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REAL INTEREST RATES, HOME GOODS AND OPTIMAL EXTERNAL BORROWING

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Abstract

The paper investigates the optimal time path of consumption and external borrowing in the dependent economy model. The small country faces given world prices and a given world real interest rates. The presence of a home goods sector implies that the relevant real interest rate appropriate to consumption decisions depends on the rate of change of the real price of home goods. The paper shows how transitory disturbances in output or in the world real interest rate affect the time profile of consumption. In particular it is shown that the presence of a home goods sector dampens the consumption effects of changes in interest rates.

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REAL INTEREST RATES, HOME GOODS AND OPTIMAL EXTERNAL BORROWING*

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This paper investigates optimal external borrowing in a small open economy that faces a given world rate of interest and experiences supply disturbances. The main point of the paper is to study the interaction of alternative disturbances--present versus future, transitory versus permanent--and the time path of equilibrium real interest rate in setting the path of consumption and external indebtedness. This question of optimal consumption choices in an open economy, intertemporal setting is receiving considerable attention recently, especially in Razin and Svensson (1980), Sachs (1981), Obstfeld (1981).

In choosing a small country setting the present paper focusses on a simpler model, but it brings in an interesting issue raised in earlier work by Bruno (1976) and, independently, by Martin and Selowsky (1981). It is argued that in a small country with a nontraded goods sector the relevant real interest is not the given world interest rate stated in terms of the domestic consumption basket. Differences between the world real interest rate and the home real interest rate arise to the extent that the relative price of home goods is changing over time. Specifically if the relative price of home goods is rising the home real interst rate is less than the world real interest rate. Conversely, if the relative price of home goods

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is falling the home real interest rate exceeds the world rate. As we will see, to the extent that the time profile of disturbances affects the relative price structure over time it affects the home real interest rate and therefore the optimal path of consumption and borrowing.

1. The Consumer

We assume a small country producing traded and nontraded goods with a fixed supply of labor that is mobile between sectors and given, specific factors in traded and home goods sectors respectively. The world rate of interest is fixed in terms of traded goods and equals r*. The home country can borrow and lend at that rate at unlimited rates, respecting of course the intertemporal budget constraint.

The representative household solves an intertemporal maximisation problem, choosing a path of consumption and debt that maximizes discounted lifetime utility:

$$(1) \qquad V = \sum_{o}^{\infty} D^{t} U_{t}(c_T, c_N) \quad ; \quad D \equiv \frac{1}{1+\delta}$$

subject to

$$(2) \qquad p_t b_{t+1} = [c_N + p_t c_T - y_t] + R\, p_t b_t \quad ; \quad R \equiv 1+r^*$$

and

$$(3) \qquad \sum_{o}^{\infty} [c_N + p_t c_T - y_t] R^{-t} + p_o b_o = 0$$

Equation (1) is the discounted value of utility of the infinitely lived household with D the discount factor and U ( ) the stationary utility funtion. Consumption of traded and nontraded goods are the arguments in

the utility function and are denoted c_T and c_N respectively. Equation (2) relates debt, b, in period t+i+1 to consumption expenditure, income y_{t+i}, and debt service in period t+i. R=1+r* denotes one plus the world real rate of interest and thus $p_t R\, b_t$ represents principal and interest on the debt issue solicited in period t-1 with p_t representing the current relative price of traded goods in terms of home goods. Equation (3) is the lifetime budget constraint.

In what follows we will assume a specific functional form of the utility function:

$$(4) \qquad U_t = \frac{1}{1-\Theta} c_t^{1-\Theta} \quad ; \quad c = c_T^a c_N^{1-a}$$

The form of the utility function shows a constant relative risk aversion function with the parameter $1/\Theta$ representing the elasticity of intertemporal substitution. The term c represents an index of current consumption and has a Cobb-Douglas form.

