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TAXES, DEFAULT RISK, AND YIELD SPREADS

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ABSTRACT

This paper represents an extension and integration of recent empirical and theoretical research on default risk and taxability. The purpose of the paper is to develop and test a model of interest rate spreads which incorporates both the effect of taxes and differences in default probabilities in a theoretically correct manner. There is an important fundamental difference between our approach to explaining yield spreads and the approach most commonly taken in literature. Unlike nearly all of the previous work, we do not begin with a yield spread model, i.e., one which begins by examining differences in yields, but rather begin with an expected return or pricing model, which can then be expressed in the yield spread format. This is a fundamental difference in approaches which we feel leads to a superior theoretical formulation which can then be tested empirically without many of the problems inherent in the alternative approach. The theoretical model is a simple extension of earlier work on default by Bierman and Hass (1975) and Yawitz (1977), altered appropriately to take explicit account of tax effects. While there is a considerable literature that analyzes the effect of taxability on rate spreads, we are unaware of any previous study that considers tax consequences in the event of default, a rather surprising omission.

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Students of financial markets can't help but be impressed by the number of new financial instruments that have been introduced successfully in recent years. The fixed income market has been particularly innovative, as "bonds" carrying a whole host of specific features have been successfully marketed. Bonds with small or zero coupons, variable coupon rates, or detachable warrants are three examples from among an expanding list of such features.

In a financial market experiencing an influx of new financing vehicles during a period of general interest rate volatility it is important for both borrowers and lenders to evaluate properly the numerous alternatives. Not surprisingly, there has developed a large and expanding literature attempting to explain the rate spreads between different fixed income securities. While this literature has addressed a broad range of issues, callability [12] and taxability [6,7,9] have been particularly important questions.

The literature on rate spreads has evolved much as one would expect. Early empirical work was in general not well founded in theory, and was not based on an underlying expected return (price) model. Later empirical research has tended to be more careful in its specification of factors effecting yield spreads and, as a consequence, better founded in valuation theory.[1]

This paper represents an extension and integration of recent empirical and theoretical research on default risk and taxability. The purpose of the paper is to develop and test a model of interest rate spreads which

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[1]An example of such an evolution begins with the important paper by Cook and Hendershott (C-H) [3] which estimates a comprehensive empirical model of yield spreads taking account of the effects of taxes, default risk, relative security supply, and through a simply proxy, for callability. Building on C-H, Yawitz and Marshall [12] reestimate their basic model, substituting a more sophisticated measure of the effect of call that is consistent with the general approach to option valuation. This more refined proxy for call enhances the overall explanatory power of the C-H equation and revises their conclusions about the effects of other factors on yield spreads.

incorporates both the effect of taxes and differences in default probabilities in a theoretically correct manner. The theoretical model is a simple extension of earlier work on default by Bierman and Hass [1] and Yawitz [11], altered appropriately to take explicit account of tax effects. While there is a considerable literature that analyzes the effect of taxability on rate spreads, we are unaware of any previous study that considers tax consequences in the event of default, a rather surprising omission.

There is an important fundamental difference between our approach to explaining yield spreads and the approach most commonly taken in the literature. Unlike nearly all of the previous work, we do not begin with a yield spread model, i.e., one which begins by examining differences in yields, but rather begin with an expected return or pricing model, which can then be expressed in the yield spread format. This is a fundamental difference in approaches which we feel leads to a superior theoretical formulation which can then be tested empirically without many of the problems inherent in the alternative approach. For example, later in the paper we develop and then test a model which relates the municipal bond yield to three factors: the tax rate, the probability of default on the municipal bond, and the yield on an equal maturity government bond. Beginning with the pricing model, it is evident that these three factors enter the regression equation in a specific multiplicative fashion that would have been very difficult to ascertain if one had instead begun with a yield spread model. This distinction will be highlighted at various points throughout our paper.

Of particular interest for our purposes are two recent papers, one by Trzcinka [9] and one by Kidwell and Trzcinka [6], which are typical of the most commonly used approach to modelling yield spreads between taxable and non-taxable bonds. These analyses provide a useful introduction to our paper. As Trzcinka correctly states, "If two bonds are identical except for tax status, an investor will be indifferent between them if the return on the tax-exempt bond, R_m, is equal to the aftertax return on the taxable bond:

$$R_m = (1-t)R_T \tag{1}$$

where t is the tax rate paid on the taxable return, R_T".

