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ABSTRACT

In the framework of continuous-time finance theory, this paper derives the optimal consumption and portfolio rules for an international investor with constant expenditure shares α_j and constant relative risk aversion $1-\gamma$ in a dynamic context. The index of value obtained from the consumption rule is used to obtain real returns on N different currencies in terms of their purchasing power over N goods. The portfolio rule is expressed in terms of the determinants of the purchasing powers, namely exchange rates and prices expressed in the numeraire currency. The optimal portfolio is interpreted as a capital position given by the expenditure shares and hedging zero net-worth portfolios depending on unanticipated inflation and risk aversion. It is shown that the minimum variance portfolio is independent of returns, but depends on expenditure patterns, while the speculative portfolio depends on risk aversion and real return differentials. When the effect of preferences on real return differentials is made explicit, it is shown that the minimum variance portfolio is affected by risk aversion. In that case, the effect of an increase in α_i on the portfolio proportions x_i will be positive when relative risk aversion is greater than one, as generally presumed.

Actual data from eight major countries is used to compute optimal portfolios based on real return differentials for different weighting schemes, degrees of risk aversion and sample periods, when exchange rates and prices are assumed to be Brownian.

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OPTIMAL CURRENCY DIVERSIFICATION FOR A CLASS OF RISK-AVERSE INTERNATIONAL INVESTORS

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The theory of finance from the perspective of continuous time, due mostly to Robert Merton, was first applied to international finance under the assumption that only the domestic good was consumed or, alternatively, that purchasing power parity obtained, so that again there was only one good.[1] Extending earlier work by Pentti Kouri,[2] these assumptions were relaxed in Kouri and Macedo (1978), where a time-invariant portfolio rule was derived for an "international investor" consuming in fixed proportions N composite goods produced in N different countries and holding a portfolio of the respective N currencies. This paper retains the convenient assumption of constant expenditure shares and relative risk aversion, also treats national outputs as composite goods and ignores nominal interest on currency holdings,[3] but derives the consumption and portfolio rules in a dynamic context (Section I). In Section II, the properties of the dynamic optimal currency portfolio are indicated and contrasted with the special cases of perfectly anticipated inflation and purchasing power parity. The effect of an increase in consumption shares on portfolio shares is also analyzed. Section III presents and interprets portfolios computed under the assumption of stationarity of the covariance between prices and exchange rates in eight major countries during the floating rate period.[4]

I Consumption and portfolio rules derived

Consider an individual consuming in fixed proportions α_j goods produced in N countries and holding a variable portfolio of the N respective currencies. This international investor takes as given the domestic currency prices of the N goods as well as the N-1 bilateral exchange rates. Defining currency N as the numeraire,[5] we will assume that exchange rates against the numeraire as well as prices in terms of the numeraire are generated by continuous stochastic processes of the Itô type:

$$\frac{dS_i}{S_i} = \pi_i(S_i, P_j)\,dt + \sigma_i(S_i, P_j)\,dz_i, \quad i=1,\ldots,N-1;$$

(1)

$$\frac{dP_j}{P_j} = \mu_j(S_i, P_j)\,dt + \delta_j(S_i, P_j)\,du_j, \quad j=1,\ldots,N;$$

where S_i is the price of currency i in terms of the numeraire (so that $S_N = 1$), P_j is the price of the good produced in country j expressed in terms of the numeraire,

$\pi_i(\mu_j)$ is the instantaneous conditional expected mean proportional change per unit of time of $S_i(P_j)$,

$\sigma_i^2(\delta_j^2)$ is the instantaneous conditional variance per unit of time of $S_i(P_j)$,

and dz_i and du_j are Wiener processes.

By assumption, the individual's instantaneous indirect utility function V is separable and can be written as the purchasing power of nominal expenditure over the N goods:

(2) $$V = E_N / \prod_{j=1}^{N} P_j^{\alpha_j}$$

where $E_N = \sum_{j=1}^{N} P_j X_j$ is nominal expenditure expressed in the numeraire, X_j being the amount of good j consumed and α_j being its share in E_N.

Similarly, the individual's real wealth W is defined as the purchasing power of the holdings of the N currencies:

(3) $$W = \sum_{j=1}^{N} N_j Q_j$$

where N_j are holdings of currency j

and $Q_j = \prod_{\ell=1}^{N} (P_\ell S_j)^{-\alpha_\ell}$ is the purchasing power of currency j over the N goods.