It is shown in the appendix that consumer maximisation leads to an optimal consumption profile that depends on the ratio of the consumption-based home real rate of interest relative to the rate of time preference, ξ:

$$(5) \qquad c_t/c_{t+1} = \xi^{-1/\Theta}$$

The term ξ in (5) is the ratio of the home real interest rate relative to the rate of time preference and is defined in (5a):

$$(5a) \qquad \xi \equiv \frac{(1+r^*)\ (p_{t+1}/p_t)^{1-a}}{1+\delta} \equiv k(p_{t+1}/p_t)^{1-a} \quad ; \quad k = DR$$

We note that in (5a) the home relative price structure over time appears as a component of the real interest rate relevant to consumption choices. The higher the future compared to the current relative price of traded goods, the higher the home real interest rate, for any given world interest rate r* fixed in terms of traded goods.

The time profile of optimal consumption has two determinants. One is the relation between the home discount rate and the world rate of interest, the term $k \equiv DR$. We assume that the discount rate equals the world interest rate so that $k = 1$. The other determinant of the optimal consumption time profile is the relative price structure over time, p_{t+1}/p_t. It is apparent from (5) that when relative prices remain constant that ratio is unity and hence the consumption profile is flat.

The coefficient of risk aversion, Θ, represents the concavity of the utility function and measures the ease of intertemporal consumption substitution. If substitutability is perfect $1/\Theta$ tends to zero. Conversely, if consumption shows very little intertemporal substitutability consumption remains an entirely flat profile and consumers smooth completely, whatever the pattern of real interest rates. The schedule cc in Figure 1 below represents equation (5).

2. The Home Goods Market

The demand for home goods can be expressed in terms of the relative price and the level of consumption:

$$c_{N,t} = \phi\, c_t p_t^a \tag{6}$$

Home goods demand is proportional to consumption, the factor of proportionality

depending positively on the relative price of traded goods.

On the supply side, home goods output is a function of the relative price and a shift parameter $\bar{q}_t$. The supply elasticity is e.

$$q_{N,t} = \bar{q}_t p_t^{-e} \tag{7}$$

Equilibrium in the home goods market then yields a relation between consumption and the relative price:

$$p_t = [\bar{q}_t/\phi c_t]^{1/(a+e)} \tag{8}$$

and

$$p_t/p_{t+1} = [\bar{q}_{t+1}/\bar{q}_t)/(c_t/c_{t+1})]^{-1/(a+e)} \tag{9}$$

Equation (9) is shown, for $(\bar{q}_{t+1}/\bar{q}_t) = 1$ as the NN schedule in Figure 1. The schedule is negatively sloped since a high current relative to future consumption level implies a high current relative to future demand for home goods, thus requiring a high current relative price of home goods. The elasticity of the relative price with respect to the consumption profile is given by -1/(a+e) and thus is smaller the higher the supply elasticity and the larger the share of traded goods in consumption.

From (9) we note that the equilibrium relative price is affected by the position of the supply curve. A rise in current relative to future supply would raise the current relative price of traded goods and would appear in Figure 1 as a rightward shift of NN.

3. Stationary Equilibrium

With output stationary and the home discount rate equal to the world interest rate point A in Figure 1 represents the stationary equilibrium. Consumers chose a flat profile of consumption since there is no incentive to tilt, because of differences between home interest rates and those prevailing in the world, consumption toward the present or the future. The flat level of consumption $\bar{c}$ will be chosen so as to satisfy (3) and (8), leading to the values of consumption and the relative price:

$$(10) \qquad \bar{c} = \bar{c}(\bar{q}, b_o, ..) \quad , \quad \bar{p} = \bar{p}(\bar{q}, b_o, ..)$$

Figure 1 shows as the schedule cc the optimal consumption profile between adjacent periods as a function of the intertemporal relative price structure. Current consumption rises relative to future consumption as today's relative price of home goods rises relative to that next period. The elasticity of substitution with respect to the intertemporal relative price structure is $(1-a)/\theta$, reflecting both the coefficient of risk aversion and the share of home goods in the consumption basket. The explanation for the positively sloped cc schedule and for the elasticity $(1-a)/\theta$ are the following. The household substitutes in response to changes in the real interest rate. Increased real interest rates lead the consumer to raise future relative to current consumption. Now suppose the relative price of home goods is expected to fall. Then a unit of traded goods borrowed today has relatively little purchasing power in terms of the consumption basket today, but costs a lot in terms of the consumption basket upon repayment of the loan next period. Since the loan adds less to consumption today than it costs to repay tomorrow it is clear that with falling

