that if the underlying stochastic process is of a

particular form for which adaptive forecasts are also rational, then the (minimum mean squared error) multiple and single span forecasts are identical. This line of argument, however, erases the distinction between adaptive and rational forecasts. An alternative way of linking single and multiple-span forecasts would be to construct an explicit model of learning in which agents do not know the true stochastic structure but form adaptive expectations which are "optimal" predictors on the basis of some subjectively assumed structure, and then somehow update their knowledge of that structure and the associated coefficient of adaptation. Models of this type, however, are not yet available in the literature and to construct one here would take us far afield. In the absence of a learning model we adopt the arbitrary assumption that a firm which forms its single-span expectation adaptively also holds that forecast for multiple-spans, that is, $P_{t,s} = P_{t,1}$ and $Q_{t,s} = Q_{t,1}$ for $s \geq 1$.[6] Although this assumption is formally identical to Muth's result, it is not assumed here that the multiple-span forecasts are minimum mean squared error predictions.

Using this assumption and (13a) and (13b), we obtain the following system of structural (14a) - (14b), reduced form (14c) and realization functions (14d) under adaptive expectations:[7]

$$(14a) \qquad R_{t,0} = -aP_t + a(1-\lambda)P_{t-1} + \frac{b\lambda}{1-\beta} Q_t + (1-\lambda)R_{t-1,0}$$

$$(14b) \qquad R_{t,1} = -a\gamma P_t + a\gamma(1-\lambda)P_{t-1} + \frac{b\lambda}{1-\beta} Q_t - \frac{b\lambda(1-\lambda)}{1-\beta} Q_{t-1}$$

$$+ (2-\gamma-\lambda)R_{t-1,1} - (1-\gamma)(1-\lambda)R_{t-2,1}$$

$$(14c) \qquad R_{t,0} = -a(1-\beta\gamma)P_t + a(2-\gamma-\lambda-\beta\gamma(1-\lambda))P_{t-1}$$
$$- a(1-\lambda)(1-\gamma)P_{t-2} + b\lambda Q_t - b\lambda(1-\lambda)Q_{t-1}$$
$$+ \beta R_{t,1} + (2-\gamma-\lambda)R_{t-1,0} - (1-\lambda)(1-\gamma)R_{t-2,0}$$
$$- \beta(2-\gamma-\lambda)R_{t-1,1} + \beta(1-\lambda)(1-\gamma)R_{t-2,1}$$

$$(14d) \qquad D_{t,1} = -aP_t + a(1+\gamma-\lambda)P_{t-1} - a\gamma(1-\lambda)P_{t-2} + \frac{b\lambda}{1-\beta}Q_t$$
$$- \frac{b\lambda}{1-\beta}Q_{t-1} + \frac{b\lambda(1-\lambda)}{1-\beta}Q_{t-2} + (1-\lambda)R_{t-1,0}$$
$$- (2-\gamma-\lambda)R_{t-2,1} + (1-\gamma)(1-\lambda)R_{t-3,1}$$

The model provides qualitative predictions on the coefficients of all variables in the unconstrained system. The unconstrained model is overidentified, and the adaptive expectations hypothesis implies a set of fifteen nonlinear parameter restrictions in (14a) - (14c) which serve to identify the five underlying parameters a, b, β, γ and λ. Estimation of the realization function (14d) is redundant since it is a linear combination of (14a) and (14b). Therefore, the basic set of estimating equations consists of (14a) - (14c). We first estimate these equations unconstrained, and then impose and test the identifying restrictions. Finally, it was noted earlier that adaptive forecasts are unbiased if the stochastic exogenous variables are (mean) stationary. This property implies the testable proposition that the realization errors have a zero mean.

2.3. Static Expectations

Under the static expectations hypothesis the firm assumes that the future values of exogenous variables will remain at their current levels, that is $P_{t,s} = P_t$ and $Q_{t,s} = Q_t$ for $s \geq 1$. It is clear from (12a) and (12b) that this hypothesis is a limiting case of adaptive expectations where $\gamma = \lambda = 1$. By substituting this condition into (14a) and (14b) we observe that, under static expectations, the structural investment equation depends only on the contemporaneous price of R&D and level of output, while the realization error depends solely on the most recent actual (not unanticipated) changes in these exogenous variables.

The most straightforward way of testing static expectations is to impose the constraints $\gamma = \lambda = 1$ in the system of equations under adaptive expectations. This procedure generates thirteen exclusion restrictions in (14a) - (14c) which can be tested directly. In addition, we estimate the realization function under static expectations (by regressing the realization error against the most recent actual change in the price of R&D and the level of output) and test the joint significance of lagged changes in these variables.

