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MEASURING RISK AVERSION FROM EXCESS RETURNS ON A STOCK INDEX

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ABSTRACT

We distinguish the measure of risk aversion from the slope coefficient in the linear relationship between the mean excess return on a stock index and its variance. Even when risk aversion is constant, the latter can vary significantly with the relative share of stocks in the risky wealth portfolio, and with the beta of unobserved wealth on stocks.

We introduce a statistical model with ARCH disturbances and a time-varying parameter in the mean (TVP ARCH-M). The model decomposes the predictable component in stock returns into two parts: the time-varying price of volatility and the time-varying volatility of returns. The relative share of stocks and the beta of the excluded components of wealth on stocks are instrumented by macroeconomic variables. The ratio of corporate profit over national income and the inflation rate are found to be important forces in the dynamics of stock price volatility.

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I. Introduction

The trade-off between risk and return is central to the theory of finance. The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) was first to provide a comprehensive framework for determining asset prices with the theme that only systematic risk is rewarded by the market. The risk premium on the market portfolio was linked to investor risk aversion by Tobin (1958), and Pratt (1964)

Merton (1969, 1973) shows that a lifetime consumption-investment model yields risk premia of the same form as the single period model when the investment opportunity set is constant and portfolios are continuously rebalanced. This result will still hold when the variance of the market portfolio varies randomly and cannot be hedged.

Bodie, Kane, and McDonald (1983) and Pindyck (1988) assume a single factor CAPM, and use a "reasonable" parameter for relative risk aversion (between 3 and 4), as defined in Pratt (1964), in an attempt to infer risk premiums from estimated variances. Inferring in the opposite direction Friend and Blume (1975) attempt to estimate the coefficient of relative risk aversion of a representative investor using estimates of relative portfolio shares of financial assets, and the ex-post excess return average and variance of these assets. While they put the estimate of relative risk aversion at about 2, their method uses the unconditional variance, which is not consistent with the model assumption of portfolio rebalancing. There, the risk premium ought to be determined by the conditional or *expected* variance.

GARCH-M models of stock returns (see Bollerslev, Chou, and Kroner (BCK, 1990)) for an extensive review and references of ARCH modeling in finance) jointly estimate the time varying conditional variance and a *constant* mean-variance ratio that represents the risk-return trade-off. BCK document the extensive use of these models (with multivariate extensions) in empirical work in financial economics.

A number of studies question the existence of a positive mean/variance ratio, directly challenging the mean-variance paradigm. In Glosten, Jagannathan and Runkle (1989), when they explicitly include the nominal risk-free rate in the conditioning information set, obtain a negative ARCH-M parameter. While Harvey (1989) finds the ratio of expected return to stock index volatility non-constant and counter cyclical, Backus and Gregory (1988) argue that the relationship between the conditional mean and the conditional variance is non-linear. Abel (1988) claims that in a general equilibrium the mean/variance relationship is not necessarily positive when the investor's preference is not logarithmic.

At the same time, there is some evidence that the static CAPM performs empirically better than the intertemporal consumption-based model. (See among others Mankiw and Shapiro (1986) and Attanasio (1989)). Moreover, the static CAPM may be attracting some new interest. As Grossman and Laroque (1987) show, explicit consideration of transaction costs in consumption technology would make the static CAPM relevant even in an intertemporal context. Others to make a case for the validity of the static CAPM are Epstein and Zin (1989), who derive an intertemporal non-expected utility model.

In a model economy where a representative agent maximizes a time-additive von Neumann-Morgenstern utility, the mean/variance ratio can still change as a result of any combination of: changing preferences toward risk, or changing investment opportunities. Only absent of any such change, with constant relative risk aversion, will the CAPM deliver a constant mean/variance ratio for the market portfolio and its components.

We begin by systematically examining the temporal instability of the mean/variance ratio, first by rolling regressions and then in a more sophisticated manner by introducing a time-varying parameter (TVP) into the ARCH-M model. Finally, we seek to identify empirical macroeconomic proxies for the unobserved components of wealth. (Related work on the sensitivity of the CAPM with respect to changes in the market portfolio is found in Stambaugh (1982)).

Section II applies the ARCH-M model to the CAPM with two risky assets, and provides further evidence of the time-varying pattern of the mean/variance ratio. A time-varying parameter model is then presented in Section III. In Section IV we examine the relation between the estimated time-varying parameter and some particular economic variables. The final section presents conclusions and suggestions for future research.

II. Estimating Risk Aversion in the CAPM Framework with the ARCH-M Model

II.1 The CAPM and the Market Portfolio

Consider an exchange economy where there are three asset classes: one risk-free asset, and two risky asset classes. The risky assets consist of a stock portfolio, whose returns are observed, and an unobserved portfolio of the remaining risky assets. The *excess* returns (over the risk-free rate) on the two risky assets are, respectively, r_S and r_N, with variances σ_S^2 and σ_N^2.

With a joint normal distribution of the excess returns, the CAPM predicts that all investors will hold the market portfolio, the value weighted portfolio of all risky assets. Individuals hold only combinations of the riskless asset and the market portfolio in relative proportions determined by individual risk aversion.

In equilibrium, the expected excess return of the market portfolio, $E(r_M)$, will be related to the means of the asset class portfolios by

$$E(r_M) = wE(r_S) + (1 - w)E(r_N) \qquad \text{(II.1)}$$

where w is the weight of the stock index portfolio in the market, and (1-w) is the proportion of the unobserved class in the value of the market portfolio. The parameter w can also be interpreted as the relative demand for stocks. (1-w) is the sum of the weights of all unobserved risky assets and r_N is the value weighted average of their excess returns.

The CAPM predicts that each risky asset will be priced to earn a risk premium that is given by,

$$E(r_i) = \delta Cov(r_i, r_M) \quad (II.2)$$

where δ is the harmonic mean of individual relative risk aversion, which may be changing over time, because of a structural change in preferences or with the distribution of wealth. Equation II.2 has to hold also for any portfolio, and so r_i may be replaced with r_S and r_N.

(II.1) and (II.2) imply that, for stocks,

$$E(r_S) = \delta[w\sigma_S^2 + (1 - w)\sigma_{SN}] \quad (II.3)$$

where $\sigma_{SN} \equiv Cov(r_S, r_N)$. Equation II.3 indicates that the expected stock-index return is proportional to the weighted average of its variance and covariance with the unobserved portfolio. That is, the stock-index risk premium depends not only on its own volatility but also on its covariances with returns of other risky assets. Thus, the relative shares of the asset classes and uncertainty about the unobserved risky portfolio will affect the stock index.

Most empirical studies of the intertemporal CAPM use broad stock indexes to proxy the market portfolio, e.g., Fama and MacBeth (1973), Black, Jensen, and Scholes (1972). This approach would be justified by either of two assumptions: w=1, that is, stocks are the only relevant risky assets, or that the unobserved assets covariance with stocks is equal to the stock variance, that is, $\sigma_{SN} = \sigma_S^2$. In each case (II.3) reduces to

$$E(r_S)=\delta\sigma_S^2 \ . \qquad \text{(II.4)}$$

None of these assumptions is supported by evidence, however, and findings demonstrate that (II.4) is not adequate to explain movements in stock-index returns.

