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CATTLE CYCLES

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ABSTRACT

U.S. beef cattle stocks are among the most periodic time-series in economics. A theory of cattle cycles is constructed, based upon rational breeding stock inventory decisions in the presence of gestation and maturation delays between production and consumption. The low fertility rates of cows and substantial lags between fertility and consumption decisions cause the demographic structure of the herd to respond cyclically to exogenous shocks in demand for beef and in production costs. Known biotechnology of cattle demographics imply sharp numerical benchmarks for the dynamic system that describes the evolution of cattle stocks and beef consumption. These compare very closely to structural econometric time-series estimates over the 1875-1990 period and prove that systematic cattle cycles have a wholly rational explanation.

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CATTLE CYCLES

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I. NATURE OF THE PROBLEM

Total stocks of U.S. beef cattle (figure 1) are among the most periodic time-series in economics. Cattle are both investment and consumption goods, and current breeding and consumption decisions have important feedbacks on future stocks. Reproductive stock accounts for about 40 percent of total stocks, and lengthy gestation and maturation lags stretch the process over several years. Economists have recognized the importance of these lags (Ezekiel (1938) is the classic reference). However, lags of biotechnology must be combined with more comprehensive economic dynamics to explain these cycles. There are many empirical studies of the supply of beef, but a more complete market account in terms of the modern economics of renewable resources and intertemporal substitution is essential to the problem.[2] The model presented here proves useful in accounting for many of the interesting cyclical properties of the data.

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[1] We are indebted to Lars Hansen and Thomas Sargent for helpful discussions, Li Hao for research assistance and the National Science Foundation for research support.

[2]Empirical studies of the supply of beef by Nerlove, et. al. (1979, 1992) are the most comprehensive available. See also Trapp (1986) and Rucker, et. al. (1986), among others. Not much is available on the market equilibrium theory of completely managed renewable resources with significant reproductive stocks. Recent work on annual storage of agricultural goods (Williams and Wright (1990), Deaton and Laroque (1992)) is related, but is inappropriate for cattle because there are no reproductive feedbacks to future stocks.

It is known that investment aspects of cattle can produce unusual supply responses. For example, an increase in demand for beef *reduces* current supply if younger animals are held back to achieve greater slaughter weight (Jarvis, 1970) or if more females are withheld for reproduction to increase the future size of the herd (Rosen, 1987). We study the breeding margin and show how rational reproductive-inventory decisions combine with production lags to produce cattle cycles.

The model is built upon elementary facts of beef cattle technology. Cattle typically are held on land that is below the margin of cultivation. Breeding stock and young animals forage in pastures, crop remnants, and rangeland. Once they are large enough, virtually all young males (steers) and about one-third of young females (heifers) are sent to corn feed lots for "finishing," to be slaughtered at approximately 2 years of age. Remaining females are bred on an annual cycle, also beginning at age 2. A cow has at most one calf after a 9-month gestation period, and an effective reproductive life of 8-10 years. These delays embody natural "time-to-build" features (Kydland and Prescott (1982)) in the age structure of stocks and cause cyclical feedbacks between current consumption and future reproduction decisions.

Trends are ignored in what follows. They include income and substitution effects in the demand for beef and many improvements in beef and substitute food technologies that are interesting in their own right, but don't contribute to understanding persistent cycles. The model has been specified with the minimum number of parameters necessary for the problem---the rates of fertility and natural deaths, the rate of interest, and demand and supply shocks. Beef cattle technology is so simple that these parameters are known (approximately) a priori and can be used to predict the time path of cattle stocks and consumption. These predictions track the actual data extremely well, and provide strong evidence that breeding stock inventory decisions are important for cattle cycles.

The next section sketches a market equilibrium model. Its dynamic properties are discussed in section III. Empirical investigation of the model specification is in section IV and some estimates appear in section V. Conclusions are in section VI.

II. THE MODEL

We focus on market equilibrium inventories of female breeding stock, assuming a one year gestation-birth delay and a fixed two year maturation lag for both breeding and consumption (slaughter). The interesting economic decision is whether to breed or consume a mature female. Males (steers) are slaughtered at maturity and are inframarginal for breeding decisions. For present purposes they can be treated as females marked for slaughter and need not be labeled as separate capital goods.[3] A number of other simplifying assumptions are made. The main ones are homogeneous and undifferentiated stocks of adult cattle, independent of age or prior fertility; an exogenously fixed slaughter age of two years[4]; and no interactions between trends and cycles.

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[3] The price of slaughter animals per unit weight is similar for steers and heifers. Beef quality is lower for cows, but they are larger so gross value is not appreciably different. These equalizing differences are ignored to simplify the dynamics.

[4] The Austrian margin is too difficult to analyze because it causes the order of the system describing market dynamics to vary with market conditions. This omission is regrettable, but may not be important, for three reasons. First, variations in slaughter age have only minor feedback on future population decisions compared to breeding stock inventory decisions. Second, veal is a very small proportion of U.S. beef consumption and almost entirely produced by the male offspring of dairy cows (a distinct breed in the U.S.). Third, slaughter age seldom varies by as much as one year, yet the total stock cycles in figure 1 are nine years or more. The Austrian margin can't affect much just as a matter of accounting.

A. Population Mechanics

Population mechanics are assumed to follow the schematic of figure 2. Adult stocks are either bred or consumed. After a one year lag each animal reserved for breeding gives birth to g calves, with $g < 1$. At birth, all surviving calves enter a "pipeline" and remain there for two years, before merging into the adult stock at maturity. Assume that beef quality is invariant to age and breeding history. Then mature adult cows are consumed on equal terms with mature steers and heifers, and it is natural to treat all adults as neoclassical capital with Poisson death rate δ. This simplification is required empirically because the stock data are total head counts, not classified by age. It also eliminates the need to keep track of the adult age distribution. The state space is compacted to only three capital goods: adults, yearlings, and calves. Empirical evidence will be brought to bear on this specification in section IV.

Population stock-flow accounts are easily expressed. Let x_t be the breeding stock, c_t the number consumed and k_t the total number of mature animals at the beginning of period t. Then

$$(1) \qquad k_t = x_t + c_t.$$

The gx_{t-1} calves born at t enter the pipeline, and the gx_{t-2} calves born one year earlier remain there as yearlings. The head count, y_t, of all stocks is the sum of adults, yearlings and calves

$$(2) \qquad y_t = x_t + gx_{t-1} + gx_{t-2}.$$

Intertemporal constraints derive from standard demographic accounts because the U.S. beef cattle industry is essentially closed. Imports and exports of cooked beef products are trivial and few live animals are shipped across international borders. Imports of foreign fresh beef have been highly restricted in the U.S. over the years. Basically none was imported until the 1960s and at their highest levels today account for less than 10% of domestic consumption. Little beef is exported from the U.S. We assume a closed system in which U.S. consumption is produced exclusively from domestic stock.

All stocks in the pipeline in figure 2 move up one place each period. A total of c_t adults are sent to slaughter (all steers and a strictly positive fraction of heifers) and δk die of natural causes. The gx_{t-3} yearlings at t-1 enter the adult herd, so breeding stock evolves as

$$(3) \qquad x_t = (1 - \delta)x_{t-1} + gx_{t-3} - c_t.$$

The three variables $[x_{t-1}, x_{t-2}, x_{t-3}]$ conveniently represent the state of the population because current stocks of calves, yearlings and adults are readily calculated from them.

B. Breeding Stock Inventory Policy

Ranchers choose between reproduction and slaughter of adults. In an ongoing market equilibrium, they do some of each and profits are equated at both margins. Assuming constant returns to scale, a rancher's calculation is independent of herd size and profits are equated on the average as well as on the margin. Let p_t be the price of an animal on the hoof and m_t the feeding cost of preparing it for slaughter. Define $q_t = p_t - m_t$ as the net return from selling an animal at t for consumption.

The return from holding an adult animal for reproduction is the opportunity to sell it and its progeny later, minus holding costs and foregone interest. A cow survives with probability $1-\delta$ and will be worth $E_t(1-\delta)q_{t+1}$ next year, where E_t is expectations given period t information. Its g progeny sit in the pipeline for two years, and will have value $E_t g q_{t+3}$ when potentially sold three years from now. Letting ι be the rate of interest and $\beta = 1/(1+\iota)$ the discount factor, the gross return from breeding is $E_t[\beta(1-\delta)q_{t+1} + \beta^3 g q_{t+3}]$. Net return subtracts expected holding costs. Let h_t be the unit holding cost for an adult and assume proportional adult equivalent holding costs of γ_0 and γ_1 for calves and yearlings. Then, discounted holding costs are

$$(4) \qquad z_t = h_t + \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}.$$

Market equilibrium requires equal value for both selling and holding, or

$$(5) \qquad q_t = E_t[\beta(1-\delta)q_{t+1} + g\beta^3 q_{t+3} - z_t].$$

Note that the coefficients on the lead terms on q in (5) are the discounted coefficients of the lag terms of x in (3). This duality property of intertemporal optimization arises because (5) is the Euler equation from maximizing discounted expected values of beef production in a perfectly competitive market.

Demand and boundary conditions complete the description of the model. Specifying a particular demand function for such a lengthy period is problematic, but since population dynamics are linear, we take demand as approximately linear too:

$$(6) \qquad c_t = \alpha_0 - \alpha p_t + d_t = \alpha_0 - \alpha q_t - \alpha m_t + d_t$$

where d_t is a demand shifter (scaled so its coefficient is unity). Initial stocks are exogenously given, with at least one of them strictly positive. Limiting future values of capital are bounded if the average rates of growth of all cost and demand shifters are less than the rate of interest.[5]

Equation (1) - (6) reduce to two linear third-order difference equations in x_t and q_t:

$$x_t - (1-\delta)x_{t-1} - gx_{t-3} = -\alpha_0 + \alpha q_t + \alpha m_t - d_t \tag{7}$$

$$E_t[q_t - \beta(1-\delta)q_{t+1} - \beta^3 g q_{t+3} + h_t + \beta g\gamma_0 h_{t+1} + \beta^2 g\gamma_1 h_{t+2}] = 0 \tag{8}$$

III. MARKET SOLUTION

We study a special case where feed, forage and land are elastically supplied to the industry. Then holding costs h_t and finishing costs m_t are exogenous, independent of x_t or c_t, and the model is recursive. The supply of beef is elastic at constant supply price q_t determined from (8), independent of x_t, consumption is demand determined at that price, and stocks evolve from the solution to (7).

