A NO-ARBITRAGE VECTOR AUTOREGRESSION OF TERM STRUCTURE DYNAMICS WITH MACROECONOMIC AND LATENT VARIABLES

Andrew Ang
Monika Piazzesi

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Andrew Ang and Monika Piazzesi
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ABSTRACT

This paper describes the joint dynamics of bond yields and macroeconomic variables in a Vector Autoregression, where identifying restrictions are based on the absence of arbitrage. Using a term structure model with inflation and economic growth factors, we investigate how macro variables affect bond prices and the dynamics of the yield curve. The setup accommodates higher order autoregressive lags for the macro factors. The macro variables are augmented by traditional unobserved term structure factors. We find that the forecasting performance of a VAR improves when no-arbitrage restrictions are imposed. Models that incorporate macro factors forecast better than traditional term structure models with only unobservable factors. Variance decompositions show that macro factors explain up to 85% of the variation in bond yields. Macro factors primarily explain movements at the short end and middle of the yield curve while unobservable factors still account for most of the movement at the long end of the yield curve.

Andrew Ang
Columbia Business School
3022 Broadway
805 Uris
New York, NY 10027
and NBER
Tel: 212-854-9154
Fax: 212-662-8474
aa610@columbia.edu
http://www.columbia.edu/~aa610

Monika Piazzesi
Anderson Graduate School of Management
UCLA
110 Westwood Plaza, Box 951481
Los Angeles, CA 90095-1481
and NBER
Tel: 310-825-3544
Fax: 310-725-5702
piazzesi@ucla.edu
http://www.personal.anderson.ucla.edu/monika.piazzesi

1   Introduction

Describing the joint behavior of the yield curve and macroeconomic variables is important for bond pricing, investment decisions and public policy. Many term structure models have used latent factor models to explain term structure movements, and although there are some interpretations to what these factors mean, the factors are not given direct comparisons with macroeconomic variables. For example, Pearson and Sun (1994)'s factors are labeled "short rate" and "inflation", but their estimation does not use inflation data. The terms "short rate" and "inflation" are just convenient names for the unobserved factors. Another example is Knez, Litterman and Scheinkman (1994), who call their factors "level," "slope" and "curvature". Similarly, Dai and Singleton (2000) use the words "level," "slope" and "butterfly" to describe their factors. These labels stand for the effect the factors have on the yield curve rather than describing the economic sources of the shocks.

In the absence of a workhorse general equilibrium model for asset pricing (see Hansen and Jagannathan (1991)), factor models have the advantage that they only impose no-arbitrage conditions and not all other conditions that characterize the equilibrium in the economy. Most existing factor models of term structure are unsatisfactory, however, because they do not model how yields directly respond to macroeconomic variables.[1] In contrast, empirical studies try to directly model the relationships between bond yields and macro variables by using Vector Autoregressive (VAR) models. Studies like Estrella and Mishkin (1997) and Evans and Marshall (1998) use VAR's with yields of various maturities together with macro variables. Using the VAR, these studies infer the relationships between yield movements and shocks in macro variables using impulse responses (IR's) and variance decomposition techniques. For example, Evans and Marshall (2000) associate shocks to economic activity and price levels with level effects across the yield curve. Another type of shock which can be identified with various schemes comes from monetary policy (see, for example, Gali (1992), Sims and Zha (1995), Bernanke and Mihov (1995), Christiano, Eichenbaum and Evans (1996a), and Uhlig (1999). For a survey, see Christiano, Eichenbaum and Evans (1998)).

Existing macro VAR studies are characterized by three features. First, only maturities whose yields which have been included in the VAR may have their behavior directly inferred by the dynamics of the VAR. As an unrestricted VAR is generally not a complete theory of the term structure, it says little about how yields of maturities not included in the VAR may move. Second, the implied movements of yields in relation to each other may not rule out arbitrage opportunities when the cross-equation restrictions implied by this assumption are not imposed in the estimation. Finally, unobservable variables cannot be included as all variables in the VAR must

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[1] The exception is Piazzesi (2001), who uses a term structure model with interest-rate targeting by the central bank and releases of macroeconomic variables such as nonfarm payroll employment.

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be observable. The VAR approach, however, is very flexible, and the Impulse Response Functions (IR's) and variance decompositions give insights into the relationships between macro shocks and movements in the yield curve.

A related asset-pricing literature beginning with Sargent (1979) has tried to estimate VAR systems of yields under the null of the Expectations Hypothesis (see Bekaert and Hodrick (2001)). While the validity of the Expectations Hypothesis as a term structure model is still being debated (see Fisher and Gilles (1998) and Longstaff (2000)), this literature has not focused on incorporating macro variables into the VAR. Our approach in this paper specifically focuses on how macro variables affect term structure dynamics, where the term structure dynamics are given by a Gaussian (VAR) term structure model with time-varying risk premia, consistent with deviations from the Expectations Hypothesis (see Fisher (1998) and Dai and Singleton (2001)).

We incorporate macro variables as factors in a term structure model by using a factor representation for the pricing kernel, which prices all bonds in the economy. The pricing kernel depends on the shocks to both observed macro factors and unobserved factors. Since macro factors are correlated with yields, incorporating these factors may lead to models whose forecasts are better than models which omit these factors. We investigate whether the purely unobservable factors of multi-factor term structure models can be explained by macro variables, and we examine how the latent factors change when macro variables are incorporated into such models. Using a pricing kernel with macro factors is a direct and tractable way of modeling how macro factors affect bond prices.

Our methodology gives us several advantages over existing empirical VAR approaches. First, it allows us to characterize the behavior of the entire yield curve in response to macro shocks rather than just the yields included in the VAR. Second, a direct comparison of macro variables with latent yield factors can be made. Third, variance decompositions and other methods can estimate the proportion of term structure movements attributable to observable macro shocks, and other latent variables. Finally, our approach retains the tractability of the VAR approaches because we estimate a VAR subject to nonlinear no-arbitrage restrictions.

The model is a discrete-time multi-factor model with time-varying risk premia. This term structure model is Gaussian, like a VAR model, and IR's and variance decompositions from the model can be easily obtained. Formally, our model is a special case of discrete-time versions of the affine class introduced by Duffie and Kan (1996), where bond prices are exponential affine functions of underlying state variables. In our model, however, some of the state variables are observed macroeconomic aggregates. With Gaussian processes, the affine model reduces to a VAR with cross-equation restrictions. Our set-up accommodates lags and moving average

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errors in the driving factors and allows us to compute variance decompositions where we can attribute the proportion of movements in the yield curve to observable and unobservable factors. We can plot IR's of shocks to various factors on any yield, since the no-arbitrage model gives us bond prices for all maturities.

Our measures of inflation and real activity are obtained from extracting principal components of two groups of variables that are selected to represent measures of price changes and economic growth. These factors are then augmented by latent variables. As term structure studies have suggested up to three latent factors as appropriate to capture most salient features of the yield curve, we estimate models with three latent factors in addition to the macro variables. Our main model has three correlated unobservable factors, together with the two macro factors (inflation and real economic activity).

The cross-equation restrictions from no arbitrage improve out-of-sample forecasts from a VAR. We find that these forecasts can be further improved by incorporating macro factors into models with latent variables. We show that a significant part of the latent factors implied by traditional models with only latent yield variables can be attributed to macro variables. In particular, "slope" and "curvature" factors can be related to macro factors, while the "level" factor survives largely intact when macro variables are incorporated.

We find that macro factors explain a significant amount of the variation in bond yields. Macro factors explain up to 85% of the forecast variance at long horizons at short and medium maturities of the yield curve. The proportion of forecast variance of yields attributable to macro factors decreases at longer yields. At the long end of the yield curve 60% of the forecast variance is attributable to macro factors at a 1-month forecast horizon, while at very long forecast horizons over 60% of the variance is attributable to unobservable factors.

This paper is organized as follows. Section 2 summarizes the data and details the construction of the macro factors as principal components of several macroeconomic variables. Section 3 motivates an affine equation for the short rate, which can be interpreted as a regression of the short rate on macro factors and an 'unexplained' component of short rate movements. This last term consists of one or more unobservable (orthogonal) factors. If the latent component is orthogonal to the macro variables, we can interpret this regression as versions of a Taylor (1993) rule, where the short rate responds to a linear combination of observable factors, and an orthogonal unobserved component. Section 4 presents the general model, discusses the estimation strategy, and describes the specific parameterization of the model to be estimated. We lay out our estimation results in Section 5, and discuss the implied IR's, variance decompositions and forecasting results. Section 6 concludes.

2 Data

2.1 Yield Data

We use data on zero coupon bond yields of maturities 1, 3, 12, 36 and 60 months from January 1952 to December 2000. The bond yields (12, 36 and 60 months) are from the Fama CRSP zero coupon files, while the shorter maturity rates (1 and 3 months) are from the Fama CRSP Treasury Bill files. Figure 1 plots some of these yields in the upper graph and Table 1 presents some sample statistics. The table shows that the data are characterized by some standard stylized facts. The average postwar yield curve is upward sloping; standard deviations of yields generally decrease with maturity; and yields are highly autocorrelated, with increasing autocorrelation at longer maturities.

The yield levels show mild excess kurtosis at short maturities which decreases with maturity, and significantly positive skewness at all maturities. Excess kurtosis is, however, more pronounced for first-differenced yields (for example, 19.44 for the 1-month yield). Although the distribution of yields in the 1990's seems to exhibit Gaussian tails, the evidence for the long series of monthly postwar yields rejects a normal distribution. For our purposes, the Gaussian assumption made in later sections is a sufficient first approximation to the dynamics of the yield curve, as we are mainly interested in the joint dynamics of yields and macroeconomic variables. The Gaussian model we present in Section 4 can be extended to incorporate heteroskedastic dynamics parameterized by discretized square-root processes.

An important stylized fact is that yields of near maturity are extremely correlated - the correlation between the 36-month and 60-month yield is 99%. In our estimations we use all five yields to estimate our models, but we specify that some of the yields are measured with error. We choose the 1, 12 and 60-month yields to be measured without error to represent the short, medium and long ends of the yield curve in our models with 3 unknown factors. (The 3-month yield has a 99% correlation with the 12-month yield, and the 36-month yield has a 99% correlation with the 60-month yield.)

2.2 Macro Variables

We use macro variables that can be sorted in two groups. The first group consists of various inflation measures which are based on the CPI, the PPI of finished goods, and spot market commodity prices (PCOM). The second group contains variables that capture real activity: the index of Help Wanted Advertising in Newspapers (HELP), unemployment (UE), the growth rate of employment (EMPLOY) and the growth rate of industrial production (IP). This list of variables includes most variables that have been used in monthly VAR's in the macro literature.

Among these variables, PCOM and HELP are traditionally thought of as leading indicators of inflation and real activity, respectively. All growth rates (including inflation) are measured as the difference in logs of the index at time t and $t - 12$, t in months.

To reduce the dimensionality of the system, we extract the first principal component of each group of variables separately. That is, we extract the first principal component from the inflation measures group, and we extract the first principal component from the real activity measures group. This leaves us with two variables which we call "inflation" and "real activity". More precisely, we first normalize the three (four) macro variables related to inflation (real activity) to zero mean and unit variance. For each group, the normalized variable vector Z_t can be represented as

$$Z_t = C f_t^o + \epsilon_t, \tag{1}$$

where $Z_t = (\text{CPI}_t\ \text{PPI}_t\ \text{PCOM}_t)$ for the inflation group or $Z_t = (\text{HELP}_t\ \text{UE}_t\ \text{EMPLOY}_t\ \text{IP}_t)$ for the real activity group. The error term ϵ_t satisfies $\text{E}(\epsilon_t) = 0$ and $\text{cov}(\epsilon_t) = \Gamma$, where Γ is diagonal. The matrices C and Γ are either 3×1 or 4×1 for the inflation group and the real activity group respectively. The extracted macro factor f_t^o has mean zero ($\text{E}(f_t^o) = 0$) and unit variance ($\text{var}(f_t^o) = 1$).

Table 2 shows the loadings of the first three (four) principal components, and the factor loadings for using only one principal component to explain the variation in each group. Over 70% (50%) of the variance of nominal variables (real variables) is explained by just the first principal component of the group. The first principal component of the inflation measures loads negatively on CPI, PPI, and PCOM. Since negative shocks to this variable represent positive shocks to inflation, we multiply it by -1 so that we can interpret it as an "inflation" factor. The first principal component of real activity measures loads negatively on HELP, EMPLOY, and IP and positively on UE. Again, we multiply this variable by -1 to interpret positive shocks to this factor as positive shocks to economic growth. We call this factor "real activity". We plot these macro factors in the bottom plot in Figure 1.

To obtain some intuition about these constructed measures of inflation and real activity, Figure 2 plots the inflation and real activity measures versus the actual inflation and real activity series. The top plot of Figure 2 graphs the inflation factor in circles versus the normalized inflation measures CPI, PCOM and PPI. The inflation factor closely tracks CPI and PPI, which roughly move together. The bottom plot of Figure 2 graphs the real activity factor in circles versus the economic growth variables HELP, EMPLOY, IP and the negative of UE. All these series have roughly the same cycles, and the real activity factor most closely corresponds with EMPLOY.

Table 3 displays the correlation between the principal components and the original macro series in each group. These correlations show that the inflation factor is most closely correlated with PPI and CPI (97% and 93% respectively) and less correlated with commodity prices (59%). The real activity factor is most closely correlated with employment growth (91%) and industrial production (87%).

The unconditional correlation between the two macro factors is tiny, one tenth of 1%, as reported in Table 3. Although the unconditional correlation is weak, the lower plot in Figure 1 of the macro factors indicates that some conditional correlations might be important. In fact, when we estimate a VAR for the macro factors, some of the conditional correlations turn out to be significant (they are not reported here). More specifically, we estimate a bivariate process with 12 autoregressive lags for the macro factors:

$$f_t^o = \rho_1 f_{t-1}^o + \ldots + \rho_{12} f_{t-12}^o + \Omega u_{t-1} \tag{2}$$

where ρ_1 to ρ_{12} and Ω are 2×2 matrices with u_t IID N(0,I). The estimation results show that the coefficient on the seventh lag of real activity in the inflation equation is significant and the coefficient on the first two lags of inflation in the equation for real activity are significant. This can also be seen from the impulse responses plotted in Figure 3. The response of inflation to shocks in real activity is positive and hump-shaped, while the response of real activity to inflation shocks is initially weakly positive, and then turns slightly negative before dying out. Since principal components are linear transformations of the data, the skewness, kurtosis and autocorrelation of the macro variables (Table 1) are inherited by the principal components f_t^o.