Equation (1) has been modified by Trzcinka and others to allow for differences in default risk by including an intercept term λ that presumably captures any differential risk premium between the two securities.

$$R_m = \lambda + (1-t)R_T \tag{2}$$

When equation (2) was estimated by Trzcinka a shifting intercept technique was employed to allow this risk premium to change over time. The Trzcinka equation can be written as follows:

$$R_{m_t} = \lambda_t + BR_{T_t} \tag{3}$$

A positive value of λ_t has been interpreted to imply that in period t the tax-free security includes a larger risk premium than the taxable security. The specific shifting parameter technique used by Trzcinka assumed that λ_t was a random walk through time. Trzcinka then interpreted the value of $\hat{B}$ as an estimate of 1-t, and used this estimate to test the Miller Hypothesis.[2]

Several comments are in order regarding the proper interpretation of this relationship. Using what we feel is a reasonable theoretical model to capture the tax consequences of default, we are able to show that if the two bonds have different probabilities of default, then B depends upon the relative magnitudes of these probabilities and cannot be interpreted simply as an estimate of 1-t. It is evident, then, that an estimation technique which allows the risk premium for default to vary stochastically is theoretically inconsistent with a stationary value of B. Stated alternatively, if the relative risk of the two bonds varies over time, then both the constant term <u>and</u> the slope coefficients in equation (2) will vary.[3]

In this paper we develop a simple theoretical model which allows us to express the relationship between tax free (municipal) and taxable bonds using a general linear equation.

$$R_{M_t} = \alpha_t + \beta_t R_{T_t} + \varepsilon_t \tag{4}$$

The key insight from this model is that the parameters α_t and β_t are specific non-linear functions of the breakeven tax rate and the default probabilities on the two securities. When equation (4) is estimated using two

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[2]Miller [7] hypothesized that in equilibrium the differential between default free taxable and non-taxable bonds will reflect the product of one minus the marginal corporate tax rate and one minus the marginal tax rate on equity income. Since the latter tax rate is usually assumed to be zero in tests of the Miller hypothesis, yield spreads between taxable and non-taxable bonds are compared to the corporate tax rate.

[3]It has come to our attention that Buser and Hess [2] attempt to modify the results of Trzcinka by incorporating the costs of leverage into their analysis. Their results and ours may be viewed as complementary in this regard.

risky bond yields, the non-linear restrictions implied by the theory allow us to infer the ratio of the default probabilities on the two securities. When a risky and a riskless yield are compared, as in this paper, one obtains a direct estimate of the default probability on the risky bond. In both cases the non-linear restrictions imposed by the theory allow one to infer the break-even tax rate.

The remainder of this paper is organized as follows. In Section I we develop the analytical model of yield spreads. Sections II and III report the results of a series of empirical tests of the basic model. In Section II default probabilities are assumed to be constant throughout the sample period, while in Section III this assumption is relaxed and default probabilities are allowed to vary with overall economic activity. The final section presents a discussion of the major implications of the paper and suggestions for future research in the area of rate spreads.

I. The Default Model

In this section we present our simple model of the default process which is then used to develop a yield spread relationship between various types of securities. We initially assume that the probability that a borrower makes the full contractual payment in the stated period, conditional on default having not previously occurred in an earlier period, is constant over time and equal to P[4]. If default occurs, no payment is made.[5] This specification is

[4]As indicated above this assumption is modified in Section III.

[5]This "zero-one" specification of the return outcomes represents a considerable simplification since partial payment or renegotiation are generally also possible outcomes. We do not feel, however that this simplification introduces a problem within the context of our risk neutral framework since the zero-one outcome can be viewed simply as capturing the expected value of the more complex payment stream.

consistent with a model in which the probability function is stationary with the conditional distribution of default uniform over time.

Using this model of the default process, Bierman and Hass [1] and Yawitz [11] have demonstrated that in a risk neutral world without taxes the yield spread between a risky bond (r) and a risk free bond (i) can be expressed as:

$$e = r - i = \frac{(1+i)(1-P)}{P} \tag{5}$$

While this result is obvious for single period bonds, it is less obvious for multiperiod securities. As demonstrated by Yawitz [8], the intuition is straightforward. The present value of a risk free dollar in year t is simply $(1+i)^{-t}$. Given the default process assumed above, the expected value of a risky dollar to be received in period t is simply $P^t(1+r)^{-t}$. Under risk neutrality these two quantities must be identical for all t. This equivalence results in equation (5), which indicates that the yield spread between risk free bonds and risky bonds with a stationary conditional probability of default is <u>independent</u> of the maturity of the bonds. Given this result, the remainder of the theoretical analysis employs one period bonds for ease of exposition.