A. Renewal Conditions

The primary task is to characterize the stochastic processes for $\{x_t,c_t\}$. Renewal requirements give rise to one unusual detail. Suppose cost and demand parameters m, h and d were constant, and breeding stock and net price, x and q, settled down to steady-state values. Inspection of (7) and (8) reveals two necessary *renewal conditions* for this problem.

$$g - \delta > 0 \tag{9}$$

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[5]We impose upper bound constraints on the amounts of cattle and money producers can borrow in order to rule out pyramiding schemes that would cause future capital values to be unbounded. Once such schemes are ruled out, it can be shown that the proposed solution is the market equilibrium (Scheinkman (1986)).

(10) $$\beta^3 g + \beta(1 - \delta) - 1 > 0$$

Births must exceed natural deaths for renewal in (9). In addition, the gross return from reproduction must exceed the return to slaughter at the steady-state price in (10). Otherwise holding costs could not be covered and the herd rationally would be driven to extinction.

B. Market Equilibrium

Assume AR(1) processes for the shocks $\{d_t, m_t, h_t\}$:

(11) $$u_t = \rho_u u_{t-1} + \epsilon_t^u \qquad \text{for } u = d, m, \text{ and } h,$$

where the ρ_u's are serial correlation parameters, ϵ_t^u are i.i.d. innovations with zero means, constant variances, and zero cross correlations at all leads and lags. Rescaling ϵ^m to include the multiplicative effect of α and suppressing constants, the full model is, in operator notation,

(12) $$(1 - \phi_1 L)(1 - \phi_2 L)(1 - \phi_3 L)x_t = \alpha q_t + [\epsilon_t^m/(1 - \rho_m L)] - [\epsilon_t^d/(1 - \rho_d L)]$$

(13) $$E_t[(1 - \lambda_1^{-1}L^{-1})(1 - \lambda_2^{-1}L^{-1})(1 - \lambda_3^{-1}L^{-1})q_t + (1 + g\gamma_0 L^{-1} + g\gamma_1 L^{-2})\epsilon_t^h/(1 - \rho_h L)] = 0,$$

where ϕ_i and λ_i are the roots of the two cubic characteristic equations[6]

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[6]The characteristic equation of (12) and (13) factors into two cubics in the recursive case. If h_t or m_t are increasing in x_t and c_t, there is rising supply price of beef and the price and quantity system is fully simultaneous. The characteristic equation remains of order 6 but has no cubic factors. The recursive form has proven useful for uncovering the essential economics of the problem. Numerical calculations reveal that the characteristic roots are insensitive to rising supply

(14) $$\phi^3 - (1 - \delta)\phi^2 - g = 0$$

(15) $$g\beta^3\lambda^3 + (1 - \delta)\beta\lambda - 1 = 0.$$

The characteristic roots have three important properties.[7]

(i) Equations (14) and (15) both have one real root and two imaginary roots. The complex roots produce cyclical responses to shocks. Let $[\phi_1, \lambda_1]$ be the real roots and $[\phi_2, \phi_3, \lambda_2, \lambda_3]$ be the imaginary roots.

(ii) The product of complementary roots across equations equals $(1 + \iota)$: $\phi_i\lambda_i = 1/\beta$, for i = 1,2,3. This familiar turnpike result is due to symmetry of coefficients in equations (3) and (5).

(iii) Three roots are explosive and three are stable. The real root of (14) is explosive, $\phi_1 > 1/\beta$, so the real root of (15) is stable, from property (ii): $\lambda_1 < 1$. The complex roots of (14) are stable and the complex roots of (15) are explosive: $|\phi_j| < 1$ and $|\lambda_j| > 1/\beta$, for j = 2,3.

Roughly speaking, the stable real root in (15) is associated with the overall speed of convergence of the system. The two complex stable roots in (14) are associated with convergence of the age structure of animals to their stable distribution. Cycles arise from demographic "echo effects" of the current age distribution on future reproductive capacity. Demand and supply shocks cause ranchers to alter how much breeding stock they carry and these decisions percolate through all future birth cohorts. The percolations oscillate unless initial stocks are accidentally lined up just right.

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price specifications in the relevant parameter range, so stock dynamics aren't affected much. This class of models easily accommodates both rising supply price and shock processes of any order and generalization is straightforward.

[7]The graphs of (14) and (15) and condition (10) prove (i) and (iii). Substituting $\phi_i\lambda_i = 1/\beta$ into either equation verifies (ii).

The model is solved by taking the unstable roots forward and the unstable roots backward (Sargent(1979), Hansen and Sargent(1990))[8]. The solution in terms of the state variables $[x_{t-1}, x_{t-2}, x_{t-3}]$ and current and past innovations $\{\epsilon_t^d, \epsilon_t^m, \epsilon_t^h\}$ can be shown to be

$$(16) \qquad (1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)x_t = \Omega_t$$

$$(17) \qquad c_t = (\phi_1 - \lambda_1)(1 - \phi_2 L)(1 - \phi_3 L)x_t - \Omega_t$$

with

$$(18) \qquad \Omega_t = [(\rho_d - \lambda_1)/(\phi_1 - \rho_d)][\epsilon_t^d/(1 - \rho_d L)] - [(\rho_m - \lambda_1)/(\phi_1 - \rho_m)][\epsilon_t^m/(1 - \rho_m L)] - [\alpha\Gamma\lambda_1/(\phi_1 - \rho_h)(1 + r^2\rho_h^2 + 2r\rho_h\cos\theta][\epsilon_t^h/(1 - \rho_h L)]$$

where Γ and ϕ are defined by $\Gamma = 1 + \beta\rho_h\gamma_0 g + \beta^2\rho_h^2\gamma_1 g$ and $\phi_j = re^{\pm i\theta}$, for $j = 2,3$. The formula for prices in terms of the state variables is similar to (17). Finally, the autonomous form for c_t results from substituting (16) into law of motion (3):

$$(19) \qquad (1 - \lambda_1 L)c_t = -(1 - \phi_1 L)\Omega_t$$

Autonomous equations for p_t or q_t also look like (19).

In this example, the c_t process (19) (and by implication the price process) is of lower order than the process for x_t (16), and contains only real roots. Consumption (and prices) do not oscillate, only stocks do. This strong restriction arises from the recursive specification--constant returns to scale and

[8]Some technical details of this specific class of models are discussed in Rosen, Murphy and Scheinkman(1989).

elastic factor supplies to the industry.[9] If factor supplies are not elastic, the stable complex roots of the characteristic equation are present in both the autonomous consumption and stock equations. Prices have predictable cycles in that case (though no profit can be made of it). Nevertheless, the recursive structure suggests that there is a sense in which cycles are more important for breeding stock than for prices or consumption and it is a convenient place from which to start.

C. Transfer Functions

Figure 3 shows how positive demand innovations ϵ_t^d work their way through the market (cost innovations look similar except for sign). They are the graphs of the power series expansions of $[(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)]^{-1}$ for x_t from (16) and of
$(1 - \phi_1 L)[(1 - \lambda_1 L)(1 - \rho_d L)]^{-1}$ for c_t from (19), multiplied by the coefficient on ϵ^d in (18), all calculated at cattle parameters $g = .85$, $\beta = .909$ and $\delta = .1$. Figure 3 indicates different market responses to transitory ($\rho_d = 0$), permanent ($\rho_d = 1$) and intermediate ($\rho_d = .6$) shocks. The coefficients on the innovation terms in (18) imply that innovations are transitory when $\rho_d < \lambda_1$: the shock dies out quicker than the market's overall response-speed. As illustrated with $\rho_d = 0$, adult stocks are sold off and consumption is increased in the period when a positive shock occurs. Consumption is reduced later so that stocks can be built back up to sustain normal demand. The initial sell-off of adult stocks disturbs the pipeline age distribution and causes the oscillatory responses of stocks.

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[9]Favarro (1989) structured a related model for Uruguayan cattle, allowing for rising supply price of land and grazing competition between cattle and sheep. Another intriguing possibility is to include rising supply price of feed grains and competition for them among cattle, hogs and poultry. Some experimentation along those lines suggests that the dynamic effects on cattle of shocks in competing industries are second-order relative to what we focus on here, but these possibilities deserve more investigation than we have given them.

Ranchers behave opportunistically and sell more when demand is high because they anticipate that it is likely to fall back to normal after a little while.

Innovations are permanent when $\rho_d > \lambda_1$. Then the shock is more persistent than the overall response-speed of the system and intertemporal substitution is opposite to that of transitory shocks. For the $\rho_d = 1$ case illustrated, consumption initially *falls* when demand increases. Ranchers know that increased demand is likely to be sustained for a long time into the future and more than the average number of females must be bred initially to achieve the greater herd size necessary to provide it. Consumption jumps up when stocks get large enough, then smoothly convergesto its new steady state. Breeding stocks increase initially, then grow cyclically to their new steady state for the same reasons as before. Figure 3 reveals a kind of backward bending initial supply response to permanent shocks, and a normal rising supply price response to transitory shocks. An intermediate case where $\rho_d = .6$ works out to be slightly on the transitory side of this example. Its responses are smaller because the two effects tend to cancel each other.

Figure 4 contrasts responses of breeding stocks to total stocks. Notice how the moving average in (2) redistributes the y-responses and stretches them out over a longer time interval. This is an important aspect for understanding the cyclicity of total stocks in figure 1.

IV. EMPIRICAL COMPARISONS

A. Estimating the Breeding Stock

The stock data[10] in figure 1 are total head counts of all beef cattle in the U.S. at the end of each year, not only the breeding stock portion in (16).

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[10]All data are from U.S. Department of Commerce, Bureau of the Census, *Historical Statistics: Colonial Times to 1970* and U.S. Department of Agriculture, *Agricultural Statistics*, annual issues.

However, equation (2) implies that breeding stocks x_t can be replaced in (16) by inverting total stock y_t

$$(20) \qquad x_t = y_t/[1 + gL + gL^2] \equiv y_t/G(L)$$

We use (20) to construct breeding stock x_t from the total herd count data y_t. The construction is important because it bears on the empirical relevance of the simplifying assumptions used to compact the state space.

Two initial conditions are needed to iterate x_t from y_t in (20). The average annual growth rate of 4.5% for the U.S. cattle population over 1875-1990 is extrapolated back to 1875 and 1876 for this purpose[11]. The following argument shows that errors do not accumulate in the iteration. The solution to (20) is

$$(21) \qquad x_t = [y_t/(1 - \xi_1 L)(1 - \xi_2 L)] + b_1\xi_1^t + b_2\xi_2^t,$$

where the b's are constants satisfying some initial conditions and the ξ's solve $\xi^2 + g\xi + g = 0$. The complex roots ξ_j are stable ($\xi_1\xi_2 = g < 1$ and $|\xi_j| = \sqrt{g}$), so the effects of measurement error decline in damped oscillations in subsequent years. Some experimentation with different initial conditions showed convergence to the same constructed x-series within 15 or 20 years.