Each instant, the level of real consumption and currency holdings are chosen simultaneously, so that the rate of real consumption is equal to income from reshuffling the portfolio:

(4) $$-Vdt = \sum_j^N dN_j Q_j + \sum_j^N dN_j dQ_j .$$

On the other hand, from (3), using Itô's Lemma, the rate of change of wealth is:

(5) $$dW = \sum_j^N N_j dQ_j + \sum_j^N dN_j Q_j + \sum_j^N dN_j dQ_j .$$

Substituting from (4), the change in wealth is equal to the difference between capital gains and consumption. The real return on the holdings of currency $i(r_i)$ is given by the proportional change in its purchasing power. It is convenient to measure currency holdings as a proportion of wealth, so that we can use the constraint the N proportions x_j sum to one to eliminate the share of the numeraire (x_N) and express real returns relative to the numeraire ($\hat{r}_i = r_i - r_N$). Then, recalling the notation of (1) and defining covariances as $\theta_{ij} = \sigma_i \delta_j \rho_{ij}$ (where ρ_{ij} is the instantaneous correlation coefficient between dz_i and du_j) and $\delta_{j\ell} = \delta_j \delta_\ell \rho_{j\ell}$, the rate of change of wealth becomes:

(6) $$dW/W = \left(\sum_i^{N-1} x_i \hat{r}_i + r_N - V/W \right) dt - \sum_i^{N-1} x_i \sigma_i dz_i - \sum_j^N \alpha_j \delta_j du_j$$

where $x_i = \frac{N_i Q_i}{W}$; $i = 1, \ldots, N-1$

$$\hat{r}_i = -\pi_i + \sigma_i^2 + \sum_j^N \alpha j\ \theta_{ij} \quad ;$$

and $$r_N = -\sum_j^N \alpha_j \mu_j + \frac{1}{2} \sum_j^N \alpha_j \delta_j^2 + \frac{1}{2} \sum_j^N \sum_\ell^N \alpha_j \alpha_\ell \delta_j \ .$$

Ignoring bequests and the discounting of future utility, the individual seeks to maximize expected utility from time 0 to time T[6]. Instantaneous utility has a constant relative risk aversion $1 - \gamma$ and is a Cobb-Douglas function of the instantaneous rate of consumption of the N goods:

(8) $$U = E_o \int_o^T \frac{1}{\gamma} \prod_j^N X_j \ (t)^{\alpha_j \gamma} dt = E_o \int_o^T \tilde{u}(t) dt$$

where E_o denotes expectation conditional upon the information available at time 0 and $\tilde{u} = \frac{1}{\gamma} \prod_j^N X_j^{\alpha j \gamma}$

To solve the maximization of (8) subject to (6), the individual chooses X_j and x_i and the state of the system is described by S_i, P_j and W. Define:

(9) $$J(W, S_i\ P_j) = \max E_t \int_t^T \tilde{u}(\tau)\ d\tau$$

Since $\tilde{U}$ is strictly concave in X_j, by Bellman's theorem there exist optimal controls, X_j^* and x_j^*, such that $F^* = 0$, F being defined as

(10) $$F(x_i, X_j, W, S_i, P_j) = \tilde{u} + \mathcal{X}(J)$$

where $\mathcal{X}$ denotes the Dynkin operator.

Denoting as subscripts the semi-elasticities of the Bellman function J with respect to its arguments[7], and substituting from (6) and (7) into the differential of J, we get its average expected change per unit of time as:

(11) $$\mathcal{L}(J) = J_W\left(\sum_i^{N-1} x_i\hat{r}_i + r_N - V/W\right) + \sum_i^{N-1} J_{S_i}\pi_i + \sum_j^{N} J_{P_j}\mu_j$$

$$+ \frac{1}{2} J_{WW}\left(\sum_i^{N-1}\sum_j^{N-1} x_i x_j \sigma_{ij} + \sum_i^{N}\sum_j^{N} \alpha_i\alpha_j\delta_{ij} + 2\sum_i^{N-1}\sum_j^{N} x_i\alpha_j\theta_{ij}\right) + \frac{1}{2}\sum_i^{N-1}\sum_j^{N-1} J_{S_iS_j}\sigma_{ij}$$

$$+ \frac{1}{2}\sum_i^{N}\sum_j^{N} J_{P_iP_j}\delta_{ij} + \sum_i^{N-1}\sum_j^{N} J_{S_iP_j}\theta_{ij} - \sum_i^{N-1} J_{S_iW}\left(\sum_j^{N-1} x_j\sigma_{ij} + \sum_j^{N}\alpha_j\theta_{ij}\right) - \sum_i^{N} J_{P_iW}\left(\sum_j \alpha_j\delta_{ij} + \sum_j^{N-1} x_j\theta_{ij}\right).$$

