Economic Rents, the Demand for Capital, and Financial Structure

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The correspondence between the demand for capital and various measures of the return on assets, the cost of capital, and Tobin's q often is tenuous (Abel and Blanchard 1986; Hayashi 1982), at times even perverse. Of a variety of possible explanations, this paper considers the consequences of allowing for declining returns to capital--a declining marginal efficiency of capital schedule (MEC). This modification not only relaxes the connection between the demand for capital and many of its traditional determinants, but it also may introduce a connection among the value of the firm, its financial structure, and its stock of assets.

When a shift of the MEC increases q on marginal assets, the demand for capital may increase greatly, negligibly, or, perhaps, even fall. The outcome depends partly on the shape of the MEC even if marginal q always equals unity for the optimal stock of assets. The demand for capital increases less, other things equal, as the MEC becomes more concave--as prospective returns on inframarginal assets increase more than those on marginal assets. The answer also depends on the determinants of q. Under plausible circumstances, the value of an enterprise may depend on its capital structure as well as its

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stock of assets. Accordingly, the demand for capital may fall even though q on marginal assets rises, because optimal leverage may change sufficiently.[1]

The first section of this paper introduces a simple, one-period investment project with declining returns to scale. When changes in business conditions do not change the project's marginal returns identically at all stocks of capital, the more concave the MEC becomes, the smaller is the subsequent change in the demand for capital, other things equal. Should the cost of capital increase with the stock of assets (the systematic risk rises as rents become less consequential), a shift of the MEC that increases the marginal return on assets might depress the demand for capital if average returns increase more than marginal returns.

In the second section, the value of the project varies with leverage because shareholders' expectations of economic rents exceed those of creditors and because the tax burden on shareholders' returns exceeds that on returns distributed to creditors. Inasmuch as q is a function of two variables, assets and leverage, a shift of the MEC that reduces marginal q (the partial derivative of q with respect to assets) at the formerly optimal stock of assets does not necessarily reduce the optimal stock of assets. Other things equal, the demand for capital may rise as leverage falls, when the MEC becomes less concave. This consequence is illustrated in a numerical example presented in the third section of this paper.

The concluding section suggests that this analysis is consistent with the relatively robust performance of accelerator or cash flow models of

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[1]Similarly, for a function of several variables, a positive partial derivative with respect to the first variable does not imply that this variable must increase if the function is to attain its maximum through the adjustment of all its variables.

investment. It also suggests a reconciliation of the pecking order view of financing investment with the existence of an optimal financial structure. This analysis also is consistent with a cyclical variation of leverage and an existence of a type of "credit crunch" not fully reflected in rates of interest. The difference between the hurdle rate of return required of investments and the rate of interest depends on the perceived slope of the MEC, especially the perceptions of creditors. Although this analysis dwells on the choice of capital structure in conjunction with the demand for capital, similar conclusions also may arise whenever the value of the firm depends on the choice of other variables--the mix of factors of production, the choice of technology, the pricing of output, or the composition of output.

I. Homogeneous Expectations: The Cost of Capital Is Independent of Leverage

The demand for investment goods may not correspond closely to changes in marginal q if marginal returns do not change commensurately at all stocks of assets when the MEC shifts. In this case, changes in the optimal stock of assets depend on changes in the slope of the MEC as well as changes in marginal q. Accordingly, when marginal q rises at the formerly optimal stock of assets, the demand for capital also rises, but this demand may increase much or little, depending on the shape of the MEC.

Unlike changes in marginal q, changes in marginal returns or the marginal cost of capital might not consistently indicate whether the demand for capital rises or falls if the enterprise's returns are correlated with those of the market portfolio. If this correlation (a measure of systematic risk) should vary with the project's stock of assets,[2] then the rate of

[2]This implies that the effective cost of capital varies with the demand for capital. In a dynamic setting, the models of Witte (1963) and Hayashi (1982) observe that the effective cost of capital also may vary with the demand for investment goods as a result of adjustment costs.

return required of marginal assets would depend on the slope of the MEC. Assuming the project's systematic risk rises with its stock of assets, then the demand for capital may fall as the MEC shifts upward, provided the slope of the MEC becomes sufficiently steep, thereby raising the rate of return required of marginal assets.

The Return on Assets, the Cost of Capital and Tobin's q

The total returns accruing to a one-year investment project, tr, depend on its stock of assets, A. Initially, expected returns, TR, may increase as A increases. But once A exceeds some critical value, A*, TR falls as A increases, due to the exhaustion of specialized resources, diseconomies of scale, the increasing cost of obtaining labor services and materials, or the increasing cost of attracting customers. Accordingly, expected returns equal

(1) $TR(A)$, where TR'' is negative when A exceeds $A*$.

Because TR" is negative, the expected rate of return on marginal assets, $\Delta TR/\Delta A - TR'$, falls as A increases above A*.

In the spirit of the Modigliani-Miller theorem, the cost of capital, ρ, is the shareholders' rate of discount when the project is financed entirely with equity (Modigliani and Miller 1958 and 1963; Miller and Modigliani 1961; Miller 1977; Myers 1984). However the project is financed, its securities are priced so that they are held in the optimal market portfolio (Lintner 1965). Whatever the blend of these securities, their composite expected return is identically TR(A).[3] Therefore, the value of the project's securities is the market valuation of its prospective total return. Whether this return is

[3]When the income that accrues to shareholders is taxed differently than the income accruing to creditors, this assertion is not true. See note 4 and section II.

conveyed entirely by equity or by another blend of securities, it warrants only one cost of capital and one valuation in the market portfolio.

Because the market value of the enterprise is independent of its financing, Tobin's q equals the replacement value of assets, A, divided into the value of equity (the present value of shareholders' expected receipts) when the project is financed entirely with equity, V.[4] Denoting the discount rate by ρ and the expected rate of return on assets (TR/A) by R,

(2) $V = A(1+R)/(1+\rho)$, and

(3) $q = (1+R)/(1+\rho)$.

Marginal q, Marginal Returns, and the Optimal Stock of Assets

For any stock of assets, marginal q is the change in the overall market value of the enterprise resulting from adding another asset, $\Delta(qA)$, divided by the replacement value of this asset, ΔA. Denoting marginal q by υ and the derivative of qA with respect to A by $D_A(qA)$,

(4) $\upsilon = D_A(qA) = q + A\, D_A q$.

Because the value of the enterprise does not depend on leverage, $\Delta(qA)$ equals ΔV when the enterprise is financed entirely with equity:

(5) $\upsilon = D_A V = q + (TR' - R - V\, D_A\rho)/(1+\rho)$.

Provided ρ does not fall as A increases, a declining MEC entails that q exceeds marginal q.