II.2 The ARCH-M Model and Some Empirical Anomalies

The ARCH-M model proposed by Engle, Lilien, and Robins (1987) consists of the system:

$$y_t = ch_t + e_t \qquad \text{(II.5)}$$

$$h_t = a_0 + a_1 e_{t-1}^2 + a_2 h_{t-1} \qquad \text{(II.6)}$$

where e_t is the prediction error assumed to be Gaussian and serially uncorrelated with mean zero and conditional variance h_t. More specifically, h_t is the conditional variance of the variable y_t given all information up to time t-1. This model characterizes the evolution of the mean and variance of a time series simultaneously.

The process specifying the conditional variance, equation II.6, is a GARCH (1,1) process. It implies that the conditional variance is driven by three factors: the autonomous component, the surprise, and last period's variance. Thus, (II.5) and (II.6) are really GARCH(1,1)-M. Richer dynamic patterns of variances can be modeled by introducing higher-order terms of past prediction errors or conditional variances, but empirical studies frequently suggest that GARCH (1,1) is adequate.[1]

[1] French, Schwert, and Stambaugh (1987), use a GARCH (2,1) in (II.6) with an intercept in the mean equation (II.5), but these variations do not make much difference. For detailed specification and estimation of the GARCH and ARCH-M models, see Bollerslev (1986) and Engle, Lilien, and Robins (1987). The CAPM does not support the inclusion of an

The ARCH-M model (II.5) and (II.6) can be used to estimate the CAPM (II.4) if the stock index is the market portfolio, and its volatility follows the GARCH process. The model will fail, however, if the estimate of relative risk aversion (the mean/variance ratio), c in (II.5), or if the GARCH parameters, a in (II.6), vary over time.

French, Schwert and Stambaugh (1987), or FSS, estimates of the risk aversion parameter are very unstable across sample periods. For the entire sample period 1928-1984 they obtain a value of 1.693 using the NYSE monthly value-weighted index. Estimates for 1928-1952 and 1953-1984 sub-periods are 1.510 and 7.220, respectively. With the Standard & Poor's daily composite index, the two sub-sample estimates are 0.598 and 7.809, even further apart. Estimates obtained in Chou (1988) seem to be more stable: 4.50, 5.05, and 6.15 for the periods 1962-1985, 1962-1973, and 1974-1985, respectively, using the weekly NYSE value-weighted index. Differences in the two studies' estimates could be attributable to the latter shorter sample period.

II.3 Rolling Sample Estimation

We use rolling samples to examine the temporal behavior of the ARCH-M coefficients. To obtain precise estimates, we need data that are more frequent than monthly. Because daily NYSE stock index data are not available until July 1962, we use the Standard & Poor's Composite Index as a proxy for the market portfolio; it is available daily from January 1928 through December 1987.[2]

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intercept in the mean equation because excess returns should be determined only by systematic risk.

[2] The authors wish to thank William Schwert for providing this data set.

We prefer weekly over daily returns to avoid documented anomalies of day-of-the-week effects, e.g., Keim (1986). Weekly excess returns are obtained by differencing the logs of weekly Tuesday closing prices. The risk-free rate used to construct the weekly excess returns is the short-term interest rate from the Ibbotson and Sinquefield database. ARCH-M coefficients were estimated for every quarter from 1933 through 1987. For each quarter estimate, the sample contains five previous years of weekly data, amounting to approximately 260 observations. The rolling estimation procedure yields a quarterly time series of the coefficient c in (II.5), with 221 quarterly observations.

The graph of this series in Figure 1 is strongly time-varying. The coefficient ranges from -0.4 to 15.6, with a mean of 5.4 and standard deviation of 4.1. Both the dynamic pattern and the magnitude of the coefficient are similar to the results in Friend and Blume (1975) who report mean/variance ratios of 0.925, 8.673, 14.165, and 1.372 for the respective four decades between 1932 and 1971. That is, the ARCH-M model, which uses the conditional distribution, confirms the instability of the mean/variance ratio. The erratic behavior of this coefficient indicates the inadequacy of the ARCH-M model to fit the stock return data.

Another empirical anomaly reported by FSS is that the ARCH-M model seems to predict risk premiums which are too high, with the average predicted excess return almost twice the average realized excess returns.[3] It is hard to accept a model that performs so badly in this respect.

To sum up, although the ARCH-M model is a useful tool in modeling the stock index return, adjustments to the model seem necessary. The instability of the estimated value for risk aversion and the inconsistent behavior of excess returns that the ARCH-M model predict are important empirical anomalies that should be resolved.

The estimated parameters from the rolling sample estimation indicates that a time-varying approach may be appropriate. The rolling sample estimated series is only an approximation because it uses relatively short sample periods (five years is arbitrary), and it is unlikely that quarter to quarter changes of the coefficient would be so large. Further, it is inconsistent to estimate a time-varying parameter, while at the same time assuming it to be constant within five-year sample periods.

The next section introduces an ARCH-M model with a time-varying mean/variance ratio, which allows formal estimation and explanation of the variation of this parameter.

III. The Time-Varying Parameter ARCH-M Model

III.1 The Model

Consider the time-varying parameter ARCH-M (henceforth TVP ARCH-M) model

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[3] FSS report the ex post mean of the index return to be 0.61% per month, while the average risk premium (the expected excess return predicted) from the ARCH-M model is 1.34%. In other words, the residual terms do not sum up to zero as the model assumes.

$$y_t = b_t h_t + e_t \tag{III.1}$$

$$b_t = b_{t-1} + v_t \tag{III.2}$$

$$h_t = a_0 + a_1 \eta_{t-1}^2 + a_2 h_{t-1} \tag{III.3}$$

where the GARCH surprise variable is

$$\eta_t \equiv y_t - E_{t-1}(y_t) \; .$$

The errors e_t and v_t are assumed to be uncorrelated Gaussians with zero means and with variances h_t and Q, respectively. This model is a direct extension of the ARCH-M model where the parameter characterizing the mean/variance trade-off is assumed as a random walk. In the literature of state space models, (III.1) and (III.2) are called the measurement and the transition equations; b_t is called the state variable. When h_t is observable, the two equations together formalize the usual time-varying regression model. As h_t measures volatility of stock returns, b_t measures the increment of the risk premium per unit of volatility and will be called the "price of volatility" of stock returns.

In our model, h_t is assumed to be driven by a "modified" GARCH(1,1) process specified by (III.3). In (III.3), the original squared prediction error, e_{t-1}^2 of (II.6), is replaced by η_{t-1}^2, a newly defined prediction error or "innovation." This replacement is necessary because both b_t and h_t are unobservable. The innovation η_t is determined by

$$\eta_t = y_t - E_{t-1}(b_t)h_t = e_t + [b_t - E_{t-1}(b_t)]h_t \tag{III.4}$$

where $E_{t-1}(b_t)$ is the optimal forecast of b_t given all information up to time t-1. As Q, the variance of the state variable b_t, becomes small, the model converges to the fixed-parameter (FP) ARCH-M model.