B. Validating the 3-State Approximation

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[11]In a deterministic system with a constant 4.5% growth rate, the ratio of breeding stock to total population, x/y, is fixed at $[1 + g/1.045 + g/(1.045)^2]^{-1}$. Given g, this factor is applied to y_{1875} to get an initial condition for x_{1875}. An initial condition for t = 1876 is obtained by applying $x_{1876} = (1.045)x_{1876}$. Recall that g is interpreted as the number of live births. The recursion formula (20) assumes that all calves survive into their second year. Calves survive at a lower rate than yearlings, but we have not experimented with alternative assumptions because this is likely to be an unimportant source of error.

Three alternative x_t series have been calculated for values of g approximating the net fertility rate of U.S. cattle under modern conditions: .80, .85 and .90. Adding consumption to both sides of equations (3)

$$k_t = c_t + x_t = (1 - \delta)x_{t-1} + gx_{t-3} \tag{22}$$

Equation (22) is an identity in the model, but not in the data. Its validity can be verified empirically by adding an intercept, a second lag in x_{t-2}, and a disturbance term to the right hand side and treating it as a regression. Actual slaughter numbers(available since 1900) are added to the constructed x_t series to estimate k_t in (1), and k_t is regressed on x_{t-1}, x_{t-2}, and x_{t-3}. If the model is valid, the regression coefficient on x_{t-1} should estimate $(1-\delta)$, in the range [.9,.95] for cattle, both the intercept and the regression coefficient on x_{t-2} should be zero, and the coefficient on x_{t-3} should give back the value of g used to construct x_t.

Regression coefficients for the entire 1900-1990 period for which k_t can be constructed and for the sub-period 1930-1990 appear in table 1. Considering the drastic simplifications, the fit is slightly astonishing. The constant term has a standard error approaching statistical significance, but a value of zero cannot be rejected and its point estimate is unreliable. The other coefficients are more sharply estimated. In all cases the estimated coefficient on x_{t-2} is small and not significantly different from zero, as required. The estimates of $(1-\delta)$ are decreasing in the assumed value of g and only mildly out of the .90-.95 range. The implied estimates of g in the last column are slightly too large, and increasingly so for larger assumed values. However, 95% confidence intervals cover the assumed value in every case. On the deficit side, F-tests formally reject the joint hypothesis $[\gamma_0=\gamma_2=0, \gamma_1=.95, \gamma_3=\text{assumed value of g}]$. And when additional lagged values of x are included in the regression, the coefficient on x_{t-4} is statistically significant (all larger lags are insignificant), indicating that the model's intertemporal specification is not strictly accurate.

Graphic evidence that the quality of approximation is good enough to warrant serious consideration is presented in figure 5. Equation (22) implies a consumption series corresponding to each $\{x\}$ series. This completely artificial consumption series imputed from $\{x\}$, with $\delta = .1$ and $g = .85$, is compared to actual slaughter numbers, a series that is collected independently of total stock counts, at the top of figure 5. Though the constructed series is noisier, its major cyclical variations compare very well both in timing and magnitude to actual consumption. It errs mostly in the trends. The actual productivity of U.S. cattle farmers in sustaining consumption is greater than the model predicts, and increasingly so after 1940. However, the comparison is quite close if trends are eliminated. This can be shown in the following way for the relationship between consumption $\{c\}$ and total stock $\{y\}$, which are independent series. Equations (20) and (22) imply the following relationship between $\{c\}$ and $\{y\}$:

$$(23) \qquad (1+gL + gL^2)c_t = -[1 - (1-\delta)L - gL^3]y_t \equiv \Phi(L)y_t$$

This are plotted ($g=.85$ and $\delta=.1$) in the lower panel of figure 5 after removing trends. The cyclical movements of $G(L)c$ and $\Phi(L)y$ are virtually identical. It is important to stress that the c and y series are independent counts of a flow and a stock and not imputed from each other.

Breeding stock $\{x\}$ imputed from $g = .85$ is plotted along with total herd size $\{y\}$ in figure 1. The y-series is smoother and more periodic than the x-series because $\{y\}$ is a three term moving average of $\{x\}$. Both change their character after 1930, due to secular improvements in the quality of the data, as well as changes in the dynamic structure of cattle technology caused by corn feeding and other agricultural innovations that are ignored here.

C. ARMA Formulation

It is pointless to test all the strong restrictions implied by this simple model, because it is known to be wrong in its details. Formal statistical rejections of the null would tell us no more than we already know. The more interesting question is "how wrong is it?" We proceed by comparing estimated autonomous ARMA forms for x_t and c_t in (16) and (19) with some theoretical benchmarks.

The benchmarks are found by multiplying both sides of (16) and (19) by $(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)$. The resulting composite error terms have zero autocorrelation after two lags, so the autonomous forms for $\{x_t, c_t\}$ can be represented as ARMA processes. The result in an ARMA(6,2) for x_t

$$(24) \qquad (1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)x_t = \omega_t + b_{x1}\omega_{t-1} + b_{x2}\omega_{t-2}$$

where ω_t is an i.i.d. random variable with zero expectation[12]. Applying the same procedure to (19) results in an ARMA(4,3) for c_t

$$(25) \qquad (1 - \lambda_1 L)(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)c_t = \tilde{\omega}_t + b_{c1}\tilde{\omega}_{t-1} + b_{c2}\tilde{\omega}_{t-2} + b_{c3}\tilde{\omega}_{t-3}$$

with i.i.d. random variable $\tilde{\omega}_t$ and MA constants b_{cj} defined by an expression similar to note 11. Write (24) as $B(L)x_t = A(L)\omega_t$, where $B(L)$ is the sixth order polynomial in L and $A(L)$ is a second order polynomial in L. Then (20) and (24) imply an ARMA(6,4) for $\{y_t\}$ of the form $B(L)y_t = G(L)A(L)\omega_t$ with

[Note Num]12. Its variance and the MA constants b_{xj} can be calculated from $\omega_t + b_{x1}\omega_{t-1} + b_{x2}\omega_{t-2} \equiv (1-\rho_m L)(1-\rho_h L)\tilde{\epsilon}_t^d + (1-\rho_d L)(1-\rho_h L)\tilde{\epsilon}_t^m + (1-\rho_d L)(1-\rho_m L)\tilde{\epsilon}_t^h$, with $\tilde{\epsilon}_j^u = [(\rho_u - \lambda_1)/(\phi_1 - \rho)]\epsilon^u$ for u = d,m,h.

the same AR part as the x-process.

Two simplifications are useful.

(i) If $\rho_d = \rho_m = \rho_h = \rho$, then $b_{x1} = b_{x2} = b_{c2} = b_{c3} = 0$ and there is effectively only one independent shock, a linear combination of the "structural" shocks ϵ_t^d, ϵ_t^m, and ϵ_t^h. Here (24) reduces to an AR(4) and (25) to an ARMA(2,1)[13]. The {y} process is ARMA(4,2).

(26) $$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho L)x_t = \omega_t$$

(27) $$(1 - \lambda_1 L)(1 - \rho L)c_t = \tilde{\omega}_t + b_c\tilde{\omega}_{t-1}$$

(28) $$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho L)y_t = \omega_t + g\omega_{t-1} + g\omega_{t-2}$$

(ii) If $\rho_m = \rho_h$ then $b_{x2} = b_{c3} = 0$, and there are two independent shocks, one to demand and another to supply. Now x_t becomes an ARMA(5,1), and c_t an ARMA(3,2):

(29) $$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)(1 - \rho_s L)x_t = \omega_t + b_x\omega_{t-1}$$

(30) $$(1 - \lambda_1 L)(1 - \rho_d L)(1 - \rho_s L)c_t = \tilde{\omega}_t + b_{c1}\tilde{\omega}_{t-1} + b_{c2}\tilde{\omega}_{t-2}.$$

Here {y} follows an ARMA(5,3) with the same autoregression coefficients as (29).

The essential elements of cattle technology are so simple that $(\lambda_1, \phi_2, \phi_3)$ in these formulas are known approximately *a priori*. They are calculated from

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[13]To economize notation it is understood that the definitions of ω and $\tilde{\omega}$ are adjusted to satisfy $\rho_d = \rho_m = \rho_h = \rho$ in (24) and (25), and to satisfy $\rho_m = \rho_h = \rho_s$ for (26) and (27).

(11) and (12) for given values of δ, β and g. In what follows we specify g = .85, δ=.10 and β=.909. These imply λ_1=.809, ϕ_2=-.2299+.7565i, and ϕ_3=-.2299-.7565i. Substituting these theoretical values into the AR parts of the ARMA processes provides benchmarks with which ARMA estimates can be compared.

V. ARMA ESTIMATES AND COMPARISONS

A. One-Shock Version.

The one-shock model in (26)-(28) accounts for the major time-series properties of y_t, x_t and c_t surprisingly well. These series have complicated trends. There was small or no growth in stocks and consumption prior to the 1930s and large growth thereafter, at least until recently. Per capita consumption of beef fell well into the 1920's, took off in the 1930's and turned down again in the late 1970's. This theory has nothing to say about trends, yet trends must be removed for statistical purposes. The ARMA's were fitted to deviations around two-piecewise linear least squares trends broken at 1930.[14] The ML exact method was used for estimation, guaranteeing that the estimated MA representation has stable roots.

The numbers in rows labeled ARJ in table 2 are estimated autoregression coefficients with the *Jth* lag of the dependent variable expressed to the right of the equals sign. Those listed in rows labeled MAJ are coefficient estimates of the *Jth* moving average term. Theoretical AR benchmarks are listed for each variable, assuming parameter values g=.85, δ=.10, β=.909 and experimenting

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[14]ARMA's first differences generally yield similar results--see appendix. If the forcing variables follow random walks, the order of the differenced ARMA should remain the same as in table 2 If they don't follow random walks, differencing adds an additional MA term. There are no reasons to expect that shocks follow random walks in this problem and we are unable to decide this point. This issue is avoided by using deviations rather than differences. Also, first differences are not quite appropriate because trends are not uniform in these data.

with different ρ-values. A value $\rho=.6$ achieved a reasonably good match between the estimates and theory for {x}, and that same benchmark is applied to the AR's for {y} and {c}. Additional support for $\rho = .6$ is provided by the fact that real prices of corn and hay follow AR(1)'s with coefficient on the order of 0.6.

