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LIQUIDITY AND INSURANCE FOR THE UNEMPLOYED

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ABSTRACT

We study the optimal design of unemployment insurance for workers sampling job opportunities over time. We focus on the timing of benefits and the desirability of allowing workers to freely access a riskless asset. When workers have constant absolute risk aversion preferences, a very simple policy is optimal: a constant benefit during unemployment, a constant tax during employment, and free access to savings using the riskless asset. Away from this benchmark, for constant relative risk aversion preferences, the optimal policy involves nearly constant benefits and the welfare gains from more elaborate policies are minuscule. Our results highlight two distinct roles for policy toward the unemployed: ensuring workers have sufficient liquidity to smooth their consumption; and providing unemployment subsidies that serve as insurance against the uncertain duration of unemployment spells.

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This paper argues that there are two distinct components in an optimal policy for the unemployed: insuring workers against uncertainty in the prospect of finding a job; and providing workers with the liquidity to smooth consumption while unemployed. In contrast to the leading papers on the optimal contracting approach to unemployment insurance (Shavell and Weiss 1979; Hopenhayn and Nicolini 1997), we distinguish between these roles by allowing workers to borrow and save. Our main conclusion is that a constant or nearly constant benefit schedule is optimal. This insures workers against unemployment risk, while their ability to dissave and borrow allows them to avoid transitory fluctuations in consumption. The dichotomy between liquidity and insurance is consistent with the spirit of Feldstein and Altman's (1998) proposal for unemployment insurance savings accounts (see also Feldstein 2005).

We represent an unemployed worker's situation using McCall's (1970) model of sequential job search. A risk-averse, infinitely lived worker periodically gets a wage offer from a known distribution. If she accepts the offer, she keeps the job at a constant wage forever. If she rejects it, she continues to search.

We compare two unemployment insurance policies. Under *constant benefits*, the worker receives a constant benefit while she is unemployed and pays a constant tax once she is employed. The worker can borrow and lend using a riskless bond. We show that the worker adopts a reservation wage that is increasing in both the unemployment benefit and the employment tax, a form of moral hazard. An insurance agency sets the level of benefits and taxes to minimize the cost of providing the worker with a given amount of utility.

We then consider *optimal unemployment insurance*. An insurance agency dictates a duration-dependent consumption level for the unemployed, funded by an employment tax that depends on the length of the jobless spell. The worker has no access to capital markets and so must consume her after-tax income. The path of unemployment consumption and employment taxes determines the worker's reservation wage, which the insurance agency cannot directly control. It sets this path to minimize the cost of providing the worker with a given amount of utility. Absent direct monitoring of wage offers or randomization schemes, we prove that this is the best unemployment insurance system, in the sense that it provides the worker with any desired level of expected utility at the lowest cost.

Our main result is that with constant absolute risk aversion (CARA) preferences, constant benefits and optimal unemployment insurance are equivalent. That is, the cost of providing the worker with a given level of utility is the same, the path of her reservation wage is the same, and the path of her consumption is the same under both insurance systems. In both cases consumption falls with the duration of the unemployment spell, both while the worker is unemployed and after she is subsequently reemployed. With optimal unemployment insurance, this is a direct feature of the policy. With constant benefits, this pattern follows from the worker's own consumption-savings decision.

We also explore optimal insurance with constant relative risk aversion (CRRA) preferences numerically. The equivalence between optimal unemployment insurance and constant benefits breaks down, but we find that our results with CARA provide an important benchmark. Optimal unemployment insurance continues to dictate a declining path of consumption for an unemployed worker and an increasing tax upon reemployment. However, just as in the CARA case, because both benefits and taxes rise, the implicit subsidy to unemployment, calculated as the net change in the worker's expected lifetime transfers if she stays unemployed for an additional period, is not declining. Indeed, while with CARA utility we show that it is perfectly flat, with CRRA preferences we find that it increases slowly during a jobless spell. In our leading example, the subsidy increases by about five percent during the first year of unemployment, so the optimal policy has quantitatively nearly-constant unemployment subsidies. Conversely, we find that restricting the unemployment insurance agency to use a constant benefit policy imposes small costs when workers have adequate liquidity. In our leading example, constant benefits costs about 10^{-5} weeks of income more than the optimal policy.

We also study an intermediate case, where the worker can freely borrow and save, while the insurance agency can set the timing of benefits optimally given this constraint. The cost of allowing the worker to engage in hidden borrowing and lending is even smaller. Interestingly, we find that the optimal path of unemployment subsidies rises with the spell's duration and is nearly indistinguishable from the one obtained when the agency controls the worker's consumption.

Our general message is that simple policies—a constant benefit and tax, combined with measures to ensure that workers have the liquidity to maintain their consumption level during a jobless spell—are optimal or nearly optimal. Our intuition for these results is the following. With CARA utility the fall in assets and consumption that occurs during an unemployment spell does not affect attitudes toward risk; as a consequence, the optimal unemployment subsidy is constant. With CRRA utility, the worker becomes more risk averse as consumption falls; this explains why the optimal subsidy increases over time. However, this wealth effect is small during a typical, or even relatively prolonged, unemployment spell, provided the worker is able to smooth her consumption.

Our result is novel along two dimensions. First, papers in the optimal contracting approach to unemployment insurance, e.g. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), have concluded that unemployment benefits should decline during an unemployment spell. When we distinguish between the timing of benefits and the consequent timing of consumption, we confirm that consumption should decline during an unemployment spell but find that benefits should be flat or slightly increasing.

Second, the literature on dynamic moral hazard models finds that it is optimal to constrain workers' savings.[1] Rogerson (1985) considers an environment in which a risk-averse worker must make an unobserved effort decision that affects her risk-neutral employer's profits. He assumes that the worker's utility is additively separable in consumption and effort and concave in consumption. In this environment, a worker confronted with the optimal path of consumption but permitted to save at the same interest rate as the employer would choose to defer some of her consumption, and so she is "savings-constrained" by optimal insurance. In contrast, in our model with CARA preferences there is no role for savings constraints. With CRRA preferences, the savings distortion is positive, but we find it to be very small in our numerical exercises.

Our paper is related to a broad literature on optimal unemployment insurance in calibrated incomplete markets economies. Hansen and Imrohoroglu (1992) find that in the absence of any moral hazard problem, optimal benefits are high, about 65 percent of past wages, and decentralize the complete markets allocation. When the authors introduce moral hazard to the model by allowing workers to turn down employment opportunities and possibly still receive benefits, they find that the optimal level of unemployment benefits quickly falls to zero. In their analysis of a general-equilibrium economy, Alvarez and Veracierto (2001) also find that the level of unemployment benefits that maximizes steady state utility is quite low. Lentz (2007) estimates a job search model with borrowing and savings and finds that the optimal constant benefit is always positive and is fairly low for all workers except those who are up against their borrowing constraint. He does not examine whether liquidity provision may be preferred to unemployment subsidies.

Using calibrated models in the spirit of Hansen and Imrohoroglu's, Abdulkadiroglu, Kuruscu, and Sahin (2002) and Wang and Williamson (2002) allow unemployment benefits to vary during the first four periods of an unemployment spell. Instead of an optimal contracting approach, these papers compute a steady state equilibrium for each policy and then use brute-force to search for the best such steady state. The computational complexity of this approach prevents them from considering less restrictive policies. In contrast to our finding that benefits should be constant or increasing, both papers find that optimal benefits may be nonmonotonic, decreasing early in an unemployment spell and then eventually increasing.

[1] A recent example is Golosov, Kocherlakota, and Tsyvinski (2003), who emphasize that capital taxation may discourage saving. Allen (1985) and Cole and Kocherlakota (2001) provide a particularly striking example of the cost of unobserved savings in a dynamic economy with asymmetric information. They prove that if a worker privately observes her income and has access to a hidden saving technology, then no insurance is possible. In our model the employment state is observable, allowing some insurance.