Some preliminary information about the relationship between the macro factors and the yield curve can be gained from the correlation matrix in Table 3. The inflation factor is highly correlated with yields. This correlation is highest for short yields (67% correlation between inflation and 1-month yields), and somewhat smaller for long yields (56% correlation between inflation and 60-month yields). Real activity is only weakly correlated with yields. This correlation does not exceed 6% for any maturity. This weak relationship is not representative for all measures of real activity. For example, the correlation of HELP and 1-month yields is 63%, but our real activity factor loads mostly on EMPLOY and IP. Hence, at least for measures of economic activity, it may matter whether the particular variable in question is a leading indicator of business cycles. This implies that in our analysis we may potentially understate the impact of real activity on the yield curve by the construction of our real activity factor.

3 A First Look at Short Rate Dynamics

3.1 Policy Rules and Short Rate Dynamics in Affine Models

According to the policy rule recommended by Taylor (1993), movements in the short rate should be traced to movements in contemporaneous macro variables f_t^o and a component which is not explained by macro variables, an orthogonal shock v_t:

$$r_t = a_0 + a_1' f_t^o + v_t \tag{3}$$

The shock v_t may be interpreted as a monetary policy shock following identifying assumptions made in Christiano, Eichenbaum and Evans (1996a,1996b). Taylor's original specification uses two macro variables as factors in f_t^o. The first variable is an annual inflation rate, similar to our inflation factor, and the second variable is the output gap. GDP data are only available at a quarterly frequency, while our real activity factor is constructed using various monthly series such as EMPLOY and IP.

Another type of policy rule that has been proposed by Clarida, Gali, and Gertler (2000) is a forward-looking version of the Taylor rule. According to this rule, the central bank reacts to *expected* inflation and the *expected* output gap. This implies that any variable that forecasts inflation or output will enter the right-hand side of (3). In the hope of capturing the information underlying macro forecasts, we add lagged macro variables as arguments in equation (3).[2] This is done by writing $X_t^o = (f_t^{o'} \ f_{t-1}^{o'} \ \dots, f_{t-p-1}^{o'})'$ for some lag length p and including the lags as arguments in the policy rule:

$$r_t = b_0 + b_1' X_t^o + v_t. \tag{4}$$

Affine term structure models (Duffie and Kan (1996)) are based on a short rate equation just like equation (3) together with an assumption on risk premia. The difference between the short rate dynamics in affine term structure models and the Taylor rule is that in affine term structure models the short rate is specified to be an affine (constant plus linear term) function of underlying latent factors X_t^u:

$$r_t = c_0 + c_1' X_t^u. \tag{5}$$

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[2] Clarida, Gali, and Gertler (2000) implement their forward-looking rule by redefining the shock term v_t to include forecast errors $f_{t+1}^o - \mathrm{E}_t(f_{t+1}^o)$. This allows them to use future values of macro variables f_{t+1}^o as arguments on the right-hand side of (3). We could in principle adopt the same approach by including these forecast errors into some latent variables, but this would mean that we would have to drop the assumption that latent and macro variables are orthogonal. Our focus is assigning as much explanatory power to macro factors as possible, so we specify the latent variables as orthogonal.

The unobserved factors themselves follow affine processes, of which a VAR is a special Gaussian case. The prices of bonds of longer maturities are explicit exponential affine functions (dependent upon parameters) of f_t^u if pricing is risk neutral. In the more general case that we consider, the risk adjustment needs to be specified carefully to obtain similar closed-form solutions for bond yields (this is explained in the next section). With or without risk adjustment, equation (5) is always an important determinant of the shape of the entire yield curve in affine term structure models.

Equations (3) through (5) are very similar: they all specify the short rate as affine functions of factors. We can combine them by writing:

$$r_t = \delta_0 + \delta_{11}' X_t^o + \delta_{12}' X_t^u, \tag{6}$$

The approach we take in this paper is to specify the latent factors X_t^u as orthogonal to the macro factors X_t^o. In this case, the short rate dynamics of the term structure model can be interpreted as a version of the Taylor rule with the errors $v_t = \delta_{12}' X_t^u$ being unobserved factors. We use the restrictions from no-arbitrage to separately identify the individual unobserved factors.

3.2   Estimating the Short Rate Dynamics

The short rate equation (6) can be estimated by ordinary least squares because of the independence assumption on X_t^o and X_t^u. Table 4 reports the estimation results from two regressions: the original Taylor rule (3) and the forward-looking version of the Taylor rule (4), which incorporates lags of the macro variables. These regression results give a preliminary view as to how much of the yield movements macro factors may explain with respect to the unobservable variables. The R^2 of the estimated Taylor rule is 45%, while the estimated forward-looking version of the Taylor raises the R^2 to 53%. These numbers suggest that macro factors should have explanatory power for yield curve movements.

The behavior of the residuals, however, provides some intuition about what to expect from a model with unobservable factors. First, the residuals from both versions of the Taylor rule are highly autocorrelated. The autocorrelation of residuals from the short rate equation with only contemporaneous macro factors is 0.945, while the autocorrelation from the equation that includes lagged macro factors is slightly lower, 0.937. The short rate itself has an autocorrelation of 0.972, indicating that macro variables do explain some of the persistent shocks to the short rate. Second, unless a variable which mimics the short rate itself is placed on the RHS of equation (3), the residuals will follow the same broad pattern as the short rate. This can be seen from Figure 4, which plots the residuals together with the de-meaned short rate. This suggests that the "level" factor found by earliest term structure studies (see Vasicek (1977)), may still

reappear when macro variables are added in a linear form to the short rate in a term structure model.

The coefficients on inflation and real activity in the simple Taylor rule are both significant and positive. This is consistent with previous estimates of the Taylor rule in the literature, and also the parameter values proposed by Taylor (1993)'s original specification. However, these coefficients are highly sensitive to the sample period selected. In particular, the sign of the Taylor-rule coefficient on real activity crucially depends on the inclusion of the two recessions in 1954 and 1958. This is evident from the plots of real activity and the 1-month yield in Figure 1. There are two major reductions in output around May 1954 and April 1958, which also correspond to NBER recessions. Both these recessions go hand in hand with decreases in the 1-month rate. These two recessions make the Taylor rule coefficient on output positive. If we start the estimation of the Taylor rule later, say in 1960 or 1970, the coefficient on real activity is negative. Only if we start the estimation after the monetary experiment of 1982 is the coefficient positive. Interestingly enough, the coefficient on output is not significant for the whole post-1982 period, but it is significant for the Greenspan years (post-1987). In contrast, Table 4 reports that most parameter estimates for the forward-looking version of the Taylor rule are not significant, except for the 11th lag on the inflation rate and current real activity. This suggests that using many lags in the Taylor rule may lead to an over-parameterized and potentially poorly behaved system.

4　A Term Structure Model with Macro Factors

Based on the macro dynamics (2) and the short rate equation (6), we now develop a discrete-time term structure model. The model combines observable macroeconomic variables with unobservable or latent factors. Risk premia in our set-up are time-varying, because they are taken to be affine in potentially all of the underlying factors. Section 4.1 presents the general model and Section 4.2 parameterizes the latent variables and risk premia. We outline our estimation procedure in Section 4.3. Section 4.4 summarizes our parameterization.

4.1　General Setup

4.1.1　State Dynamics

Suppose there are K_1 observable macro variables f_t^o and K_2 latent variables f_t^u. The vector $F_t = (f_t^{o'}, f_t^{u'})'$ is assumed to follow a Gaussian VAR(p) process:

$$F_t = \Phi_0 + \Phi_1 F_{t-1} + \ldots + \Phi_p F_{t-p} + \theta_0 u_t \tag{7}$$

with $u_t \sim$ IID $N(0, I)$. The state of the economy is described by a K vector of state variables X_t. We partition the state vector X_t into observable variables X_t^o and unobservable variables X_t^u. The observable vector contains current and past levels of macroeconomic variables $X_t^o = (f_t^{o'}\ f_{t-1}^{o'}\ \dots, f_{t-p}^{o'})'$, while $X_t^u = f_t^u$ contains latent yield factors. We take the bivariate VAR(12) in equation (2) as the process for inflation and real activity so set $p = 12$. Moving average terms can be accommodated by including q lagged error terms $\theta_1 u_{t-1}, \dots, \theta_q u_{t-q}$ on the right hand side of (7). In this case, the vector of observable state variables also includes past innovations to the macro variables $X_t^o = (f_t^{o'}\ f_{t-1}^{o'}\ \dots, f_{t-p}^{o'}\ u_{t-1}^{o'}\ \dots u_{t-q}^{o'})'$.[3]

We write the dynamics of $X_t = ((X_t^o)'\ (X_t^u)')'$ in compact form as a first order Gaussian VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t \tag{8}$$

with $\epsilon_t = (u_t'\ 0\ \dots\ 0)'$. In the first order companion form, there are blocks of zeros in the $K \times K$ matrix Σ to accommodate higher order lags in F_t.

4.1.2 Short Rate Equation

The one-period short rate r_t is assumed to be an affine function of all state variables:

$$r_t = \delta_0 + \delta_1' X_t. \tag{9}$$

We work with monthly data, so that we can use the one-month yield y_t^1 as an observable short rate r_t. By constraining the coefficient δ_1 to depend only on contemporaneous factor values, we can obtain strict versions of the Taylor rule (3). We call this the "Macro Model." We also consider the case where δ_1 is unconstrained, which correspond to the forward-looking Taylor rule incorporating lags. We refer to this formulation as the "Macro Lag Model," because it uses lags of macro variables in the short rate equation.

Structural changes (or regime shifts) in the economy may cause the relationships in the factor dynamics (8) and the short rate equation (9) to change over time (Ang and Bekaert (1998)). We will assume that during our sample period, these relationships are stable, just as in Gali (1992), Christiano, Eichenbaum and Evans (1996a) and Cochrane (1998).

4.1.3 Pricing Kernel

To develop the term structure model, we use the assumption of no-arbitrage (Harrison and Kreps (1979)) to guarantee the existence of an equivalent martingale measure (or risk-neutral measure) Q such that the price of any asset V_t that does not pay any dividends at time $t + 1$ satisfies $V_t =$

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[3] In the case of one lag $p = 1$ and no MA components $q = 0$, then $K = K_1 + K_2$.

$E_t^Q(\exp(-r_t)V_{t+1})$, where the expectation is taken under the measure Q. The Radon-Nikodym derivative (which converts the risk-neutral measure to the data-generating measure) is denoted by ξ_{t+1}. Thus, for any $t+1$ random variable Z_{t+1} we have that $E_t^Q(Z_{t+1}) = E_t(\xi_{t+1}Z_{t+1})/\xi_t$. The assumption of no-arbitrage, or equivalently the assumption of the existence of ξ_{t+1}, allows us to price any asset in the economy, in particular all nominal bond prices.

Assume that ξ_{t+1} follows the log-normal process:

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{10}$$

where λ_t are the time-varying the market prices of risk associated with the sources of uncertainty ϵ_t. We parameterize λ_t as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \tag{11}$$

for a K-dimensional vector λ_0 and a $K \times K$ matrix λ_1. This specification has been used by Constantinides (1992), El Karoui, Myneni and Viswanathan (1992), and Liu (1999), among many others. Fisher (1998) and Dai and Singleton (2001) argue that this specification can explain deviations from the Expectations Hypothesis. Equations (10) and (11) relate shocks in the underlying state variables (macro and latent factors) to ξ_{t+1} and therefore determine how factor shocks affect all yields.

We define the pricing kernel m_{t+1} as:

$$m_{t+1} = \exp(-r_t)\,\xi_{t+1}/\xi_t. \tag{12}$$

Substituting $r_t = \delta_0 + \delta_1' X_t$ we have:

$$m_{t+1} = \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \delta_0 - \delta_1' X_t - \lambda_t'\epsilon_{t+1}\right) \tag{13}$$

4.1.4   Bond Prices

We take equation (13) to be a nominal pricing kernel which prices all nominal assets in the economy. This means that the total gross return process R_{t+1} of any nominal asset satisfies:

$$E_t(m_{t+1}R_{t+1}) = 1. \tag{14}$$

If p_t^n represents the price of an n-period zero coupon bond, then equation (14) allows bond prices to be computed recursively by:

$$p_t^{n+1} = E_t(m_{t+1}p_{t+1}^n). \tag{15}$$

The state dynamics of X_t (equation (8)) together with the dynamics of the short rate r_t (equation (9)) and the Radon-Nikodym derivative (equation (10)) form a discrete-time Gaussian K-factor model with K_1 observable factors and K_2 unobservable factors. It falls within the affine class of term structure models because bond prices are exponential affine functions of the state variables. More precisely, bond prices are given by:

$$p_t^n = \exp(\bar{A}_n + \bar{B}_n' X_t), \qquad (16)$$

where the coefficients $\bar{A}_n$ and $\bar{B}_n$ follow the difference equations:

$$\bar{A}_{n+1} = \bar{A}_n + \bar{B}_n'(\mu - \Sigma'\lambda_0) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n - \delta_0$$
$$\bar{B}_{n+1}' = \bar{B}_n'(\phi - \Sigma\lambda_1) - \delta_1' \qquad (17)$$

with $\bar{A}_1 = -\delta_0$ and $\bar{B}_1 = -\delta_1$.

These difference equations can be derived by induction using equation (15).[4] For a one-period bond, $n = 1$, we have:

$$\begin{aligned} p_t^1 &= \mathrm{E}_t[m_{t+1}] = \exp\{-r_t\} \\ &= \exp\{-\delta_0 - \delta_1' X_t\}. \end{aligned}$$

Matching coefficients leads to $\bar{A}_1 = -\delta_0$ and $\bar{B}_1 = -\delta_1$. Suppose that the price of an n-period bond is given by $p_t^n = \exp(A_n + B_n X_t)$. Now we show that the exponential form also applies to the price of the n period bond:

$$\begin{aligned} p_t^{n+1} &= \mathrm{E}_t[m_{t+1}p_t^n] \\ &= \exp\left\{-r_t - \frac{1}{2}\lambda_t'\lambda_t + A_n\right\}\mathrm{E}_t[\exp\{-\lambda_t'\epsilon_{t+1} + B_n'X_{t+1}\}] \\ &= \exp\left\{-r_t - \frac{1}{2}\lambda_t'\lambda_t + A_n\right\}\mathrm{E}_t[\exp\{-\lambda_t'\epsilon_{t+1} + B_n'(\mu + \phi X_t + \Sigma\epsilon_{t+1})\}] \\ &= \exp\left\{A_n - \delta_0 + B_n'\mu + (B_n'\phi - \delta_1')X_t - \frac{1}{2}\lambda_t'\lambda_t\right\}\mathrm{E}_t[\exp\{-(\lambda_t' + B_n'\Sigma)\epsilon_{t+1}\}] \\ &= \exp\left\{-\delta_0 + A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n - \delta_1'X_t + B_n'\phi X_t - B_n'\Sigma\lambda_1 X_t\right\} \end{aligned}$$

Matching coefficients results in the recursive relations in equations (17).