Proposition 1: If q is independent of leverage and shareholders maximize their aggregate wealth, then:

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[4]Personal and corporate income tax rates are not considered in this section. An explicit consideration of a simple corporate income tax is deferred until section II. For a review and analysis of much of the literature covering taxes, corporate finance, and investment incentives see Auerbach (1983) and Poterba and Summers (1983).

(a) the optimal stock of assets equates marginal q with unity, but neither the marginal return on assets nor the marginal cost of capital equals the discount rate unless the discount rate does not vary with the project's stock of assets;
(b) a shift of the MEC that increases (decreases) marginal q also increases (decreases) A, but the degree to which A changes depends on the shape of the MEC;
(c) when the cost of capital depends on A, a shift of the MEC that increases (decreases) the marginal return on assets may decrease (increase) A.

Because q does not depend on leverage, the optimal choice of A also does not depend on leverage. Suppose the project is financed entirely with equity. Should shareholders change their investment in the project, the change in their wealth would equal the change in the value of the enterprise, ΔV, less the change in the replacement value of assets, ΔA. At the optimal stock of assets, ΔV equals ΔA,[5] implying that

(6) $1 = \upsilon$.

Substituting (5) into (6) and (3) for q in the first term of (5),

(7) $\rho = TR' - qA\, D_A\rho$.

At the optimal stock of assets, the expected rate of return on the marginal asset equals the cost of capital only if the cost of capital does not vary with assets.[6] From (7), the marginal cost of capital may be defined as $\rho + qA\, D_A\rho$.

<u>The Demand for Capital: The Cost of Capital Is Constant</u>

In Figure 1, the optimal scale of the project (A_0) is defined initially by the intersection of TR' with the horizontal cost of capital function.

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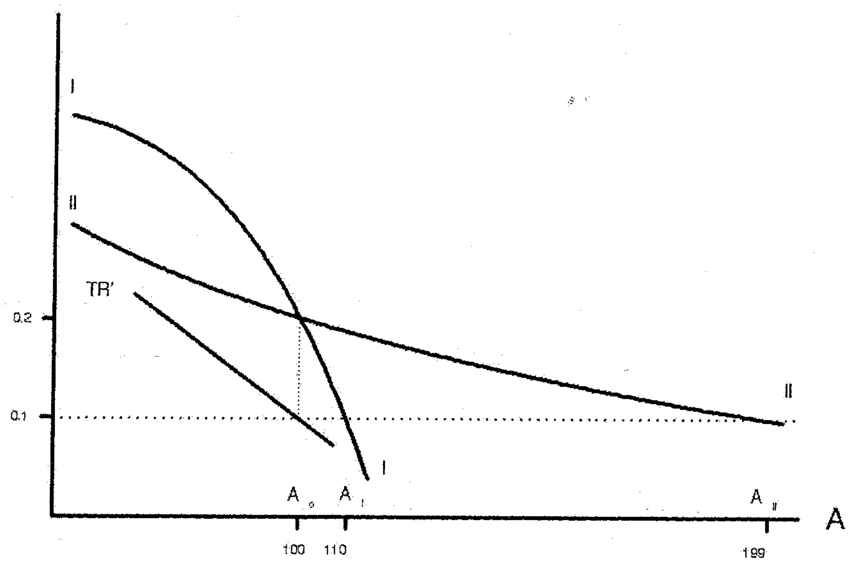
[5]This arbitrage between capital goods and equities is essentially that described by Keynes (1936, chapter 12) and Tobin (1969, 1982).

[6]TR' equals ρ also because the personal and corporate income tax rates are zero (King 1977; Auerbach 1979).

Because alternatives I and II increase TR' by the same amount at A_0, they also increase $\upsilon = ((1+TR')/(1+\rho))$ by the same amount at A_0. For I (TR' becomes more concave) the optimal stock of assets is less than that for II (TR' becomes more convex). q is greatest for alternative I, which offers the greatest area under its curve, the greatest economic rents and average returns.

Figure 1



I: $TR' = .415 - 2.15 * 10^{-9} A^{4}$ II: $TR' = 1.594 - 0.88 * A^{1}$

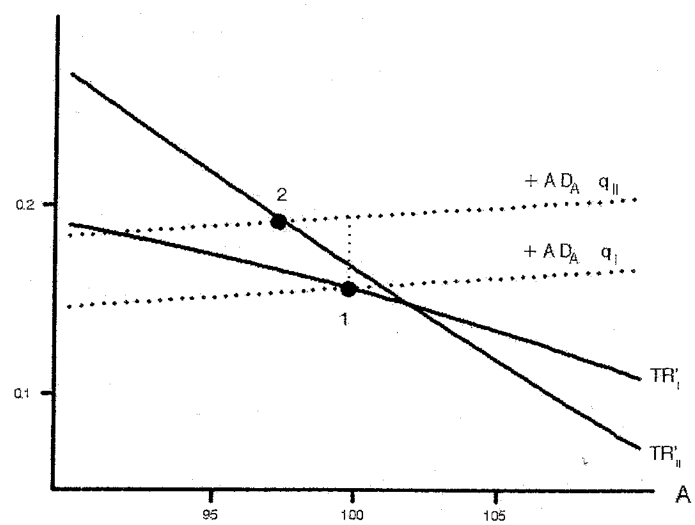
$R = .415 - 4.3 * 10^{-10} A^{4}$ $R = 1.594 - 0.8 * A^{1}$

Although an increase in expected marginal returns or marginal q correctly foretells a greater demand for capital, these statistics are not sufficient for determining the magnitude of this demand without knowledge of the shape of the MEC schedule.

<u>The Demand for Capital: The Cost of Capital Increases with Assets</u>

When the MEC is declining, the uncertainty regarding the rate of return on marginal investments may increase as the expected return on marginal investments falls. Furthermore, the covariance between the return on assets and the return on the market portfolio may increase as the project's unique features dwindle along with the rent. When the downward-sloping MEC is accompanied by a rising covariance between the project's returns and market returns, ρ increases with A for risk-averse investors. In this case, the distinction between the MEC and the cost of capital becomes much less manifest, because ρ depends on the characteristics of the project's returns.

Figure 2



$TR' = a - bA^c$

$R = a - bA^c/(c+1)$

$= .05 + .0005A$

$q = (1+R)/(1+\rho)$

I: $a = .255$, $b = 10^{-9}$, $c = 4$

II: $a = 10.0$, $b = 6.205$, $c = .1$

The demand for capital depends on the slopes of the functions TR' and ρ. According to (7), demand tends to fall as $D_A\rho$ increases or as average returns (q) increase relative to marginal returns (marginal q).

Suppose the demand for the project's output shifts, increasing both the rents on inframarginal assets and the return on marginal assets (Figure 2). Initially, TR' intersects the marginal cost of capital schedule at point 1. If the rents on inframarginal assets rise more than the return on marginal assets, then the marginal cost of capital schedule may shift upward more than TR' at point 1. The optimal stock of assets falls even though $(TR'-\rho)$ increases, because the increase in q is sufficiently great compared to the increase in TR' at point 1 -- TR'_{II} is more concave than TR'_{I}.