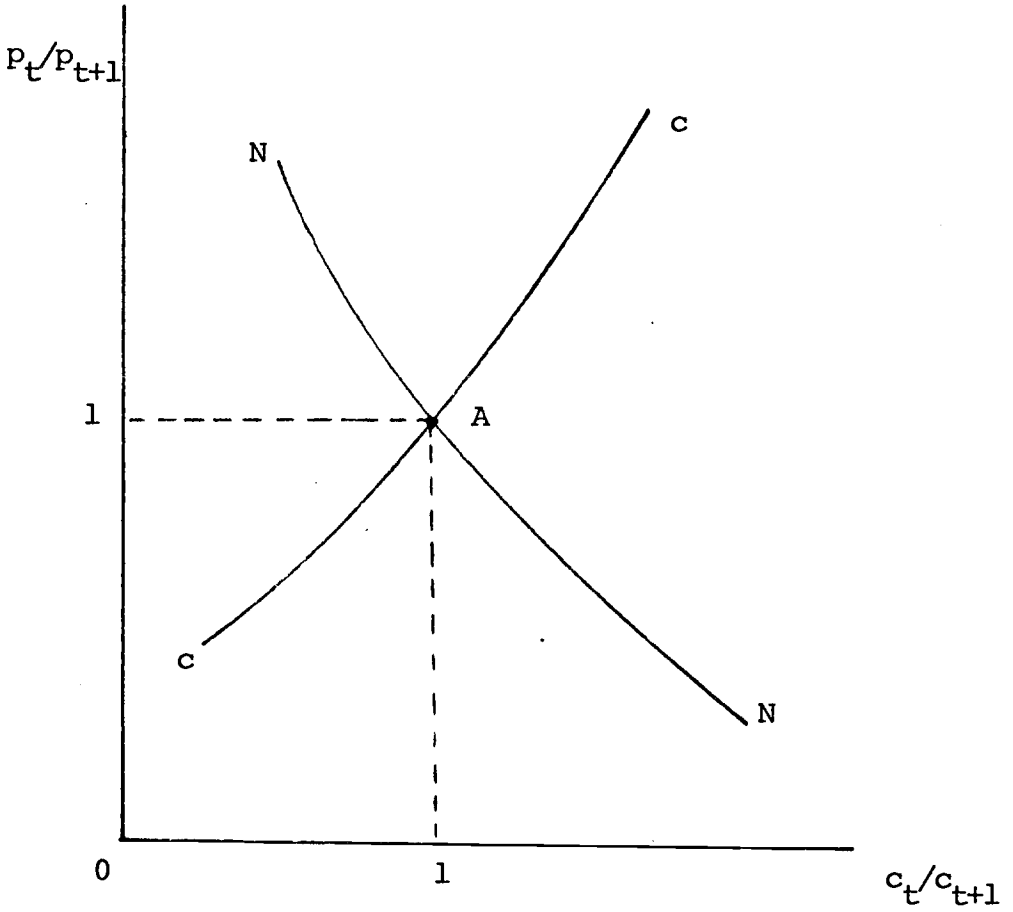

Figure 1

relative prices the real interest rate in terms of our consumption basket exceeds the world real rate. Conversely, if the relative price of home goods were rising the home real interest rate would be less than the world rate, changing relative prices induce consumers to choose a profile of consumption that is tilted toward the present or the future.

Two factors determine the extent to which changing relative prices of home goods affect the consumption profile. One is the share of home goods in consumption, 1-a, the other is the coefficient of risk aversion. It is clear that if the home goods share is negligible the home real interest rate is practically equal to the given world rate. Variations in relative prices over time are only important if the share of home goods in consumption is significant. The larger the share the flatter the cc schedule because now even a small change in the relative price structure over time involves a relative large change in the home real interest rate.

In the stationary equilibrium spending, $c_N + \bar{p}c_T$ will fall short of or exceed income in every period depending on the initial debt position. With an initial indebtedness the country would run an indefinite trade surplus, spending falling short of output thus transferring the debt service. We consider next how this equilibrium is affected by transitory or permanent disturbances in output.

