NBER WORKING PAPER SERIES

JUNIOR MUST PAY: PRICING THE IMPLICIT PUT IN PRIVATIZING SOCIAL SECURITY

George M. Constantinides
John B. Donaldson
Rajnish Mehra

Working Paper 8906
http://www.nber.org/papers/w8906

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2002

We thank Henning Bohn for his insightful comments. We also thank Malcolm Baker, Jean Boivin, Laurent Calvet, John Campbell, Rick Mishkin, Andrei Shleifer, Kent Smetters, Jeremy Stein, Luis Viceira, Steve Zeldes, the seminar participants of the Columbia University macro workshop and the Harvard University financial economics workshop for helpful discussions. We remain responsible for errors. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

Junior Must Pay: Pricing the Implicit Put in Privatizing Social Security
George M. Constantinides, John B. Donaldson and Rajnish Mehra
NBER Working Paper No. 8906
April 2002
JEL No. D91, E2, E60, G11, G13, H55

ABSTRACT

Proposals that portion of the Social Security Trust Fund assets be invested in equities entail the possibility that a severe decline in equity prices renders the Fund assets insufficient to provide the currently mandated level of benefits. In this event, existing taxpayers may be compelled to act as insurers of last resort. The cost to taxpayers of such an implicit commitment equals the value of a put option with payoff equal to the benefit's shortfall. We calibrate an OLG model that generates realistic equity premia and value the put. With 20 percent of the Fund assets invested in equities, the highest level currently under serious discussion, we value a put that guarantees the currently mandated level of benefits at one percent of GDP, or a temporary increase in Social Security taxation of at most 25 percent. We value a put that guarantees 90 percent of benefits at merely .03 percent of GDP. In contrast to earlier literature, our results account for the significant changes in the distribution of security returns resulting from Trust Fund purchases. We also explore the inter-generational welfare implications of the guarantee.

George M. Constantinides
Graduate School of Business
The University of Chicago
1101 East 58th Street
Chicago, IL 60637
and NBER
gmc@gsb.uchicago.edu

John B. Donaldson
Columbia University

Rajnish Mehra
Department of Economics
University of California
Santa Barbara, CA 93106
and NBER

I Introduction

This paper is a contribution to the current debate on the privatization of Social Security. The term *privatization* covers a gamut of policy definitions and implications. We use the term to refer to the proposal to invest in the stock market funds that are earmarked for Social Security payments. Currently, the Social Security Trust Fund (hereafter, SSTF) is restricted to invest only in United States Treasury securities.

Unlike other capital market investments, Social Security is a form of social insurance, in that it *implicitly guarantees a minimum consumption level* for its participants. Suppose that a large fraction of Social Security taxes (Trust funds) is invested in equities. Given the volatility of the United States stock market over the past 70-odd years, there is a distinct possibility that Social Security funds invested in the stock market decline in value to a level such that they are inadequate to provide a subsistence level of benefits. In such a situation, the government may be compelled to remedy the shortfall by raising taxes on the younger working generations.

We argue that any *time-consistent* discussion on privatizing Social Security must take into account this *de facto* role of the younger working generations as insurers of last resort. By providing a consumption floor for the retired cohort, current wage earners effectively write a put option on their risky portfolio, to be honored by succeeding generations. In this paper, we construct a model that allows us to explicitly price this put option. We do this to give planners and policy makers an idea of the order of magnitude of the costs involved.

When 20 percent of Trust Fund assets are invested in equities, the highest level currently under serious discussion, we find that a put that guarantees the currently mandated level of benefits is priced at one percent of GDP for reasonable model parameterizations. This corresponds to a temporary increase in the rate of social security taxation vis-à-vis the current level of, at most, 20 percent. A put that guarantees only 90 percent of the currently mandated benefits is priced at roughly .03 percent of GDP, which is proportionately much less. In contrast to the earlier literature, our results fully take into account the non-trivial changes in the distributions of security returns resulting from Trust Fund equity purchases. The equilibrium returns we report also achieve non-trivial equity premia and non-trivial equity and risk-free return standard deviations. We also explore the inter-generational welfare implications of implementing the guarantee.

The direct antecedents of our work are the models in Feldstein et al. (2001), Smetters (1998, 2001), and Pennacchi (1999). These papers evaluate a "benefits guarantee" or put in the context of partial equilibrium models that take as fixed the distribution of equilibrium returns. In related work, Abel (2001) and Diamond and Geanakoplos (2001) explore the implications of investing SSTF assets in the stock market on the aggregate investment and the time path of the capital stock. Our exchange economy shares many features with Abel's (2001) production economy, which, however, focuses on capital stock dynamics. Our conclusions are broadly consistent with those of Diamond and Geanakoplos (2001). In particular, we also find that diversification by the SSTF into equities reduces the equity premium and presents the opportunity for welfare enhancements. Diamond and Geanakoplos (2001) do not present numerical estimates of the effects they detail. Albeit in a simpler context, this estimation is the primary focus of our work and, in this sense, it complements many of their conclusions. While recognizing the nature of the risks involved, they also do not seek to value the Social Security put. In a series of papers, Bohn (1997, 1998, 1999) and Campbell et al. (2001) study the risk reallocation characteristics of various Social Security financing and payout arrangements, especially defined benefit versus defined contribution systems. Excellent discussions of many of the practical issues surrounding Trust Fund stock market investing and Social Security reform in general can be found in the volumes edited by Campbell and Feldstein (2001) and Mitchell et al. (1999). Many of the issues underlying the need for Social Security reform of some type are well laid out in Geanakoplos et al. (1999) Kotlifkoff et al (1998) and Mitchell and Zeldes (1996).

The paper is organized as follows. In Section II, we describe the economy and define equilibrium. In Section III, we explain how the Social Security put is valued. We calibrate the economy in Section IV and present our results in Section V. Our conclusions are presented in Section VI.

II The Economy

We consider a one-good overlapping-generations, pure exchange economy. It is a version of the economy studied in Constantinides et al. (2002), generalized to account for Social Security transfers. Each generation consists of a continuum of consumers and lives for three periods as *young, middle-aged* and *old*. Three is the minimal number of periods that captures the heterogeneity of consumers across the age groups that we wish to emphasize: the *borrowing-constrained* young, the *wage-earning* middle-aged, and the *dis-saving* old. In the calibration, each period is taken to represent 20 years. By restricting our attention to an exchange setting, we focus on the purely re-distributive effects. That is, we assume that the technological or other mechanisms that give rise to the yearly aggregate income process of the United States economy are invariant to the manner by which Social Security payments are financed. Abel (2001) provides a different perspective.

There is one consumption good in each period and it perishes at the end of the period. Wages, Social Security taxes, consumption, payments from, and prices of all securities traded in the model are denominated in units of the consumption good.

We admit two types of securities in positive net supply, a consol *bond* and *equity*. Consumers may purchase these securities but realistic borrowing and short sale restrictions are in effect. We consider various scenarios under which the SSTF may purchase either security or a combination of them.

The bond is default-free and pays one unit of consumption in each period, in perpetuity. We think of the bond as a proxy for long-term government debt. Its supply is fixed at B units in perpetuity.[1] The *ex coupon* bond price in period t is denoted by q_t^b and is the price of the claim to the unit coupon paid in perpetuity, beginning with period $t+1$.

Equity in this model is a claim to a residual dividend stream and pays a net dividend d_t^e in period t. We think of equity as the sum total of the claims to firms including net corporate debt payments, net rental payments, etc. The supply of equity is fixed at one share in perpetuity. The

[1]We focus on consol bonds for two reasons. First, it is specious to introduce a short-term (say, one-year) bond in this economy because the length of one period is assumed to be 20 years. Second, the Trust Fund is invested primarily in coupon bearing Treasury securities rather than discount T-bills, and we would like the model to reflect this fact.

issue and repurchase of equities and bonds is implicitly accounted for by the fact that the equity is defined as the claim to the *net* dividend. We do not model the process by which firms finance the net dividend—firms are exogenous to our exchange economy. The *ex dividend* price of equity in period t is denoted by q_t^e. This equity security represents ownership of the dividend stream in perpetuity, beginning with period $t+1$.

II.1 Wage and income profiles

Consumers born in period t receive deterministic wage income $w_0 > 0$, when young. We interpret w_0 to be net of all income (as opposed to social insurance) taxes. Without loss of generality, the Social Security taxes levied on the young are divided into three components, τ_0, τ_0^e, and τ_0^b, according to the manner by which they are invested. The tax τ_0 is that portion paid out immediately to the current old generation under a pay-as-you-go system. This is largely the form of current Social Security finance. The second and third components, τ_0^e and τ_0^b, represent the taxes invested in equity and bonds, respectively. The actual quantities purchased of each security are τ_0^e / q_t^e (equity) and τ_0^b / q_t^b (bonds).

We assume that the young do not privately participate in the financial markets.[2] Their budget constraint is

$$c_{t,0} \leq w_0 - \tau_0 - \tau_0^e - \tau_0^b \tag{2.1}$$

where $c_{t,0}$ denotes the consumption of the young in period t.

When entering middle age, a young consumer either becomes a high-wage earner with probability h or a low wage earner with probability $1-h$. Consumers become aware of their high or low wage status only upon entering middle age. Since we assume a continuum of consumers, h also represents the fraction of the middle-aged cohort who are high-wage earners; similarly, the fraction of low wage earners is $1-h$. A high-wage middle-aged consumer

[2]This assumption reflects an implicit borrowing constraint in the following sense: under our calibration, consumers experience a very steep consumption profile while passing from young to middle age. The young would not wish to save under these conditions. Consumption smoothing considerations suggest that they would wish to borrow against their future (higher) middle-aged income. In practice, this is difficult to execute without holding other assets (collateral). As we exclude bequests, there is no provision for an accumulation of collateral. We summarize these

receives wage income $\tilde{w}_{H,t+1}$, out of which he/she pays Social Security taxes, invests privately in equity and bonds, and consumes.

We denote by $c_{t,1}^{H}$ the consumption in period $t+1$ of a high-wage middle-aged consumer born in period t. We denote by $z_{t,1}^{e}$ and $z_{t,1}^{b}$ this consumer's private purchases of equity and bonds in period $t+1$. The three Social Security tax components, $\tau_{H,1}$, $\tau_{H,1}^{e}$ and $\tau_{H,1}^{b}$ are directly analogous to the corresponding quantities levied on the young: $\tau_{H,1}$ denotes the pay-as-you-go component while $\tau_{H,1}^{e}$ and $\tau_{H,1}^{b}$ denote the amounts invested in equity and bonds, respectively. The actual quantities purchased of each security are $\tau_{H,1}^{e} / q_{t+1}^{e}$ (stock) and $\tau_{H,1}^{b} / q_{t+1}^{b}$(bonds). The budget constraint for the high-wage middle-aged consumer is

$$c_{t,1}^{H} + q_{t+1}^{e} z_{t,1}^{e} + q_{t+1}^{b} z_{t,1}^{b} \leq \tilde{w}_{H,t+1} - \tau_{H,1} - \tau_{H,1}^{e} - \tau_{H,1}^{b} \tag{2.2}$$

Circumstances are similar for the low-income middle-aged consumers, except that we assume that they do not privately participate in financial markets. The notation is analogous to that detailed above, except for the superscript/subscript L which identifies this group of consumers. We assume that the wage income of this group, $w_{L,t+1}$, is deterministic. We allow for the possibility that $\tau_{H,1}^{e} \neq \tau_{L,1}^{e}$ and $\tau_{H,1}^{b} \neq \tau_{L,1}^{b}$, and that consumers of different income levels pay different Social Security taxes and receive different levels of benefits, which is indeed the case in the United States. The budget constraint for the low-wage middle-aged consumer is

$$c_{t,1}^{L} \leq w_{L,t+1} - \tau_{L,1} - \tau_{L,1}^{e} - \tau_{L,1}^{b} \text{ [3]} \tag{2.3}$$

For simplicity, we rule out bequests. For a period-t-born consumer of low middle-aged income status, the period $t+2$ retirement consumption, $c_{t,2}^{L}$, is limited to the wage in retirement, w_2, plus the Social Security benefits

constraints by the indicated assumption.