It is unclear whether this is due to subtle differences in model assumptions, e.g. both these papers allow for saving but not borrowing and the source of the moral hazard problems are different, or whether it is a consequence of the restriction on the policy space. In any case, and closer to our results, both papers emphasize the importance of distinguishing unemployment benefits from consumption; although benefits may be non-monotone, consumption is always monotonically declining during an unemployment spell. Both also report relatively modest gains to allowing for time-varying benefits in an environment with savings. Our contracting approach is closer to Hopenhayn and Nicolini (1997) and allows us to study optimal unemployment insurance without the ad hoc restrictions imposed by numerical considerations.

Our use of a sequential search departs from Hopenhayn and Nicolini (1997), which assume that there is only a job search effort decision.[2] There are three reasons for this modeling choice. First, our model produces stark results on optimal policy in a straightforward way, which we believe is intrinsically useful. On the other hand, the sequential search model is not critical for these results. Indeed, the paper most closely related to ours is Werning (2002), which introduces hidden borrowing and savings into the Hopenhayn and Nicolini (1997) search effort model, and some of his results are analogous to ones we report here. For example, he proves that constant benefits and taxes are optimal under CARA preferences if the cost of search is monetary. Despite this, and in contrast to our results here, in Werning (2002) constant benefits are not equivalent to optimal unemployment insurance, even with CARA utility, since it is always desirable to exclude the worker from the asset market.

Second, the sequential search model is empirically relevant. Starting with the work of Feldstein and Poterba (1984), a number of authors have documented that an increase in unemployment benefits raises workers' reservation wage and consequently reduces the rate at which they find jobs. The sequential search model is a natural one for thinking about this fact. Third, the sequential search model is the backbone of most research on equilibrium unemployment. At the heart of the Lucas and Prescott (1974) equilibrium search model and of versions of the Pissarides (1985) matching model with heterogeneous firms are individual sequential search problems. More recently, Ljungqvist and Sargent (1998) examine an economy in which each individual engages in sequential job search from an exogenous wage distribution.

This paper proceeds as follows. Section I describes the model's environment and the two policies we consider. Section II then establishes the equivalence between the two systems under CARA preferences. Section III quantitatively evaluates optimal unemployment insurance and optimal constant benefits with CRRA preferences, highlighting the relationship between

[2] Shavell and Weiss (1979) allow for both hidden search effort and hidden wage draws. See also exercise 21.3 in Ljungqvist and Sargent (2004). However, both of these models assume that employed workers cannot be taxed, and neither examines optimal benefits when workers have access to liquidity.

unemployment insurance and liquidity. Section IV concludes.

I. Two Policies for the Unemployed

We begin by describing the common physical environment of the model. We then discuss the two policies we consider, *constant benefits* and *optimal unemployment insurance*.

A. The Unemployed Worker

There is a single risk-averse worker who maximizes the expected present value of utility from consumption,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c(t)) dt,$$

where $\rho > 0$ represents the discount rate and $u(c)$ is the increasing, concave utility function.

At each instant, a worker can be employed at a wage w or unemployed. A worker employed at w produces w units of the consumption good per unit of time. Following Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), we assume that jobs last forever. An unemployed worker receives a single independent wage draw from the cumulative distribution function F according to a Poisson process with arrival rate $\alpha > 0$. We assume that F is continuous and has finite expectation and that there is some chance of drawing a positive wage, so $F(w) < 1$ for some $w > 0$. The worker observes the wage and decides whether to accept or reject it. If she accepts w, she immediately becomes employed at that wage. If she rejects w, she remains unemployed and continues to search for a job. The worker cannot recall past wage offers.

We assume that an unemployment agency only observes whether the worker is employed or unemployed. In particular, it does not observe the worker's wage, even after she decides to take a job.[3] The objective of the unemployment insurance agency is to minimize the cost of providing the worker with a given level of utility, discounted at rate r. We assume that $\rho = r$ throughout the paper.

B. Policy I: Constant Benefits

The policy we call *constant benefits* is defined by a constant unemployment benefit $\bar{b}$, a constant employment tax $\bar{\tau}$, and perfect access to a riskless asset with net return r. That is,

[3] If the wage were observable, an unemployment insurance agency could tax employed workers 100 percent and redistribute the proceeds as a lump-sum transfer. Workers would be indifferent about taking a job and hence would follow any instructions on which wages to accept or reject. This makes it feasible to obtain the first best, complete insurance with the maximum possible income. Private information is a simple way to prevent the first best, but other modeling assumptions could also make the first best unattainable, e.g., moral hazard among employed workers.

the worker faces the budget constraint $\dot{a}(t) = ra(t) + \bar{b} - c(t)$ if unemployed, and $\dot{a}(t) = ra(t) + w - \bar{\tau} - c(t)$ if employed at wage w. In addition, the no-Ponzi condition $\lim_{t\to\infty} e^{-rt}a(t) \geq 0$ must be satisfied.

Since the worker's problem is stationary, we present it recursively. Given policy parameters $\bar{b}$ and $\bar{\tau}$, start by considering a worker who is employed at wage w and has assets a. Since the worker's discount rate is equal to the interest rate, she consumes her after tax-income plus the interest on her assets $c^e(a, w) = ra + w - \bar{\tau}$, so that assets are kept constant, $\dot{a} = 0$. This means that her lifetime utility is

$$V^e(a, w) = \frac{u(ra + w - \bar{\tau})}{r}. \tag{1}$$

Next consider an unemployed worker with assets a and let $V^u(a)$ denote her expected lifetime utility. This must satisfy the Hamilton-Jacobi-Bellman equation

$$rV^u(a) = \max_c \big(u(c) + V^{u\prime}(a)(ra + \bar{b} - c)\big) + \alpha \int_0^\infty \max\{V^e(a, w) - V^u(a), 0\}\, dF(w). \tag{2}$$

The first term on the right hand side describes an unemployed worker's consumption choice when she does not have an offer in hand; she chooses consumption to set the marginal utility of consumption equal to the marginal value of assets. The second term describes the choice of whether to accept a job paying w, in which case she gets a capital gain $V^e(a, w) - V^u(a)$, or to reject it and get nothing. Since $V^e(a, w)$ is increasing in w, the worker will accept offers above some reservation wage and reject the rest.

The solution to the two maximizations in the Hamilton-Jacobi-Bellman equation gives the worker's unemployment consumption $c^u(a; \bar{b}, \bar{\tau})$ and reservation wage $\bar{w}(a; \bar{b}, \bar{\tau})$ policies as a function of current assets a. Using $\dot{a} = ra + \bar{b} - c^u(a; \bar{b}, \bar{\tau})$, this gives a time path for assets $a(t)$.

An unemployment insurance agency chooses $\bar{b}$ and $\bar{\tau}$ to maximize the worker's utility given some available resources and an initial asset level. Equivalently, we consider the dual problem of minimizing the total resource cost, equal to benefits net of taxes plus initial assets, of delivering a certain utility for the worker. The optimal constant benefit policy solves

$$C^c(v_0, a_0) \equiv \min_{\bar{b},\bar{\tau}} \int_0^\infty e^{-\int_0^t \left(r + \alpha(1 - F(\bar{w}(a(s); \bar{b}, \bar{\tau})))\right) ds} \left(\bar{b} - \alpha(1 - F(\bar{w}(a(t); \bar{b}, \bar{\tau})))\frac{\bar{\tau}}{r}\right) dt + a_0 \tag{3}$$
$$\text{subject to } V^u(a_0; \bar{b}, \bar{\tau}) = v_0.$$

A standard Ricardian-equivalence argument implies that $V^u(a; \bar{b}, \bar{\tau}) = V^u(a + x/r; \bar{b} - x, \bar{\tau} + x)$ for any x. The same is true for total resources, so it follows that $C^c(v_0, a_0)$ is independent of

a_0. Abusing notation, we write $C^c(v_0)$.

C. Policy II: Optimal Unemployment Insurance

Under optimal unemployment insurance, a worker who is unemployed at time t consumes $b(t)$, while a worker who finds a job at time t pays a tax $\tau(t)$, depending on when she finds a job, for the remainder of her life. One can conceive of more complicated insurance policies where the agency asks the worker to report her wage draws, advises her on whether to take the job, and makes payments conditional on the worker's entire history of reports. That is, one can model unemployment insurance as a revelation mechanism in a principal-agent problem. We prove in Appendix A that the policy we consider here does as well as any deterministic mechanism as long as absolute risk aversion is non-increasing.