In this paper we modify the above result by introducing taxes into the analysis of yield spreads between risky and risk free bonds. As we indicated earlier, we were surprised to find that the tax implications of default have not previously been incorporated into analytical or empirical models of interest rate levels and spreads. In this paper the tax environment for fixed income securities is modelled as follows:

(1) the coupon payment is either taxable as ordinary income <u>or</u> it is tax free, depending on the type of bond, and

(2) in the event of default, no payment is made and the foregone principal is immediately deductible from taxable income. Whether a complete deduction is applicable (ordinary loss) or the deduction is treated as a capital loss depends upon the tax status of the investor and the length of time the bond was held.[6]

In the theoretical model below we allow for a distinction between the ordinary income tax rate, τ, and the rate applicable to this default loss, whether this is the ordinary or capital gain rate, by specifying that the latter is a fixed proportion, α, of the former, where $0 < \alpha < 1$.[7] Taking account of taxes, equations (6) and (7) express the after-tax payment outcomes from investing a dollar in risky bonds that have either a taxable or a non-taxable coupon, respectively. Thus, equation (6) would describe a corporate bond, while (7) would describe a municipal bond.

$$\theta_C = \begin{cases} [1 + r_C(1-\tau)] & \text{with probability } P_C \\ \alpha\tau & \text{with probability } (1-P_C) \end{cases} \tag{6}$$

$$\theta_M = \begin{cases} (1+r_M) & \text{with probability } P_M \\ \alpha\tau & \text{with probability } (1-P_M) \end{cases} \tag{7}$$

Throughout this discussion r's denote loan rates (yields) and P's denote the

[6]For individuals default results in a capital loss if the bond was purchased more than one year prior to default. However, banks are always allowed to treat default as an ordinary loss for tax purposes. The particular tax treatment in the event of default that is impounded in the yields on risky bonds will depend upon the tax status of the marginal investor in those bonds. The empirical section of this paper attempts to identify whether default is treated as a capital loss or an ordinary loss at the margin.

[7]This simple view is consistent with the treatment of capital gain under the U.S. tax system.

probability of payment.

The expected after tax rate of return on the corporate bond can be expressed as

$$\bar{R}_C = E(\theta_C)-1 = P_C r_C(1-\tau) - (1-P_C)(1-\alpha\tau), \tag{8}$$

while the expected rate of return on the municipal bond is

$$\bar{R}_M = E(\theta_M)-1 = P_M r_M - (1-P_M)(1-\alpha\tau). \tag{9}$$

These expressions for $\bar{R}_M$ and $\bar{R}_C$ can be interpreted as follows: the first term is the expected after tax coupon (interest) payment; the second term is the expected after tax <u>loss</u> due to default. The absolute value of the second term varies <u>inversely</u> with the tax rate, indicating that for a given market yield the possibility of default results in a larger reduction in expected after tax return, the smaller the tax rate.

Yield spreads among the various securities can be derived easily if the assumption of risk neutrality is maintained. We consider two other bonds, one taxable, and one tax free, and both riskless. The default free, tax free rate of return is denoted by i, while the default free, taxable rate is denoted by r_g (government bonds). Note that for the risk free, tax free security, its expected return, $\bar{R}_i$, equals i, while the after tax return on the risk free government bond, $\bar{R}_g$, equals $r_g(1-\tau)$. One particularly interesting exercise is to derive the equilibrium relationship between the promised yields on government, municipal, and corporate bonds, and the risk free, tax free rate. Equations (10) - (12) express these relationships.

$$r_g = \frac{i}{1-\tau} \tag{10}$$

$$r_M = \frac{i}{P_M} + \frac{(1-P_M)}{P_M}(1-\alpha\tau) \tag{11}$$

$$r_c = \frac{i}{P_c(1-\tau)} + \frac{(1-P_c)}{P_c}\frac{(1-\alpha\tau)}{(1-\tau)} \tag{12}$$

Consider first the municipal rate r_M. If $\alpha\tau=1$ then $r_M = \frac{i}{P_M}$, indicating that principal is completely insured by the tax code in the event of default, but that the coupon payment is not insured. As $\alpha\tau$ falls toward zero, r_M must increase reflecting the additional compensation required for the possibility that a portion of the principal will not be recovered in the event of default. That is, the lower the tax rate that is applicable to a default, the lower the "insurance coverage" provided by the tax code and the greater the yield compensation required by investors.

Figure 1 indicates how the tax rate affects the locus of yield-default probability combinations for which the expected after-tax return is a constant i. The figure is drawn assuming a constant value for α. Note that for a given probability of default, the yield required by an investor to be indifferent between a risky and a riskless <u>tax free</u> bond varies <u>inversely</u> with the investor's tax rate. Stated alternatively, the expected after-tax rate of return on a given risky tax free bond varies <u>directly</u> with the investor's tax rate. The intuition here is straightforward. If a tax-free bond makes its promised interest payment, the realized after-tax return is obviously independent of the investor's tax rate. On the other hand, if the bond defaults, the investor's loss is $(1-\alpha\tau)$ per dollar of principal since the government bears $\alpha\tau$ of the loss. The larger the tax rate, the smaller the after tax loss in the event of default, and hence, the lower the required yield on the bond. The fact that the investor's tax rate is relevant for making comparisons among tax free bonds with different default probabilities is due to the asymmetric effect of taxes on the two return outcomes. If payment is made, the tax system is irrelevant. If default occurs, the capital gains rate determines the size of the loss to the investor.