[3] Our assumption of non-market participation for this class of consumers is fully exogenous, as their income profile looking forward to old age is rapidly diminishing under our calibration. This is also an empirical fact for a large subset of the United States population, yet they do not save. We offer no explanation for this phenomenon but model it as an exogenous fact in the manner indicated.

$$c_{t,2}^{L} \leq w_2 + \tau_{L,1} + \tau_0 + \left(q_{t+2}^{e} + d_{t+2}^{e}\right)\left(\frac{\tau_{L,1}^{e} + \tau_{o}^{e}}{q_{t+1}^{e}}\right) + \left(q_{t+2}^{b} + 1\right)\left(\frac{\tau_{L,1}^{b} + \tau_{0}^{b}}{q_{t+1}^{b}}\right) \qquad (2.4)$$

The latter includes not only the elements arising from the pay-as-you-go aspects of the system ($\tau_{L,1} + \tau_0$) but also the proceeds of the sales of securities held on the consumer's behalf under the proposed financing alternatives.

In the case of a previously high-income middle-aged consumer, consumption in old age, $c_{t,2}^{H}$, is further augmented by the proceeds of the sale of the consumer's privately held securities:

$$c_{t,2}^{H} \leq w_2 + \tau_0 + \tau_{H,1} + \left(q_{t+2}^{e} + d_{t+2}^{e}\right)\left(z_{t,1}^{e} + \frac{\tau_{H,1}^{e} + \tau_{0}^{e}}{q_{t+1}^{e}}\right) + \left(q_{t+2}^{b} + 1\right)\left(z_{t+2}^{b} + \frac{\tau_{H,1}^{e} + \tau_{0}^{b}}{q_{t+1}^{b}}\right)^{4} \qquad (2.5)$$

Equations (2.4) and (2.5) imply that the taxes on the young which are invested in equity and bonds benefit the generation that is old in the next period rather than the young themselves, when they become old in two periods. Thus, we do not have the feature of generation-specific accounts that are held and augmented for multiple periods, whereby a consumer's benefits are determined by his/her contributions alone. Rather, our model is one of aggregate equity investment by the Trust Fund on behalf of all beneficiaries collectively. Since the level of public debt outstanding is \$3.5 trillion, but the assets of the fund are expected to peak at \$7.5 trillion, it is clear that other assets besides bonds must be purchased by the fund at large and equities are a natural first choice. We thus focus on modeling this aggregate phenomenon and retain the current formulation.

We rule out negative consumption and personal bankruptcy by imposing the following constraints that are easily satisfied under our parameterization:

$$z_{t,1}^{e} \geq 0, \quad z_{t,1}^{b} \geq 0, \quad c_{t,1}^{H} \geq 0, \quad c_{t,1}^{L} \geq 0, \quad c_{t,2}^{H} \geq 0, \text{ and } c_{t,2}^{L} \geq 0. \qquad (2.6)$$

[4]Equations (2.4) and (2.5) suggest that taxes paid by the low-income middle-aged exclusively fund the retirement of the low-income old, and analogously for the high-income middle-aged, as though the low and high-income "accounts" were segregated from one another. This is not the case in the model; the current representation rather is intended to capture the fact that under the present Social Security system, the level of benefits is, at least in part,

We study the equilibrium security prices and the value of the implied Social Security put as we alter the relative mix of $\{\tau_0^e, \tau_0^b, \tau_{H,1}, \tau_{H,1}^e, \tau_{H,1}^b\}$ and $\{\tau_0^e, \tau_0^b, \tau_{L,1}, \tau_{L,1}^e, \tau_{L,1}^b\}$, maintaining constant their respective total values.

A consumer born in period t maximizes expected utility

$$\max E\left[\sum_{i=0}^{2} \beta^i u(c_{t,i}) \middle| \Im_t \right] \tag{2.7}$$

subject to conditions (2.1), (2.2), and (2.4), if of low-income in middle age; or subject to conditions (2.1), (2.3), and (2.5), if of high-income in middle age. $\Im_t$ is the information available to the consumer at date t, β is the subjective discount factor, and the direct utility function is of the standard form $u(c) = (1-\gamma)^{-1} c^{1-\gamma}, \gamma > 0$. It is clear that there are no decisions to be made in the first and final periods of a consumer's life, only the high-income consumers have any decision to make, and then only when middle-aged. Problem (2.7) effectively reduces to a one-period problem.[5]

The aggregate income is given by

$$\tilde{y}_t = w_0 + h\tilde{w}_{H,t} + (1-h)w_{L,t} + w_2 + B + d_t^e. \tag{2.8}$$

Note that B captures the aggregate coupon on government debt.

We assume that there are J, $j = 1,2,...,J$, states of nature with transition matrix $\Pi = \{\pi_{ij}\}$. We model the joint stochastic process of aggregate income, $\tilde{y}_t \equiv y(j)$, and the high middle-aged wage, $\tilde{w}_{H,t} \equiv w(j)$, as a time-stationary Markov chain with a unique stationary probability

determined by the level of prior contributions.

[5]In particular, the consumer is unable while young to insure against the event that he/she will receive a low-income realization when in middle age. The likelihood of this event is thus not present in (2.7), and equilibrium is invariant to whether or not the consumer knows the future income state.

distribution.[6] Since the process is time stationary, it turns out that there exists an equilibrium with time stationary pricing and security demand functions.

The aggregate dividend is the residual

$$d_t^e = \tilde{y}_t - w_0 - h\tilde{w}_{H,t} - (1-h)w_{L,t} - B \tag{2.9}$$

As a function of the state *j*, the aggregate dividend is denoted as $d_t^e = d(j)$.

A number of comments are in order. First, our model identifies the middle-aged consumers with a high wage income as the only cohort that invest in equity and bonds. We acknowledge, however, that private ownership of a large amount of securities need not necessarily be coincident with high wage income. Second, the assumption of a non-stochastic wage income for the non-stockholding class reflects the empirical observation that the consumption of stockholders is more variable than that of non-stockholders; see Mankiw and Zeldes (1991).

The quantities $\tau_{H,1}^e$ and $\tau_{H,1}^b$ are determined by legislative fiat. By the nature of the budget constraint affecting the high-income middle-aged, however, it is possible for that group to negate the effects of these taxes, at least in part, by reducing their private security demands. This observation reflects the occasional criticism that the proposal to allow stock market investments by the Trust Fund will simply allow the well-to-do to substitute one form of tax advantaged investment (401K plans, Keogh plans) for another (Social Security equity investments), with no net increase in savings. More significantly, it also suggests that the equity investment proposal will have aggregate consequences for security markets principally in so far as it undertakes equity investments on behalf of the current non-stockholding class.

Stock market investing by the Trust Fund, with its attendant positive expected returns, should help reduce income inequality among the old. This is a defense sometimes offered on its behalf. Our calibration reflects this feature as well: the degree of income inequality of the old declines with the introduction of Trust Fund stock market investing.

[6] We implicitly interpret d_t^e as the after tax income to corporate capital. McGrattan and Prescott (2000) provide data that suggest that over 90 percent of corporate capital is equity capital; we use the indicated approximation (100 percent of corporate capital is equity). We assume that income taxes exactly cover government expenditures so that neither appear in the economy-wide budget constraint.

II.2 Equilibrium

We consider the set of stationary rational expectations equilibria for which the consumption and investment policies of the consumers born in each period and the bond and equity prices $q^b(j)$ and $q^e(j)$ are measurable with respect to the current state (j) and are such that: (a) each consumer's consumption and investment policy maximizes the consumer's expected utility from the set of admissible policies while taking the price processes as given; and (b) bond and equity markets clear in all periods.

Specifically, let $z_1^e(j)$ and $z_1^b(j)$ denote the holdings of equity and bonds, respectively, by a high-income middle-aged consumer in state j. Let $w_{H,1}(j)$ denote the consumer's wage income. The first-order necessary conditions are also sufficient and are stated below:

$$\begin{aligned}
&u_1(w_{H,1}(j) - z_1^e(j)q^e(j) - z_1^b(j)q^b(j) - \tau_{H,1}^e - \tau_{H,1}^b - \tau_{H,1})q^b(j) \\
&= \beta\sum_{k=1}^{J} u_1((q^e(k) + d^e(k))[z_1^e(j) + (\tau_{H,1}^e + \tau_0^e)/q^e(j)] \\
&\quad + (q^b(k)+1)[z_1^b(j) + (\tau_{H,1}^b + \tau_0^b)/q^b(j)] + w_2 + \tau_0 + \tau_{H,1})[q^b(k)+1]\pi_{jk}
\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}
&u_1(w_{H,1}(j) - z_1^e(j)q^e(j) - z_1^b(j)q^b(j) - \tau_{H,1}^e - \tau_{H,1}^b - \tau_{H,1})q^e(j) \\
&= \beta\sum_{k=1}^{J} u_1((q^e(k) + d^e(k))[z_1^e(j) + (\tau_{H,1}^e + \tau_0^e)/q^e(j)] \\
&\quad + (q^b(k)+1)[z_1^b(j) + (\tau_{H,1}^b + \tau_0^b)/q^b(j)] + w_2 + \tau_0 + \tau_{H,1})[q^e(k)+d^e(k)]\pi_{jk}
\end{aligned} \tag{2.11}$$

The market clearing conditions are

$$B = hz_1^b(j) + h(\tau_{H,1}^b + \tau_0^b)/q^b(j)) + (1-h)(\tau_{L,1}^b + \tau_0^b)/q^b(j)) \tag{2.12}$$

and

$$1 = hz_1^e(j) + h(\tau_{H,1}^e + \tau_0^e)/q^e(j)) + (1-h)(\tau_{L,1}^e + \tau_0^e)/q^e(j)) \tag{2.13}$$

Definition: An equilibrium for the economy described by equations (2.1) – (2.8) is a set of functions $\{q_1^e(j), q_1^b(j), q^e(j), q^b(j)\}$, $j = 1,2,...,J$ which satisfy equations (2.9)–(2.13).

Under standard conditions, equilibrium exists and is easily computable.[7]

III The Value of the Social Security Put Option

We consider the scenario in which high-income middle-aged retirees are guaranteed a level of benefits M_H and low-income middle-aged retirees are guaranteed a level of benefits M_L, where $M_L \le M_H$. We argue that the current system is effectively pay-as-you-go because the assets of the Social Security Trust Fund were \$798 billion on December 31, 2000, while the present value of all liabilities is estimated to be in excess of \$9 trillion. This system is reasonably captured in our scenario by requiring that

$$\tau_0 + \tau_H^1 = M_H \; (\tau_H^e = \tau_H^b = 0)$$

and (3.1)

$$\tau_0 + \tau_L^1 = M_L \; (\tau_L^e = \tau_L^b = 0).$$

The current proposals to privatize Social Security can be broadly classified under two operational definitions of the term "privatize".

