Tobin's q, Economic Rents, and the Optimal Stock of Capital

by Richard W. Kopcke

No. 92-3 February 1992

Working Paper Series

Federal Reserve Bank of Boston

Tobin's q, Economic Rents, and the Optimal Stock of Capital

by Richard W. Kopcke

February 1992
Working Paper No. 92-3

Federal Reserve Bank of Boston

Tobin's q, Economic Rents, and the Optimal Stock of Capital

by

Richard W. Kopcke

February 3, 1992

Federal Reserve Bank of Boston

Tobin's q, Economic Rents, and the Optimal Stock of Capital

The correspondence between the demand for capital and various measures of Tobin's q often is tenuous (Abel and Blanchard 1986; Hayashi 1982), at times even perverse. Among the possible explanations for this apparent challenge to the q theory of investment, this paper considers the consequences of allowing the return on capital to vary with the scale of production. When enterprises earn economic rents on inframarginal investments, the q theory of investment does not claim that changes in the optimal stock of capital must correspond consistently to changes in marginal q.

Recent theoretical refinements of the theory of investment conclude that marginal q need not equal average q and that marginal q need not always equal unity at the optimal stock of capital.[1] Because estimates of marginal q are much less accurate than estimates of average q, the questionable performance of empirical models could be attributed to errors in estimating marginal q when average and marginal q diverge. Moreover, the optimal value of marginal q may vary with adjustment costs or the value of implicit options, thereby weakening the link between the demand for capital and empirical measures of q or marginal q.

This paper examines the relationship between marginal q and the optimal stock of capital when enterprises earn economic rents. Suppose theory requires marginal q to equal unity at the optimal stock of capital;

---

[1]See, for example, Hayashi (1982, 1985). The implications of adjustment costs and the value of the option of waiting to invest are examined by Myers (1977), McDonald and Seigel (1986), Pindyck (1988, 1991), Greenwald and Stiglitz (1989), and Galeotti and Schiantarelli (1991). Wildasin (1984) and Hayashi and Inoue (1990) discuss the complications of multiple capital inputs. The consequences of imperfections in capital and goods markets are explored by Chirinko (1987) and Fazzari, Hubbard and Petersen (1988).

adjustment costs and the value of implicit options are negligible. Then, as shown below, theory also allows the optimal stock of capital to fall greatly, negligibly, or, perhaps, even to rise when lower expected returns reduce marginal q by a specific amount at the formerly optimal stock of capital. The outcome depends partly on the change in rents on inframarginal investments. For a given change in marginal q, the magnitude of the change of the optimal stock of capital tends to be greater as rents become smaller relative to marginal returns. The result also depends on the determinants of q. q may depend on an enterprise's choice of leverage as well as the magnitude of its stock of capital. In this case, reductions in expected returns that depress marginal q may increase the optimal stock of capital if the change in expected rents also alters optimal leverage sufficiently.

The first section of this paper introduces a simple, one-period project with one factor of production, capital, for which returns decline with increasing scale. After defining the concept of marginal q, this section describes the relationships among the optimal stock of capital, marginal q, and the marginal return on investment for this project. When changing returns on investments reduce (increase) marginal q at the formerly optimal stock of capital, the optimal stock of capital also falls (rises), but, as discussed in this section, the change in marginal q does not indicate the magnitude of the change in the optimal stock of capital. The change in the scale of the project depends on the returns on inframarginal investments as well as those on marginal investments, a distinction that does not arise with constant returns. A numerical example illustrates this conclusion.

If, contrary to the assumption of the first section, q does not vary solely with the scale of the project, marginal q and the optimal stock of capital can change in opposite directions. In the second section of this paper, q is a function of the stock of capital and leverage: shareholders' expectations of the project's rents exceed creditors' expectations, and the tax burden on shareholders' returns exceeds that on returns distributed to creditors. In this case, a change in expected returns that reduces marginal q at the formerly optimal stock of capital may increase the optimal stock of capital, provided the optimal choice of leverage changes sufficiently, as is likely when returns on inframarginal investments fall relative to those on marginal investments. This result is analogous to the potential conflict between the conclusions of partial and general equilibrium analysis in microeconomics.

The third section comprises two numerical examples. The first illustrates the possibility that marginal q and the optimal stock of capital may move in opposite directions. The second shows how constraints on shareholders' access to equity financing may enforce a tendency for marginal q and capital to change in the same direction. With these constraints, however, marginal q tends to exceed unity at the optimal stock of capital.

The conclusion of this paper notes that the potential for marginal q and the optimal stock of capital to change in opposite directions arises whenever, in addition to capital, q is a function of at least one other variable that is to be chosen by the enterprise. Although this paper stresses q's potential dependence on leverage, q also depends on the mix of factors of production, the technology of production, the composition of output, and the promotion of output. Consequently, the potential for a seemingly perverse correspondence between changes in marginal q and the optimal stock of capital does not rest solely on conditions that violate the assumptions of the Modigliani-Miller theorem.

I. q Depends Only on the Stock of Capital

This section examines a simple enterprise for which marginal q equals unity at the optimal stock of capital. When changing conditions alter expected returns in this model, the resulting changes in the optimal stock of capital do not necessarily correspond to changes in marginal q if the enterprise earns an economic rent. When marginal q falls (rises) at the formerly optimal stock of capital, the optimal stock of capital also falls (rises), but the magnitude of this change depends on the size of the rents on inframarginal investments.

The Return on Capital, the Cost of Capital, and Tobin's q

The expected net revenue accruing to a one-year enterprise depends on its stock of capital, its only factor of production. Given its capital, K, the enterprise produces $Q(K)$ units of output.[2] This production function exhibits diminishing returns. Because of uncertainties regarding the supply schedule for raw materials and the demand schedule for output, the enterprise's net revenues are not known when its capital is installed. The enterprise expects to receive a net price (value-added) of $P(Q(K))$ for each unit of its output. This expected net price falls as Q increases because of an upward-sloping supply schedule for raw materials or a downward-

---

[2] K also may be regarded as an index of the enterprise's scale of operation, indicating the distance along its expansion path through its isoquants in factor space.

sloping demand schedule for the enterprise's output. Accordingly, expected net revenue is

$$(1) \quad REV(K) = P(Q(K))\ Q(K)$$

$$Q' > 0$$
$$Q'' < 0$$
$$P' < 0.$$

The investors who finance the enterprise pay $1 for each unit of capital, and, after one period, sell the undepreciated capital for the same price. The function P, consequently, represents the expected value of the net price of the enterprise's output relative to the price of capital goods. The investors' expected rate of return is defined by

$$(2) \quad 1+r(K) = \frac{K + REV(K)}{K}$$

$$r(K) = REV(K)/K .$$

Tobin's q is the ratio of the value of this enterprise in financial markets to the replacement value of its capital goods. In the spirit of the Modigliani-Miller theorem, the value of the enterprise does not depend on investors' arrangements for sharing its cash flow among themselves (Modigliani and Miller 1958 and 1963; Miller and Modigliani 1961; Miller 1977; Myers 1984). However the enterprise is financed, its securities are priced so that they are held in the optimal market portfolio (Lintner 1965), and their composite return is always the same as that of the enterprise.[3] Accordingly, the market valuation of the enterprise is the same whether its cash flow is conveyed entirely by equity or by another blend of securities.

The enterprise's cost of capital is the discount rate implicit in the unique market valuation of its expected cash flow. For a given value of K, the correlation between the enterprise's returns and either the returns on the market portfolio (CAPM) or the potential states of nature (APT) entails a cost of capital of $\rho(K)$.[4] Consequently, Tobin's q equals

$$(3)\quad q(K) = \frac{(K+REV(K))}{(1+\rho(K))} / K = \frac{(1+r(K))}{(1+\rho(K))}.$$

For the enterprise to be viable for any value of K,

$$(4)\quad r(K) \geq \rho(K).$$

The cost of capital does not exceed the rate of return on capital.