4. An Output Increase

Suppose that currently households learn about a permanent increase in output starting next period. Thus $\bar{q}_{t+1} = \bar{q}'$ starting next period. We want to investigate the effect of the output increase on the profile of

consumption, relative prices and borrowing.[1] In Figure 2 we show that the rise in future home goods output leads to a shift of the NN schedule to N'N'. At each level of relative consumption the present relative price of traded goods will fall relative to that prevailing next period. The new equilibrium is at point A'. Thus a future rise in output will reduce current consumption relative to the future because the output increase raises the home real rate of interest.

The equilibrium relative price structure and the equilibrium consumption profile can be derived by equating (5) and (9) to yield:

$$(11) \qquad p_t/p_{t+1} = (\bar{q}_{t+1}/\bar{q}_t)^{-\theta\beta}\, k^{\beta} \quad ; \quad \beta \equiv 1/[\theta(a+e)+(1-a)]$$

and

$$(12) \qquad c_t/c_{t+1} = (\bar{q}_{t+1}/\bar{q}_t)^{-(1-a)\beta}\, k^{-(a+e)\beta}$$

Consider now the extent to which the consumption profile is moved by a change in future output. Two reference points are A_1 and A_2 corresponding respectively to the cases of zero and infinite intertemporal substitution elasticities. Consider first the case where the elasticity of substitution is infinite, point A_2. In this case relative prices do not change and hence the composition of demand between home goods and traded goods is unchanged. In the case of perfect substitutability the consumption profile exactly mirrors the profile of real income or output. Starting from a zero debt equilibrium there is no change in consumption until the increase in real

[1] A future increase in output in the traded goods sector that leaves home goods supply unchanged will lead to a rise in the level of consumption. There will be current borrowing to finance the higher, flat consumption profile with the debt serviced and paid once income rises. Because the relative price of non-traded goods remains unchanged over time, rising once and for all, there are no interest rate effects on the time profile of consumption.