Plan 1. Leave the current level of Social Security taxation unchanged while diverting some fraction of the tax revenues to equities and/or long-term bonds. This amounts to requiring

$$\tau_0 + \tau_o^e + \tau_0^b + \tau_{H,1} + \tau_{H,1}^e + \tau_{H,1}^b = M_H$$

and (3.2)

$$\tau_0 + \tau_o^e + \tau_0^b + \tau_{L,1} + \tau_{L,2}^e + \tau_{L,2}^b = M_L,$$

[7]Note that we define (and subsequently compute) steady state equilibria. We do not consider equilibrium transition paths between steady states. Feldstein et al. (2001) and Smetters (2001) examine this issue.

with at least one of $\{\tau_H^e, \tau_H^b\}$ and at least one of $\{\tau_L^e, \tau_L^b\}$ being strictly positive[8]. Under this plan, the resources available for benefits disbursement vary across states. In particular, if the current state is j and the state next period is k, then next period the payments to the formerly high- and formerly low-income retirees are, respectively,

$$\tau_0 + \tau_{H,1} + \frac{(\tau_o^e + \tau_{H,1}^e)}{q^e(j)}[q^e(k) + d^e(k)] + \frac{(\tau_0^b + \tau_{H,1}^b)}{q^b(j)}[q^b(k) + 1]$$

and (3.3)

$$\tau_0 + \tau_{L,1} + \frac{(\tau_o^e + \tau_{L,1}^e)}{q^e(j)}[q^e(k) + d^e(k)] + \frac{(\tau_0^b + \tau_{L,1}^b)}{q^b(j)}[q^b(k) + 1].$$

If the model is calibrated to reflect the high mean equity returns observed in the United States over the past 70 years, in most states the available resources exceed the mandated benefit levels. In the event of a severe downturn in the securities markets, however, the assets of the Trust Fund may fall short of the level necessary to fund promised benefits. This discrepancy would need to be offset with additional taxes. Opponents to privatization emphasize this latter possibility.

For simplicity of presentation, let us temporarily focus on the formerly high-wage old consumers, and let k^* denote a disaster state, if the preceding state is j. The shortfall in state k is

$$\tau_0 + \tau_{H,1} + \frac{(\tau_{H,1}^e + \tau_0^e)}{q^e(j)}[q^e(k^*) + d^e(k^*)] + \frac{(\tau_{H,1}^b + \tau_0^b)}{q^b(j)}[q^b(k^*) + 1] - M_H < 0. \qquad (3.4)$$

A tax surcharge to cover the shortfall is essentially the payoff to a put option written by the young and middle-aged and given to the old, with exercise price $M_H - \tau_0 - \tau_{H,1}$, when the underlying portfolio is $\{(\tau_{H,1}^e + \tau_0^e)/q^e(j), (\tau_{H,2}^b + \tau_0^b)/q^b(j)\}$. From the perspective of the high-income middle-aged (the consumers most likely to have to cover the shortfall), the value of this

[8] At the aggregate level M_H and M_L must be weighted by the respective fraction of the population to which they apply. See the section on calibration, especially equation (4.2).

implicit guarantee is the value of the implied Social Security put.[9] Its value differs across states because different current states give different conditional expectations on income and security price levels next period.

For the high-wage middle-aged consumers, the value in state j of this implied benefit, $V_{SSP}^{H}(j)$, is given by

$$\begin{aligned} V_{SSP}^{H}(j) = \beta \sum_{k=1}^{J} \pi_{jk} \frac{u_1(c_2^H(k))}{u_1(c_1^H(j))} \max\{0, M_H - \tau_0 - \tau_{H,1} \\ - \frac{(\tau_0^e + \tau_{H,1}^e)}{q^e(j)}[q^e(k) + d^e(k)] - \frac{(\tau_0^b + \tau_{H,1}^b)}{q^b(j)}[q^b(k) + 1]\}, \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} c_2^H(k) = [q^e(k) + d^e(k)]\{z_1^e(j) + (\tau_0^e + \tau_{H,1}^e) / q^e(j)\} \\ + [q^b(k) + 1]\{z_1^b(j) + (\tau_0^b + \tau_{H,1}^b) / q^b(j)\} + w_2 + \tau_0 + \tau_{H,1} \end{aligned} \tag{3.5a}$$

and

$$c_1^H(j) = w_H(j) - \tau_{H,1} - \tau_{H,1}^e - \tau_{H,1}^b - z_1^e(j) q^e(j) - z_1^b(j) q^b(j). \text{ [10]} \tag{3.5b}$$

Note that equation (3.5) implies that we undertake a marginal analysis from the perspective of a middle-aged consumer when the put is not yet in place (i.e., his/her $c_1^H(j)$ and $c_2^H(k)$ do not reflect put payments). Since the put increases consumption when the consumer is old and reduces it when the consumer is middle-aged, the put reduces the marginal rate of substitution in the relevant states. Therefore, our computations overstate the true value of the put.

By way of contrast, we also value the Social Security put extended to the formerly low-wage old by the currently low-wage old. If the current state is j, this quantity, $V_{SSP}^{L}(j)$, is given by

[9]We argue that this burden would fall on the high-income consumers because it would most likely be financed by dramatically increasing the range of income subject to Social Security taxes. For low-income consumers, their entire wage income is already subject to the tax.
[10]Note that (3.5a) implies that if SSTF assets should exceed the mandated benefits, the incremental difference is paid

$$
\begin{aligned}
V_{SSP}^{L}(j) = \beta \sum_{k=1}^{J} \pi_{jk} \frac{u_1(c_2^L(k))}{u_1(c_1^L(j))} \max\{0, M_L - \tau_0 - \tau_{L,1} \\
- \frac{(\tau_{L,1}^e + \tau_0^e)}{q^e(j)}(q^e(k) + d^e(k)) - \frac{(\tau_{L,1}^b + \tau_0^b)}{q^b(j)}(q^b(k) + 1)\}
\end{aligned} \tag{3.6}
$$

with $c_2^L(k)$ and $c_1^L(j)$ defined in equations (2.3) and (2.4).[11,12]

Our formulation allows us to value the put for all possible Social Security financing schemes; that is, for all possible combinations of $\{\tau_0, \tau_0^e, \tau_0^b, \tau_{H,1}, \tau_{H,1}^e, \tau_{H,1}^b\}$ and $\{\tau_0, \tau_0^e, \tau_0^b, \tau_{L,1}, \tau_{L,1}^e, \tau_{L,1}^b\}$. It also allows us to see how these alternatives influence equilibrium returns. However, we require that $M^H - \tau_0 - \tau_{H,1} = M^L - \tau_0 - \tau_{L,1}, \tau_L^e = \tau_H^e$ and $\tau_L^b = \tau_H^b$. This standardization has two implications. First, the put extended to the low-income old is valued by the high-income middle-aged identically as their own put. The same is true from the perspective of the low-income middle-aged consumer. Together these identifications allow us to report only two put values for each set of parameters rather than four. The second implication is that the low-income consumers have a higher proportion of their Social Security Trust Fund assets invested in securities. This reduces the inequality of income in old age. We maintain these conventions throughout the paper.

If either of the puts is to have value, there must exist a depression state that the economy can enter into with positive probability. This follows from the fact that the value of taxes invested in equity alone will increase on average by nearly a factor of four over 20 years (the length of our period), if the average annual real return on equity is seven percent. Under typical normality assumptions on returns, the probability of a twenty-year negative return realization would be exceedingly small. We avoid this unrealistically optimistic scenario by introducing the low-probability depression state. There is, however, another perspective on the put which

to the old. To assume otherwise would be to add a 'pseudo-bequest', something we have ruled out previously.

11 Expressions (3.5) and (3.6) would appear to suggest that the high-income middle-aged insure only the high-income old and similarly for the low-income groups. Since the put payoffs are identical in structure, however, if we set $M_L = M_H$, $\tau_{H,1}^e = \tau_{L,1}^e$ and $\tau_{H,1}^b = \tau_{L,1}^b$, expression (3.5) can be used to value a put extended by the high-income middle-aged to the low-income old. In a like fashion we could use (3.6), properly parameterized, to compute the value of a put extended by the low-income middle-aged to the high-income old.

12 Note that in a complete market $V_{SSP}^L(j) = V_{SSP}^H(j)$ if $M_L - \tau_0 - \tau_{L,1} = M_H - \tau_0 - \tau_{H,1}$, our markets are clearly incomplete

obviates the need for introducing a depression state, and one that is perhaps more typical of the current privatization debate.

Plan 2. Rather than distribute a surplus, reduce Social Security taxes. Investing Social Security tax revenues in positive return investments would require, on average, less to be taxed today vis-à-vis the full pay-as-you-go system where the return is implicitly zero.[13] The crucial question here is what mean return we may assume. Given the practical difficulties in forecasting conditional returns, it is reasonable to assume that the government would take the mean unconditional returns under the full pay-as-you-go system as the benchmark. After all, the pay-as-you-go system is the only one for which unconditional historical return data is available on which to base an estimate. Under this plan, the rates $\tau_0^e, \tau_0^b, \tau_{H,1}^e$, and $\tau_{H,1}^b$ are chosen to satisfy

$$E\left[\tau_0 + \tau_{H,1} + (\tau_{H,1}^e + \tau_0^e)(1 + r^e(j,k)) + (\tau_{H,1}^b + \tau_0^b)(1 + r^b(j,k))\right] = M_H, \tag{3.7}$$

and the rates $\tau_0^e, \tau_0^b, \tau_{L,1}^e$ and $\tau_{L,1}^b$ are chosen to satisfy

$$E\left[\tau_0 + \tau_{L,1} + (\tau_{L,1}^e + \tau_0^e)(1 + r^e(j,k)) + (\tau_{L,1}^b + \tau_0^b)(1 + r^b(j,k))\right] = M_L. \tag{3.8}$$

The expectation is the unconditional one under the pure pay-as-you-go system while $r^e(j,k)$ and $r^b(j,k)$ are, respectively, the equilibrium equity and bond returns when the pay-as-you-go economy passes from state j to state k. The two Social Security put values are then computed exactly as in equations (3.5) and (3.6), using equilibrium return data appropriate for the actual level of Trust Fund taxation and securities investment. Under this scenario, it is unnecessary to

because of the "limited participation" phenomena.

[13] This presumes that the new steady state has been achieved. During the transition from the full pay-as-you-go system to the invest-something-in securities regime, however, taxes will have to be increased as the working consumers will have to provide for the then-current old as well as financing security purchases for themselves when they are old in the subsequent period. In our calibration (to follow), we limit ourselves to the case in which a maximum 20 percent of benefits is financed by Trust Fund security purchases. For this scenario, taxes imposed on the middle-aged would have to rise by a maximum of 1 percent of their income during the transition year, assuming that year did not correspond to a disaster state. In the following year, taxes imposed on the middle-aged would experience a one-time decrease of 1 percent. See Feldstein et al. (2001) for a careful analysis of these issues.

introduce a disaster state. In our numerical work, we consider both cases, with and without a disaster state.

IV Calibration

We present results for values of the risk aversion coefficient $\gamma = 4$ and 6. We set the (twenty-year) subjective discount factor at $\beta = .44$, which corresponds to an annualized value of .96, as is typically assumed in business cycle studies. We set the fraction of the population owning stock at $h = .45$, which implies that 30 percent of the population, the high-income middle-aged and the high-income old, have non-trivial investment in the stock market.

The economy-wide state variables are the level of output, $\tilde{y}$, and the wage of the high-income middle-aged, $\tilde{w}_{H,1}$. Calibration is considerably simplified by the observation that equilibrium security prices are linear scale multiples of these wage and income variables and economy wide parameters. This follows from the homogeneity introduced by the constant-relative-risk-aversion preferences and implies that the equilibrium joint probability distribution of the bond and equity *returns* is invariant to the *level* of the exogenous macro-economic variables *for a fixed* $\tilde{w}_H, \tilde{y}$ *joint probability structure*. The scale of the economy is thus irrelevant. Measured as a fraction of the expected output, the value of the Social Security put is also scale invariant. Accordingly, we parameterize the model around the following fundamental ratios:

(i) The average share of income to labor,

$$\frac{E\left[w_0 + h\tilde{w}_{H,t} + (1-h)w_L + w_2\right]}{E[\tilde{y}]} \tag{4.1}$$

For the United States economy, this ratio lies in the range [.66, .75], depending on the historical period and the manner of adjusting capital income. For most cases, we choose a value of .7, but also undertake a sensitivity analysis.