Having substituted (11) into (10), the first order conditions are obtained by differentiating F with respect to the control variables and equating to zero. Take good j:

(12) $$\frac{\partial F}{\partial X_j} = \gamma\alpha_j\,\tilde{u}/X_j - J_W P_j Q_N/W = 0$$

Add up the N conditions to eliminate γ and write the demand function as:

(13) $$X_j^* = \alpha_j E_N/P_j$$

Substituting (13) back into the utility function to get the separable instantaneous indirect utility function in (2), adjusted for risk, we see that the purchasing power of the numeraire, adjusted for the marginal utility of real income, measures the utility of an extra unit of currency N:

(14) $$\frac{\partial[V^\gamma/\gamma]}{\partial E_N} = V^{\gamma-1} Q_N$$

Take now asset m:

(15) $$\frac{\partial F}{\partial x_m} = J_W\hat{r}_m + J_{WW}\left(\sum_i^{N-1} x_i\sigma_{mi} + \sum_j^{N}\alpha_j\theta_{mj}\right) - \sum_i^{N-1} J_{S_iW}\sigma_{im}$$

$$-\sum_j^{N} J_{P_jW}\theta_{jm} = 0 \quad .$$

To obtain the portfolio rule, we write the N-1 conditions in (15) in matrix form and solve for $\underset{\sim}{x}$, the N-1 column vector of portfolio proportions. First define adjusted "semi-elasticites" of the Bellman function with respect to exchange rates and prices:

$$J_{S_i} = \frac{S_i \partial^2 J}{\partial S_i \partial W} / \frac{\partial J}{\partial W} \qquad i = 1, \ldots, N-1$$

$$J_{Pj} = \frac{P_j \partial^2 J}{\partial P_j \partial W} / \frac{\partial^2 JW}{\partial W^2} \qquad j = 1, \ldots, N$$

The column vector of semi-elasticities is denoted by dropping i subscripts for J_{Si} and j subscripts for J_{Pj} and the same applies to the vectors of α_j and $\hat{r}_m$. Also, relative risk aversion is defined as:

$$-\frac{J_{WW}}{J_W} = 1 - \gamma .$$

Finally, the N-1 by N matrix of covariances between exchange rates and numeraire prices, θ_{ij} (which we denote by Θ), can be decomposed into a matrix of covariances between exchange rates and domestic prices, denoted by Ψ, and an augmented variance covariance matrix of exchange rate changes, σ_{ij}, denoted by $\tilde{S}$:

(16) $$\Theta = \Psi - \tilde{S},$$

where $\tilde{S} = [S \mid \underset{\sim}{0}]$, S being the variance covariance matrix of exchange rate changes and $\underset{\sim}{0}$ a N-1 column vector of zeros.

Using this notation, the portfolio rule can be expressed as:

(17) $$\underset{\sim}{x} = (\tilde{I} - S^{-1}\Psi)(\alpha - J_P) + \frac{1}{1-\gamma}(S^{-1}\hat{r} - J_S),$$

where $\tilde{I} = [\underset{\sim}{I} \mid \underset{\sim}{0}]$, $\underset{\sim}{I}$ being the N-1 identity matrix.

II Properties of the optimal currency portfolio

The portfolio rule for the N-1 assets in (17) can be related to some special cases analyzed in the literature and used to find out the effect of changes in α_i on x_i.

Suppose first that purchasing power parity holds continuously and therefore that relative numeraire prices cannot change. The covariance matrix between numeraire prices and exchange rates reduces to a vector of covariances between the domestic good of country N and the N-1 exchange rates. In other words, the Ψ matrix defined in (16) decomposes into:

(18) $$\Psi = \tilde{S} - \Psi_N e'$$

where $\Psi_N = [\sigma_1 \delta_N \rho_{1N} \quad \sigma_{N-1} \delta_N \rho_{N-1N}]'$

and e is a N column vector of ones.