3. Data and Empirical Results

3.1. Description of Data

The data set used in this study is drawn from annual surveys of actual and planned investment expenditures on plant and equipment and R&D by firms, conducted by the McGraw-Hill Publishing Company (for a fuller description see Eisner 1978 and Rasmussen 1969). There was a problem of sporadic missing observations in the data for different firms. Using some

supplementary information, we were able to construct a set of data on actual and one-span planned R&D for the period 1959-1969 and on sales for 1954-1969 for forty-nine manufacturing firms, subject to the requirement that no firm have more than two missing observations. Because the missing data vary by firm and by variable, the usable sample depends on the model being estimated. It is not entirely clear whether the reported data on planned R&D should be interpreted as expressed in current or anticipated prices. Since the McGraw-Hill surveys request information on planned R&D expenditures and do not indicate that these should be in present prices, we interpret them as in anticipated (one-year ahead) prices (which is consistent with the definition of $R_{t,s}$ in the model; see section 1). The sales data are deflated by the Wholesale Price Index for total manufacturing. We also require (as an independent variable) a price index for R&D investment goods. To construct a firm-specific index would require information on the firm's composition of R&D expenditures, which is not available. We therefore chose to use an aggregate index for manufacturing constructed on the basis of the mix of R&D inputs at the (roughly two-digit) industry level (Schankerman 1979). This is essentially equivalent to using time dummies in the regressions.

Estimation of the model under rational expectations is conducted on detrended variables. Each variable is regressed on a constant, a linear trend and trend squared (for each firm separately), and the residuals from these regressions are used as data in estimating the R&D investment model. This is frequently done (Sargent 1978, 1979a; Meese 1979) to ensure stationarity of the stochastic variables in the model and on the argument that the theory under rational expectations predicts that the deterministic components (presumed to be known) of the process linking endogenous and

exogenous variables will not be characterized by the same distributed lag model as their indeterministic components. Detrending prior to estimation is an attempt to isolate the indeterministic components. We also estimated the model without detrending and the major conclusions reported later did not change. These arguments in favor of detrending do not apply to the model under adaptive and static expectations because these forecasting devices are not based on the underlying stochastic processes generating the exogenous variables, and hence they do not distinguish between the deterministic and indeterministic components. We therefore estimate the model under adaptive and static expectations without prior detrending. This means of course that the fits of the equations under rational expectations cannot be compared directly, since the dependent variables are measured differently.

3.2. Empirical Results Under Rational Expectations

All models were estimated by Zellner's seemingly unrelated equations technique (Zellner 1962), which is generalized least squares allowing for free correlation in the errors across equations. Table 1 presents the unconstrained estimates of the model under rational expectations using a third order autoregressive specification for the price of R&D and the level of output.[8] Because the means were removed in the detrending procedure, the results in Table 1 represent within-firm, over-time regressions. We first note that the estimated autoregressions imply both real and complex roots which satisfy the stationarity condition that the largest modulus be less than unity. The low R^2 in the autoregression for output indicates a large unanticipated component in the prediction of output. The much higher R^2 in the autoregression for the price of R&D is not a statistical artifact reflecting the use of the same aggregate price index for all firms

Table 1

Table 1. Empirical Results Under Rational Expectations

Equation/ Dependent Variable	Independent Variable												
	P_t	P_{t-1}	P_{t-2}	P_{t-3}	Q_t	Q_{t-1}	Q_{t-2}	Q_{t-3}	$R_{t,1}$	S_t^{P*}	$S_t^{Q\S}$	R^2	DW
Structural $R_{t,0}$	.11[a] (.13)				.071 (.027)	.16 (.03)	-.17 (.03)					.13	2.45
Reduced Form $R_{t,0}$	-.27 (.15)	.45 (.24)	-.15[a] (.14)		.020 (.010)	.039 (.011)	-.011[a] (.011)		.85 (.002)			.88	1.44
Structural $R_{t,1}$	.70 (.22)	-.92 (.28)	.37 (.17)		.060 (.029)	.140 (.032)	-.19 (.034)					.11	2.51
Realization $D_{t,1}$										1.62[a] (1.13)	.23 (.05)	.05	2.10
Autoregression P_t		1.08 (.03)	.12 (.05)	-.55 (.03)								.95	2.38
Autoregression Q_t						.33 (.05)	-.27 (.05)	.053[a] (.055)				.11	1.82

Notes: Estimated standard errors are in parentheses. A superscript "a" denotes statistical insignificance at the 0.05 level.

* $S_t^P = P_t - b_1 P_{t-1} - b_2 P_{t-2} - b_3 P_{t-3}$ where the b's are the estimated coefficients in the autoregression for P_t.