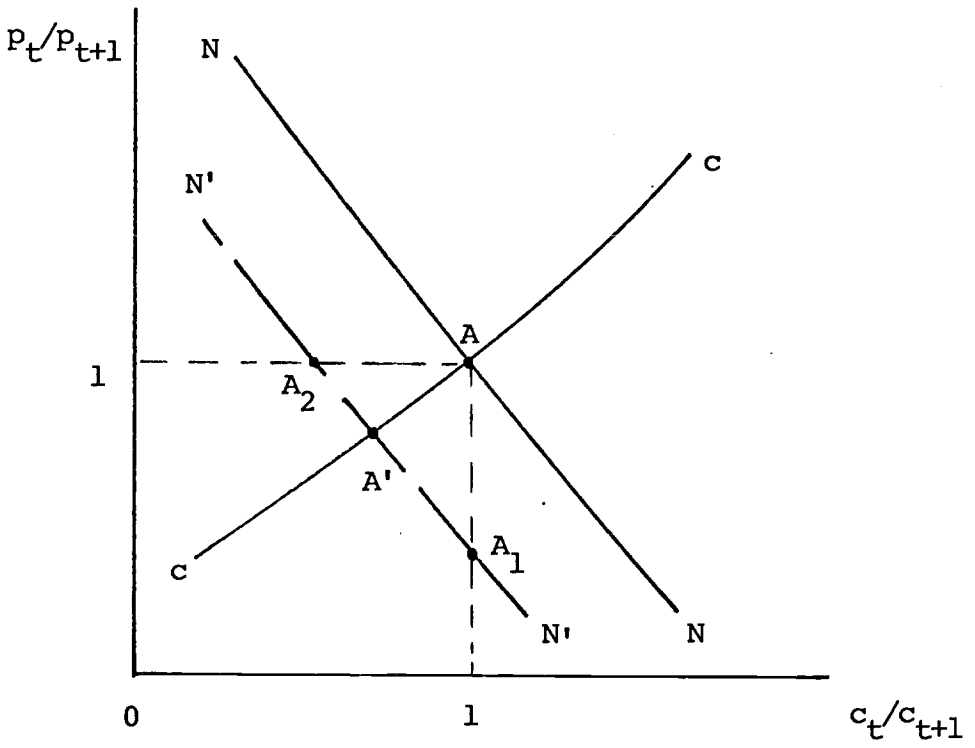

Figure 2

income materializes. At that time consumption rises in proportion to (and by the same amount) as real income. The intuition of this result is the following: With unchanged output and current consumption there is no change in the current relative price of traded goods. Next, period, as (8) shows, a proportional rise of output ($\bar{q}$) and consumption again leaves the relative price unchanged. Therefore p_t/p_{t+1} remains unchanged as shown in Figure 2. The solution, as can be verified from (3) satisfies the lifetime budget constraint.

Consider next the case where consumption levels are proportional, with a zero intertemporal elasticity of substitution, as shown by point A_1. With the consumption profile entirely flat and with wealth or the present value of income increased there will be a rise in the level of consumption from $\bar{c}$ to $\bar{c}'$. Consumption rises in anticipation of higher future level of income as shown in Figure 3. But because consumption rises relative to present income there is current borrowing with the debt serviced and repaid through subsequent trade surpluses.

From the examination of the extreme cases it is clear that the response to a future, permanent increase in income is never one of a trade surplus. Either the home country remains in trade balance or consumption rises in anticipation of the future income increase, leading to external borrowing. The tendency toward a deficit and borrowing will be larger the more important the consumption smoothing. The share of nontraded goods likewise affects the outcome. If the share is near zero the cc schedule in Figure 2 is close to vertical and the consumption smoothing dominates because the home real interest rate in unchanged.

The alternative exercise to be studied is a current, transitory rise in income. In this event there is a rightward shift in NN and the current period equilibrium involves a rise in present relative to future consumption induced

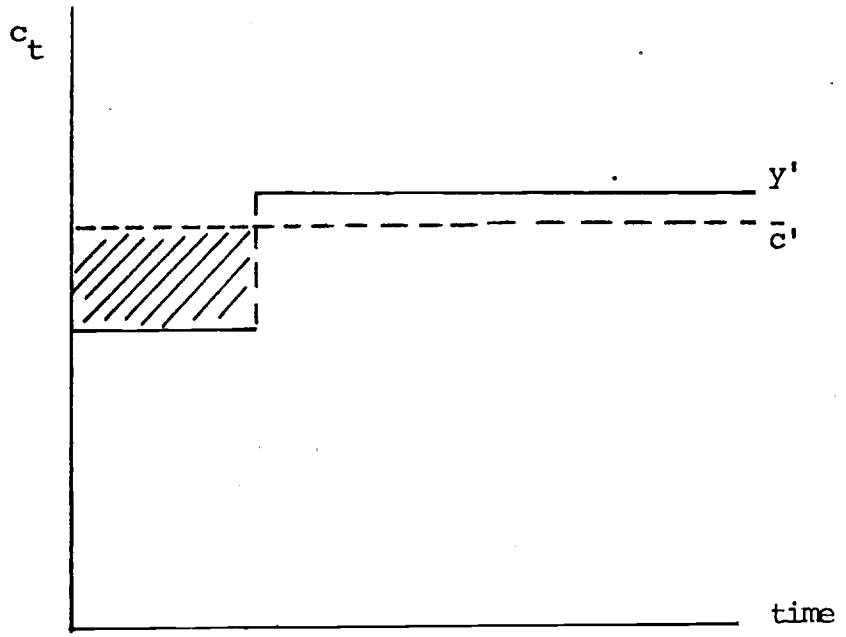

Figure 3

by the fall in the home equilibrium real interest rate. Again consider the cases where there is perfect or zero intertemporal substitutability in consumption. With zero substitutability, the consumption profile remains flat and rises uniformly, though by less than the increase in current income. Therefore a present trade surplus develops financing a longrun trade deficit.