II. Heterogeneous Expectations: The Cost of Capital Increases with Leverage

When the value of an enterprise depends on its capital structure, the stock of assets that equates marginal q with unity also may vary with its choice of leverage. Because the enterprise's optimal capital structure depends on the slope of its MEC under these circumstances, its optimal stock of assets also may depend on the slope of its MEC.

In this section q depends on leverage as a result of asymmetries in the income tax laws and differences among investors' expectations.[7] Accordingly, when a shift of the MEC decreases marginal q, the demand for capital may rise if the MEC becomes less concave. In other words, when the value of an enterprise is a function of more than one variable (here, assets and leverage), a change in conditions that reduces the partial derivative of this value with respect to assets (evaluated at the formerly optimal stock of assets and leverage) does not necessarily imply that the optimal stock of assets falls if the optimal choice of leverage also changes.

[7] These examples are not the only sources of heterogeneity or asymmetry. Suppose income is not taxed, as was the case before 1909. If shareholders lacked sufficient resources to finance the project themselves (to purchase a stock of assets even as great as A*), they might pay a premium to obtain external financing. Regulations, contracts, and conventions governing the eligible assets of banks, insurance companies, pension funds, and other institutional investors tend to favor debt over equity, thereby increasing the cost of external equity financing compared to that of debt financing. Even if outsiders were no less optimistic than shareholders, external financing may command a premium, because those who control access to external financing may attempt to extract a participation in the project's rents. See Navin and Sears (1955), Carosso (1970), Baskin (1988), and Bernanke and Gertler (1990).

Leverage, Homogeneous Expectations, and q

The proportion of the project's assets financed by equity is φ; that financed by debt is $(1-\varphi)$. Denoting the rate of interest on debt by i, the rate of return to creditors is i as long as the rate of return on assets $(tr/A = r)$ plus shareholders' equity is great enough to pay debt service obligations; otherwise, the rate of return to creditors is only $(r+\varphi)/(1-\varphi)$. This rate of return on debt is r_c, and its expected value is R_c. Given φ, i equates the expected return on debt with the creditor's discount rate, ρ_c.[8] Denoting the probability density function for the return on assets by pdf(r),

$$(8) \qquad \rho_c = i \int_{(1-\varphi)i-\varphi}^{\infty} pdf(r)\ dr + \int_{-100\%}^{(1-\varphi)i-\varphi} \{(r+\varphi)/(1-\varphi)\}\ pdf(r)\ dr, \text{ or}$$

$$(9) \qquad i-\rho_c = \int_{-100\%}^{(1-\varphi)i-\varphi} \{i-(r+\varphi)/(1-\varphi)\}\ pdf(r)\ dr.$$

The "risk premium" embedded in i equals the expected value of the potential losses on debt contracts. Other things equal, a lower R entails a greater i.

Both the interest rate and the discount rate on debt may vary with the enterprise's leverage. If the return on assets is not correlated with that on the market portfolio or if creditors are risk-neutral, ρ_c equals the risk-free rate of return for all φ. When r is positively correlated with market returns and creditors are risk-averse, ρ_c rises as φ falls, because

[8]The descriptions of creditors' and shareholders' discount rates and their relationship to the cost of capital are discussed in more detail below.

the covariance between r_c and market returns increases with leverage.[9] When φ is near unity, neither i nor ρ_c exceeds the risk-free rate of return significantly. As φ approaches zero, $(i-\rho_c)$ increases as the distribution of r_c becomes more diffuse and the coverage ratio, $R/(i(1-\varphi))$, falls.

The returns on assets and debt are related to the return on equity, r_e, and its expected value, R_e, according to the following accounting identities:

(10) $r = \varphi r_e + (1-\varphi) r_c$, and

(11) $R = \varphi R_e + (1-\varphi) R_c$.

Denoting the shareholders' discount rate as ρ_e,

(12)

$$q = \varphi(1+R_e)/(1+\rho_e) + (1-\varphi)(1+R_c)/(1+\rho_c)$$

$$= \varphi \{1+ \int_{(1-\varphi)i-\varphi}^{\infty} (r - (1-\varphi)i)/\varphi \; pdf(r)\; dr - \int_{-100\%}^{(1-\varphi)i-\varphi} pdf(r)\; dr\} \;/\; (1+\rho_e) \; +$$

$$(1-\varphi)\; \{1 + (\int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i \; pdf(r)\; dr + \int_{-100\%}^{(1-\varphi)i-\varphi} (r+\varphi)\; pdf(r)\; dr)/(1-\varphi)\}/(1+\rho_c)$$

<u>Proposition 2</u>: If q is independent of capital structure, then (12) equals (3); and if i equates R_c with ρ_c, then:

(13) $\varphi\rho_e + (1-\varphi)\rho_c = R - (R-\rho)(1+\rho)^{-1}(1+\rho_e)$.

(a) When the MEC is declining and R exceeds ρ (q exceeds unity), the weighted discount rate is less than the expected rate of return on assets. Unless ρ_e is less than or equal to ρ, (13) implies that the

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[9]The distribution of r_c collapses on i as φ approaches unity, whereas the distribution of r_c approaches that of r as φ approaches zero.

weighted discount rate also is less than the cost of capital.[10]

(b) If the return on assets is correlated positively with market returns and investors are risk-averse, then the covariance between market returns and r_e increases with leverage, ρ_e increases with leverage, and (13) implies that the difference between R and the weighted discount rate also increases with leverage when assets are fixed.

(c) If the return on assets is not correlated with market returns or investors are risk-neutral, then the cost of capital and the discount rates of shareholders and creditors equal the risk-free rate of return.

Leverage, Heterogeneous Expectations, and q

Other things equal, q falls with increasing leverage when shareholders' expectation of economic rents exceeds that of creditors. Shareholders may be more optimistic about their project's returns for at least two reasons. First, the uncertainty inherent in a project's returns may be reflected in the distribution of assessments among investors even when the same information is available to all. Second, not all investors possess the same information or are equally able to extract rents from a specific project. In either case, equity is most valuable to those who foresee the opportunity for the most profit; others are less eager to acquire a residual claim on the enterprise's returns (Navin and Sears 1955; Jensen and Meckling 1976; Myers and Majluf 1984; Baskin 1988; Bernanke and Gertler 1990).[11]

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[10] ρ_e cannot be less than ρ unless r is negatively correlated with the rate of return on the market portfolio. See cases (b) and (c) below.