§ $S_t^Q = Q_t - c_1 Q_{t-1} - c_2 Q_{t-2} - c_3 Q_{t-3}$ where the c's are the estimated coefficients in the autoregression for Q_t.

in the sample. Estimation of this autoregression on a single time series yields an $R^2 = .98$. There is in fact only a very small unanticipated component in the measured price of R&D.

Most of estimated coefficients in the investment equations are statistically significant. The sum of the output coefficients is positive in two of the three investment equations, which is expected since a sustained increase in the level of output should raise the shadow price and hence the optimal level of R&D. By analogous reasoning we expect the sum of the price coefficients to be negative, but it is essentially zero in the empirical results. Not much can be deduced from the particular pattern of coefficients since under rational expectations this pattern is related in a highly nonlinear way to the eigenvalues from the autoregressions for price and output. We formally test these restrictions later. Also note that the structural investment equations account for only about ten percent of the within-firm variance in actual and planned R&D. The much better fit of the reduced form equation for actual R&D is due to the presence of the leading anticipation, $R_{t,1}$, as a regressor.

One notable result in Table 1 is the coefficient on $R_{t,1}$ in the reduced form equation for $R_{t,0}$. We showed in section 1 that this coefficient should equal the gross discount factor $\beta = (1 - \delta)/(1 + r)$. Assuming $r = .10$ and $\delta = .10$ we expect to obtain $\beta = 0.8$, which is close to the actual estimated value $\hat{\beta} = .85$.[9] As we will see later, however, the estimate of β is robust to different specifications of expectations formation and hence the result in Table 1 should not be interpreted as evidence in favor of rational expectations.

The realization function in Table 1 relates the (one-span) realization error to the contemporaneous unanticipated components in the price of R&D and the level of output (S_t^P and S_t^Q). These components are defined within the estimation procedure to ensure that they are consistent with the estimated autoregressions for price and output (see notes to Table 1). The "surprise" in output has a significantly positive effect on the difference between actual and planned R&D, which is the expected result since a positive surprise in output raises the shadow price of R&D and hence the optimal R&D investment. The expected effect of a surprise in the price of R&D is negative since an unexpected rise in its price shifts the marginal cost of R&D schedule upward and hence lowers the optimal investment in R&D. The estimate in Table 1 has the wrong sign but is statistically insignificant.

We turn next to various tests of the rational expectations hypothesis. The first, and least stringent, test concerns the realization errors. It was pointed out in section 1 that the mean of the realization errors will be zero if the firm forms unbiased predictors of the price of R&D and the level of output. Since rational forecasts are unbiased, this is an implication of the rational expectations hypothesis. The mean of the realization errors (based on data prior to detrending) for the entire sample is not significantly different from zero (-0.83 with a standard error of 2.18). When computed separately for each firm, only three of the forty-nine firms exhibit non-zero means and each of these cases is only marginally significant. We conclude that the rational expectations hypothesis passes this weak necessary condition, but it is important to reiterate that any unbiased forecasting device would also satisfy this requirement.

The formal parametric tests are considered next. First, rational expectations implies a set of nonlinear restrictions on the parameters of the system of investment equations. These restrictions are expressed in terms of the eigenvalues of the autoregressive structures generating the price of R&D and the level of output. We use the following two-stage testing procedure: First the unconstrained system [(9) - (10) and (11a) - (11b)] is estimated and the eigenvalues are computed. The nonlinear restrictions embodied in (11a) - (11b) are then computed numerically, and the constrained system is estimated. We do not iterate on this procedure (using the new estimates for the autoregressions), but the second-stage constrained estimates are consistent in any case. The test requires an assumed value for the gestation lag, θ. The reported results are based on $\theta = 2$ (from Pakes and Schankerman, this volume) but they are not sensitive to different values (we experimented with $1 \leq \theta \leq 4$).

Table 2

The results are summarized in the first row of Table 2. The parameter restrictions are strongly rejected. The computed F of 21.4 greatly exceeds the critical value of 1.62. Imposition of the restrictions reduces the total mean squared error by 11.2 percent. However, one may object to a simple F-test at a fixed level of significance in a sample as large as ours (1444 observations in the system as a whole). The reason is that any null hypothesis (viewed as an approximation) will be rejected with certainty as the sample size goes to infinity if the Type I error is held constant. Leamer (1978, Chapter 4) argues forcefully that the critical value of the F-statistic should rise with sample size to avoid this interpretive problem. He proposes an alternative measure of the critical value (which we call the Bayesian F) which has the property that, given a diffuse prior