(ii) The coefficient of variation of twenty-year aggregate income, $\sigma(\tilde{y}) / E(\tilde{y})$.

The first major challenge to our calibration exercise is the estimation of this unconditional moment. Unfortunately, a century-long time series provides only five non-overlapping observations, resulting in large standard errors of the point estimates. Standard econometric methods designed to extract more information from the time series, such as the utilization of overlapping observations only marginally increase the effective precision and still leave large standard errors. We thus consider a wide range of potential values in the range $[.10,.30]$.

(iii) The coefficient of variation of twenty-year wage income for the middle-aged,

$$\frac{\sigma(h\tilde{w}_{H,t} + (1-h)w_L)}{E(h\tilde{w}_{H,t} + (1-h)w_L)}.$$

This vital statistic represents another calibration challenge, for the same reasons mentioned above. Ideally, we would like our calibration to reflect the fact that the young experience large idiosyncratic uncertainty in their future labor income, but this is captured only in the most limited way ($\tilde{w}_{H,t}$ versus w_L). Accordingly, we invoke consumer heterogeneity as the justification for being liberal in estimating these moments. In particular, we assume that the coefficient of variation lies in the range $[.10,.25]$ and conduct a sensitivity analysis.

(iv) The average share of income going to interest on government debt, $B / E(\tilde{y})$.

The United States government interest expense in 1999 was $230 billion, which corresponds to 2.5 percent of GDP ($9254 billion). Since our calibration is normalized at $E(\tilde{y}) = 122,000$ in the absence of a disaster state, we choose $B = 3,000$, which matches this statistic almost exactly, for current levels of government debt. Recall that our consol bonds are a proxy for long term debt, and that each bond pays one unit of consumption every period; hence aggregate interest payments are also 3000 and 3000/122,000 ≈ .0247. For this same period, Federal Social Security, Medicare, and other income security payments totaled $818 billion, which represented 8.8 percent of GDP. We match this figure by setting the respective exercise prices (promised benefit levels) at $M_H = 12,000$ and $M_L = 10,000$. The level of benefits expressed as a fraction of national income then works out to be

$$\{(12,000)h + (10,000)(1-h)\} / 122,000 = .089 \tag{4.2}$$

or 8.9 percent, with $h = .45$.

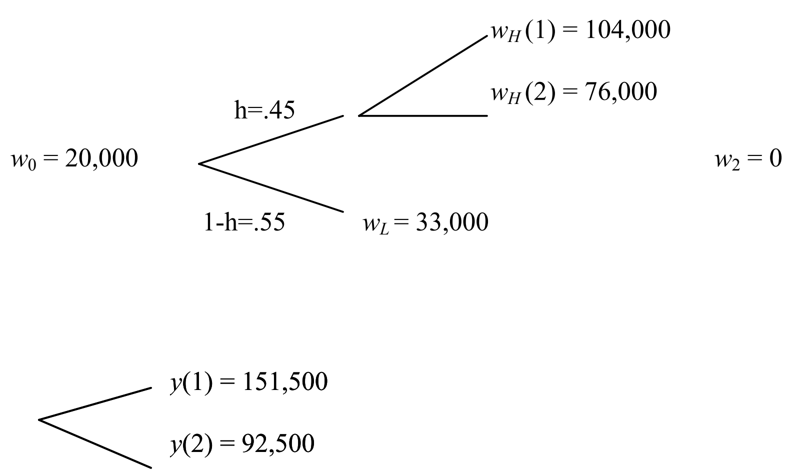
(v) The average share of income going to the labor of the young, $\frac{w_0}{E(\tilde{y})}$.

Somewhat arbitrarily, we fix the income of the young as $w_0 = 20,000$, which corresponds to the ratio $w_0 / E(\tilde{y}) \approx .16$[14].

(vi) The ratio of the average income of the high-income middle-aged to the low-income middle-aged, $E\left[\tilde{w}_{H,t}\right] / w_L$.

This ratio is fixed at 2.25. In order to compute upper bounds for the put values we assume that the old receive no wage income, $w_2 = 0$. Their entire income comes either as Social Security payments, (for those who were low income as middle-aged), or as Social Security payments plus private security holdings (in the case of the high-income middle-aged). Lastly, the ratio of the average income of the high-income middle-aged to the low-income middle-aged was fixed at 2.25.

Our base-case calibration is summarized as follows:



(4.3)

[14] In a private communication John Campbell has pointed out that this figure may be too low. (See Campbell et al. (2001)). Our results, however, are quite insensitive to w_0 as long as w_0 is not high enough that individuals participate in equity markets without borrowing..

For these values, $E\left[w_0 + h\tilde{w}_{H,t} + (1-h)w_L + w_2\right] / E[\tilde{y}] = .695 \approx .7$, $\sigma(\tilde{y}) / E(\tilde{y})$ = .30, and $\sigma(h\tilde{w}_{H,t} + (1-h)w_L) / E\left[h\tilde{w}_{H,t} + (1-h)\, w_L\right] = .18$.

It remains to consider the probability structure. Of special relevance to security pricing are **(vii) the auto-correlation** $corr(\tilde{y}_t, \tilde{y}_{t-1})$**, (viii) the auto-correlation** $corr(\tilde{w}_{H,t}, \tilde{w}_{H,t-1})$**,** and **(ix) the cross-correlation** $corr(\tilde{y}_t, \tilde{w}_{H,t})$**.** Lacking sufficient time-series data to estimate these statistics, we present results for a variety of correlation structures. In particular, we consider four possible structures: $corr(\tilde{y}_t, \tilde{y}_{t-1}) = corr(\tilde{w}_{H,t}, \tilde{w}_{H,t-1}) = .1$ or .8 in conjunction with $corr(\tilde{y}_t, \tilde{w}_{H,t}) =$ .1 or .8.

There are enough degrees of freedom for the above possibilities to be captured by the following 4×4 transition matrix Π:

$$\Pi: \quad \begin{array}{c} \\ (y_1, w_{H,1}) \\ (y_1, w_{H,2}) \\ (y_2, w_{H,1}) \\ (y_2, w_{H,2}) \end{array} \begin{array}{c} \begin{array}{cccc} (y_1, w_{H,1}) & (y_1, w_{H,2}) & (y_2, w_{H,1}) & (y_2, w_{H,2}) \end{array} \\ \begin{bmatrix} \phi & \pi & \sigma & H \\ \pi + \Delta & \phi - \Delta & H & \sigma \\ \sigma & H & \phi - \Delta & \pi + \Delta \\ H & \sigma & \pi & \phi \end{bmatrix} \end{array} \tag{4.4}$$

Given the assumed symmetry of the transition matrix, there are only five parameters to be determined in this matrix: $\phi, \pi, \sigma, \Delta, \text{and } H$, subject to the condition that the row sums equal one, **(x)** $\phi + \pi + \sigma + H = 1$.

In total, there are *twelve* parameters to be determined: the five matrix parameters $\phi, \pi, \sigma, \Delta, \text{and } H$ plus the seven parameters $w_{0,} w_{L,} w_{H,1}, w_{H,2}, y_1, y_2,$ and B. (Recall that the income of the old is set equal to zero, $w_2 = 0$.) The parameters are chosen to satisfy the following *eleven* conditions: the ten target moments, (i)-(x), and the normalization $E[y] =$ 122,000. That leaves one extra degree of freedom that is chosen to ensure that all the elements of the transition matrix are positive. The precise values of $\phi, \pi, \sigma, \Delta, \textit{and } H$ are given below. In the discussion and tables, we uniquely identify each transition structure with the corresponding value of the parameter ϕ.

Parameter Values

Corresponding to Various Correlation Structures

$corr(\tilde{y}_t, \tilde{y}_{t-1})$ and $corr(\tilde{w}_{H,t}, \tilde{w}_{H,t-1})$	$corr(\tilde{y}_t, \tilde{w}_{H,t})$	ϕ	Π	σ	H	Δ
.1	.1	.5298	.0202	.0247	.4253	.01
.1	.8	.8393	.0607	.0742	.0258	.03
.8	.1	.5496	.0004	.0034	.4466	.03
.8	.8	.8996	.0004	.0034	.0966	.03

The preceding probability structure lacks a disaster state. As we have noted under Plan 1, the Social Security put is likely to come into force most significantly in a disaster state comparable to the Great Depression of the 1930s, where output fell to 2/3 of its 1929 level. During such episodes, security prices are likely to be low and to persist in that state. We accommodate the potential for this sort of event by modifying our stochastic process on the output and high-income wage to admit the disaster state, $(y_3, w_{H,3})$, where $y_3 = .66\, y_2$ and $w_{H,3} = .66 w_{H,2}$. The probability transition matrix is modified as

$$\begin{array}{c} \\ (y_1, w_{H,1}) \\ (y_1, w_{H,2}) \\ (y_2, w_{H,1}) \\ (y_2, w_{H,2}) \\ (y_3, w_{H,3}) \end{array} \begin{array}{c} \begin{array}{ccccc} (y_1, w_{H,1}) & (y_1, w_{H,2}) & (y_2, w_{H,1}) & (y_2, w_{H,2}) & (y_3, w_{H,3}) \end{array} \\ \begin{bmatrix} \phi - \eta_1 & \pi & \sigma & H & \eta_1 \\ \pi + \Delta & \phi - \Delta - \eta_2 & H & \sigma & \eta_2 \\ \sigma & H & \phi - \Delta - \eta_3 & \pi + \Delta & \eta_3 \\ H & \sigma & \pi & \phi - \eta_4 & \eta_4 \\ A_1 & A_2 & A_3 & A_4 & 1 - \sum_{j=1}^{4} A_j \end{bmatrix} \end{array} \qquad (4.5)$$

The η_i parameters govern the likelihood of entering the disaster state while the A_i parameters describe the likelihood of exiting from it. The values of the puts are relatively insensitive to the patterns of A_i and η_i, provided they result in the same stationary probability of disaster. This is not surprising as (European-style) put options depend only on the distribution of the states at expiration, rather than the paths to these states. In all our cases, we choose $\{A_i\}$ and $\{\eta_i\}$ such that the stationary probability of the disaster state is approximately ten percent.

In Table 1, we report historical estimates of the mean and standard deviation of the annualized, 20-year holding-period-return on the CRSP value-weighted index of NYSE, AMEX, and NASDAQ stocks; and on the Ibbotson US Government Treasury Long Term bond file. The mean (equity or bond) return is defined as 100 x [{sample mean of the 20-year holding period return}$^{1/20}$ - 1]. The standard deviation of the (equity or bond) return is defined as 100 x [sample std {(20-year holding period return)$^{1/20}$}]. We also report the mean equity premium, defined as the difference of the mean return on equity and the mean return on the bond; and the standard deviation of the premium, defined as 100 x [sample std {(20-year equity return)$^{1/20}$ - (20-year bond return)$^{1/20}$}].

In Table 1 the (real) mean equity return is 5% - 6% with a std of 3% - 4%; the mean bond return is about 1% ; and the mean equity premium is 5% - 6% . (We stress that what we refer to here as the standard deviation of equity return is the std of the *annualized 20-year equity return*, as defined above, and is very different than the standard deviation of the 1-year equity return which is typically of the order of 16% annual).

V **Results**

First, we consider the case $\tau_0^e = \tau_0^b = 0$: the benefits received by the old from security purchases, whether privately or publicly, are exclusively determined by their own contributions as middle-aged consumers. We consider each of the proposed plans in turn, and focus not only on the value of the put, but also on the effects of such policies on the properties of the equilibrium security returns.