Taking (18) into account, and the fact that J is invariant to a change in all prices, we see that the vector of expenditure shares and the J_p terms drop out:

(19) $$\underset{\sim}{x} = S^{-1} \Psi_N + \frac{1}{1-\gamma} [S^{-1} \hat{r} - J_S]$$

Since there are no relative price changes, the N goods can be aggregated into a single good when purchasing power parity holds. If, in addition, inflation in the Nth country is known (so that $\delta_N = 0$ and $\Psi_N = \underset{\sim}{0}$), the portfolio only depends on real return differentials, as in the case analyzed by Kouri (1977) in the framework of a model with a richer menu of assets.[8] As shown in Section III, however, using actual data to compute optimal portfolios suggests not only that inflation is not known with certainty but also that preferences do matter.

Indeed, preferences also enter real return differentials. Writing the first equation in (7) in vector form, we have:

(20) $$\hat{r} = -\underset{\sim}{\pi} + \hat{S}\underset{\sim}{e} + \Theta\alpha$$

where $\underset{\sim}{\pi} = [\pi_1 \quad \pi_{N-1}]'$

and $\hat{S} = \{\sigma_i^2\}$ is a diagonal matrix of exchange rate variances

and $\underset{\sim}{e}$ is a N-1 column vector of ones.

Then substitute (20) into (17) and use (16) to obtain:

(21) $$\underset{\sim}{x} = - \frac{\gamma}{1-\gamma} (\tilde{I} - S^{-1}\Psi)\ (\alpha - J_P) + \frac{1}{1-\gamma} (S^{-1}\hat{S}\underset{\sim}{e} - S^{-1}\underset{\sim}{\pi} - J_S)$$

Thus, even when inflation is perfectly anticipated and Ψ is a zero matrix, the effect of an increase in α_i will only be a one to one increase in x_i if risk aversion is infinite, so that, from (17) we have $x_i = \alpha_i$ for all i. Otherwise, the effect will be given by

(22) $$\frac{\partial x_i}{\partial \alpha_i} = -\frac{\gamma}{1-\gamma} \quad .$$

In particular, if risk aversion is unity ($\gamma = 0$) portfolio shares are insensitive to expenditure shares; if risk aversion is one half, an increase in α_i leads to a one-to-one decline in x_i. For the effect in (22) to be positive we have, therefore, to accept the common presumption that $\gamma < 0$. so that risk aversion is greater than one.[9]

We now derive the portfolio rule for the N assets, recalling that, by definition,

(23) $$x_N = 1 - \underset{\sim}{e}'\underset{\sim}{x},$$

where $\underset{\sim}{e}$ is a N-1 column vector of ones.

Using (17) and (23), we define as the vector of minimum variance portfolio proportions for the N assets, x^m, the capital position adjusted for the effects of inflation on utility:

(24) $$x^m = (I - \Phi)\ (\alpha - J_P)$$

where I is the N identity matrix ;

$$\Phi = \begin{bmatrix} S^{-1}\Psi \\ \text{-------} \\ -\underset{\sim}{e}'S^{-1}\Psi \end{bmatrix} \quad :$$

The minimum variance portfolio proportions sum to unity. Given that the α's are the capital position, inflation is hedged against by zero net worth portfolios and, in particular we have:

$$e'\Phi = \underline{0}'$$

where $\underline{0}$ is a N column vector of zeros.

In turn we define a zero net-worth portfolio based on real return differentials, adjusted for the effects of exchange rates on utility. The vector of speculative portfolio proportions for the N assets, x^s, is constructed by imposing the requirement that the augmented matrix of own and cross effects is symmetric. It can be written as:

(25) $$x^s = \frac{1}{1-\gamma}(\Sigma r - \tilde{J}_S),$$

where $$\Sigma = \left[\begin{array}{c|c} S^{-1} & -S^{-1}\underset{\sim}{e} \\ \hline -\underset{\sim}{e}'S^{-1} & \underset{\sim}{e}'S^{-1}\underset{\sim}{e} \end{array}\right]$$

$$r = [r_1 \qquad r_N]' \text{ .}$$

and $$\tilde{J}_S = [J_{S_1} \qquad J_{S_{N-1}} - \underset{\sim}{e}'J_S]'.$$

Since the speculative portfolio proportions sum to zero, we have by the symmetry of Σ that:

$$e'\Sigma = \underline{0}', \ \Sigma e = \underline{0} \text{ and } e'\tilde{J}_S = 0.$$

The total portfolio is then given by adding x^m and x^s:

(26) $$x = (I - \Phi)(\alpha - J_P) + \frac{1}{1-\gamma}(\Sigma r - \tilde{J}_S)$$