The estimated AR(4) coefficients on {x} resemble their benchmarks, though better at the first two lags than at the last two. The virtue of using {x} for estimatation is that its ARMA has the fewest number of MA terms, and MA terms are difficult to estimate. But there is a potential problem: its 3-period lagged recursive construction from (20) might induce spurious autocorrelations in the estimates due to inaccuracies of specification. The higher order MA processes for {y} was estimated to check this point. As noted above {y} has the same AR benchmark as {x} and its MA benchmark is $1+.85L+.85L^2$. The estimates for {y} in the second column confirm that the AR estimates for {x} are not spurious consequences of the way {x} was constructed. The AR's match closely those of {x} and the MA terms are in the right ballpark relative to their benchmarks (though too large--they imply roots perilously close to mod 1). The estimates for {c} compare fairly well in the AR part, but not in the MA part. Similar results were obtained on series constructed with g=.8 and g=.9 and are not reported.

Table 2 does not conveniently indicate how well this model fits the data in many respects. Some pictures show that the quality of fit is exceptional.

First, the sample paths of {x}, {y} and {c} were predicted by applying the theoretical ARMA benchmark coefficients to estimated innovations. The residuals from the estimated ARMA's were used as innovations. Figure 6 compares the predicted and actual series. In each case solid lines are data (in deviation form) and simulations are marked by either circles or diamonds. The circles apply the theoretical benchmark ARMA coefficients in table 2 directly to the residuals estimated from corresponding table 2 regressions. The diamonds are indirect simulations: those for x apply $G^{-1}(L)$ to the directly simulated y-series and those for y apply $G(L)$ to the directly simulated x-series. Finally, the indirect simulation for c applies (22) to directly simulated $\{x\}$.[15] Differences between simulations and realizations are negligible except for the indirect prediction of c, which, unlike the others, compounds specification and estimation errors. The artificially simulated series are very close to the data, especially considering the complexity of the phenomena and the utter simplicity of the model.

Another visualization is provided by spectral variance decompositions of estimated and theoretical ARMA's. These appear (over the entire circle) in Figure 7. Spectra implied by the theoretical benchmarks in table 2 are marked by circles. The solid lines are spectra implied by the estimated ARMA's in the table. Again, the comparisons are very close. These calculations are not independent of those underlying figures 6, but they reveal an important new point. The accumulation of mass near 2 radians in the theoretical spectrum of x implies a 3+ period cycle in breeding stock. The mass in the empirical spectrum of x calculated from the ARMA estimates occurs at a slightly higher frequency than predicted, but the difference is minor. It is possible that the three period recursive construction of $\{x\}$ causes the extra weight at 2 radians in its empirical spectrum in figure 7. However, the spectral comparison for y proves that the empirical mass at 2 radians for x is not artificial. Theory predicts an absence of mass near 2 radians in the y-spectrum, just as the ARMA

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[15]Let B_i and $\mathbf{B}_i$ represent the theoretical and estimated AR portion for series $i=x,y$ and let G and $\mathbf{G}$ denote theoretical and estimated MA parts for y. Then the circles for x and y respectively are $B_x^{-1}(L)\mathbf{B}_x(L)x_t$ and $B_y^{-1}(L)\mathbf{B}_y(L)G(L)G^{-1}(L)y_t$. The simulation of $\{y\}$ has more chances to go wrong than that of $\{x\}$. The indirect simulation for y is $B_x^{-1}(L)\mathbf{B}_x(L)G(L)x_t$ and for x is $B_y^{-1}(L)\mathbf{B}_y(L)G^{-1}(L)x_t$. Comparable expressions apply to c.

estimates show. The heavier mass at the neighboring longer frequency (this is even more pronounced in the difference estimates--see appendix) is not implied by this theory. Finally the constant supply price assumption predicts no systematic cycles in {c} and none are found in this simple specification.

Similar conclusions follow from the AR characteristic roots calculated at the bottom of table 2. The two stable complex roots ϕ_2 and ϕ_3 arising from cattle biotechnology and demographics in (14) is what causes the 3+ year cycle in breeding stock dynamics. In all cases the estimated and predicted values of these two roots are reasonably close to each other. The comparisons for λ_1 and ρ are less close.

B. Two-Shock Version

Estimates of the two-shock model in table 2 reinforce these conclusions. Here theoretical benchmarks are harder to establish. We have little prior knowledge of the roots of the shock processes for the AR part. The .6 value of ρ used in the one-shock model was arbitrarily retained for the two-schock benchmark, leaving one free autoregression parameter for the second shock. A value of .95 matches up reasonably well, but by no means perfectly. Note the three theoretical AR roots λ_1, ϕ_2, and ϕ_3 remain unchanged. MA benchmarks are very complicated now (see equations (29) and (30)) and require much more detailed technical knowledge of the costs, production structure and shock processes than is available to us. These demanding refinements are left for future work.

The estimated AR coefficients are in the right ballpark in each case and the implied empirical spectra (including the MA part) in figure 8 are similar to figure 7. The two-shock model confirms the 3+ year cycle in {x} and the absence of 3 year cycles in {c}. However, the empirical spectrum of {c} shows a small mass point at about .66 radians, or a tendency toward a 9.5 year cycle, which is not predicted by the model. Finally, the implied roots for ϕ_2 and ϕ_3

are close to their benchmarks. However, other complex roots appear here (and also in the one-shock model) that remain to be explained (the two shock model in differenced form has similar properties as those in table 2). Finally, the three-shock model is too complicated to yield reliable estimates. The estimates we managed to produce are very imprecise, but their implied spectral decompositions are much like those in figures 7 and 8.

VI. CONCLUSION.

The evidence suggests that models of this kind contribute substantially toward understanding cattle cycles. Shocks to demand and supply have persistent long term effects on future stocks by changing farmer's incentives to carry breeding stock and altering the age composition of herds. The large weight of breeding stock in the total cattle population and their lengthy gestation and maturation periods probably makes cattle a pathological case compared to other managed animal populations. However, pathologies sometimes are useful to study because their properties are exaggerated and easier to observe. The fully rational model investigated here is capable of explaining important cyclical aspects of these data. Appeals to cobweb theory or to ignorance of cattle farmers are unnecessary to account for them.

The findings also suggest that a recursive structural model with long run constant returns to scale is a good approximation. In a fully simultaneous model with rising supply price, the reduced form AR parts of the autonomous forms in (16) and (19) for $\{c\}$ and $\{x\}$ would be exactly the same, and *both* would exhibit period 3 cycles. A 3-cycle is observed in stocks, but not in consumption. Empirical experimentation reveals that the AR part of the consumption ARMA definitely is of lower order than those for stocks, as a constant supply price, recursive structure requires. In this respect also the empirical representations are inconsistent with the kinds of cobweb expectational models commonly suggested as causing cattle cycles. We cannot pursue this point here because a constant supply price model cannot nest static (cobweb) price expectations into breeding inventory decisions--static expectations and constant supply price implies corner solutions in which all adult stocks are either held or slaughered, contrary to what the data show. Still, only a little reflection is needed to recognize that static cobweb price expectations wedded to this technology and rising supply price must generate 3 year cycles in consumption not observed in the data. Some longer cycles in consumption and stocks not explained by this model appear to be in the data. Perhaps some of these can be better understood with improved specifications of cattle dynamics and by serious investigation of the shock processes that drive these kinds of models.

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TABLE 1

Regression Estimates:
$k_t = x_t + c_t = b_0 + b_1x_{t-1} + b_2x_{t-2} + b_3x_{t-3}$

g	PERIOD	Regression Coefficient (Std. Errors)				$\bar{R}^2$
		b_0	b_1	b_2	b_3	
.80	1900-1990	-1544 (849)	1.11 (.12)	.04 (.15)	.83 (.12)	.978
	1930-1990	1785 (1153)	.97 (.12)	.04 (.15)	.88 (.12)	.977
.85	1900-1990	-1524 (859)	1.01 (.11)	.09 (.14)	.92 (.11)	.976
	1930-1990	1855 (1162)	.87 (.11)	.08 (.13)	.96 (.11)	.976
.90	1900-1990	-1509 (863)	.91 (.11)	.14 (.13)	1.00 (.10)	.977
	1930-1990	1895 (1160)	.80 (.11)	.13 (.12)	1.03 (.10)	.975

Notes: x_t is constructed for indicated value of g from total stock series by methods described in text. c_t is slaughter numbers, USDA sources.

TABLE 2

ARMA ML ESTIMATES AND BENCHMARKS*

LAGS	ONE-SHOCK MODEL					TWO-SHOCK MODEL				
	ESTIMATES (S.E.)			BENCHMARKS		ESTIMATES (S.E.)			BENCHMARKS	
	x	y	c	x,y	c	x	y	c	x,y	c
AR1	.96 (.07)	.90 (.07)	1.22 (.31)	.95	1.41	1.48 (.28)	1.51 (.22)	2.38 (.14)	1.90	2.36
AR2	-.35 (.08)	-.31 (.06)	-.37 (.28)	-.46	-.49	-.91 (.28)	-.95 (.20)	-2.18 (.23)	-1.36	-1.82
AR3	.91 (.08)	1.00 (.07)	—	.66	—	1.12 (.13)	-1.24 (.10)	.78 (.12)	1.10	.46
AR4	-.69 (.08)	-.76 (.07)	—	-.30	—	-1.24 (.26)	1.43 (.23)	—	-.93	—
AR5	—	—	—	—	—	.49 (.19)	-.58 (.17)	—	.29	—
MA1	—	.96 (.05)	-.04 (.32)	0 .85	.74	-.39 (.31)	.46 (.24)	-1.28 (.18)	—	—
MA2	—	.99 (.04)	—	0 .85	—	—	.55 (.22)	.59 (.18)	—	—
MA3	—	—	—	—	—	—	-.48 (.24)	—	—	—
RMSE	1663	1575	1431	—	—	1589	1531	1431	—	—
AR ROOTS	-.38 ± .86i .86 ± .21i	-.42 ± .89i .87 ± .21i	.66 .56	-.23 ± .76i .82, .6	.81 .6	-.4 ± .86i .9 .69 ± .35i	-.42 ± .89i .93	.95 .72 ± .56i	-.23 ± .76i .81, .6	.81, .6 .95
IMPLIED CYCLES	3.16, 13.1	3.12, 13.1	None	3.05	None	3.13, 13.4	3.12, 13.9	9.5	3.4	None

* All data are deviations from piecewise linear trend with 1930 breakpoints.