[11] Investors' expectations of returns may span the range extending from the very optimistic to the very pessimistic. In the eyes of creditors, shareholders are most likely to experience "winners' curses." The marginal creditor ordinarily is less optimistic than the inframarginal creditor. Similarly, marginal shareholders tend to be less optimistic than inframarginal shareholders. In this continuous spectrum of opinions, some "outsiders" (creditors) are more optimistic than others. Only after leverage is so great and the marginal creditors are sufficiently pessimistic, does the firm offer equity or mezzanine stakes to the more optimistic outsiders.

Suppose, for simplicity, that a project's return on assets is not correlated with market returns or that investors are risk-neutral, implying that ρ_e and ρ_c equal ρ for all A and for all φ. Denoting the creditors' assessment of the distribution of the rate of return on assets as $pdf_c(r)$ and the shareholders' assessment as pdf(r), (12) becomes,

$$(14)\quad q = \{ \varphi + \int_{(1-\varphi)i-\varphi}^{\infty} (r - (1-\varphi)i)\, pdf(r)\, dr + \int_{-100\%}^{(1-\varphi)i-\varphi} (-\varphi)\, pdf(r)\, dr \} / (1+\rho) +$$

$$\{ (1-\varphi) + \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\, pdf_c(r)\, dr + \int_{-100\%}^{(1-\varphi)i-\varphi} (r+\varphi)\, pdf_c(r)\, dr \} / (1+\rho)$$

$$= (1+R)/(1+\rho) + \{ \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\, (pdf_c(r)-pdf(r))\, dr + \int_{-100\%}^{(1-\varphi)i-\varphi} (r+\varphi)\, (pdf_c(r)-pdf(r))\, dr \} / (1+\rho)$$

$$= (1+R)/(1+\rho) +$$

$$\{ \rho(1-\varphi) - \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\, pdf(r)\, dr - \int_{-100\%}^{(1-\varphi)i-\varphi} (r+\varphi)\, pdf(r)\, dr \} / (1+\rho).$$

If $pdf_c(r)$ were identical to pdf(r), q would be independent of φ. The last equality in (14) obtains because the rate of interest equates the expected return on debt with the creditors' discount rate (see (9)).[12]

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[12]In this model, the terms on debt compensate creditors for the risks they bear (the interest rate floats), the market value of debt equals its face value, and shareholders have no opportunity to benefit from strategies that could reduce the market value of debt relative to its face value.

Suppose that $pdf_c(r)$ matches pdf(r) in all respects, except that the mean of $pdf_c(r)$ is less than R, the mean of pdf(r), for all A.[13] From (9), the rate of interest that equates r_c with ρ is greater for $pdf_c(r)$ than for pdf(r). Denoting this difference by δ,[14] for shareholders the interest rate on debt need be only $i-\delta$ to equate its expected rate of its expected rate of return with ρ:

(15)

$$\rho(1-\varphi) = \int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} (1-\varphi)(i-\delta)\, pdf(r)\, dr + \int_{-100\%}^{(1-\varphi)(i-\delta)-\varphi} (r+\varphi)\, pdf(r)\, dr$$

$$= \{ \int_{(1-\varphi)i-\varphi}^{\infty} (1-\varphi)i\, pdf(r)\, dr + \int_{-100\%}^{(1-\varphi)i-\varphi} (r+\varphi)\, pdf(r)\, dr \} -$$

$$\{ \int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} (1-\varphi)\delta\, pdf(r)\, dr + \int_{(1-\varphi)(i-\delta)-\varphi}^{(1-\varphi)i-\varphi} (r+\varphi-(1-\varphi)i)\, pdf(r)\, dr \}.$$

Substituting (15) into (14)

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[13]If pdf and pdf_c did not have the same variance or covariance with market returns, then they might entail different discount rates for the debt.

[14]If, for given A, creditors' expected rate of return on assets equals ρ, then i increases without limit as φ approaches zero, provided $pdf_c(r)$ places no upper bound on r: when φ is zero, the expected return on debt would be less than ρ for any finite value of i, because removing any portion of the upper tail of pdf_c yields a distribution whose mean is less than the expected return on assets. If the expected return on assets exceeds ρ, then i is bounded as φ approaches zero: the maximal value for i solves (9) when φ equals zero. Therefore, if creditors expect the return on assets to equal ρ, while shareholders believe R exceeds ρ, then δ increases without limit as φ approaches zero. If both creditors and shareholders expect the project to earn rents, then δ is bounded as φ approaches zero.

(16)

$$q=(1+R)/(1+\rho)-\{(1-\varphi)\delta\int_{(1-\varphi)(i-\delta)-\varphi}^{\infty} pdf(r)\;dr+\int_{(1-\varphi)(i-\delta)-\varphi}^{(1-\varphi)i-\varphi}(r+\varphi-(1-\varphi)i)\;pdf(r)\;dr\}/(1+\rho)$$

$$=(1+R)/(1+\rho)\;-\;\lambda(\varphi,A)\;.$$

λ equals the expected value of returns forgone by sharholders due to the additional compensation, $(1-\varphi)\delta$, required by less optimistic creditors.

Leverage, Heterogeneous Expectations, Asymmetric Taxation, and q

The net return on assets tends to rise with leverage when the corporate income tax rate on the returns to creditors is less than that on the returns to equity.[15] Suppose the tax rate on earnings distributed as interest is zero and that on the return to equity is τ when this return is positive.[16] Accordingly, (16) becomes

$$(17)\quad q\;=\;(1+R)/(1+\rho)\;-\;\lambda(\varphi,A)\;-\;\tau\int_{(1-\varphi)i}^{\infty}(r\;-\;(1-\varphi)i)\;pdf(r)\;dr\;/\;(1+\rho)$$

$$=\;(1+R)/(1+\rho)\;-\;\lambda(\varphi,A)\;-\;\tau\;\mu(\varphi,A)\;.$$

[15]Although this section assumes that returns are not correlated with those of the market portfolio and that the cost of capital increases with leverage for other reasons, a lower effective tax rate on returns also tends to increase the variance of these returns and, perhaps, the covariance between these returns and those on the market portfolio.

[16]The project is organized as a C corporation rather than an S corporation, for example.

μ equals the expected value of shareholders' returns subject to taxation, which decreases either as leverage increases or as A rises, thereby depressing the return on assets.

q initially may increase with leverage when a project is financed almost entirely by equity, because after-tax returns increase with this change in leverage. However, q eventually falls with increasing leverage as creditors, who are less optimistic than shareholders, require substantial additional compensation for bearing the risk of financing the project.

