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# Optimal Tax Policy and Expected Longevity: A Mean and Variance Utility Approach<sup>1</sup>

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### Abstract

This paper studies the normative problem of redistribution between agents who can influence their survival probability through private health spending, but who differ in their attitude towards the risks involved in the lotteries of life to be chosen. For that purpose, a two-period model is developed, where agents' preferences on lotteries of life can be represented by a mean and variance utility function allowing, unlike the expected utility form, some sensitivity to what Allais (1953) calls the 'dispersion of psychological values'. It is shown that if agents ignore the impact of their health spending on the return of their savings, the decentralization of the first-best utilitarian optimum requires intergroup lump-sum transfers and group-specific taxes on health spending. Under asymmetric information, we find that subsidizing health expenditures may be optimal as a way to solve the incentive problem.

*Keywords:* longevity, risk, lotteries of life, non-expected utility theory, moments of utility theory, health spending.

*JEL codes:* D81, H21, I12, I18, J18.

# 1 Introduction

Whereas human longevity depends on factors of various natures - genetic, environmental or sociocultural -, a large demographic literature also emphasizes the crucial influence of the individuals' lifestyles on their longevity.<sup>1</sup> Clearly, *how long* one lives is not independent from *how* one lives. Individual longevity depends on the extent to which one is willing to 'make an effort' to improve or preserve his health, and differences in the amount of efforts carried out by individuals tend to be reflected by longevity differentials.<sup>2</sup>

What should a utilitarian government do in front of such a heterogeneity of lifestyles and longevities? The answer clearly depends on the source of the heterogeneity and on whether longevity is exogenous or not. However, it also depends crucially on the form of the individual's preferences. For instance, Bommier *et al.* (2007 a,b) show that if longevity depends on exogenous health endowments, it is optimal to redistribute from short-lived toward long-lived individuals only when individuals have additively separable preferences, while relaxing this latter assumption, agents should be compensated for their poor longevity. As they explain in their work, assuming additively separable preferences leads to an implicit assumption of net *risk neutrality toward the length of life* which leads to strong (and disputable) conclusions in terms of redistribution.<sup>3</sup>

Starting from the works of Bommier *et al.* (2007 a,b), we might, on the opposite, examine the redistributive consequences of another potential source of heterogeneity in preferences: *the attitude of agents towards risk* and, more precisely, towards *risk on longevity*. This source of heterogeneity is generally ignored since most economic models assume both expected utility and additive lifetime welfare which, as we already mentioned, presupposes that *all* agents exhibit risk-neutrality with respect to the length of life.<sup>4</sup> But it is not difficult to see that the attitude towards risk plays a crucial role here, so that assuming a generalized net risk-neutrality with respect to longevity is a

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<sup>1</sup>See Vallin *et al* (2002).

<sup>2</sup>Health-improving efforts can take various forms: the effort can be either *temporal* (e.g. physical activity, see Surault 1996 and Kaplan et al. 1987), *physical* (e.g. abstinence of food, see Solomon and Manson, 1997), or *monetary* (e.g. health services, see Poikolainen and Eskola, 1986).

<sup>3</sup>See Bommier (2005) on the notion of risk neutrality toward the length of life.

<sup>4</sup>One of such models is Leroux (2007).

quite strong postulate.

Clearly, when an individual chooses how much to invest in his health, he does not choose a certain length of life, but, rather, expresses a preference for a particular *lottery of life*, whose different scenarii involve different lengths of life.<sup>5</sup> The chosen level of health-improving effort will not be a guarantee of a longer life, but only of a longer *expected* length of life with also some possible consequences on the *variance* of the length of life.<sup>6</sup> Thus, in the context of risk about the length of life, individual choices of health-improving efforts may reflect their attitudes towards risk about the length of life, so that the making of a *uniform* assumption on the attitude towards risk may oversimplify the problem of the optimal public intervention.

Let us illustrate this with the following example (see Figure 1). A person of age 50, who has a disease, can choose between two possible lotteries of life: either lottery A, ‘no medical treatment’, or lottery B, ‘medical treatment’ (assumed to be costless).<sup>7</sup> Under no medical treatment, the patient is certain to live the next 10 years for sure, but not longer. On the contrary, under the medical treatment, the patient can die during the intervention with a probability  $1/2$ , but can, if the intervention is a success, live until the age of 70 years with a probability  $1/2$ . What will the patient choose?

It is not straightforward to see what the patient will decide. Actually, each lottery exhibits the same expected length of life, equal to 60 years, but different degrees of risk about the length of life: whereas lottery A is risk-free, lottery B is risky regarding the length of life.<sup>8</sup>

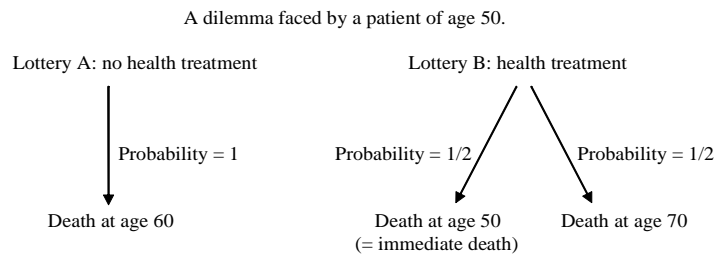
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<sup>5</sup>Strictly speaking, we should talk of a lottery on the length of life rather than of a lottery of life.

<sup>6</sup>To be precise, the chosen level of health-improving effort is likely to influence the dispersion of the ages at death.

<sup>7</sup>Consumptions per life-period are assumed to be the same under the two lotteries.

<sup>8</sup>Note that, in general, the choice of an effort level influences not only the expected length of life and the variance of the age at death but also per period utility.



### Choice between two lotteries of life

Under net risk-neutrality with respect to the length of life, a patient would be totally indifferent between lotteries A and B, and would toss a coin to decide whether he will undergo the medical treatment or not. However, such an indifference is highly unlikely, because the degree of risk about the length of life is a non-neutral information for decision-makers. Thus, it is likely that individuals differ largely regarding their attitude towards risk, and do not all exhibit risk-neutrality with respect to the length of life. Obviously, some patients, who are risk-averse with respect to the length of life, will choose no medical treatment (lottery A), while some others, who are risk-lover, will choose the medical treatment (lottery B).<sup>9</sup> As this example illustrates, the observed inequality in health-influencing efforts is likely to reflect the heterogeneity of preferences, and, in particular, the heterogeneity of individual attitudes towards risk. But this raises the difficult question of the optimal public policy in that context: what should a utilitarian government do in front of such a heterogeneity in the attitude towards risk?

The goal of this paper is to examine the optimal public policy in an economy where agents can influence their survival by exerting some monetary effort, but differ in their attitude towards risk with respect to the length of life. For simplicity, we assume that individuals live for a maximum of two periods, the first one being certain while the second one is conditional on survival. Naturally, by choosing their health expenditures, agents choose a specific lottery of life, which reflects their attitude toward risk.

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<sup>9</sup> Alternatively, if the treatment had the virtue not to raise, but to reduce the variance of the age at death, risk-averse agents would *ceteris paribus* opt for the treatment.

So as to account for individuals' attitude toward risk on longevity, we model individual preferences using a 'mean and variance' utility function, and we assume that individuals have different sensitivities to the variance of lifetime welfare.<sup>10</sup> As this is well-known since Bommier's (2005) work, there exist two broad ways to depart from net risk-neutrality with respect to the length of life. One way is to relax additive lifetime welfare, as in Bommier's (2005) works; the alternative solution is to relax the expected utility hypothesis. The former approach has the advantage to keep on relying on the - convenient - expected utility theory, but suffers from a lack of intuition behind non-additive lifetime welfare. This is why, in this paper, we prefer to keep additive lifetime welfare but to relax the expected utility hypothesis. Thus, lifetime welfare is still assumed to be additive in temporal welfare (without pure time preferences), but the expected utility hypothesis is here replaced by a less restrictive postulate.