TABLE A1

ARMA ONE-SHOCK ESTIMATES ON FIRST DIFFERENCES*

LAG	x		BENCHMARK	y		c		BENCHMARK
AR1	.17 (.09)	.57 (.18)	.95	1.06 (.14)	.58 (.15)	.95 (.22)	1.58 (.07)	1.41
AR2	-.43 (.07)	-.43 (.07)	-.46	-1.15 (.17)	-.44 (.06)	-.34 (.10)	-.94 (.07)	-.49
AR3	.62 (.07)	.75 (.05)	.66	.85 (.13)	.86 (.08)			
AR4	-.32 (.09)	-.58 (.10)	-.30	-.44 (.09)	-.66 (.11)			
MA1		-.44 (.31)		-.11 (.14)	.40 (.18)	-.72 (.23)	-1.56 (.12)	
MA2				.78 (.12)	.52 (.17)		.82 (.13)	
MA3					-.52 (.18)			
RMSE	1678	1604		1583	1552	1494	1407	
AR ROOTS	-.42±.86i .50±.32i	-.41±.87i .69±.4i	-.23±.76i .82, .6	-.08±.88i .61±.45i	-.43±.9i .72±.39i	.48±.34i	.74±.56i	
CYCLES	3.11,11.0	3.1,12.0	3.1	3.8,9.9	3.1,12.7	10.2	9.7	None

* ML exact method coefficient estimate (Standard Error).

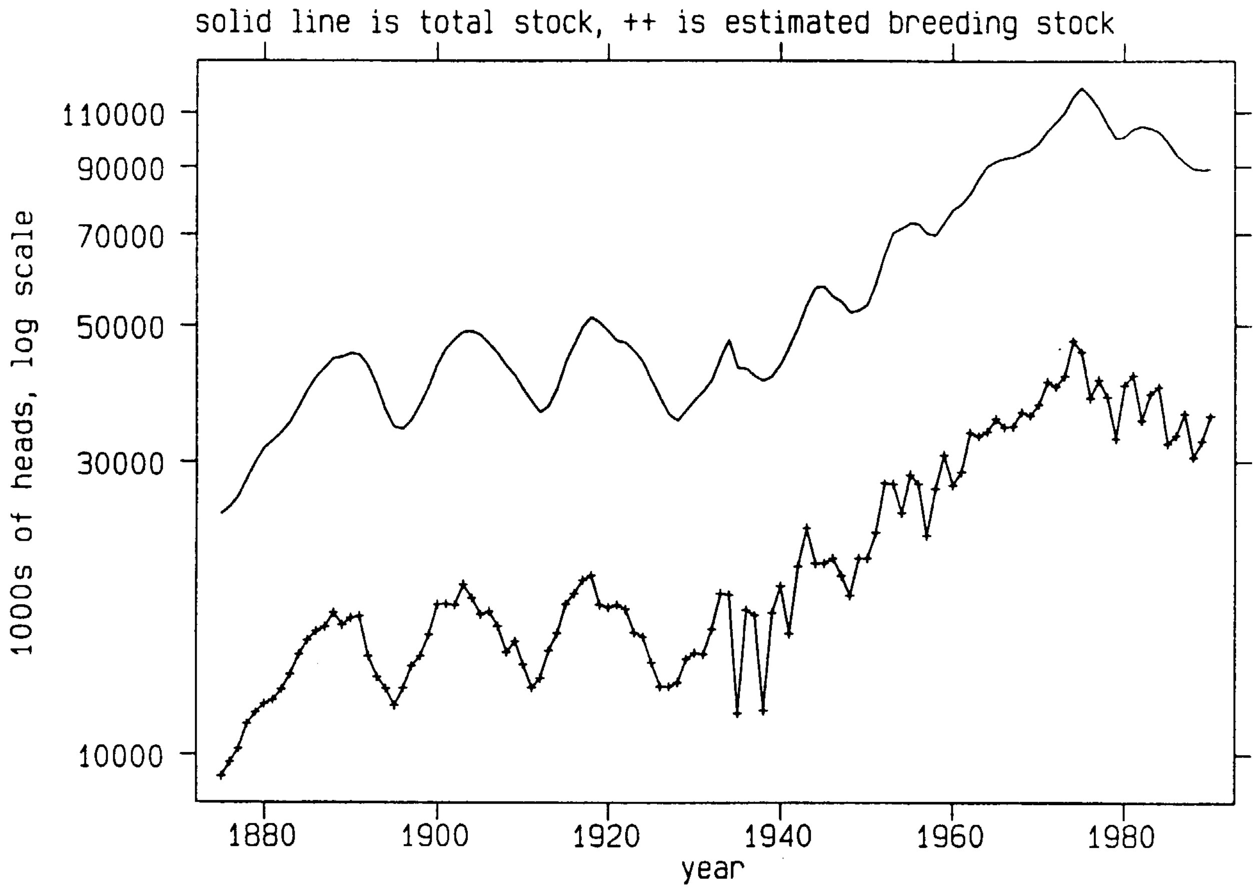


Figure 1: Beef Cattle Stocks

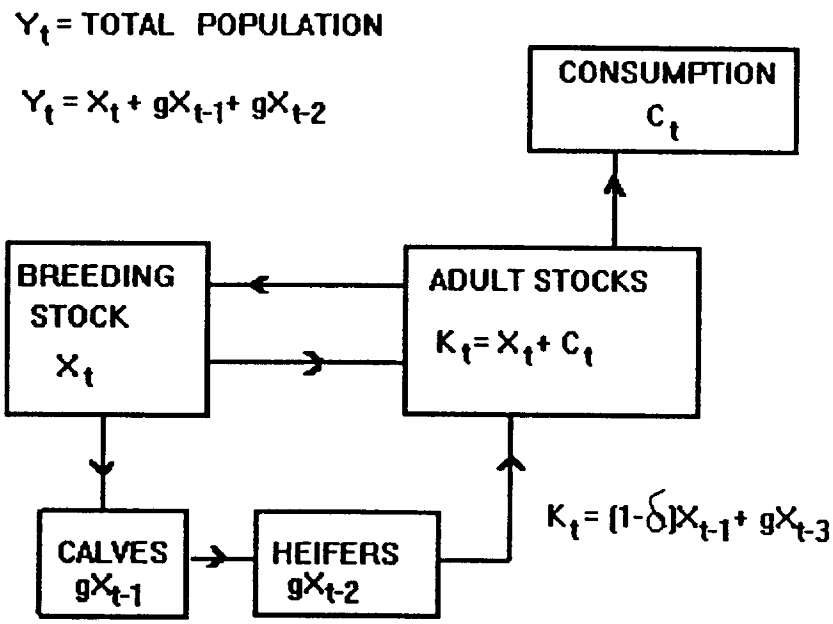
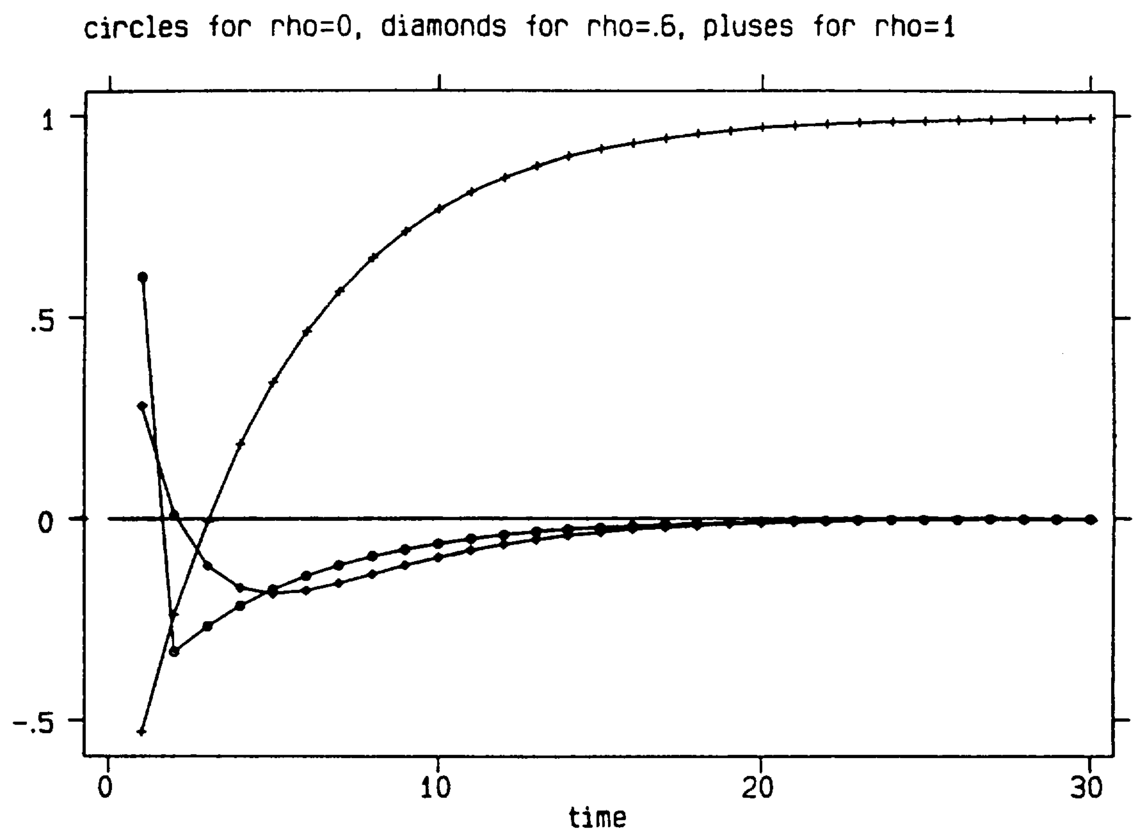
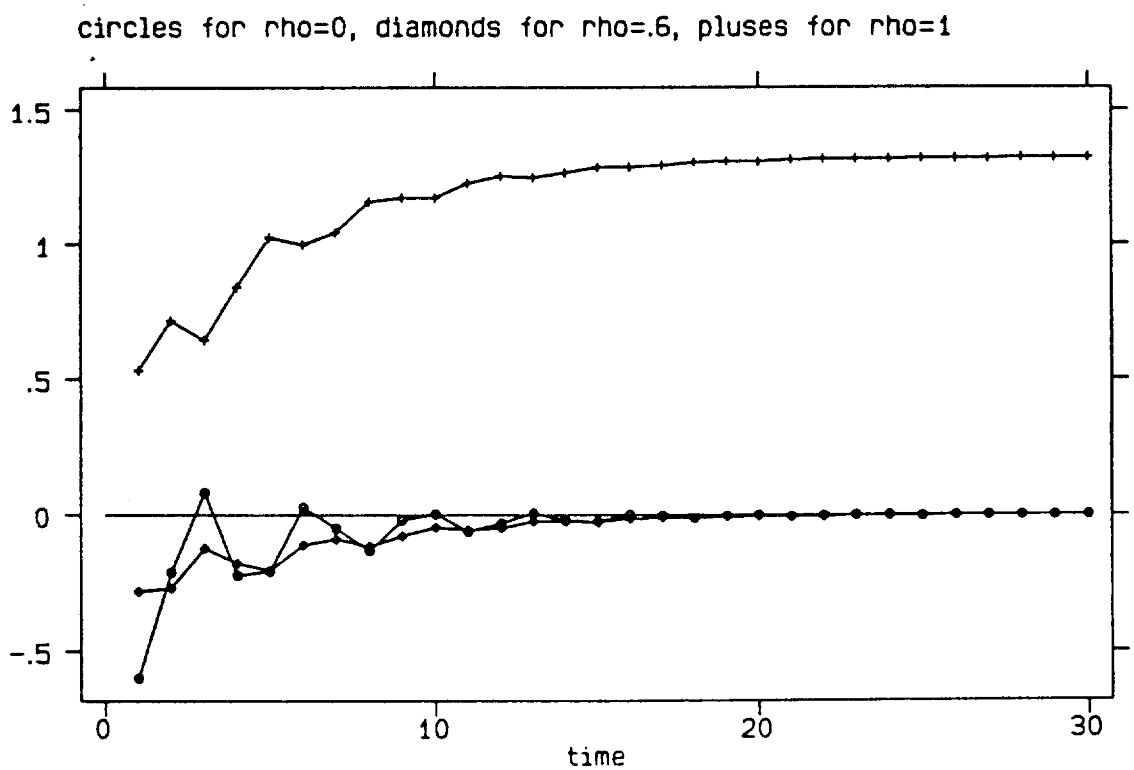