Table 2. Tests of Expectations Hypotheses

	Computed F	Critical $F_{.05}$	%Δ MSE	Bayesian F[a]
Rational				
(1) Investment Equations	21.4	F(18,1426) = 1.62	11.2	7.54
(2) Realization Function	10.5	F(4,376) = 2.39	11.0	6.05
Adaptive				
(3) Investment Equations	4.32	F(15,1201) = 1.67	4.4	7.31
Static				
(4) Investment Equations $\gamma = 1$	3.84	F(5,1201) = 2.22	1.5	7.22
(5) Investment Equations $\gamma = \lambda = 1$	189.0	F(13,1201) = 1.73	201.0	7.29
(6) Realization Function	12.8	F(4,439) = 2.39	10.4	6.23

[a] Bayesian $F = \frac{T - k}{p} (T^{p/T} - 1)$ where T is the sample size, T − k denotes degrees of freedom and p is the number of restrictions.

distribution, the critical value is exceeded only if the posterior odds favor the alternative hypothesis. The Bayesian F is reported in the last column of Table 2. In the case of rational expectations, the Bayesian F is 7.54 which is far below the computed F of 21.4. We conclude that the parameter restrictions under rational expectations are rejected even after this adjustment for sample size.

The second row in Table 2 summarizes the test of the joint significance of two lagged surprises in the price of R&D and the level of output in the realization function. Under rational expectations only the contemporaneous surprises should affect the realization error since earlier surprises were known when the R&D plan was formulated. Again, the computed F statistic of 10.5 exceeds both the conventional and the Bayesian critical values (2.39 and 6.05, respectively), and the null hypothesis is rejected.

We conclude from these results that the evidence does not support the rational expectations formulation of the model, at least one based on a third-order autoregressive representation of the price of R&D and the level of output. Various qualifications and explanations for this negative finding will be discussed later, but first we examine the empirical results under alternative expectations hypotheses.

3.3. Empirical Results Under Adaptive and Static Expectations

Table 3

The unconstrained estimates of the model under adaptive expectations are reported in Table 3. The fits of the regression are very good, especially since the data contain both cross sectional and time series variation (the cross sectional variance comprises about 75 percent of the total variance in the sample). On the whole, the pattern of estimated coefficients is consistent with the adaptive expectations hypothesis.

Table 3. Empirical Results Under Adaptive Expectations

Equation/ Dependent Variable	Independent Variable												R^2	DW
	P_t	P_{t-1}	P_{t-2}	Q_t	Q_{t-1}	Q_{t-2}	$R_{t-1,0}$	$R_{t-2,0}$	$R_{t,1}$	$R_{t-1,1}$	$R_{t-2,1}$	$R_{t-3,1}$		
Structural $R_{t,0}$	-.29[a] (.27)	.29[a] (.28)		.024 (.004)			.74 (.02)						.80	1.98
Reduced Form $R_{t,0}$	-.15[a] (.11)	.20[a] (.19)	-.05[a] (.10)	.035 (.009)	-.033 (.010)		.49 (.05)	-.085[a] (.05)	.84 (.01)	-.34 (.04)	.037[a] (.053)		.99	2.03
Structural $R_{t,1}$	-.25[a] (.31)	.25[a] (.32)		-.018[ab] (.017)	.045[b] (.017)					.66 (.04)	.095[b] (.038)		.75	2.24

Notes: Estimated standard errors are in parentheses. A superscript "a" denotes statistical insignificance at the 0.05 level. A superscript "b" denotes an estimated coefficient which has the wrong sign according to the model.

The estimated coefficients on the price variables are uniformly insignificant, which may reflect the inadequacy of the aggregate price index used in the estimation.[10] However, all but two of the other coefficients are statistically significant and seventeen of the twenty estimated parameters have the sign predicted by the model. Also note that the point estimate of the coefficient of $R_{t,1}$ in the reduced form equation for $R_{t,0}$ is 0.84, which is very close to its predicted value. This is almost identical to the estimate under rational expectations, and as we indicated earlier it should be interpreted more as support for the cost of adjustment formulation of the model than for either specific expectations mechanism. The magnitudes of the other parameter estimates in Table 3, however, do tend to support the adaptive expectations hypothesis. A comparison of these results with the corresponding parameters in (14a) - (14c) indicates that many of the parameter restrictions implied by adaptive expectations are satisfied approximately by the unconstrained point estimates.

Before turning to the formal tests of adaptive expectations, we first note that this hypothesis is not consistent with the zero mean of the realization errors. The reason is that the observed price of R&D and the level of output are not mean stationary and hence adaptive forecasts as formulated in (13a) - (13b) are not unbiased. This violation should be qualified by two considerations. First, we have only single-span realization errors to test the hypothesis. Second, and more important, the adaptive forecasting device in (13a) - (13b) can be modified easily to account for (known) trends in the variables and the modified version will produce unbiased forecasts (see Note 5 for discussion).