Marginal Returns, Marginal q, and the Optimal Stock of Capital

The expected marginal return on capital is the expected change in revenue, ΔREV, resulting from a change in the stock of capital, ΔK. Denoting the derivative of y with respect to x as $D_x y$ and the elasticity

---

[3]When the income that accrues to shareholders is taxed differently than the income accruing to creditors, this assertion is not true. The analysis of section II introduces a simple corporate income tax. See King (1977), Auerbach (1979, 1983), DeAngelo and Masulis (1980), and Poterba and Summers (1983).

[4]With increasing K, the importance of the enterprise's rents dwindle along with its average return on capital (see (5) and (6) below). For risk-averse investors, the discount rate may depend on K because the uncertainty regarding the rate of return on capital may change as the rents are diluted. Furthermore, the covariance between the return on capital for the enterprise and the return on the market portfolio may change as increasing K diminishes the importance of the enterprise's unique features.

of y with respect to x ($(x/y)D_x y$) as $\eta_{y,x}$, the expected marginal return is

$$(5) \quad \frac{\Delta REV(K)}{\Delta K} \rightarrow REV'(K) = PQ'(1+\eta_{P,Q}) > 0.$$

Assuming that the elasticity of demand does not rise as K and Q rise, the expected marginal return on capital falls as K rises:[5]

$$(6) \quad REV''(K) = REV'(K) \left(\frac{P'}{P} + \frac{Q''}{Q} + \frac{D_K(\eta_{P,Q})}{1+\eta_{P,Q}}\right) < 0.$$

For any stock of capital, marginal q is the change in the overall market value of the enterprise resulting from the addition of another capital good, $\Delta(qK)$, divided by the replacement value of this capital good, ΔK. Accordingly, marginal q is

[5]This restriction on the elasticity of demand is sufficient, but not necessary for REV' to be a declining function of K, provided the magnitudes of P' and Q'' are sufficiently great. Because P' is negative and P is positive, there exists a threshold for K such that for all K greater than this threshold, the elasticity of demand falls as K increases.

$$(7)\quad \upsilon \triangleq \frac{\Delta(qK)}{\Delta K} \rightarrow D_K(qK) = q + K\, D_K q$$
$$= q\,(1+\eta_{q,K})\ .$$

$$(8)\quad \upsilon = q + (REV' - r - qK\,\rho')\ /\ (1+\rho)$$
$$= (1 + (REV' - qK\,\rho')\ /\ (1+\rho)\ .$$

Marginal q tends to fall as K increases if the marginal return on investment also falls as K increases:[6]

$$(9)\quad \upsilon' = (REV'' - 2\upsilon\,\rho' - qK\,\rho'')\ /\ (1+\rho)\ .$$

If investors choose K to maximize their wealth, then marginal q equals unity at the optimal stock of capital. When investors alter K, the resulting change in their wealth equals the change in the value of the enterprise, $\Delta(qK)$, less their expenditures on new capital goods, ΔK.[7] From the description of marginal q, investors become wealthier by increasing K if υ exceeds unity. Once υ drops to unity, investors do not alter their wealth by undertaking further investment. After υ falls below unity, investors reduce their wealth by increasing K.

---

[6]A negative REV'' is not a necessary condition for this result. The contribution of the second term in the numerator may suffice. On the other hand, the rate of change of the cost of capital may decrease as K increases (the third term of the numerator), so a negative REV'' is not a sufficient condition. Nevertheless, a lower bound on the cost of capital implies that once the stock of capital becomes sufficiently great, marginal q is a declining function of K.

[7]This arbitrage between capital goods and securities is essentially that described by Keynes (1936, chapter 12) and Tobin (1969, 1982).

Substituting unity for υ in (8),

$$(10) \quad REV' = \rho + qK\rho'.$$

At the optimal stock of capital, the expected return on marginal capital equals the marginal cost of capital as defined on the right side of (10). Equation (10) may not be consistent with the familiar first-order condition resulting from the maximization of profit with respect to K. ρ' must be zero to reconcile (10) with profit maximization when profit is defined as $REV(K) - \rho(K)K$. If ρ' is not zero, then this reconciliation requires that q equals unity at the value of K for which υ equals unity, implying that the maximal value of q also is unity (see (7)).

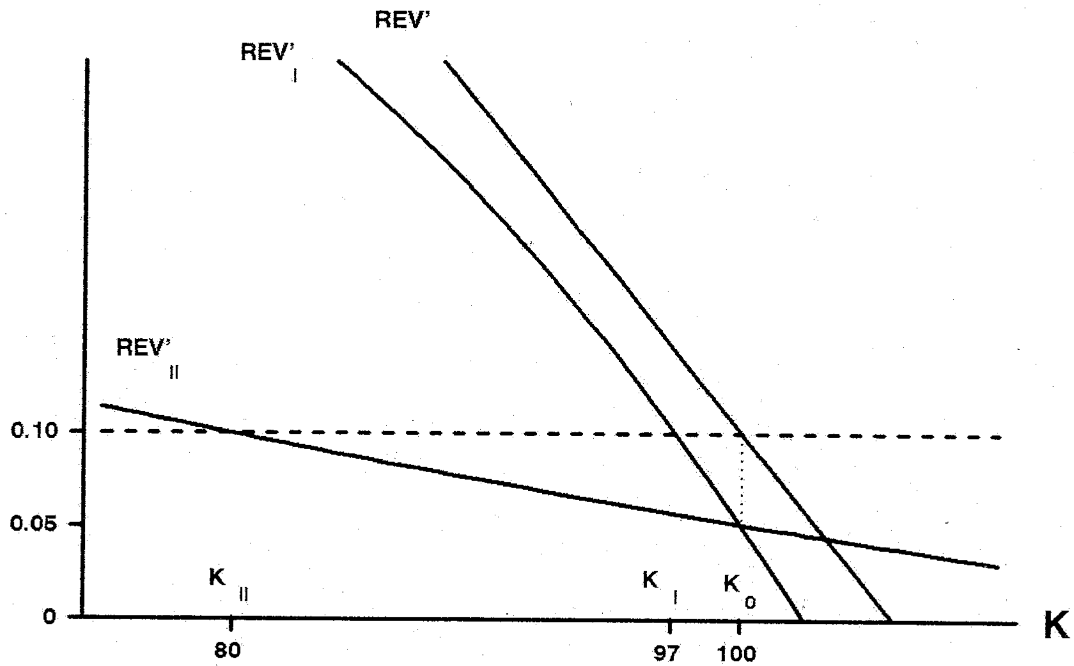
<u>Proposition 1</u>: If an enterprise earns economic rents, q is independent of leverage, and investors maximize their aggregate wealth,

then:
- (i) the optimal stock of capital equates marginal q with unity;
- (ii) if the cost of capital varies with K, the optimal K does not maximize profit or the return on capital; and
- (iii) any shift of the expected revenue function, $REV(K)$, that increases (decreases) marginal q also increases (decreases) the optimal K, but the change in the optimal K depends on the shape of REV.

