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THE SIMPLE ANALYTICS OF DEBT-EQUITY SWAPS

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ABSTRACT

Recent attempts to resolve the international debt crisis have lead some countries to engage in debt-equity swaps. The paper explores conditions under which such transactions are beneficial to the debtor as well as the creditors. It identifies a market failure that may prevent the emergence of mutually beneficial swaps and analyzes the effects of swaps on the investment level in the debtor country. The latter helps to evaluate the contribution of this policy to future difficulties with debt service payments.

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THE SIMPLE ANALYTICS OF DEBT-EQUITY SWAPS

by

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The debt crisis of the 1980's has inspired search for innovative solutions to the debt problem. Amongst the many proposals that have emerged so far, debt forgiveness and debt conversion schemes play a central role. One of the proposed mechanisms for debt forgiveness is to establish an international corporation that will buy back the debt of developing countries and forgive part of it (see Peter B. Kenen (1983)). The proposal has been debated, but so far no action has been taken toward its implementation.

Contrary to the Corporation's proposal, debt conversion schemes have been implemented in a number of countries, including major debtors such as Argentina, Brazil, Chile, Mexico and the Philippines, and they are actively considered by a number of other countries, including Honduras, Morocco and Nigeria. A typical debt conversion scheme specifies conditions for the exchange of debt for domestic assets, the entities that may participate in it, and longer-term rights and obligations. In some cases the schemes are designed for foreign creditors or multinational corporations, in others they are designed for domestic residents, where the intention of the latter is to retrieve flight capital. In many cases debt for conversion purposes is aquired on the secondary market. So far only a small share of debt has been converted by means of these schemes--about 2% of the debt of countries that have engaged in them--but they may become much more important in the future. When debt is exchanged for equity it is called a debt-equity swap (see Lewis S. Alexander (1987a) for more factual details).

One central idea behind the Corporation and the debt conversion schemes is to take advantage of the high discounts on debt on the secondary market. It is quite common for debts to be traded at 50 cents to the dollar, with some debts being traded at even higher discounts. Hence, it is argued, debt forgiveness may be not very costly and the debtor may gain from the conversion of cheap debt into holdings of other domestic assets. In both cases the debt burden is eased.

This paper is concerned with debt-equity swaps in which foreign residents are a party to the exchange (i.e., it does not deal with flight capital). As a byproduct, I also provide two results on debt forgiveness. I suggest an approach for dealing with these problems and demonstrate its usefulness by addressing a number of key questions to which it can provide an answer. These questions include the following: What type of resource reallocations between debtor and creditor can be achieved by debt-equity swaps? What are the conditions under which there exist swaps that are beneficial to both parties? What are the effects of debt equity swaps and debt forgiveness on investment in the debtor country? By dealing with these issues one can examine the usefulness of the framework. It should, however, be made clear from the start that I suggest a simple framework that can be extended or modified for particular applications. Here the concern is with the clarification of some fundamental issues rather than with particular applications. For example, I assume the existence of capital controls in the debtor country up to the point at which there are no private capital movements. This is convenient for analytical purposes and it represents a good approximation for some countries (with effective quantative restrictions). One may want to modify it for applications to other countries.

A minimal framework for dealing with some basic issues is developed in the following section. It is based on the assumption that the debtor's real income is a random variable, and that its foreign debt is government owned. As a result of output fluctuations and limits on its ability to tax, the government's capacity to service debt is also random. Consequently, it cannot make the required debt payments in all states of nature. A debt-equity swap consists of an exchange of debt for claims to the random output.

A characterization of feasible reallocations of the transfer of resources from debtor to creditor across states of nature by means of debt-equity swaps is provided in Section II. Given the current situation in which swaps are small relative to the stock of debt, the emphasis is on small swaps. I show that small Pareto-improving swaps do not always exist. I derive a necessary and sufficient condition for their existence and clarify its economic content.

The implications of the existence of many creditors are explored in Section III. I show that due to the fact that debt of the type considered in this paper (i.e., which is fully repaid in some states but only partially repaid in others, with the subset of states of full repayment depending on the debt's size) is priced nonlinearly on international financial markets. This generates an externality across creditors. Consequently, there may exist small Pareto-improving swaps that will not eventuate.

In Section IV the model is extended in order to deal with investment. It is shown that in this framework debt forgiveness reduces share prices and investment, and that a debt-equity swap raises share prices and investment only if the cost of the swap in terms of equity is sufficiently high. The negative effect of debt forgiveness on investment results from a positive income effect in the second period (the analysis is conducted in a two-period framework). As income increases the demand for equity declines as a result of a decline in desired savings. Consequently, share prices fall and so does investment (for a more thorough analysis of debt forgiveness see Elhanan Helpman (1988b)). Concluding comments are provided in Section V.

I. Minimal Framework

The Debtor's output is given by $\tilde{\theta}E$, where $\tilde{\theta}$ denotes a random productivity shock and E represents its constant activity level in production. In the market interpretation of the model E also represents the number of equities issued by domestic firms. Due to controls on international capital movements E is owned by domestic residents. States of nature are identified with productivity shock levels. Thus, state θ is the state in which the productivity shock obtains the value θ.