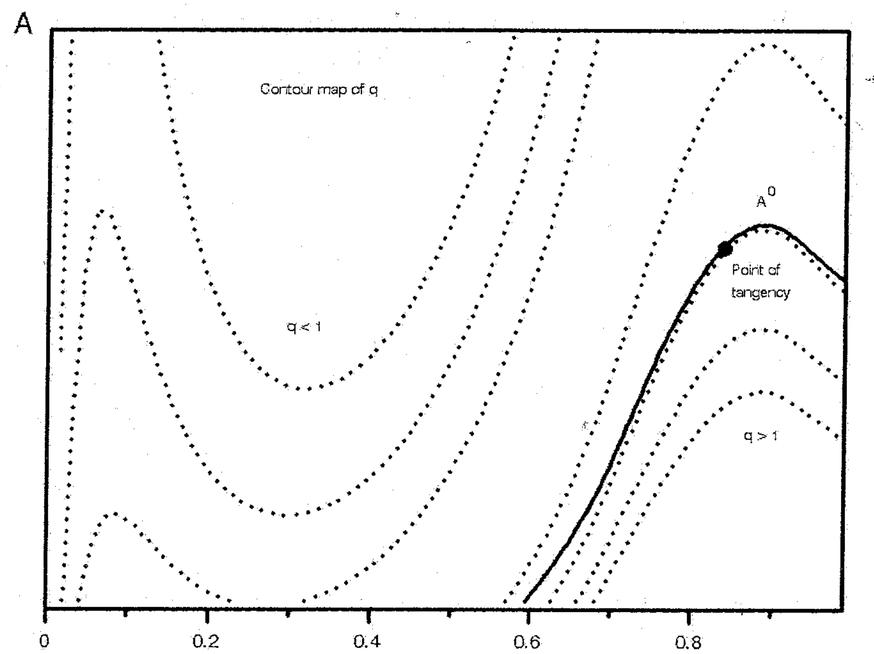
The Optimal Stock of Assets and the Optimal Capital Structure

When q depends on the enterprise's capital structure as well as its stock of assets, selecting the optimal A and φ may be separated into two steps. First, for any value of φ, shareholders maximize their wealth by choosing A so that marginal q equals unity, thereby defining the optimal A as a function of $\varphi, A^0(\varphi)$ (Figure 3).[17] Second, q is maximized with respect to φ, subject to the constraint that A equals $A^0(\varphi)$: φ maximizes $q(\varphi, A^0)$ at the point where the graph of A^0 is tangent to a contour of $q(\varphi, A)$.[18]

[17]Because A^0 is not horizontal, the optimal value of A varies with φ (see the Appendix).

[18]These steps imply that q may be written as a function of φ alone, but q cannot be expressed as a function of A alone. This ranking of decisions cannot be reversed by first choosing φ to maximize q given A, then choosing A so that υ equals unity (see the Appendix). Because the ranking of decisions cannot be reversed, no alternative definition of marginal equity values might take into account the change in leverage by defining $\upsilon=1$ along a path passing through points 1 and 2 in Figure 4.

Figure 3



<u>Proposition 3</u>: If the MEC is declining, shareholders maximize their aggregate wealth, the return on the project's assets is uncorrelated with market returns (or investors are risk-neutral), and q is a function of A and φ as described in (17), then:

(a) the marginal value of q equals unity at the optimal stock of assets, but the return on marginal assets and the marginal cost of capital exceed the cost of capital;[19]

(b) the value of A that equates marginal q with unity varies with φ;

[19]This conclusion is reinforced when the project's returns are positively correlated with market returns and ρ increases with A (see the discussion following Proposition 1). This conclusion may be contradicted when the project's returns are negatively correlated with market returns.

(c) although a shift of the MEC may reduce (increase) marginal q, A may rise (fall), provided the shape of the MEC changes sufficiently as it shifts;
(d) given A, the optimal choice of φ ordinarily does not minimize the cost of capital.

Given φ, applying (4) to (17) defines marginal q,[20]

(18) $\upsilon = (1+R+AR')/(1+\rho) - \tau(\mu + AD_A\mu) - (\lambda + AD_A\lambda)$.

For fixed φ, the optimal capital stock equates υ with unity, according to logic similar to that of Proposition 1. Therefore, from (18), the function $A^0(\varphi)$ is implicitly defined by

(19) $\rho = TR' - \{\tau(\mu + AD_A\mu) + (\lambda + AD_A\lambda)\}(1+\rho)$.

The marginal return on assets exceeds the cost of capital for the optimal choice of A, because the two terms inside the braces are positive.[21] From (19), the marginal cost of capital is

$\rho + \{\tau(\mu + AD_A\mu) + (\lambda + AD_A\lambda)\}(1+\rho)$.

Because μ and λ are different functions of φ, the slope of A^0 can be positive or negative, but it is not zero over any interval of φ (see the Appendix). When φ is near unity, for example, an increase in φ may increase

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[20]Because the project's returns are not correlated with market returns in this example, the cost of capital does not depend on A.

[21]That ρ is less than TR' when corporate income is taxed is a familiar result. For fixed φ, $\mu(\varphi, A_1)$ may be represented as:

$$\int^{A_1} m\, dA/A_1,$$

an average of "marginal μ." Differentiating this expression for μ with respect to A_1 reveals that m equals $\mu + AD_{A\mu}$. By definition, for fixed φ, neither μ nor $AD_{A\mu}$ increase as A increases; consequently, if m were negative for A_1, then m would be negative for all A greater than A_1, and for some A greater than A_1, μ also would be negative. Because μ is positive for all A, m also must be positive for all A. A similar logic applies to λ.

μ (returns subject to taxation) more than it diminishes λ (the additional interest that shareholders must pay to creditors). In this case, the slope of A^0 is negative: as φ increases, A^0 falls, thereby increasing the marginal rate of return on assets (TR') in order to satisfy (19). Conversely, for values of φ nearer zero the slope of A^0 may be positive: an increase in φ may increase the tax burden less than it diminishes the rate of interest on debt.

When the enterprise earns economic rents (q exceeds unity for some choice of A and φ), $D\varphi q$ ordinarily is not zero at the optimal choice of φ and A (see the Appendix). Therefore, conditional on A, the choice of φ does not necessarily minimize the effective cost of capital (that is, maximize q in (17)): on the margin, the increase in shareholders' tax burden ($\tau D_{\varphi}\mu$) does not equal the reduction in compensation required by creditors ($-D_{\varphi}\lambda$) when equity financing increases. A marginal adjustment of leverage from its optimal value may diminish the cost of capital (increase q, given A), but in doing so shareholders would reduce their wealth.[22]

The demand for capital depends on the shape of the MEC. In Figure 4, a shift of the MEC displaces the graph of A^0 downward at point 1. But the optimal value of A does not fall, because the slopes of A^0 and the contours of q become steeper. From (17) the slope along a contour of q is

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[22]Suppose, given A, a greater value of φ raises q (sets $\tau D_{\varphi}\mu - D_{\varphi}\lambda$ equal to zero). Then marginal q would increase with φ (moving to the right from optimum shown in Figure 3), and A must also increase to equate marginal q with unity (to return to the graph of A^0 once again). But these new values of (φ, A) do not produce as much wealth for stockholders as the point where the graph of A^0 is tangent to a contour of q.