When risk aversion is zero and intertemporal substitutability is infinite we have the other limiting case. Present consumption rises to match fully the present increase in income. Future consumption remains at the level of permanent income and there is no change in the trade balance. Thus once more we find that the consumption smoothing is not offset by real interest rate movements. Income and consumption move in the same direction, a present rise in income leading to lending and a future rise in income leading to borrowing, never to lending.

What is the role of the home goods sector in influencing the magnitude of the trade balance effects and the tilting of the consumption profile? From (12) it can be seen that a rise in either the elasticity of home goods supply or in the share of traded goods, a, will lower the elasticity of the consumption profile with respect to relative output levels. A high home goods supply elasticity reduces the impact of disturbances both on intertemporal relative prices and on the consumption profile. A higher traded goods share will reduce the elasticity of the consumption profile although it may raise the movement in relative prices.

4. Transitory Changes in the World Interest Rate

So far we have assumed a given world rate of interest r^* and furthermore equality of the world interst rate and the domestic discount rate, $(1+r^*)/(1+\delta)=k=1$.

We now assume that there is a transitory increase in the world rate of interest in the present period. From (5) a rise in the current world rate of interest tilts the consumption profile toward the future. In Figure 4 this is shown as the leftward shift of cc to c'c'. The rise in the world rate of interest thus leads a rise in the present relative to the future relative price of traded goods.

Consider first the case of zero intertemporal substitution. Starting with a zero initial debt and hence the equality of income and spending along the stationary path, higher interest rates have no effect on the consumption profile. The consumption profile remains flat at the unchanged level of income. The existence of an initial debt or net foreign assets modifies the result in the following manner. With external net assets the higher interest receipts raise current income and therefore raise the consumption path. Conversely with initial debt there is a reduction in the consumption profile.

The case of some substitutability, as shown in Figure 4, leads to a reduction in present relative to future consumption. Starting from zero debt that implies a reduction in present consumption and a rise in the entire future level of consumption. Thus the home country will run a trade surplus in the high interest period and a trade deficit when interest rates return to their initial levels. The extent of intertemporal substitution can be seen from the expression for the equilibirum relative consumption levels:

$$c_t/c_{t+1} = k^{-\frac{a+e}{\theta(a+e)+(1-a)}} \tag{13}$$

Equation (11) shows the role of the nontraded goods sector. If either the share of traded goods approaches unity or the elasticity of home goods supply tends