The government taxes output at the rate t, so that output owners receive income $(1-t)\theta E$ in state θ. In particular, the owner of one unit of E is entitled to $(1-t)\theta$ units of output in state θ. The government has an external debt D. Required service payments on this debt, which consist of principal plus interest, are RD units of output in every state, where R stands for one plus the interest rate. Tax revenue is used to service the debt. I assume that there exist realizations of the productivity shock at which tax revenue is insufficient to cover the required debt service payment. This is supposed to represent the situation of some major debtors who will not be able to repay their debt in all states of nature. Formally,

$$t\tilde{\theta}E < RD \text{ with positive probability.}$$

This implies that there exists a critical value θ_c, defined by

$$(1') \qquad \theta_c = RD/(tE),$$

such that debt is fully repaid in the high-productivity states $\theta \geq \theta_c$, but cannot be fully repaid in the low-productivity states $\theta < \theta_c$. It is assumed that in the low-productivity states, in which tax revenue falls short of debt service payments, creditors receive the entire tax revenue. I also assume that t represents the highest possible tax rate, and that the government has no other sources of income (the case in which some domestic firms are government owned will be discussed at a later stage).

It is clear from this specification that apart from states in which tax revenue is insufficient to cover debt repayment ($\theta < \theta_c$) there typically also exist states in which tax revenue exceeds debt repayment ($\theta > \theta_c$). One needs therefore to state explicitly what is done in these states with tax revenue in excess of debt repayment. For the purpose of this study I assume that it is redistributed to the public as lump-sum transfers.[1] Under these assumptions state-contingent consumption of Debtor residents is

$$(2') \qquad c(\theta) = \begin{cases} (1 - t)\theta E & \text{for} \quad \theta \le \theta_c, \\ (1 - t)\theta E + (t\theta E - RD) & \text{for} \quad \theta \ge \theta_c, \end{cases}$$

where consumption in low productivity states consists of after-tax output and cunsumption in high productivity states consists of after-tax output plus the lump-sum transfer $t\theta E$-RD. Creditors receive the state-contingent payments

$$(3') \qquad d^{*}(\theta) = \begin{cases} t\theta E & \text{for} \quad \theta \le \theta_c, \\ RD & \text{for} \quad \theta \ge \theta_c. \end{cases}$$

In this setup Debtor residents have no explicit decision problem; they consume their after-tax output plus government transfers. Creditor residents recieve full debt repayment in high productivity states and the tax revenue in low productivity states.

Now consider a debt-equity swap. Suppose that $\Delta>0$ units of debt are swapped for $\in>0$ units of equity, where equity is measured in units of E. I assume that the Creditor cannot take a short position in equity. For the swap to take place the government has to acquire the equity or to provide the Creditor with the resources needed for its acquisition. There are several mechanisms by means of which this can be done; I will discuss some of them in Section IV. At this juncture the reader may find it easiest to assume that the government taxes away or confiscates the needed equity. After the swap, required debt repayments are $R(D-\Delta)$ in every state and the Creditor receives $(1-t)\theta\in$ in state θ on account of equity holdings. Naturally, for sufficiently small values of Δ there still exist states in which the government cannot fully repay the remaining debt. Assuming that foreign-owned income from equity ownership is also taxed at the rate t, debt is fully repaid in states that satisfy

$$t\theta E \geq R(D - \Delta),$$

so that the critical value θ_c, which now depends on Δ, becomes

(1) $$\theta_c(\Delta) = R(D - \Delta)/(tE),$$

i.e., debt is fully repayed in states $\theta \geq \theta_c(\Delta)$ and the Creditor receives the tax revenue in states $\theta \leq \theta_c(\Delta)$. In this case the consumption of Debtor residents equals

(2) $$c(\theta;\Delta,\epsilon) = \begin{cases} (1 - t)\theta(E - \epsilon) & \text{for} \quad \theta \leq \theta_c(\Delta), \\ (1 - t)\theta(E - \epsilon) + [t\theta E - R(D - \Delta)] & \text{for} \quad \theta \geq \theta_c(\Delta), \end{cases}$$

and the Creditor receives payments

(3) $$d^*(\theta;\Delta,\epsilon) = \begin{cases} (1 - t)\theta\epsilon + t\theta E & \text{for} \quad \theta \leq \theta_c(\Delta), \\ (1 - t)\theta\epsilon + R(D - \Delta) & \text{for} \quad \theta \geq \theta_c(\Delta). \end{cases}$$

Thus, the swap reduces the Debtor residents' income from claims to output in all states and it increases their income from government transfers in high productivity states as a result of the easing of the debt service burden.

Moreover, it increases the set of states in which debt is fully repaid. These three factors need to be properly weighed in order to evaluate the desirability of the swap from the point of view of the Debtor. The Creditor too has to weigh three factors. The swap increases his income in all states on account of equity holdings, it reduces his income in high productivity states as a result of lower debt service payments, and it increases the set of states in which he receives full repayment on the remaining debt.

In order to evaluate the desirability of swaps, I assume that a representative resident of the Debtor has a strictly concave von Neumann-Morgenstern utility function $u(c)$; i.e., the Debtor is risk averse. His subjective probability distribution of states--i.e., productivity shocks--is represented by the cumulative distribution function $G(\theta)$, defined on the interval $[0,+\infty)$. Hence, his expected utility from a given swap, $(\Delta,\in)$, is our welfare criterion, given by

$$(4) \qquad U(\Delta,\in) = \int_0^\infty u[c(\theta;\Delta,\in)]dG(\theta).$$

Equations (1), (2) and (4) provide a valuation of every swap from the point of view of the Debtor.