The formal tests of adaptive expectations are presented in the third row of Table 2.[11] There are fifteen nonlinear restrictions implied by the

hypothesis. The computed F statistic is 4.32, compared to a critical value of 1.67, and the hypothesis is rejected formally. However, imposition of the constraints raises the mean squared error by only 4.4 percent. This suggests that the restrictions may not be a bad approximation in view of the large sample size. A testing procedure using the Bayesian F supports this view. The critical value is 7.31 and the adaptive expectations restrictions are not rejected. It is worth reiterating that the proper interpretation of this result is that, given a diffuse prior distribution on the parameters, the posterior odds favor the null hypothesis that the restrictions hold.

As indicated in section 2.3, static expectations are a special case of adaptive expectations where $\gamma = \lambda = 1$. Inspection of the unconstrained estimates in Table 3 suggested that the constraint $\gamma = 1$ is more reasonable than $\lambda = 1$ and we therefore test the former separately. The results are summarized in rows 4 and 5 of Table 2. The computed F for the five restrictions implied by $\gamma = 1$ is 3.84, while the critical value is 2.22. The restrictions are marginally rejected but the change in the mean squared error is a negligible 1.5 percent. When judged against the Bayesian F of 7.22, the hypothesis $\gamma = 1$ is easily accepted. However, the restrictions implied by the joint hypothesis $\gamma = \lambda = 1$ (completely static expectations) are strongly rejected. The computed F of 189.0 greatly exceeds both the conventional and Bayesian critical values, and the mean squared error more than doubles when the constraints are imposed. As an additional check, we also estimated the realization function under full static expectations and tested the joint significance of two lagged changes in the price of R&D and the level of output. Under static expectations only the

contemporaneous changes in these variables should influence the realization error. As row 6 in Table 2 indicates, the hypothesis is rejected at both conventional and Bayesian critical values.

We conclude from these tests that the evidence generally supports the adaptive expectation hypothesis and decisively rejects the strong version of static expectations. Actually, the hypothesis most favored by the data is a mixed one with static expectations on the price of R&D and an adaptive mechanism on the level of output.

We can use the constrained estimates from the adaptive version to identify the underlying parameters in the model. The estimates (standard error) are: $\hat{a} = -.003$ (.009), $\hat{\beta} = .85$ (.015), $\hat{\lambda} = .17$ (.032), $\hat{\gamma} = 1.28$ (.080), and $\hat{b} = .013$ (.017). The estimate $\hat{a}$ has the right sign but is insignificant, and $\hat{\gamma}$ lies outside the required range $0 < \gamma \leq 1$ but not substantially so. (This violation can occur because the restrictions are rejected under classical testing criteria, but accepted after a Bayesian adjustment for sample size.) The $\hat{\lambda}$ implies an average lag of about five years in the formation of output expectations ($(1 - \hat{\lambda})/\hat{\lambda} = 4.9$). The estimate $\hat{b}$ can be used to compute the elasticity of R&D with respect to the shadow price of R&D, η_{rq}. Using equations (6) and (7) we can write $\eta_{rq} = b(\sum_{j=s+\theta}^{\infty} \beta^j Q_{t,j})/R$. Evaluating at the sample means (denoted by bars) and letting $Q_{t,j} = Q$, $\eta_{rq} = [b/(1-\beta)]\,\bar{Q}/\bar{R}$. This yields the point estimate (standard error) $\hat{\eta}_{rq} = 1.45$ (0.82). The point estimate is imprecise (which may not be surprising since $\hat{\eta}_{rq}$ is a nonlinear function of estimated parameters), but it indicates that a ten percent increase in the shadow price of R&D raises the optimal level of R&D by about fifteen percent. It is interesting to note that this estimate of η_{rq} is broadly similar to estimates of the elasticity of the investment-capital ratio with respect to Tobin's q for traditional capital (Abel 1979;

Cicollo 1975). Also note that our model of investment in R&D is based on cost minimization and as a result the shadow price of R&D is proportional to the expected levels of output in the future. Therefore, η_{rq} may also be interpreted as the elasticity of R&D with respect to a sustained increase in all future levels of output. The estimate $\hat{\eta}_{rq} = 1.45$ then implies that R&D rises somewhat more than proportionally to the ("permanent" or sustained) level of output. Given its statistical imprecision, this finding is not inconsistent with the empirical literature on the relationship between R&D and output (for a review see Scherer 1980).