FIGURE 2: Population Flow Chart

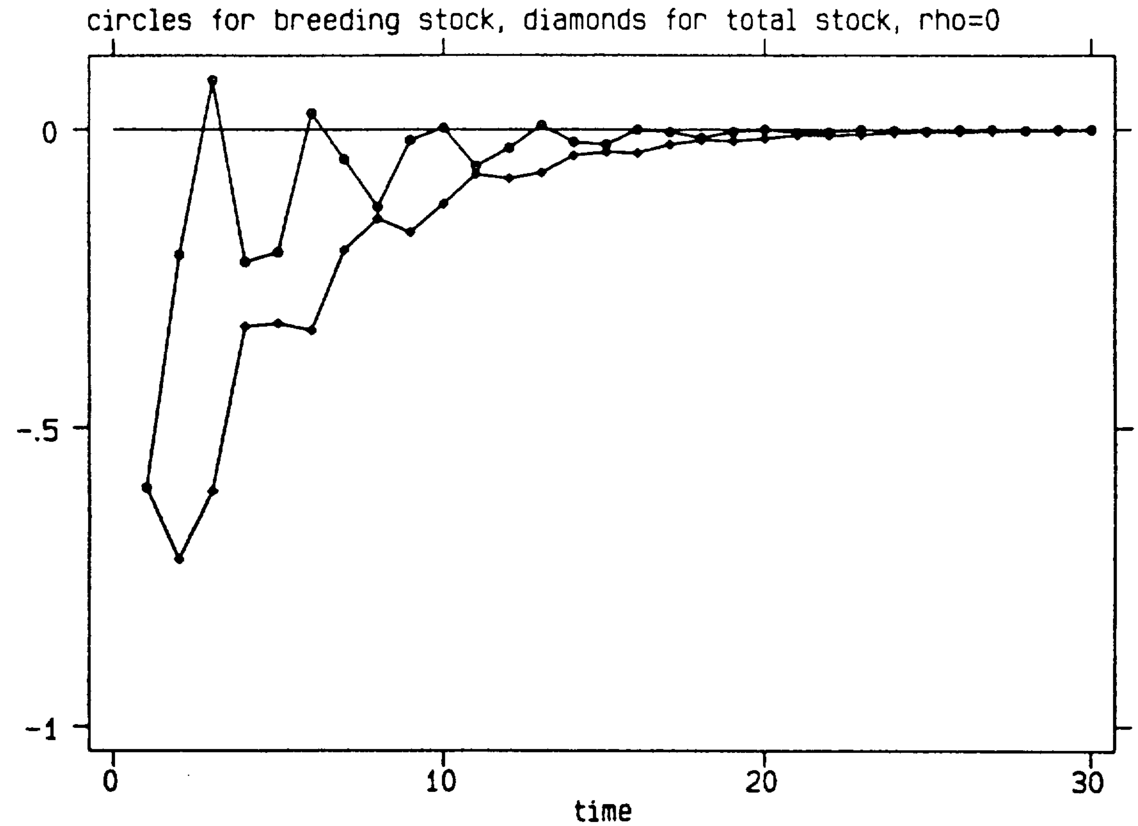


CONSUMPTION RESPONSE

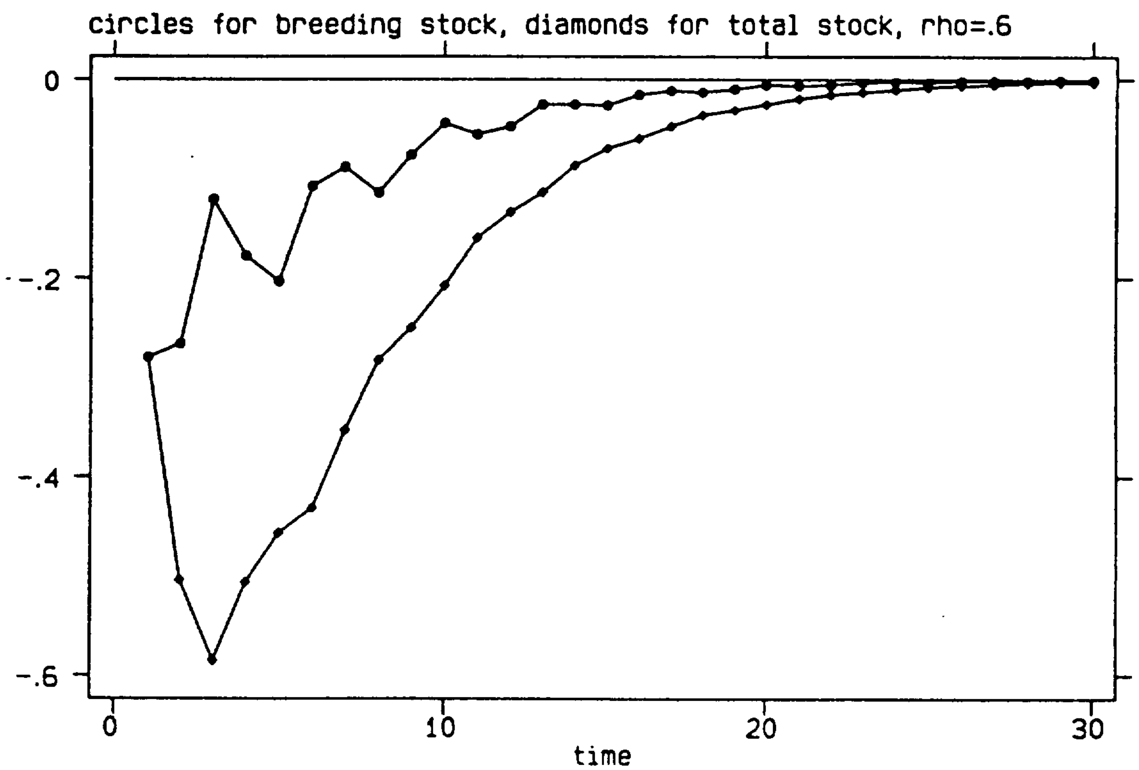


BREEDING STOCK RESPONSE

FIGURE 3: DISTRIBUTED LAGS

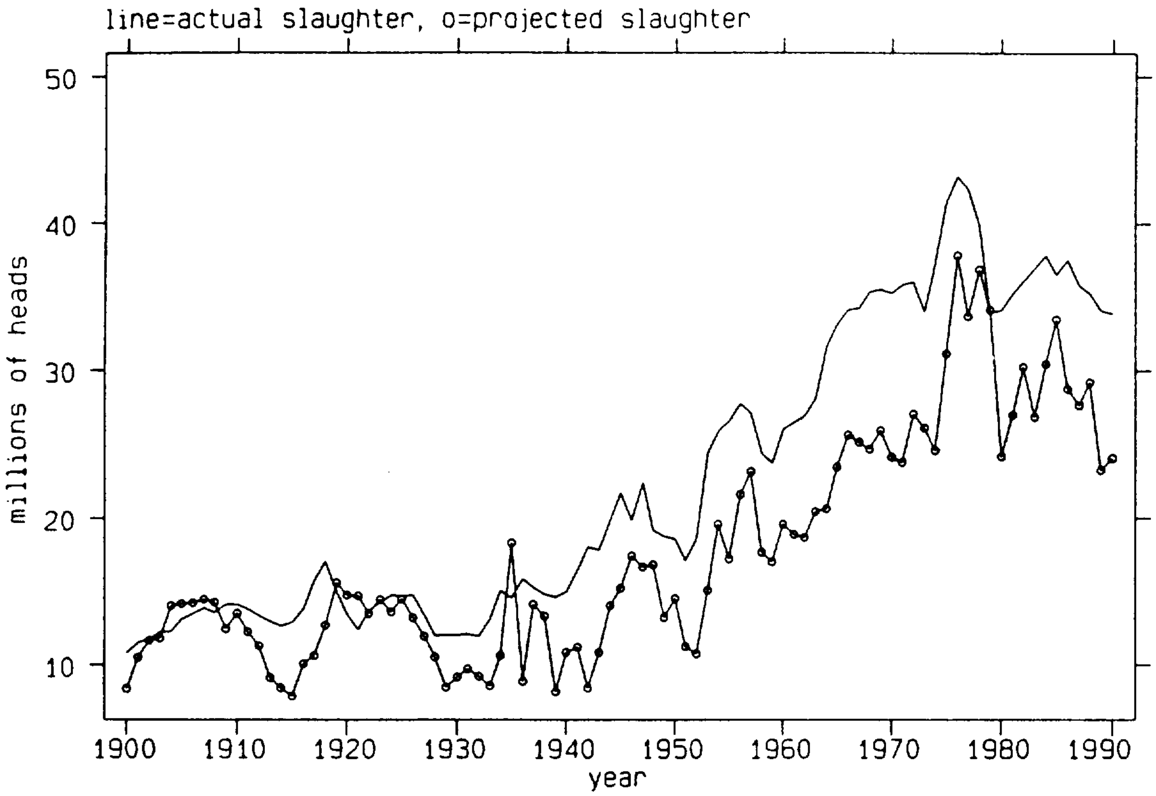


BREEDING STOCK AND TOTAL STOCK RESPONSES

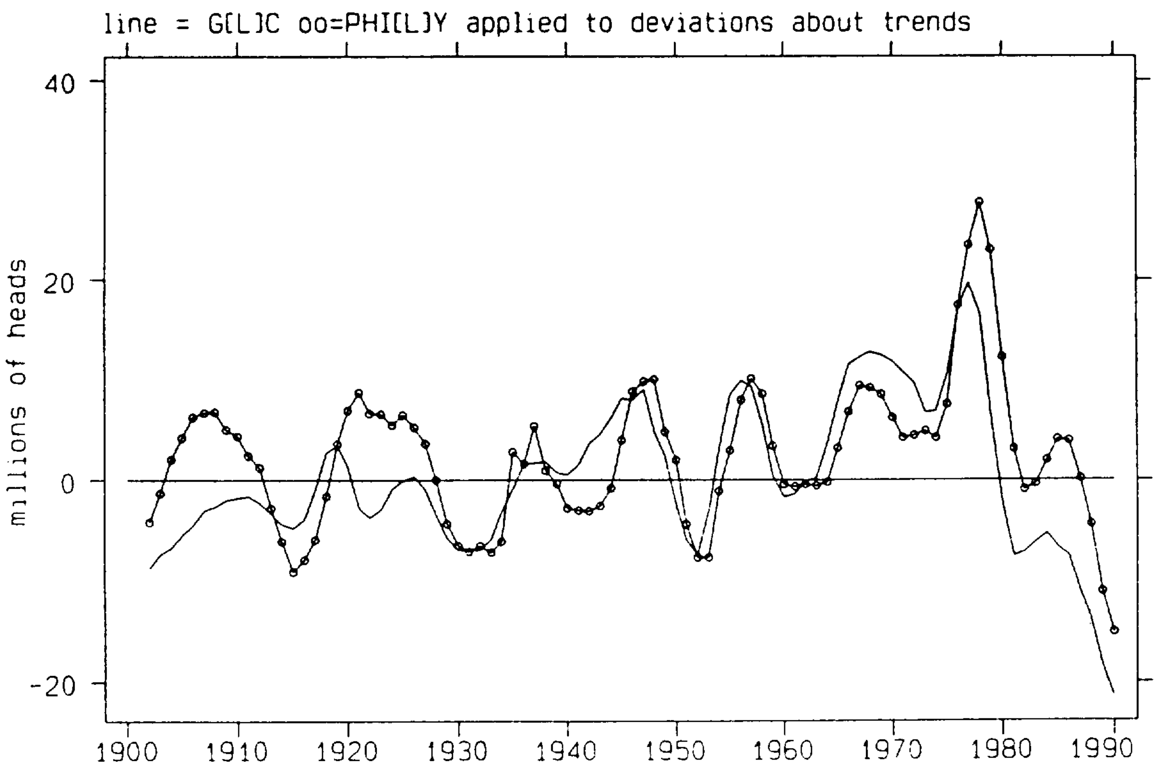


BREEDING STOCK AND TOTAL STOCK RESPONSES

FIGURE 4: DISTRIBUTED LAG COMPARISONS



Actual and Model-Projected Consumption



Cumulated Slaughter/Total Stock Comparison

FIGURE 5