As far as the Creditor is concerned, I assume that he has access to international financial markets which enable him to hold a well diversified portfolio. Consequently, his marginal utility of state-contingent payments by the Debtor are not affected by the swap. Let $\mu^*(\theta)$ denote his marginal utility of state-θ payments. Then his expected utility of a swap is

$$(5) \qquad U^{*}(\Delta,\epsilon) = \int_{0}^{\infty} \mu^{*}(\theta) d^{*}(\theta;\Delta,\epsilon) dG^{*}(\theta),$$

where $G^{*}(\theta)$ denotes his subjective probability distribution function. The Creditor's valuation of a swap is represented by (1), (3) and (5).

II. Are Swaps Desirable?

There are several methods of analysis that can be used to answer the question posed in the title of this section. I have chosen to start with a description of the effects of swaps on state contingent peyments of Debtor to Creditor in order to gain insight into their possible role as risk sharing devices. This is followed by a derivation of asset indifference curves that will be used in subsequent analysis. In particular, they will be used to derive a necessary and sufficient condition for the existence of small mutually beneficial swaps. As I have explained in the introduction, this paper focuses on small swaps.[2] These tools are then used to shed light on the question: How likely is the existence of mutually beneficial swaps?

First, consider the effect of swaps on the state contingent transfers from debtor to creditor. Schedule OAB in Figure 1 describes this profile prior to a swap. The Debtor consumes the difference between OC and OAB, where the slope of OC is E (the stock of equity). Now, given $\Delta>0$ and $\epsilon=0$, the transfer of resources from Debtor to Creditor shifts to $OA_{\Delta}B_{\Delta}$, with the Debtor consuming the difference between OC and $OA_{\Delta}B_{\Delta}$. This describes the effect of debt forgiveness. In the case of a debt-equity swap it is necessary to add to the resource transfer the return on equity $(1-t)\theta\epsilon$. For sufficiently small values of ϵ the resource transfer profile becomes $OA_{S}B_{S}$.

The resulting change in the Debtor's consumption profile is described in Figure 2. The Debtor gains in states which lie in the interval (θ_1, θ_2) and loses in all other states (except, of course, in θ_1 and θ_2). The Creditor gains in states in which the Debtor loses and vice versa.

Now consider the case in which the productivity shock in the Debtor country is idiosyncratic; i.e., it is statistically independent of economic conditions in the rest of the world. Then one expects the Creditor's marginal utility $\mu^*(\theta)$ to be the same in all states. In this case the Creditor will not agree to swaps which raise the Debtor's expected consumption level. Hence, if (i) both parties agree on the probability distribution of the productivity shock; and (ii) the entire mass is concentrated on two points, say θ_L and θ_M in Figure 2; then there do not exist mutually beneficial swaps. This result stems from the fact that in this case a swap reduces the Debtor's consumption in a low consumption state θ_L, in which the marginal utility of consumption is high, and raises it in a high consumption state θ_M, in which the marginal utility of consumption is low (see the figure). Suppose that $(\Delta,\in)$ is chosen so as to make the expected value of this consumption change equal to zero, so as to make the Creditor indifferent to the swap. Then the Debtor will lose from it, as we know from the standard theory of choice under risk (see the discussion following Proposition 1). This demonstrates that there exist conditions under which there do not exist mutually beneficial small swaps.[3]

The general case is more easily treated in asset space $(\Delta,\in)$. For this reason I present in Figure 3 the asset indifference curves

$$U(\Delta,\in) = U(0,0) \quad \text{and} \quad U^*(\Delta,\in) = U^*(0,0)$$

for the case of smooth distribution functions. Every swap that leads to points above the Debtor's indifference curve and below the Creditor's indifference curve is beneficial to both parties. Hence, the figure represents a situation in which there exist mutually beneficial swaps. The Creditor's indifference curve is concave and the Debtor's indifference curve can be concave or convex, ralative to the horizontal axis (see Helpman (1988a, Appendix) for a formal proof). It is instructive to understand the reasons for the particular curveture of these curves. It is clear from Figure 1 that every increase in Δ increases the set of states in which the remaining debt is fully repaid as well as repayment per unit debt in the other states. These changes make the remaining debt a more valuable asset. For this reason the larger Δ, the larger the Creditor's loss from giving up an additional unit of debt. Consequently, he requires a larger marginal compensation in terms of equity in order to maintain a constant expected utility level. This explains the concave shape of the Creditor's indifference curve. Similarly for the Debtor; the larger Δ, the more he stands to gain from a marginal debt reduction. Therefore, at the margin he has to give up more equity per unit debt in order to maintain a constant expected utility level. If he was risk neutral, his asset indifference curve would have been concave, just as the Creditor's. However, risk aversion introduces convexity into the indifference curve. For these reasons his indifference curve can be concave or convex, depending on the degree of risk aversion.