V.1 Social Security Plan 1

We present the results for Plan 1 when the SSTF revenues are invested in either equity or bonds.[15] In all cases, the amount invested is 2,000 ($\tau^e_{H,1} = \tau^e_{L,1}$ = 2,000, or $\tau^b_{H,1} = \tau^b_{L,1}$ i.e., $\tau^b_{H,1} = \tau^b_{L,1}$= 2,000), which is approximately 18 percent of SSTF revenues. The latter figure is an upper bound of any proposal that is likely to be accepted in the immediate future. Note that 2000 also represents the maximum possible put payoff at expiration; this is 1.6 percent of the average output in normal times, $E[y]$= 122,000. In this scenario, the source of value for the put is the presence of the disaster state. For all the cases reported, the stationary probability of a disaster state is about ten percent and represents a 45 percent drop in output relative to the mean level.[16]

When the SSTF revenues are invested in stock (Table 2, with $\tau^e_L = \tau^e_H = 2000$, $\tau^b_H = \tau^b_L = 0$), the average value of the high-income put, EV^H_{SSP}, is slightly more than one percent of the expected national income, with a standard deviation of 0.75 percent across all the cases when the risk aversion coefficient is 4. For the low-income consumers, the corresponding values are as high as 2.24 percent of the expected national income, with a standard deviation of 1.34 percent. The results are relatively uniform across all the probability structures because the value of the put depends upon the stationary distribution of income, which is fairly constant across all the cases. Note that the calculations of the put use the marginal rate of substitution (MRS) of the corresponding income groups. If all the calculations use the MRS of the high-income middle-aged, then V^H_{SSP} and V^L_{SSP} coincide in every state.

Since the disaster state is extreme, these figures provide an upper bound on the value of the put. These figures suggest a 95 percent upper bound probability that $V^H_{SSP} \leq 3$ percent and $V^L_{SSP} \leq$ 6 percent. The former figure is more realistic as any shortfall is more likely to be made up with higher taxes on the middle-aged high income population cohort. We have conducted a wide class of comparative dynamics exercises to support these assertions.[17]

The value of the put is drastically reduced if the old generation is guaranteed only 90 percent of the mandated benefits rather than 100 percent of the benefits. The mean value of the

---

[15] Combinations of stock and long-term bond financing lead to put values intermediate between the entries reported here.
[16] We say 'approximately' because the different matrix structures give stationary probability distributions, which differ slightly from one another.
[17] There is another sense in which these estimates are upper bounds. In computing the relevant MRS we do not take

put decreases to a maximum (across all probability structures) of .028 percent, from the perspective of the high-income middle-aged. If we take the viewpoint of the low-income middle-aged consumer, the put declines to roughly $1/10^{th}$ of its full benefits value.

These figures ignore the fact that in states of high security prices the fund would be able to provide benefits in excess of the mandated minimums, a surplus surely available to be carried over to future periods at least in part.[18] This would reduce future put costs. At 10 percent, our stationary probability of disaster is also high; if this probability is reduced to 5 percent, the corresponding mean values of the puts decrease by one half.

When the SSTF revenues are invested in bonds, the mean value of the put for the high-income middle-aged declines relative to equity investment, partly because bonds are less risky. However, for the low-income middle-aged, the opposite is true. This observation reflects the fact that the shift to bond investment results in a lower return (see Table 3). The MRS of the low-income middle-aged is, on average, higher since their old age income is, on average, less than under equity investing. For the low-income middle-aged, this latter effect (which increases put values) dominates the former one (less risky investment vehicle) to create significant increases in V_{SSP}^{L}. For the high-income middle-aged, however, the same effect is much less strong because they have other investing alternatives (in particular, they hold all the equity) for maintaining the smoothness of their income stream.

In Table 3, we present the effect of Plan 1 on equilibrium security returns. For all securities, the mean return is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$ - 1]. The standard deviation of the (equity, bond or consol) return is defined as 100 x [std {(20-year holding period return) $^{1/20}$}]. The mean equity premium return over the bond return, "MEAN PRM/BOND", is defined as the difference between the mean return on equity and the mean return on the bond. The standard deviation of the premium of equity return over the bond return, "STD PRM/BOND", is defined as 100 x [sample std {(20-year equity return)$^{1/20}$ - (20-year bond return)$^{1/20}$}]. The mean premium of equity return over the consol return, "MEAN PRM/CONSOL", and the standard deviation of the premium of equity

into account the implicit smoothing provided by the put.

[18] This surplus could be priced as the corresponding call with the same exercise priced as the put, $M_H - \tau_0 - \tau_{H,1}$ in the case of the high-income middle-aged consumer. We cannot explicitly account for this carry-over, however, as we do not admit a storage technology.

return over the consol return, “STD PRM/CONSOL”, are defined in a similar manner.

Plan 1 increases prices and substantially reduces expected returns and return standard deviations. The effect is greatest on equity returns when the SSTF revenues are invested in equity ($\tau_H^e = \tau_L^e = 2000$) and greatest on bond returns, when the SSTF revenues are invested in bonds ($\tau_H^b = \tau_L^b = 2000$). When the SSTF invests exclusively in equity, in particular, mean equity returns decline by about 3 percent[19], long term bond and risk free returns simultaneously decline by 1.5 percent and .8 percent, respectively. When the SSTF invests exclusively in bonds, the mean risk free rate declines by about 1.4 percent, the mean long-term bond rate by 1.75 percent and the mean equity rate by 2.3 percent. In fact, for the reported parameterization, the real risk free rate becomes negative. In all cases, the premium declines compared to the pay-as-you-go system. The fact that the qualitative impact of the SSTF’s forays into the market is similar irrespective of the investment vehicle chosen reflects the considerable substitutability of equity and bonds. While the magnitude of Trust Fund participation in the financial markets is not large when measured as a percentage of national income, this simple model suggests that its influence on equilibrium returns may be substantial[20]. The counterfactually high level of mean security returns is due largely to the extreme income, dividend, and wage uncertainty we have imposed upon the model. It represents an unattractive feature of seeking upper bound estimates for put values.

The results obtained above are due almost exclusively to the securities market participation by the Fund on behalf of low-income consumers who would otherwise not choose to hold stock or long-term debt. High-income consumers can largely undo the securities purchases made on their behalf by correspondingly reducing their private purchases, a conclusion that is apparent from an inspection of their budget constraint. Low-income consumers benefit from these purchases in that, on an expected basis, they are better off as old persons relative to their situation under pure lump sum taxation. For the parameterizations of Tables 2 and 3, under pure lump sum taxation, the consumption of the low-income old is \$10,000; if $\tau_L^e = 2000$, its expected value is roughly \$26,000 (the result is obviously not true state by state, however). It is on this

[19] Recall in our calibration we are investing only 18% of the SSTF in securities. The decline will be considerably more if 100% of the Trust Fund is invested in securities.

[20] We stress that any privatization scheme explicitly recognize this when estimating the expected rate of return on Trust Fund investments.

basis that some have argued for SSTF stock market participation as a mechanism for reducing old age income inequalities.

Mention should be made of the sensitivity of our results to alternative parameter specifications. First, we consider changes in the CRRA and the results across all the probability structures are essentially the same. In Table 4, we report results for both the high and low correlation cases. If the RRA coefficient increases from $\gamma=4$ to $\gamma=6$, the expected value of the put to the high-income middle-aged, EV_{SSP}^{H}, approximately doubles to two percent of national income; the standard deviation approximately doubles as well. For the low-income middle-aged, the increase is much larger and exceeds ten. The general explanation for these results is straightforward: as consumers become more risk averse, the income insurance represented by the put becomes more valuable to them. The fact that the change is so much greater for the low-income middle-aged is attributable to the higher MRS attendant to their much greater inter-temporal income inequality.

Significant changes are also manifest in the pattern of security returns. In the presence of greater risk aversion, all security returns decline and the risk free rate becomes negative. Consumers that are more risk-averse desire smoother inter-temporal consumption profiles and the only way to accomplish this objective is to demand more securities of both types (their returns being less than perfectly positively correlated allows for some diversification). Therefore, prices increase, but do so differentially, and the equity premium increases. These remarks and those of the preceding paragraph above represent a brief summary of the results presented in Table 4.

Of additional interest is the effect on put values and equilibrium return distributions of changes in the incidence of privately held stockholdings (the parameter h). These are presented in Table 5, in this case of a single representative probability structure. It is seen that as the fraction of the population participating in the securities markets rises, equilibrium returns decline sharply. Simultaneously, the expected value and standard deviation of the put rise. The former result is due, in part, to the enhanced volatility of equilibrium security (underlying asset) returns and, in part, to the lower mean returns which afford less opportunity for consumption smoothing and thus create a higher average MRS for consumers of both income categories.

V.2 Social Security Plan 2

Recall that the idea underlying this plan is for the SSTF to take advantage of the potentially higher returns afforded by stocks (if history repeats itself), by lowering Social Security taxes rather than enhancing benefits. Compared with a pay-as-you-go system (with the implicit zero rate of return), less needs to be set aside today in order to create the required level of benefits in the future.[21] The portion of the Social Security tax devoted to security purchases must have a value equal not to the associated benefits but to their present value. The issue is only the *rate* at which the benefits are to be discounted, recognizing that the very institution of such a policy will have equilibrium effects on the rates themselves.

For the purposes of our calculation, we set the discount rate equal to the prevailing (in the model) rate under the pay-as-you-go system. This choice is based on the following two arguments. First, it reflects 'historical' experience and can be argued on that basis, and (at least in our context) justifies the greatest tax relief. Second, general equilibrium effects are almost surely too subtle to be computed with any confidence in a real world context. By adopting this convention, we maintain our objective of seeking reasonable upper bound estimates for the put values. That we would obtain an upper bound follows from the fact that the public purchase of securities on behalf of the Trust Fund in general serves to lower equilibrium returns vis-à-vis their pay-as-you go levels. The present value of future benefits should thus be higher than we assume, thereby increasing the likelihood the put will be in the money at its expiration.

In Table 6, we present the results of this exercise when we eliminate the disaster state (i.e., $\eta_i \equiv 0 \, \forall_i$, and $A_1 = .5$, $A_2 = .5$, $A_3 = 0$, and $A_4 = 0$) for a representative pair of cases. Under Plan 2, there is no longer a need to retain the possibility of disaster to guarantee that the put has value since the plan at best guarantees only the expected level of benefits and not their level state by state. Subsequently we reintroduce the disaster state. For economy of presentation, only the results for the probability structure corresponding to two correlation structures are presented; the results are similar across the other probability structures.

Both put values are very small—at least from the perspective of the high-income middle-aged. In this case, the value of the put is less than .5 percent of National Income. If it were

[21] For example, Feldstein and Samwick (1997) argue that an 18.75 percent payroll tax rate necessary to sustain promised benefits as the United States population ages could be replaced with a 2 percent tax rate in the long run under all equity investing.

guaranteed by the high-income middle-aged (the most likely scenario), this same estimate would also apply to the put insurance of the low-income elderly. As before, the substantially higher values of V^L_{SSP} are due to the much greater average MRS attendant to their less smooth inter-temporal income profiles under this plan.[22] Although we do not report them, the corresponding values are almost insignificant if the guarantee is again reduced to 90 percent of currently mandated benefits.

Security returns reflect the by-now expected pattern: Trust Fund purchases increase net demand and prices, and lower returns. That our estimates are not too far off the corresponding rational expectations values (that is, where the rates assumed in determining the $\tau^e_H, \tau^e_L, \tau^b_H, \tau^b_L$ values coincide with the actual prevailing rates) is confirmed by the relatively modest decline in equilibrium rates. The fact that rates are in general lower than in Tables 2-4 reflects the reduced income and dividend uncertainty in the absence of a disaster state.