<u>The Optimal Stock of Capital: the Cost of Capital is Constant</u>

The two cases in figure 1 illustrate the third conclusion of proposition 1. Suppose, for simplicity, that the cost of capital is constant, at 10 percent. From (10), the optimal scale of the enterprise (K_o) is defined initially by the intersection of REV' with the horizontal

Figure 1



I: $R = 0.55 - 1.0 * 10^{-9} K^{4}$

$REV' = 0.55 - 5.0 * 10^{-9} K^{4}$

II: $R = 2.27 - 1.27 * K^{-1}$

$REV' = 2.27 - 1.40 * K^{-1}$

cost of capital function. Suppose potential changes in the demand schedule for the enterprise's output could shift the schedule REV' to either REV'_I or REV'_{II}. Because both alternatives reduce expected marginal returns by the same amount at K_o, they also reduce marginal q by the same amount at K_o (see (8)). Yet, the optimal stock of capital falls more for the second alternative than it does for the first, because the slope of REV'_I exceeds that of REV'_{II} -- the returns on inframarginal investments for the first alternative increase more rapidly as K falls than do those for the second.

q is greater for REV'_I than for REV'_{II} at K_o, but this does not imply that q, by itself, is an accurate indicator of the optimal stock of capital. Suppose the two schedules of expected marginal returns shift up, instead of down, by the same amount at K_o. In this case, the optimal stock of capital increases more for the second alternative than for the first, even though q for the second alternative is less than q for the first.

The change in the optimal stock of capital depends on the shape of the function representing the marginal return on investment. As suggested in figure 1, when REV' is more concave -- rents on inframarginal investments are relatively great -- the optimal stock of capital changes less for a given change in marginal q. Although an increase in expected marginal returns or marginal q correctly indicates an increase in the optimal stock of capital under the conditions of proposition 1, these statistics are not sufficient for determining the magnitude of this increase without knowledge of the rents on inframarginal investments.

II. q Depends on the Stock of Capital and Leverage

In this section *q* depends on the enterprise's leverage as well as its stock of capital as a result of asymmetries in the income tax law and differences among investors' expectations. Because the stock of capital that equates marginal q with unity varies with leverage under these circumstances, a change in conditions that reduces marginal q (evaluated at the formerly optimal stock of capital and leverage) does not necessarily imply that the optimal stock of capital falls if optimal choice of leverage also changes.

Leverage, Heterogeneous Expectations, and q

Not all investors are equally optimistic about the enterprise's returns for at least two reasons. First, the uncertainty inherent in returns is reflected in the distribution of assessments among investors. Some investors, for example, believe that states of nature favorable to the enterprise are very likely to occur, while others believe these states are not so probable. Second, not all investors possess the same information about the enterprise's returns in the various states of nature, and not all are equally able to extract its rents. Equity is most valuable to those who foresee the greatest rent, while other investors do not value a residual claim on the enterprise's returns so greatly.[8]

[8]These examples are not the only sources of heterogeneity or asymmetry. If shareholders lacked sufficient resources to finance the project themselves, they might pay a premium to obtain external financing. Regulations, contracts, and conventions governing the eligible assets of banks, insurance companies, pension funds, and other institutional investors tend to favor debt over equity, thereby increasing the cost of external equity financing compared to that of debt financing. Even if outsiders were no less optimistic than shareholders, external

The enterprise's investors comprise shareholders and creditors. Although shareholders are able to finance the enterprise entirely by themselves, they may borrow funds from creditors. The proportion of the enterprise's capital financed by shareholders is φ; that financed by creditors is $(1-\varphi)$. For any stock of capital, all shareholders describe the rate of return on capital by the same probability distribution, $pdf_s(\tilde{r}|K)$. The corresponding distribution for creditors is $pdf_c(\tilde{r}|K)$. The shareholders' expected value of the enterprise's rate of return on capital, $r_s(K)$, exceeds the creditors' expectation.

To simplify the following analysis, assume either that, for any K, investors believe the enterprise's returns are not correlated with those of the market portfolio or that investors are risk-neutral. Under these circumstances, the discount rate for shareholders and creditors alike is the risk-free discount rate, hereafter denoted by ρ, which does not vary with φ or K.

The rate of interest on the enterprise's debt equates creditors' expected return on debt with their discount rate.[9] Denoting the rate of interest as i, the yield on debt also is i provided the sum of the enterprise's return and shareholders' equity is sufficiently great to pay interest obligations, $\tilde{r} + \varphi \geq i(1-\varphi)$. Otherwise, the rate of return to

---

financing may command a premium, because those who control access to external financing may attempt to extract a share of the project's rents. See Navin and Sears (1955), Carosso (1970), Jensen and Meckling (1976), Myers and Majluf (1984), Baskin (1988), and Bernanke and Gertler (1990).

[9]Because the terms on debt compensate creditors for the risks they bear and the market value of debt equals its face value, shareholders have no opportunity to benefit from strategies that could reduce the market value of debt relative to its face value.

creditors is only $(\tilde{r}+\varphi)/(1-\varphi)$. Equating the discount rate with the creditors' expected rate of return on debt,

$$(11)\quad \rho = \int_{(1-\varphi)i-\varphi}^{\infty} i\, pdf_c(\tilde{r})\, d\tilde{r} + \int_{-100\%}^{(1-\varphi)i-\varphi} [(\tilde{r}+\varphi)/(1-\varphi)]\, pdf_c(\tilde{r})\, d\tilde{r},$$

$$(12)\quad i-\rho = \int_{-100\%}^{(1-\varphi)i-\varphi} [i-(\tilde{r}+\varphi)/(1-\varphi)]\, pdf_c(\tilde{r})\, d\tilde{r}.$$

The "risk premium" embedded in i equals the expected value of the potential losses on debt contracts. Other things equal, a lower expected rate of return on capital entails a greater i.

q equals the sum of the values of equity shares and debt divided by the replacement value of capital goods. The value of debt is the present value of creditors' expected receipts, which from (11) equals $(1-\varphi)K$. The value of equity is the present value of the receipts that shareholders expect to receive one period in the future. Shareholders receive a payment (which includes some return of their initial investment) only when $\tilde{r}+\varphi$ exceeds $(1-\varphi)i$; otherwise, they lose their entire investment.

$$(13)\quad q = \{ K \int_{(1-\varphi)i-\varphi}^{\infty} [\tilde{r}+\varphi-(1-\varphi)i]\ pdf_s(\tilde{r})\, d\tilde{r}/(1+\rho) \quad + \quad K(1-\varphi)\} \,/\, K$$

$$= \{ \varphi + \int_{(1-\varphi)i-\varphi}^{\infty} (\tilde{r}-(1-\varphi)i)\ pdf_s(\tilde{r})\, d\tilde{r}$$

$$- \int_{-100\%}^{(1-\varphi)i-\varphi} \varphi\ pdf_s(\tilde{r})\, d\tilde{r}\ \} \,/\, (1+\rho) \ +$$

$$\{ (1-\varphi) + \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\ pdf_c(\tilde{r})\, d\tilde{r}$$

$$+ \int_{-100\%}^{(1-\varphi)i-\varphi} (\tilde{r}+\varphi)\ pdf_c(\tilde{r})\, d\tilde{r}\ \} \,/\, (1+\rho) \ .$$

$$= (1+r_s)/(1+\rho) \quad + \{ \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\ (pdf_c(\tilde{r}) - pdf_s(\tilde{r}))\ d\tilde{r}$$

$$+ \int_{-100\%}^{(1-\varphi)i-\varphi} (\tilde{r}+\varphi)(pdf_c(\tilde{r}) - pdf_s(\tilde{r}))\ d\tilde{r}\ \} \,/\, (1+\rho)$$

If pdf_c were identical to pdf_s or if φ were unity, then (13) would be identical to (3). Using (11) the last equality of (13) can be rewritten:

$$(14)\quad q = (1+r_s)/(1+\rho) \quad + \{ \rho(1-\varphi) - \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\ pdf_s(\tilde{r})\, d\tilde{r}$$

$$- \int_{-100\%}^{(1-\varphi)-\varphi} (\tilde{r}+\varphi)\ pdf_s(\tilde{r})\, d\tilde{r}\ \} \,/\, (1+\rho) \quad .$$