The above discussion has important implications regarding the types of investors that will be attracted to risky, tax-free bonds. Suppose risky bonds initially offer yields which plot on the locus of municipal yields with $\tau = 0$ in Figure 1. While the investor with a zero tax rate would then be indifferent among all tax-free bonds, an investor with a positive tax rate would have a preference for risky bonds, since the expected after tax return for taxable investors would be higher, the greater the bond's default probability. This demand for risky bonds would increase their prices, reducing their yields. In Figure 1 this would tend to rotate the locus of municipal yields clockwise. If equilibrium were to settle at the point where the relationship between r_M and (1-P) reflects a tax rate of .4, then investors with tax rates lower than .4 would purchase risk free tax free bonds, while investors with tax rates higher than .4 would purchase risky tax free bonds.[8]

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[8]This clientele effect can be demonstrated easily using equations (9) and (11). Suppose equilibrium in the municipal market settles at a point where the breakeven tax rate is τ^*. From equation (11) the yield default relationship will be as follows:

$$r_M^* = \frac{i}{P_M} + \frac{1-P_M}{P_M}(1 - \alpha\tau^*) \qquad \text{(F1)}$$

Substituting into equation (9) yields the expected after tax return from investing in municipal bonds to any investor.

$$\bar{R}_M = P_M r_M^* + (1-P_M)(\alpha\tau-1) = i + \alpha(1-P_M)(\tau-\tau^*) \qquad \text{(F2)}$$

According to the above equation the expected after tax return from investing in a municipal bond will be greater (less) than i when τ is greater (less) than τ^*. In addition, since the default probability is multiplied by the difference between the two tax rates, it is clear that high tax investors gain more the higher the default probability.

Figure 1 also portrays the effect of default on taxable bond yields. As indicated by (10), the required yield on a riskless taxable bond varies directly with the investor's tax rate. This effect is straightforward and can be seen by moving upward along the vertical axis in Figure 1. The introduction of the possibility of default, however, complicates the analysis considerably. From Equation (12) it can be demonstrated that for given values of i and P_c, the required yield on a risky, taxable bond varies directly with the investor's tax rate, i.e.,

$$\frac{\partial r_c}{\partial \tau} = \frac{1}{P_c(1-\tau)^2}\left[i + (1-P_c)\ (1-\alpha)\right] \tag{13}$$

Two separate effects can be noted here. The higher the tax rate the greater the yield required to make the expected after tax coupon payments on a taxable bond equal to that on a risk free, tax free bond. This compensation is greater, the greater the probability of default since the compensation for default risk (higher coupon) is itself taxable in this case. The second effect of taxes on the yield of a risky taxable bond occurs through the expected after tax loss of principal in the event of default. The higher the investor's tax rate, the lower is the expected after tax loss from default due to the "insurance" aspect of the tax code. However, since the former effect must dominate even if the write-off is at the ordinary income tax rate (α=1), the net effect on yields of a higher tax rate remains positive ($\partial r_c/\partial\tau$) > 0. Thus, in contrast to the case of a tax-free security, for taxable securities the required yield varies <u>directly</u> with the investor's tax rate.

As in the case of tax-free securities, the tax implications of default lead to a clientele effect for taxable securities as well. However, in this case the clientele effect interacts with the default probability in the opposite direction. To illustrate this, suppose the breakeven tax rate for all probabilities of default is τ^*. The yield-default relationship for taxable securities will be as follows:

$$r_c^* = \frac{i}{P_c(1-\tau^*)} + \frac{(1-P_c)\quad(1-\alpha\tau^*)}{P_c(1-\tau^*)} \tag{14}$$

Substituting equation (14) into equation (8), one obtains an expression for the expected after tax return for an investor with a tax rate τ.

$$\bar{R}_c = P_c r_c^* (1-\tau) + (1-P_c)\ (\alpha\tau - 1) = i\ \frac{(1-\tau)}{(1-\tau^*)} + (1-P_c)\ \left[\frac{(1-\tau)(1-\alpha\tau^*)}{(1-\tau^*)} - (1-\alpha\tau)\right]$$

$$= \frac{[i(1-\tau) + (1-P_c)\ (1-\alpha)\ (\tau^* - \tau)]}{(1-\tau^*)} \qquad (15)$$

Equation (15) implies that the expected after tax return will be greater (less) than i when the investor's tax rate is less (greater) than the breakeven tax rate. The interesting result, however, is the implication of different default probabilities. Note that as long as capital losses are not fully taxed (i.e., $\alpha < 1$) investors with tax rates less than the breakeven tax rate will prefer less risky taxable securities. This preference disappears if capital losses are fully taxed.