In Table 7, we present the corresponding results when the disaster state is reintroduced. The natural comparison of Plan 1 and Plan 2 can be found by matching the respective panels of Tables 2 and 3 with the corresponding ones in Table 7. Under Plan 2, the V^H_{SSP} rises as high as 2.5 percent of average national income; for V^L_{SSP} it is as high as 30 percent. This is not entirely surprising: under Plan 2, the likelihood of a Social Security shortfall, and, should it occur, the magnitude of the shortfall, are both greater than under Plan 1 because the amounts invested are much lower (227.6 vs. 2000 in the case of equity, 818.25 vs. 2000 in the value of long term bonds for the low correlation scenario). The huge increase in the V^L_{SSP} is due again to the high average MRS value for the low-income middle-aged; their consumption here is much less effectively smoothed than in Tables 2 and 3 above. There is also a difference in the pattern of equilibrium returns across the various financing options. Under Plan 2, all rates decline monotonically as we pass from a full pay-as-you-go to partial equity to partial long term debt financing.

In our search for reasonable upper bounds on the put, there is one more experiment open to us: use the rates for a non-disaster economy to determine the level of Plan 2 investments in a

---

[22] In particular, for the states in which the security returns are negative, the MRS of the low-income middle-aged consumers is much greater than under the pure pay-as-you-go system.

disaster state economy. The idea here is to examine the consequences of the Trust Fund ignoring the possibility of a disaster state.

These results are presented in Table 8 for the representative case. Under equity financing, V_{SSP}^{H} achieves a value of 2.44 percent of average National Income, which is less than the corresponding figure under disaster anticipation (2.49 percent). If long-term bond financing is used, however, V_{SSP}^{L} rises to 33.51 percent of average income, thereby vastly exceeding the 23.65 percent under disaster anticipation (Table 7). Why the conflicting results? Let us consider V_{SSP}^{L} first, as its explanation is most straightforward. There are two effects. First, the equilibrium risk free rate under the disaster scenario (3.17 percent) is lower than under the no disaster one (5.92 percent) since risk free securities are more desirable in the latter environment. The Fund thus anticipates earning a higher return on its investments than actually turns out to be the case. In addition, by anticipating higher rates, the Fund invests less (633.1) than a proper equilibrium analysis would presume (818.5). These two effects reinforce one another to reduce the value of Trust Fund assets and substantially increase the value of the put.

Under equity financing, the analysis is somewhat more complex. Since the amount invested is determined under a no disaster scenario, the assumed rate (10.52 percent) is lower than in the corresponding disaster (11.48 percent) scenario and more is invested (270 vs. 227). Ceteris paribus, this should reduce the value of the put. Working in the opposite direction is the rate effect: the higher equity purchases on behalf of the low-income middle-aged lowers equilibrium returns (10.03 percent) below the disaster level. Thus, rates are not as high, an effect that, ceteris paribus, should increase the value of the put. For this set of cases, the latter effect dominates the former to reduce V_{SSP}^{H} to 2.44 percent of output. The general pattern of security returns does not much differ from what has been presented earlier.

V.3 Extending Security Investing to the Young

A slight generalization of our model can be effected if we admit Social Security investments in stock and risky debt financed by taxes imposed on the young generation as well as the middle-aged. Given the high average returns on these securities it is natural for the Fund to wish to invest as much as possible in these vehicles. To do so breaks the direct connection, however, between a middle-aged consumer's Social Security investments in risky securities and his/her own level of retirement benefits that is present in the current formulation. Rather, under this formulation, both young and old contribute to a general pool of securities, the aggregate value of which in part determines the welfare of the generation that will be old in the subsequent period. Therefore, the young do not directly benefit from their *own* Social Security purchases when they themselves are old.

The results of this exercise are contained in Tables 9 and 10, which are, respectively, the direct analogues of Tables 2 and 3, except that the total Social Security investment in stock and/or debt is split evenly between the young and middle-aged ($1000 to each) holding each generation's (and each middle-aged income category's) aggregate tax payment constant. A quick comparison of the results in Tables 2 and 9 reveals that this modification produces virtually no change in the value of the Social Security put, across the various probability transition matrices. A comparison of Tables 3 and 10 leads to an even stronger result of security returns: they are identical vis-à-vis their statistical summaries.

The intuition behind this finding is straightforward: conditional expected security returns in all states are sufficiently high that the high-income middle-aged do not wish to reduce their private holdings of securities even as the Trust Fund invests on their behalf. In effect, the transfer does not affect overall demand from the perspective of this group of consumers. Similarly, the reduction in exogenous demand by the Trust Fund on behalf of the low-income middle-aged is exactly offset by the corresponding increase for the young. As a result, total demand is unaffected on a state-by-state basis, leaving security return patterns largely unchanged.

V.4 Welfare Comparisons

The welfare implications of the Social Security put critically depend on the consumer's age and income status. The welfare measures are defined in Appendix 1. For all the parameterizations considered in this paper, the high-income middle-aged suffer a loss in welfare if the put program is implemented. This follows from the fact that their income profiles, for all equilibria we consider, are highly upward sloping because of their private purchases of securities. The presence of the Social Security put payment has the effect of making it more so, with the consequent welfare reduction.

For the low-income middle-aged, however, the situation is different. Their age-based income profile, in particular, is downward sloping; for this group the institution of put protection represents real consumption smoothing, and their welfare consequently improves.

From the perspective of a young consumer who is uncertain as to his/her income status when achieving middle age, these two alternatives must be weighted by their respective probabilities. Since the marginal utility, when old, of a low-income middle-aged consumer is so much higher than that of a high-income consumer, the net effect of adding the put is to increase welfare. For a young consumer the percentage increase in welfare, relative to the base no-put case is found in Table 11, Panel A, for the scenario of all equity investing (the case in which the put payments are greatest). Measuring welfare as the expected utility of a young consumer looking forward, the percentage welfare increase when the put is introduced is in the neighborhood of 3.4 percent.

Recognizing the problems inherent in utility comparisons, we also compute the maximum an old consumer would be willing to pay to the young in order to create the put mechanism. As shown in Panel B, this amount represents approximately a one percent increase in young consumption when averaged across our various probability structures.

V.5 Comparison with Earlier Literature

We compare our results with those of Smetters (2001) and Feldstein and Samwick (1997). Smetters (2001) considers the case of complete privatization and fixes the Social Security tax rate such that the assets of the Fund fully cover, on an expected basis, the ex ante promised benefits (the present value of the expected shortfall is zero). When the value of the put is added, however, the reduction in unfunded liabilities is only 21.1 percent (see Smetters (2001), Table 3.1, $\psi = \chi = 1$, $\bar{e} = .07$). Since the current level of unfunded benefits is approximately \$8 trillion, or 80 percent of GDP, this calculation implicitly values the put at 63 percent of GDP.

Our earlier estimate of the value of the put is significantly smaller than that of Smetters (2001) partly because we do not consider complete privatization. We estimate the value of the put under complete privatization by setting parameter values $\tau_0 = \tau_{L,1} = \tau_{H,1} = 0$, $\tau^e_{H,1} = (1.1052)^{-20} 12{,}000 = 1623$, $\tau^e_{L,1} = (1.1052)^{-20} 10{,}000 = 1353$, and $\phi = .5298$ in our Plan 2. Other correlation structures produce similar results. For the parameterization otherwise as in

Table 6, this calculation yields $EV_{SSP}^{H} \approx .03E[y]$, which is still small. If we retain the indicated parameter values, yet admit a disaster state in the manner of Table 7, then $EV_{SSP}^{H} = .19E[y]$, which is almost one third of the Smetters (2001) value[23].

It is also of interest to compare our results with the "two percent rule" propounded in Feldstein and Samwick (1997). These authors demonstrate that a two percent contribution level, when invested in equities, would be able to replace fully the current pay-as-you-go system. In Table 6 (Plan 2) mean return levels are such as to suggest that the corresponding tax levels imposed on the high and low-income middle-aged, respectively, under full equity investing (no-pay-as-you-go or bond components) would be 1623 and 1353 (see calculation above). As a proportion of the average underlying wage for that same case, these figures represent an average tax rate of 2.28 percent, which is broadly consistent with Feldstein and Samwick (1997).

VI Concluding Remarks

If a large fraction of Social Security taxes is invested in equities, there is a distinct possibility that Social Security funds decline in value to a level such that they are inadequate to provide a subsistence level of benefits to the old generation. Since Social Security is a form of social insurance that implicitly provides a guarantee on a minimum consumption level of the older generation, the government may be compelled to remedy a shortfall by raising taxes on the younger working generations. We argue that any time-consistent discussion on privatizing Social Security must take into account this *de facto* role of the younger working generations as insurers of last resort. We price this implicit insurance provided by the younger working generations as a put option on the value of the Social Security Trust Fund with strike price equal to the implicit guaranteed level.

From the perspective of the high-income middle-aged—the group most likely to have to cover shortfalls of the Social Security Trust Fund—the value of the put is estimated to be slightly in excess of one percent of GDP under our most realistic scenario (Plan 1). This corresponds to

[23] This leaves open the question of why we achieve a put valuation of, at best, one-third the Smetters value.

$100 billion, or at most a 25 percent increase in Social Security taxes, if it were to be honored. A put that guarantees only 90 percent of the currently mandated benefits is priced at roughly .03 percent of GDP, which is proportionately much less. We do not regard the cost of even the 100 percent guarantee as insurmountably large. Instituting such privatization policies is further seen to give rise to a substantial increase in security prices with the consequent reduction in returns. It is also seen to substantially reduce income inequality across the old consumers. This latter consequence may ultimately be the greatest argument in its favor.

REFERENCES

Abel, Andrew B. "The Effects of Investing Social Security Funds in the Stock Market When Fixed Costs Prevent Some Households from Holding Stocks." *American Economic Review,* 2001, 91, pp. 128-48.

Abel, Andrew B. "The Social Security Trust Fund, the Riskless Interest Rate, and Capital Accumulation," in John Y. Campbell and Martin Feldstein, eds., *Risk Aspects of Investment-Based Social Security Reform*, Chicago: The University of Chicago Press, 2001, Chapter 5, pp. 153–93.

Bohn, Henning. "Should the Social Security Trust Fund hold Equities? An Intergenerational Welfare Analysis." *Review of Economic Dynamics*, July 1999, 2, pp. 666-97.

Bohn, Henning. "Social Security Reform and Financial Markets," in Steven Sass and Robert Triest, eds., *Social Security Reform*, Boston: Federal Reserve Bank of Boston, 1998, pp. 193-227.

Bohn, Henning. "Risk Sharing in a Stationary Overlapping Generations Economy." Working paper, University of California, Santa Barbara, 1997.

Campbell, John Y., Cocco, Joao F., Gomes, Francisco J., and Maenhout Pascal J. "Investing Retirement Wealth: A Life-Cycle Model" in *Risk Aspects of Investment-Based Social Security Reform*, John Y. Campbell and Martin Feldstein, eds., University of Chicago Press, 2001.

Campbell, John Y., and Feldstein, Martin, eds. *Risk Aspects of Investment-Based Social Security Reform*. University of Chicago Press. 2001.

Constantinides, George M., Donaldson, John B., and Mehra, Rajnish. "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." *Quarterly Journal of Economics*, February 2002, 17, 269-96.

Diamond, Peter A., and Geanakoplos, John. "Social Security Investment in Equities I: Linear Case." Working paper, 2001.

Feldstein, Martin, Ranguelova, M. Elena, and Samwick, Andrew. "The Transition to Investment-Based Social Security When Portfolio Returns and Capital Profitability Are Uncertain" in *Risk Aspects of Investment-Based Social Security Reform*, John Y. Campbell and Martin Feldstein, eds., University of Chicago Press, 2001.