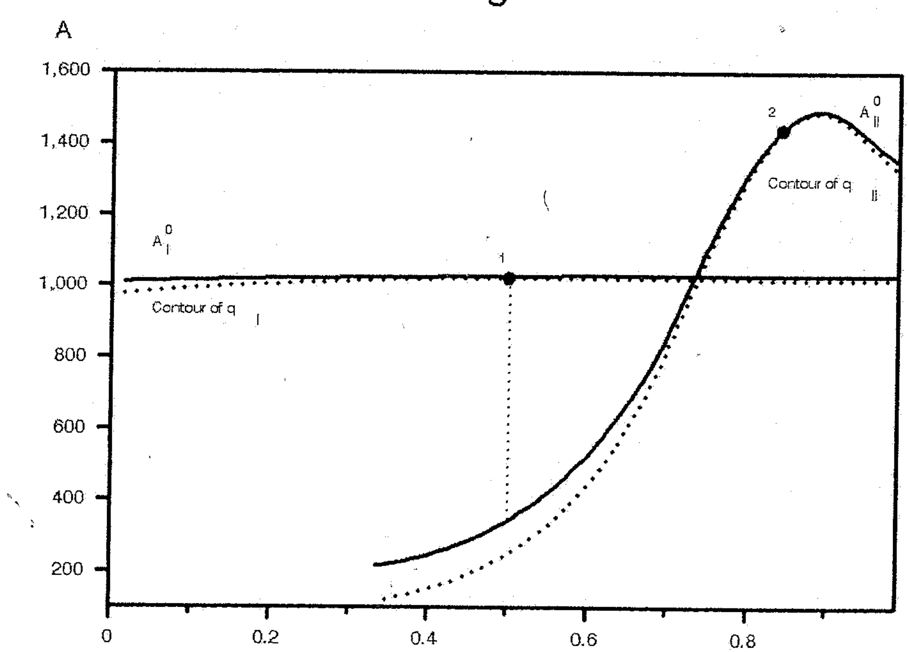
$$(20) \quad \frac{D\varphi\{\tau\mu + \lambda\}}{(TR' - R)/(A(1+\rho)) - D_A\ \{\tau\mu + \lambda\}},$$

and from (19) the slope of A^0 is

$$(21) \quad \frac{D\varphi\{\ \tau(\mu + AD_A\mu) + (\lambda + AD_A\lambda)\ \}}{TR'' - D_A\ \{\ \tau(\mu + AD_A\ \mu) + (\lambda + AD_A\lambda)\ \}}.$$

As the MEC becomes less concave, both $(TR' - R)$ and TR'' become less negative. If, as shown in the figure (and as illustrated by the example in the next section), the slopes of A^0 and the contour of q are positive at points 1 and 2 because the numerators and denominators of (20) and (21) are negative, then the slopes of A^0 and the contours of q will tend to increase as the MEC given becomes less concave.

Figure 4



I: Specified in expressions (22), (23), (24), (9), (17), and (19). See Table 1 column 1.

II: Specified in expressions (25), (23), (24), (9), (17) and (19). See Table 1 column 3.

III. Heterogeneous Expectations and the Supply of Equity Financing

This section comprises two numerical examples. According to Proposition 3, marginal q and the demand for capital goods can change in opposite directions. This section's first example illustrates this possibility. The discussion in the previous sections of this paper assumes that shareholders' endowments do not constrain their equity investments. Should shareholders' potential contributions of equity be constrained, then the demand for capital goods can only increase (decrease) when marginal q rises above (falls below) unity. Nevertheless, in this case as in Propositions 1(b) and 3, the change in marginal q does not determine the magnitude of the change in the demand for capital goods. This section's second example illustrates this conclusion.

<u>The Supply of Equity Financing Is Not Constrained</u>

For shareholders, the initial distribution of the project's rate of return on assets, pdf(r), is rectangular: the range of the distribution is 200 percentage points, and the mean equals

$$(22) \quad R(A) = 4 - 7*10^{-13}*A^4.$$

For creditors, the distribution $\mathrm{pdf}_c(r)$ is the same as that for shareholders except that its mean is lower

$$(23) \quad R_c(A) = R(A) - .9*(R(A) - \rho).$$

The remaining parameters of (9), (17), and (19) are

$$(24) \quad \rho = \rho_e = \rho_c = .05$$
$$\tau = .5$$

Under the conditions stated in Proposition 3, the optimal choice of (φ, A) for the functions specified above is point 1 in Figure 4 and in

Table 1. The marginal return on assets and the marginal cost of capital are more than three times greater than the investors' discount rate (5 percent) or the interest rate on debt, because the marginal rate of change of interest expense is relatively great (see (19)). For this reason, the rate of interest on debt, after corporate taxes, is less than the discount rate.

Suppose investors revise their perceptions of the project's returns so that the MEC becomes less concave and marginal q at point 1 is lower. For shareholders, the new pdf(r) is identical to the first except that the mean return becomes

$$(25)\quad R(A) = .5 - 9.157*10^{-2}*A^{1}.$$

Similarly, the new $pdf_c(r)$ is identical to the first, except that its mean is defined by (23) and (25).

With this revised MEC, q falls substantially at point 1, and marginal q is less than unity. Although the marginal return on assets increases, the marginal cost of funds exceeds the marginal return on assets. Despite these consequences, the new optimal choice of (φ,A) is point 2 in Figure 4 and in Table 1, at which the demand for assets is greater than it is at point 1. The marginal return on assets and the marginal cost of capital also are greater at point 2 than at point 1.

Table 1
The Demand for Capital and the Optimal Choice of Financial Structure

	Initial MEC Evaluated at Point 1	Revised MEC Evaluated at Point 1	Point 2	Point 3
Stock of Assets (A)	\$1019.8	\$1019.8	\$1474.1	\$600.0
Equity Financing (φ)	50%	50%	86%	85%
Tobin's q				
Average (q)	2.48	1.00	1.01	1.02
Marginal (υ)	1.00	.99	1.00	1.01
	- - - - - - - - -	In Percent	- - - - - - - - -	
Marginal Return on Assets (TR')	21.4	30.0	29.1	32.6
Marginal Cost of Capital	21.4	32.2	39.1	31.1
less Discount Rate (ρ)	16.4	27.2	24.1	26.1
Interest Rate on Debt				
Before Taxes (i)	6.3	18.2	5.9	6.1
After Taxes ($(1-\tau)i$)	3.2	9.1	3.0	3.1
Interest Premium Paid by Shareholders (δ)	.6	10.8	.9	1.1
$D_\varphi q$	2.2	3.4	.9	1.1
$D_\varphi \lambda$	8.6	15.7	3.7	4.1
$D_\varphi \mu$	-6.3	-12.1	-2.7	-2.9

The Initial MEC case is described in (22), (23), and (24). The Revised MEC case is described in (23), (24), and (25). The column headings refer to points in Figures 4 and 5. For columns 1 and 3, the discussion before Proposition 3 describes the optimal strategy; for column 4, the discussion before (26) describes the optimal strategy, given that equity financing cannot exceed \$510. The marginal cost of capital (discussed after (19)) equals TR' when marginal q equals unity; otherwise, it equals
$\rho + (\tau(\mu + AD_{A\mu}) + (\lambda + AD_A\lambda))(1+\rho)$.