More precisely, it is assumed that agents's preferences on lotteries of life can be represented by a 'mean and variance' utility function of the kind defended by Allais (1953) in his seminal paper. Actually, Allais emphasized that, given that the dispersion of psychological values is 'the specific element of the psychology of risk' (Allais, 1953, p. 512), it follows that '[...] even in a first approximation, one should take into account the second order moment of the distribution of psychological values' (1953, p. 513).<sup>11</sup> Moreover, it was also quite clear in Allais's mind that '[...] one cannot regard as irrational a psychological attitude in front of risk that takes the dispersion of psychological values into account.' (see Allais, 1953, p. 520).<sup>12</sup> We shall thus postulate a mean and variance utility function, which is a simple generalization of the EU form accounting for Allais's intuition. Naturally, other forms could be chosen instead (see Stigum and Wenstop, 1983; Schmidt, 2004), but the mean and variance utility function has the advantage of

<sup>10</sup> Actually, the 'mean and variance' utility form is a special case of what Machina (2007) calls the 'Moments of Utility' approach in non-EU theory, followed by Hagen (1979) and Munera and de Neufville (1983).

<sup>11</sup> Original version: '[...] même dans une première approximation, on doit tenir compte du moment d'ordre deux de la distribution des valeurs psychologiques'.

<sup>12</sup> Original version: '[...] on ne saurait considérer comme irrationnelle une attitude psychologique devant le risque qui tient compte de la dispersion des valeurs psychologiques.'

simplicity.<sup>13</sup>

The rest of the paper is organized as follows. Section 2 presents the model and derives the laissez-faire equilibrium. Section 3 studies the first-best social optimum and its decentralisation. The second-best problem is considered in Section 4. A numerical illustration is developed in Section 5. Conclusions are drawn in Section 6.

## 2 The model

### 2.1 Environment

Let us consider a population of individuals who live a first period of life (whose length is normalized to one) with certainty, but survive to the second period only with a probability  $\pi$ . This probability depends positively on some monetary investment  $m$ :

$$\pi = \pi(m)$$

equivalently,  $m$  can be regarded as a private health expenditure made by the agent in the first period of his life, so as to increase his survival probability. We assume here that agents have the same survival function  $\pi(\cdot)$  with  $\pi'(\cdot) \geq 0$  and  $\pi''(\cdot) \leq 0$ . Note that life expectancy equals  $1 + \pi(m)$ .

However, agents are assumed to differ in their preferences. In order to introduce these differences in preferences, we assume that individual preferences can be represented by a function having the ‘mean and variance’ utility form (see Allais, 1953), and that agents exhibit different degrees of sensitivity to the volatility of welfare:<sup>14</sup>

$$U_i = \bar{u}_i - \gamma_i \text{var}(u_i)$$

where  $\bar{u}_i$  is the expected lifetime welfare of an agent with type  $i$ , while  $\text{var}(u_i)$  is the variance of his lifetime welfare.<sup>15</sup> The parameter  $\gamma_i$  reflects the sensitivity to the variance

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<sup>13</sup>Moreover, that functional form shall, unlike the expected utility function, allow some risk-aversion with respect to the length of life, even under additive lifetime welfare.

<sup>14</sup>Note that this function, although more general than the usual expected utility function, could still be generalized by taking into account higher moments of the distribution of lifetime welfare across scenarios of lotteries of life. For more general functions, see Hagen (1979) and Machina (1983).

<sup>15</sup>Note that preferences represented by a function of that form do not necessarily satisfy the independence axiom, as the initial ordering between two lotteries may be inverted by a convex combination of those lotteries.



of lifetime welfare exhibited by a lottery of life. Under complete insensitivity,  $\gamma_i$  equals 0 and we are back to standard expected utility (EU) theory. On the contrary, if  $\gamma_i$  is positive, the agent prefers, *ceteris paribus*, lotteries with a lower variance of lifetime welfare across scenarios, while a negative  $\gamma_i$  reflects the tastes of ‘variance-lover’ agents.<sup>16</sup>

For simplicity, we shall, in the rest of this paper, concentrate on two types of agents, who differ only in their degree of sensitivity to the variance of lifetime welfare:

- Agents of type 1, with  $\gamma_1 > 0$ ;
- Agents of type 2, with  $\gamma_2 \geq 0$  and  $\gamma_2 < \gamma_1$ .

In other words, we shall focus here on two types of agents whose preferences on lotteries of life cannot be represented by a standard expected utility function, i.e. on two types of quasi expected utility maximizers. However, the extent to which those agents depart from EU theory varies across the types: type-1 agents depart more from EU theory than type-2 agents.

Under a zero utility from death and additive lifetime welfare (with no pure time preferences), the expected lifetime welfare  $\bar{u}_i$  is, for  $i \in \{1, 2\}$ :

$$\begin{aligned}\bar{u}_i &= \pi(m_i) [u(c_i) + u(d_i)] + (1 - \pi(m_i)) [u(c_i)] \\ &= u(c_i) + \pi(m_i)u(d_i)\end{aligned}$$

where  $c_i$  and  $d_i$  denote, respectively, first and second period consumptions of an individual with type  $i$ . The function  $u$  is increasing and strictly concave.

In this two-scenarios world, the variance of lifetime welfare takes a simple form:

$$\begin{aligned}var(u_i) &= \pi(m_i) [(u(c_i) + u(d_i)) - (u(c_i) + \pi(m_i)u(d_i))]^2 \\ &\quad + [1 - \pi(m_i)] [u(c_i) - (u(c_i) + \pi(m_i)u(d_i))]^2 \\ &= [u(d_i)]^2 [(1 - \pi(m_i))\pi(m_i)]\end{aligned}\tag{1}$$

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<sup>16</sup>By *abus de langage*, it could also be said that agents with a higher  $\gamma_i$  are more ‘risk-averse’ than agents with lower  $\gamma_i$  levels.

The variance of lifetime welfare depends on second-period consumption and on the variance of lifetime, equal to  $(1 - \pi(m_i))\pi(m_i)$ .<sup>17</sup> Hence, we shall, for ease of notation, use the notation  $var(u_i) \equiv var(d_i, m_i)$  in the rest of this paper.

It should be stressed that second-period consumption tends, *ceteris paribus*, to increase the variance of lifetime welfare: the higher second-period consumption is, the higher the welfare gain of survival is, and the higher the welfare loss from death is too. Second-period consumption is thus a major source of welfare volatility.

However, the probability of survival  $\pi(m_i)$  has a non-monotonic relationship with the variance of lifetime welfare. Actually,

$$\frac{\partial var(d_i, m_i)}{\partial \pi(m_i)} = [u(d_i)]^2 [1 - 2\pi(m_i)] \begin{matrix} > \\ = 0 \\ < \end{matrix} \iff \pi(m_i) \begin{matrix} < \\ = \\ > \end{matrix} \frac{1}{2}$$

Hence, a higher survival probability raises the variance of lifetime welfare if  $\pi(m_i)$  is smaller than  $1/2$ , whereas it tends to lower it if  $\pi(m_i)$  is larger than  $1/2$ . Moreover, given that  $\frac{\partial var(d_i, m_i)}{\partial m_i} = \frac{\partial var(d_i, m_i)}{\partial \pi(m_i)} \pi'(m_i)$ , it follows that a higher health effort tends to raise the variance of lifetime welfare if  $\pi(m_i)$  is smaller than  $1/2$ , whereas it lowers it if  $\pi(m_i)$  is higher than  $1/2$ .