From equations (1)-(5) one can calculate the slopes of these asset indifference curves

$$(6) \qquad \rho(\Delta,\epsilon) = [-U_\epsilon(\Delta,\epsilon)/U_\Delta(\Delta,\epsilon)] = \frac{\int_0^\infty \mu[c(\theta;\Delta,\epsilon)](1-t)\theta dG(\theta)}{\int_{\theta_c(\Delta)}^\infty \mu[c(\theta;\Delta,\epsilon)]RdG(\theta)}$$

$$(7) \qquad \rho^*(\Delta,\epsilon) = [-U^*_\epsilon(\Delta,\epsilon)/U^*_\Delta(\Delta,\epsilon)] = \frac{\int_0^\infty \mu^*(\theta)(1-t)\theta dG^*(\theta)}{\int_{\theta_c(\Delta)}^\infty \mu^*(\theta)RdG^*(\theta)},$$

where $\mu(\cdot)$ is the Debtor's marginal utility of consumption. For convenience of exposition let $M\equiv\rho(0,0)$ and $M^*\equiv\rho^*(0,0)$ denote the slopes of the asset indifference curves at the origin. It is clear from Figure 3 that there exist mutually beneficial small swaps if and only if the Crditor's indifference curve is steeper at the origin than the Debtor's indifference curve. Hence,

<u>Proposition 1</u>. There exist small Pareto-improving debt-equity swaps if and only if $M^*>M$.

In order to gain a better understanding of the circumstances in which there exist Pareto-improving small swaps, consider again the case in which the Creditor's marginal utility of resources is state-independent and both parties have the same probability assesments. Assume also that the entire mass of the distribution is concentrated on three states; a low productivity state θ_L, a medium productivity state θ_M, and a high productivity state θ_H, as depicted in Figure 2 (i.e., $\theta_L<\theta_c<\theta_M<\theta_H$). The probability of state i is

π_i, i=L,M,H. Under these circumstances (6) and (7) imply:

$$M = (1 - t)(\pi_L\theta_L\mu_L + \pi_M\theta_M\mu_M + \pi_H\theta_H\mu_H)/R(\pi_M\mu_M + \pi_H\mu_H),$$

$$M^* = (1 - t)(\pi_L\theta_L + \pi_M\theta_M + \pi_H\theta_H)/R(\pi_M + \pi_H),$$

where μ_i is the Debtor's marginal utility of consumption in state i. Due to risk aversion $\mu_L > \mu_M > \mu_H$. These expressions imply that $M^* > M$ if and only if

$$(8) \qquad \pi_L\theta_L[\pi_M(\mu_L - \mu_M) + \pi_H(\mu_L - \mu_H)] < \pi_M\pi_H(\theta_H - \theta_M)(\mu_M - \mu_H).$$

Hence, when $\pi_H = 0$; i.e., there are in effect only two states, (8) implies $M > M^*$, so that there do not exist mutually beneficial small swaps, as I have argued before. This suggests that existence of Pareto-improving swaps is possible only when there exists a third high productivity state in which the debtor gains from the swap. In particular, it is easy to see that for every triple of positive probabilities and values of the low and medium productivity levels there exists a sufficiently high value of θ_H which ensures $M^* > M$, and thereby the existence of mutually beneficial small swaps (μ_H is declining in θ_H). It is also clear that (8) is more likely to be satisfied the closer μ_L is to μ_M; i.e., when there is low risk aversion at low to medium consumption levels. There do not exist mutually beneficial swaps when $\mu_M = \mu_H$; i.e., when there is risk neutrality at medium to high consumption levels.

To summerize, there do not exist mutually beneficial swaps whenever there exist only two states or the Debtor is risk neutral at medium to high consumption levels. In the other cases mutually beneficial swaps exist only when there are sufficiently high productive states.

III. Many Creditors

My analysis of conditions that ensure the existence of mutually beneficial swaps is readily applicable to cases in which there exists a single creditor. It is, however, often the case that there exists a large number of creditors. If they operate in concert, we can use the single creditor results. However, in this case it is also necessary to specify a mechanism that determines the way in which they split the benefits of swaps. Which swap is agreed upon depends then on the bargaining game played by the Debtor and the consortium of Creditors. Every bargaining procedure yields its own result.

In this section I take up the case in which there exists a competitive fringe of Creditors. In this case it is still true that $U^*(\Delta,\epsilon)$ represents the valuation of a swap on international financial markets; i.e., $U^*(\Delta,\epsilon)$ is the value of the remaining debt plus the value of the aquired equity. In fact, the linearity of (5) together with (3) imply that this value can be decomposed into $U^*(\Delta,\epsilon)=U^*(\Delta,0)+U^*(0,\epsilon)$, where the first component represents the value of remaining debt and the second represents the value of equity. The second component is linear in equity holdings; $U^*(0,\epsilon)=U^*_\epsilon(0,0)\epsilon$. Hence, the price of equity, which by a suitable choice of units equals $U^*(0,\epsilon)/\epsilon=U^*_\epsilon(0,0)$, is constant. On the other hand, the price of a remaining unit of debt, which equals $U^{*}(\Delta,0)/(D-\Delta)$, is an increasing function of Δ. The latter point follows from my explanation of the curveture of the indifference curves in Figure 3. To repeat, when Δ increases the remaining debt is fully repaid in more states and the repayment per-unit debt in other states increases as well. Therefore the unit value of remaining debt increases (see Helpman (1988a, Section 5) for a diagramatical exposition of this result).[4]