Given $\{b(t)\}$ and $\{\tau(t)\}$, consider a worker who chooses a sequence of reservation wages $\{\bar{w}(t)\}$. Her utility from time $t' \geq 0$ onward is

$$(4) \quad U\big(t', \{\bar{w}(t), b(t), \tau(t)\}\big) \equiv \int_{t'}^{\infty} e^{-\int_{t'}^{t}\left(r+\alpha(1-F(\bar{w}(s)))\right)ds} \left(u(b(t)) + \alpha \int_{\bar{w}(t)}^{\infty} \frac{u(w-\tau(t))}{r} dF(w)\right) dt.$$

The worker remains unemployed at time $t \geq t'$ with probability $e^{-\int_{t'}^{t}\alpha(1-F(\bar{w}(s)))ds}$, in which case she gets utility $u(b(t))$. If she draws a wage above $\bar{w}(t)$, she takes the job and gets $u(w-\tau(t))$ forever.

Now consider an unemployment insurance agency that sets the sequence of unemployment consumption and employment taxes $\{b(t), \tau(t)\}$ to minimize the cost of providing the worker with utility v_0:

$$(5) \qquad C^*(v_0) \equiv \min_{\{\bar{w}(t), b(t), \tau(t)\}} \int_0^{\infty} e^{-\int_0^t \left(r+\alpha(1-F(\bar{w}(s)))\right)ds} \left(b(t) - \alpha(1-F(\bar{w}(t)))\frac{\tau(t)}{r}\right) dt,$$

subject to two constraints. First, the worker's utility must equal v_0 if she uses the recommended reservation wage sequence, $v_0 = U\big(0, \{\bar{w}(t), b(t), \tau(t)\}\big)$. And second, she must do at least as well using the recommended reservation wage sequence as any other sequence $\{\hat{\bar{w}}(t)\}$, $U\big(0, \{\bar{w}(t), b(t), \tau(t)\}\big) \geq U\big(0, \{\hat{\bar{w}}(t), b(t), \tau(t)\}\big)$. That is, the agency recognizes that the worker will choose her reservation wage sequence $\{\bar{w}(t)\}$ to maximize her utility given $\{b(t), \tau(t)\}$. The solution to this problem describes optimal unemployment benefits.

It is useful to express this problem recursively, using utility $v(t') = U\big(t', \{\bar{w}(t), b(t), \tau(t)\}\big)$

as a state variable. Equation (4) implies that utility evolves according to

$$\dot{v}(t) = rv(t) - u(b(t)) - \alpha \int_{\bar{w}(t)}^{\infty} \left(\frac{u(w - \tau(t))}{r} - v(t) \right) dF(w), \tag{6}$$

while the incentive constraint $U\big(0, \{\bar{w}(t), b(t), \tau(t)\}\big) \geq U\big(0, \{\hat{\bar{w}}(t), b(t), \tau(t)\}\big)$ is equivalent to $v(t) = u(\bar{w}(t) - \tau(t))/r$. The cost function $C^*(v)$ defined above must then solve the Hamilton-Jacobi-Bellman equation

$$rC^*(v) = \min_{\bar{w},b,\tau} \Bigg(b + C^{*\prime}(v) \left(rv - u(b) - \alpha \int_{\bar{w}}^{\infty} \left(\frac{u(w-\tau)}{r} - v \right) dF(w) \right) - \alpha(1 - F(\bar{w})) \left(\frac{\tau}{r} + C^*(v) \right) \Bigg) \tag{7}$$

$$\text{subject to } v = \frac{u(\bar{w} - \tau)}{r}, \tag{8}$$

Moreover, the optimal sequence $\{\bar{w}(t), b(t), \tau(t)\}$ must be generated by the Hamilton-Jacobi-Bellman equation's policy functions.

II. Equivalence for a Benchmark: CARA Utility

There are two disadvantages to constant benefits relative to optimal unemployment insurance. First, there is a restriction on the time path of unemployment benefits and taxes, so $b(t)$ and $\tau(t)$ are constant. Second, the planner does not directly control the worker's consumption and so is constrained by her savings choices. This can be thought of as an additional dimension of moral hazard. Thus, constant benefits are more costly than optimal unemployment insurance: $C^c(v) \geq C^*(v)$. However, in this section we prove analytically that constant benefits achieve the same outcome as optimal unemployment insurance for the case with CARA preferences, $u(c) = -\exp(-\gamma c)$, where $\gamma > 0$ denotes the coefficient of absolute risk aversion and we allow $c \in \mathbb{R}$. A key feature is that all workers have the same attitude towards lotteries over future wages, which makes the model particularly tractable. We later show that these results provide a good benchmark for other preference specifications.

A. *Constant Benefits*

We characterize constant benefits in two steps. First, we characterize individual behavior given unemployment benefits $\bar{b}$, employment taxes $\bar{\tau}$, and assets a. Then we discuss how to choose these parameters optimally. It is convenient to define the net benefit or unemployment subsidy by $\bar{B} \equiv \bar{b} + \bar{\tau}$. We can equivalently think of constant benefits as a lump-sum tax $\bar{\tau}$, paid during both employment and unemployment, together with an unemployment subsidy $\bar{B}$.

The first step follows from solving the Hamilton-Jacobi-Bellman equation (2).

Proposition 1 *Assume CARA preferences. Given a constant benefits policy $(\bar{b}, \bar{\tau})$ with $\bar{B} = \bar{b} + \bar{\tau}$, the reservation wage, consumption and utility of the unemployed satisfy*

$$\gamma(\bar{w} - \bar{B}) = \frac{\alpha}{r} \int_{\bar{w}}^{\infty} \big(1 + u(w - \bar{w})\big) dF(w) \tag{9}$$

$$c^u(a) = ra - \bar{\tau} + \bar{w} \tag{10}$$

$$V^u(a) = \frac{u(ra - \bar{\tau} + \bar{w})}{r} \tag{11}$$

Proof. The worker's sequence problem implies that the value function must have the form

$$V^u(a) = \frac{u(ra - \bar{\tau} + k_1)}{r} \tag{12}$$

for some constant k_1. We determine this constant, and the rest of the solution along with it. The maximization with respect to consumption in equation (2) delivers

$$c^u(a) = ra - \bar{\tau} + k_1 \tag{13}$$

Substituting this and the functional form for V^u back into the value function equation (2) gives

$$u'(ra - \bar{\tau} + k_1)(k_1 - \bar{b} - \bar{\tau}) = \frac{\alpha}{r} \int_0^{\infty} \max\big\{u(ra - \bar{\tau} + w) - u(ra - \bar{\tau} + k_1), 0\big\} dF(w). \tag{14}$$

The right hand side implies that workers accept any wage above k_1, i.e. $\bar{w} = k_1$. Substituting this into equation (13) delivers equation (10), while substituting it into equation (12) delivers equation (11). Now use $k_1 = \bar{w}$ and the identities $u'(c) = -\gamma u(c)$ and $u(c_1 - c_2) = -u(c_1)/u(c_2)$ to write equation (14) as equation (11). ■

Equation (9) indicates that the reservation wage $\bar{w}$ is increasing in the net unemployment subsidy $\bar{B}$. This is the essence of the moral hazard problem in our model—the more one tries to protect the worker against unemployment by raising unemployment benefits and funding the benefits by an employment tax, the more selective she becomes. The equation also shows that a worker's assets a do not affect her reservation wage, so it is constant during a spell of unemployment.

The consumption policy function in equation (10) has a permanent-income form with a constant precautionary savings component. Assets evolve as $\dot{a} = ra + \bar{b} - c^u(a) = \bar{B} - \bar{w}$. If there is some chance of getting a wage in excess of the unemployment subsidy, $F(\bar{B}) < 1$, equation (9) implies $\bar{w} > \bar{B}$, and so assets and consumption decline over time. Unemployed workers face uncertainty: a wage draw above $\bar{w}$ is good news leading to a jump up in consumption, while no wage offer, or a draw below $\bar{w}$, is bad news leading to a gradual decline in consumption.