Given that, in industrialized economies, the life expectancy is high and the variance of lifetime is low (as the survival curve is close to the rectangular), we shall, in the rest of this paper, concentrate on values of  $\pi$  larger than  $1/2$  for any level of health efforts. A natural corollary of this postulate is that, when an individual spends more on his health, he does not only increase his life expectancy, but reduces also the variance of his lifetime.<sup>18</sup>

<sup>17</sup>Note that the variance of lifetime has a minimum at  $\pi = 0$  and  $\pi = 1$ , and reaches its maximum at  $\pi = 1/2$ .

<sup>18</sup>Note also that our focus on  $\pi > 1/2$  has another intuitive justification. Actually, when  $\pi > 1/2$ , individual welfare is increasing in life expectancy for any level of  $\gamma_i$ , as one expects. To see this, note that imposing  $\frac{\partial U_i}{\partial \pi} \geq 0$  amounts to assume:

$$\frac{\partial U_i}{\partial \pi} = u(d_i) - \gamma_i [u(d_\gamma)]^2 [1 - 2\pi(m_i)] \geq 0$$

which, under  $\gamma_i > 0$ , is true for all levels of  $d_i$  if and only if  $\pi(m_i) \geq 1/2$  for all levels of  $m_i$ .

## 2.2 The laissez-faire

Agents of type  $i \in \{1, 2\}$  choose first period and second period consumptions, as well as health expenditure so as to maximize their objective function subject to their budget constraint:

$$\begin{aligned} & \max_{c_i, d_i, m_i} U_i(c_i, d_i, m_i) \\ \text{s.to } & \begin{cases} c_i = w - s_i - m_i \\ d_i = R_i s_i \end{cases} \end{aligned}$$

where lifetime utility takes the following form

$$U_i(c_i, d_i, m_i) = u(c_i) + \pi(m_i)u(d_i) - \gamma_i \text{var}(d_i, m_i) \quad (2)$$

and  $\text{var}(d_i, m_i)$  is equal to (1). We assume that savings are entirely invested in private annuities and that  $R_i$  is the return of an annuity. The wealth endowment  $w$  is exogenous and identical for all agents. Note also that there is no pure time preference, and that the interest rate is zero.

Assuming actuarially fair prices (i.e.  $R_i = 1/\pi(m_i)$ ), the laissez-faire allocation for an agent of type  $i \in \{1, 2\}$  satisfies the following conditions:

$$u'(c_i) = u'(d_i) - \gamma_i \text{var}_d(d_i, m_i) / \pi(m_i) \quad (3)$$

$$u'(c_i) = \pi'(m_i)u(d_i) - \gamma_i \text{var}_m(d_i, m_i) \quad (4)$$

where  $\text{var}_x(x, y)$  and  $\text{var}_y(x, y)$  are partial derivatives of the variance of lifetime welfare with respect to  $x$  and  $y$ . Condition (3) characterizes the optimal saving decision. In the absence of any sensitivity to the variance of lifetime welfare ( $\gamma_i = 0 \forall i \in \{1, 2\}$ ), each agent would choose to smooth consumption over time (i.e.  $c_i = d_i \forall i \in \{1, 2\}$ ), because of the conjunction of no pure time preference, an actuarially fair annuity price and a zero interest rate. However, under a positive  $\gamma_i$ ,  $c_i > d_i \forall i$ , since  $\text{var}_d(d_i, m_i)$  is always positive; thus the sensitivity of agents to the variance of lifetime welfare makes them consume more in the first period. Actually, consuming more during the first period is a simple way to insure oneself against undergoing a big loss of welfare if one dies at

the end of the first period. Thus, concentrating consumption in the first period is a straightforward way to protect oneself against a too large variation of lifetime welfare across scenarios of the lottery of life. Note also that the higher  $\gamma_i$  is, the steeper the intertemporal consumption profile will be *ceteris paribus*, because the more variance-sensitive the agent is, the more he will use that trick to avoid big welfare losses. This result is presented in the proposition below:

**Proposition 1** *If the market of annuities is actuarially fair,  $c_i > d_i$  for any individual with type  $\gamma_i > 0$ .*

Condition (4) characterizes the level of health expenditure chosen by the individual in the equilibrium. Under traditional expected utility theory, this condition would collapse to  $u'(c_i) = \pi'(m_i)u(d_i)$ , stating that the optimal health expenditure is such that the marginal welfare gain due to health expenditure (in terms of the second period of life) should equalize the marginal welfare cost of such an effort. However, under a positive  $\gamma_i$ , the marginal lifetime utility from health expenditure depends also on its impact on the variance of lifetime welfare, which is always positive since we assume that  $\pi(m_i)$  is higher than 1/2. Thus, under positive sensitivity to the variance in welfare, the level of health investment is always greater than under expected utility theory. Note also that in the *laissez-faire*, the individual does not take into account the impact of health expenditures on the return of his savings,  $R_i = 1/\pi(m_i)$  so that the individual chooses a level of health expenditures which is too high compared to its optimal level.<sup>19</sup>

We can now study the equilibrium levels of consumptions and of health expenditure of individuals with different sensitivities to the variance in welfare. Our results are summarized in the following proposition:

**Proposition 2** *Provided the market for annuities is actuarially fair, the *laissez-faire* allocation is such that*

(i) *if  $c_1 = c_2$ , then  $d_1 < d_2$  and  $m_1 \geq m_2$  [or  $d_1 \leq d_2$  and  $m_1 > m_2$ ],*

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<sup>19</sup>This result is highlighted in Becker and Philipson (1998). Actually, each agent tends to consider that his own health effort will not affect the return of the annuity whereas at the aggregate level it does. This is also emphasized in Sheshinski (2007, chapter 7).

(ii) if  $d_1 = d_2$ , then  $c_1 > c_2$  and  $m_1 \leq m_2$  [or  $c_1 \geq c_2$  and  $m_1 < m_2$ ].

Note first that, given the postulated general functional forms for  $u(c)$  and  $\pi(m)$ , it is not possible, in the present model, to fully describe the optimal levels of consumptions and health expenditures for the two types of agents. Depending on the particular functional forms chosen for  $u(c)$  and  $\pi(m)$ , agents's consumptions and efforts  $(c_i, d_i, m_i)$  may *a priori* vary in different ways. We will come back on the importance of functional forms in Section 5, where we present some numerical examples. This is why we shall here equalize first or second period consumptions between individuals so as to determine how the chosen variables  $(c_i, d_i, m_i)$  differ across agents.

If first-period consumption is equal for the two types of agents, then it is necessarily the case that an agent who has a larger sensitivity to the dispersion of psychological values chooses a lower second-period consumption and more health spending than an agent with a lower  $\gamma_i$ . The intuition behind that result is the following. For a more sensitive agent, having a lower second-period consumption and spending more on health is a rational way to reduce the variance of lifetime welfare since the potential loss (i.e. second-period utility) would be smaller and this would happen with a lower probability. Agents with a lower  $\gamma_i$  do not have the same concerns, and thus choose, for an equal first period consumption, a higher second-period consumption and a lower health effort.

If, alternatively, it is second-period consumption that is equal for both types of agents, then, without surprise, agents who are more sensitive to the variance will consume more in the first period in comparison to less-variance-sensitive agents, so that, given the budget constraint faced, they will also invest less in health in comparison with agents with a lower  $\gamma_i$ . That result is not surprising, as consuming more in the first period is a standard way to insure oneself against a too large volatility of lifetime welfare.