What is the price of debt (in terms of equity) that the Debtor pays for a swap of size Δ? In the presence of a competitive fringe of creditors it is equal to the post-swap value of a unit of remaining debt. For suppose it is higher, then every remaining creditor agrees to swap his debt at a lower price. And if it is lower, every creditor refuses to swap, because the resale value of his asset is higher than the offered swap price. Therefore, the equilibrium swap exchange rate (i.e., relative price) is

$$x(\Delta) = \frac{U^{*}_{\epsilon}(0,0)}{U^{*}(\Delta,0)/(D-\Delta)}, \tag{9}$$

where the denominator on the right hand side represents the equilibrium price of debt and the numerator represents the equilibrium price of equity. Since the numerator declines in Δ, (9) shows that

<u>Proposition 2</u>. The larger the debt to be swapped, the lower the price the Debtor will receive for his equity.

The nonlinearity in the pricing of debt on international financial markets introduces an externality across creditors. It stems from the fact that when a single creditor reaches a swap agreement the remaining creditors make a capital gain on their debt. The same argument applies to debt forgiveness. An attempt to buy debt on the secondary market in order to forgive it--as in, say, Kenen's proposal-- raises its price. For this reason secondary market discounts prior to the availability of information on buy-backs or purchases of debt in order to forgive it overestimate the discounts that will prevail when the purchases actually take place (see Michael P. Dooley (1988) and Helpman (1988a, Section 6) on this point). In addition, the free rider problem that was discussed by Paul R. Krugman (1985) can also be cast in terms of this externality.

The Debtor's optimal decision is presented in Figure 3. The curve $\epsilon=\Delta/x(\Delta)$ represents equilibrium market opportunities. It is shown in Helpman (1988a, Appendix) that its slope is flatter than the slope of the indifference curve $U^*(\cdot)$, and it is located to the right of the indifference curve. This reflects the above discussed externality. Taking advantage of market opportunities, the Debtor's optimal policy is to swap Δ_o units of debt for equity. This will raise his expected utility to U_o. Observe, however, that now the existence of small beneficial swaps to the Debtor requires $M<x(0)$, where $x(0)$ is the slope of the $\epsilon=\Delta/x(\Delta)$ curve at the origin. Clearly, since $x(0)<M^*$, it might happen that $x(0)<M<M^*$. In this case small Pareto-improving swaps exist, but will not eventuate. Hence,

Proposition 3. In the presence of a competitive fringe of creditors there exist circumstances in which no small swap will take place despite the availability of Pareto-improving small swaps.

This result points out a market failure that may result from this externality.

IV. Investment

This section deals with the effects of debt-equity swaps on investment in the Debtor country. In the process of this analysis I clarify some taxation issues and the role of government ownership of domestic companies. In order to deal with these problems it is necessary to somewhat extend the model. Assume therefore that there are two periods. The discussion in the previous sections applies to the second period, except for the debt-equity swap that takes place in the first period. Residents of the Debtor country choose in the first period consumption c_0 and the amount of domestic equity holdings e. As a result of restrictions on international capital movements they cannot hold foreign assets or borrow abroad, so that equity provides the only instrument by means of which they transfer purchasing power from the first to the second period.[5] In this case (2) implies that second period consumption can be written as

$$c(\theta;\Delta,I,e) = (1 - t)\theta e + T(\theta,\Delta,I),$$

where I equals the investment level, $e=E-\epsilon$ represents Debtor residents' holdings of domestic equity, and $T(\theta,\Delta,I)$ represents government transfers. Transfers are given by

$$(10) \qquad T(\theta,\Delta,I) = \begin{cases} 0 & \text{for} \quad \theta \le \theta_c(\Delta,I), \\ t\theta E(I) - R(D-\Delta) & \text{for} \quad \theta \ge \theta_c(\Delta,I), \end{cases}$$

where the number of real equities E is an increasing concave function of the investment level; i.e., $E=E(I)$. For this reason the critical state $\theta_c(\cdot)$ also depends on the investment level. The individual investor in equity chooses e taking Δ and I as given.

Now the representative individual's preferences over first-period consumption and equity holdings can be written as

$$(11) \qquad V(c_0,e;\Delta,I) = v(c_0) + \delta\int_0^{\infty} u[(1-t)\theta e + T(\theta,\Delta,I)]dG(\theta),$$

where the right hand side is equal to the utility from first-period consumption plus the discounted expected utility from second-period consumption, with δ being the subjective discount factor. The individual's budget constraint is

$$(12) \qquad c_0 + qe \le y + qE(I) - I - qC(\Delta,I),$$

where q denotes the price of equity, y denotes first-period output, $qE(I)-I$ represents the net value of domestic firms on the stock market (as in Peter A. Diamond (1967)), and $C(\Delta,I)$ represents the cost of the swap in terms of equity, with $qC(\Delta,I)$ being the tax imposed by the government in the first period in order to aquire the resources needed for the swap. In the case of debt forgiveness $C(\Delta,I)\equiv 0$. Thus, the left hand side represents spending on consumption and equity, while the right hand side represents resources available to the private sector. The cost of the swap is allowed to depend on I. For example, in the presence of a competitive fringe of creditors the swap exchange rate $x(\cdot)$ is a function of I, because it depends on E. I assume that $C(\cdot)$ increases in Δ.