The next step is to minimize the cost of providing the worker with initial utility v_0. Using the result that the reservation wage is constant and the functional form for V^u in equation (11) to solve out $\bar{\tau}$, we can rewrite the optimization problem (3) as

$$C^c(v_0) = \min_{\bar{B},\bar{w}} \left(\frac{\bar{B}}{r + \alpha(1 - F(\bar{w}))} + \frac{u^{-1}(rv_0) - \bar{w}}{r} \right),$$

subject to equation (9). The first term is the expected present value of unemployment subsidies, while the second term is $a_0 - \bar{\tau}/r$, set at a level to ensure the worker gets utility v_0. Using equation (11) to eliminate $\bar{B}$ gives us our characterization of optimal constant benefits:

Proposition 2 *Assume CARA preferences. Then the optimal constant benefits policy is such that $\bar{B}^* = \bar{b}^* + \bar{\tau}^*$ is independent of worker's initial utility v_0 and initial assets a_0. The reservation wage satisfies $\bar{w}^* \in \arg\max_{\bar{w}} \Phi(\bar{w})$, where*

$$\Phi(\bar{w}) = \frac{\int_{\bar{w}}^{\infty} \big(1 + \gamma\bar{w} + u(w - \bar{w})\big) dF(w)}{r + \alpha(1 - F(\bar{w}))}. \tag{15}$$

The minimum cost is

$$C^c(v_0) = \frac{u^{-1}(rv_0)}{r} - \frac{\alpha}{\gamma r}\Phi(\bar{w}^*). \tag{16}$$

Given $\bar{w}^*$, the promised utility v_0, and initial assets a_0, we can compute $\bar{b}^*$ and $\bar{\tau}^*$ from equations (9) and (11). Of course, Ricardian equivalence implies that arbitrary combinations of initial assets and lump-sum taxes deliver the same allocations. Optimality only pins down the unemployment subsidy $\bar{B}^* = \bar{b}^* + \bar{\tau}^*$.

B. Optimal Unemployment Insurance

Next we characterize optimal unemployment insurance when the insurance agency has access to time-varying benefits and taxes and can control workers' consumption. To do so, it is convenient to first deduce the shape of the cost function directly from the sequence problem.

Lemma 1 *Assume CARA preferences. The cost function satisfies*

$$C^*(v) = \frac{u^{-1}(rv)}{r} + C_0. \tag{17}$$

Moreover, let $\{\bar{w}^*(t), b^*(t), \tau^*(t)\}$ *denote the optimum for initial promised utility* $u(0)/r$*. Then* $\{\bar{w}^*(t), b^*(t) + u^{-1}(rv), \tau^*(t) - u^{-1}(rv)\}$ *is optimal for any initial promise* v*.*

Proof. Equation (4) and CARA preferences imply that adding x to unemployment consumption and subtracting x from the employment tax simply multiplies lifetime utility by the positive constant $-u(x)$ without affecting her reservation wage. That is, $U\big(t', \{\bar{w}(t), b(t), \tau(t)\}\big) = -u(x)U\big(t', \{\bar{w}(t), b(t) + x, \tau(t) - x\}\big)$ for all x.

Now let $C_0 \equiv C^*(u(0)/r)$. If the policy $\{\bar{w}^*(t), b^*(t), \tau^*(t)\}$ is optimal for initial promised utility $u(0)/r$, the policy $\{\bar{w}^*(t), b^*(t)+u^{-1}(rv), \tau^*(t)-u^{-1}(rv)\}$ is feasible and delivers utility v. A standard revealed preference argument establishes it is optimal. The cost of this policy is $u^{-1}(rv)/r$ plus the cost of the policy that delivers utility $u(0)/r$, completing the proof. ■

The optimal path for consumption shifts in parallel with promised utility, while the path for the reservation wage is unchanged. The cost function reflects these two features. Indeed, since promised utility is a state variable for the problem, the lemma implies that the optimal reservation wage path will be constant. These results are implications of the absence of wealth effects with CARA preferences.

To find the constant C_0, substitute the cost function from equation (17) into the Hamilton-Jacobi-Bellman equation (7) and evaluate it at $v = u(0)/r$. Use the incentive constraint (8) to eliminate the employment tax τ; and use the first order condition $u'(b)C^{*\prime}(v) = 1$ from equation (7) to eliminate unemployment consumption b. Solving for C_0 gives

$$C_0 = \min_{\bar{w}} \frac{-\frac{\alpha}{r\gamma}\int_{\bar{w}}^{\infty}\left(1 + \gamma\bar{w} + u(w - \bar{w})\right) dF(w)}{r + \alpha(1 - F(\bar{w}))} = -\frac{\alpha}{r\gamma}\max_{\bar{w}} \Phi(\bar{w}), \tag{18}$$

where $\Phi(\bar{w})$ is defined by equation (15).

Equation (18) reveals that the reservation wage is constant and identical to that in the case of constant benefits. Substituting equation (18) into equation (17) proves that the cost to the agency with optimal unemployment insurance, $C^*(v)$, is identical to the cost with constant benefits, $C^c(v)$ in equation (16). Our main result follows:

Proposition 3 *Assume CARA preferences. The allocation obtained with constant benefits is optimal.*

In other words, given a level of promised utility, the path of consumption contingent on the history of wage draws is the same under constant benefits and optimal unemployment insurance. There is no loss in allowing the worker to borrow and lend, nor any loss in restricting the planner to the simple policy. Under both policies, consumption falls while a worker remains unemployed and remains lower once the worker is reemployed. In the case with constant benefits, the worker achieves this by decumulating assets during an unemployment spell.

When workers can freely borrow and lend the constant benefit policy is optimal, but, by Ricardian equivalence, there are many other transfer policies that achieve the same allocation. For example, suppose we set a worker's initial assets to zero and use the optimal unemployment insurance path $\{b^*(t), \tau^*(t)\}$ for benefits and taxes. Note that $b^*(t)$ is decreasing and $\tau^*(t)$ is increasing as a function of unemployment duration t. Although the worker can borrow and save, she optimally consumes her after-tax income and keeps her assets at zero.

In what sense then can we say that constant benefits characterize the optimum? To answer this, we define the subsidy to unemployment at some time t in a way that is invariant to all Ricardian-equivalent policies. Suppose that if a worker takes a job at t, she must pay taxes equal to $\frac{\tau(t)}{r}$ in present value terms. If she remains unemployed until t', she receives a transfer $\int_t^{t'} e^{-r(t''-t)} b(t'') dt''$ and then pays taxes valued at $e^{-r(t'-t)} \frac{\tau(t')}{r}$. The difference is the unemployment subsidy between t and t'. To obtain the instantaneous subsidy at t, divide by $t' - t$ and take the limit as t' converges to t:

$$B(t) \equiv b(t) + \tau(t) - \frac{\tau'(t)}{r}. \tag{19}$$

Any optimal policy $\{b(t), \tau(t)\}$ must set $B(t) = \bar{B}^*$. In particular, the unemployment subsidy is the same with constant benefits or optimal unemployment insurance.

III. Wealth Effects: CRRA Utility

The closed form results we obtained so far were derived under an assumption of CARA preferences. This section analyzes an alternative case numerically, constant relative risk aversion (CRRA). The period utility function is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion.

We again consider our two alternative policies: *optimal unemployment insurance* and *constant benefits*. The equivalence between these two policies breaks down with CRRA preferences. In our numerical examples, optimal unemployment insurance involves a subsidy that gradually rises during an unemployment spell. In addition, the unemployment insurance agency distorts the worker's savings behavior: if she had access to a storage technology with the same rate of return as the agency's, a worker would choose to save some of her income. Nevertheless, we find little welfare gain in moving from constant benefits to optimal unemployment insurance.

We also analyze an intermediate case, where benefits may change during an unemployment spell but the worker is free to borrow and lend at the same interest rate as the insurance agency. We find that the optimal path of unemployment subsidies is quantitatively indistinguishable from the case where savings is observable. In particular, the subsidy again rises slowly during an unemployment spell.