The optimal currency portfolio in (26) can equivalently be interpreted as showing the difference between expenditure shares and portfolio shares. Where inflation rates are known with certainty ($\delta_j = 0$ and the Φ matrix vanishes) and when risk aversion is very large ($\gamma \to \infty$) the two shares are equal. Larger variance of inflation and lower risk aversion are the two factors contributing to the zero net-worth hedging portfolios. Thus $\Phi\alpha$ hedges against the covariance of inflation and exchange rate changes, and is independent of real returns and risk aversion while Σr incorporates real return differentials and is scaled by risk aversion. The other determinants of optimal currency diversification, J_P and J_S, are dynamic adjustments to the direct effects of price and exchange rate changes on utility. Using the presentation in (21), on the other hand, we could define a modified speculative portfolio based on nominal return differentials, π_i, for the unit risk aversion case (γ = 0) and call it the Bernouilli portfolio, x^B:

(25')
$$x^B = \hat{I} - \Sigma\pi - \tilde{J}_S$$

$$\hat{I} = \left[\frac{S^{-1}\hat{S}\underset{\sim}{e}}{1 - \underset{\sim}{e}'S^{-1}\hat{S}\,\underset{\sim}{e}}\right]$$

and
$$\pi = (\underset{\sim}{\pi} \quad 0)'$$

Note that $e'x^B = 1$ because $e'\hat{I} = 1$. Using (25'), the portfolio rule for the N assets is a weighted average of x^m and x^B, with weights depending on risk aversion, just like in (21):

(26')
$$x = \frac{-\gamma}{1-\gamma}\, x^m + \frac{1}{1-\gamma}\, x^B$$

Rather than analysing the time-variant portfolio rules in (26) and (26') by an explicit solution of the Bellman function, we will now focus on the popular Brownian motion case.

III <u>Optimal currency diversification in the Brownian motion case</u>

If prices and exchange rates are stationary and lognormally distributed, so that π_i, μ_j, and σ_i and δ_j in (1) above are given constants, wealth becomes the only state variable in (9), all of the elements of J_p and $\tilde{J}_z$ are zero and the rule in (26) can be written as:

(27) $$x = (I - \Phi)\,\alpha + \frac{1}{1-\gamma}\,\Sigma r.$$

As an illustration of the usefulness of a rule like (27), consider an international investor who holds a portfolio of Canadian, French, German, Italian, Japanese, Swiss, British and American currencies and consumes the basket of goods included in these countries' consumer price indices. Using the average quarterly percentage changes in the purchasing powers of these eight currencies to deflate call money rates and to generate the matrices Φ and Σ in (27) and taking a given capital position, we can compute optimal portfolios. This is done in Table 1, using as weights the share of each country in U.S. dollar income in the mid-seventies, for two sample periods and two degrees of risk aversion.

Table 1 suggests that inflation was not fully anticipated because, even under the assumption of stationarity, in both sample periods the portfolios differ from the capital position more than they differ from each other (given the same degree of risk aversion). Note also that when risk aversion is infinite the optimal portfolio is the minimum variance portfolio but when risk aversion is two ($\gamma = -1$) it is the sum of the minimum variance portfolio and one-half of the speculative portfolio defined on real return differentials. Furthermore, the comparison of the two sample periods suggests the usefulness of a time-invariant rule like (27). In fact, except for sterling and lire, where real returns changed substantially in 1979 and 1980, the shares do not change "too much." [11]

Table 1

Optimal Currency Diversification

Country (α Weights %)	1973; 4 - 1980; 12 Portfolio (%)		1973; 4 - 1978; 4 Portfolio (%)	
	$\gamma = -1$	$\gamma = -\infty$	$\gamma = -1$	$\gamma = -\infty$
Canada (5)	2	17	4	18
France (9)	12	12	11	12
Germany (13)	12	9	13	14
Italy (5)	-5	5	2	6
Japan (16)	13	13	11	8
Switzerland (2)	6	2	7	0
United Kingdom (7)	14	9	5	9
United States (43)	46	33	47	33

Source: End of month exchange rates, consumer price indices and call money rates from <u>IFS</u>. Matrices in (27) reported in Macedo (1982) for the 1973; 4 - 1978; 4 period and available from the author for the 1973;4 - 1980; 12 period.

Now if inflation were perfectly anticipated, an increase in the share of goods from country i in expenditure would imply an equal increase in the share of currency i in the minimum variance portfolio. The larger the variance of inflation relative to the variance of exchange rate changes the less an increase in α_i will be reflected in an increase in x^m_i.