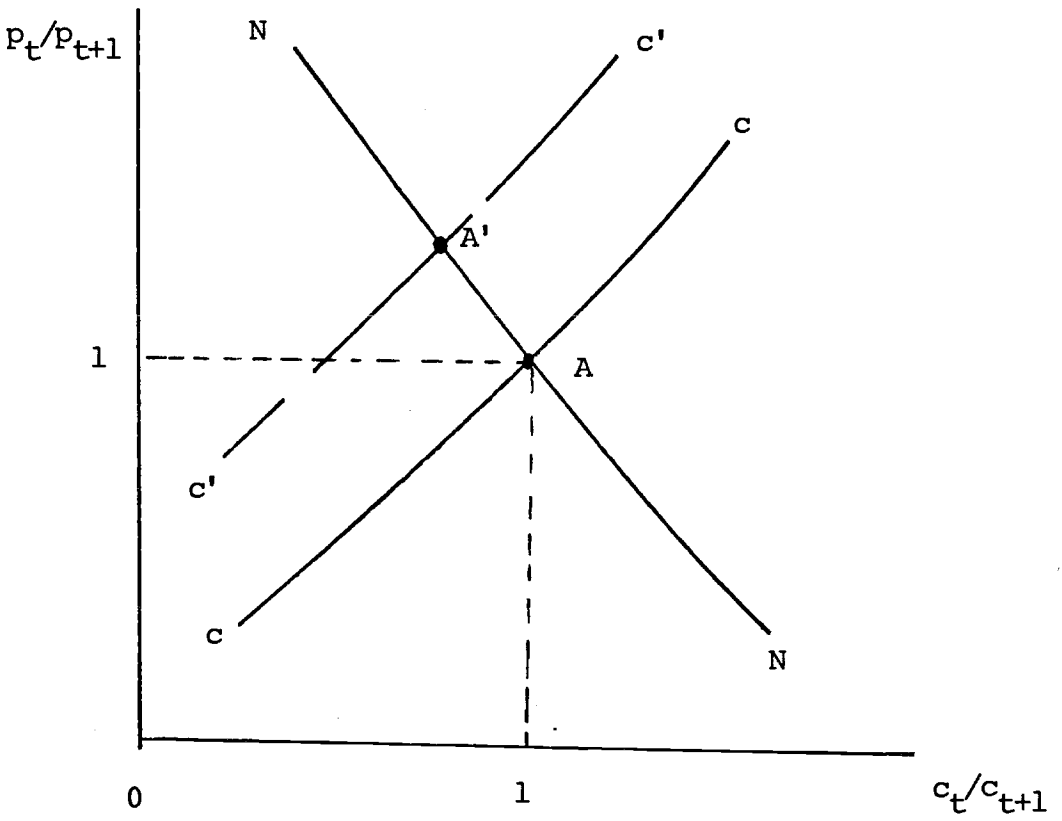

Figure 4

toward infinity the elasticity in (11) tends to $-1/\theta$ which is the result of the small open economy without home goods. The presence of the home goods sector tends to dampen the effect of international interest rate changes on the consumption profile and the trade balance. Current consumption (hence home goods demand) rises relative to future consumption when the current relative price of traded goods rises relative to that in the future. The change in the term structure of relative prices in turn implies that the home component of the real interest rate--due to relative price changes--moves in the opposite direction to the increase in the world rate thus dampening the adjustment in consumption. This dampening effect is directly apparent from the fact that the absolute value of the elasticity in (11) is less than $1/\theta$ which is the appropriate elasticity in the absence of home goods.

Concluding Remarks

We have studied the role of the nontraded goods sector in influencing the consumption and trade balance response to changes in output or in the world rate of interest. The finding is that the presence of a nontraded goods sector, as already noted by Bruno (1976) and Martin and Selowsky (1981), introduces a discrepancy between the home real rate of interest and that prevailing in the rest of the world. The distinction implies that changes in the time profile of home goods output will affect the profile of consumption. The extent to which this occurs is inversely related to the share of traded goods in consumption and to the elasticity of home goods supply. Transitory changes in the world rate of interest likewise affect the time profile of consumption. This time, the home goods market serves to dampen the movements in the consumption profile as the international and domestic components of the real interest rate move in offsetting directions.

The present model is oversimplified for a complete analysis of trade balance issues. For that purpose it would be interesting to introduce also capital accumulation to study the interaction of saving and investment on relative prices and the balance of trade, but that is the topic of further work.

Appendix

We use equations (1), (2) and (4) to derive the first order conditions establishing the optimal composition of current consumption between home goods and traded goods as well as the optimal consumption profile. Differentiating (1), using (2), with respect to consumption of traded goods yields:

$$A\text{-}1 \qquad U_t^T/U_t^N = p_t$$

where U^i is the partial derivative of U ( ) with respect to the ith argument.

Maximizing (1), using (2), with respect to the optimal borrowing, b_{t+1}, yields:

$$A\text{-}2 \qquad U_t^N/U_{t+1}^N = DR(p_{t+1}/p_t)$$

Using the functional form $U = \frac{1}{1-\Theta} c^{1-\Theta}$ with $c = c_T^a c_N^{1-a}$ we derive

$$A\text{-}3 \qquad c_N/c_T = \phi p_t \quad ; \ \phi \equiv (1-a)/a$$

and

$$A\text{-}4 \qquad U_t^N = (1-a)\, c_t^{-\Theta}\, (c_T/c_N)^a = (1-a)c_t^{-\Theta}(\phi p_t)^{-a}$$

Therefore A-2 can be rewritten as:

$$A\text{-}5 \qquad DR(p_{t+1}/p_t) = (c_t/c_{t+1})^{-\Theta}(p_t/p_{t+1})^{-a}$$

or

$$A\text{-}6 \qquad c_t/c_{t+1} = [k\,(p_{t+1}/p_t)^{1-a}]^{-1/\Theta} \qquad ; \qquad k \equiv DR$$

which is the equation of the cc schedule.

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