3.4. Adaptive versus Rational Expectations

The statistical tests conducted in sections 3.2 and 3.3 yield two main conclusions. First, the data do not support a rational expectations formulation based on third-order autoregressive representations of the exogenous variables (price of R&D and level of output). Second, the evidence is generally consistent with adaptive expectations and especially favors adaptive forecasting on output and static expectations on the price of R&D. Why would a firm employ two different forecasting devices for the two exogenous variables? The simple answer that the empirical confirmation of this mixed hypothesis is weak and should not be taken too seriously seems at odds with the statistical tests. A more interesting explanation might argue that this finding reflects rational forecasting for the true stochastic processes generating the exogenous variables and that the rejection of rational forecasting in section 3.2 is due to a misspecification of these processes. Is the mixed static-adaptive expectations hypothesis consistent with rational expectations?

As indicated earlier (note 8 in section 3.2), there is some evidence that a moving average specification of the stochastic processes might be more appropriate than a third-order autoregressive one. However, in order for this alternative explanation to work the true stochastic processes must be of a particular form: (1) Q_t must be an IMA (1,1) (integrated moving average) process $Q_t = Q_{t-1} + \zeta_t - \psi\zeta_{t-1}$, where ζ_t is a white noise error, since Muth (1960) shows that for this process rational forecasts are also adaptive, and (2) P_t must be a random walk process $P_t = P_{t-1} + v_t$ where v_t is a white noise error, since for this model static expectations are rational.

We cannot test this explanation rigorously with the available data but several pieces of indirect evidence are worth noting. First, Muth (1960) shows that for an IMA(1,1) process the adaptation coefficient in the rational forecast (λ in our notation) equals the ratio of the variance of the permanent component to the total variance. A consistent estimator of this ratio is given by the R^2 from the fitted IMA(1,1) regression. Under this hypothesis the estimated autoregression on Q_t in section 3.2 is of course misspecified, but it is interesting to note that the $R^2 = .11$ from that regression is quite close to (and within two standard errors of) the constrained estimate of the adaptation coefficient $\hat{\lambda} = .17$. Similarly, the $R^2 = .98$ from the autoregression on P_t is very close to the restricted value $\gamma = 1$ which was accepted by the data. These observations lend some credence to this alternative explanation.

On the other hand, if this alternative were true one would expect the adaptive expectations formulation to be confirmed on detrended data (where the nonstationarity in the observed price and output series has been removed). However, re-estimation of the model under adaptive expectations

on detrended data indicates that the parameter restrictions are rejected both at conventional and Bayesian critical values of the F statistic.[12] As a further check, we estimated a first order autoregressive process for detrended P_t. Under this explanation, the coefficient on lagged P_t should be unity and the errors should be serially uncorrelated. The estimated coefficient is essentially unity, but there is strong evidence of serial correlation (Durbin Watson = 0.57) and in this respect the first-order specification is distinctly worse than higher-order autoregressive processes.

We conclude that the evidence is mixed on whether rational expectations can be reconciled with the empirically supported adaptive-static expectations scheme.

Concluding Remarks

This paper proposes a framework which integrates convex costs of adjustment and expectations formation in the determination of actual and multiple-span planned investment decisions in R&D at the firm level. The framework is based on cost minimization subject to the firm's expectations of the future stream of output and the price of R&D. The model results in equations for actual and multiple-span planned R&D investment and for the realization error as a function of these expectations. One of the unique features of the model is that it accommodates alternative mechanisms of expectations formation and provides a methodology for testing these hypotheses empirically. In order to give the model empirical content, a specific mechanism of expectations formation must be specified. We investigate the three leading forecasting hypotheses--rational, adaptive and static expectations. Estimable equations and a set of testable parameter restrictions are derived under each of these three hypotheses.

The models are estimated on a set of pooled firm data covering the period 1959-1969. The empirical results indicate that the parameter restrictions implied by both the rational and (fully) static expectations hypotheses are strongly rejected. The evidence generally supports adaptive expectations, both in terms of qualitative consistency of the unconstrained estimates with the predictions of the model and the formal tests of the parameter restrictions. Actually, it appears that the hypothesis most favored by the data is a mixed one, with adaptive forecasting on the level of output and static expectations on the price of R&D. We also investigated whether this basic empirical finding could be reconciled with rational expectations and the formal rejection of this hypothesis explained by a misspecification of the stochastic processes generating the exogenous variables in the model. The available evidence for this interpretation is mixed. We emphasize that the basic empirical conclusion of this paper is that adaptive (or mixed adaptive-static) expectations are confirmed by the data. The appropriate interpretation of this result, however, remains an open question.

The theoretical framework and the empirical findings suggest directions for future research. The model could be improved by endogenizing the level of output and proceeding from profit maximization rather than cost minimization, and by treating both R&D and physical capital as quasi-fixed assets subject to costs of adjustment. On the empirical side, richer data sets are needed to explore the formation of expectations more fully, and specifically to establish whether the adaptive expectations hypothesis constitutes a substantive alternative to or simply a guise for rational expectations.