Thus, one cannot say, under general functional forms, whether agents with a higher  $\gamma_i$  will spend more or fewer resources in health. A higher sensitivity to the dispersion of psychological value may imply that an agent spends more on health (at the cost of second-period consumption) or on the contrary, spends less on health (to favour first-period consumption). That indeterminacy can be explained as follows. The two ways

to protect oneself against a high volatility of lifetime welfare are either to spend a lot on health or to spend a lot in first-period consumption. Which solution dominates depends on the curvatures of  $u(c_i)$  and  $\pi(m_i)$ . If  $\pi'(m_i)$  is large and  $u'(c_i)$  is low, then agents of type 1 will opt for the first way to avoid lifetime welfare variance; on the contrary, if  $\pi'(m_i)$  is low and  $u'(c_i)$  is large, agents of type 1 will opt for the second as a more efficient way to avoid lifetime welfare volatility.

### 3 The first-best problem

#### 3.1 The social optimum

In this section, we assume that the social planner is utilitarian and that he perfectly observes individuals' types.<sup>20</sup> The social planner can lend or borrow at a zero interest rate in order to balance the budget at any given period. The resource constraint of the economy is thus:

$$\sum_{i=1,2} n_i (c_i + \pi(m_i) d_i + m_i) \leq w \quad (5)$$

where  $n_i$  denotes the number of agents of type  $i \in \{1, 2\}$  in the population. Thus, the social planner chooses consumption paths as well as health investments levels for each type of individuals in order to maximize

$$\sum_{i=1,2} n_i (u(c_i) + \pi(m_i)u(d_i) - \gamma_i var(d_i, m_i))$$

subject to (5).

The first order conditions yield:

$$u'(c_i) = \lambda \quad (6)$$

$$u'(d_i) - \gamma_i \frac{var_d(d_i, m_i)}{\pi(m_i)} = \lambda \quad (7)$$

$$\pi'(m_i)u(d_i) - \gamma_i var_m(d_i, m_i) = \lambda [1 + \pi'(m_i) d_i] \quad (8)$$

Combining (6) and (7), we obtain the optimal trade-off between present and future consumptions; this is identical to our laissez-faire condition (3) when the price of the

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<sup>20</sup>Note that the standard Benthamite utilitarian criterion exhibits various limitations in general, and in the particular context of endogenous longevity (see Broome, 2004). Thus, it is used here on the mere grounds of analytical convenience.

annuity is actuarially fair. Thus, first-period consumption is still preferred to future consumption in the first-best. On the contrary, (8) together with (6) differs from (4) by a term  $-\lambda\pi'(m_i)d_i$ . In the first-best, the social planner realizes that the level of health expenditure also modifies the budget set. Indeed, a higher level of effort  $m_i$  not only increases direct utility through higher survival but also decreases consumption possibilities as  $\pi(m_i)$  increases in (5). Thus, in the first-best optimum, the social planner induces the individual to exert lower effort so as to limit the negative impact of  $m_i$  over the individual's budget set; this was not the case in the laissez-faire as the individual was taking the annuity return,  $R_i$  as given. These first results are summarized in the following proposition:

**Proposition 3** *The first-best allocation is such that, for agents of type  $i \in \{1, 2\}$ :*

- (i)  $m_i^{FB} < m_i^{LF}$ ,
- (ii)  $c_i > d_i$ .

where  $m_i^{FB}$  and  $m_i^{LF}$  are the level of health expenditures in the first-best and laissez-faire respectively. We now turn to the allocation of consumptions and of health expenditure according to individuals types. Obviously, first-period consumption is equalized across individuals. However, considering (7) and (8), there is no reason for second-period consumptions and health expenditure to be identical across individuals (since  $\gamma_i$  enters into those equations). A priori, it is impossible to rank health expenditures and consumptions depending on individuals' types, unless some additional assumptions are made. Therefore, we assume two individuals with types  $\gamma_1$  and  $\gamma_2$  such that one is sensitive to the variance in lifetime welfare and the other is not and obtain the following results:<sup>21</sup>

**Proposition 4** *Consider two types of individuals with sensitivity to the variance in welfare such that  $\gamma_1 > 0$  and  $\gamma_2 = 0$ . The first-best yields:*

- (i)  $c_1 = c_2 = \bar{c}$ ,
- (ii)  $d_1 < d_2$ ,

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<sup>21</sup>This proposition is proven in Appendix B.

(iii)  $m_1 \geq m_2$ .

In the first-best, first period consumption is equalized across individuals while second period consumption and health expenditures are differentiated between individuals.

The individual with a zero sensitivity to the variance obtains a higher level of second-period consumption than a variance-sensitive agent. This result is not surprising, as second-period consumption tends necessarily to raise lifetime welfare variance.<sup>22</sup> Given that lifetime welfare variance enters type 1's utility negatively, it does not come as a surprise that the social optimum implies  $d_1 < d_2$ .

However, it is not clear whether health expenditures should be higher or lower for agents with higher sensitivity to the variance. This indeterminacy can be explained as follows. On the one hand, higher health investment reduces the lifetime welfare variance of variance-sensitive agents, which matters for *those* agents (unlike for agents of type 2), and, as such, is justified on the grounds of social welfare maximization. On the other hand, dedicating more resources to the health of variance-sensitive agents has, given  $d_1 < d_2$ , a *smaller* impact on the expected lifetime welfare of agents of type 1 than on the expected lifetime welfare of agents of type 2. Hence, whether  $m_1$  exceeds  $m_2$  or not depends on which effect dominates, and, thus, on the level of  $\gamma$  and on the shapes of the utility function and the survival function.<sup>23</sup> As shown in the Appendix B, if  $\gamma_1$  is extremely large, one necessarily has  $m_1 > m_2$ , because the social welfare gain from dedicating more resources to the health of agents of type 1 is here large (given the extreme sensitivity of those agents to the variance of lifetime welfare) and thus largely compensates the social welfare loss due to the lower second-period utility exhibited by the life of agents of type 1. But, as we shall show in the numerical section, the condition  $\gamma_1$  large is only sufficient (but not necessary) to have  $m_1 > m_2$ .

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<sup>22</sup>Clearly, in the extreme case where second-period consumption equals 0, and  $u(0) = 0$ , there is a zero lifetime welfare variance despite the risk about the length of life.

<sup>23</sup>For instance, if the utility function is strongly concave in consumption, we have, at the first-best,  $u(d_1) \approx u(d_2)$ , so that the second effect is minor and dominated by the first one, leading to  $m_1 > m_2$  at the social optimum. Similar examples can be used to show the sensitivity of the social optimum to the shape of the survival function  $\pi(m_i)$ .