The expected rents on inframarginal assets (at point 1) fall with the revision of the MEC, and creditors (who are not as optimistic as shareholders under any circumstances) require greater rates of interest on loans to prevent their expected rate of return from falling. With the revised MEC, interest expense also rises at a greater rate as assets increase than under the initial MEC. Consequently, both leverage and the interest rate on debt are lower at point 2 (column 3) than they are at point 1 (column 1). Because the terms of credit have deteriorated (column 1 versus column 2), shareholders reduce their leverage in order to reduce the cost of debt financing.

Despite the lower rate of interest at point 2, the hurdle rate required of investments (the marginal return) increases, because the marginal cost of capital increases. With the decline in expected rents, $(\lambda + AD_A\lambda)$ in (19) increases. Because both the optimal hurdle rate of return and the difference between this return and debt yields may change when the MEC shifts, debt yields may be a poor proxy for the cost of capital or hurdle rate of return that determine the demand for capital assets.

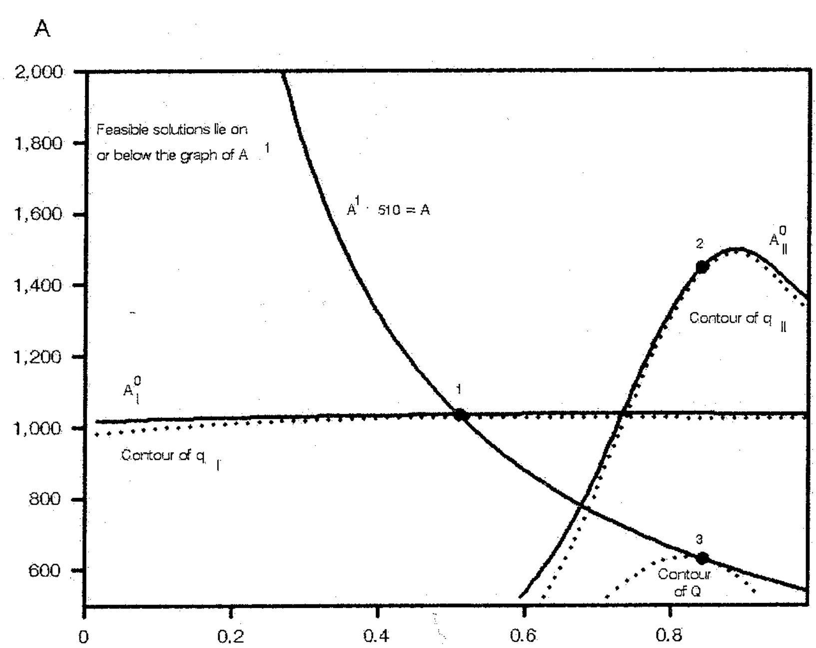
Given A, the choice of φ does not minimize the cost of capital (maximize q) for either point 1 or point 2. In both cases, a small reduction in leverage would reduce interest expenses (due to the premium paid to creditors) more than it would reduce the value of the tax shelter (due to the tax treatment of debt financing). Nevertheless, points 1 and 2 are optimal: instead of choosing φ to minimize the cost of capital, shareholders choose φ and A jointly to maximize their wealth.

The Supply of Equity Financing Is Constrained

If shareholders possess only $510, so that their equity investment absorbs all their funds at point 1, then they are unable to provide $1,268 of equity in order to reach point 2 after the revision of the MEC (Figure 5).[23] Their best strategy, under these circumstances, selects the φ that maximizes the value of their \$510 equity investment. Taking into account the equity constraint, the function $A^1(\varphi)$ shows the enterprise's maximum stock of assets for each value of φ. Shareholders select the point on $A^1(\varphi)$ that maximizes:

(26) $Q(\varphi) = (q-(1-\varphi))/\rho$.

Figure 5



See Figure 4 and Table 1

[23]In the spirit of Modigliani-Miller, if shareholders write personal loans to obtain the required funds, their leverage is no less than it would be if they had purchased \$1,474 of assets with only \$510 of equity. In fact, the terms on personal loans, including the loss of limited liability, make this source of financing less attractive than leveraging the project.

When equity constraints are binding, the optimal choice of A (point 3) lies on the section of A^1 beneath its intersection with A_{II}^0 ; above this intersection, marginal q is less than unity. Consequently, any shift of the MEC that reduces marginal q below unity (at the former value of A) must also reduce the demand for assets. If the value of marginal q were greater than unity initially, then a shift of the MEC that reduces marginal q, but leaves it above unity at the former value of A, may not reduce the demand for assets.

The optimal stock of assets declines approximately 40 percent (from point 1 to point 3) after the shift of the MEC if the supply of equity financing is limited to $510 (Table 1). Marginal q exceeds unity at the optimal choice of A, and the marginal return on assets exceeds the marginal cost of capital.

IV. Conclusion

When enterprises earn economic rents that vary with their stocks of assets, their demand for capital depends on the slope of their MEC schedules as well as marginal measures of their return on assets, their cost of capital, or Tobin's q. Even though a shift of the MEC may increase marginal q or marginal returns relative to the marginal cost of capital, the demand for capital tends to be smaller as the average return on assets rises relative to the returns on marginal assets. Accordingly, as expectations of economic rents shift, the correspondence between the demand for capital and its traditional determinants may change.

According to Propositions 1 and 3, not all models of investment are equally robust. In many circumstances, descriptions of the demand for capital that assume the marginal rate of return on assets equals either the (user) cost of capital or the weighted cost of funds are less stable than models that assume marginal q equals unity. Models assuming that investment spending or changes in the demand for capital correspond to changes in marginal q at any given stock of capital also may be relatively unstable.[24]

Horse races among models of investment using ex post data typically find that the accelerator model finishes in the money, while the performance of other models generally is less consistent (Berndt 1991). Even though the demand for capital may not be stably related to many of its traditional determinants when the pattern of economic rents shifts, the ex post rate of output may be a reliable indicator of an enterprise's scale of operation and its demand for capital, provided the ratio of capital to other factors of production is sufficiently stable. Consequently, the accelerator model might be expected to carry less of a handicap when the course of output is known or can be predicted with sufficient accuracy.