It is clear from this description that for a given investment level a swap involves a substitution of second-period for first-period taxes. The government increases taxes in the first period (in which the swap is performed) and reduces net taxes in the second period (in some states). First-period taxes are needed in order to obtain the resources required for the swap. These can be imposed directly, or indirectly by means of printing money (inflation tax) or issuing domestic debt. The current model cannot deal with the monetary aspects of the problem. But it should be clear that resources for the swap have to be extracted one way or another. There is an alternative to taxation if the government owns domestic companies. Suppose, for example, that there are no taxes in the second period, but instead the government owns a proportion t of the domestic companies. In this case an equity provides θ units of output in state θ. Now suppose that the government swaps its own equity for debt. Then, as long as the investment level is constant, the foregoing analysis goes through with every share in the previous case replaced by $(1 - t)$ shares in the current case, provided $tE(I)\geq(1-t)\epsilon$. The last condition states that the government has enough shares to perform the swap. Combinations of partial ownership and partial taxation are also possible. The essential point is that if the government owns equity, it is required to redistribute the excess of its income from equity over debt repayment to the private sector. The public is indirectly the owner of government companies and its debt.

The individual chooses c_0 and e so as to maximize (11) subject to (12). Denoting by $s(c_0,e;\Delta,I) \equiv V_e(c_0,e;\Delta,I)/V_{c_0}(c_0,e;\Delta,I)$ his marginal rate of substitution between equity and first period consumption, the first order conditions of this problem yield

$$(13') \qquad q = s(c_0,e;\Delta,I).$$

However, due to the restrictions on capital movements, the clearing of the first-period commodity market requires $c_0+I=y$ and the clearing of the equity market requires $e=E-C(\Delta,I)$. Therefore, (13') and the market clearing conditions imply that the demand price of equity is

$$(13) \qquad q(\Delta,I) \equiv s[y - I, E(I) - C(\Delta,I);\Delta,I].$$

Differentiation of (13) yields

$$(14) \qquad q_\Delta = - s_e C_\Delta + s_\Delta.$$

It shows that the demand price for shares is affected by two considerations. First, the larger the swapped debt the more equity has to be relinquished, which leaves domestic stockholders with less equity holdings. Since the demand price $s(\cdot)$ declines in equity holdings, this element brings about an increase in share prices. Second, the swap has a direct effect on the demand price for equity, which is represented by the second term on the right hand side of (14). This effect stems from government transfers. The larger Δ, the larger the transfers that the individual receives in the second period, and therefore the less he values equity which is used to transfer purchasing power from the first to the second period (i.e., s_{Δ} is negative). This reduces the demand price for equity. Consequently, the change in the demand price depends on which one of these considerations extracts a stronger influence. If the former is stronger, the demand price rises. If the latter is stronger, the demand price declines. The former is zero in the case of debt forgiveness. Therefore, in the case of debt forgiveness the demand price declines.

Given an equity price q, the net value of firms equals

$$qE(I) - I.$$

I assume that firms choose the investment level so as to maximize their net value (see Diamond (1967)). Their equilibrium condition is

$$(15) \qquad qE'(I) = 1.$$

This condition describes demand for investment as a function of share prices, or alternatively, the supply price of equity as a function of the investment level.

For every Δ conditions (15) and (16) determine equilibrium share prices and investment. The equilibrium determination of these variables is described in Figure 4. Curve S describes the supply price of equity while curve D describes the demand price in the absence of swaps. The intersection point A describes equilibrium investment and share prices. Now suppose that a swap takes place. It does not affect the supply curve. However, the demand curve at point A increases if and only if the condition for a demand price increase applies. Namely, if and only if the cost of a swap is sufficiently high. If it increases, investment and share prices go up; if it declines, investment and share prices decline. Therefore,

Proposition 4. A debt-equity swap raises investment and share prices if and only if the equity cost of the swap is sufficiently high.

Since debt forgiveness does not involve giving up equity, this result also implies:

Proposition 5. Debt forgiveness reduces investment and share prices.

The last proposition has important implications. It shows that debt forgiveness can bring about a reduction in the capacity to repay debt. The decline in investment reduces the set of states in which debt is fully repaid and payments per unit debt in states in which it is only partially repaid. For a fuller discussion of debt forgiveness see Helpman (1988b).

V. Conclusions

Debt-equity swaps and debt forgiveness are practical issues that require careful analysis. The results of this paper demonstrate that there do not exist simple and clear-cut answers to a number of major questions, but they also show how to identify relevant considerations. Some of the conclusions are:

1. Small debt-equity swaps can be beneficial to both parties, but this is not always the case.
2. In the presence of a competitive fringe of creditors there is a uniqe price at which a swap of a given size can be performed, with the price being higher the larger the swap.
3. Under these circumstances small swaps may fail to take place despite the existence of Pareto-improving deals.
4. Debt forgiveness may reduce investment in the debtor country, thereby imposing a secondary cost via a reduction of debt service payments.
5. A debt-equity swap may increase or reduce investment.

In all cases with ambiguous answers, we have identified the conflicting elements that have to be assessed empirically.