### 3.2 Decentralisation

We now study how to decentralise the above optimum through a tax-and-transfer scheme. In the following, we assume that instruments available for the social planner are a tax on savings, a tax on health expenditures and lump sum transfers. We still assume that the annuity market is actuarially fair so that  $R_i = 1/\pi(m_i)$  at equilibrium. The individual's problem is then to maximize:

$$u(w - s_i(1 + t_i) - m_i(1 + \theta_i) + T_i) + \pi(m_i)u(Rs_i) - \gamma_i \text{var}(Rs_i, m_i)$$

where  $t_i$  is the tax on savings,  $\theta_i$  the tax on health expenditures and  $T_i$  is a monetary transfer for any individual with sensitivity  $\gamma_i$ . Deriving first order conditions with respect to  $s_i$  and  $m_i$  and rearranging them, we obtain

$$\frac{\pi(m_i)u'(d_i) - \gamma_i \text{var}_d(d_i, m_i)}{u'(c_i)} = \pi(m_i)(1 + t_i) \quad (9)$$

$$\frac{\pi'(m_i)u(d_i) - \gamma_i \text{var}_m(d_i, m_i)}{u'(c_i)} = (1 + \theta_i) \quad (10)$$

Comparing these conditions with both (6), (7) and (8), we find that the optimal tax on savings is always zero for any type of individual, e.g.  $t_i = 0$  but the optimal level of the tax on health expenditures  $\theta_i$  should be equal to  $\pi'(m_i)d_i > 0$ . This can be related to Becker and Philipson (1998); by implementing a positive tax on health expenditures, one limits health expenditures and make it tend toward its first-best level. Note that this tax on health spending is type-specific, that is, it depends on the degree of variance-sensitivity  $\gamma_i$ .

To explore how the optimal tax on health spending  $\theta_i$  varies with  $\gamma_i$ , let us turn back to the special case of Proposition 4, where  $\gamma_1 > 0$  and  $\gamma_2 = 0$ . In that specific case, we have, under a large  $\gamma_1$ ,  $\theta_1 = \pi'(m_1)d_1 < \theta_2 = \pi'(m_2)d_2$ , so that  $m_1 > m_2$ . Otherwise, we have  $\theta_1 \leq \theta_2$  if  $m_1 < m_2$ . We also find that if  $m_1 < m_2$ , the level of expected consumption, defined by

$$\bar{c} + \pi(m_i)d_i + m_i$$

is always larger for type-2 agents than for type-1 agents. In this case, the first-best optimum transfers resources from the individual with higher sensitivity to the variance

to the individual with the lowest one and  $T_1 < T_2$ . On the opposite, if  $m_1 \geq m_2$ , the direction of transfers is ambiguous and  $T_1 \geq T_2$ .

## 4 The second-best problem

Whereas the above optimal policy discussion assumed a perfect observability of agents' preferences on lotteries of life, such a perfect observability is hardly plausible. Actually, while a government would like agents to reveal their preferences (i.e. their  $\gamma_i$ ), these may be tempted to lie on their type to benefit from an allocation that better coincides with their preferences. Hence, we need to relax the assumption of perfect observability of types, and turn to the analysis of the second-best.

In order to simplify the analysis, we shall concentrate, in the rest of the paper, on the two types of agents of Proposition 4: variance-sensitive agents ( $\gamma_1 > 0$ ) and expected utility maximisers ( $\gamma_2 = 0$ ). *A priori*, the first-best may redistribute from agents of type 2 to agents of type 1, or, the contrary, from agents of type 1 to agents of type 2, depending on the functional form of the survival probability and the utility function. For instance, referring to Proposition 4, it is possible that  $m_1 < m_2$  at the social optimum, so that it is likely that agents of type 1 will be tempted to pretend to be of type 2 (as  $c_1 = c_2$  and  $d_1 < d_2$ ).<sup>24</sup> However, if  $\gamma_1$  is sufficiently large, implying  $m_1 > m_2$  at the social optimum, it may be the case that variance-insensitive agents mimic variance-sensitive agents. Therefore, we need to consider here the two possible cases.

The problem faced by the social planner takes the following general form:

$$\begin{aligned} & \sum_{i=1,2} n_i U_i(c_i, d_i, m_i) \\ s.to & \begin{cases} \sum_{i=1,2} n_i (c_i + \pi(m_i) d_i) \leq w \\ U_2(c_2, d_2, m_2) \geq U_2(c_1, d_1, m_1) \\ \text{or } U_1(c_1, d_1, m_1) \geq U_1(c_2, d_2, m_2) \end{cases} \end{aligned}$$

In the case where type-1 agents are tempted to copy type-2 agents, the first incentive compatibility constraint is binding, and the second one is not (see Subsection 4.1). On

<sup>24</sup>But the mimicking behavior of type-2 agents is not certain, as a large second-period consumption raises also the variance of lifetime welfare.

the contrary, in the case where type-2 agents are tempted to mimic type-1 agents, the second incentive compatibility constraint is binding, and the other is not (see Subsection 4.2).<sup>25</sup>

#### 4.1 Case 1: type 1 mimicking type 2

The Lagrangian of the problem has the following expression

$$\begin{aligned} \mathcal{L} = & n_1 [u(c_1) + \pi(m_1)u(d_1) - \gamma_1 var(d_1, m_1)] + n_2 [u(c_2) + \pi(m_2)u(d_2)] \\ & + \lambda \left[ w - \sum_{i=1,2} n_i (c_i + m_i + \pi(m_i) d_i) \right] \\ & + \mu \left[ \begin{array}{l} u(c_1) + \pi(m_1)u(d_1) - \gamma_1 var(d_1, m_1) \\ -u(c_2) - \pi(m_2)u(d_2) + \gamma_1 var(d_2, m_2) \end{array} \right] \end{aligned}$$

where  $\lambda$  and  $\mu$  are the Lagrange multipliers associated to, respectively, the resource constraint and the incentive compatibility constraint. Solving the Lagrangian and rearranging FOCs, we find, for the mimicker (type-1 individual), the usual result of no distortion at the top:

$$\begin{aligned} \frac{u'(d_1) - \gamma_1 \frac{var_{d_1}(d_1, m_1)}{\pi(m_1)}}{u'(c_1)} &= 1 \\ \frac{\pi'(m_1)u(d_1) - \gamma_1 var_{m_1}(d_1, m_1)}{u'(c_1)} &= 1 + \pi'(m_1) d_1 \end{aligned}$$

so that, comparing these equations with (9) and (10), second-best optimal taxes level are

$$\begin{aligned} t_1 &= 0 \\ \theta_1 &= \pi'(m_1) d_1 \end{aligned}$$

As in the first-best, savings are not taxed, while health spending are taxed in such a way as to correct for the Becker-Philipson effect. We also find that the following trade-offs

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<sup>25</sup>Note that one cannot exclude double copying. For simplicity, we concentrate on one directional mimicking behaviour.

for type-2 agents

$$\frac{u'(d_2)}{u'(c_2)} = 1 - \frac{\mu}{\lambda n_2} \frac{\gamma_1 \text{var}_d(d_2, m_2)}{\pi(m_2)}$$

$$\frac{\pi'(m_2)u(d_2)}{u'(c_2)} = [1 + \pi'(m_2) d_2] - \frac{\mu}{\lambda n_2} \gamma_1 \text{var}_m(d_2, m_2)$$

and taxes have now the following forms:

$$t_2 = -\frac{\mu}{\lambda n_2} \frac{\gamma_1 \text{var}_d(d_2, m_2)}{\pi(m_2)} < 0$$

$$\theta_2 = \pi'(m_2) d_2 - \frac{\mu}{\lambda n_2} \gamma_1 \text{var}_m(d_2, m_2) > 0$$

Thus, in the second-best, it is optimal to subsidize savings and to tax health expenditures for the type-2 agent. The rationale behind  $t_2 < 0$  is that in a second-best, it is optimal to encourage second-period consumption for the type-2 agent as a way to discourage a type-1 individual from mimicking a type-2 (by doing so, he would obtain too high a level of second period consumption). Regarding the tax on health spending, the incentive compatibility constraint tends to reinforce the internalisation of the Becker-Philipson effect. This additional constraint prevents the individual 1 from mimicking individual 2 as individual 1 would not invest enough in health if he was pretending to be of type 2.