There are three sets of unknowns to be estimated: b_t, the states; h_t, the variances of e_t; and a_1, a_2, a_3, and Q, the fixed parameters. The estimation of these unknowns is carried out simultaneously by a Kalman filter and maximum likelihood. Estimates of the states are produced by the Kalman filter conditional on the parameter values. Given values of the parameters, the variance of the measurement errors can be obtained through the GARCH equation. After each pass of the Kalman filter and the GARCH equation, the value of the likelihood can be computed, and nonlinear routines can then be used to maximize the likelihood. These steps are repeated until convergence is reached.[4]

At each point in time, the contemporaneous variance of η_t (denoted H_t) is obtained from the values of the parameters. The log likelihood function for this model can be written in terms of the innovations (see Schweppe (1965)), as

$$L = \sum_t L_t = \sum_t -\frac{1}{2}(\log H_t + \frac{\eta_t^2}{H_t}) \quad . \tag{III.5}$$

The quasi Gauss-Newton algorithm is used to maximize the likelihood function. Non-negativity constraints are imposed on h_t by restricting a_0 to be non-negative and a_1 and a_2 to be between 0 and 1. Numerical

4 The Kalman filter is widely used in systems engineering. It has been applied also to economic models with time-varying coefficients and unobservable components. Basically it is a recursive algorithm that produces optimal estimates of the state variable. It is optimal in the sense that it produces the minimum mean square error estimates of the states, conditional on the newly available information. Anderson and Moore (1971) give a comprehensive exposition of Kalman filter methods, and Engle and Watson (1985) provide a survey of applications of the Kalman filter in economics.

derivatives are used to compute the gradient using the IMSL sub-routine "BCONF."

Initial values are required for both state and variance variables, b_0, h_0, as well as for the parameters a_i and Q. Values from estimating a FP (fixed-parameter) ARCH-M are natural candidates for the a_i's and h_0, and indeed turn out to be quite efficient in approaching the final estimated values. A diffuse prior distribution is assumed for the initial value of the state, b_0, i.e., we assign a large value (1000) to its variance.

III.2 Results

The data used for estimation are the monthly excess returns (in percents) of the NYSE value-weighted index for 1926-1985. There are 720 observations. The FP ARCH-M model estimates (with t-statistics in brackets) are:

$$r_{S,t} = \underset{(5.24)}{3.00}h_t + e_t \tag{III.6}$$

$$h_t = \underset{(3.40)}{0.996} + \underset{(5.97)}{0.129}e^2_{t-1} + \underset{(38.89)}{0.835}h_{t-1} \tag{III.7}$$

These parameter estimates are used for initial values in estimating the TVP ARCH-M model. The final converged values for the parameters in the variance equation, a_0, a_1, and a_2 are, respectively, 0.989, 0.127, and 0.836, very close to the estimates in the fixed-parameter model. The estimated value of Q is 0.032, much smaller than that of h_t (average of 31.995). The average value of h_t corresponds to a standard deviation of 19.6% per year, which is close to that of Ibbotson and Sinquefield.

The fact that the parameters in the variance equation are so close to that of the FP ARCH-M model implies that the estimates of h_t in the two models will also be close.

Figure 2 plots the variance estimates from the two models. They seem to be indistinguishable. Note that the expected risk premium predicted by the model is $b_t h_t$. Although the conditional variances from the two models are similar, the implied risk premiums (expected excess returns) for the two can still be quite different if the price of volatility, b_t, varies significantly over time.

Indeed, the data suggest that b_t varies significantly over time. Figure 3 plots estimates of b_t together with its upper and lower 95% confidence bounds at each point in time. The wide intervals of the earlier sample periods are natural consequences of the Kalman filter estimation technique when a diffuse prior is imposed on the initialization of the state variable. At each point in time, only past information (which includes the large variance set for the initial state) is incorporated in estimating the state variable. Imprecise estimates are obtained during earlier periods of the sample, because little information from the data is used, leaving only the effect of the diffuse prior. This phenomenon explains the initial broad confidence intervals of b_t which gradually narrow to a reasonably stationary level.

Except for the earlier periods, b_t is mostly significantly positive, confirming the existence of time-varying risk premiums. For some periods the significance levels are greater than 5%, but except for a few early periods the point estimates are always positive. Excluding the first ten years, b_t ranges during the five decades 1936-1985 from 0.17 to 5.99 with an average of 3.04 and standard deviation of 1.68. The average b_t is virtually identical to the estimate using the fixed-parameter model, 3.00.

It's interesting to compare this b_t series with the rolling sample result (see Figure 4). The general patterns of these two series are quite similar. They are low in the thirties and gradually increase during the forties. They remain high during the fifties and sixties, then drop back to a lower level after the oil shocks and recession of the mid-seventies. The correlation coefficient of these two series for their overlapping sample periods (quarterly 1933-1985) is 0.87. The TVP series is notably smoother than the rolling sample estimates, which suggests that the extreme fluctuations of the rolling sample estimates may be partly due to sampling errors.[5]

As we noted earlier, although the volatility series from the FP ARCH-M and the TVP ARCH-M models are indistinguishable, the implied equity premiums or the expected excess returns can be quite different, as is evident from variations in the price of volatility. Comparison of these two series provides an opportunity to resolve the "puzzle" reported by FSS that the ARCH-M model gives an average risk premium that is twice as high as the average realized excess return.

Table 1 shows that the average risk premium for the TVP ARCH-M model is .54 or .60, depending on the treatment of negative values,

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[5] Some fluctuation in the rolling sample estimates may be attributable to shifts in parameters in the variance equation that the time-varying parameter model, which assumes constancy for all these parameters, cannot capture.

while the FP ARCH-M average is .96. The sample average excess return is .64 close to the TVP-ARCH-M average risk premium. Figure 5 graphs equity premiums (predicted excess returns) from both models. During highly volatile periods, the fixed-parameter model seems to overestimate the level of risk premiums. For less erratic periods, the difference between these two series is not obvious.

To compare the predictive powers of the two methods, we regress the realized excess return on each of the predicted premiums. Regressions with both predicted premiums as explanatory variables are also estimated. Table 2 presents the regression results for the full sample period and for two sub-periods. Both ordinary standard errors and White's consistent standard errors are given. The R^2 of the regression with a regressor from the TVP ARCH-M model is significantly higher than the regression using FP ARCH-M in all samples. When both regressors are included in the regression, the premium predicted by the TVP ARCH-M model has a higher t-value than that predicted by the fixed parameter model in all samples.

IV. Explaining Variations in the Price of Volatility

IV.1 Economic Variables Affecting the Price of Volatility

Application of a TVP ARCH-M model appears to correct the biased forecasts of risk premiums that are generated by the FP ARCH-M model. Here we try to explain variations in b_t, estimates of the price of volatility, by examining its relation with some macroeconomic variables under the assumption that the true model is a CAPM with a constant price of risk. As b_t is the mean/variance ratio of the stock-index excess return, dividing both sides of (II.3) by σ_S^2 yields

$$b_t = \delta[w_t + (1-w_t)\frac{\sigma_{SN}}{\sigma_S^2}] \qquad \text{(IV.1)}$$

The sensitivity of the return on the unobserved portfolio to the returns of stocks, that is, the beta coefficient of the unobservable assets on the stock index is $\beta \equiv \sigma_{SN}/\sigma_S^2$. Letting β and δ be time varying,

$$b_t = \delta[w_t + (1-w_t)\beta_t] \qquad \text{(IV.2)}$$

Thus, the price of volatility of stock returns depends not only on the risk aversion parameter, δ, but is also affected by the portfolio weight w_t and the sensitivity parameter β_t. b_t will be identical to δ_t in two extreme cases: $w_t=1$ or $\beta_t=1$. We use economic variables that proxy changes in w_t, β_t, and δ_t to test the validity of the TVP model in explaining variations in the price of volatility.