Finally, it is necessary to point out that in practical evaluations of debt-equity swaps there exist additional considerations that have to be borne in mind. For example, capital controls are typically not as tight as assumed in the model, and it is therefore important to consider the effect of a swap on net capital inflow. This is the more so the larger the deviation of the shadow price of capital from the market price. And there exists the problem of "round-tripping", which results from loopholes that enable investors to obtain subsidies embedded in existing swap arrangements without fulfilling all other obligations. Naturally, these problems exist on top of the problems discussed in the paper and they deserve separate treatment.

References

Alexander, Lewis S., "Economic Issues for Debtor Countries Raised by Debt-for-Equity Swaps," mimeo, September 1987.

———, "Debt-for-Equity Swaps: A Formal Analysis," mimeo, September 1987.

Bulow, Jeremy I. and Rogoff, Kenneth, "A Constant Recontracting Model of Sovereign Debt," forthcoming in the Journal of Political Economy.

Diamond, Peter, A., "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty," American Economic Review, 57:1967, 759-776.

Dooley, Michael P., "Buy-Backs and Market Valuation of External Debt," International Monetary Fund Staff Papers, 35:1988, 215-229.

Helpman, Elhanan, "The Simple Analytics of Debt-Equity Swaps and Debt Forgiveness," International Monetary Fund Working Paper No. 88/30, 1988a.

———, "Voluntary Debt Reduction: Incentive and Welfare," International Monetary Fund Working Paper 88/84, 1988b.

Kenen, Peter B., *New York Times*, 6 March 1983.

Krugman, Paul R., "International debt strategies in an uncertain world," in Gordon W. Smith and John T. Cuddington (eds.), International Debt and the Developing Countries, IBRD, Washington, DC, 1985.

Merton, Robert C., "On the pricing of corporate debt: The risk structure of interest rates," Journal of Finance, 24:1974, 449-470.

Footnotes

* Tel-Aviv University. My work on this paper began when I was a visiting scholar in the Research Department of the International Monetary Fund, and continued at MIT and Tel-Aviv University. Many colleagues and seminar participants at the IMF, MIT, Harvard, Boston University, Princeton, Columbia, Duke, and Chicago, contributed to the clarification of various points. I thank them all, and especially Eduardo Borensztein, Guillermo Calvo, Max Corden, Michael Dooley, Rudiger Dornbusch, Assaf Razin, Kenneth Rogoff, and Lars Svensson. Two referees provided useful comments. The Horowitz Institute at Tel-Aviv University funded part of the research.

1 Other alternatives, such as the provision of public goods, are also possible. The important point is to specify a mechanism for the valuation of these resources. It should, however, be clear that the choice of a specification affects some of the results. An example of an alternative tax structure and its implications are presented in Helpman (1988a, Section 9).

2 One may also consider situations in which the Debtor becomes a creditor. If, for example, the Debtor is risk averse, the Creditor's marginal

utility is the same in every state, and both have the same subjective probability distribution, then Pareto-Optima consist of allocations in which the Debtor obtains the same consumption level in every state. These allocations provide perfect insurance, and they can be attained by means of a swap in which the entire debt D plus some bonds issued by the Creditor are exchanged for the entire stock of equities E. Since my interest is mainly in small swaps, this possibility is not considered further.

3 However, in this case there exists a mutually beneficial small negative swap. My assumption that excludes short positions in equity holdings makes negative swaps non-feasible.

4 This is similar to the well known result that the value of a unit of a firm's debt declines with its financial leverage. See, for example, Robert C. Merton (1974). See also Jeremy I. Bulow and Kenneth Rogoff (forthcoming) and Alexander (1987b).

5 It is easy to add a domestic bond market to the model. However, in the absence of capital movements this market has to clear at zero indebtedness. Consequently the following analysis would not be affected by this modification. In fact, one can calculate from what follows the equilibrium interest rate on this bond market.