Our findings are summarized in the proposition below:

**Proposition 5** *Consider two types of individuals with sensitivity to the variance in welfare such that  $\gamma_1 > 0$  and  $\gamma_2 = 0$ . Under asymmetric information,*

- *The “no distortion at the top” result holds.*
- *A subsidy on savings is optimal for type-2 individuals and is equal to  $-\mu\gamma_1 \text{var}_d(d_2, m_2) / \lambda n_2 \pi(m_2)$ .*
- *A tax on health expenditures for type-2 individuals is optimal and equal to  $\pi'(m_2) d_2 - \mu\gamma_1 \text{var}_m(d_2, m_2) / \lambda n_2$ .*

## 4.2 Case 2: type 2 mimicking type 1

We now study the case where type-2 agents mimic type-1 agents. The Lagrangian of this problem has the following expression

$$\begin{aligned} \mathcal{L} = & n_1 [u(c_1) + \pi(m_1)u(d_1) - \gamma_1 \text{var}(d_1, m_1)] + n_2 [u(c_2) + \pi(m_2)u(d_2)] \\ & + \lambda \left[ w - \sum_{i=1,2} n_i (c_i + m_i + \pi(m_i) d_i) \right] \\ & + \mu [u(c_2) + \pi(m_2)u(d_2) - u(c_1) - \pi(m_1)u(d_1)] \end{aligned}$$

Note that the incentive compatibility constraint has here a simple expected utility form, as  $\gamma_2 = 0$ .

Proceeding as before, we find the trade-offs between two period consumptions and between consumption and health spending for the mimicker (here individual 2):

$$\begin{aligned} \frac{u'(c_2)}{u'(d_2)} &= 1 \\ \frac{\pi'(m_2)u(d_2)}{u'(c_2)} &= [1 + \pi'(m_2) d_2] \end{aligned}$$

The usual result of no distortion at the bottom holds; for the individual with  $\gamma_2 = 0$ , the trade-offs between two-period consumption and between consumption and health expenditures are equivalent to the first-best ones. Thus, the tax on savings is zero in this case ( $t_2 = 0$ ) and health expenditures are taxed in the same way as in the first-best, i.e.  $\theta_2 = \pi'(m_2) d_2$ .

Rearranging first order conditions, individual-1 trade-offs are now distorted:

$$\begin{aligned} \frac{u'(d_1) - \gamma_1 \text{var}_{d_1}(d_1, m_1) / \pi(m_1)}{u'(c_1)} &= 1 + \frac{\mu}{\lambda n_1} \gamma_1 \frac{\text{var}_d(d_1, m_1)}{\pi(m_1)} \\ \frac{\pi'(m_1)u(d_1) - \gamma_1 \text{var}_{m_1}(d_1, m_1)}{u'(c_1)} &= [1 + \pi'(m_1) d_1] + \frac{\mu}{\lambda n_1} \gamma_1 \text{var}_m(d_1, m_1) \end{aligned}$$

Comparing them with (9) and (10) of the decentralized problem, we find that the second-best optimum could be decentralized by implementing taxes on savings and on health expenditures equal to

$$\begin{aligned} t_1 &= \frac{\mu}{\lambda n_1} \gamma_1 \frac{\text{var}_d(d_1, m_1)}{\pi(m_1)} > 0 \\ \theta_1 &= \pi'(m_1) d_1 + \frac{\mu}{\lambda n_1} \gamma_1 \text{var}_m(d_1, m_1) \geq 0 \end{aligned}$$

Let now study the level of these taxes. We find that type-1 agents should face a tax on savings. The intuition behind this result is related to the incentive constraint: since agents of type-2 always prefer higher levels of second-period consumption, it is optimal to tax savings of the type-1 agents in order to prevent the former from pretending to be the latter.

However, the sign of  $\theta_1$  is uncertain and might be positive or negative depending on the size of two countervailing effects. On the one hand,  $\pi'(m_1)d_1 > 0$  corresponds to the Becker-Philipson effect; as we already mentioned in the first-best, imposing a tax on health expenditures is a way to limit individuals' investment in health and thus its negative impact on their budget set. On the other hand,  $\mu\gamma_1\text{var}_m(d_1, m_1)/\lambda n_1 < 0$  and is related to the incentive effect; in order to prevent mimicking behavior of type 2, the social planner would like to subsidize health expenditures of type 1. indeed, a type-2 agent mimicking a type-1 agent would obtain a too high level of health spending. Note also that if individuals could perfectly see the impact of their health investment on their budget set, the Becker-Philipson effect would be absent and in the second-best, subsidization of health expenditures would be optimal. Thus, the overall effect on the sign of  $\theta_1$  is ambiguous and depends on the magnitude of both the Becker-Philipson and the incentive effects.

Our findings are summarized in the proposition below:

**Proposition 6** *Consider two types of individuals with sensitivity to the variance in welfare such that  $\gamma_1 > 0$  and  $\gamma_2 = 0$ . Under asymmetric information,*

- *The “no distortion at the bottom” result holds.*
- *A tax on savings is optimal for type-1 individuals and is equal to  $\mu\gamma_1\text{var}_d(d_1, m_1)/\lambda n_1\pi(m_1)$ .*
- *A positive or negative tax on health expenditures for type-1 individuals is optimal and equal to  $\pi'(m_1)d_1 + \mu\gamma_1\text{var}_m(d_1, m_1)/\lambda n_1$ .*

## 5 A numerical illustration

Let us now illustrate our results by some numerical simulations. For that purpose, we shall, for simplicity, remain in the case where  $\gamma_1 > 0$  and  $\gamma_2 = 0$ .

Throughout this section, it is assumed that the temporal utility function  $u(c)$  takes a simple constant-elasticity form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + \alpha$$

where  $\sigma$  and  $\alpha$  are preference parameters.<sup>26</sup> As this is well-known, the intercept parameter  $\alpha$  captures the relative importance, in welfare terms, of the mere survival in comparison with utility from material consumption.<sup>27</sup>

The survival function  $\pi(m)$  takes the following simple form:

$$\pi(m) = \frac{1/2 + \beta m}{1 + \beta m}$$

where  $\beta > 0$ . Note that this functional form satisfies the assumptions made in the model:  $\pi'(m) > 0$ ,  $\pi''(m) < 0$  and  $\pi(m) \geq 1/2 \forall m$ .

We shall use those following benchmark values for key parameters.<sup>28</sup>

parameters	$\alpha$	$\sigma$	$\beta$	$w$
values	10	0.83	1	10

Table 1: Parameter values

In order to illustrate the sensitivity of our theoretical results to the heterogeneity of agents's attitudes towards risk, Table 2 presents the laissez-faire, the first-best and second-best allocations under various levels of preference parameter  $\gamma_1$ , while  $\gamma_2 = 0$ .

Regarding the laissez-faire, let us first note that, as  $\gamma_1$  rises, first and second-period consumptions for type-1 agents fall, and health spending  $m_1$  grow. This is in conformity

<sup>26</sup>That functional form is also used by Becker *et al* (2005). Note that  $\sigma$  can no longer be interpreted here as the coefficient of relative risk aversion with respect to  $c$ , because individual utility function takes a non-expected utility form under  $\gamma \neq 0$ .

<sup>27</sup>The parameter  $\alpha$  can be estimated, together with other preference parameters  $\sigma$  and  $\gamma_i$ , from empirical estimates of the 'neutral' consumption flow (making a person indifferent between a period of life with that consumption and death).