Furthermore, the effect of the covariance of inflation and exchange rates can be estimated from the elements of Φ: if risk aversion is two, for example, $\partial x_j/\partial\alpha_i = \phi_{ij}/2$ while if risk aversion is four, $\partial x_j/\partial\alpha_i = .8\phi_{ij}$. Using the same data as in Table 1, left panel, we can thus estimate the size of the effect before it is scaled down by risk aversion, provided that risk aversion is larger than one. It turns out that about half of the entries of the computed Φ matrix are less than .1 in absolute value, but that the effects of the Japanese share on the Canadian dollar and on the U.S. dollar shares, the effect of the British share on the French franc share or the own effect of Italy are larger and should not be neglected.[11]

This suggests that even national infinitely risk-averse investors will hold a diversified currency portfolio, so that the usual presumption that the domestic currency is the "preferred monetary habitat" should be used with caution. On the other hand, as risk aversion decreases, the effect of consumption preferences on optimal currency diversification also decreases, until it vanishes when relative risk aversion is unity.

Conclusion

This paper has derived the optimal consumption and portfolio rules for an international investor with constant expenditure shares α_j and constant relative risk aversion $1-\gamma$ in a dynamic contest. The index of value obtained from the consumption rule was used to obtain real returns on different currencies in terms of their purchasing power over goods. The portfolio rule was expressed in terms of the determinants of the purchasing powers, exchange rates and prices expressed in the numeraire currency. The optimal portfolio was interpreted as a capital position given by the expenditure shares and hedging portfolios depending on unanticipated inflation and risk aversion. It was shown that, when exchange rates and prices are Brownian motion there are only two zero net worth portfolios, the minimum variance portfolio independent of risk aversion and real return differentials but depending on expenditure patterns and the speculative portfolio independent of preferences and inflation, but depending on risk aversion and real return differentials. When the effect of preferences on real return differentials (and hence on the speculative portfolio) is made explicit, this separation is blurred and the minimum variance portfolio is affected by risk aversion. In that case, the effect of an increase in α_i on the portfolio proportions x_i will be positive when relative risk aversion is greater than one, as generally presumed.

Actual data from eight major countries is used to compute optimal portfolios based on real return differentials for different weighting schemes, degrees of risk aversion and sample periods and thereby show the empirical usefulness of this application of the theory of international finance from the perspective of continuous time. In particular, the results presented cast doubt on two popular hypotheses, purchasing power parity and the preferred monetary habitat.

Notes

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1. See Solnik (1974). A survey of Merton's work can be found in Merton (1975) and in Chow (1979).

2. Kouri (1976) derived the forward premium in a two-country model with different consumption preferences in each country and exchange rate and price changes generated by Brownian motion. He also used Poisson processes to analyze the adjustable peg regime.

3. The case where national outputs are not composite goods and where assets yield a known nominal return is analyzed in Macedo (1982).

4. Dornbusch (1980a) has a survey of the Brownian motion model.

5. The definition of the numeraire is, of course, arbitrary. Also, the analysis could equally well be conducted expressing the prices of the N goods in domestic currency rather than in the numeraire. See Meerschwam (1982).

6. A bequest function with elasticity γ with respect to terminal wealth would not change the results. It would also be easy to introduce a discount factor on instantaneous utility. Finally, we take T to be "very large" so that we have an approximately infinite horizon.

7. For example, $J_W = W\partial J/\partial W$ and $J_{S_iP_j} = S_iP_j\partial^2 J/\partial S_i \partial P_j$.

8. See also Fama and Farber (1979) and Hodrik (1981).

9. On the presumption about γ see Macedo (1980), Krugman (1981) and, relaxing the assumption of constant expenditure shares, Stulz (1981).

10. See further discussion of weighting schemes and relative risk aversion in Macedo (1981). A five-currency portfolio for a Bernoulli international investor is reported in Kouri and Macedo (1978). Dornbusch (1980b) computes an optimal dollar and D-mark portfolio.

11. Systematic evidence on the time-invariance of computed portfolios can be found in Macedo, Goldstein and Meerschwam (1982).

12. These effects are $\partial x^m_{CA}/\partial\alpha_{JA} = -.3$; $\partial x^m_{US}/\partial\alpha_{JA} = .4$; $\partial x^m_{FR}/\partial\alpha_{UK} = .2$ and $\partial x^m_{IT}/\partial\alpha_{IT} = .8$ (so that $\phi_{44} = .2$).

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