Notes

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1. This assumption implies that the marginal savings in variable costs due to R&D is a constant, i.e., $\partial^2 C/\partial K^2 = 0$. This violates the standard second order condition for restricted cost functions that $\partial^2 C/\partial K^2 < 0$ and, in a static context, generates as infinitely elastic demand for R&D (and hence an indeterminate level of R&D). In a cost of adjustment framework the analogue is an infinitely elastic shadow price of R&D, but an optimal level of R&D is ensured by an upward sloping marginal cost of investment schedule (see Figure 1).

2. The following setup is based on Sargent (1978) but we extend the argument to planned investment and realization functions. The assumption that u_t and ε_t are contemporaneously uncorrelated simplifies the prediction formulas for P_t and Q_t. This assumption is subjected to an empirical test (see note 8).

3. The procedure to derive (11a) - (11c) is as follows. From the assumption $E_t(\varepsilon_{t+j}) = E_t(u_{t+j}) = 0$ for $j > 0$ we obtain $P_{t,s} = dM\Lambda^s M^{-1} x_t$ and $Q_{t,s} = eN\Omega^s N^{-1} z_t$. Substitutions of these expressions into (6) and (7) and some manipulation yields (11a) and (11b). To derive (11c) note from (9) - (10) that $x_t = B^s x_{t-s} + B^{s-1}\varepsilon^*_{t-s+1} + \ldots + \varepsilon^*_t$ and $z_t = C^s z_{t-s} + C^{s-1} u^*_{t-s+1} + \ldots + u^*_t$. Using these and the expressions for $P_{t,s}$ and $Q_{t,s}$ in (8) yields (11c).

4. Similar implications appear in the literature on the efficient market hypothesis (Fama 1970) and recent work on the permanent income hypothesis under rational expectations (Bilson 1980; Hall 1978).

5. If the forecasted variable, say P_t, is trended then the adaptive forecast in (13a) will be biased. If the series is growing at the rate g, then an unbiased predictor is obtained from the modified adaptive forecast $P_{t,1} = (1 + g)\gamma \sum_{i=0}^{\infty} (1 - \gamma)^i P_{t-i}$. Given that the agent forecasts adaptively and g is ascertainable, it is reasonable to assume the agent uses the modified formula.

6. If P_t and Q_t are growing at rate g_r and g_q and the firm uses the unbiased modified version of adaptive forecasting (note 5), we have $P_{t,s} = (1 + g_p)^{s-1} P_{t,1}$ and $Q_{t,s} = (1 + g_q)^{s-1} Q_{t,1}$. Then the coefficients in the system of equations in (14a) - (14d) are slightly modified.

7. Equation (14a) is obtained by substituting (13b) into (6) for $s = 0$ and performing a Koyck transformation on Q_t (to remove the infinite past series on Q_t). To obtain (14b) substitute (13a) - (13b) into (6) for $s = 1$ and perform two sequential Koyck transformations on P_t and Q_t. Equation (14c) is derived by a similar procedure using (13a) - (13b) in (7). Finally, (14d) is obtained by lagging (14b) and subtracting it from (14a).

8. Two points should be noted. First, we checked the assumption that the disturbances ε_t and u_t in (9) and (10) are contemporaneously uncorrelated by testing the univariate autoregressive representations against a general bivariate specification. This involves testing the joint significance of three lagged values of Q_t in the autoregression for P_t and three lagged values of P_t in the autoregression for Q_t. The computed F statistics are

1.42 and 1.60, respectively, compared to the critical level $F(3,548) = 2.60$. The simplifying assumption $E(\varepsilon_t u_t) = 0$ is accepted. Second, there is evidence that a higher order autoregression is appropriate, but including more than three lagged values of output and price would reduce the sample size unacceptably. These higher order terms affect only the last coefficient in the AR(3) representation and they do not improve the equations in terms of serial correlation. Still, they probably do indicate that a moving average or mixed process is more appropriate, but the structure of our data does not permit use of such specifications. In section 3.4 we discuss the implications of these considerations for the interpretation of the empirical findings under rational expectations.

9. The assumed $\delta = .10$ is much lower than the rate estimated by Pakes and Schankerman (this volume). However, in our model δ is the rate of decline in the ability of R&D to "produce" cost reductions, not the rate of decline in appropriable revenues considered by Pakes and Schankerman. For more on the distinction see Schankerman and Nadiri (1982).

10. One problematic result is that the price coefficients in each equation sum to zero. This suggests that the true model should relate the stock of knowledge to the price of R&D, since the first-differenced version (involving R&D flow) would then yield the observed result. On the other hand, the result may just reflect the rather poor price index which we use.