Inferences about the CAPM are sensitive to the set of assets used in the test. Stambaugh (1982) examines the effect of moving from narrow to broader stock indexes. But even if we could compile an index of all the incorporated enterprises in the U.S., it would account for less then 10% of wealth if we included human capital, and less than one third of the total wealth of U.S. citizens excluding human capital (See Ibbotson and Brinson (1987, ppp.18-35)). We choose to treat the aggregate of all assets other than equities as the unobservable complement of total wealth.

We use four different proxies for w_t. The first two proxies are the broadest in that they refer to all U.S. assets; real (including human capital), and financial. The flow of income from ownership of stocks is approximately measured by corporate profits while the income from all wealth is simply national income. If each is I(1), and each is discounted at the same rate, their ratio will approximately equal the ratio of the value of stocks to total wealth. Hence the share of corporate profit in national, w_1, income is a possible proxy for w.

The second proxy, w_2, is the ratio of the value of all NYSE stocks to gross consumption. The single-factor CAPM with a constant opportunity set (which is equivalent to the consumption beta model) implies that changes in gross consumption reflect changes in total wealth. At best, this measure can only be proportional to the share of equities in total wealth. Both consumption figures and the value of NYSE stocks are available monthly from 1959 to 1985; quarterly observations are available from 1946 to 1985.

Two more proxies for w correspond to narrower definitions of wealth. For w_3, total wealth is an estimate of physical wealth which includes all financial and tangible assets for the total U.S. economy, while for w_4 only financial assets are included. Total equity value is used for the numerator instead of the aggregate value of NYSE-listed stocks.[6] These data come from the "Balance Sheets for the U.S. Economy" (1987) published by the Federal Reserve Board. Only annual observations are available.

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[6] As of December 1985, the total value of stocks listed on the NYSE was about 79% of the value of all U.S. corporate equity.

The total wealth portfolio is dominated by human capital and real estate. There is little doubt that the beta of real estate on stocks is less than one, indeed, it may very well be negative (See Ibbotson and Brinson (1987, pp.35-43)). The beta of human capital on stocks is also most likely small. While business cycles affect labor income and corporate profits similarly, they affect highly skilled labor less than unskilled labor. Indeed, investment in human capital may very well be counter cyclical. While it is impossible to compute β_t directly, time-varying elements of β_t may be captured nevertheless by economic variables. We use the rate of inflation and the real interest rate. It is plausible that the sensitivity of wealth-asset prices to the prices of stocks differ in period of different levels of inflation and real interest rates.

The third source of variations in b_t comes from δ_t, the risk aversion parameter. For a broad class of stylized utility functions, e.g., HARA, relative risk aversion will depend on the level of wealth, and consequently may be correlated with changes in the level of consumption. There is neither evidence nor stylized fact on whether relative risk aversion is increasing, decreasing, or constant in wealth, although it is a stylized fact that *absolute* risk aversion decreases with wealth (see Machina (1987)).

IV.2 Correlation of b_t with Economic Variables

Table 3 presents regressions of b_t and h_t on the four proxies for the stock-portfolio weight, the two economic variables that are expected to be correlated with the beta of unobserved wealth on stocks (the real rate of interest and the rate of inflation), and the instrument for risk aversion (real per-capita consumption). We report results for the value weighted series only, since they are almost identical to the equally weighted series. All the variables in Table 3 are estimated to be I(1), and hence differenced. These estimates are approximations for the variables in equation (IV.2) and are estimated with quarterly data.

The economic variables, particularly the proxies for the relative portfolio shares of stocks, are, by design, contemporaneously correlated with the stock returns. As a result they will also be highly correlated with the estimated b_t and, but not with the h_t series. To minimize the effect of this spurious correlation, the second panel of Table 3 presents identical regressions with lagged values of the economic variables. Each panel in Table 3 presents estimates from three regressions on the economic variables, two for the price of volatility and one for the volatility itself. The first regression of the price of volatility excludes the volatility itself, the second includes it.

The regression results clearly support the hypothesis that the price of volatility can be varying due to changes in the relative value of stocks, and the beta of unobserved assets, even if risk aversion remains constant. The next to last row of Table 3 gives the χ^2 statistic (with 4 degrees of freedom) for the hypothesis that the coefficients of all four proxies for portfolio share of stocks are zero. The critical value for $\alpha=.001$ is 14.86, while the test statistic is greater than 30 in all four regressions.

The positive coefficient of all w_t's is consistent with (IV.2) for beta less than one. When the value of stocks rises relative to other components of wealth, a rise in the price of volatility can be attributable to the increased *marginal* risk of stocks, rather than to higher risk aversion. At the same time, the variance of the *rate of return* of stocks is actually lower, as suggested by the negative coefficients of the proxies (in 7 out of 8 cases) in the regression of conditional variance on the economic variables (and as might be predicted by a leverage argument).

Both proxies for the beta of unobserved assets with stocks, the rate of inflation and the real rate of interest, have a significant (negative) impact on the price of volatility. Equation IV.2 predicts that beta will be negatively correlated with the price of volatility. Our results agree if we assume that the beta of unobserved assets on stocks is greater in periods of high inflation and real interest rates.

Real per-capita consumption, the proxy for the coefficient of risk aversion, shows a contemporaneous strong (negative) impact on b_t, and hardly any impact when lagged one quarter. With constant relative risk aversion, the result of the contemporaneous regression is attributable to the positive correlation of changes in consumption with changes in wealth, and hence with rates of return on stocks. The absence of significant impact of consumption in the lagged equation is consistent with this explanation. On the other hand, if taken at face value, the positive coefficient of consumption in the contemporaneous regression suggests that risk aversion is increasing with wealth. Risk aversion may be changing for other reasons that are not proxied here, however.

The results presented in Table 3 make a case for our argument that risk aversion may not easily be inferred from rates of return on stocks and that economic variables affect the price of volatility.

IV.3 Further Tests of the Dependence of b_t on Economic Variables

The regression analysis so far has been descriptive since it uses as dependent variable estimates from the entire sample. We now substitute (IV.2) for (III.2) and recognize that the economic variables must interact with the volatility. If only a single variable is relevant, the model becomes

$$r_{S,t} = c_1\sigma^2_{S,t} + c_2(Z^*_t\sigma^2_{S,t}) + e_t \qquad \text{(IV.3)}$$

$$\sigma^2_{S,t} = a_0 + a_1 e^2_{S,t-1} + a_2\sigma^2_{S,t-1} \qquad \text{(IV.4)}$$

where r is the excess return on stocks, and Z_t is one of the proxies or related variables of w, β, and δ. This is the ARCH-M model with cross-product terms in the mean equation, and can be estimated by maximum likelihood.