When shareholders' expectations of an enterprise's economic rents exceed the expectations of others, the value of the enterprise depends on its capital structure as well as its stock of capital. This discrepancy between the assessments of shareholders and creditors also may create the appearance of a "pecking order" in the enterprise's sources of funds. When business conditions are promising, entrepreneurs expect relatively great returns, and if creditors tend to concur, the optimum degree of leverage is relatively great. Accordingly, an expanding capital stock absorbs both internal funds

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[24]This conclusion is strengthened when marginal q (using the value of the firm's <u>marketable</u> securities) is not necessarily equal to unity at the optimal stock of assets. The value of implicit options, such as waiting to invest, must be taken into account for a more comprehensive concept of q. See Myers (1977), McDonald and Siegel (1986), Pindyck (1988), and Greenwald and Stiglitz (1989).

and a relatively large amount of debt financing. When conditions are less promising, the difference between shareholders' and creditors' expectations may be great; consequently, the optimal degree of leverage falls. With the diminished demand for capital, the volume of new debt financing falls more than the retention of earnings. On the margin, changes in the demand for capital correspond comparatively closely with changes in the volume of debt financing, suggesting that enterprises rely first on internal sources and then on external sources to fund their capital budgets.

Behind this apparent pecking order is an optimal capital structure. When an enterprise's cash flow is sufficiently great compared to its rate of investment in assets, the substitution of credit for internal funds prevents leverage from falling to zero. Accordingly, relatively mature enterprises distribute dividends to shareholders even as they increase their obligations to creditors. When cash flow is not so great compared to the demand for new assets, the enterprise reinvests all of its internal funds to prevent leverage from rising to unity. Consequently, enterprises that are growing relatively rapidly retain all earnings. If the demand for assets is very great compared to cash flow, the enterprise must issue new equity to achieve its optimal capital structure.

When the discrepancy between the expectations of shareholders and creditors increases greatly, creditors are unwilling to maintain the previous degree of debt financing at interest rates that entrepreneurs consider reasonable. To the degree that enterprises offer creditors more acceptable coverage ratios by cleaning up their balance sheets and by curtailing their capital budgets, the change in interest rates on debt may understate the change in creditors' discount rates as perceived by shareholders. In these cases, the resulting reduction in the volume of debt financing or of investment may seem to be especially great compared to the rise in interest rates, and enterprises that are highly levered given creditors' expectations of rents may be "shut out" of credit markets at reasonable rates of interest, giving the appearance of a rationing of credit.

A change in conditions that increases marginal q may diminish the demand for capital any time the value of enterprises earning economic rents depends on the choice of variables other than the stock of capital--variables such as the capital-labor ratio, the choice of technology, the pricing of products, or the mix of outputs. For example, when q varies with the employment of labor services, the stock of assets for which marginal q equals unity also varies with labor services (see the Appendix). A lower tax on the income of laborers may diminish the wage rate, thereby increasing rents, other things equal. But the demand for capital may increase comparatively little or even subside if the lower relative wage rate induces management to substitute labor for capital to a great degree.

Appendix

Given that $q(\varphi, A)$ exceeds unity for some feasible choice of (φ, A) and that q is a continuous function, the region over which $q(\varphi, A)$ exceeds unity, A exceeds A*, and φ is positive but less than unity is an open set. Within this set, the function A^0 assigns to each value of φ that value of A for which marginal q equals unity. The projection of this region onto the φ axis is the relevant domain for A^0. The only points where A^0 may intersect the contour $q=1$ are those where both υ and q equal unity, implying that $D_A q$ is zero (see (4)) and that the tangent to the contour is of q perpendicular to the φ axis at these points.

$A^0(\varphi)$ is not parallel to the φ axis (horizontal) over any open interval for φ unless the corporate tax rate is zero and creditors' assessment of the return on assets matches that of stockholders. From the total differential of (4), assuming υ is constant at unity,

$$(A1) \quad D_\varphi A^0 = -(A\, D_\varphi D_A q + D_\varphi q) \,/\, (2D_A q + A D_A^2 q) .$$

The denominator of (A1) is negative. If the numerator equals zero, then (from the continuity of q)

$$(A2) \quad A\, D_A(D_\varphi q) = -(D_\varphi q) .$$

Satisfying equation (A2) over an open interval of φ requires that q be independent of φ -- τ and δ equal zero.

Although A^0 is not horizontal over an open interval in the domain of φ under the conditions of this paper, $D\varphi A^0$ may be zero at one or possibly more points. The optimum choice of φ may correspond to one of these points only under special circumstances. If A^0 is tangent to a contour of q at a point where both are horizontal to the φ axis:

$$(A3) \quad D_\varphi q = -(\tau D_\varphi \mu + D_\varphi \lambda) = 0$$

$$(A4) \quad D_A D_\varphi q = -D_A(\tau D_\varphi \mu + D_\varphi \lambda).$$

Consequently, $\tau D_\varphi \mu$ and $D_\varphi \lambda$ must be equal in magnitude but of opposite signs, (A3); furthermore, these slopes must change at the same rate (in opposite directions) when A changes, (A4). These two conditions generally will not be satisfied at the same point for independent specifications of the functions pdf(r) and $\mathrm{pdf}_c(r)$, as is illustrated by the example in section III of the paper.

Because A^0 is not necessarily horizontal at the optimal choice of φ: given A, $D_\varphi q$ does not necessarily equal zero, and the marginal tax saving associated with a small alteration of leverage does not equal the marginal change in the cost of debt financing.

Because A^0 and the contour of q to which it is tangent are not necessarily horizontal at the optimal choice of (φ, A), the optimal choice of (φ, A) ordinarily is not attained by: first, maximizing q with respect to φ given A, thereby defining $\varphi^0(A)$; second, choosing the pair(s) for which $\upsilon(\varphi^0, A)$ equals unity. With this approach, φ^0 intersects the contours of q only where their tangents are parallel to the φ axis. Therefore, this alternative cannot yield the optimal choice of (φ, A), unless, perhaps, A^0 is horizontal at the optimal choice of φ.

Instead of dwelling solely on leverage, q may depend on the enterprise's choice of labor as well as capital. Suppose

$$(A5)\quad q(L,A) = (1+R(L,A)) \;/\; (1+\rho),\ \textit{where}$$

$$(A6)\quad R(L,A) = (p(Q(L,A))\, Q(L,A) - w(L)\, L) \;/\; A.$$

If the functions p (the price of output, from a downward-sloping demand curve), Q (the quantity of output, from a neoclassical production function), and w (the wage rate, from an upward-sloping supply schedule) have the usual properties and if q exceeds unity for some choice of (L,A), then the region over which q exceeds unity is an open set. If the optimal choice of A, given L, equates marginal q with unity, then the function A^0(L), the analysis of the choice of the optimal (L,A), and the correspondence between changes in marginal q and the demand for capital are analogous to those for (φ, A).

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