A. Parameterization

To proceed numerically, we need to make choices for the discount rate $r = \rho$, the arrival rate of job offers α, the coefficient of relative risk aversion σ, the wage distribution $F(w)$, and the worker's initial utility v_0. Given our model's simplifying assumptions, we do not seek a definitive calibration. Instead, we explored the more qualitative results, which can be compared to the CARA benchmark, for a range of parameter values.

We normalize the time unit to be a week and set $r = 0.001$, equivalent to an annual discount factor of 0.949. We fix the coefficient of relative risk aversion at $\sigma = 2$ but later consider the robustness of our results to a higher value, $\sigma = 6$.[4]

We adopt a Fréchet wage distribution, $F(w) = \exp(-zw^{-\theta})$ with support $(0, \infty)$, and parameters $z, \theta > 0$. We set $\alpha = 1$ but find little sensitivity of our results to this choice. With CRRA preferences, the parameter z acts as an uninteresting scaling factor on wages, and so we normalize $z = 1$ without loss of generality. The mean log wage draw is then $\frac{\bar{\gamma}}{\theta}$, where $\bar{\gamma} \approx 0.577$ is Euler's constant, and the standard deviation of log wages is $\frac{\pi}{\sqrt{6}\theta} \approx \frac{1.28}{\theta}$. We set $\theta = 100$, which implies that, in the absence of unemployment benefits, the weekly job finding rate is about 0.1 (Meyer 1990). Figure 1 plots the density function $F'(w)$, in particular highlighting the positive skewness of the distribution. We also consider the robustness of our results to changes in the wage distribution, to $\theta = 20$, which increases the dispersion in wages, raising the option value of job search and the expected duration of unemployment.

Finally, we set the worker's initial utility so the cost of optimal unemployment insurance is zero, $C^*(v_0) = 0$.

[4] Hopenhayn and Nicolini (1997) use $\sigma = 1/2$ in their baseline calibration. They argue that over short horizons, a high intertemporal elasticity of substitution may be appropriate. In our view, this remark resonates introspectively, but is at the same time misleading since it confounds attitudes regarding consumption and net income paths. In their model, consumption and net income are equivalent; but our model allows saving and borrowing, and as a result a worker displays an infinite elasticity of substitution with respect to the timing of transfers.

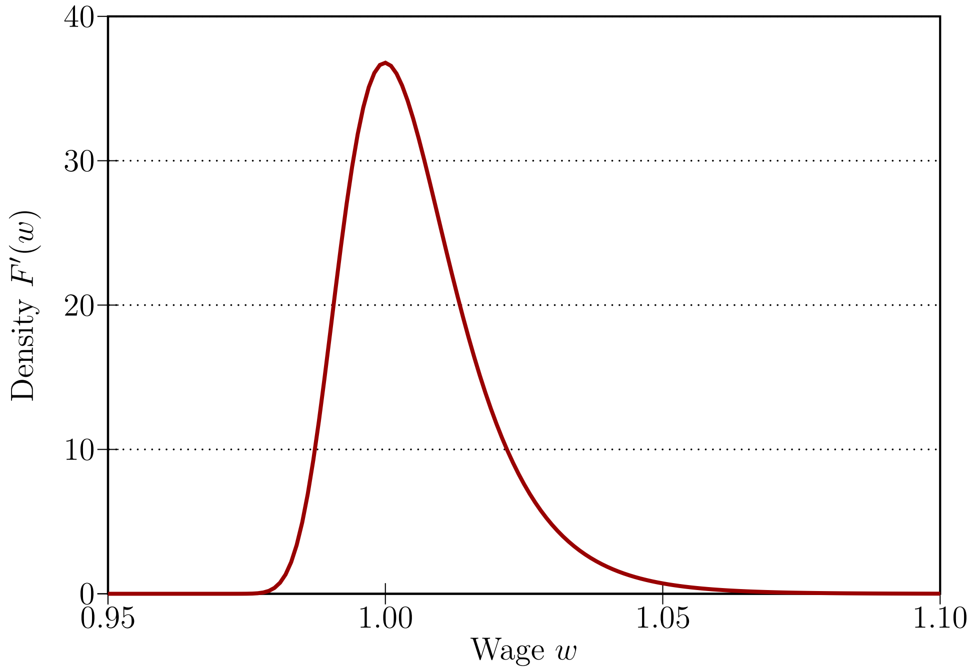

Figure 1: Wage density: $F'(w) = \theta w^{-\theta-1} e^{-w^{-\theta}}$, $\theta = 100$.

B. Optimal Unemployment Insurance

Optimal unemployment insurance is obtained using the policy functions from the Hamilton-Jacobi-Bellman equations (7) and (8). It is straight-forward to solve this as a system of differential equations. In our baseline parameterization, $C^*(v_0) = 0$ implies $v_0 = -977.6$, which is equivalent to a certain consumption of 1.023 forever. We find that the cost function $C^*(v)$ is nearly linear in consumption equivalent units $u^{-1}(rv)$, with a slope of approximately $1/r$ when v is large. We obtained this slope as an exact result with CARA utility; see equation (17).

With optimal unemployment insurance, a worker accepts 9.65 percent of job offers at the start of her unemployment spell. The job finding hazard declines monotonically but gradually to 9.58 percent after 1000 weeks of unemployment, so the reservation wage is virtually constant. It is worth noting that the chance of a twenty year unemployment spell is vanishingly small; 99.3 percent of unemployment spells end in the first year, with a similar hazard rate in each subsequent year.

Turning to the policy that implements this reservation wage, the left panel in Figure 2 shows that a worker's unemployment consumption $b(t)$ is falling over time (dashed brown line), while her reemployment tax $\tau(t)$ is increasing (dash-dot orange line). Putting these together, a worker's expected utility $v(t)$ declines over time. The right panel plots the optimal allocation against utility. Since the optimal policy is time-consistent, this panel also

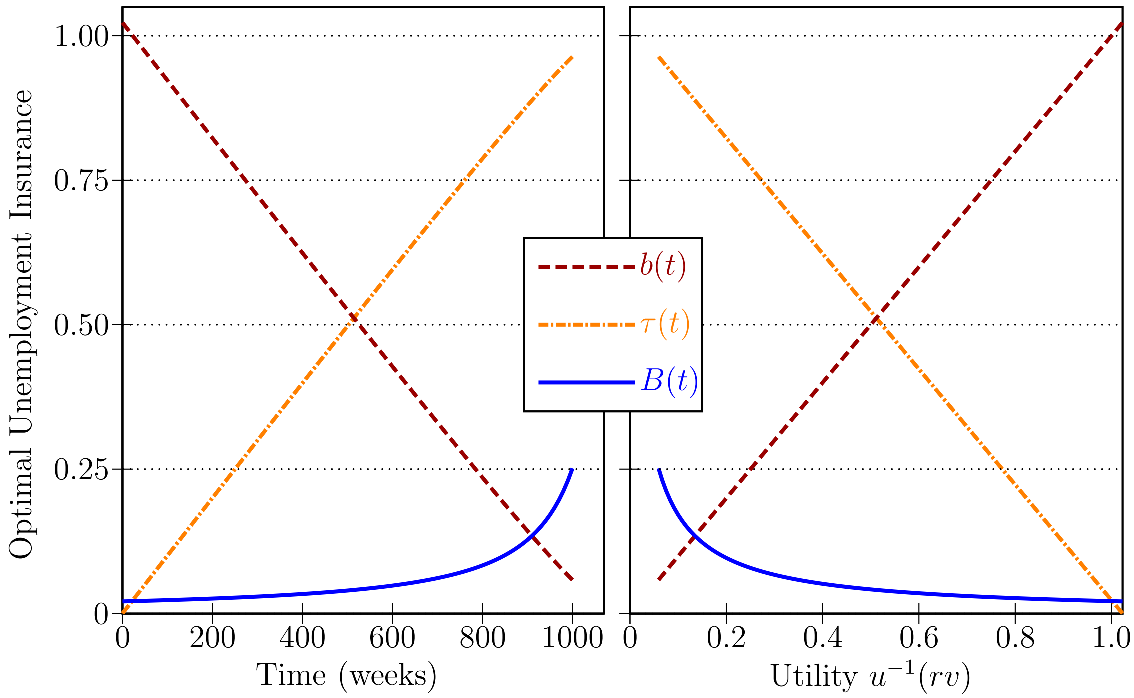

Figure 2: Optimal unemployment consumption $b(t)$, employment taxes $\tau(t)$, and unemployment subsidy $B(t) = b(t) + \tau(t) - \tau'(t)/r$ during the first 1000 weeks of unemployment.

represents optimal unemployment consumption and reemployment taxes for an arbitrary level of initial utility v_0.