<sup>28</sup>Note that the preference parameter  $\alpha$  is fixed to a strictly positive level to guarantee the non-negativity of health expenditures.

with the scenario (i) of Proposition 2 above: as the degree of variance-sensitivity grows, agents reduce their second-period consumption and raise their health effort. Note also that, as expected, we have  $d_1 < d_2$ . In this example, variance-sensitive agents consume less in the first period, and spend more on health in comparison with EU maximizers.

At the first-best, first-period consumptions are equalized across all agents. Note that, as  $\gamma_1$  rises, it is socially optimal to raise  $m_1$  and to decrease  $m_2$ . This result was not guaranteed at the general level: as this was discussed in Section 3, two effects are here at work: as  $\gamma_1$  grows, the reduction of the variance of lifetime welfare of type-1 agents becomes more important for the social planner, but, given a decreasing  $d_1$ , the direct utility gain from investing in the survival of type-1 agents is lower. In the present case, the high intercept  $\alpha$  (fixed to 10) makes the first effect dominate the second one, so that a rise in the degree of variance sensitivity of type-1 agents leads to a larger health spending for those agents.

	$\gamma_1$	<b>0.01</b>	<b>0.1</b>	<b>0.5</b>
<b>Laissez-faire</b>	$c_1$	3.19	2.60	1.62
	$c_2$	3.27	3.27	3.27
	$d_1$	3.07	1.90	0.28
	$d_2$	3.27	3.27	3.27
	$m_1$	4.05	5.64	8.12
	$m_2$	3.80	3.80	3.80
<b>first-best</b>	$c_1$	1.46	1.29	1.03
	$c_2$	1.46	1.29	1.03
	$d_1$	1.38	0.78	0.006
	$d_2$	1.4	1.29	1.03
	$m_1$	2.42	3.59	5.32
	$m_2$	2.23	2.05	1.77
<b>Who mimicks who</b>		2 $\rightarrow$ 1	2 $\rightarrow$ 1	no one
<b>Decentralisation</b>	tax on savings ( $t_1$ )	0.0006	0.035	
<b>second-best</b>	tax on health ( $\theta_1$ )	0.0574	-0.018	

Table 2: Sensitivity to gamma 1

Whereas there can be, in theory, a mimicking behavior from type-1 to type-2 agents or from type-2 to type-1 agents, the numerical simulations carried out tend all to point to a copying behavior from the variance-insensitive agents to the variance-sensitive agents,



so as to benefit from higher health expenditures. Here again, that result is driven by the assumption of a high intercept of the temporal utility function (i.e.  $\alpha = 10$ ). That high intercept means that it is survival that matters above all, so that type-2 agents tend to copy type-1 agents in order to benefit from higher health spending at the social optimum.<sup>29</sup> Hence, in order to prevent EU maximizers from pretending to be variance-sensitive, the second-best policy involves a tax on savings of type-1 agents. Note, however, that the sign of the optimal second-best tax on health spending varies with  $\gamma_1$ : for  $\gamma_1 = 0.01$ , type-1 agents health spending should be taxed, whereas these should be subsidized under  $\gamma_1 = 0.1$ . That sign inversion is in line with our theoretical discussions in Section 4: under a low  $\gamma_1$ , the Becker-Philipson effect still dominates the incentive compatibility component within the optimal  $\theta_1$  formula, while the opposite holds for a larger  $\gamma_1$ .

Given the importance of the preference parameter  $\alpha$  for the issue at stake, it makes sense to explore the sensitivity of our conclusions to its level. That sensitivity analysis is carried out in Table 3, which shows the laissez-faire, the first-best and second-best under  $\alpha$  varying between 10 and -3.<sup>30</sup> Not surprisingly, the laissez-faire level of health expenditures tends to fall monotonically as  $\alpha$  falls: when survival *per se* becomes less important, agents tend to invest less in their health.

In terms of public policy, we can notice that, as  $\alpha$  falls, there is a fall in optimal health spending, and a rise in first-period and second-period consumptions for all agents. As far as the first-best is concerned, it should be stressed that, as  $\alpha$  falls, the gap between  $m_1$  and  $m_2$  falls, as, for a given  $\gamma_1$ , the gain, in welfare terms, from spending more in the survival of type-1 agents relative to type-2 agents decreases.<sup>31</sup> Regarding the second-best, it is crucial to notice that, whereas there is a tendency of EU maximizers to copy type-1 agents when  $\alpha$  is large, the opposite holds when  $\alpha$  becomes strongly

<sup>29</sup>However, for  $\gamma_1 = 0.5$ , type-2 agents do no longer want to copy type-1 agents, as second-period consumption for type-1 agents is far too low for type-2 agents.

<sup>30</sup> $\gamma_1$  is here fixed to 0.1.

<sup>31</sup>Intuition: the first effect (reducing welfare variance of type-1 agents) tends, as  $\alpha$  falls, to become less and less sizeable than the second effect (related to the higher direct utility from the survival of type-2 agents), explaining that trend.

negative. The intuition for this is merely that, under a low  $\alpha$ , survival does no longer matter *per se*, so that type-2 agents do no longer want the high health spending of type-1 agents. On the contrary, type-1 agents are tempted to copy type-2 agents to benefit from their larger second-period consumption, explaining the inverse copying behavior. As a consequence, while the second-best policy involves the taxation of type-1 agents' savings and the subsidization of type-1's health spending when  $\alpha$  is large, we observe the opposite for  $\alpha$  low.

	$\alpha$	<b>10</b>	<b>5</b>	<b>0</b>	<b>-1</b>	<b>-3</b>
<b>Laissez-faire</b>	$c_1$	2.60	3.21	3.96	4.14	4.54
	$c_2$	3.27	3.60	4.07	4.19	4.48
	$d_1$	1.90	2.47	3.25	3.45	3.90
	$d_2$	3.27	3.60	4.07	4.19	4.48
	$m_1$	5.64	4.55	3.18	2.86	2.18
	$m_2$	3.80	3.22	2.46	2.27	1.82
<b>first-best</b>	$c_1$	1.29	1.61	2.09	2.21	2.55
	$c_2$	1.29	1.61	2.09	2.21	2.55
	$d_1$	0.78	1.07	1.52	1.64	1.98
	$d_2$	1.29	1.61	2.09	2.21	2.55
	$m_1$	3.59	2.76	1.71	1.47	0.90
	$m_2$	2.05	1.77	1.26	1.11	0.72
<b>Who mimics who</b>		2 $\rightarrow$ 1	2 $\rightarrow$ 1	no one	no one	1 $\rightarrow$ 2
<b>Decentralisation</b>	tax on savings ( $t_i$ )	0.035	0.01			-0.0018
<b>second-best</b>	tax on health ( $\theta_i$ )	-0.018	0.03			0.434

Table 3: Sensitivity to the intercept value

For completeness of analysis, Table 4 shows also the laissez-faire, the first-best and second-best optima for different values of the preference parameter  $\sigma$ .<sup>32</sup> Here again, it appears that the most plausible case is the one where variance-insensitive agents are tempted to copy variance-sensitive agents in order to benefit from higher health spending.<sup>33</sup> Hence, it does not come as a surprise that the optimal second-best policy involves the taxation of type-1 agents' savings and a subsidy on type-1 health spending, which turns into a tax as  $\sigma$  falls.<sup>34</sup>

<sup>32</sup> $\gamma_1$  is here fixed to 0.1 and  $\alpha$  equals 10.

<sup>33</sup>The reason for this is the high intercept  $\alpha = 10$ .