Monthly data are used for formal tests on the significance of the coefficient c_2 because higher frequency data provide better estimates of conditional variances. The sample covers the period 1959-1985, corresponding to the availability of the consumption data. Estimation results are also reported for sub sample periods 1959-1972, and 1973-1985. Table 4 presents the results.

Except for per capita real consumption, estimation results support the argument that the price of volatility is affected by the economic variables, consistent with the regression result of Table 3. Once the cross product term $w_2\sigma^2_{S,t}$ is included in the model, the coefficient of $\sigma^2_{S,t}$ becomes insignificant and even assumes the wrong sign. The low t-statistic values of the coefficient of the cross-product term may reflect multicollinearity. Estimation with only the cross-product term yields a t-statistic of 2.82 for the full sample, with 2.67 and 1.00 for sub sample periods.

The role of inflation appears to be the most important. When the inflation rate is used for Z in the estimation, the t-values of $\pi_t\sigma^2_{S,t}$ are always significant at the 5% level.

In an attempt to estimate a structural model we assume that the sensitivity factor, β_t, is linearly dependent on the rate of inflation, i.e., $\beta_t = A_0 + A_1\pi_t$, and that the stock return is driven by a process with mean given by (II.3) with a GARCH(1,1) variance specification. The model can then be written as

$$r_{S,t} = \delta[w_{t-1} + A_0(1 - w_{t-1}) + A_1(1 - w_{t-1})\pi_{t-1}]\sigma^2_{S,t} + e_t \qquad \text{(IV.5)}$$

$$\sigma^2_{S,t} = a_0 + a_1e^2_{t-1} + a_2\sigma^2_{S,t-1} \qquad \text{(IV.6)}$$

Equation (IV.5) can also be rewritten as

$$r_{S,t} = C_1w_{t-1}\sigma^2_{S,t} + C_2(1 - w_{t-1})\sigma^2_{S,t} + C_3(1 - w_{t-1})\pi_{t-1}\sigma^2_{S,t} + e_t \qquad \text{(IV.7)}$$

where $C_1=\delta$, $C_2=A_0\delta$, and $C_3=A_1\delta$. Note that all explanatory economic variables are lagged once to ensure that the expected return depends only on predetermined variables. The portfolio weight of stocks, w_t, is measured by the ratio of corporate profit to national income because it is available from 1946.

It turns out that very large standard errors are obtained for estimates of C_1 and C_2. The reason is that w_t is very smooth compared with $\sigma^2_{S,t}$, so the collinearity between $w_t\sigma^2_{S,t}$ and $(1-w_{t-1})\sigma^2_{S,t}$ is high. The model is also estimated assuming $A_0=1$, i.e., $\beta_t=1+A_1\pi_t$, implying $C_1=C_2$ and that β equals one with no inflation. Table 5 gives the results, including estimation with $w_t=1$, which corresponds to the usual fixed-parameter ARCH-M model.

As the likelihood function values indicate, both models with and without unit restriction for A_0 outperform the usual ARCH-M model. The restricted version ($A_0=1$) cannot be rejected and reduces the standard error of the coefficient C_1, which is also an estimate of the risk aversion parameter δ. In this procedure, the point estimate of this parameter becomes positive for the full period and the second sub sample period. The estimates of C_1 and C_3 are reasonably stable across the two sub sample periods, although both are less significant for the period 1966-1985. The likelihood ratio test for model stability across sample periods suggests that the restricted model is stable, while the fixed-parameter ARCH-M model is not.

To investigate the robustness of our model we perform some further diagnostic tests. We have restricted our variance specification to GARCH(1,1). But there are no theories to exclude other economic variables that may be important in driving the conditional variances as well as the conditional mean. It is possible therefore that the effects of economic variables on the price of volatility are attributable to this relationship through the second moment.

Researchers who follow this strategy in modeling stock variances include Campbell and Shiller (1989), Harvey (1989), Attanasio (1989), and Attanasio and Wadhwani (1989) among others. Attanasio and Wadhwani (1989), for example, find that the predictability of expected stock returns given lagged dividend yields reported in Fama and French (1988) can be explained by a risk measure estimated by an ARCH with lagged dividend yields.

We hence re-estimate our final model (with the restriction $A_0=1$) while allowing lagged inflation rates and lagged portfolio weights to enter into the variance equation. A likelihood ratio test of our original model against this general model gives a test statistic of 6.86, which is significant at the 5% level but not at the 1% level. This result indicates that a better forecast of the volatilities may be obtained by including economic variables in the GARCH(1,1) model.

Our original conclusion concerning the mean effect, however, is not much affected by this re-estimation. The estimates of C_1 and C_3 are 7.32 (with t-value of 4.92) and -7.04 (with t-value of -3.26), which are fairly close to the original estimates in Table 5. Further, the significance of the two economic variables (w_2 and π) added to the variance equation is weaker according to the Wald test (t-values 1.67 and -0.75, respectively, not significant at the 5% level).

According to the result of the restricted model for the full sample period, the estimate of the risk aversion parameter is 7.8, which is most likely too high. The two standard error low bound for this parameter is 4.67, which is more in accord with other estimates.

V. Conclusion

Analysis of an econometric model estimating a time-varying risk/return relation of the stock market indicates that the TVP ARCH-M model provides more precise estimates of the expected return of the stock market index than the fixed-parameter ARCH-M model. The model takes explicit account of the role of risky assets other than stocks in explaining the time variation of the mean/variance ratio for stocks. Proxies for portfolio weights and the beta of the unobserved assets on stocks are found to be important in determining expected stock returns.

Although our empirical work is only preliminary, further studies in this vein seem promising. More detailed investigations may explain the role of the rate of inflation in stock price movements. The relationship of our findings with the recent literature on predictability of excess stock returns may be fruitful for future research.

Appendix: Notation

r_M : excess return of the market portfolio = total return − r_f

r_f : riskless rate

r_S : excess return of a comprehensive stock index

r_N : excess return of the unobserved risky asset other than stocks

w : portfolio weight of stocks in the market portfolio (or the relative demand for stocks)

w_i : proxies for w

w_1 : corporate profit/national income

w_2 : total NYSE value/gross consumption

w_3 : total stock value/value of total financial assets + tangible assets

w_4 : total stock value/value of total financial assets

β : sensitivity of returns of the unobserved risky asset to stocks = $Cov(r_N, r_S)/var(r_S)$

π : inflation rate measured by the Consumer Price Index

R : real interest rate = $r_f - \pi$

C : real per capita consumption

b : time-varying parameter measuring price of volatility of stocks

δ : relative risk aversion

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Table 1: Summary Statistics of Realized and Predicted Risk Premiums

Risk Premium*	Mean	Std Dev	Minimum	Maximum
r_s	0.64	5.60	-29.03	38.16
r_s^{TVP}	0.54	0.63	-3.51	4.73
r_s^{TVP} **	0.60	0.47	0.00	4.73
r_s^{FP}	0.96	1.28	0.24	9.86

Notes:

* r_s^{TVP} is the predicted risk premium using the TVP ARCH-M model, and r_s^{FP} the predicted risk premium according to the fixed-parameter model

**This series is non-negativity corrected, i.e., all negative values are replaced by zeros.

Table 2: Comparison of ARCH-M Models in Predicting Risk Premiums

$$r_s = \alpha_0 + \alpha_1 r_s^{TVP} + \alpha_2 r_s^{FP} + \varepsilon^*$$

Sample	α_0	α_1	α_2	D.W.	R^2
1926-1985	-0.044	1.268	-	1.75	0.020
	(0.277)**	(0.333)			
	[0.421]	[0.574]			
	0.342	-	0.311	1.77	0.005
	(0.265)		(0.166)		
	[0.326]		[0.402]		
	-0.724	1.576	0.533	1.75	0.032
	(0.351)	(0.345)	(0.171)		
	[0.449]	[0.555]	[0.377]		
1926-1955	0.313	1.171	-	1.72	0.021
	(0.416)	(0.425)			
	[0.538]	[0.628]			
	0.567	-	0.235	1.74	0.003
	(0.464)		(0.214)		
	[0.459]		[0.423]		
	-0.430	1.446	0.453	1.71	0.032
	(0.551)	(0.444)	(0.221)		
	[0.567]	[0.611]	[0.400]		
1956-1985	-1.165	2.633	-	1.85	0.030
	(0.514)	(0.786)			
	[0.562]	[0.823]			
	-0.711	-	1.916	1.88	0.016
	(0.503)		(0.783)		
	[0.624]		[1.112]		
	-1.33	2.34	1.45	1.87	0.039
	(0.628)	(0.800)	(0.791)		
	[0.687]	[0.886]	[1.142]		

Notes:

* r_s is the excess return of the monthly NYSE value weighted index, r_s^{TVP} is the predicted risk premium using the TVP ARCH-M model and r_s^{FP} the predicted risk premium according to the fixed-parameter model.

** Numbers in parentheses and in brackets are standard errors by OLS and White's robust standard errors, respectively.

Table 3: Multiple Regression of b_t and h_t on Economic Variables *

	With contenporaneous explanatory variables			With explanatory variables lagged one period		
Dependent variable	b^{VWR}	b^{VWR}	h^{VWR}	b^{VWR}	b^{VWR}	h^{VWR}
w_1	3.92 (1.75)**	3.89 (1.73)	-10.77 (-0.14)	4.77 (1.22)	5.02 (1.33)	-79.48 (-1.04)
w_2	3.05 (14.51)	3.06 (14.55)	4.48 (0.60)	0.74 (2.00)	0.69 (1.91)	-38.32 (-5.26)
w_3	-4.59 (-1.55)	-4.88 (-1.65)	-111.48 (-1.06)	1.21 (0.24)	2.56 (0.52)	-6.39 (-0.06)
w_4	10.27 (3.30)	10.10 (3.24)	-63.50 (-0.58)	7.66 (1.43)	8.74 (1.68)	-15.96 (-0.15)
π	-0.11 (-2.19)	-0.12 (-2.33)	-3.14 (-1.78)	-0.28 (-3.22)	-0.24 (-2.79)	2.11 (1.22)
R	-0.13 (-2.48)	-0.14 (-2. 61)	-2.85 (-1.53)	-0.29 (-3.10)	-0.25 (-2.74)	2.46 (1.36)
C	0.60 (2.65)	0.54 (2.28)	-24.81 (-3.08)	-0.65 (-1.69)	-0.34 (-0.88)	-3.81 (-0.51)
h^{VWR}	-	-0.003 (-1.08)	-	-	0.01 (3.25)	-
$CONST.$	-0.04 (-2.57)	-0.03 (-2.36)	0.93 (1.93)	0.01 (0.60)	0.003 (0.12)	0.14 (0.30)
R^2	0.77	0.78	0.24	0.30	0.35	0.28
D.W.	2.21	2.21	1.85	2.03	2.09	2.03
$\chi^2(4)$***	105.35	104.13	15.65	32.23	39.64	35.19
p-value	< 0.01%	<0.01%	< 1%	< 0.01%	< 0.01%	< 0.01%

Notes:

* w_i's are portfolio shares of stocks with different wealth measures, π is the inflation rate, R is the real interest rate, and C is the real per capita consumption in thousands of dollars. The sample period is quarterly 1951.I - 1985.IV with 140 observations. All variables in the regressions are first differenced.

** Numbers in the parentheses are t-values.

*** $\chi^2(4)$ is the test statistic for the joint hypothesis that all coefficients for w_i's, i = 1,2,3,4 are zero. This is an omitted variable test of the Lagrange Multiplier type .

Table 4: Tests of Explanatory Powers of Economic Variables

$$r_{s,t}=c_1\sigma^2_{s,t}+c_2(Z_t^*\sigma^2_{s,t})+e_t \quad (4.3)$$

$$\sigma^2_{s,t}=\alpha_0+\alpha_1 e^2_{t-1}+\alpha_2\sigma^2_{s,t-1} \quad (4.4)$$

Sample	*Z*	c_1	c_2	α_0*10^{-4}	α_1	α_2	LR*
1959-85	none	3.60 (2.77)**	-	1.35 (1.58)	0.11 (2.45)	0.82 (11.49)	-
	w_2	-3.89 (-0.84)	9.60 (1.54)	1.23 (1.61)	0.11 (2.52)	0.82 (12.14)	2.2
	π	9.49 (4.07)	-13.56 (-3.97)	1.22 (1.59)	0.11 (2.52)	0.82 (11.63)	12.88
	R	2.75 (2.09)	9.38 (2.36)	1.35 (1.58)	0.04 (2.43)	0.98 (10.92)	4.90
	C	6.32 (0.73)	-0.38 (-0.35)	1.44 (1.56)	0.11 (2.43)	0.81 (10.91)	0.16
1959-72	none	5.39 (2.47)	-	1.20 (0.97)	0.13 (1.41)	0.78 (4.97)	-
	w_2	-24.23 (-1.18)	31.20 (1.46)	1.08 (1.04)	0.12 (1.50)	0.79 (5.65)	5.66
	π	10.14 (1.05)	-19.69 (-7.12)	1.28 (0.88)	0.12 (4.58)	0.78 (1.61)	6.9
	R	2.40 (0.94)	17.10 (1.34)	1.15 (0.91)	0.10 (1.31)	0.81 (5.25)	2.87
	C	12.67 (0.64)	-1.15 (-0.39)	1.22 (0.93)	0.13 (1.42)	0.78 (4.93)	0.22
1973-85	none	1.95 (1.13)	-	3.24 (0.88)	0.08 (1.56)	0.78 (4.02)	-
	w_2	-0.89 (-0.08)	4.28 (0.25)	3.29 (0.74)	0.05 (1.35)	0.79 (3.45)	0.36
	π	10.05 (2.39)	-13.41 (-2.55)	2.92 (0.84)	0.07 (1.43)	0.79 (4.03)	7.9
	R	1.37 (0.78)	6.98 (1.43)	3.20 (0.88)	0.07 (1.43)	0.78 (3.94)	2.46
	C	-22.13 (-0.83)	2.78 (0.90)	3.44 (0.88)	0.08 (1.41)	0.77 (3.80)	0.97

Notes:

* The likelihood ratio test statistics testing the significance of the inclusion of the cross product term. The 5 percent critical value for this statistic (χ^2 with one degree of freedom) is 3.84).