The continuously compounded yield y_t^n on an n-period zero coupon bond is given by:

$$\begin{aligned} y_t^n &= -\frac{\log p_t^n}{n} \\ &= A_n + B_n'X_t \qquad (18) \end{aligned}$$

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[4] See the techniques in Campbell, Lo and MacKinlay (1997), Bekaert and Grenadier (2001) and Backus, Foresi, Mozumdar and Wu (2001).

where $A_n = -\bar{A}_n/n$ and $B_n = -\bar{B}_n/n$. Note that yields are affine functions of the state X_t, so that equation (18) can be interpreted as being the observation equation of a state space system. Additional observation equations will come from the observable variables X_t^o. Most examples of discrete-time affine models have not incorporated lagged state variables or moving average errors. However, by treating both the lagged variables and moving average errors as state variables in X_t, the affine form is still maintained. Despite time-varying risk premia, our system is still Gaussian, and IR's, variance decompositions and other techniques can be handled as easily as an unrestricted VAR.

4.2   Choice of Parameterization

4.2.1   Latent Variables

Empirical studies have concluded that three unobserved factors explain much of yield dynamics (see Knez, Litterman and Scheinkman (1994)). To compare models with only latent variables with models incorporating both latent and macro factors we use three unobservable factors. Hence our most comprehensive model consists of two macro and three latent factors.

Since there are unobservable variables present, normalizations can be made that give observationally equivalent systems. The idea behind these normalizations in a VAR setting is that affine transformations and rotations of the unobservable factors lead to observationally equivalent yields. These normalizations are discussed in detail in Dai and Singleton (2000). We estimate the most general parameterization for the unobserved variables in this paper, and then re-estimate the system a second time while setting any insignificant parameters to zero. This is more efficient, ensures identification, allows comparison across models, and gives sufficient freedom to capture yield curve dynamics.

We estimate the following system for the unobservable factors:

$$f_t^u = \rho f_{t-1}^u + u_t^u, \tag{19}$$

where u_t^u IID N(0,1) and the 3×3 companion matrix ρ is lower triangular. This is the most general identified representation for a Gaussian specification. A multi-factor Vasicek (1977) model with correlated unobservable factors consists of (19), an affine short rate equation (5), and the assumption that $\lambda_1 = 0$. In a Vasicek model, specifying the companion form and holding fixed the covariances is equivalent to holding the companion form fixed and specifying the covariances. As the latent factors are AR(1) processes, the coefficients $\Phi_2 \ldots \Phi_p$, in equation (7) corresponding to $X_t^u = f_t^u$ are zero. Numerous papers in the term structure literature have used independent factors as a first-cut modeling approach, including Longstaff and Schwartz (1992) and Chen and Scott (1993). At the estimated parameters, however, the latent factors usually

turn out to violate the independence assumption. We therefore estimate a correlated latent factor model to give the latent variables a fair chance to explain the yield curve by themselves, without the inclusion of macro variables.

We impose independence between latent and macro factors, so that the upper-right 24×3 corner and the lower-left 3×24 corner of Φ in the compact form in (8) contains only zeros. This approach to including observed macro factors in a pricing kernel specifies all uncertainties arising in the latent factors as orthogonal to the macro variables and can answer how yields respond to pure macro shocks. However, by assuming independence of latent and macro factors we cannot ask how macro factors respond to latent yield factors. This contradicts empirical evidence that the term structure predicts movements in macro economic activity (see Harvey (1988) and Estrella and Hardouvelis (1991)). Extensions of this model where this question can be addressed can be done by freeing up the companion matrix to allow feedback (so Φ does not contain zero corner blocks), and looking at contemporaneous correlations of macro and latent factors (θ_0 does not contain zero corner blocks). We leave extensions that free up correlations between factors for future research and focus on the impact of macro variables on yields.

4.2.2 Risk Premia

The data-generating and the risk neutral measures coincide if $\lambda_t = 0$ for all t. This case is usually called the "Local Expectations Hypothesis," which differs from the traditional Expectations Hypothesis by Jensen inequality terms (see Cochrane (2001), Chapter 19). Macro models, such as Fuhrer and Moore (1995), usually impose the Expectations Hypothesis to infer long term yield dynamics from short rates. The dynamics of the term structure in the real measure depend on the risk premia parameters λ_0 and λ_1 in equation (11). A non-zero vector λ_0 affects the long-run mean of yields because this parameter affects the constant term in the yield equation (18). A non-zero matrix λ_1 affects the time-variation of risk-premia, since it affects the slope coefficients in the yield equation (18). In a Vasicek (1977) model λ_0 is non-zero and λ_1 is zero, which allows the average yield curve to be upward sloping, but does not allow risk premia to be time-varying.

Estimating prices of risk is difficult. Many estimations of term structure models with latent factors cannot reject the hypothesis that the market prices of risk are zero (for example, see the low t-statistics in Dai and Singleton (2000)). Parameter estimates of λ_0 and λ_1 are therefore interesting in themselves, because the evidence against the expectations hypothesis is still being debated. Although there is strong traditional evidence against it (Campbell and Shiller (1991)), newer evidence finds the expectations hypothesis much harder to reject in international data (Hardouvelis (1994)), or taking into account small-sample biases (see Bekaert and Hodrick (2001) and Bekaert, Hodrick, and Marshall (2001)).

The number of λ parameters to estimate is very large: λ_0 has 5 and λ_1 has 25 parameters in the case of the models with macro variables. To avoid over-fitting, we fix some of these parameters before estimation. First, we set any λ_1 parameters corresponding to the latent variables to zero in estimations of models with macro variables, if they were already insignificant in the Yields-Only estimation. We also specify the λ_1 matrix to be block-diagonal, with zero restrictions on the upper-right and lower-left corner blocks. This assumption is in the spirit of trying to let the macro variables characterize as much of the term structure dynamics as possible without resorting to orthogonal latent variables. Finally, we set all λ_1 parameters corresponding to lagged macro variables to zero. This leaves two non-zero matrices on the diagonal of λ_1 to estimate: a 2×2 matrix for current macro variables and a 3×3 matrix for the latent variables.

4.3 Estimation Method

To estimate the model, we transform a system of yields and observables (Y_t, X_t^o) into a system of observables and unobservables $X_t = (X_t^o, X_t^u)$. The yields themselves are analytical functions of the state variables X_t, which allow us to infer the unobservable factors from the yields. The estimation method is maximum likelihood, and we derive the likelihood function in the Appendix. In traditional VAR approaches, yields and macro variables are used directly as inputs into a VAR after specifying the autoregressive lag length. The likelihood for the VAR is a function of (Y_t, X_t^o), and inferences about yield curve movements and macro shocks can be drawn from the parameters in the companion form coefficients and covariance terms. Our approach amounts to estimating a VAR of (Y_t, X_t^o), with assumptions that (i) identify an unobservable component orthogonal to macro shocks and (ii) guarantee no arbitrage.

We use a two-step consistent estimation procedure. In the first step, we estimate the macro dynamics (2) and the coefficients δ_0 and δ_{11} of the macro factors in the short rate dynamics equation (6). In a second step, we estimate the remaining parameters of the term structure model holding all pre-estimated parameters fixed. One reason to do this is the difficulties associated with estimating a large number of factors simultaneously with maximum likelihood when yields are highly persistent.[5] This procedure also avoids the estimation of a large number of lag coefficients $(\rho_1, \ldots \rho_{12})$ in the bivariate VAR for the macro variables by maximizing a computationally intensive likelihood function.

Both the macro dynamics (2) and the short rate coefficients of the macro variables in equation (6) are estimated by ordinary least squares, as reported in Sections 2 and 3. Since our

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[5] We tried to estimate various versions of the model in a single step with maximum likelihood. These estimations typically produced explosive yield dynamics. Fixing the parameters that characterize the dependence of the short rate on the observable factors in a (consistent) first-step estimation turned out to be a tractable way to avoid the problem of nonstationary dynamics.

constructed macro factors have zero mean and unit variance, the constant δ_0 in the short rate equation represents the unconditional mean of the 1 month yield, which equals 5.10% on an annualized basis. This number has to be divided by 12 to obtain an estimate for δ_0 at a monthly frequency. The regression coefficients δ_{11} of the short rate equation give the maximal proportion of short rate movements explained by the macro factors, with all remaining orthogonal factors being unobservable. We use the term structure model to identify the unexplained proportion.

Holding δ_0, δ_{11}, and the parameters entering equation (2) fixed, we estimate all other parameters of the term structure model including the remaining coefficients in δ_{12} corresponding to the latent factors in the term structure estimation. To obtain starting values for ρ in equation (19) we estimate the model with λ_0 and λ_1 equal to zero. We then estimate λ_1 still holding λ_0 fixed at zero. In the next estimation round, we estimate λ_0 while setting parameters in λ_1 that turned out to be insignificantly different from zero. We finally set insignificant λ_0 parameters to zero and re-estimate. This method gives more efficient estimates than a one-step estimation under the null that the insignificant parameters are equal to zero.

Finally, our likelihood construction solves for the unobservable factors from the joint dynamics of the zero coupon bond yields and the macro factors. To do this, we follow Chen and Scott (1993) and assume that as many yields as unobservable factors are measured without error, and the remaining yields are measured with error. In particular, for our models with three latent factors we assume the 1, 12 and 60-month yields are measured without error, and the 3 and 36-month yields are measured with error.

4.4 Summary of Parameterization

To summarize, we estimate the following special case of the general model. The bivariate system of macro factors f_t^o follows the process:

$$f_t^o = \rho_1 f_{t-1}^o + \ldots + \rho_{12} f_{t-12}^o + \Omega u_t, \tag{20}$$

with u_t IID N(0,I). The 2×2 matrices $\rho_1, \ldots, \rho_{12}, \Omega$ consist of free parameters.

The trivariate system of latent factors f_t^u follows the process:

$$f_t^u = \rho f_{t-1}^u + \epsilon_t \tag{21}$$

with ϵ_t IID N(0,1). The 3×3 matrix ρ is lower triangular to ensure identification. The shock processes ϵ_t and u_t are independent.

The short rate equation is:

$$r_t = \delta_0 + \delta_{11}' X_t^o + \delta_{12}' X_t^u, \tag{22}$$

where the parameters δ_0 and δ_{11} are consistently estimated by least squares in a first-step procedure prior to maximizing the likelihood. The observable factors are $X_t^o = (f_t^{o\prime} \ f_{t-1}^{o\prime} \ \ldots, f_{t-p}^{o\prime})$ and the latent factors are $X_t^u = f_t^u$. The full set of state variables is $X_t = ((X_t^o)', (X_t^u)')'$.

Market prices of risk are affine in the state vector:

$$\lambda_t = \lambda_0 + \lambda_1 X_t. \tag{23}$$

The matrix λ_1 has an upper-left 2×2 matrix and a lower-right 3×3 matrix corresponding to f_t^o and f_t^u, while the remaining parameters are set to zero. The parameters in λ_0 corresponding to f_t^o and f_t^u are free, and all remaining parameters are restricted to be zero.

Our most comprehensive model contains two macro factors and three unobservable factors, which we denote as the "Macro Lag" model. The estimation of δ_{11} that restricts the parameters on lagged parameters to be zero as in equation (22) is denoted the "Macro" model. The estimation without any macro variables we call the "Yields-Only" model.

5 Estimation Results

Section 5.1 interprets the parameter estimates of the Macro and Yields-Only models. To determine the effect of the addition of macro factors into term structure models, we look at the IR's of each factor in Section 5.2. The variance decompositions in Section 5.3 allow us to attribute the forecast variance at a particular horizon to shocks in macro and latent factors. We compare the latent factors from the different models in Section 5.4 and find that macro factors do account for some of the latent factors from the Yields-Only model. In Section 5.5, we find that imposing the cross-equation restrictions from no-arbitrage forecasts better than the unrestricted VAR's common in the macro literature. Moreover, incorporating macro variables into a term structure model helps us obtain even better forecasts. Derivations for the IR's and variance decompositions are presented in the Appendix.

5.1 Parameter Estimates

5.1.1 Yields-Only Model

Table 5 presents the estimation results for the Yields-Only Model. The order of the latent factors in Table 5 is unspecified, but we present the estimation results by ordering the latent factors by decreasing autocorrelation. The model has one very persistent factor, one less persistent but still very strongly persistent factor, and the last factor is strongly mean-reverting. This is consistent with previous multi-factor estimates in the literature such as Chen and Scott (1993).

These unobservable factors have been labeled "level," "slope," and "curvature" respectively because of the effects of these factors on the yield curve. In Figure 5 we plot the normalized factors against three normalized transformations of the 1 month, 12 month and 60 month yields (y_t^1, y_t^{12} and y_t^{60} respectively). The first latent variable, Unobs 1, closely corresponds to a "level" effect, which is defined as $1/3(y_t^1 + y_t^{12} + y_t^{60})$. The correlation between Unobs 1 and the level transformation is 92%. The second latent variable, Unobs 2, closely corresponds to a "spread" transformation, defined as $y_t^{60} - y_t^1$. Unobs 2 and the spread have a correlation of 58%. Finally, the third latent variable, Unobs 3, is related to a "curvature" transformation, defined as $y_t^1 - 2y_t^{12} + y_t^{60}$. Unobs 3 has a 77% correlation with curvature.[6]

In Table 5, the estimated vector λ_0 has one significant parameter corresponding to the most highly autocorrelated factor. The parameter is negative, so that the unconditional mean of the short rate under the risk-neutral measure is higher than under the data-generating measure. Since bond prices are computed under the risk-neutral measure, negative parameters in λ_0 induce long yields to be on average higher than short yields. Time-variation in risk premia is mainly driven by the first and third unobservable factor. In other words, risk premia in bond yields mainly depend on the level and the curvature of the yield curve.

5.1.2 Models with Yields and Macro Variables

Tables 6 and 7 contain estimation results of the Macro Model and the Macro Lag Model. The autocorrelations of the unobservable factors are comparable across all models, with the exception of Unobs 3. The autocorrelation of Unobs 3 is approximately the same in the Yields-Only and Macro Model (0.7646 and 0.7728 respectively) but is more persistent in the Macro Lag Model (0.8210). The δ_1 coefficients corresponding to latent factors are also approximately the same across the three models.

Turning to the risk premia parameters in Tables 6 and 7, the λ_1 coefficients corresponding to inflation and real activity are significant. This implies that time-variation in risk premia significantly depends on observable macro factors. However, the estimates of the λ_1 risk parameters differ across the Macro and Macro Lag Model. In particular, the inflation-real activity cross-terms ($\lambda_{1,12}$ and $\lambda_{1,21}$), where the additional two subscripts denote matrix elements, are much larger in absolute magnitude in the Macro Model than in the Macro Lag Model. Similarly, the inflation and real activity diagonal terms ($\lambda_{1,11}$ and $\lambda_{1,22}$) are smaller in the Macro Model estimation than in the Macro Lag Model. This implies that the behavior of inflation and real activity on the term structure may be potentially quite different across the Macro and Macro

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[6] For comparison, the standard Knez, Litterman and Scheinkman (1994) three principal components of the 1 month, 12 month and 60 month yields have correlations 100%, 99.8% and 88.6% with the level, spread and curvature yield transformations.