Feldstein, Martin, and Samwick, Andrew. "The Economics of Prefunding Social Security and Medicare Benefits." *NBER Macroeconomics Annual,* 1997, pp. 115-48.

Geanakoplos, John, Mitchell, Olivia S., and Zeldes, Stephen P. "Would a Privatized Social Security System Really Pay a Higher Rate of Return?" in Olivia Mitchell, Robert J. Meyers, and Howard Young, eds., *Prospects for Social Security Reform*, Pension Research Council for the Wharton School of the University of Pennsylvania, 1999.

Kotlifkoff, Lawrence J., Smetters, Kent and Wallises, John. "Social Security: Privatization and Progressivity." *American Economic Review*, May 1998.

Mankiw, N. Gregory, and Zeldes, Stephen P. "The Consumption of Stockholders and Non-Stockholders." *Journal of Financial Economics* 1991, 29, pp. 97-112.

McGrattan, Ellen, and Prescott, Edward C. "Is the Stock Market Overvalued?" *Federal Reserve Bank of Minneapolis Quarterly Review,* 2000, 24, pp. 20-40.

Mitchell, Olivia S., Myers, Robert J., and Young, Howard, eds., Prospects *for Social Security Reform*. University of Pennsylvania Press, 1999.

Mitchell, Olivia, and Zeldes, Stephen P. "Social Security Privatization: A Structure for Analysis." *The American Economic Review,* May 1996, 86 (2), pp. 363-67.

Pennacchi, George. "The Value of Guarantees on Pension Fund Returns." *Journal of Risk and Insurance,* June 1999, 66 ().

Smetters, Kent. "The Effect of Pay-When-Needed Benefit Guarantees on the Impact of Social Security Privatization," in *Risk Aspects of Investment-Based Social Security Reform*, John Y. Campbell and Martin Feldstein, eds., 2001.

Smetters, Kent. "Privatizing Versus Prefunding Social Security." Working paper, University of Pennsylvania, 1998.

Appendix: Definitions Used in the Welfare Analysis

1. Expected utility (welfare) of a representative consumer in the absence of the put:

$$EU(\;)^{\text{No Put}} = u(c_{t,0}) + \beta h\left[\sum_{j=1}^{5}\pi(j)\left\{u(c_{t,1}^{H}(j) + \beta\sum_{k=1}^{5}\pi_{jk}u(c_{t,2}^{H}(k))\right.\right]+$$

$$\beta(1-h)\left[\sum_{j=1}^{5}\pi(j)\left\{u(c_{t,1}^{L}(j)) + \beta\sum_{k=1}^{5}\pi_{jk}u(c_{t,2}^{L}(k))\right.\right],$$ $\pi(j)$ stationary probabilities, j = 1,2

2. Expected utility (welfare) of a middle-aged high-income representative consumer in the absence of the put:

$$EU_{H}^{M}(\;)^{\text{No Put}} = \sum_{j=1}^{5}\pi(j)\left\{u(c_{t,1}^{H}(j) + \beta\sum_{k=1}^{5}\pi_{jk}u(c_{t,2}^{H}(k))\right\}$$

3. Expected utility (welfare) of a middle-aged low-income representative consumer in the absence of the put:

$$EU_{L}^{M}(\;)^{\text{No Put}} = \sum_{j=1}^{5}\pi(j)\left\{u(c_{t,1}^{L}(j)) + \beta\sum_{k=1}^{5}\pi_{jk}u(c_{t,2}^{L}(k))\right\}$$

4. The payoff to the put in state j when the preceding state is ℓ, for the high-income old aged is given by:

$$P^{H}(\ell,j) = \max\left\{0, M_{H} - \tau_{0} - \tau_{H,1} - \left(\frac{\tau_{0}^{e} + \tau_{H,1}^{e}}{q^{e}(\ell)}\right)[q^{e}(j) + d^{e}(j)] - \left(\frac{\tau_{0}^{b} + \tau_{H,1}^{b}}{q^{b}(\ell)}\right)[q^{b}(j)+1]\right\}$$

For the low-income old aged, the analogous quantity, denoted $P^{L}(\ell, j)$, is given by:

$$P^{L}(\ell,j) = \max\left\{0, M_{L} - \tau_{0} - \tau_{L,1} - \left(\frac{\tau_{0}^{e} + \tau_{L,1}^{e}}{q^{e}(\ell)}\right)[q^{e}(j) + d^{e}(j)] - \left(\frac{\tau_{0}^{b} + \tau_{L,1}^{b}}{q^{b}(\ell)}\right)[q^{b}(j)+1]\right\}$$

Accordingly, the introduction of the put modifies consumption for the middle-aged and old of each income type:

$$\hat{c}_{t,1}^{H}(\ell,j) = c_{t,1}^{H}(j) - P^{H}(\ell,j)$$

$$\hat{c}_{t-1,2}^{H}(j,k) = c_{t-1,2}^{H}(k) + P^{H}(j,k)$$

$$\hat{c}_{t,1}^{L}(\ell,j) = c_{t,1}^{L}(j) - P^{L}(\ell,j)$$

$$\hat{c}_{t-1,2}^{L}(j,k) = c_{t-1,2}^{L}(k) + P^{L}(j,k)$$

5. The Expected utility (welfare) of a middle-aged high-income representative consumer in the presence of the put is:

$$Eu_H^M(\ )^{\text{Put}} = \sum_{\ell} \pi(\ell) \left\{ \sum_{j=1}^{5} \pi_{\ell j}\, u\{\hat{c}_{t,1}^H(\ell, j)) + \beta \sum_{k=1}^{5} \pi_{j,k}\, u(\hat{c}_{t,2}^H(j,k)) \right\}$$

The analogous quantity for the middle-aged low-income representative consumer in the presence of the put is:

$$Eu_L^M(\ )^{\text{Put}} = \sum_{\ell} \pi(\ell) \left\{ \sum_{j=1}^{5} \pi_{\ell j} u(\hat{c}_{t,1}^L(\ell, j)) + \beta \sum_{k=1}^{5} \pi_{jk}\, u(\hat{c}_{t,2}^L(j,k)) \right\}$$

6. The expected utility (welfare) of a representative consumer in the presence of the put:

$$Eu(\ )^{\text{Put}} = u(c_{t,0}) + \beta[hEu_H^M(\ )^{\text{Put}} + (1-h)Eu_L^M(\ )^{\text{Put}}]$$

7. The gain in expected utility (welfare) of a representative consumer if the put is incorporated:

$$\text{Gain} \equiv Eu(\ )^{\text{Put}} - Eu(\ )^{\text{No Put}}$$

8. Maximum amount, x_L that would be transferred voluntarily from an old low-income consumer in exchange for introducing the Social Security put:

x_L satisfies $$\sum_{\ell} \pi(\ell) \left\{ \sum_{j=1}^{5} \pi_{\ell,j}\, u(\hat{c}_{t,1}^L(\ell, j)) + \beta \sum_{k=1}^{5} \pi_{jk} u(\hat{c}_{t,2}^L(j,k) - x_L) \right\} = Eu_L^M(\ )^{\text{No Put}}.$$

Note that the transfer x_L is the same across all the states. The analogous quantity for the high-income individuals results in a negative x_H.

TABLE 1

Real Returns   2/1947 - 12/1996

	EQUITY	BOND	PREMIUM
MEAN	5.5%	0.12%	5.38%
STD	3.26%	2.14%	3.33%

Nominal Returns   1/1926 - 12/1996

	EQUITY	BOND	PREMIUM
MEAN	11%	4.02%	6.98%
STD	3.18%	2.68%	4.19%

We report empirical estimates of the mean and standard deviation of the annualized, 20-year holding-period-return on the CRSP value-weighted index of NYSE, AMEX and NASDAQ stocks; and on the Ibbotson US Government Treasury Long Term bond file. The mean return (on equity or the bond) is defined as 100 x [{sample mean of the 20-year holding period return}$^{1/20}$ - 1]. The standard deviation of the (equity or bond) return is defined as 100 x [sample std {(20-year holding period return)$^{1/20}$ }]. The mean premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as 100 x [sample std {(20-year equity return)$^{1/20}$ - (20-year bond return)$^{1/20}$ }]. Estimates on *real* returns cover the sample period 2/1947 - 12/1996, with 358 overlapping observations. Estimates on *nominal* returns cover the sample period 1/1926 - 12/1996, with 611 overlapping observations.

TABLE 2

Social Security Put Valuation: Plan 1

Panel A

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.1				
	LOW SERIAL AUTOCORR. OF y AND OF w^1 (0.1)		HIGH SERIAL AUTOCORR. OF y AND OF w^1 (0.8)	
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$
MEAN V_{SSP}^H	1.0	0.9	1.0	0.9
STD OF V_{SSP}^H	0.8	0.8	0.8	0.8
MEAN V_{SSP}^L	2.2	2.4	2.2	1.9
STD OF V_{SSP}^L	1.3	2.3	1.3	1.8

Panel B

CORRELATION $(\tilde{y}, \tilde{w}_H)$= 0.8				
	LOW SERIAL AUTOCORR. OF y AND OF w^1 (0.1)		HIGH SERIAL AUTOCORR. OF y AND OF w^1 (0.8)	
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$
MEAN V_{SSP}^H	1.1	0.9	1.1	0.9
STD OF V_{SSP}^H	0.8	0.8	0.8	0.9
MEAN V_{SSP}^L	2.2	2.4	2.3	1.9
STD OF V_{SSP}^L	1.3	2.2	1.2	1.8

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.

TABLE 3

Equilibrium Security Returns

Panel A

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.1

	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)		
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$
MEAN r^e	11.5	8.5	9.2	10.9	8.0	8.6
STD OF r^e	9.2	5.8	6.2	9.4	5.9	6.4
MEAN r^b	4.6	3.2	2.7	4.6	3.2	2.7
STD OF r^b	7.8	4.4	3.8	7.6	4.2	3.6
MEAN r^f	0.8	0.0	-0.6	0.9	0.0	-0.6
STD OF r^f	7.4	4.4	3.5	7.4	4.4	3.4
MEAN r^p	10.6	8.5	9.8	10.0	7.9	9.2
STD OF r^p	6.0	5.8	5.6	6.3	6.0	5.8

Panel B

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.8

	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)		
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 2000$
MEAN r^e	11.5	8.6	8.6	11.0	8.0	8.7
STD OF r^e	9.1	5.7	6.4	9.3	5.8	6.2
MEAN r^b	4.6	3.2	2.7	4.6	3.2	2.7
STD OF r^b	7.8	4.4	3.6	7.7	4.2	3.6
MEAN r^f	0.8	0.0	-0.6	0.9	0.0	-0.6
STD OF r^f	7.4	4.4	3.4	7.4	4.4	3.5
MEAN r^p	10.7	8.5	9.2	10.1	8.0	9.3
STD OF r^p	6.0	5.8	5.8	6.2	5.9	5.7

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.
The mean security return is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$ - 1]. The standard deviation of the return is defined as 100 x [std {(20-year holding period return)$^{1/20}$}].