Suppose that, for any K, pdf_s matches pdf_c in all respects, except that the shareholders' expectation of the enterprise's rate of return on capital exceeds the expectation of creditors.[10] Therefore, for given φ and K, the rate of interest that equates the expected return on debt with the discount rate is greater for creditors than it would be if shareholders were purchasing the debt. Denoting this difference by δ, for shareholders the interest rate need be only $(i-\delta)$ to equate the expected return on debt with ρ.[11] From the shareholders' viewpoint (from (11)):

$$(15)\quad \rho(1-\varphi) = \{ \int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} (1-\varphi)(i-\delta)\ pdf_s(\tilde{r})\,d\tilde{r} + \int_{-100\%}^{(1-\varphi)(i-\delta)} (\tilde{r}+\varphi)\ pdf_s(\tilde{r})\,d\tilde{r} \}$$

$$= \{ \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\ pdf_s(\tilde{r})\,d\tilde{r} + \int_{-100\%}^{(1-\varphi)i-\varphi} (\tilde{r}+\varphi)\ pdf_s(\tilde{r})\,d\tilde{r} \}$$

$$- \int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} (1-\varphi)\delta\ pdf_s(\tilde{r})\,d\tilde{r}$$

$$- \int_{(1-\varphi)(i-\delta)-\varphi}^{(1-\varphi)i-\varphi} (\tilde{r}+\varphi-(1-\varphi)i)\ pdf_s(\tilde{r})\,d\tilde{r}\ .$$

---

[10]If the two distributions were not otherwise identical, then they might entail different discount rates for debt and equity.

[11]If, for given K, the creditors' expected rate of return on capital equals ρ, then i and δ increase without limit as φ approaches zero, provided pdf_c places no upper bound on $\tilde{r}$: when φ is zero, the creditors' expected return on debt would be less than ρ for any finite value of i. If the creditors' expected return on capital exceeds ρ, then i and δ are bounded as φ approaches zero.

Substituting (15) for $\rho(1-\varphi)$ in (14),

$$(16) \quad q = (1+r_s)/(1+\rho) - \Big\{ (1-\varphi)\delta \int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} pdf_s(\tilde{r})\,d\tilde{r} + \int_{(1-\phi)(i-\delta)-\varphi}^{(1-\varphi)i-\varphi} (\tilde{r}+\varphi-(1-\varphi)i)\; pdf_s(\tilde{r})\,d\tilde{r} \Big\} / (1+\rho)$$

$$= (1+r_s)/(1+\rho) - \lambda(\varphi, K).$$

λ, a positive function, reflects the expected value of the revenues forgone by shareholders in order to compensate the less optimistic creditors.

Asymmetric Taxation

The net return on the enterprise's stock of capital goods tends to rise with leverage when the enterprise pays a tax on returns that are distributed to shareholders, while paying no tax on returns distributed to creditors.[12] Assuming the return to shareholders is taxed as corporate income at rate τ when this return is positive, then (16) becomes

$$(17) \quad q = (1+r_s)/(1+\rho) - \lambda(\varphi, K) - \tau \int_{(1-\varphi)i}^{\infty} (\tilde{r}-(1-\varphi)i)\; pdf_s(\tilde{r})\,d\tilde{r} \,/\, (1+\rho)$$

$$= (1+r_s)/(1+\rho) - \lambda(\varphi, K) - \tau\,\mu(\varphi, K) \;..$$

μ, a positive function, reflects the expected value of shareholders'

[12]If total net returns were correlated with the market portfolio or investors were risk-averse, then the discount rate might vary with leverage.

returns subject to corporate income taxation. This tax liability decreases as either leverage or K increases.

Marginal q and the Optimal Choice of Capital and Leverage

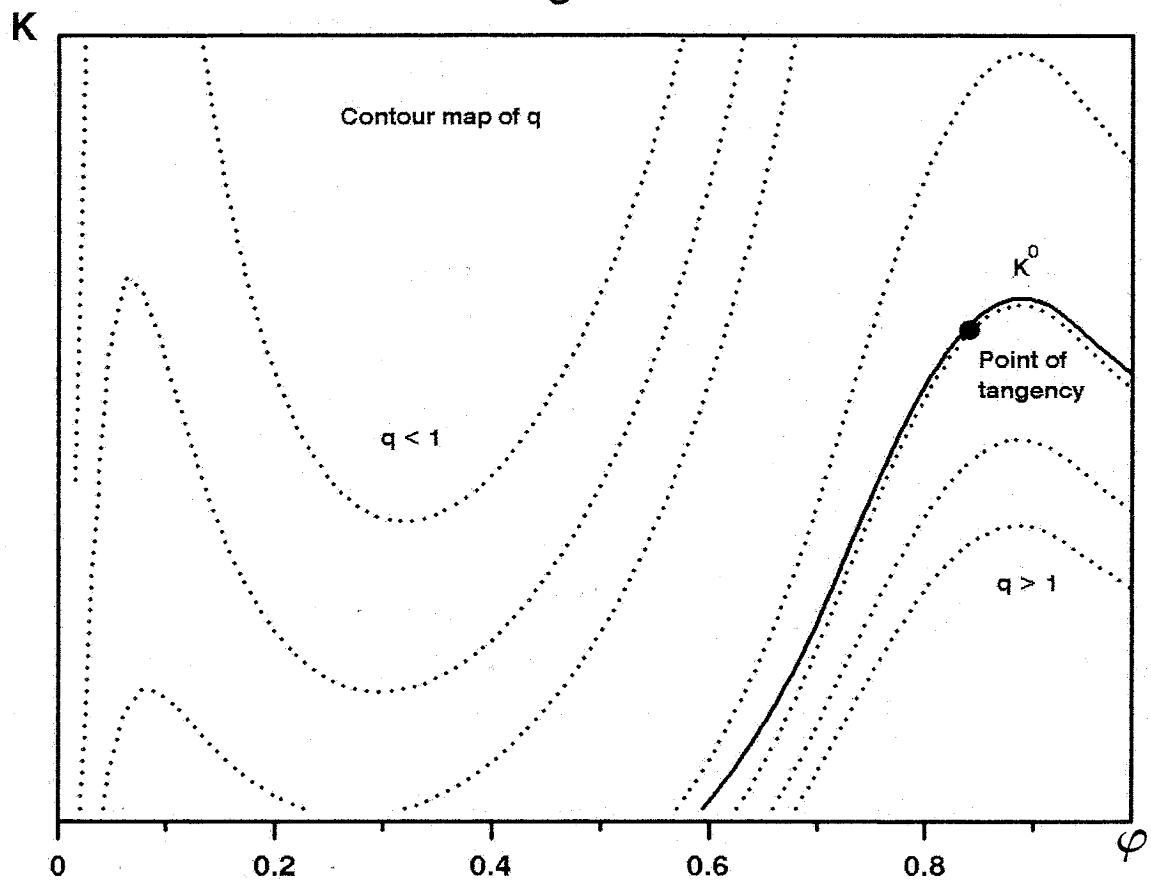
When q depends on the composition of the enterprise's financing as well as its stock of capital, selecting the optimal K and φ may be separated into two steps. First, for any value of φ, shareholders maximize their wealth by choosing K so that marginal q equals unity, thereby defining the optimal K as a function of φ, $K^o(\varphi)$ (see figure 2). For reasons discussed below, the optimal choice of K varies with φ, because K^o is not horizontal over any open interval of φ. Second, q is maximized with respect to φ, subject to the constraint that K equals K^o: φ maximizes $q(\varphi, K^o)$ at the point where the graph of K^o is tangent to a contour of q. These steps imply that q may be written as a function of φ alone, but q cannot be expressed as a function of K alone.[13]

If shareholders chose K to maximize their wealth, for fixed φ the optimal K equates υ with unity. When shareholders alter K, the resulting change in the value of their shares equals the change in the value of the enterprise less the change in value of its debt: $\Delta(qK) - (1-\varphi)\Delta K$. The net change in shareholders' wealth equals the change in the value of their shares less $\varphi\Delta K$, their expenditures on new capital goods: $\Delta(qK) - \Delta K$. From the definition of marginal q, shareholders benefit by increasing K whenever υ exceeds unity; they are indifferent about altering K when υ equals unity; and they benefit by decreasing K when υ is less than unity.