II. Empirical Results: Constant Probabilities

In this section we empirically investigate the relationship between two particular bond types, municipals and governments. Our selection of a risky and a riskless bond allows us to obtain a point estimate of the probability of default for the risky (municipal) security. These particular yield series are employed to minimize the possible complications from callability. Governments are either non-callable in the case of bills and intermediate maturity bonds or callable only in the last few years in the case of long term issues. Municipals with an initial maturity of ten years or less are not callable. The "industry" norm is for long term bonds to be callable at $102, but with 10 years of call protection.

Finally this choice of yield series minimizes the measurement error problem common to yield spread studies. As is well-known, random measurement error in the independent variable leads to coefficients biased toward zero in any least squares estimation while measurement error in the dependent variable does not result in bias though it does lead to less efficient estimates. Yield spread studies, e.g., Trczinka [9], often regress municipal and corporate rate series for equivalent ratings and terms to maturity on each other. Since both municipals and corporates are heterogeneous and lightly traded, these series undoubtedly contain measurement error and yield biased parameter estimates. Since U.S. government issues are more homogeneous and more heavily traded, this measurement error is minimized and the use of governments as the independent variable minimizes measurement error bias in the parameters.

Using equations (10) and (11) from the previous section, we can express the appropriate equilibrium relationship between the yield on a risky municipal bond and a default free government bond as follows,

$$r_M = \frac{(1-P)(1-\alpha\tau)}{P} + \frac{(1-\tau)}{P} r_g \qquad (16)$$

where τ is the breakeven tax rate for this probability of default, α is the proportion of default losses that are written off as ordinary income, and P is the probability of payment for the municipal. Note that although there is a linear relationship between r_M and r_g according to the theoretical model of the previous section, the slope and intercept of that relationship are both non-linear functions of the probability of payment and the breakeven tax rate. In particular, it should be noted that the slope coefficient does not equal one minus the breakeven tax rate, as commonly assumed, unless the tax free security is also default free. Thus, previous studies that make this inference underestimate that tax rate when the tax free security is subject to a non-zero probability of default.

In this section we estimate equation (16) two ways. A regression analysis of one variable on another can provide only two unique parameter estimates. Since equation (16) has three parameters, one parameter must be fixed throughout the estimation. The empirical results that follow include one of two alternative assumptions. In the first case α is assumed to equal unity for all maturities. This implicitly assumes that the marginal investor is allowed to fully deduct capital losses from income for tax purposes (e.g. banks). These results are contained in the unstarred columns of Table I. The alternative assumption is that for all maturities greater than one year α equals the percentage of capital gains (losses) that are included in taxable income.[9] These results are contained in the starred columns of Table I. A comparison of the results in the two cases should shed some light on the appropriate assumption about this parameter.

Before turning to the empirical results some brief comments on the data and estimation technique are in order. The data used in this study consist of monthly observations of the yields required to sell new municipal and government securities at par.[10] The data sample begins in August, 1965, and ends in March, 1981, and includes observations with maturities of 20, 10, 5, and 1 years, respectively. In addition, the municipal bond data includes observations from three different grades, prime, good, and medium. Thus, there are twelve separate regressions that can be performed.

The estimation technique employed in this paper is an iterative non-linear least squares procedure developed by Ralston and Jennrich [8]. This procedure produces consistent (under weak conditions) estimates of all model parameters by minimizing the sum of the squared errors. These are maximum likelihood estimates if the error term is normally distributed. It also produces an asymptotic variance-

[9]Throughout most of the sample period that percentage was .4, but for some of the earlier periods it equalled .5. Note that if α had not varied over the sample period the least squares estimate of the slope and intercept and the sum of squares error would have been equal to those for $\alpha = 1$ (or any other constant α). However, estimates of P and τ would have varied with α in a known manner.

[10]The government yield series were obtained from the Citibank Economic Database, while the municipal yield series were obtained from the Analytical Record of Yields and Yield Spreads, published by Salomon Brothers.

covariance matrix for the parameter estimates which was used to obtain t-statistics for each estimated parameter.

The results in Table I provide strong evidence that this model of the relationship between a risky, tax free bond and a riskless taxable bond is appropriate. In every equation estimated the R^2 exceeds .987 and the t-statistics are significant at the 1% level.[11] In addition the implied values for P and τ are extremely plausible. Closer inspection of the results also yields the following conclusions and conjectures.

(1) In most cases the estimated probability of payment for equal maturity bonds is lower the lower the bond's rating. The exceptions occur for maturities of 20 and 10 years. In those cases the estimated P values for good and medium bonds are virtually identical.

(2) The behavior of the probability of payment for different maturity, equal rated bonds also tends to conform to the assumption of the model. In most cases the estimated values of P for any grade move very little as maturity changes.