Lag Models. We confirm this in the following sections where we examine IR's and variance decompositions. In contrast, the λ_1 parameters corresponding to the latent factors are roughly similar across the Macro and Macro Lag Models, with the exception of the Unobs 3 diagonal term ($\lambda_{1,55}$).

5.2 Impulse Responses

5.2.1 Factor Weights Across the Yield Curve

From equation (18), the effect of each factor on the yield curve is determined by the weights B_n that the term structure model assigns on each yield of maturity n. These weights B_n also represent the initial response of yields to movements in the various factors. Figure 6 plots these weights as a function of yield maturity for the Yields-Only model in the upper graph, and the Macro Model in the lower graph. The B_n coefficients have been scaled to correspond to movements of one standard deviation of the factors, and have been annualized by multiplying by 1200.

Turning first to the Yields-Only model, the weight on the most persistent factor (Unobs 1) is almost horizontal. This means that it affects yields of all maturities the same way, so we can call it a level factor. The coefficient of the second factor (Unobs 2) is upward sloping. It mainly moves the short end of the yield curve relative to the long end, so Unobs 2 is therefore a slope factor. The coefficient on the least persistent factor (Unobs 3) is hump-shaped. Movements in this factor affect yields at the short-end of the yield curve and middle and long-end of the yield curve with different signs. Hence, the B_n weights corresponding to Unobs 3 have a twisting effect, so Unobs 3 is thus a curvature factor. The inverse hump in the coefficient of this factor cannot be accommodated in a model with independent factors and constant risk premia, where yield coefficients are monotone functions of maturity.

The corresponding coefficients of the Macro model in the lower plot in Figure 6 look very similar. We again find that Unobs 1 though 3 represent level, slope and curvature factors. We find the same correspondence in the B_n coefficients of the Macro Lag model (which we do not graph here). The B_n coefficients corresponding to inflation and real activity we represent as stars and circles, respectively. The effect of inflation is hump-shaped but mostly affects short yields and less so long yields. The magnitude of the inflation weights are higher than the level factor weights at short maturities, and about half the magnitude of the slope factor weights. Real activity has a much weaker hump-shaped effect on the yield curve. This suggests that macro factors have much explanatory power for yield curve dynamics. To trace out the long-term responses of the yield curve from shocks to the macro variables after the yield curve's initial response, we now compute IR's.

5.2.2 Impulse Responses from Macro Shocks

We look at IR's to yields of maturities 1, 12 and 60 months. Our term structure model allows us to obtain the movements of the yield curve in response to driving shocks at all horizons, including maturities omitted in estimation. The IR's for *all* maturities are known analytical functions of the parameters. This is in contrast to estimations with VAR's where IR's can only be calculated for yields included in the VAR. Our estimation also guarantees that the movements of yields are arbitrage-free.

Figure 7 shows IR's of 1, 12 and 60 month yields from the Macro model and the Macro Lag model. In addition, we calculate the IR's from a simple unrestricted VAR(12), with macro factors and 5 yields similar to Campbell and Ammer (1993). We order the variables with macro factors first, and then yields with increasing maturities. The x-axis on each plot is in months and the IR's are given in terms of annualized percentages for a shock of one standard deviation.

In all models, shocks to inflation and real activity raise the yield curve across all maturities, with a larger response to inflation shocks. The IR's of macro shocks are hump-shaped in all models. The hump in the unrestricted response to real activity shocks occurs after one year, while the hump in the response to inflation shocks occurs later, after two years. The magnitudes and the exact location of the humps differ across models.

Turning first to the IR's of the unrestricted VAR in the first column of Figure 7, a one-standard deviation shock to inflation initially raises the 1-month yield about 10 basis points. The response peaks after about two years at 30 basis points and then slowly levels off. The response of longer yields has the same overall shape. The initial response of the 1-year yield (5-year yield) is only 8 basis points (5 basis points). The response increases to around 25 basis points (22 basis points) after two years, and then dies off slowly. The response of yields to real activity shocks in the unrestricted VAR is slightly smaller than the response to inflation shocks. The response is again hump-shaped with the hump occurring after one year. The unrestricted response of the 1-month yield to a 1 standard deviation output shock is around 15 basis points initially. The response increases to 30 basis points after two years, and then dies off. The initial responses of the 1-year yield (5-year yield) is 15 (10) basis points. The response increases to 25 (18) basis points after one year and then dies off.

The last two columns of Figure 7 list IR's in the Macro and Macro Lag models. The hump-shape of the IR's are similar to the shape of the IR's from the unrestricted VAR, but the IR's are much larger. For example, the initial response of the 1-year yield to a 1 standard deviation inflation shock is 50 basis points in the Macro model. The response then increases to almost 1 percentage point after one year, where it peaks. The hump in the IR to inflation shocks in the Macro Lag model occurs later, after 2 years, and is therefore similar to the pattern in the

unrestricted response. The IR's from the Macro Lag model, however, are more erratic than those of the Macro model. This is because the contemporaneous and 11 lags of the short rate equation contain many insignificant parameters, which cause poor behavior due to over-parameterization.

Our results are different from the empirical VAR studies such as Evans and Marshall (2000). They associate shocks to price levels and economic growth as having a level-shift effect across the yield curve. We find that shocks to these factors generally affect the short yield end of the yield curve more than the long end of the yield curve. The structure in Evans and Marshall (2000) is to construct VAR's for yields with macro variables and a monetary policy proxy (the federal funds rate). In our model, most of the movement in the yield curve at long horizons is explained by unobserved factors, whereas in the VAR most movements in the term structure are unexplained and are absorbed by the VAR as covariance error. When estimating the model, the first unobservable factor mimics the level of the term structure because this is the linear combination that looks like the first principal component of yields, which the macro factors cannot replicate. This factor is very persistent, and affects the long end of the yield curve more than the short end.

5.3   Variance Decompositions

To gauge the relative contributions of the macro and latent factors to forecast variances we construct variance decompositions. These show the proportion of the forecast variance attributable to each factor, and are closely related to the IR's of Section 5.2. Table 8 summarizes our results. The proportion of unconditional variance accounted for by macro factors is decreasing with the maturity of yields: highest at the short and middle-ends of the yield curve, and smallest for the long-end. The largest effect is on the 1-month yield where macro factors account for 83% of the unconditional variance (where the forecasting horizon is infinite). The proportion of forecast variance explained by macro factors displays an interesting pattern for different parts of the yield curve. For short and intermediate maturities, this pattern is hump-shaped. For example, macro factors account for 50% of the 1-step ahead forecast variance of the 1-month yield. This percentage rises to 78% at 12 months and 85% at 60 months, but then converges to 83% for extremely long horizons. For long yields, the explanatory power of the macro variables decreases with forecasting horizon. Macro variables only account for 40% of the unconditional variance of long yields, while the rest is attributed to latent factors. The low variance decomposition of long yields is due to the dominance of persistent unobserved factors (the near unit-root factor). Overall, Table 8 shows that the macro factors explain a large amount of term structure movements, particularly at the short and middle parts of the yield curve.

More detailed variance decompositions are listed in Table 9 for 1, 12 and 60 month maturities. To interpret the top row of Table 9, for the Yields-Only model, 13.81% of the 1-step ahead

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forecast variance of the 1-month yield is explained by the first unobserved factor, 33.03% by the second unobserved factor and 53.14% by the third unobserved factor. In the row labeled $h = 1$ of the Macro Model in the first panel corresponding to the 1-month yield, 48.87% of the 1-step ahead forecast variance is attributable to inflation, 0.74% to real activity and the remainder to the latent factors.

Focusing on the Macro Model, inflation has more explanatory power for forecast variances than real activity at all points of the yield curve and for all forecast horizons. For example, at the 12-month horizon 68.60% (9.46%) of the forecast variance of the 1-month yield is accounted for by inflation (economic growth). The explanatory power of real activity generally rises with the forecast interval h. At the long end of the yield curve the explanatory power of inflation decreases with h. Inflation and real activity remain important in the Macro Lag Model, but the proportion attributable to macro variables is much smaller for the 60 month yield than the Macro Model.

Turning now to the latent factors in Table 9, Unobs 1, corresponding to the first highly persistent latent factor, dominates the variance decompositions for all the yields at long horizons. Its importance increases for yields with long maturity. This effect mirrors the flat B_n yield factor weights in Figure 6. The second unobserved factor (Unobs 2) has greatest effect on short-run flucations of yields with intermediate maturities. The third unobserved factor (Unobs 3) is the strongly mean-reverting factor and acts only on the short end of the yield curve. In Table 9 it accounts for 53% of the forecast variance for the 1-month yield at a one month horizon, but has little effect on longer yields. These patterns are mirrored in both the Yields-Only model and models with macro variables, but with different magnitudes. These variance decompositions suggest that the role of the "level" factor (Unobs 1), "butterfly" factor (Unobs 2) and "slope" factor (Unobs 3) remain roughly the same with the addition of macro factors. The next section seeks to quantify the change in the behavior of these unobserved factors in the presence of macro variables.

5.4   Comparison of Factors

The addition of macro factors into a term structure model is shown quantatively in Table 10. In this table we regress the latent factors from the Yields-Only model onto the macro and latent factors from the Macro and Macro Lag Models. We run three series of regressions, first only on the macro variables (Panel A), and then onto the macro and latent variables of the Macro Model (Panel B), and then onto the macro and latent variables of the Macro Lag Model (Panel C). All the variables in the regressions are normalized.

Turning first to Panel A of Table 10, the traditional level factor loads significantly onto

inflation and real activity, with an adjusted R^2 of 22%. In particular, the loading on inflation is positive and large (0.46). This suggests that the traditional level factor captures a strong inflation effect. When the second latent factor, labeled "slope," is regressed onto the macro factors, we obtain a high R^2 of 49%, with significant negative loadings particularly on inflation (-0.67). Hence, much of the traditional slope factor is also related to the dynamics of inflation. Finally, the third latent factor ("curvature") is poorly accounted by macro factors $R^2 = 3\%$. However, the traditional curvature factor does load significantly onto real activity.

Panel B of Table 10 reports the regression from the traditional Yields-Only factors onto the macro and latent factors implied by the Macro Model. The level factor from the Yields-Only model translates almost one for one with the level factor of the Macro Model. The magnitude of the coefficient on Unobs 1 of the Macro model is very close to 1, and the full regression produces an R^2 of 99%. However, the loadings on the macro factors are significant suggesting that macro variables do account for some of the level factor.

The reason why the level factor survives almost intact when macro factors are introduced is because the level factor proxies for the first principal component of the yield curve, as shown in Figure 5. The unobservable factors are linear combinations of the yields, and the best linear combination of yields which explains term structure movements is the first principal component. When macro factors are added, these factors still do not resemble the level of the yield curve, and so this factor is still necessary to explain the movements across the term structure.

When we regress the Yields-Only slope factor (Unobs 2) onto the Macro Model factors the loading of the Unobs 2 factor from the Macro Model is significantly smaller than 1, while the coefficient on inflation is very large and negative, and the coefficient on real activity is also significant. This means that a large part of the traditional slope factor can be attributed to inflation movements. In particular, when inflation is high, the slope narrows because the short rate increases relative to the long rate. Turning finally to the regression of the Yields-Only curvature factor (Unobs 3), this regression still has a significant negative coefficient on real activity, but most of the correspondence is with the Unobs 3 factor from the Macro Model (the coefficient is 0.91).

Panel C of Table 10 reports the regression coefficients of the latent factors from the Yields-Only model onto the macro and latent factors of the Macro Lag Model. We see that the level effect again survives almost one for one and there is still a large loading on the inflation factor by the Yields-Only model's Unobs 2. However, the R^2's of the Unobs 2 and Unobs 3 regressions are much smaller than the Macro Model regressions in Panel B.

In summary, Table 10 shows that the traditional level and slope factors are markedly associated with and accounted by observable macro factors. In particular, inflation accounts for

large amounts of the dynamics of the traditional slope factor. However, the level effect survives almost intact when macro factors are added to a term structure model.

5.5 Forecasts

The variance decompositions hint that term structure models with observable macro variables may help in forecasting future movements in yields. However, these are statements based on assuming a particular model as the true model after estimation, and may not hold in a practical setting where more parsimonious data representations often out-perform sophisticated models, particularly if these more sophisticated models are over-parameterized. To determine if this is actually the case we conduct an out-of-sample forecasting experiment.

Our procedure for examining out-of-sample forecasts over the last 5 years of our sample is as follows. We examine forecasts for all the five yields used in estimation. At each date t, we estimate the models using data up to and including time t, and then forecast the next month's yields at time $t + 1$. The macro factor data is formed using the principal components of the macro data up to time t, and we estimate the short rate equation and the bivariate VAR of the macro dynamics only using data up to time t for the Macro and Macro Lag Models. Hence, we only use data available in the information set at time t in making the forecast at time $t + 1$.

We perform a comparison of out-of-sample forecasts for six models. First, we use a simple random walk. Second, we investigate out-of-sample forecasts for the corresponding VAR(12)'s which do not impose cross-equation restrictions. Our first VAR uses only yields, and we use a second VAR which incorporates yields and macro variables. Our last three models are the Yields-Only model, the Macro model and the Macro Lag model. We use two criteria to compare our forecasts across the models. The first is the Root Mean Squared Error, RMSE, of actual and forecasted yields, and the second is the Mean Absolute Deviation, MAD.

Table 11 lists the results of the out-of-sample comparisons. Lower RMSE and MAD values denote better forecasts. We note the following points regarding the forecasting performance of the models. First, a random walk easily beats an unconstrained VAR. The result holds independently of whether the VAR's only contain yields, or are augmented with macro variables. The bad performance is due to the high persistence of yields and small sample biases in the estimation of autoregressive coefficients in over-parameterized VAR's.

Second, imposing the cross-equation restrictions from no-arbitrage helps in forecasting. The improvement in forecasting performance is substantial, generally about 25% of the RMSE and 30% of the MAD for all yields. These constrained VAR's perform in line with, and slightly better, than a random walk (except for the 3-month yield). Duffee (2001) remarks that beating

a random walk with a traditional affine term structure model is difficult. From forecasting exercises without risk premia (not reported here), we know that this result crucially depends on the type of risk adjustment. Linear risk premia, not considered by Duffee (2001), seem to do well in this regard.

Third, the forecasts of the Macro model are far better than those of the Macro Lag model. While the forecasts of the Macro Lag model are comparable to those of unconstrained VAR's, the Macro model slightly outperforms a random walk (except for the 3-month yield). Both the Macro model and the Macro Lag model impose cross-equation restrictions on a VAR with yields and macro variables. The Macro Lag model, however, has a large number of insignificant coefficients entering the short rate equation. This over-parameterization causes poor out-of-sample performance.