We also look at the subsidy to unemployment, the additional resources that a worker gets by remaining unemployed for one more period (equation 19). The solid blue line in Figure 2 shows that the unemployment subsidy is initially about 0.021. It rises to 0.022 if the worker stays unemployed for a year and continues to rise thereafter as utility falls during a prolonged spell of unemployment.

The picture of $b(t)$ and $\tau(t)$ in Hopenhayn and Nicolini (1997) is qualitatively similar to our Figure 2, but they do not compute the distinctive path of the unemployment subsidy $B(t)$.[5] $B(t)$ is much lower than unemployment consumption $b(t)$ because the rising employment tax $\tau(t)$ acts as an implicit debt that finances unemployment consumption; likewise, the subsidy $B(t)$ is flatter because as $b(t)$ falls, $\tau(t)$ rises so that the net disincentive to find a job is kept near constant. In this way, the transfers and taxes $b(t)$ and $\tau(t)$ can be seen as an unemployment subsidy combined with a loan from the insurance agency. This distinction between unemployment consumption and subsidies is crucial for understanding the difference between the results of this paper on the one hand, and Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) on the other.

[5] Werning (2002) computes the net subsidy to unemployment from Hopenhayn and Nicolini (1997) allocation and finds that it is nearly constant, starting quite low and rising very slowly.

C. Constant Benefits

We now turn to the solution of equation (3), characterizing the optimal constant benefit policy. We have already observed that optimal unemployment insurance entails nearly constant benefits during the first year of unemployment and that the probability of an unemployment spell lasting longer than a year is very small. This suggests that there may be little loss in restricting the unemployment insurance agency to use constant benefits. We confirm this result, at least when workers have access to adequate liquidity.

In our baseline parameterization, we find that $C^c(v_0) = 0.02$. That is, the cost of restricting the unemployment insurance agency to using constant benefits is equal to about two percent of average weekly earnings. Despite this small cost, the optimal policy with constant benefits is quite different than optimal unemployment insurance. The agency sets $\bar{B} = \bar{b} + \bar{\tau} = 0.073$, more than three times the initial level of the optimal unemployment subsidy $B(0)$ shown in Figure 2.

To understand why $\bar{B}$ is larger than $B(0)$, it is important to think about borrowing constraints. With CRRA preferences, the no-Ponzi condition $\lim_{t\to\infty} e^{-rt}a(t) \geq 0$ is equivalent to Aiyagari's (1994) natural borrowing constraint: assets must stay sufficiently high to ensure that, with probability 1, a worker can pay the interest on her debt while maintaining nonnegative consumption. Since we have assumed that there is always a chance of not getting a job offer during an arbitrarily long time period, an unemployed worker's labor income may be zero over any finite horizon with positive (though vanishing) probability. The natural borrowing constraint then requires that $a \geq \underline{a} = -\bar{b}/r$. Higher unemployment benefits then serve two roles: they insure workers against unemployment risk, and they provide a guaranteed source of income, which allows a worker borrow more against her future income.

To disentangle the insurance and liquidity roles of unemployment benefits, we introduce an additional policy instrument. At any time, an unemployed worker may go on welfare. If she chooses to do this, she is paid $\underline{w} > 0$ forever (at a cost $\underline{w}/r$) and she must continue to pay the interest on her debt. On the other hand, she forgoes the possibility of getting a job. With this policy, the worker's borrowing constraint is

$$\underline{a} = -\frac{\max\{\underline{w}, \bar{b}\}}{r}.$$

Setting $\underline{w} = 0.2$ makes the cost of constant benefits negligible, on the order of 10^{-5}, while it lowers the optimal constant benefit to $\bar{B} = 0.022$, in line with the initial optimal unemployment subsidy. Under this policy, a worker can spend about 4 years unemployed before she goes on welfare; the probability that she has to do so is about 10^{-15}. Thus welfare offers little insurance and so has a small direct effect on incentives; but it provides liquidity by allowing a worker to borrow far more than she could with constant benefits.

We now briefly analyze an intermediate case: optimal time-varying unemployment insurance with hidden savings (Werning 2002). Relative to constant benefits, this is a less restrictive policy because unemployment benefits and reemployment taxes may change arbitrarily during an unemployment spell. Relative to optimal unemployment insurance, hidden savings constrains the unemployment insurance agency because it cannot distort the worker's consumption-savings decision.

Formally, we look for unemployment benefits and reemployment taxes which ensure that the worker maintains zero assets when she can borrow and lend at rate $r = \rho$; by Ricardian equivalence, there is no loss of generality in this timing of transfers. Also without loss of generality, the employment tax $\tau(t)$ depends only on the time t when the worker found a job. While unemployed, consumption $c(t) = b(t)$ must maximize

$$\int_0^\infty e^{-\int_0^t \left(r+\alpha(1-F(\bar{w}(s)))\right)ds} \left(u(c(t)) + \alpha \int_{\bar{w}(t)}^\infty \frac{u(w-\tau(t))}{r} dF(w)\right) dt$$

subject to $\dot{a}(t) = ra(t)+b(t)-c(t)$ and the no-Ponzi game condition, with $a(0) = 0$ given. One implication of this is that the worker's consumption Euler equation must hold, so marginal utility is a Martingale:

$$u''(b(t))\dot{b}(t) = -\alpha \int_{\bar{w}(t)}^\infty \left(u'(w-\tau(t)) - u'(b(t))\right) dF(w). \tag{20}$$

Following Werning (2002), we introduce the marginal utility of consumption as an additional state variable and solve the cost minimization problem (7)–(8) with the additional constraint (20), which can be seen as a law of motion for the new state variable. The Euler equation (20) together with (8) are necessary for the worker's optimum. Since they may not be sufficient, we are solving a relaxed planning problem. Therefore, after finding a solution to this relaxed problem, the last step is to verify that under the proposed policy the worker's optimum involves no saving: $a(t) = 0$ for all t. If this is the case, then the solution to the relaxed problem and the original hidden-savings planning problem coincide.

When we implement this program, we find that allowing for hidden savings has a negligible effect on the cost of unemployment insurance; this is not surprising since constant benefits augmented with welfare already reduced the cost to 10^{-5}. More interestingly, we find that the optimal path of unemployment subsidies is quantitatively indistinguishable from the model with observable savings; at 0 and at 52 weeks, the difference in optimal unemployment subsidies is of the order 10^{-6}. This reinforces our two main conclusions: the optimal policy entails nearly constant benefits; and the cost of restricting policy to use constant benefits while allowing the worker to borrow and lend is minuscule.

D. Robustness

This section discusses the robustness of our results. We first ask the extent to which our results depend on the wage distribution, in particular on the assumption that a worker finds a job in ten weeks on average. There are a few reasons to explore this assumption. First, our results indicate that constant unemployment benefits and constant employment taxes do almost as well as a fully optimal unemployment insurance policy. It could be that this result would go away if unemployment spells tended to last longer and therefore presented a bigger risk to individuals. Second, in many countries, notably much of Europe, unemployment duration is substantially longer, although this is at least in part a response to unemployment benefits that are high compared to workers' income prospects (Ljungqvist and Sargent 1998; Blanchard and Wolfers 2000). And third, workers typically experience multiple spells of unemployment before locating a long-term job (Hall 1995). Although modeling this explicitly would go beyond the scope of this paper, we have found in other work that raising unemployment duration has similar effects on optimal benefit levels to modeling repeated unemployment spells (Shimer and Werning 2007).