---

[13]This ranking of decisions cannot be reversed by first choosing leverage to maximize q given K, then choosing K so that marginal q equals unity (see appendix). Therefore, the definition of marginal q implied by (17) must be conditional on leverage.

Figure 2



Given φ, applying (7) to (17) defines marginal q,

$$(18) \quad \upsilon = (1 + r_s + K\, r_s')/(1+\rho) - (\lambda + K\, D_K\lambda) - \tau(\mu + K\, D_K\mu) \ .$$

Setting marginal q equal to unity, K^o is implicitly defined by

$$(19) \quad REV_s' = \rho + (1+\rho)\{\ \tau(\mu + K\, D_K\mu) + (\lambda + K\, D_K\lambda)\ \}$$
$$= \rho + (1+\rho)\{\ \tau\mu(1 + \eta_{\mu,K}) + \lambda(1 + \eta_{\lambda,K})\ \} \ .$$

The expression on the right side of (19) defines the marginal cost of capital. Together, (7) and (19) show that when marginal q does not equal average q, for the same reasons the marginal cost of capital also does not equal the average cost of capital. If the premium that shareholders pay creditors (δ) is sufficiently great or rises sufficiently rapidly as K increases, then the shareholders' expected marginal return on capital exceeds ρ at the optimal stock of capital and the marginal cost of capital increases with K.[14]

K^o is not horizontal (parallel to the φ axis) over any open interval for φ unless the corporate tax rate is zero and creditors' assessment of the return on capital matches that of shareholders. From the total differential of (7), assuming υ is constant at unity, the slope of

[14] λ, $D_K\lambda$, and μ are not negative; $D_K\mu$ is not positive. Consequently, the marginal cost of capital is less than ρ when the magnitude of $\tau K\, D_K\mu$ exceeds the sum of the other terms in the braces. That ρ is less than REV_s' when corporate income is taxed is a familiar result: $\mu(1+\eta_{\mu,K})$ ordinarily exceeds zero. If ρ exceeds shareholders' expectation of the marginal return on capital, ρ exceeds creditors' expectation of this marginal return by a greater amount. Under these conditions, $\lambda(1+\eta_{\lambda,K})$ likely exceeds the magnitude of $\tau K\, D_K\mu$.

K^o is

$$(20)\quad D_\varphi K^o = -\frac{D_\varphi \upsilon}{D_K \upsilon} = -\frac{K\, D_\varphi D_K q + D_\varphi q}{2\, D_K q + K\, D_K^2 q} .$$

The denominator of (20) is negative. If the numerator equals zero, then (from the continuity of q)

$$(21)\quad K\, D_K(D_\varphi q) = -(D_\varphi q) .$$

In general, satisfying (21) over an open interval of φ requires that q be independent of φ -- τ and δ are zero.[15]

The slope of K^o can be positive or negative. When φ is near unity, for example, an increase in φ may increase returns subject to taxation more than it diminishes the additional interest that shareholders must pay to creditors. In this case, the slope of K^o is negative: as φ increases, K^o falls thereby increasing the marginal rate of return on capital to match the increase in the marginal cost of capital in order to satisfy (19). Conversely, for values of φ nearer zero the slope of K^o may be positive: an increase in φ may increase the tax burden less than it diminishes the rate of interest on debt.

When the function K^o is tangent to a contour of q, their slopes are equal, implying (from (20) and the total differential of q)

---

[15](21) also would be satisfied if $D_\varphi q$ took the functional form $f(\varphi)/K$. But this essentially requires that Q, pdf_s, and pdf_c be independent of K.

(22) $\frac{D_\varphi \upsilon}{D_K \upsilon} = \frac{D_\varphi q}{D_K q}$.

When the enterprise earns economic rents (q exceeds unity), $D_\varphi q$ ordinarily is not zero when $D_\varphi \upsilon$ is zero (see appendix). Therefore, the slopes of K^o and the contour of q ordinarily are not zero at their point of tangency. When $D_\varphi q$ is not zero, the choice of φ does not necessarily minimize the effective cost of capital -- maximize q given K. When equity financing increases, the marginal increase in tax liabilities does not equal the marginal reduction in compensation required by creditors. Changing leverage from its optimal value may diminish the cost of capital (increase q, given K), but in doing so shareholders would reduce their wealth.[16]

Proposition 2: If q is a function of leverage as described in (17), the shareholders' ability to invest in the enterprise is not constrained, shareholders maximize their aggregate wealth, and the discount rate for all investors is identical and constant,

then:
(i) marginal q equals unity at the optimal stock of capital;
(ii) both the marginal return and the marginal cost of capital exceed the discount rate;[17]
(iii) given K, the optimal choice of leverage

[16] Suppose, given K, a greater value of φ increases q. Marginal q would increase with φ (moving to the right from the optimum shown in figure 2), and K must also increase to equate marginal q with unity (to return to the graph of K^o). But this new point does not produce as much wealth for shareholders as does the point where K^o is tangent to a contour of q.

[17] This conclusion regarding the marginal return on capital and the cost of capital is reinforced when the cost of capital increases with K (the enterprise's returns are positively correlated with market returns or investors are risk-averse). This conclusion may not obtain should the cost of capital decrease with K.

ordinarily does not minimize the cost of capital;

(iv) the optimal K varies with leverage; and

(v) the optimal K may vary inversely with the change in marginal q if the shape of revenue function, $REV'_s(K)$, should change as it shifts.

The optimal stock of capital depends on the shape of the shape of REV_s. In figure 3, a shift of REV_s displaces the graph of K^o downward at point 1. But the optimal value of K does not fall, because the slopes of K^o and the contours of q become steeper, displacing the tangency to point 2. From the total differential of (17), the slope along a contour of q is

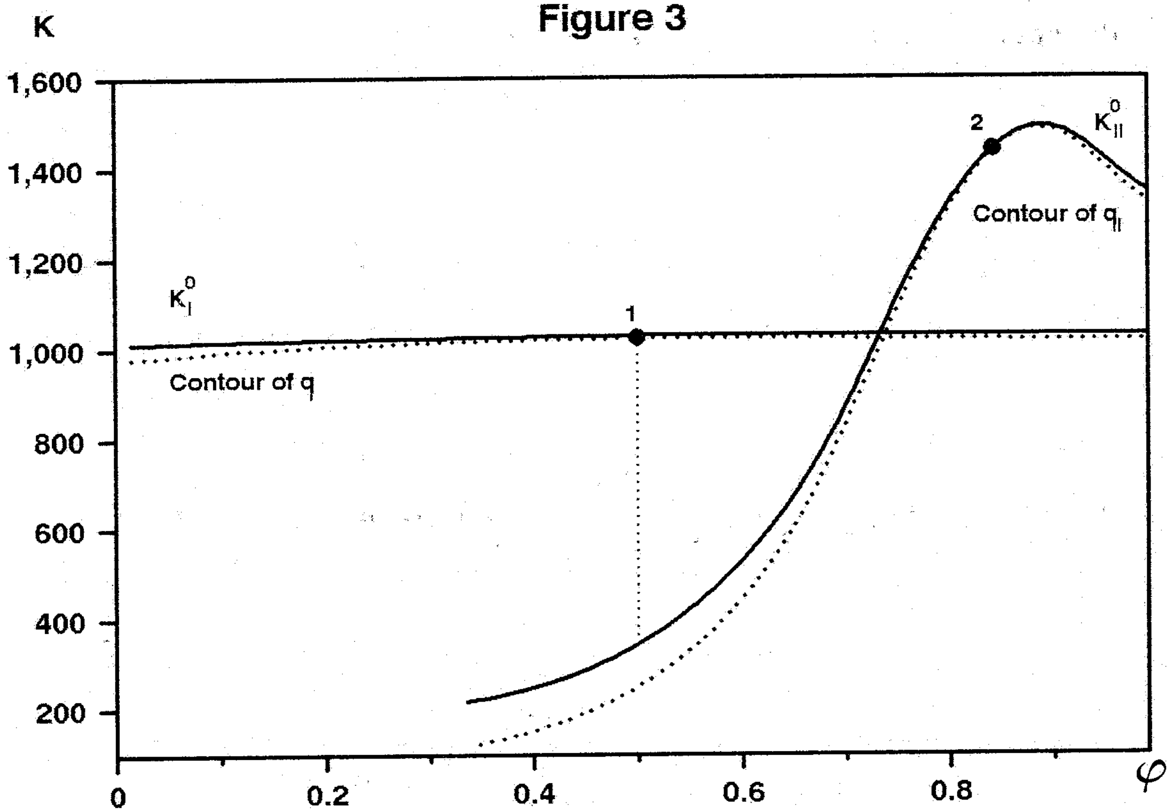
$$(23) \quad D_\varphi K|_{q=const} = \frac{D_\varphi(\tau\mu + \lambda)}{(REV'_s - r_s)/(K(1+\rho)) - D_K(\tau\mu + \lambda)} ,$$