TABLE 4

Value of Put and Security Returns for Different Risk Aversion

CORRELATION $(\tilde{y},\tilde{w}_H)$= 0.1												
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)						HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)					
	$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=0$		$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=2000$		$\tau_H^e=\tau_L^e=0$ $\tau_H^b=\tau_L^b=2000$		$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=0$		$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=2000$		$\tau_H^e=\tau_L^e=0$ $\tau_H^b=\tau_L^b=2000$	
	$\gamma=4$	$\gamma=6$	$\gamma=4$	$\gamma=6$	$\gamma=4$	$\gamma=6$	$\gamma=4$	$\gamma=6$	$\gamma=4$	$\gamma=6$	$\gamma=4$	$\gamma=6$
MEAN V_{SSP}^H	0	0	1.0	2.3	0.9	2.2	0	0	1.1	2.4	0.9	2.3
STD OF V_{SSP}^H	0	0	0.7	1.4	0.8	1.4	0	0	0.8	1.5	0.9	1.4
MEAN V_{SSP}^L	0	0	2.2	24.4	2.4	36.5	0	0	2.3	25.5	1.9	27.3
STD OF V_{SSP}^L	0	0	1.3	13.5	2.3	27.5	0	0	1.2	13.9	1.8	17.3
MEAN r^e	11.5	11.4	8.5	7.9	9.2	8.7	11.0	10.9	8.0	7.4	8.7	8.2
STD OF r^e	9.2	11.1	5.8	5.9	6.2	6.3	9.3	11.0	5.8	5.7	6.2	6.2
MEAN r^b	4.6	4.5	3.2	2.7	2.7	2.2	4.6	4.5	3.2	2.7	2.7	2.2
STD OF r^b	7.8	10.2	4.4	5.3	3.8	4.4	7.7	10.1	4.2	5.1	3.6	4.1
MEAN r^f	0.8	-1.3	0.0	-2.4	-0.6	-2.8	0.9	-1.3	0.0	-2.4	-0.6	-2.8
STD OF r^f	7.4	9.6	4.4	5.2	3.5	3.9	7.4	9.6	4.4	5.2	5.7	3.9
MEAN r^p	10.6	12.7	8.5	10.3	9.8	11.4	10.1	12.1	8.0	9.8	9.3	10.9
STD OF r^p	6.0	6.0	5.8	5.9	5.6	5.6	6.2	5.7	5.9	5.7	5.7	5.3

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.
The mean security return is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$ - 1].The standard deviation of the return is defined as 100 x [std {(20-year holding period return)$^{1/20}$}].

TABLE 5

Security Returns and Participation (h)

CORRELATION $(\tilde{y},\tilde{w}_H)$= 0.1 LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 2000$		
	h = 0.3	h = 0.45	h = 0.5
MEAN V_{SSP}^H	0.7	1.0	1.1
STD OF V_{SSP}^H	0.5	0.8	0.8
MEAN V_{SSP}^L	1.5	2.2	2.3
STD OF V_{SSP}^L	1.0	1.3	1.4
MEAN r^e	11.7	8.5	7.6
STD OF r^e	5.8	5.8	5.6
MEAN r^b	4.0	3.2	2.9
STD OF r^b	4.4	4.4	4.5
MEAN r^f	1.2	0.0	-0.3
STD OF r^f	4.7	4.4	4.3
MEAN r^p	10.5	8.5	7.9
STD OF r^p	6.8	5.8	5.8

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.
The mean security return is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$ - 1].The standard deviation of the return is defined as 100 x [std {(20-year holding period return)$^{1/20}$}].

TABLE 6

Value of Put and Security Returns—Plan 2

CORRELATION $(\tilde{y}, \tilde{w}_H) = 0.1$						
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)		
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 270.5^a$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 633^b$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 210.3^c$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 352.3^d$
MEAN V_{SSP}^H	0.0	0.4	0.2	0.0	0.2	0.1
STD OF V_{SSP}^H	0.0	0.3	0.2	0.0	0.2	0.2
MEAN V_{SSP}^L	0.0	20.0	12.5	0.0	19.0	8.4
STD OF V_{SSP}^L	0.0	10.0	14.2	0.0	18.0	13.3
MEAN r^e	10.5	9.9	9.7	11.9	11.2	11.1
STD OF r^e	5.0	4.8	4.7	4.4	4.2	4.2
MEAN r^b	5.9	5.4	5.2	9.1	8.4	8.2
STD OF r^b	3.0	2.8	2.6	2.0	1.9	1.8
MEAN r^f	6.4	5.9	5.7	9.4	8.8	8.6
STD OF r^f	2.6	2.3	2.2	2.1	2.0	1.9
MEAN r^p	4.2	4.0	4.0	2.5	2.4	2.5
STD OF r^p	4.5	4.4	4.3	2.9	2.8	2.9

We set $\gamma = 4$, $\beta = 0.44$, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in (4.3).The transition matrix is per (4.4) . In both cases $\sigma(y) / E(y) = 0.25$, $\sigma(w_H) / E(w_H) = 0.10$.

[a] $270.5 \approx \dfrac{2000}{(1.1052)^{20}}$

[b] $633 \approx \dfrac{2000}{(1.0592)^{20}}$

[c] $210.3 \approx \dfrac{2000}{(1.1192)^{20}}$

[d] $352.3 \approx \dfrac{2000}{(1.0907)^{20}}$

TABLE 7

Value of Put and Security Returns—Plan 2 and Anticipated Disaster State

	CORRELATION $(\tilde{y},\tilde{w}_H)$ = 0.1					
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)		
	$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=0$	$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=227.6^a$	$\tau_H^e=\tau_L^e=0$ $\tau_H^b=\tau_L^b=818.3^b$	$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=0$	$\tau_H^b=\tau_L^b=0$ $\tau_H^e=\tau_L^e=249.0^c$	$\tau_H^e=\tau_L^e=0$ $\tau_H^b=\tau_L^b=810.5^d$
MEAN V_{SSP}^H	0.0	2.5	1.9	0.0	2.6	2.0
STD OF V_{SSP}^H	0.0	1.3	1.1	0.0	1.4	1.2
MEAN V_{SSP}^L	0.0	29.4	23.7	0.0	30.6	24.1
STD OF V_{SSP}^L	0.0	13.0	15.9	0.0	23.3	12.0
MEAN r^e	11.5	10.1	9.7	11.0	9.5	9.2
STD OF r^e	9.2	7.4	6.8	9.3	7.3	6.8
MEAN r^b	4.6	3.5	3.1	4.6	3.5	3.1
STD OF r^b	7.8	5.9	4.7	7.7	5.6	4.5
MEAN r^f	0.9	-0.5	-0.6	0.9	-0.4	-0.6
STD OF r^f	7.4	5.6	4.3	7.4	5.5	4.4
MEAN r^p	10.6	10.6	10.4	10.1	10.0	9.9
STD OF r^p	6.1	5.9	5.8	6.2	6.0	5.9

We set $\gamma = 4$, $\beta = 0.44$, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.
In both cases $\sigma(y) / E(y) = 0.25$, $\sigma(w_H) / E(w_H) = 0.10$.

(a) $227.6 \approx \frac{2000}{(1.1148)^{20}}$

(b) $818.3 \approx \frac{2000}{(1.0457)^{20}}$

(c) $249.0 \approx \frac{2000}{(1.1098)^{20}}$

(d) $810.5 \approx \frac{2000}{(1.0462)^{20}}$

TABLE 8

Value of Put and Security Returns—Plan 2 and Unanticipated Disaster State.

CORRELATION $(\tilde{y}, \tilde{w}_H) = 0.1$ LOW SERIAL AUTOCORR.OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)			
	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 0$	$\tau_H^b = \tau_L^b = 0$ $\tau_H^e = \tau_L^e = 270.5^a$	$\tau_H^e = \tau_L^e = 0$ $\tau_H^b = \tau_L^b = 633^b$
MEAN V_{SSP}^H	0.0	2.4	2.1
STD OF V_{SSP}^H	0.0	1.3	1.2
MEAN V_{SSP}^L	0.0	26.1	33.5
STD OF V_{SSP}^L	0.0	12.5	18.6
MEAN r^e	11.5	10.0	9.9
STD OF r^e	9.2	7.3	7.0
MEAN r^b	4.6	3.5	3.2
STD OF r^b	7.8	5.8	5.0
MEAN r^f	0.9	-0.5	-0.6
STD OF r^f	7.4	5.5	4.6
MEAN r^p	10.6	10.5	10.5
STD OF r^p	6.0	5.9	5.8

We set $\gamma = 4$, $\beta = 0.44$, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1 = 0.10$, $\eta_i = 0.09$, i=2,3,4 and $A_1 = A_3 = 0.05$, $A_2 = A_4 = 0.35$.
In both cases $\sigma(y) / E(y) = 0.25$, $\sigma(w_H) / E(w_H) = 0.10$.

(a) $270.5 \approx \frac{2000}{(1.1052)^{20}}$

(b) $633 \approx \frac{2000}{(1.0592)^{20}}$

TABLE 9

Value of Put when the Young are Taxed

Panel A

CORRELATION $(\tilde{y},\tilde{w}_H)$= 0.1				
	LOW SERIAL AUTOCORR. OF AND $\tilde{y}$ OF $\tilde{w}_H$ (0.1)		HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)	
	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$
MEAN V_{SSP}^H	1.02	0.86	1.05	0.87
STD OF V_{SSP}^H	0.74	0.76	0.75	0.83
MEAN V_{SSP}^L	2.15	2.42	2.24	1.89
STD OF V_{SSP}^L	1.32	2.25	1.27	1.78

Panel B

CORRELATION $(\tilde{y},\tilde{w}_H)$= 0.8				
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)		HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)	
	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$
MEAN V_{SSP}^H	1.04	0.86	1.07	0.88
STD OF V_{SSP}^H	0.76	0.77	0.77	0.85
MEAN V_{SSP}^L	2.18	2.37	2.26	1.86
STD OF V_{SSP}^L	1.34	2.24	1.24	1.78

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.

TABLE 10

Security Returns when the Young are Taxed

Panel A

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.1				
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)		HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)	
	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$
MEAN r^e	8.5	9.2	8.0	8.6
STD OF r^e	5.8	6.2	5.9	6.4
MEAN r^b	3.2	2.7	3.2	2.7
STD OF r^b	4.4	3.8	4.2	3.6
MEAN r^f	0.0	-0.6	0.0	-0.6
STD OF r^f	4.4	3.5	4.4	3.4
MEAN r^p	8.5	9.8	7.9	9.2
STD OF r^p	5.8	5.6	6.0	5.8

Panel B

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.8				
	LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)		HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)	
	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$	$\tau_0^b = \tau_H^b = \tau_L^b = 0$ $\tau_0^e = \tau_H^e = \tau_L^e = 1000$	$\tau_0^e = \tau_H^e = \tau_L^e = 0$ $\tau_0^b = \tau_H^b = \tau_L^b = 1000$
MEAN r^e	8.6	9.3	8.0	8.7
STD OF r^e	5.7	6.1	5.8	6.2
MEAN r^b	3.2	2.7	3.2	2.7
STD OF r^b	4.4	3.8	4.2	3.6
MEAN r^f	-0.0	-0.6	0.0	0.6
STD OF r^f	4.4	3.5	4.4	3.5
MEAN r^p	8.6	9.9	8.0	9.3
STD OF r^p	5.8	5.6	5.9	5.7

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.

Table 11

Social Security Put Induced Welfare Changes

Panel A

Percentage Increase in Welfare, EU( ) of a Representative
Young Person as a Consequence of Introducing the Social Security Put

CORRELATION $(\tilde{y}, \tilde{w}_H) = 0.1$	
LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)	HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)
3.4%	3.26%

CORRELATION $(\tilde{y}, \tilde{w}_H) = 0.8$	
LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)	HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)
3.5%	3.37%

Panel B

Maximum Voluntary Transfer from Old to Young in Exchange for Introducing the Social Security Put Expressed as a Percentage of Young Income.

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.1	
LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)	HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)
1.105%	0.89%

CORRELATION $(\tilde{y}, \tilde{w}_H)$ = 0.8	
LOW SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.1)	HIGH SERIAL AUTOCORR. OF $\tilde{y}$ AND OF $\tilde{w}_H$ (0.8)
1.073%	0.95%

We set $\gamma = 4$, β =0.44, h=0.45. $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with η_1= 0.10, η_i= 0.09, i=2,3,4 and A_1=A_3= 0.05, A_2=A_4= 0.35.