Finally, incorporating macro variables helps in forecasting. More precisely, the forecasts of the Macro model are uniformly better than the Yields-Only model (except for the 3 month yield). Hence, we can conclude that (i) adding term structure restrictions improves forecasts relative to unconstrained VAR's, even beating a random walk, and (ii) forecasts can be further improved by including macro variables. Note, however, that we have shown this improvement is only in incrementally adding macro factors to a given number of latent factors.

6   Conclusion

This paper presents a Gaussian model of the yield curve with observable macroeconomic variables and traditional latent yield variables. The model takes a first step towards understanding the joint dynamics of macro variables and bond prices in a factor model of the term structure. Risk premia are time-varying; they depend on both observable macro variables and unobservable factors. Our approach extends the existing empirical VAR work by imposing no-arbitrage assumptions which allow identification of unobservable factors, and allows the movements of the entire yield curve to be derived consistent with no-arbitrage.

We find that macro factors explain a significant portion (up to 85%) of movements in the short and middle parts of the yield curve, but explain only around 40% of movements at the long end of the yield curve. The effects of inflation shocks are strongest at the short end of the yield curve. Comparing the latent factors from traditional three latent factor models of term structure, the "level" factor survives almost intact when macro factors are incorporated, but a significant proportion of the "level" and "slope" factors are attributed to macro factors, particularly to inflation. Incorporating macro factors in a term structure model also improves out-of-sample forecasts.

In future research, we plan to extend our empirical specification to allow non-diagonal terms in the companion form for the factors. This introduces correlations among the latent factors and feedback from latent factors to macro variables. This allows inference of how latent yield factors drive macro variables, along the lines of Estrella and Hardouvelis (1991) but with the dynamics of the yield curve modeled in a no-arbitrage pricing approach.

Appendix

A   Likelihood Function

We have data on an N vector of zero coupon yields Y_t. Our approach to estimation is to solve for the unobserved factors f_t^u from the yields Y_t and the observed variables X_t^o, which includes observed macro variables f_t^o and latent variables f_t^u and lagged terms of the driving factors.

Suppose first that we have $N = K_2$ yields of different maturity $n_1, \ldots, n_{K_2}$, as many yields as we have unobserved factors, f_t^u. Stacking the equations for the K_2 yields, with $Y_t = (y_t^{n_1} \ldots y_t^{n_{K_2}})'$, we can write:

$$Y_t = A + B X_t, \tag{A-1}$$

where A is K_2x1 and B is K_2xK. Partition the matrix B into $B = [B^o \, B^u]$ where B^o is a $K_2 \times (K - K_2)$ matrix which picks up the observable factors and B^u is a K_2xK_2 invertible matrix that picks up the unobservable factors. Then we can infer the unobservable factors in $X_t^u \equiv f_t^u$ from Y_t and the pricing matrices A and B using an inversion from the equation:

$$Y_t = A + B^o X_t^o + B^u X_t^u. \tag{A-2}$$

However, the term structure model will only price exactly the yields used to invert the latent factors. To increase the number of yields to $N \geq K_2$ in the estimation, we follow Chen and Scott (1993), and others, in assuming that some of the yields are observed with measurement error. There will be K_2 yields from which we invert to obtain the latent variables, and the other $N - K_2$ yields are measured with error. We assume this measurement error is IID, and the measurement error is uncorrelated across the yields measured with error. Let B^m denote a $N \times (N\text{-}K_2)$ measurement matrix and u_t^m be an $(N\text{-}K_2)$-dimensional Gaussian white noise with a diagonal covariance matrix independent of X_t. We can then write:

$$Y_t = A + B^o X_t^o + B^u X_t^u + B^m u_t^m. \tag{A-3}$$

In equation (A-3) the yields measured without error will be used to solve for X_t^u, and the yields measured with error have non-zero u_t^m. For a given parameter vector $\theta = (\mu, \Phi, \Sigma, \delta_0, \delta_1, \lambda_0, \lambda_1)$, we can invert equation (A-3) to obtain X_t^u and u_t^m. The variance of the measurement error in our estimations are very small and choosing different bonds to be measured without error do not affect our results.

Denoting the normal density functions of the state variables X_t and the errors u_t^m as f_X and f_{u^m} respectively, the joint likelihood $\mathfrak{L}(\theta)$ of the observed data on zero coupon yields Y_t and the observable factors X_t^o is given by:

$$\mathfrak{L}(\theta)) = \prod_{t=2}^T f(Y_t, X_t^o | Y_{t-1}, X_{t-1}^o)$$

$$\log(\mathfrak{L}(\theta)) = \sum_{t=2}^T -\log|\det(J)| + \log f_x(X_t^o, X_t^u | X_{t-1}^o, X_{t-1}^u) + \log f_{u^m}(u_t^m)$$

$$= -(T-1)\log|\det(J)| - (T-1)\frac{1}{2}\log(\det(\Sigma\Sigma'))$$

$$- \frac{1}{2}\sum_{t=2}^T (X_t - \mu - \Phi X_{t-1})'(\Sigma\Sigma')^{-1}(X_t - \mu - \Phi X_{t-1})$$

$$- \frac{T-1}{2}\log\sum_{i=1}^{N-K_2}\sigma_i^2 - \frac{1}{2}\sum_{t=2}^T\sum_{i=1}^{N-K_2}\frac{(u_{t,i}^m)^2}{\sigma_i^2} \tag{A-4}$$

where σ_i^2 is the variance of the i-th measurement error and the Jacobian term is given by

$$J = \begin{pmatrix} I & 0 & 0 \\ B^o & B^u & B^m \end{pmatrix}.$$

Note that the Jacobian terms of the likelihood in equation (A-4) do not involve A_n, and hence the constant prices of risk λ_0 but do involve the linear prices of risk λ_1.

B Impulse Responses

To derive the IR's of the yields from shocks to the macro variables and latent yield factors $F_t = (f_t^{o'}, f_t^{u'})'$ consider the VAR(12) form of F_t in equation (7), repeated here:

$$F_t = \Phi_0 + \Phi_1 F_{t-1} + \ldots + \Phi_p F_{t-12} + \theta_0 u_t. \tag{B-5}$$

The Φ_i coefficients take the following form in our parameterization:

$$\Phi_0 = 0 \quad \Phi_1 = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho \end{pmatrix} \quad \Phi_i = \begin{pmatrix} \rho_i & 0 \\ 0 & 0 \end{pmatrix} \text{ for } i = 2, \ldots, 12.$$

We write this as an implied Wold MA(∞) representation:

$$F_t = \sum_{i=0}^{\infty} P_i u_{t-i}, \tag{B-6}$$

where $u_t = (u_t^{o'} u_t^{u'})'$ are the shocks to F_t. Note that a Choleski adjustment is needed to take into account the contemporaneous correlation of the shocks.

The yield on an n-period zero coupon bond y_t^n is a linear combination of current and lagged values of u_t from equation (18), which we can write as:

$$y_t^n = A_n + \sum_{i=0}^{\infty} \psi_i^n u_{t-i}, \tag{B-7}$$

where the row vectors ψ_i^n are functions of B_n. Note that this is just a linear transformation of the original MA(∞) form, and the B_n are closed-form from equation (17).

For example, for the Macro Model, the state-space X_t is given by:

$$X_t = (f_t^{o'} \; f_{t-1}^{o'} \; \ldots \; f_{t-12}^{o'} \; f_t^{u'})',$$

where f_t^o are the two macro factors, and f_t^u are the three unobservable factors. The yields for maturity n, y_t^n, can be written as:

$$\begin{aligned} y_t^n &= A_n + B_n' X_t \\ &= A_n + B_{n1}' f_t^o + \cdots + B_{n12}' f_{t-12}^o + B_{n13}' f_t^u \\ &= A_n + \bar{B}_{n1}' F_t + \cdots + \bar{B}_{n12}' F_{t-12} \end{aligned} \tag{B-8}$$

where we partition as $B_n = [B_{n1} \ldots B_{n13}]$, where B_{ni} corresponds to f_{t-i}^o for $i = 0, \ldots, 12$ and B_{n13} corresponds to f_t^u, and $\bar{B}_{n1} = [B_{n1} B_{n13}]$, and $\bar{B}_{ni} = [B_{ni} 0]$ for $i = 2, \ldots 12$.

Then substituting the MA(∞) representation for F_t we have:

$$\begin{aligned} \psi_0^n &= \bar{B}_{n1}' P_0 \\ \psi_1^n &= \bar{B}_{n1}' P_1 + \bar{B}_{n2}' P_0 \\ &\vdots \\ \psi_i^n &= \bar{B}_{n1}' P_{i-1} + \cdots + \bar{B}_{n12}' P_{i-12}, \text{ for } i \geq 12. \end{aligned} \tag{B-9}$$

and so on.

The vector ψ_i^n is the IR for the n-period yield at horizon i for shocks to the driving variables F_t at time 0. For k yields of maturities $n_1, \ldots n_k$, we can stack the coefficients of each yield to write:

$$Y_t = A + \sum_{i=0}^{\infty} \Psi_i u_{t-i}, \tag{B-10}$$

where $Y_t = (y_t^{n_1} \ldots y_t^{n_k})'$ and the j-th row of Ψ_i is ψ_i^n.

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C   Variance Decompositions

Working with the MA(∞) representation of the yields in equation (B-10), the error of the optimal h-step ahead forecast at time t, $\hat{Y}_{t+h|t}$ is:

$$\hat{Y}_{t+h|t} - Y_{t+h} = \sum_{i=0}^{h-1} \Psi_i u_{t+h-i} \tag{C-11}$$

Let the j-th component of a vector be denoted by a superscript j and $\Psi_{jk,i}$ denote the element in row j, column k of Ψ_i. Then:

$$\hat{Y}^j_{t+h|t} - Y^j_{t+h} = \sum_{k=1}^{K} (\Psi_{jk,0} u^k_{t+h} + \ldots \Psi_{jk,h-1} u^k_{t+1}) \tag{C-12}$$

Denote the mean squared error of $\hat{Y}^j_{t+h|t}$ as $\text{MSE}(\hat{Y}_{t+h|t})$. Then

$$\text{MSE}(\hat{Y}_{t+h|t}) = \sum_{k=1}^{K} (\Psi^2_{jk,0} + \cdots + \Psi^2_{jk,h-1}). \tag{C-13}$$

The contribution $\Omega_{jk,h}$ of the k-th factor to the MSE of the h-step ahead forecast of the j-th yield is:

$$\Omega_{jk,h} = \frac{\sum_{i=0}^{h-1} \Psi^2_{jk,i}}{\text{MSE}(\hat{Y}_{t+h|t})}, \tag{C-14}$$

which decomposes the forecast variance at horizon h of the j-th yield to the various factors.

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Table 1: Summary Statistics of Data

	Central Moments				Autocorrelations		
	mean	stdev	skew	kurt	lag 1	lag 2	lag 3
1 mth	5.1316	2.7399	1.0756	4.6425	0.9716	0.9453	0.9323
3 mth	5.4815	2.8550	1.0704	4.5543	0.9815	0.9606	0.9419
12 mth	5.8849	2.8445	0.8523	3.8856	0.9824	0.9626	0.9457
36 mth	6.2241	2.7643	0.7424	3.5090	0.9875	0.9739	0.9620
60 mth	6.4015	2.7264	0.6838	3.2719	0.9892	0.9782	0.9687
CPI	3.8612	2.8733	1.2709	4.3655	0.9931	0.9847	0.9738
PCOM	0.9425	11.2974	1.0352	6.0273	0.9684	0.9162	0.8600
PPI	3.0590	3.6325	1.4436	4.9218	0.9863	0.9705	0.9521
HELP	66.7517	22.0257	-0.1490	1.8665	0.9944	0.9900	0.9830
EMPLOY	1.6594	1.5282	-0.4690	3.2534	0.9378	0.8954	0.8410
IP	3.4717	5.3697	-0.5578	3.6592	0.9599	0.8889	0.7972
UE	5.7344	1.5650	0.4924	3.2413	0.9906	0.9777	0.9595

The 1, 3, 12, 36 and 60 month yields are annual zero coupon bond yields from the Fama-Bliss CRSP bond files. The inflation measures CPI, PCOM and PPI refer to CPI inflation, spot market commodity price inflation, and PPI (Finished Goods) inflation respectively. We calculate the inflation measure at time t using $\log(P_t/P_{t-12})$ where P_t is the inflation index. The real activity measures HELP, EMPLOY, IP and UE refer to the Index of Help Wanted Advertising in Newspapers, the growth rate of employment, the growth rate in industrial production and the unemployment rate respectively. The growth rate in employment and industrial production are calculated using $\log(I_t/I_{t-12})$ where I_t is the employment or industrial production index. For the macro variables, the sample period is 1952:01 to 2000:12. For the bond yields, the sample period is 1952:06 to 2000:12.

Table 2: Principal Component Analysis

Principal Components: Inflation

	1st	2nd	3rd	Factor Loading
CPI	-0.6343	-0.3674	0.6802	-0.9286
PCOM	-0.4031	0.9080	0.1145	-0.5901
PPI	-0.6597	-0.2015	-0.7240	-0.9657
% variance explained	0.7143	0.9775	1.0000	

Principal Components: Real Activity

	1st	2nd	3rd	4th	Factor Loading
HELP	-0.3204	-0.7365	-0.5300	0.2719	-0.4622
UE	0.3597	-0.6283	0.6871	0.0612	0.5188
EMPLOY	-0.6330	-0.1648	0.2444	-0.7158	-0.9131
IP	-0.6060	0.1886	0.4327	0.6403	-0.8742
% variance explained	0.5202	0.7946	0.9518	1.0000	

We take the three (four) macro variables representing inflation (real activity) and normalize them to unit variance. Then the normalized data Z_t has the following 1 factor model:

$$Z_t = C f_t^o + \epsilon_t$$

where C is the factor loading vector, $\mathrm{E}(f_t^o) = 0$, $\mathrm{cov}(f_t^o) = I$, $\mathrm{E}(\epsilon_t) = 0$, and $\mathrm{cov}(\epsilon_t) = \Gamma$, where Γ is a diagonal matrix. The columns titled "principal components" list the principal components corresponding to the first to smallest eigenvalue. The % variance explained for the nth principal component gives the cumulative proportion of the variance explained by the first up to the nth eigenvalue. IP refers to the growth in industrial production, CPI to CPI inflation, PCOM to commodity price inflation and PPI to PPI inflation, HELP refers to the Index of Help Wanted Advertising in Newspapers, UE to the unemployment rate, EMPLOY to the growth in employment. The sample period is 1952:01 to 2000:12

Table 3: Selected Correlations

	CPI	PCOM	PPI	
Inflation	0.9286	0.5901	0.9657	

	HELP	UE	EMPLOY	IP
Real Activity	0.4622	-0.5188	0.9131	0.8742

	Inflation	Real Activity	1 mth	12 mth
Real Activity	0.0017			
1 mth	0.6666	0.0627		
12 mth	0.6484	0.0510	0.9771	
60 mth	0.5614	-0.0270	0.9191	0.9639

The table reports selected correlations for the inflation factor extracted from the first principal component of PCI, PCOM and PPI, the real activity factor extracted from the first principal component of HELP, UE, EMPLOY and IP, and the 1, 12 and 60 month bond yields, which are used in the estimation. IP refers to the growth in industrial production, CPI to CPI inflation, PCOM to commodity price inflation, PPI to PPI inflation, HELP refers to the Index of Help Wanted Advertising in Newspapers, UE to the unemployment rate, EMPLOY to the growth in employment. The sample period is 1952:06 to 2000:12.