(3) There seems to be little systematic relationship between the estimated tax rates and bond rating. The estimated tax rates often increased as one moved from prime to good rated bonds, as predicted by our analysis of clientele effects, but decreased as one moved from good to medium rated bonds.

(4) There is a strong relationship between the estimated tax rates and the maturity of the bonds. In every case the estimated tax rate for bonds

[11]It should be noted that the residuals exhibited first order autocorrelation. Therefore, the estimated standard errors used to calculate the t-statistics are biased downward and the t-statistics are overstated.

of a given grade increased as the maturity was decreased. This result, which is consistent with a market segmentation hypothesis in the municipal market,[12] could also be due to the effect of callability which *ceteris paribus* increases the 20 year municipal yield.

(5) The evidence regarding the appropriate assumption about α is somewhat mixed. A comparison of the starred and unstarred entries in Table I indicates that for 10 and 20 year maturities the assumption of fully deductible capital losses ($\alpha = 1$) results in slightly lower R^2's and slightly higher standard errors than the alternative assumption. For the 5 year maturity the results are virtually identical.[13] Thus, there is weak evidence that the marginal investors in long-term securities are individuals rather than institutions. It should also be noted that the assumption of full deduction for capital losses results in lower estimates of the probability of payment than the alternative assumption.

III. Empirical Results: Variable Probabilities

In this section we modify the assumption that the conditional probability of payment is constant through time by allowing that parameter to vary with the overall level of economic activity. Previous studies have argued that default probabilities should vary with the business cycle. In this section we operationalize this argument by assuming that the conditional probability of payment at any given time t can be expressed as follows,

$$P_t = P_o + P_1 \% \Delta \, IP_t + \mu_t, \qquad (17)$$

[12] See Hendershott and Kidwell [5] for a discussion of this hypothesis.

[13] Recall that for maturities of 1 year or less capital losses are by definition short-term and are fully deductible.

where $\%\Delta IP$ is the percentage change in the industrial production index and μ_t is a white noise error term. Since investors are assumed in this study to be expected return maximizers, the expected value of P_t will be a determinant of the yield on a risky bond. If we make the additional assumption that either expectations are static or that the percentage change in industrial production is a stationary process, then the appropriate empirical relationship between equal maturity municipal and government yields will be as follows.[14]

$$r_{Mt} = ((1-\alpha\tau)\ [(1/(P_o + P_1\ \%\Delta IP_t)) - 1]) + (1 - \tau)r_{gt}/\ (P_o + P_1\ \%\Delta IP_t) \quad (18)$$

Table II presents the results of the estimation of equation (18) using the Ralston-Jennrich nonlinear least squares procedure. As in the previous section two alternative assumptions about the parameter α are employed. These results are contained in the starred and unstarred rows of Table II, respectively.

The results in Table II are very similar to those in the previous section. Again the R^2's are quite high and the standard errors are quite low which attests to the appropriateness of this specification. In general the specification of a variable probability of payment improved the explanatory power of the overall model, particularly for the shorter maturities. In fact, the coefficient and t-statistic for the parameter P_1 is higher the shorter the maturity.[15] Thus, this

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[14]Either of these assumptions is sufficient to ensure that

$$E(\%\Delta IP_{t+j}) = \%\Delta IP_t \text{ for all } j \quad (F3)$$

[15]This probably reflects the fact that the assumption that $E(\%\Delta IP_{t+j}) = \%\Delta IP_t$ does not hold over long periods of time.

specification seems to be better suited to maturities of five years or less. The behavior of the estimated values for τ and P_0 are consistent with the results of the previous section. In particular, a strong relationship between maturity and the estimated tax rate is still evident in the results. The relationship between the parameter P_0 and grade and maturity also mirrors its counterpart from the previous section.

IV. Implications and Conclusions

The results of this study have a number of interesting implications. One particularly interesting implication of the empirical results is that the yield spreads among different grade municipal bonds are much smaller than the spread between a prime grade municipal bond and a government bond, even after adjustment for taxes. To see this consider the implications of the results in Table I for 5 year bonds. Suppose for simplicity that the breakeven tax rate is constant across probabilities of default at .5, that the government rate r_g is 5%, and that α is .4. Using the estimated P values in the starred columns, the following yields are implied.

	Pre-tax Yield	After tax yield
Government Bond	5%	2.5%
Prime Grade Municipal	3.46%	3.46%
Good Grade Municipal	3.56%	3.56%
Medium Grade Municipal	3.65%	3.65%

Note that the spread between the after tax yield on a government bond and a prime grade municipal is approximately 4 times as large as the spread between the yields on the prime and medium grade municipal bonds. Since the after tax yield on the government bond in this example corresponds to the rate on a hypothetical risk-free municipal security, this exercise indicates that prime grade municipal bonds do have significant risk premiums embodied in their yields.