To explore this possibility, we choose $\theta = 20$, so that expected unemployment duration of a risk-neutral worker with no unemployment benefits is about one year. This raises the unconditional standard deviation of wages by a factor of five, which increases the option value of job search. We revisit our main conclusions under this alternative parameterization:

- *Under optimal unemployment insurance, the subsidy $B(t)$ rises slowly.* We start a worker with utility v_0 such that $C^*(v_0) = 0$. The optimal subsidy is 0.113 initially and rises to 0.118 after one year of unemployment and 0.2 after ten years, relative to a mean wage draw of 1.03.

- *The optimal job finding rate changes slowly.* In the same experiment, it rises from 1.898 percent per week to 1.901 percent per week during the first ten years of unemployment.

- *There is little cost to constant benefits.* The optimal constant benefit that delivers utility v_0 sets $\bar{B} = 0.25$ and costs $C^c(v_0) = 0.59$, about half a week's income.

- *Providing liquidity further reduces costs.* Augmenting constant benefits with a welfare program that delivers income 1 reduces the cost of constant benefits to less than 0.001 and lowers the optimal constant benefit to $\bar{B} = 0.124$. Allowing for time-varying benefits with hidden saving has a negligible effect on costs and on optimal policy compared to optimal unemployment insurance with observable savings.

We have also examined the robustness of our results to higher risk aversion by setting $\sigma = 6$. Optimal unemployment subsidies are higher than the benchmark with $\sigma = 2$, as the CARA results suggest. Otherwise this change in preferences has little effect on our results.

Finally, we have focused on the savings distortions of unemployed workers. This makes sense since in our model, it is not optimal to distort the savings of employed workers. However, in a version of the model with repeated unemployment spells, savings distortions during employment may be part of a comprehensive optimal tax system. While the extension to repeated spells and the study of optimal taxation of employed workers is beyond the scope of this paper on unemployment insurance, we believe that our results on savings distortions during unemployment are unlikely to be affected.

IV. Conclusion

This paper characterizes optimal unemployment insurance in a sequential search model. Our main result is that with CARA preferences, constant benefits coupled with free access to borrowing and lending of a riskless asset is optimal. With CRRA preferences, the exact optimality of constant benefits breaks down. In numerical explorations, we find that the optimal unemployment subsidy *rises* very slowly over time. However, we find little loss to a constant unemployment subsidy if workers are given enough liquidity. This quantitative result is robust to the key parameters of the model. These conclusions contrast with Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) because these papers do not separate insurance from liquidity.

Our distinction between liquidity and insurance resonates with recent policy proposal by Feldstein and Altman (1998) and Feldstein (2005). These papers discuss replacing unemployment benefits with unemployment saving accounts, which provide workers with the liquidity needed to help smooth consumption through unemployment spells. Workers must save into these accounts while employed and can draw them down during unemployment. At retirement workers can keep the positive balance, while some debt may be forgiven, the only insurance component of the plan. Although our results do not support zero unemployment subsidies, the optimal level of benefits may be low once liquidity is ensured; they turned out to be so in our simulations. Of course, our model has nothing to say about the forced-saving element of this policy proposal.

We have deliberately written a stark model of job search in order to keep the analysis relatively simple and comparable to previous work on the timing of unemployment benefits. While the model lends itself to various extensions, our results suggest that it will be important to evaluate the relative efficiency of simple benefit policies coupled with free access to the asset market and to distinguish between insuring workers against uncertainty in the duration of a jobless spell and ensuring they have the liquidity to smooth consumption while unemployed.

Appendix

A. General Mechanisms

This section uses the revelation principle to set up the most general deterministic mechanism that an unemployment insurance agency might contemplate given the assumed asymmetry of information. We allow the worker to make reports on the privately observed wage and we allow taxes to vary during an employment spell. We show that neither of these capabilities is useful: the planner does just as well by offering unemployment benefits that depend on the duration of unemployment, and setting employment taxes that depend on the duration of the previous unemployment spell, not on employment tenure.

A. The Recursive Mechanism

For notational convenience, we present the general mechanism directly in its recursive form—this can be justified along the lines of Spear and Srivastava (1987). Our general mechanism involves the following:

1. At any point in time the unemployed worker has some promise for expected lifetime utility v. The policy objects b, $\tau(w, t)$, and $v^u(w)$, explained below, depend implicitly on this value.

2. At any point in time the unemployed worker can claim to have received a wage offer and make a report $\hat{w}$ to the planner:

   (a) If the worker reports $\hat{w} < \bar{w}$, she rejects the job and is promised a new continuation utility $v^u(\hat{w})$.

   (b) If the worker reports $\hat{w} \geq \bar{w}$, she accepts the job and pays a tax $\tau(\hat{w}, t)$ in each subsequent period $t \geq 0$.

   Note that since employment is observable, a worker without a wage offer cannot claim to have an offer $w \geq \bar{w}$, only an offer $w < \bar{w}$.

3. While unemployed the worker receives an unemployment benefit b and lifetime utility

evolves according to

$$\dot{v} = (\rho + \alpha)v - u(b) - \alpha \int\limits_{[\underline{w},\bar{w})} v^u(w)\, dF(w) - \alpha \int\limits_{[\bar{w},\infty)} \int_0^\infty e^{-\rho t} u(w - \tau(w,t))\, dt\, dF(w)$$

B. The Planner's Problem

Let $C^*(v)$ denote the cost to the planner of having a worker with promised utility v. The planner's problem may be expressed recursively as follows:

$$\begin{aligned}(r+\alpha)C^*(v) = \min_{\bar{w},b,v^u(w),\tau(w,t)} \Bigg(& b + \alpha \int\limits_{[\underline{w},\bar{w})} C(v^u(w))\, dF(w) - \alpha \int\limits_{[\bar{w},\infty)} \int\limits_0^\infty e^{-rt}\tau(w,t)\, dt\, dF(w) \\ & + C^{*\prime}(v)\Big((\rho+\alpha)v - u(b) - \alpha \int\limits_{[\underline{w},\bar{w})} v^u(w)\, dF(w) \\ & - \alpha \int\limits_{[\bar{w},\infty)} \int\limits_0^\infty e^{-\rho t} u(w - \tau(w,t))\, dt\, dF(w)\Big)\Bigg)\end{aligned}$$

subject to a set of truth telling constraints for all w, $\hat{w}$:

$$\int_0^\infty e^{-\rho t} u(w - \tau(w,t))\, dt \geq \int_0^\infty e^{-\rho t} u(w - \tau(\hat{w},t))\, dt \qquad w, \hat{w} \geq \bar{w} \tag{21}$$

$$\int_0^\infty e^{-\rho t} u(w - \tau(w,t))\, dt \geq \max\{v^u(\hat{w}), v\} \qquad w \geq \bar{w} > \hat{w} \tag{22}$$

$$v^u(w) \geq \int_0^\infty e^{-\rho t} u(w - \tau(\hat{w},t))\, dt \qquad \hat{w} \geq \bar{w} > w \tag{23}$$

$$v^u(w) \geq v^u(\hat{w}) \qquad \bar{w} > w, \hat{w} \tag{24}$$

$$v^u(w) \geq v \qquad \bar{w} > w \tag{25}$$

$$v \geq v^u(\hat{w}) \qquad \bar{w} > \hat{w} \tag{26}$$

Inequality (21) ensures that workers that take a job do not misrepresent their wage. Inequality (22) ensures that they are willing to report a wage $w \geq \bar{w}$ rather than report a lower wage or claim not to have gotten an offer at all. Inequality (23) ensures that workers who get an offer $w < \bar{w}$ do not claim to have an acceptable offer. Inequality (24) ensures that workers who do not take a job do not misrepresent the rejected wage. Inequality (25) ensures that workers that get an offer but should not take a job do not prefer to claim they did not receive an offer. Finally, inequality (26) ensures that workers that do not have an offer do not wish to claim that they actually do, but that the offer is below $\bar{w}$ and should be rejected.

We now proceed to simplify the planner's problem.