Table 4: The Dependence of the Short Rate on Macro Variables

Panel A: y_t^1 on Constant, Inflation and Real activity

Coeff	Inflation	Real Activity	Constant	Adj R^2
t	0.0143	0.1535	0.4250	0.4523
	(0.0070)*	(0.0070)†	(0.007)†	

Panel B: y_t^1 on Constant, 12 lags of Inflation and Real activity

Coeff	Inflation	Real activity	Constant	Adj R^2
t	0.0037	0.0398	0.4296	0.5337
	(0.0534)	(0.0065)†	(0.0306)†	
$t-1$	0.0659	0.0150		
	(0.0828)	(0.0452)		
$t-2$	-0.0435	0.0105		
	(0.0830)	(0.0450)		
$t-3$	0.0062	-0.0054		
	(0.0833)	(0.0444)		
$t-4$	0.0233	-0.0172		
	(0.0828)	(0.0441)		
$t-5$	-0.0088	0.0145		
	(0.0825)	(0.0442)		
$t-6$	-0.0245	-0.0213		
	(0.0825)	(0.0438)		
$t-7$	0.0175	0.0062		
	(0.0821)	(0.0435)		
$t-8$	0.0080	0.0196		
	(0.0825)	(0.0438)		
$t-9$	-0.0049	0.0121		
	(0.0821)	(0.0441)		
$t-10$	-0.0079	0.0005		
	(0.0820)	(0.0439)		
$t-11$	0.1427	-0.0069		
	(0.0522)†	(0.0299)		

In Panel A we regress the 1 month yield y_t^1 on a constant, the inflation factor and the real activity factor. In Panel B we regress y_t^1 on a constant, inflation, real activity and 11 lags of inflation and real activity. We report OLS standard errors in parenthesis. Standard errors significant at the 5% (1%) level are denoted * (†). Sample period is 1952:01 to 2000:12.

Table 5: Yields-Only Model Estimates

Companion Form Φ

0.9924	0.0000	0.0000
(0.0039)		
0.0000	0.9548	0.0000
	(0.0062)	
0.0000	-0.0021	0.7646
	(0.0001)	(0.0210)

Short Rate Parameters δ_1 ($\times$ 100)

Unobs 1	Unobs 2	Unobs 3
0.0136	-0.0451	0.0237
(0.0020)	(0.0005)	(0.0015)

Prices of Risk λ_0 and λ_1

		λ_1 matrix		
	λ_0	Unobs 1	Unobs 2	Unobs 3
Unobs 1	-0.0033	-0.0069	0.0000	0.0000
	(0.0004)	(0.0040)		
Unobs 2	0.0000	0.0445	0.0000	-0.2585
		(0.0050)		(0.0197)
Unobs 3	0.0000	-0.0049	0.0000	0.0241
		(0.0090)		(0.0026)

Measurement Error ($\times$ 100)

3 month	36 month
0.0203	0.0090
(0.0003)	(0.0002)

The table reports parameter estimates and standard errors in parenthesis for the 3-factor Yields-Only model $X_t = \Phi X_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, I)$, Φ lower triangular and the short rate equation given by $r_t = \delta_0 + \delta_1' X_t$. All factors $X_t \equiv f_t^u$ are unobservable. The coefficient δ_0 is set to the sample unconditional mean of the short rate, 0.0513/12. The prices of risk λ_t are given by $\lambda_t = \lambda_0 + \lambda_1 X_t$. The system is first estimated with $\lambda_0 = 0$ and λ_1 unconstrained. In a second estimation, the insignificant coefficients in λ_1 are set to zero. The sample period is 1952:06 to 2000:12.

Table 6: Macro Model Estimates

Companion Form Φ for Latent Factors

0.9915	0.0000	0.0000
(0.0042)		
0.0000	0.9392	0.0000
	(0.0122)	
0.0000	0.0125	0.7728
	(0.0146)	(0.0217)

Short Rate Parameters δ_1 for Latent Factors ($\times$ 100)

Unobs 1	Unobs 2	Unobs 3
0.0138	-0.0487	0.0190
(0.0021)	(0.0007)	(0.0022)

Prices of Risk λ_0 and λ_1

	λ_0	λ_1 matrix				
		Inflation	Real Activity	Unobs 1	Unobs 2	Unobs 3
Inflation	0.0000	-0.4263	0.1616	0.0000	0.0000	0.0000
		(0.1331)	(0.0146)			
Real Activity	0.0000	1.9322	-0.1015	0.0000	0.0000	0.0000
		(0.3893)	(0.0329)			
Unobs 1	-0.0039	0.0000	0.0000	-0.0047	0.0000	0.0000
	(0.0003)			(0.0043)		
Unobs 2	0.0000	0.0000	0.0000	0.0459	0.0000	-0.2921
				(0.0055)		(0.0205)
Unobs 3	0.0000	0.0000	0.0000	-0.0035	0.0000	0.0200
				(0.0001)		(0.0028)

Measurement Error ($\times$ 100)

3 month	36 month
0.0207	0.0091
(0.0003)	(0.0002)

The table reports parameter estimates and standard errors in parenthesis for the Macro model with the short rate equation specified with only current inflation and current real activity, as reported in Panel A of Table 4. The short rate equation is given by $r_t = \delta_0 + \delta_1' X_t$, where δ_1 only picks up current inflation, current real activity and the latent factors. The dynamics of inflation and real activity are given by a 12 lag VAR (not reported). The model is $X_t = \Phi X_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, I)$. X_t contains 12 lags of inflation and real activity and three latent variables, which are independent at all lags to the macro variables. In a pre-estimation we find the inflation and real activity VAR(12), and the coefficients on inflation and real activity in the short rate equation. The coefficient δ_0 is set to the sample unconditional mean of the short rate, 0.0513/12. We first estimate the latent factor parameters and the prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$, restricting λ_1 to be block diagonal and using the same form of the prices of risk for the latent factors as the Yields-Only estimation in Table 5. In a second estimation, the insignificant coefficients in λ_0 and λ_1 are set to zero. The sample period is 1952:06 to 2000:12.

Table 7: Macro Lag Model Estimates

Companion Form Φ for Latent Factors

0.9922	0.0000	0.0000
(0.0039)		
0.0000	0.9431	0.0000
	(0.0118)	
0.0000	-0.0189	0.8210
	(0.0135)	(0.0216)

Short Rate Parameters δ_1 for Latent Factors ($\times$ 100)

Unobs 1	Unobs 2	Unobs 3
0.0130	-0.0438	0.0256
(0.0020)	(0.0010)	(0.0025)

Prices of Risk λ_0 and λ_1

	λ_0		Real	λ_1 matrix		
		Inflation	Activity	Unobs 1	Unobs 2	Unobs 3
Inflation	0.0000	-0.8442	-0.0017	0.0000	0.0000	0.0000
		(0.2397)	(0.0582)			
Real Activity	0.0000	1.1209	0.2102	0.0000	0.0000	0.0000
		(0.1375)	(0.0275)			
Unobs 1	-0.0050	0.0000	0.0000	-0.0048	0.0000	0.0000
	(0.0003)			(0.0040)		
Unobs 2	0.0000	0.0000	0.0000	0.0483	0.0000	-0.2713
				(0.0068)		(0.0195)
Unobs 3	0.0000	0.0000	0.0000	-0.0248	0.0000	0.1624
				(0.0078)		(0.0292)

Measurement Error ($\times$ 100)

3 month	36 month
0.0251	0.0107
(0.0005)	(0.0003)

The table reports parameter estimates and standard errors in parenthesis for the Macro Lag model with the short rate equation specified with 12 lags of inflation and current real activity, as reported in Panel B of Table 4. The short rate equation is given by $r_t = \delta_0 + \delta_1' X_t$, where δ_1 only picks up 12 lags of inflation and real activity and the latent factors. The dynamics of inflation and real activity are given by a 12 lag VAR (not reported). The model is $X_t = \Phi X_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, I)$. X_t contains 12 lags of inflation and real activity and three latent variables, which are independent at all lags to the macro variables. In a pre-estimation we find the inflation and real activity VAR(12), and the coefficients on inflation and real activity in the short rate equation. The coefficient δ_0 is set to the sample unconditional mean of the short rate, 0.0513/12. We first estimate the latent factor parameters and the prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$, restricting λ_1 to be block diagonal and not picking up any lagged variables. We us the same form of the prices of risk for the latent factors as the Yields-Only estimation in Table 5. In a second estimation, the insignificant coefficients in λ_0 and λ_1 are set to zero. The sample period is 1952:06 to 2000:12.

Table 8: Proportion of Variance Explained by Macro Factors in the Macro Model

	Horizon h			
	1 mth	12 mth	60 mth	∞
short end	50%	78%	85%	83%
middle	67%	79%	78%	73%
long end	61%	63%	48%	38%

We list the contribution of the macro factors to the h-step ahead forecast variance of the 1 month yield (short end), 12 month yield (middle) and 60 month yield (long end). These are the variance decompositions from the Macro model outlined in more detail in Table 9.

Table 9: Variance Decompositions

	h	Macro Factors	Real Activity	Latent Factors Unobs 1	Unobs 2	Unobs 3
		Inflation				
1 month yield						
	1			0.1381	0.3303	0.5314
Yields-Only	12			0.3081	0.4342	0.2577
	60			0.5579	0.3116	0.1305
	∞			0.6771	0.2279	0.0950
	1	0.4887	0.0074	0.0328	0.4088	0.0622
Macro	12	0.6860	0.0946	0.0251	0.1836	0.0108
	60	0.7139	0.1372	0.0390	0.1050	0.0048
	∞	0.6985	0.1343	0.0597	0.1027	0.0047
	1	0.0005	0.1081	0.0549	0.6235	0.2130
Macro	12	0.2242	0.3478	0.0446	0.3357	0.0477
with Lags	60	0.6885	0.1805	0.0315	0.0900	0.0095
	∞	0.6749	0.1768	0.0507	0.0882	0.0093
12 month yield						
	1			0.5972	0.3517	0.0511
Yields-Only	12			0.7116	0.2752	0.0132
	60			0.8594	0.1361	0.0045
	∞			0.9103	0.0869	0.0028
	1	0.6343	0.0332	0.0733	0.2112	0.0480
Macro	12	0.7066	0.0848	0.0709	0.1272	0.0105
	60	0.6615	0.1208	0.1271	0.0851	0.0054
	∞	0.6173	0.1128	0.1854	0.0794	0.0051
	1	0.0166	0.2107	0.2114	0.4659	0.0955
Macro	12	0.3268	0.1895	0.2001	0.2588	0.0249
with Lags	60	0.5879	0.1249	0.1905	0.0899	0.0067
	∞	0.5237	0.1113	0.2790	0.0801	0.0060
60 month yield						
	1			0.7507	0.1963	0.0530
Yields-Only	12			0.8443	0.1427	0.0130
	60			0.9319	0.0641	0.0040
	∞			0.9578	0.0397	0.0025
	1	0.5950	0.0170	0.2756	0.0804	0.0319
Macro	12	0.5690	0.0615	0.3054	0.0561	0.0080
	60	0.4027	0.0755	0.4848	0.0333	0.0037
	∞	0.3164	0.0593	0.5953	0.0262	0.0029
	1	0.0021	0.0199	0.8060	0.1184	0.0535
Macro	12	0.0597	0.0170	0.8376	0.0703	0.0153
with Lags	60	0.0906	0.0169	0.8619	0.0262	0.0045
	∞	0.0582	0.0108	0.9112	0.0168	0.0029

The table lists the contribution of factor i to the h-step ahead forecast of the 1 month yield. To interpret the top row, for the Yields-Only model, 13.81% of the 1-step ahead forecast variance is explained by the first unobserved factor, 33.03% by the second unobserved factor and 53.14% by the third unobserved factor. The Yields-Only model only has three latent factors. The macro models have inflation, real activity and three latent factors. The Macro model has no lags of inflation and real activity in the short rate equation, while the Macro with Lags model does.

Table 10: Comparison of Yields-Only and Macro Factors

Dependent Variable	Independent Variables					
	Inflation	Real Activity	Unobs 1	Unobs 2	Unobs 3	Adj R^2

Panel A: Regressions on Macro Factors

Dependent Variable	Inflation	Real Activity	Unobs 1	Unobs 2	Unobs 3	Adj R^2
Unobs 1	0.4625	-0.0726				0.2180
"level"	(0.0735)	(0.0860)				
Unobs 2	-0.6707	-0.1890				0.4902
"spread"	(0.0716)	(0.0611)				
Unobs 3	0.0498	-0.1794				0.0343
"curvature"	(0.0629)	(0.0714)				

Panel B: Regressions on Factors from Macro Model

Dependent Variable	Inflation	Real Activity	Unobs 1	Unobs 2	Unobs 3	Adj R^2
Unobs 1	0.1118	0.0307	0.9507	-0.0174	0.0038	0.9971
	(0.0054)	(0.0056)	(0.0055)	(0.0056)	(0.0047)	
Unobs 2	-0.9364	-0.1026	0.0199	0.7624	0.0279	0.9981
	(0.0037)	(0.0037)	(0.0042)	(0.0032)	(0.0029)	
Unobs 3	0.0427	-0.1238	0.1656	-0.1455	0.9071	0.9256
	(0.0262)	(0.0260)	(0.0289)	(0.0241)	(0.0233)	

Panel C: Regressions on Factors from Macro Lag Model

Dependent Variable	Inflation	Real Activity	Unobs 1	Unobs 2	Unobs 3	Adj R^2
Unobs 1	-0.0580	-0.0207	1.0248	0.0035	0.0058	0.9979
	(0.0049)	(0.0040)	(0.0044)	(0.0047)	(0.0036)	
Unobs 2	-0.7069	-0.1132	-0.2955	0.5700	0.1306	0.8715
	(0.0393)	(0.0313)	(0.0356)	(0.0376)	(0.0315)	
Unobs 3	0.1112	-0.0081	0.2059	0.0228	0.8119	0.7470
	(0.0458)	(0.0386)	(0.0507)	(0.0365)	(0.0424)	

Regressions of the latent factors from the Yields-Only model with only latent factors (dependent variables) onto the macro factors and latent factors from the Macro and Macro Lag model (independent variables). All factors are normalized, and standard errors, produced using 3 Newey-West (1987) lags, are in parentheses. Panel A lists coefficients from a regression of the Yields-Only latent factors onto only macro factors. Panel B lists coefficients from a regression of Yields-Only latent factors on the macro and latent factors from the Macro model with only contemporaneous inflation and real activity in the short rate equation. Panel C lists coefficients from a regression of Yields-Only latent factors on the macro and latent factors from the Macro Lag model with contemporaneous inflation and real activity and 11 lags of inflation and real activity in the short rate equation.