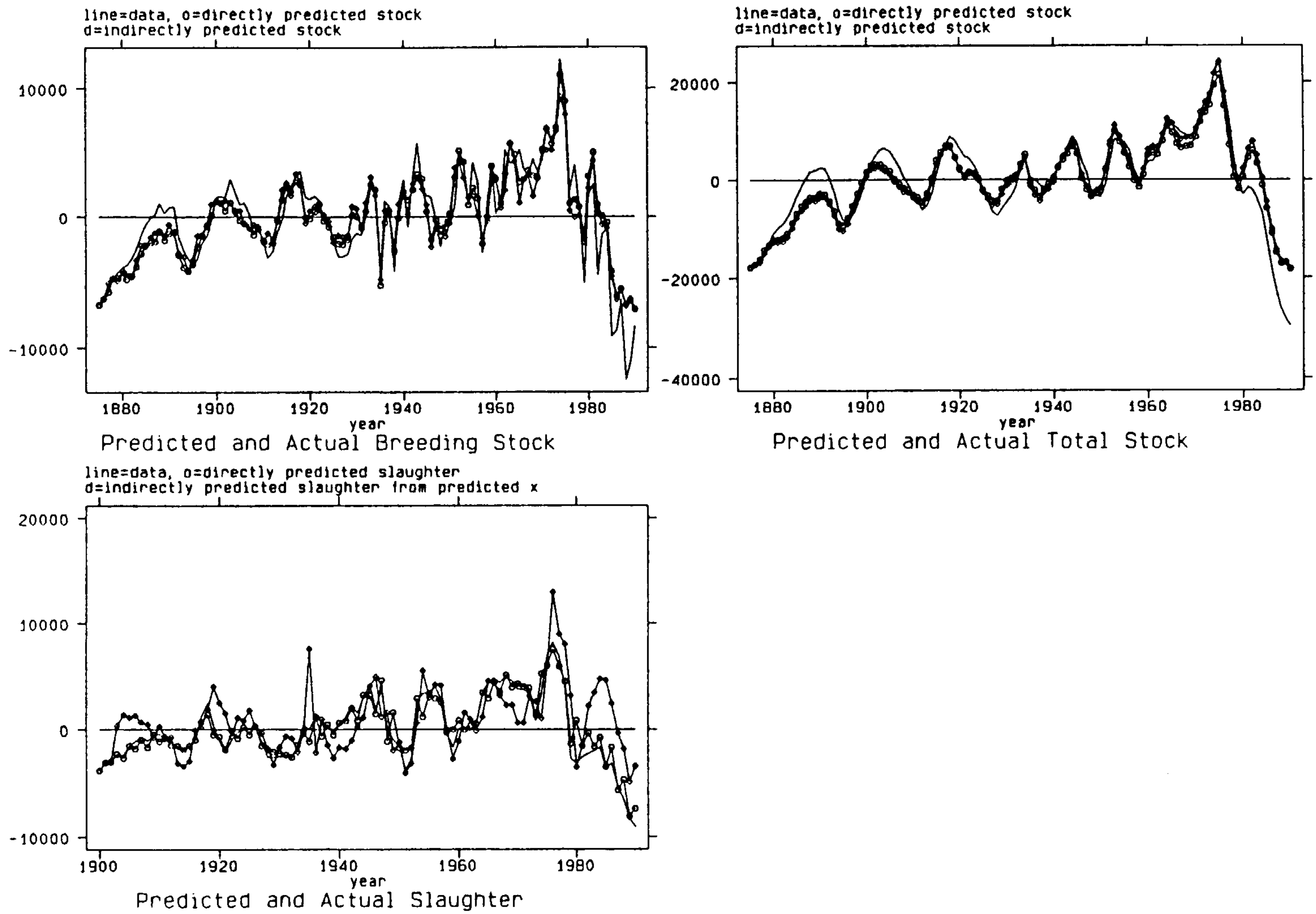

line=data, o=directly predicted stock
d=indirectly predicted stock
10000
0
-10000
1880
1900
1920
1940
1960
1980
year
Predicted and Actual Breeding Stock
line=data, o=directly predicted stock
d=indirectly predicted stock
20000
0
-20000
-40000
1880
1900
1920
1940
1960
1980
year
Predicted and Actual Total Stock
line=data, o=directly predicted slaughter
d=indirectly predicted slaughter from predicted x
20000
10000
0
-10000
1900
1920
1940
1960
1980
year
Predicted and Actual Slaughter