and from (20) the slope of K^o is

$$(24) \quad D_\varphi K^o = \frac{D_\varphi \; (\; \tau(\mu + KD_K\mu) \; + \; (\lambda + KD_K\lambda) \;)}{REV''_s \; - \; D_K \; (\; \tau(\mu + KD_K\mu) \; + \; (\lambda + KD_K\lambda) \;)} .$$

As REV_s becomes less concave, both $(REV'_s - r_s)$ and REV''_s become less negative. If, as shown in the figure (and as illustrated by the example in the next section), the slopes of K^o and the contours of q are positive at points 1 and 2 because the numerators and denominators of (23) and (24) are negative, then the slopes of K^o and the contours of q will tend to increase as REV_s becomes less concave.

Figure 3



I: Specified in expressions (25), (26), (27), (11), (17), and (19). See Table 1 column 1.

II: Specified in expressions (26), (27), (28), (11), (17) and (19). See Table 1 column 3.

III. Heterogeneous Investors and the Correspondence between Marginal q and the Optimal Stock of Capital

This section comprises two numerical examples. According to proposition 2, marginal q and the demand for capital goods can change in opposite directions. This section's first example illustrates this possibility. The discussion in the previous sections of this paper assumes that shareholders' endowments do not constrain their equity investments. Constraining shareholders' potential contributions of equity may reinforce the tendency for marginal q and the optimal stock of capital to change in the same direction. This section's second example illustrates this conclusion. Nevertheless, in this second case, as in propositions 1 and 2, the change in marginal q does not determine the magnitude of the change in the optimal stock of capital.

The Supply of Equity Financing Is Not Constrained

For shareholders, the initial distribution of the project's rate of return on capital, pdf_s, is rectangular: the range of the distribution is 200 percentage points, and the mean equals

$$(25) \quad r_s(K) = 4. - 7.\times10^{-13} K^4 .$$

For creditors, the distribution pdf_c, is the same as that for shareholders except that its mean is lower

$$(26) \quad r_c(K) = r_s(K) - .9 \; (r_s(K) - \rho) .$$

The remaining parameters of (9), (17), and (19) are

$$\text{(27)}\quad \begin{aligned} \rho &= .05 \\ \tau &= .5 \quad . \end{aligned}$$

Under the conditions stated in proposition 2, the optimal choice of (φ, K) for the functions specified above is point 1 in figure 3 and in table 1. The marginal return on capital and the marginal cost of capital exceed both the investors' discount rate (5 percent) and the interest rate on debt, because the marginal rate of change of interest expense, $D_K\lambda$, is relatively great (see (19)). For this reason too, the rate of interest on debt, after corporate taxes, is less than the discount rate.

Suppose investors revise their perceptions of the enterprise's returns so that *REV* becomes less concave and marginal q falls below unity at point 1. For shareholders, the new pdf_s is identical to the first except that the mean rate of return becomes

$$\text{(28)}\quad r_s(K) = .5 - 9.157\times10^{-2}\, K^{.1} \ .$$

Similarly, the new pdf_c is identical to the first, except that its mean is defined by (26) and (28).

With this revision, q falls substantially at point 1, and marginal q is less than unity. Although this revision increases the marginal cost of funds more than the marginal return on capital at the formerly optimal choice of (φ, K) (table 1, column 2), the new optimal choice of (φ, K) is point 2 in figure 3 and in table 1, at which the optimal stock of capital is greater than it is at point 1.

Table 1

The Demand for Capital and the Optimal Choice of Financial Structure

	Initial *REV*	Revised *REV*		
	Evaluated at	Evaluated at		
	Point 1	Point 1	Point 2	Point 3
Stock of Capital (K)	\$1019.8	\$1019.8	\$1474.1	\$600.0
Equity Financing (φ)	50%	50%	86%	85%
Tobin's q				
Average (q)	2.48	1.00	1.01	1.02
Marginal (υ)	1.00	.99	1.00	1.01
	- - - - - - - - in percent - - - - - - - -			
Marginal Return on Capital (*REV'*)	21.4	29.9	29.1	30.9
Marginal Cost of Capital	21.4	30.9	29.1	29.9
Interest Rate on Debt				
Before Taxes (i)	6.3	18.2	5.9	6.1
After Taxes ($(1-\tau)i$)	3.2	9.1	3.0	3.1
Interest Premium Paid by Shareholders (δ)	0.6	10.8	0.9	1.1
$D_\varphi q$ ($= -(D_\varphi\lambda + D_\varphi\tau\mu)$)	2.2	3.4	0.9	1.1
$D_\varphi\lambda$	-8.6	-15.7	-3.7	-4.1
$D_\varphi\tau\mu$	6.3	12.1	2.7	2.9

The Initial REV case is described in (25), (26), and (27). The Revised REV case is described in (26), (27), and (28). The column headings refer to points in figures 3 and 4. For columns 1 and 3, the discussion preceding proposition 2 describes the optimal strategy; for column 4, the discussion before (29) describes the optimal strategy, given that equity financing cannot exceed \$510. The marginal cost of capital is defined by (19).

The expected rents on inframarginal investments fall with the revision of expected returns, and creditors, who are not as optimistic as shareholders, require greater rates of interest on loans to prevent their expected rate of return from falling (column 2 versus column 1). With the revised returns, interest expense also rises at a greater rate as leverage increases -- $D_\varphi \lambda$ becomes more negative and $D_\varphi q$ increases. For these reasons, both leverage and the interest rate on debt are lower at point 2 than they are at point 1. Because the terms of credit have deteriorated, the hurdle rate required of investments also increases -- both λ and $D_K \lambda$ increase, raising the marginal cost of capital in (19). Accordingly, the marginal cost of capital and the marginal return on capital are greater at point 2 than at point 1.

Given K, the choice of φ does not minimize the cost of capital (maximize q) at either point 1 or point 2. In both cases, a small reduction in leverage would reduce the premium paid to creditors more than it would increase the tax burden on the return to capital. Nevertheless, points 1 and 2 are optimal: instead of choosing φ to minimize the cost of capital, shareholders choose φ and K jointly to maximize their wealth.