11. We also reformulated the model in (14a) - (14c) using the modified version of adaptive expectations, and estimated the unconstrained and restricted systems. This required estimates of the trends in P_t and Q_t which were obtained from regressions of the logs of these variables against time. The formal tests of the parameter restrictions were qualitatively similar to those reported in the text.

12. The computed F is 8.59, compared to the conventional $F(15,1171) = 1.67$ and the Bayesian $F = 7.29$. Imposition of the restrictions raised the mean squared error by 10.0 percent.

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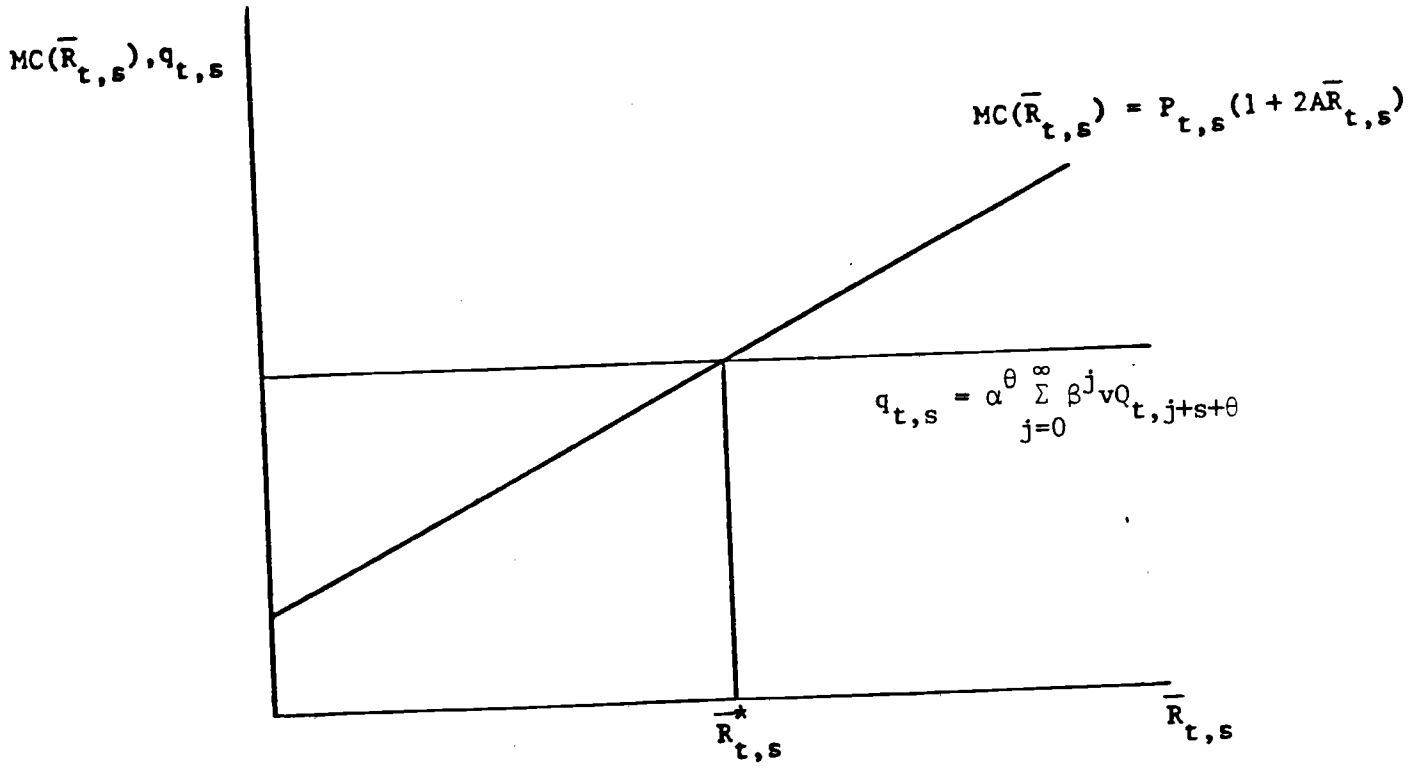
Figure Legend

1. Determination of Optimal Planned R&D


$MC(\bar{R}_{t,s}), q_{t,s}$
$MC(\bar{R}_{t,s}) = P_{t,s}(1 + 2A\bar{R}_{t,s})$
$q_{t,s} = \alpha^{\theta} \sum_{j=0}^{\infty} \beta^{j} vQ_{t,j+s+\theta}$
$\bar{R}^{*}_{t,s}$
$\bar{R}_{t,s}$