** Numbers in parentheses are t-values.

Table 5: Estimation of the Final Model*

$$r_{s,t}=C_1(x_{t-1}\sigma^2_{s,t})+C_2[(1-x_{t-1})\sigma^2_{s,t})]+C_3[(1-x_{t-1})\pi_{t-1}\sigma^2_{s,t}]+e_t$$

$$\sigma^2_{s,t}=\alpha_0+\alpha_1 e^2_{t-1}+\alpha_2\sigma^2_{s,t-1}$$

Sample	C_1	C_2	C_3	α_0*10^{-4}	α_1	α_2	likelihood
1946-85	-5.6 (-0.12)**	9.5 (1.54)	-9.0 (-3.34)	1.27 (1.78)	0.09 (2.67)	0.83 (13.00)	-1331.23
	7.81 (4.98)	-	-8.69 (-3.61)	1.27 (1.98)	0.09 (2.69)	0.83 (13.08)	-1331.26
w=1	4.62 (4.12)	-	-	1.47 (1.74)	0.09 (2.63)	0.82 (11.46)	-1337.32
1946-65	110.8 (0.96)	-4.7 (-0.28)	-7.5 (-2.02)	1.63 (1.05)	0.10 (1.45)	0.76 (4.38)	-633.00
	9.81 (4.77)	-	-7.96 (-2.19)	1.69 (1.03)	0.10 (1.41)	0.76 (4.16)	-633.34
w=1	7.96 (4.19)	-	-	2.22 (1.23)	0.13 (1.65)	0.69 (3.47)	-636.07
1966-85	-119.7 (-1.59	20.2 (2.26)	-9.9 (-2.22)	2.06 (1.22)	0.08 (1.78)	0.82 (7.43)	-693.10
	6.22 (2.04)	-	-8.35 (-1.92)	2.07 (1.20)	0.08 (1.77)	0.82 (7.50)	-694.63
w=1	2.2 (1.52)	-	-	2.37 (1.13)	0.07 (1.83)	0.81 (6.37)	-696.16

Notes:

* Monthly data are used, w_{t-1} is the ratio of corporate profit over national income, and π_{t-1} is the inflation rate measured by CPI.

** Numbers in parentheses are t-values.

Figure 1 : Estimates of Volatility Prices Using Previous 5-years Rolling Samples (1933-1987)

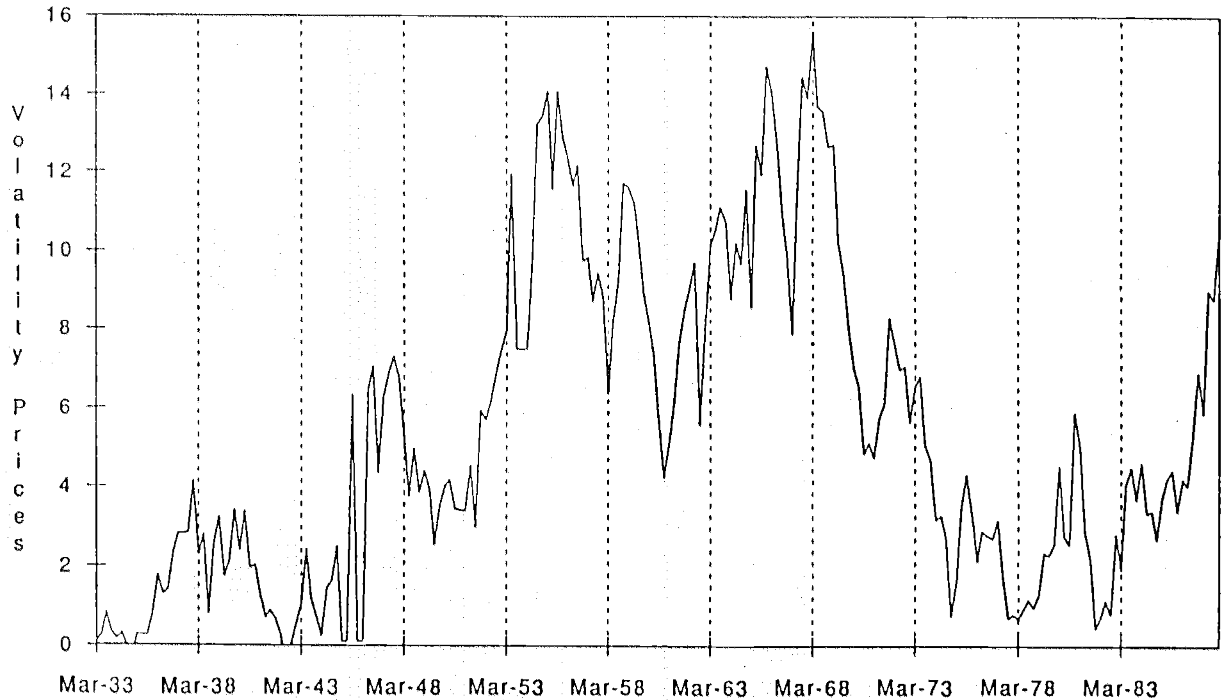

Figure 2 : Volatility Estimates Using Fixed-Parameter vs. Time-Varying Parameter ARCH-M Models (1926-1985)