We would anticipate that one would find our conclusion that even prime rated municipal bond yields contain a significant premium for default risk to be contrary to actual default experience on such securities. In fact, the historical evidence indicates that defaults on State and Local government bonds are not as rare as one might think. We direct the interested reader to Hempel's [4] study of bond defaults over the period 1839-1965 for evidence on this question.[16]

In summary, in this paper we have developed and tested a model of the interaction of default and taxation on yield spreads. The theoretical model, although quite simple, seems to have a high degree of explanatory power. The relationship between government yields and municipal yields was investigated empirically. The parameters representing default risk and tax rates from the empirical analysis are entirely believable. This strong evidence implies that this model is potentially useful for the determination of the interaction of default risk and taxation in studies of the yield spreads between other types of securities. In addition, future tests of the Miller hypothesis could be developed using this framework.

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[16]It has also been suggested to us that the seemingly large default probabilities may be due in part to the market's expectation that municipal securities could lose their tax exempt status before reaching maturity. To the extent that this is true our estimate of the default probability will be biased upward.

Dependent Variable	τ	P	SE	R^2
Prime 20	.425 (19.56)	.981 (301.91)	.0054	.9905
Prime 20*	.439 (19.61)	.984 (467.98)	.0053	.9907
Good 20	.433 (19.36)	.978 (287.43)	.0055	.9905
Good 20*	.448 (19.43)	.982 (446.89)	.0054	.9907
Medium 20	.393 (15.37)	.978 (264.06)	.0063	.9889
Medium 20*	.406 (15.31)	.982 (399.56)	.0063	.9891
Prime 10	.500 (29.19)	.979 (327.29)	.0042	.9927
Prime 10*	.507 (28.43)	.985 (567.02)	.0041	.9928
Good 10	.496 (27.33)	.977 (305.53)	.0044	.9922
Good 10*	.503 (26.58)	.984 (532.45)	.0044	.9923
Medium 10	.459 (21.43)	.978 (280.59)	.0052	.9905
Medium 10*	.466 (20.88)	.984 (461.78)	.0052	.9905
Prime 5	.521 (34.85)	.981 (370.20)	.0037	.9932
Prime 5*	.521 (33.45)	.988 (654.85)	.0037	.9931
Good 5	.516 (33.21)	.980 (366.47)	.0039	.9930
Good 5*	.517 (31.72)	.987 (624.24)	.0039	.9929

Table I
Estimation Results: Constant Probability

(t-statistics in parentheses)

Dependent Variable	τ	P	SE	R^2
Medium 5	.492 (26.87)	.979 (311.63)	.0046	.9915
Medium 5*	.494 (25.94)	.986 (538.20)	.0046	.9914
Prime 1	.568 (39.51)	.981 (346.12)	.0045	.9879
Prime 1*	---same as above---			
Good 1	.567 (40.32)	.978 (345.68)	.0044	.9891
Good 1*	---same as above---			
Medium 1	.553 (34.39)	.977 (307.42)	.0050	.9872
Medium 1*	---same as above---			

Table I

(Cont'd.)

Dependent Variable	τ	P_0	P_1	SE	R^2
Prime 20	.429 (19.79)	.980 (295.66)	.122 (1.90)	.0053	.9907
Prime 20*	.444 (19.85)	.984 (463.21)	.088 (1.96)	.00529	.9910
Good 20	.437 (19.58)	.976 (273.74)	.113 (1.71)	.00551	.9907
Good 20*	.452 (19.61)	.981 (442.12)	.082 (1.76)	.0054	.9909
Medium 20	.396 (15.46)	.977 (257.98)	.096 (1.34)	.0063	.9890
Medium 20*	.409 (15.41)	.982 (394.03)	.072 (1.37)	.0062	.9892
Prime 10	.505 (30.18)	.977 (324.30)	.189 (3.35)	.0040	.9931
Prime 10*	.512 (29.38)	.984 (572.32)	.122 (3.38)	.0040	.9932
Good 10	.501 (28.14)	.975 (301.76)	.184 (3.11)	.0043	.9927
Good 10*	.508 (27.34)	.983 (534.83)	.120 (3.12)	.0043	.9927
Medium 10	.464 (21.85)	.977 (275.28)	.160 (2.43)	.0051	.9908
Medium 10*	.470 (21.27)	.983 (459.72)	.109 (2.44)	.0051	.9908

Table II

Estimation Results: Variable Probability

(t statistics in parentheses)