The Supply of Equity Financing Is Constrained

If shareholders possess only $510, so that their equity investment absorbs all their funds at point 1, then they are unable to provide $1,268 of equity in order to reach point 2.[18] Their best strategy, under these

[18] In the spirit of Modigliani-Miller, if shareholders write personal loans to obtain the required funds, their leverage is no less than it would be if they had purchased $1,474 of assets with only $510 of equity. In fact, the terms on

circumstances, selects the φ that maximizes the value of their $510 equity investment (figure 4). Taking into account this equity constraint, the function $K^1(\varphi)$ shows the enterprise's maximum stock of capital for each value of φ. Shareholders select the point on K^1 that maximizes the ratio of the market value of the enterprise's equity to the shareholders' initial investment:

$$(29) \quad v(\varphi) = (q - (1-\varphi))/\varphi .$$

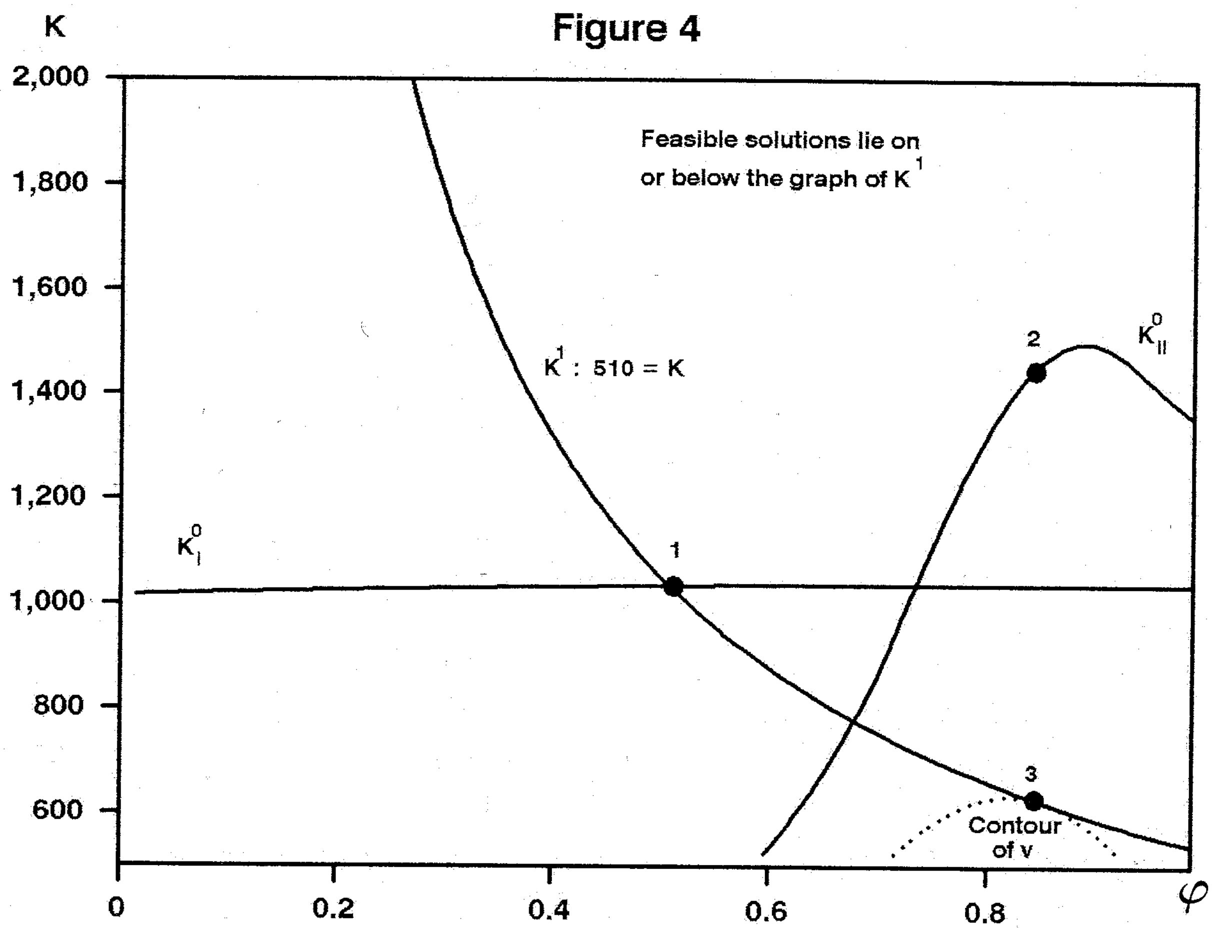
When equity constraints are binding, the optimal choice of K (point 3) lies on the section of K^1 beneath its intersection with K^o_{II}; above this intersection, marginal q is less than unity. Marginal q exceeds unity at the optimal choice of K, and the marginal return on capital exceeds the marginal cost of capital.

Any shift of expected returns that reduces marginal q below unity (at the formerly optimal value of K) must also reduce the optimal stock of capital when equity constraints are binding. If the value of marginal q were greater than unity initially, then a shift of expected returns that reduces marginal q, but leaves it above unity at the formerly optimal value of K, may not reduce the optimal stock of capital.

---

personal loans, including the loss of limited liability, make this source of financing less attractive than leveraging the project.

Figure 4



See Figure 4 and Table 1.

IV. Conclusion

When enterprises earn economic rents that vary with their scale of operation, their optimal stock of capital depends on the rate at which these rents erode with increasing capital. When a shift of expected returns depresses marginal q, the optimal stock of capital tends to decline less when the rents on inframarginal investments rise relative to the returns on marginal investments. Accordingly, as expectations of economic rents shift, the correspondence between the demand for capital and q may change. Marginal q and the optimal stock of capital can even move in opposite directions when q depends on the composition of the enterprise's financing as well as its stock of capital.[19]

This last conclusion does not depend solely on the failure of the Modigliani-Miller theorem. Marginal q and the optimal stock of capital also may move in opposite directions when the magnitude of enterprise's rents depend on variables such as the capital-labor ratio, the choice of technology, the pricing of its products, or the mix of outputs. For instance, when q varies with the employment of labor services, the stock of capital for which marginal q equals unity also varies with labor services (see the appendix). A lower tax on the income of laborers may diminish the wage rate, thereby increasing rents, other things equal. But the optimal stock of capital could fall (or at least increase comparatively little) if the lower relative wage rate induces management to substitute labor for capital.

---

[19]These conclusions are strengthened when marginal q is not necessarily equal to unity at the optimal stock of assets. See the citations mentioned in the introduction to this paper.

Appendix

Given that $q(\varphi, K)$ exceeds unity for some feasible choice of (φ, K) and that q is a continuous function, the region over which q exceeds unity, marginal q declines with increasing K, and φ is positive but less than unity is a compact set. Within this set, the function K^o assigns to each value of φ that value of K for which marginal q (υ) equals unity. The projection of this region onto the φ axis is the relevant domain for K^o. The only points where K^o may intersect the contour q=1 are those where both υ and q equal unity, implying that $D_K q$ is zero (see (7)) and that the tangent to the contour of q is perpendicular to the φ axis at these points. Elsewhere the graph of K^o remains strictly inside the contour q=1.

Although K^o generally is not horizontal over an open interval in the domain of φ under the conditions of this paper, $D_\varphi K^o$ may be zero at one or possibly more points. The optimum choice of (φ, K^o) may correspond to one of these points only under special circumstances. If K^o is tangent to a contour of q at a point where both are horizontal to the φ axis (from (23) and (24), interchanging the order of differentiation in the numerator of (24)):

$$(A1) \quad D_\varphi q = -(\tau D_\varphi \mu + D_\varphi \lambda) = 0$$

$$(A2) \quad D_K D_\varphi q = -D_K\ (\tau D_\varphi \mu + D_\varphi \lambda) = 0.$$

$\tau D_\varphi \mu$ and $D_\varphi \lambda$ must be equal in magnitude but of opposite signs, and these slopes must change at the same rate (in opposite directions) when K changes. These two conditions generally will not be satisfied at the same point for independent specifications of the functions *pdf*, as is illustrated by the example in section III.