Lemma 2 *(a) Suppose an optimum has the schedule $v^u(w)$, then the mechanism that replaces it with a constant schedule $v^u(w) = v$ for any $w < \hat{w}$ is also optimal. (b) The incentive constraints (21)–(26) can be replaced with the single equality condition,*

$$v = \int_0^\infty e^{-\rho t} u(\bar{w} - \tau(\bar{w}, t))\, dt, \tag{27}$$

and constraint (21).

Proof. (a) Condition (25) and (26) imply that $v^u(w) = v$ for all $w < \bar{w}$.

(b) For a constant v the constraints (24)–(26) are trivially satisfied. Since the right-hand side of constraint (23) is increasing in w, it is equivalent to

$$v \geq \int_0^\infty e^{-\rho t} u(\bar{w} - \tau(\hat{w}, t))\, dt$$

for all $\hat{w} \geq \bar{w}$. Constraint (21) implies $\hat{w} = \bar{w}$ maximizes the right-hand side of this inequality, so it reduces to

$$v \geq \int_0^\infty e^{-\rho t} u(\bar{w} - \tau(\bar{w}, t))\, dt. \tag{28}$$

Next, note that inequality (22) is now equivalent for all $w \geq \bar{w}$

$$\int_0^\infty e^{-\rho t} u(w - \tau(w, t))\, dt \geq v.$$

If $w > \bar{w}$, then

$$\int_0^\infty e^{-\rho t} u(w - \tau(w, t))\, dt \geq \int_0^\infty e^{-\rho t} u(w - \tau(\bar{w}, t))\, dt > \int_0^\infty e^{-\rho t} u(\bar{w} - \tau(\bar{w}, t))\, dt,$$

where the first inequality uses (21) and the second uses monotonicity of the utility function. Therefore the preceding inequality is tightest when $w = \bar{w}$, so inequality (22) is equivalent to

$$\int_0^\infty e^{-\rho t} u(\bar{w} - \tau(\bar{w}, t))\, dt \geq v. \tag{29}$$

Inequalities (28) and (29) hold if and only if equation (27) holds, completing the proof. ∎

C. Constant Absolute Risk Aversion

So far we have not made any assumptions about the period utility function u except that it is increasing. This section examines the implications of having constant absolute risk aversion preferences.

Lemma 3 *With CARA utility, an optimum must feature the tax on the employed $\tau(w,t)$ independent of w and t.*

Proof. With exponential utility w on both sides of equation (21) cancels, implying that the remaining term $\int_0^\infty e^{-rt}u(-\tau(w,n))$ must be some value independent of w. Let x denote this value. It follows that an optimum must solve the subproblem

$$\min_{\{\tau\}} \left(- \int_0^\infty e^{-rt}\tau(w,t)\,dt \right)$$
$$\text{subject to } x = \int_0^\infty e^{-\rho t} u\big(-\tau(w,t)\big)\,dt.$$

Using $r = \rho$, the first order condition for this problem reveals that an $\tau(w,t)$ must be independent of (w,t). ■

Lemmas 2 and 3 allow us to rewrite the planning problem as in (7)–(8). Private information prevents "employment insurance," so the tax rate τ is independent of the wage. With CARA preferences and jobs that last forever, the wage effectively acts as a permanent multiplicative taste shock. This ensures that all employed workers have the same preferences over transfer schemes, which makes it impossible to separate workers according to their actual wages. Since workers have concave utility, introducing variability in taxes is not efficient.

With non-CARA utility, workers with different wages rank tax schedules differently. In some cases, it may be possible to exploit these differences in rankings to separate workers according to their wage; see Prescott and Townsend (1984) for an example. If workers have decreasing absolute risk aversion (DARA), including CRRA preferences, those earning lower wages are more reluctant to accept intertemporal variability in taxes. One can therefore induce these workers to reveal their wage by giving them a choice between a time-varying employment tax with a low discounted cost and a constant tax with a high cost. High wage workers would opt for the time-varying schedule. This does not, however, reduce the planner's cost of providing an unemployed worker with a given level of utility, since it transfers income from low wage to high wage workers. It is therefore not optimal.

References

Abdulkadiroglu, Atila, Burhanettin Kuruscu, and Aysegul Sahin, 2002. "Unemployment Insurance and the Role of Self-Insurance." *Review of Economic Dynamics*. 5 (3): 681–703.

Aiyagari, S. Rao, 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics*. 109 (3): 659–684.

Allen, Franklin, 1985. "Repeated Principal-Agent Relationships with Lending and Borrowing." *Economic Letters*. 17: 23–31.

Alvarez, Fernando, and Marcelo Veracierto, 2001. "Severance payments in an economy with frictions." *Journal of Monetary Economics*. 47 (3): 477–498.

Blanchard, Olivier, and Justin Wolfers, 2000. "The Role of Shocks and Institutions in the Rise of European Unemployment: the Aggregate Evidence." *Economic Journal*. 110 (462): C1–C33.

Cole, Harold, and Narayana Kocherlakota, 2001. "Efficient Allocations with Hidden Income and Hidden Storage." *Review of Economic Studies*. 68: 523–542.

Feldstein, Martin, 2005. "Rethinking Social Insurance." *American Economic Review*. 95 (1): 1–24.

Feldstein, Martin, and Daniel Altman, 1998. "Unemployment Insurance Savings Accounts." NBER Working Paper 6860.

Feldstein, Martin, and James Poterba, 1984. "Unemployment Insurance and Reservation Wages." *Journal of Public Economics*. 23 (1-2): 141–167.

Golosov, Mikhail, Narayana Kocherlakota, and Aleh Tsyvinski, 2003. "Optimal Indirect and Capital Taxation." *Review of Economic Studies*. 70 (3): 569–587.

Hall, Robert E., 1995. "Lost Jobs." *Brookings Papers on Economic Activity*. 1 (1): 221–273.

Hansen, Gary D., and Ayse Imrohoroglu, 1992. "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard." *Journal of Political Economy*. 100 (1): 118–142.

Hopenhayn, Hugo, and Juan Pablo Nicolini, 1997. "Optimal Unemployment Insurance." *Journal of Political Economy*. 105 (2): 412–438.

Lentz, Rasmus, 2007. "Optimal Unemployment Insurance in an Estimated Job Search Model with Savings." University of Wisconsin Mimeo.

Ljungqvist, Lars, and Thomas Sargent, 1998. "The European Unemployment Dilemma." *Journal of Political Economy*. 106 (3): 514–550.

Ljungqvist, Lars, and Thomas Sargent, 2004. *Recursive Macroeconomic Theory*, MIT Press, second edn.

Lucas, Robert E. Jr., and Edward C. Prescott, 1974. "Equilibrium Search and Unemployment." *Journal of Economic Theory*. 7 (2): 188–209.

McCall, John, 1970. "Economics of Information and Job Search." *Quarterly Journal of Economics*. 84 (1): 113–126.

Meyer, Bruce, 1990. "Unemployment Insurance and Unemployment Spells." *Econometrica*. 58 (4): 757–782.

Pissarides, Christopher A., 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *The American Economic Review*. 75 (4): 676–690.

Prescott, Edward C., and Robert Townsend, 1984. "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard." *Econometrica*. 52 (1): 21–45.

Rogerson, William, 1985. "Repeated Moral Hazard." *Econometrica*. 53 (1): 69–76.

Shavell, Steven, and Laurence Weiss, 1979. "Optimal Payment of Unemployment-Insurance Benefits over Time." *Journal of Political Economy*. 87 (6): 1347–1362.

Shimer, Robert, and Iván Werning, 2007. "Reservation Wages and Unemployment Insurance." *Quarterly Journal of Economics*. 122 (3): 1145–1185.

Spear, Stephen, and Sanjay Srivastava, 1987. "On Repeated Moral Hazard with Discounting." *Review of Economic Studies*. 54 (4): 599–617.

Wang, Cheng, and Stephen D. Williamson, 2002. "Moral Hazard, Optimal Unemployment Insurance, and Experience Rating." *Journal of Monetary Economics*. 49 (7): 1337–1371.

Werning, Iván, 2002. "Optimal Unemployment Insurance with Unobservable Savings." Mimeo.