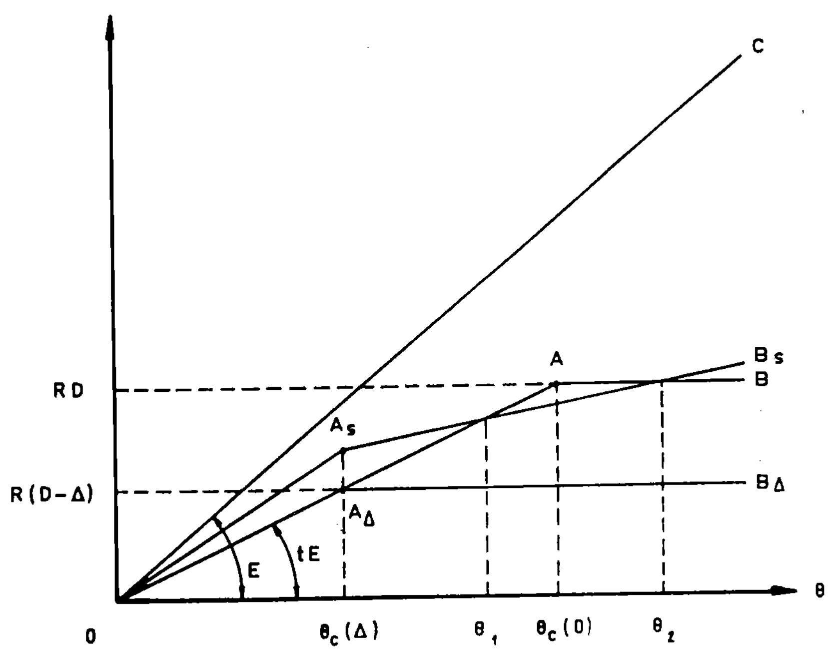

Figure 1

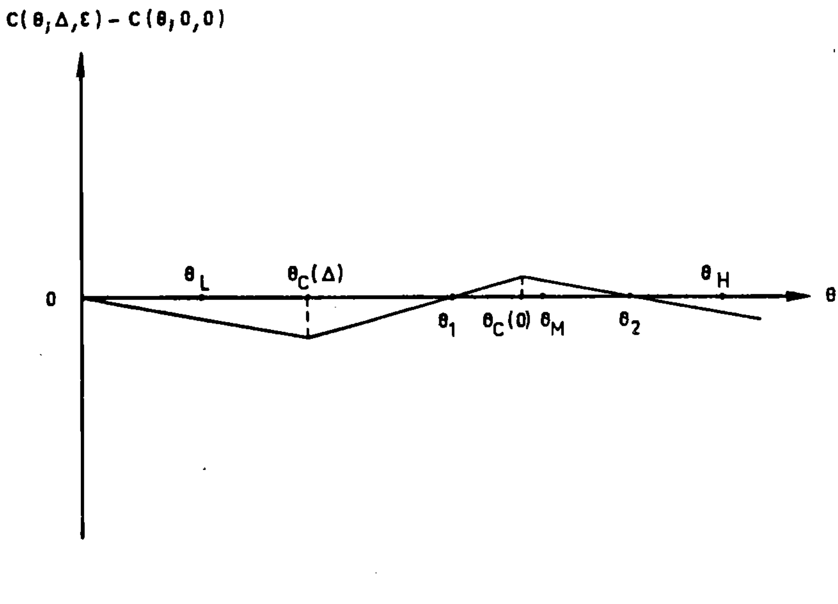

Figure 2

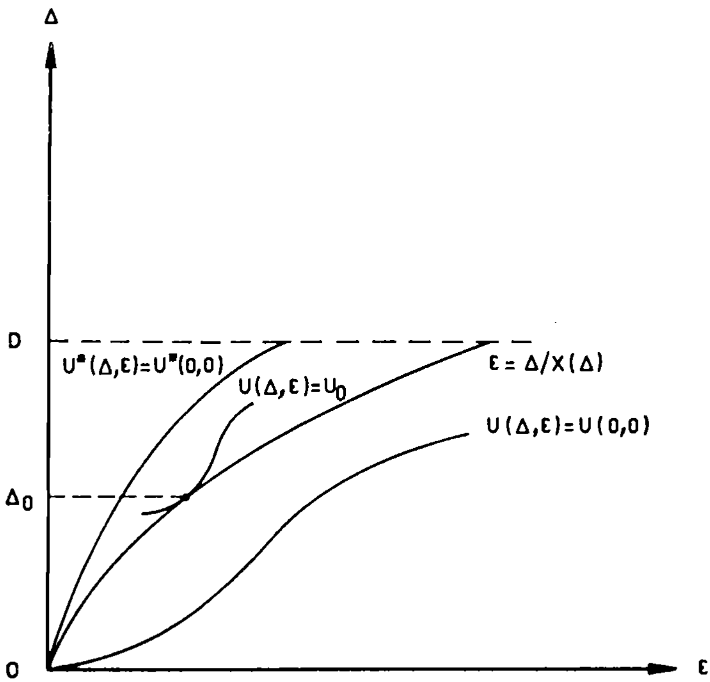

Figure 3

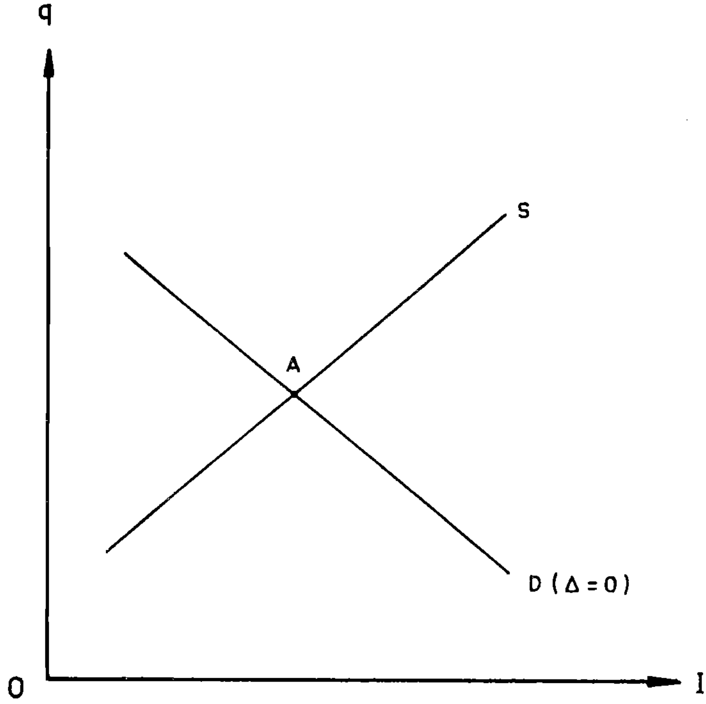

Figure 4