Table 11: Forecast Comparisons

RMSE Criteria

| Yield (mths) | RW | Unconstrained VAR's | | VAR's with Cross-Equation Restrictions | | |
		VAR Yields Only	VAR with Macro	Yields Only	Macro Model	Macro Lag Model
1	0.3160	0.3905	0.3990	0.3012	0.2889	0.3906
3	0.1523	0.2495	0.2540	0.1860	0.2167	0.2876
12	0.1991	0.2776	0.2722	0.1914	0.1851	0.2274
36	0.2493	0.3730	0.3644	0.2489	0.2092	0.2665
60	0.2546	0.3793	0.3725	0.2497	0.2333	0.2530

MAD Criteria

| Yield (mths) | RW | Unconstrained VAR's | | VAR's with Cross-Equation Restrictions | | |
		VAR Yields Only	VAR with Macro	Yields Only	Macro Model	Macro Lag Model
1	0.2252	0.3076	0.3242	0.2155	0.2039	0.2981
3	0.1159	0.1987	0.2056	0.1442	0.1693	0.2344
12	0.1639	0.2176	0.2204	0.1616	0.1559	0.1870
36	0.1997	0.2991	0.2924	0.1974	0.1604	0.2111
60	0.2054	0.2957	0.2930	0.2017	0.1883	0.2064

We forecast over the last 60 months (the out-sample) of our sample and record the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual values. Lower RMSE and MAD values denote better forecasts. Forecasts are 1-step ahead. We first estimate models on the in-sample, and update the estimations at each observation in the out-sample. RW denotes a random walk forecast, VAR Yields Only denotes a VAR(12) only with 5 yields, VAR with Macro denotes a VAR(12) fitted to the macro factors and all 5 yields, Yields-Only denotes the 3 factor latent variable model without macro variables, the Macro model has only contemporaneous inflation and real activity in the short rate equation, and the Macro Lag model has contemporaneous and 11 lags of inflation and real activity in the short rate equation. The first three of these models are thus unconstrained estimations, while the last three impose the cross-equation restrictions derived from the absence of arbitrage.
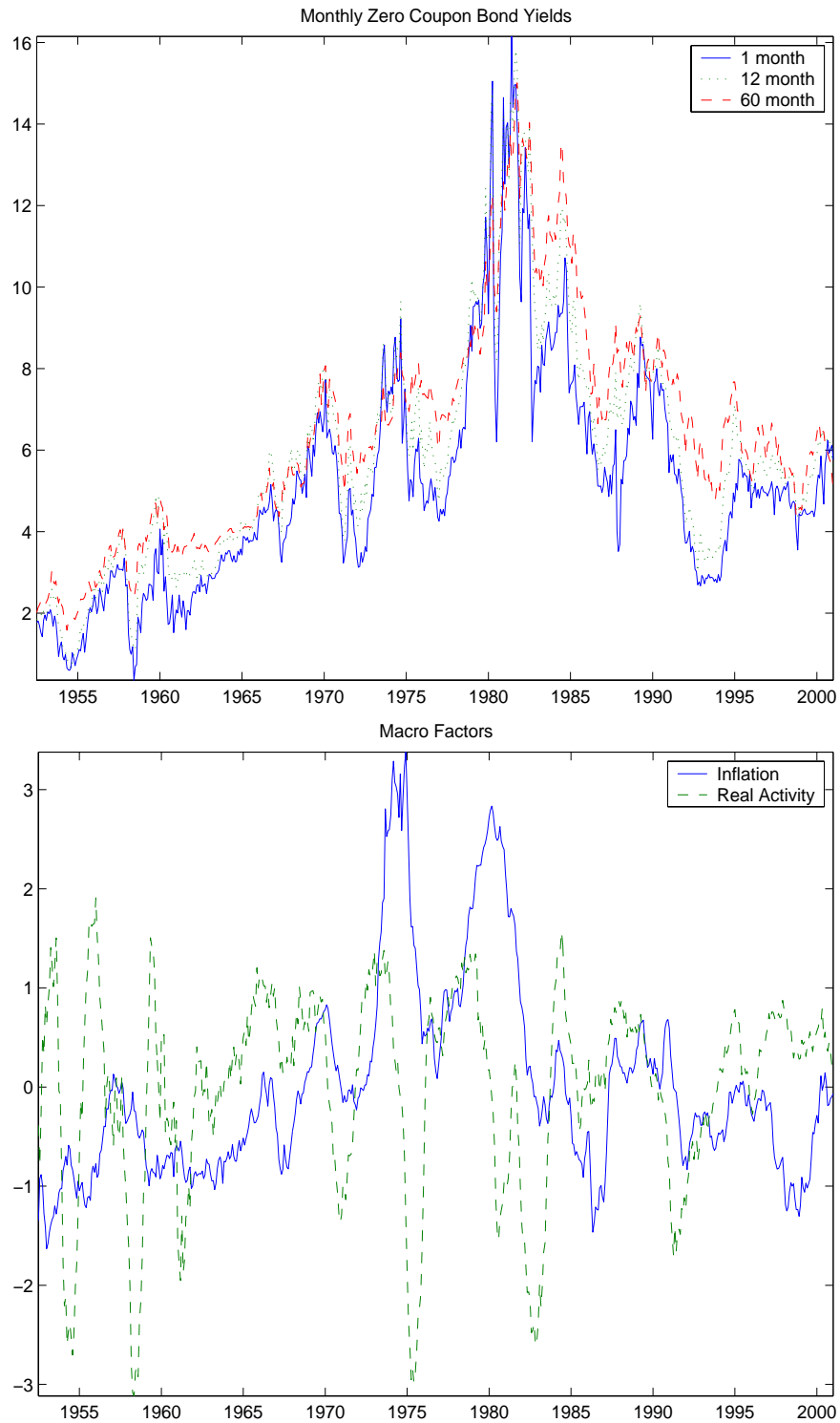
The top panel shows a plot of (annualized) monthly ZCB yields of maturity 1 month, 12 months and 60 months. The bottom panel plots the two macro factors representing inflation and real activity. The sample period is 1952:06 to 2000:12.

Figure 1: Bond Yields and Macro Principal Components
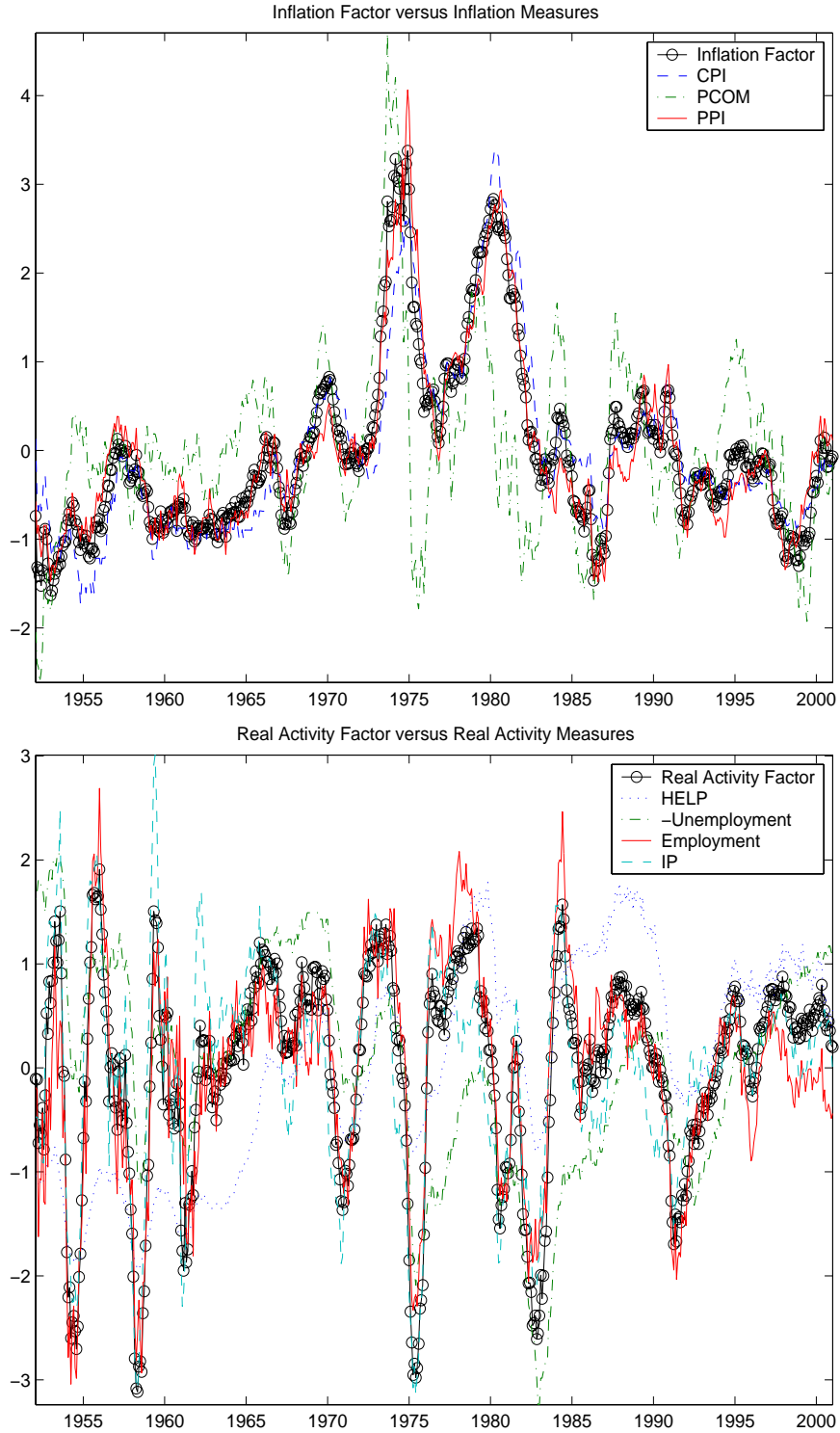
The top panel shows a plot of the inflation factor with CPI, PCOM and PPI measures of inflation. The bottom panel shows a plot of the real activity factor with HELP, the negative of unemployment, employment and IP measures of real activity. All variables are standardized to have zero mean and unit variance. The sample period is 1952:01 to 2000:12.

Figure 2: Inflation and Real Activity
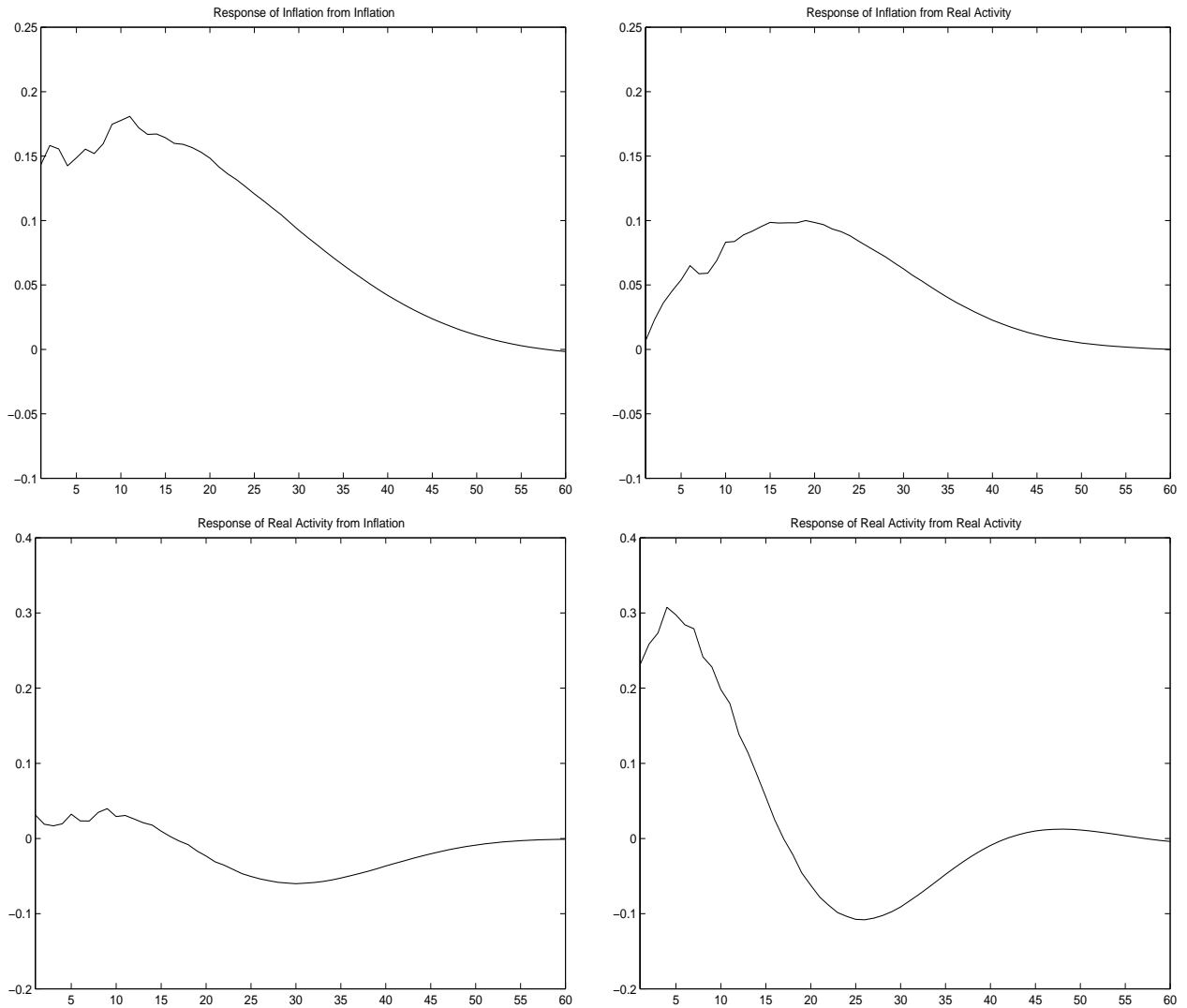
We fit a VAR(12) to the inflation and real activity macro factors. The plot shows the impulse responses to a Cholesky one standard deviation innovation to each variable. Time is in months on the x-axis.