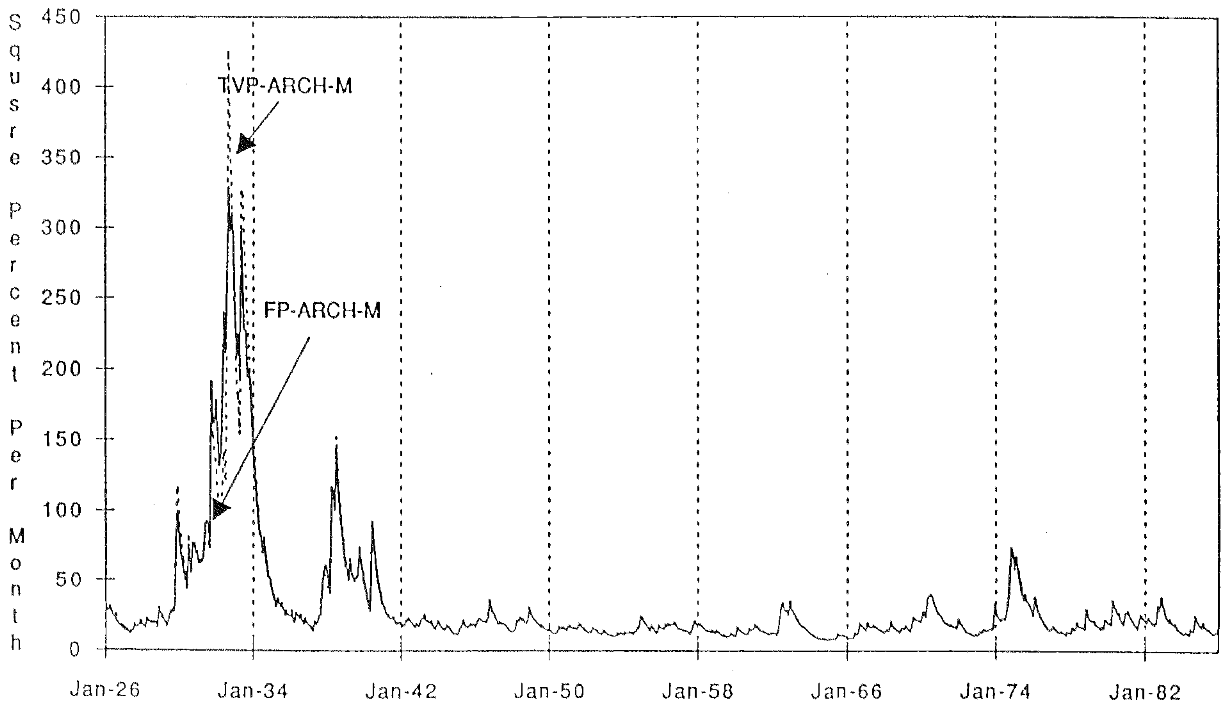

Figure 3 : Mean and 95% Confidence Interval for Volatility Prices
Using TVP-ARCH-M Model (1926-1985)

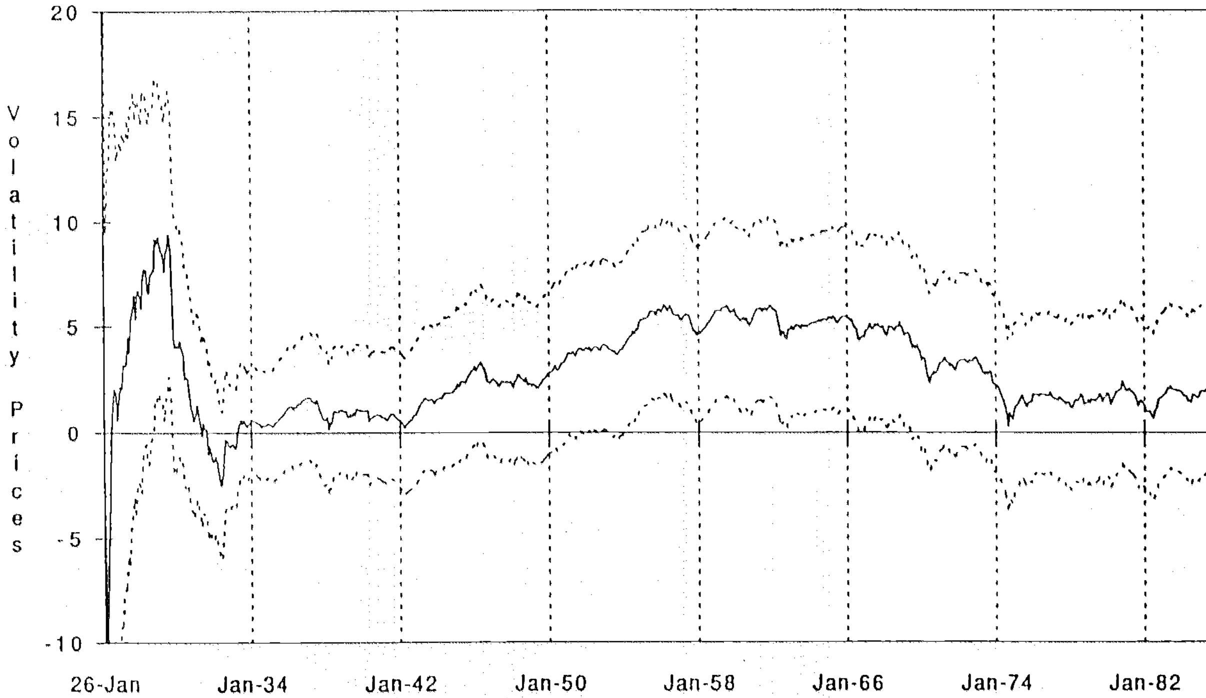

Figure 4 : Estimates of Volatility Prices Using TVP-ARCH-M Model vs. FP-ARCH-M with Rolling Samples (1933.I-1985.IV)

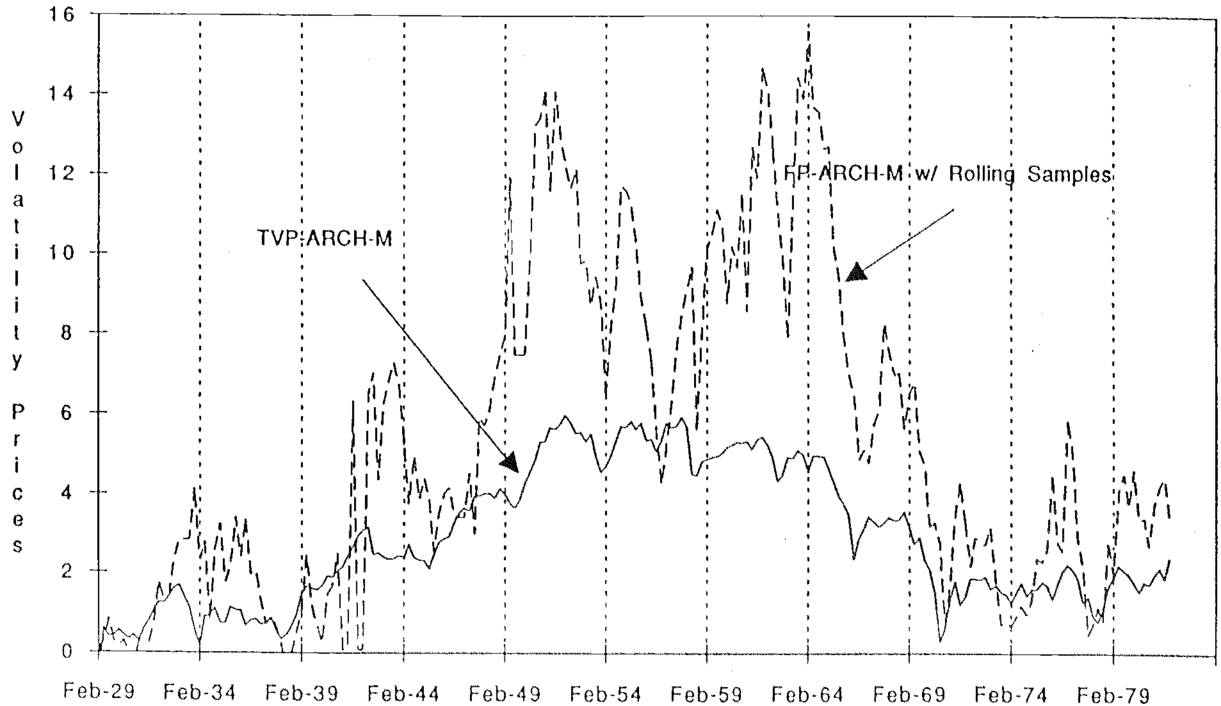

Figure 5 : Predicted Equity Premiums Using Fixed-Parameter vs. Time-Varying-Parameter ARCH-M Models (1926-1985)