FIGURE 6

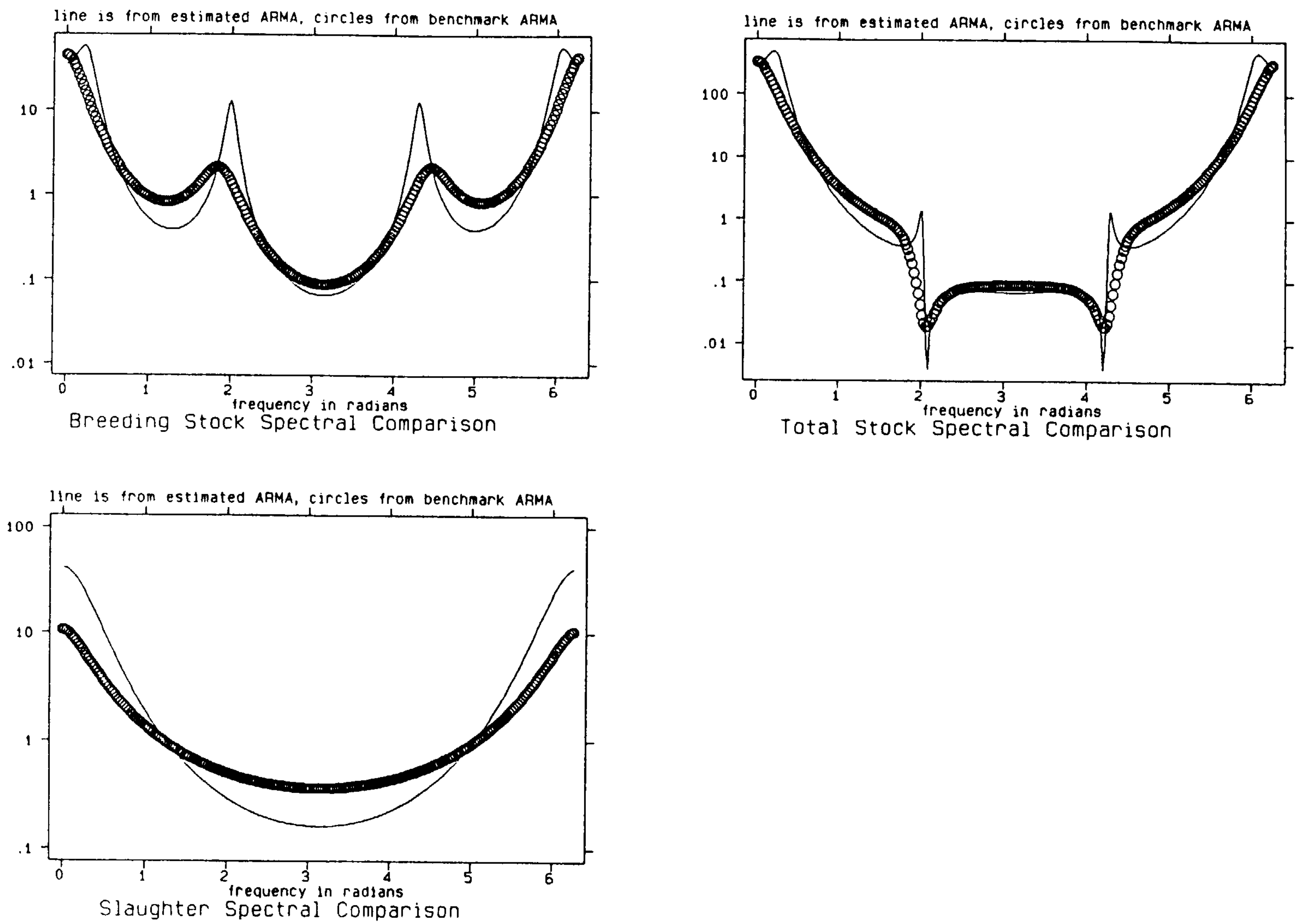

line is from estimated ARMA, circles from benchmark ARMA
10
1
.1
.01
0 1 2 3 4 5 6
frequency in radians
Breeding Stock Spectral Comparison
line is from estimated ARMA, circles from benchmark ARMA
100
10
1
.1
.01
0 1 2 3 4 5 6
frequency in radians
Total Stock Spectral Comparison
line is from estimated ARMA, circles from benchmark ARMA
100
10
1
.1
0 1 2 3 4 5 6
frequency in radians
Slaughter Spectral Comparison

FIGURE 7

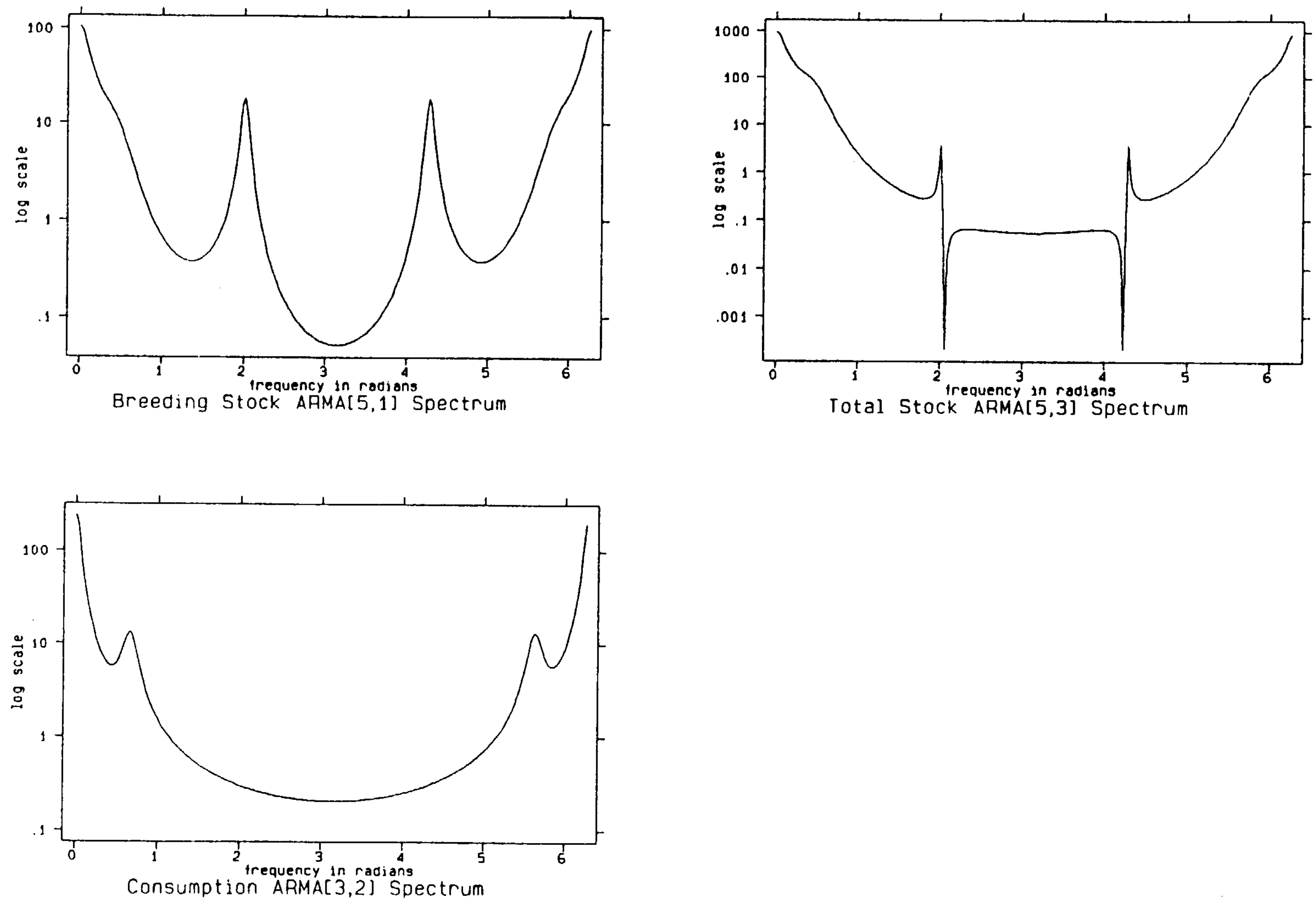
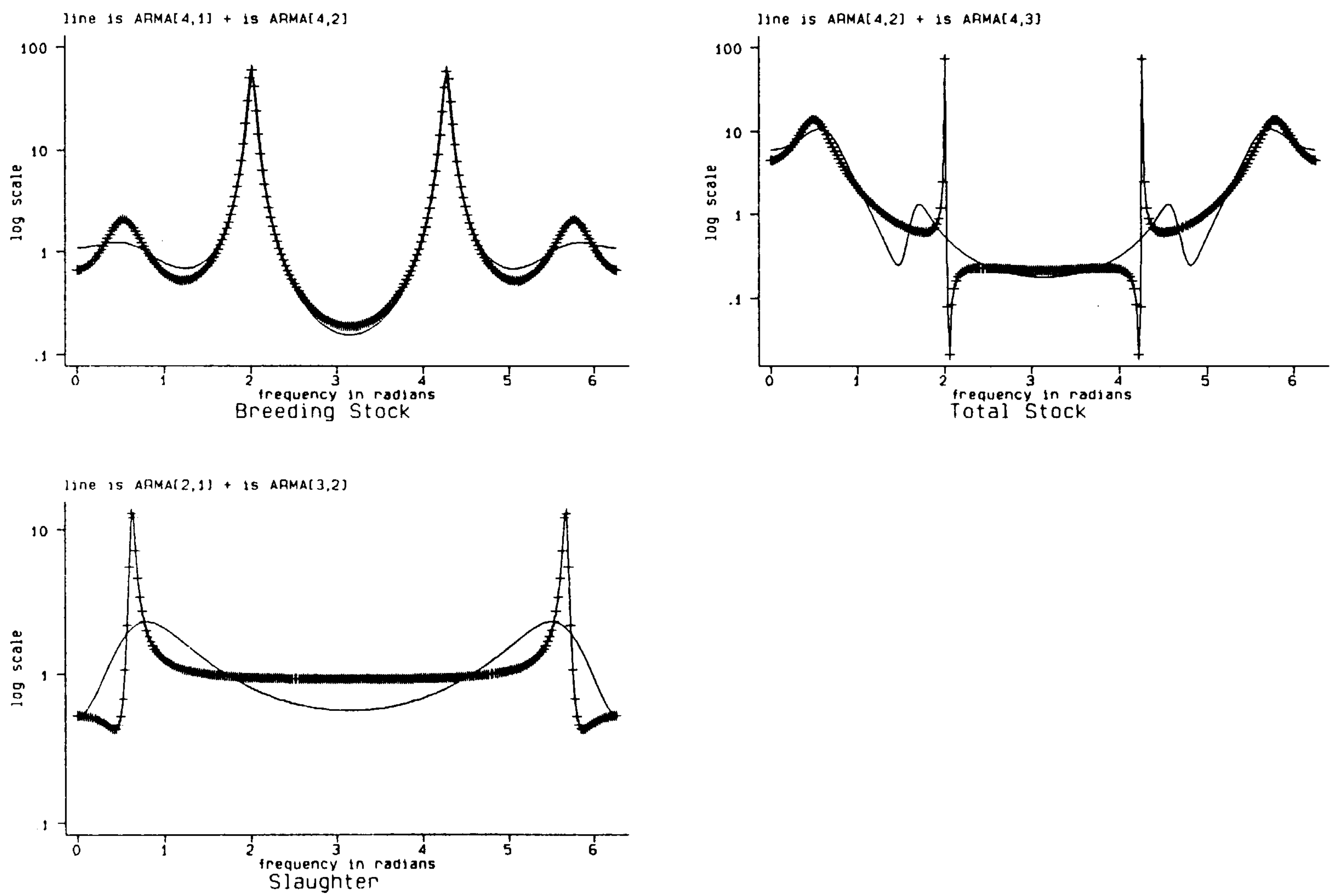

log scale
100
10
1
.1
0 1 2 3 4 5 6
frequency in radians
Breeding Stock ARMA[5,1] Spectrum
log scale
1000
100
10
1
.1
.01
.001
0 1 2 3 4 5 6
frequency in radians
Total Stock ARMA[5,3] Spectrum
log scale
100
10
1
.1
0 1 2 3 4 5 6
frequency in radians
Consumption ARMA[3,2] Spectrum

FIGURE 8



Spectra from Differenced Estimates

FIGURE A1