Figure 3: Impulse Responses from the VAR(12) on Macro Factors

Residuals from Taylor Rule Specifications
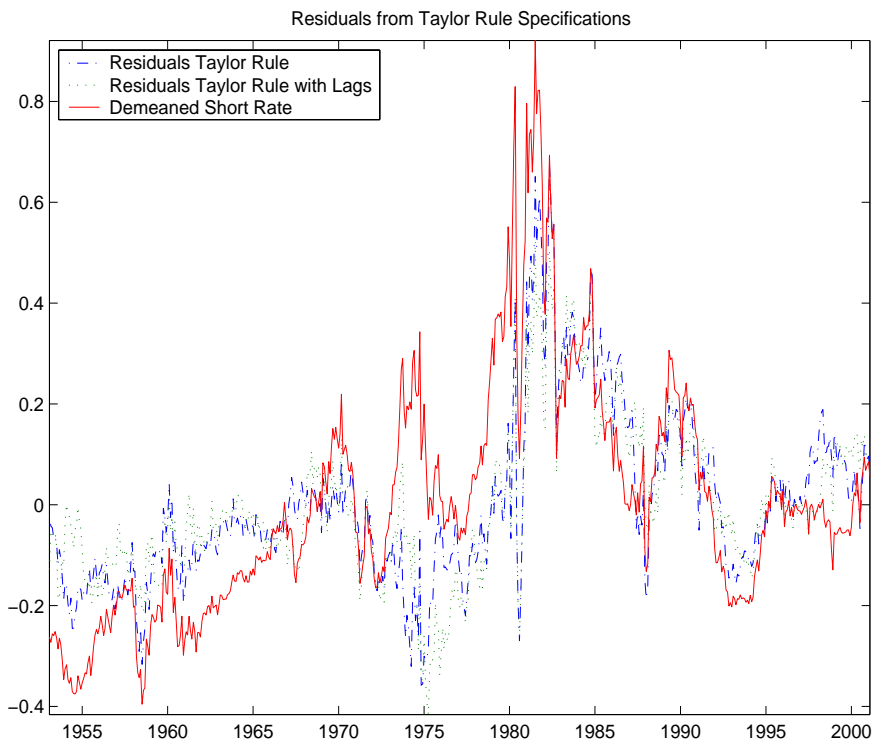
We show the residuals from the Taylor rule regressions, together with the de-meaned short rate (1 month yield). We show the residuals from the Taylor rule with no lags, which have 0.9458 autocorrelation, and the residuals from the Taylor rule with 11 lags, which have 0.9370 autocorrelation. For comparison, the autocorrelation of the short rate is 0.9716.

Figure 4: Residuals from the Taylor Rule Regressions
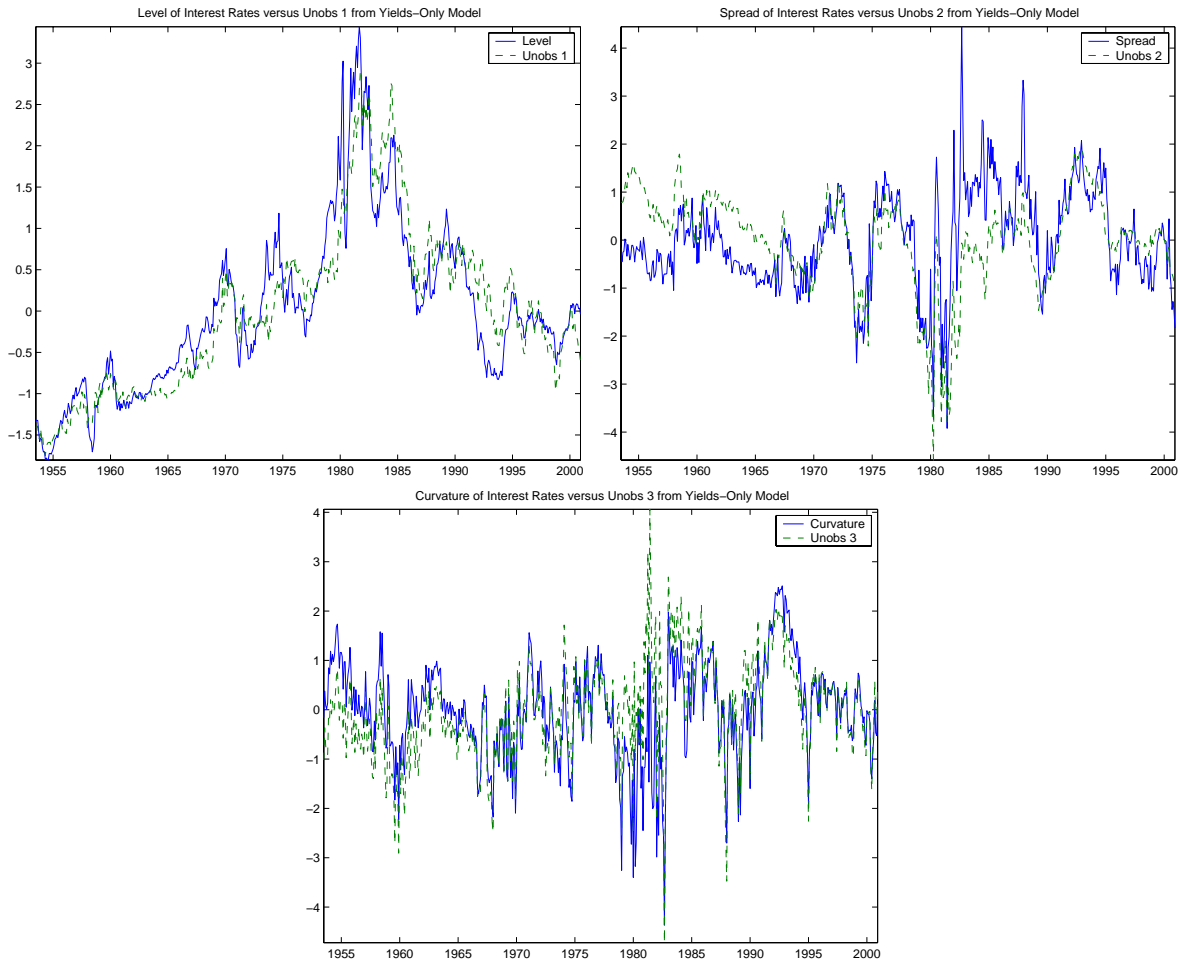
We show the unobserved yield factors from the Yields-Only model versus the level of interest rates (top left plot), spread (top right plot) and curvature of interest rates (bottom plot). All variables are normalized to have zero mean and unit variance. The level of interest rates is defined as $1/3(y_t^1 + y_t^{12} + y_t^{60})$ where y_t^1, y_t^{12} and y_t^{60} are the one-month yield, 12-month yield and 60-month yield respectively. The spread of interest rates is defined as $y_t^{60} - y_t^1$. The curvature of interest rates is defined as $y_t^1 - 2y_t^{12} + y_t^{60}$. The correlation between the level of interest rates and Unobs 1 is 92%, the correlation between the spread and Unobs 2 is 58% and the correlation between curvature and Unobs 3 is 77%.

Figure 5: Unobserved Yield Factors versus Level, Slope and Curvature
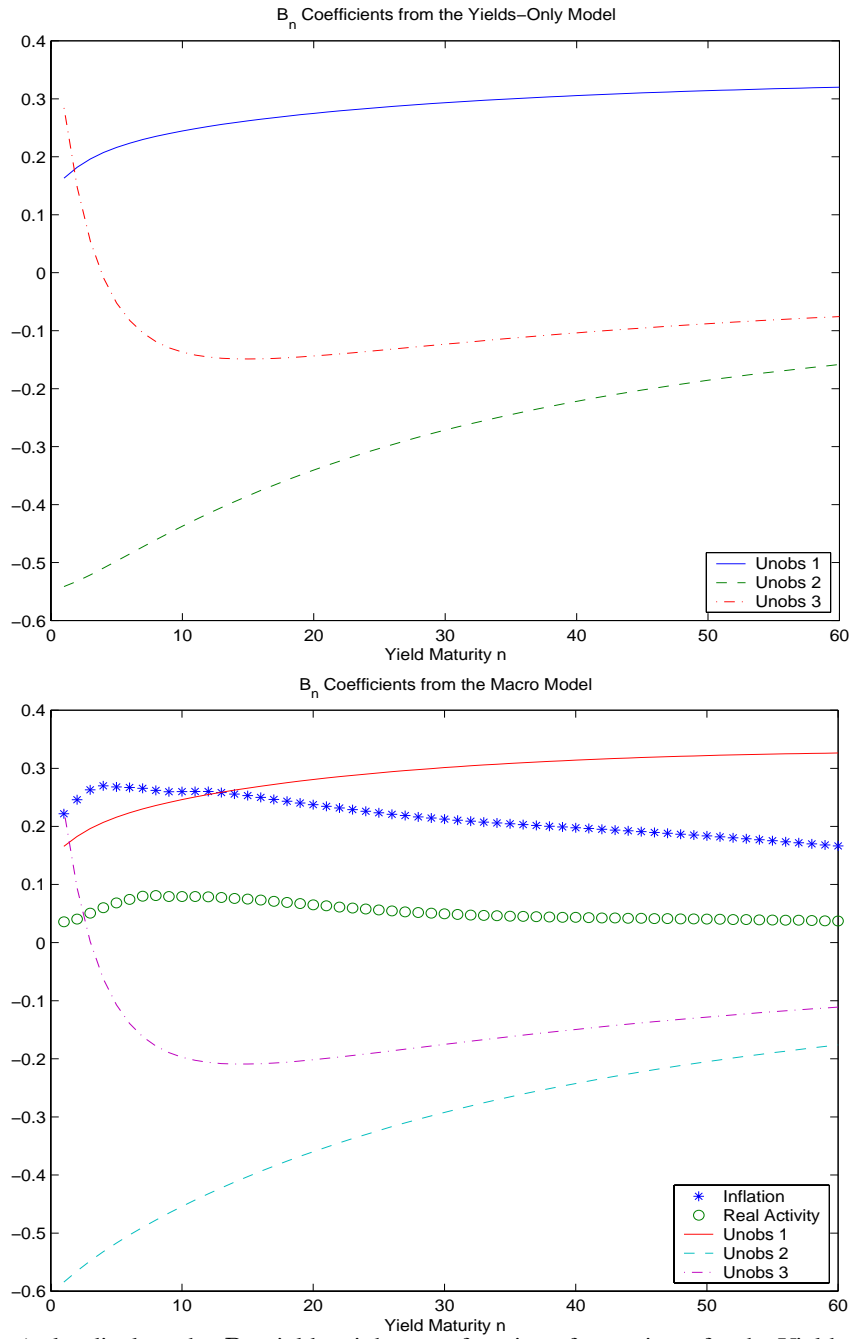
The top (bottom) plot displays the B_n yield weights as a function of maturity n for the Yields-Only (Macro) model. The weights have been scaled to correspond to one standard deviation movements in the factors and annualized by multiplying by 1200.

Figure 6: B_n Yield Weights for the Yields-Only and Macro Model
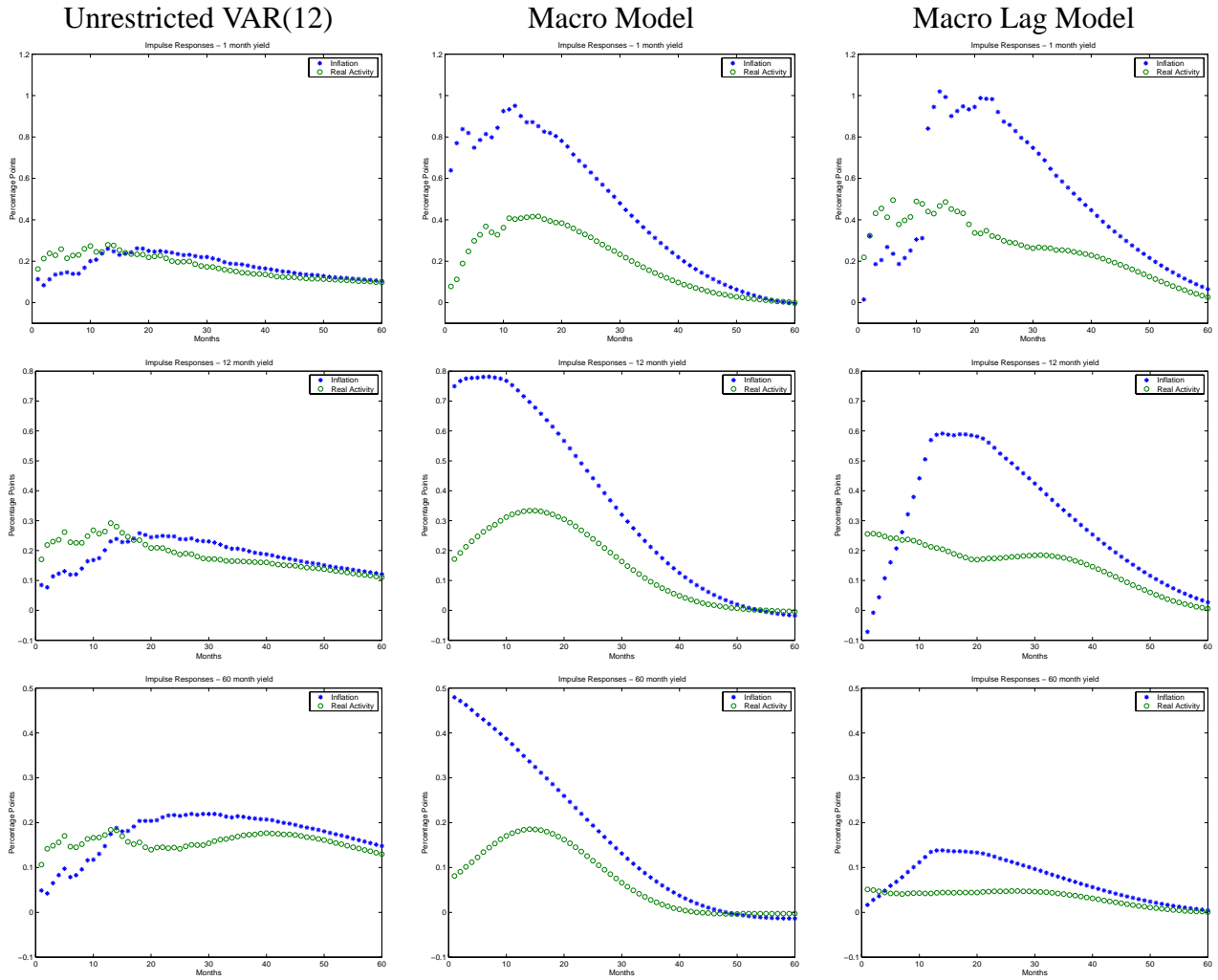
Impulse Responses (IR's) for 1 month (top row), 12 month (middle row) and 60 month (bottom row) yields. The first column presents IR's from an unrestricted VAR(12) fitted to macro variables and yields ; the middle column presents IR's from the Macro model; and the last column presents IR's from the Macro Lag model. The IR's from the latent factors are drawn as lines, while the IR's from inflation (real activity) are drawn as stars (circles). All IR's are from a one standard deviation shock.

Figure 7: Impulse Response Functions