Dependent Variable	τ	P_0	P_1	SE	R^2
Prime 5	.528 (37.06)	.980 (372.58)	.241 (4.64)	.0035	.9940
Prime 5*	.528 (35.49)	.987 (674.46)	.150 (4.63)	.0035	.9938
Good 5	.524 (35.15)	.978 (352.98)	.246 (4.57)	.0037	.9937
Good 5*	.524 (33.61)	.986 (641.79)	.154 (4.55)	.0037	.9936
Medium 5	.499 (27.84)	.977 (307.23)	.208 (3.40)	.0044	.9926
Medium 5*	.500 (26.84)	.985 (541.81)	.135 (3.39)	.0044	.9919
Prime 1	.578 (42.84)	.978 (347.62)	.374 (5.38)	.0042	.9896
Prime 1*	-----same as above-----				
Good 1	.577 (44.19)	.975 (350.18)	.385 (5.76)	.0040	.9908
Good 1*	-----same as above-----				
Medium 1	.563 (37.24)	.974 (306.98)	.392 (5.25)	.0047	.9889
Medium 1*	-----same as above-----				

Table II

(Cont'd)

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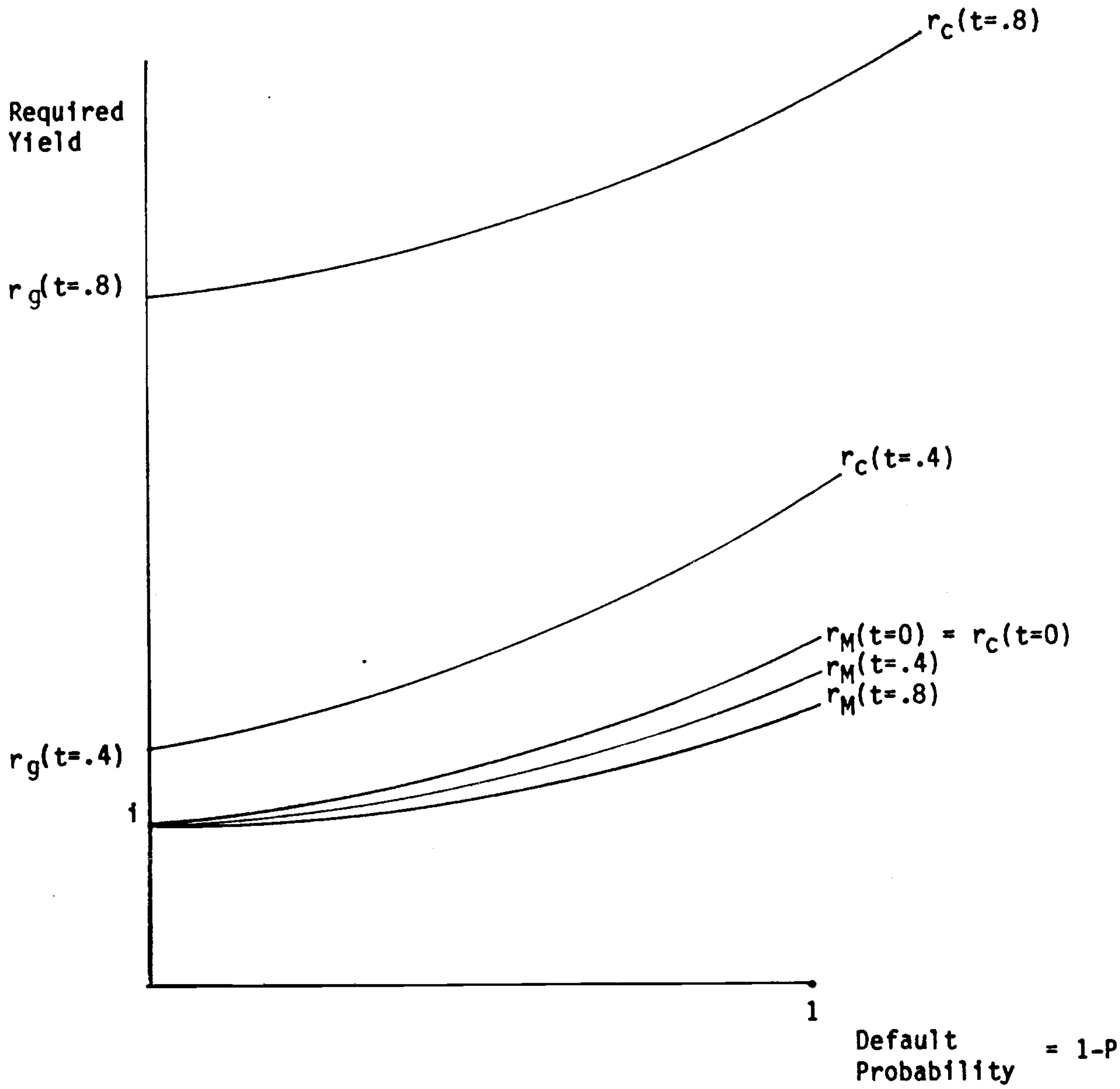

FIGURE I

The Effect of Default and Tax Rates on Yields