Because K^o is not necessarily horizontal at the optimal choice of φ: given K, $D_\varphi q$ does not necessarily equal zero, and the marginal tax saving associated with a small alteration of leverage does not equal the marginal change in the cost of debt financing.

The optimal choice of (φ, K) ordinarily is not attained by: first, maximizing q with respect to φ given K, thereby defining $\varphi^o(K)$; second, choosing the pair(s) for which υ equals unity. With this approach, $\varphi^o(K)$ intersects the contours of q only where their tangents are parallel to the φ axis. Therefore, this alternative cannot yield the optimal choice of (φ, K), unless K^o happens to be horizontal at the optimal choice of φ.

q may depend on the enterprise's choice of labor as well as capital. Suppose

$$(A3) \quad r(L,K) = \frac{P(Q(L,K))\ Q(L,K) - W(L)\ L}{K} = \frac{REV(L,K)}{K}$$

$$(A4) \quad q(L,K) = \frac{1 + r(L,K)}{1 + \rho}$$

$$(A5) \quad \upsilon(L,K) = q(L,K) + K\, D_K q(L,K) = \frac{1 + D_K REV(L,K)}{1 + \rho} \quad .$$

For simplicity, ρ is a constant, and the price of capital goods is \$1. If the functions P (the net price of output, from a downward-sloping demand curve), Q (a production function), and W (the wage rate, from an upward-sloping supply schedule) have the usual properties and if q exceeds unity for some choice of (L,K), then the region over which q exceeds unity is a compact, possibly convex set.

The optimal choice of K, given L, equates marginal q with unity, $K^o(L)$. Because q is not independent of L, K^o is not horizontal over any open interval of L, unless (following the logic of (20), (21), and footnote 14)

$$(A6) \quad D_L q = \frac{D_L REV}{K\ (1+\rho)} = \frac{f(L)}{K} \quad ,$$

which essentially requires that Q be independent of K. Analogous to (23) and (24), the slopes of contours of q and K^o are, respectively,

$$(A7)\quad D_L K|_{q=const} = -\frac{D_L q}{D_K q} = -\frac{D_L REV}{D_K REV - r}$$

$$(A8)\quad D_L K^o = -\frac{D_L \upsilon}{D_K \upsilon} = -\frac{D_L D_K REV}{D_K^2 REV} .$$

At the optimal choice of K, both denominators are negative (the marginal return on capital falls with increasing K, and it is less than the average return on capital). Therefore, both numerators are positive in order for these slopes to be equal at the optimal choice of K.

A change in conditions that diminishes the average return on capital more than the marginal return, thereby making *REV* less concave, tends to increase both of these slopes, because both denominators become less negative. In this case, as in the example in section III, marginal q might fall while the optimal stock of capital rises. Consequently, the analysis of the choice of the optimal (L,K) and the correspondence between changes in marginal q and the optimal stock of capital for (L,K) are similar to those for (φ, K).

References

Abel, Andrew B. and Olivier J. Blanchard. 1986. "The Present Value of Profit and Cyclical Movements in Investments." Econometrica, 54(2) March, pp. 249-74.

Auerbach, Alan J. 1979. "Wealth Maximization and the Cost of Capital." Quarterly Journal of Economics, 93(3) August, pp. 433-46.

______________. 1983. "Taxation, Corporate Financial Policy, and the Cost of Capital." Journal of Economic Literature, September, 21(3), pp. 905-40.

Baskin, Jonathan Barron. 1988. "The Development of Corporate Financial Markets in Britain and the United States, 1600-1914: Overcoming Asymmetric Information." Business History Review, 62 Summer, pp. 199-237.

Bernanke, Ben and Mark Gertler. 1990. "Financial Fragility and Economic Performance." Quarterly Journal of Economics, 105 (1) February, pp. 87-114.

Carosso, Vincent P. 1970. Investment Banking In America. Cambridge, MA: Harvard University Press.

Chirinko, Robert. 1987. "Tobin's Q and Financial Policy." Journal of Monetary Economics, 19, pp. 69-87.

DeAngelo, Harry and Ronald W. Masulis. 1980. "Optimal Capital Structure Under Corporate and Personal Taxation." Journal of Financial Economics, 8(1) March, pp. 3-30.

Fazzari, Steven, Glenn Hubbard, and Bruce C. Petersen. 1988. "Financing Constraints and Corporate Investment." Brookings Papers on Economic Activity, 1, pp. 141-95.

Greenwald, Bruce C. and Joseph E. Stiglitz. 1989. "Financial Market Imperfections and Productivity Growth," National Bureau of Economic Research, Working Paper no. 2945, April.

Galeotti, Marzio and Fabio Schiantarelli. 1991. "Generalized Q Models for Investment." Review of Economics and Statistics, 73(3) August, pp. 383-92.

Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation." Econometrica, January, 50(1), pp. 213-24.

______________. 1985. "Corporate Finance Side of the Q Theory of Investment." Journal of Public Economics, 27, pp. 261-88.

Hayashi, Fumio and Tohru Inoue. 1990. "The Relationship of Firm Growth and Q with Multiple Capital Goods: Theory and Evidence from Panel Data on Japanese Firms." Econometrica, 59, pp. 731-5.

Jensen, Michael C. and William H. Meckling. 1976. "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure." Journal of Financial Economics, 3, pp. 305-60.

Keynes, John Maynard. 1936. The General Theory of Employment, Interest, and Money, Harcourt, Brace, and World (1965).

King, Mervyn A. 1977. Public Policy and the Corporation. Chapman and Hall.

Lintner, John. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." Review of Economics and Statistics, February, pp. 13-37.

McDonald, Robert S. and Daniel R. Siegel. 1986. "The Value of Waiting to Invest." Quarterly Journal of Economics, 101(4) November, pp. 707-28.

Miller, Merton H. 1977. "Debt and Taxes," Journal of Finance, 32(3) May, pp. 261-75.

Miller, Merton H. and Franco Modigliani. 1961. "Dividend Policy, Growth, and the Valuation of Shares." Journal of Business, 34(3) October, pp. 411-33.

______________. 1958. "The Cost of Capital, Corporation Finance, and the Theory of Investment." American Economic Review, 48(3) June, pp. 261-97.

______________. 1963. "Corporate Income Taxes and the Cost of Capital: A Correction." American Economic Review, 53(3) June, pp. 433-43.

Myers, Stewart C. 1977. "Determinants of Corporate Borrowing." Journal of Financial Economics, 5(2), pp. 147-75.

______________. 1984. "The Capital Structure Puzzle." Journal of Finance, 39(3) July, pp. 575-92.

Myers, Stewart C. and Majluf, N. 1984. "Corporate Financing and Investment Decisions When Firms Have Information Investors Do Not Have." Journal of Financial Economics, 13(2) June, pp. 187-221.

Navin, Thomas R. and Marian V. Sears. 1955. "The Rise of a Market for Industrial Securities, 1887-1902." Business History Review, 30, pp. 105-138.

Pindyck, Robert S. 1988. "Irreversible Investment, Capacity Choice, and the Value of the Firm." American Economic Review, 78(5) December, pp. 969-85.

______________. 1991. "Irreversibility, Uncertainty, and Investment." Journal of Economic Literature, 29(3) September, pp. 1110-48.

Poterba, James J. and Lawrence H. Summers. 1983. "Dividend Taxes, Corporate Investment, and 'q'." Journal of Public Economics, November.

Tobin, James. 1969. "A General Equilibrium Approach to Monetary Theory." Journal of Money, Credit, and Banking, 1(1) February, pp. 15-29.

_______________. 1982. "Money and Finance in the Macroeconomic Process." Journal of Money, Credit, and Banking, 14(2) May, pp. 171-204.

Wildasin, David E. 1984. "The Q Theory of Investment with Many Capital Goods." American Economic Review, 74, pp. 203-10.