<sup>34</sup>This sign inversion is due to the change in the size of the Becker-Philipson effect with respect to the

	$\sigma$	<b>0.83</b>	<b>0.75</b>	<b>0.6</b>	<b>0.5</b>	<b>0.3</b>
<b>Laissez-faire</b>	$c_1$	2.60	2.95	3.47	3.84	4.93
	$c_2$	3.27	3.48	3.75	3.91	3.91
	$d_1$	1.90	2.11	2.27	2.28	1.91
	$d_2$	3.27	3.48	3.75	3.91	3.91
	$m_1$	5.64	5.11	4.46	4.11	3.38
	$m_2$	3.80	3.44	2.97	2.71	2.71
<b>first-best</b>	$c_1$	1.29	1.43	1.61	1.72	1.98
	$c_2$	1.29	1.43	1.61	1.72	1.98
	$d_1$	0.78	0.85	0.85	0.81	0.55
	$d_2$	1.29	1.43	1.61	1.72	1.98
	$m_1$	3.59	3.26	2.92	2.74	2.41
	$m_2$	2.05	1.95	1.80	1.72	1.56
<b>Who mimics who</b>		2 $\rightarrow$ 1	2 $\rightarrow$ 1	2 $\rightarrow$ 1	no one	no one
<b>Decentralisation</b>	tax on savings ( $t_1$ )	0.035	0.018	0.003		
<b>second-best</b>	tax on health ( $\theta_1$ )	-0.018	0.003	0.024		

Table 4: Sensitivity to sigma

Finally, Table 5 explores the sensitivity of our conclusions to the shape of the survival function  $\pi(m)$ . For that purpose, we make the parameter  $\beta$  vary between 0.5 and 2. As  $\beta$  grows, the productivity of health spending grows, implying here a fall of health spending under the laissez-faire for all agents (but the fall is stronger for variance-sensitive agents), as well as a rise in first- and second-period consumptions. In the first-best, the rise in  $\beta$  implies also a rise in first- and second-period consumptions, as well as a fall of health spending. For each of the values of  $\beta$  under study, EU-maximizers are here tempted to copy type-1 agents, because of the importance of survival *per se*. Hence, the second-best policy involves a tax on type-1 agents' savings, as well as a subsidy on their health spending (the Becker-Philipson component being here dominated by the incentive compatibility component of the optimal tax on health).

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incentive compatibility constraint. As  $\sigma$  falls, the degree of concavity of temporal welfare in consumption also falls, which reinforces the strength of the Becker-Philipson effect.

	$\beta$	<b>0.5</b>	<b>1</b>	<b>1.5</b>	<b>2</b>
<b>Laissez-faire</b>	$c_1$	2.28	2.6	2.84	3.01
	$c_2$	2.98	3.27	3.47	3.61
	$d_1$	1.39	1.90	2.22	2.46
	$d_2$	2.98	3.27	3.47	3.61
	$m_1$	6.49	5.64	5.07	4.65
	$m_2$	4.50	3.80	3.35	3.04
<b>first-best</b>	$c_1$	1.22	1.29	1.38	1.45
	$c_2$	1.22	1.29	1.38	1.45
	$d_1$	0.52	0.78	0.95	1.07
	$d_2$	1.22	1.29	1.38	1.45
	$m_1$	3.98	3.59	3.28	3.05
	$m_2$	2.21	2.05	1.90	1.77
<b>Who mimics who:</b>		2 $\rightarrow$ 1	2 $\rightarrow$ 1	2 $\rightarrow$ 1	2 $\rightarrow$ 1
<b>Decentralisation</b>	tax on savings ( $t_1$ )	0.060	0.035	0.023	0.017
<b>second-best</b>	tax on health ( $\theta_1$ )	-0.013	-0.018	-0.015	-0.012

Table 5: Sensitivity to form of the survival

In sum, this numerical application confirms the findings of previous sections: the form of the optimal tax and transfer policy depends on the degree of variance-sensitivity of agents, and on the shapes of utility and survival functions. Whereas the case where EU maximizers are tempted to copy variance-sensitive agents to benefit from higher health spending seems more plausible, whether health expenditures of the mimickee should be taxed or subsidized depends on the precise parametrization of the model.

## 6 Conclusions

This paper aims at studying the optimal taxation policy in an economy where agents can influence their longevity through health efforts but who differ in their attitude towards lotteries of life. For that purpose, we set up a two-period model in which the agents influence their survival probability by means of first-period health spending. Moreover, the heterogeneity of agents is captured by assuming that preferences on lotteries of life can be represented by various sensitivities to the variance of lifetime utilities.

It is shown that, in the laissez-faire, a higher sensitivity to the dispersion of psychological value implies that an agent spends more on health (at the cost of second-period

consumption), or, on the contrary, spend less on health (to favour first-period consumption). The choice between those two ways to protect oneself against a high volatility of lifetime welfare depends on the specific functional forms for  $u(c_\gamma)$  and  $\pi(m_\gamma)$ .

At the social optimum, first-period consumptions are equalized across agents, whereas agents with a higher sensitivity to the variance should have a lower second-period consumption. It is not obvious to see whether more variance-sensitive agents should benefit from higher or lower health expenditures, as, from a social point of view, the welfare gain from reducing the variance of their lifetime welfare is to be compared with the lower expected welfare associated with the survival of those agents (given their lower second-period consumption). The social optimum can be decentralized by means of group-specific taxes on health spending (to internalize the Becker-Philipson effect) and by adequate lump sum transfers.

Under asymmetric information, it is not obvious to see which form the second-best policy should take, as the mimicking may occur from agents with a low-variance-sensitivity to agents with a high-variance-sensitivity or the opposite. In the former case, the second-best policy requires a tax on the mimickers' savings and a tax or a subsidy on their health expenditures, as subsidizing health expenditures is a way to guarantee incentive-compatibility in that case. In the opposite case, the second-best policy requires a subsidy on the mimickers' savings, and a tax on their health spending. Whether it is the former or the latter case that prevails depends on the intercept of the temporal utility function. Numerical simulations emphasized also some other determinants of the second-best policy, including parameters of the survival functions.

To conclude, it should be stressed that the present study, by focusing exclusively on one source of heterogeneity across agents (their sensitivity to variance of utilities) only covers *one* aspect of the design of the optimal taxation policy under endogenous (differentiated) longevity. Undoubtedly, other sources of heterogeneity exist, regarding, for instance, the genetic background, the degree of rationality/myopia, the impatience or the disutility of efforts. Hence, one could hardly hope to provide a complete answer to that problem without considering what the optimal policy becomes when those various

sources of heterogeneity coexist. To answer that question, a crucial point will concern how those different characteristics are correlated across individuals types. Thus, in the light of the difficulties faced in the present study where agents differed in only one aspect, one could hardly overestimate the problems raised by a more complete study of optimal taxation in an economy where agents differ in several characteristics influencing longevity.

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# Appendix

## A laissez-faire: proof of Proposition 2

Replacing for the expressions of  $var_d(d_i, m_i)$  and  $var_m(d_i, m_i)$ , expressions (3) and (4) become, for an individual of type  $i \in \{1, 2\}$ :

$$\begin{aligned} u'(c_i) &= u'(d_i)\pi(m_i) [1 - 2\gamma_i u(d_i)\pi(m_i)(1 - \pi(m_i))] \\ u'(c_i) &= \pi'(m_i)u(d_i) [1 - \gamma_i u(d_i) (1 - 2\pi(m_i))] \end{aligned}$$

Suppose now that  $\gamma_1 > \gamma_2$ . If  $c_1 = c_2 = \bar{c}$ , we have:

$$\begin{aligned} u'(\bar{c}) &= u'(d_i)\pi(m_i) [1 - 2\gamma_i u(d_i)\pi(m_i)(1 - \pi(m_i))] \\ u'(\bar{c}) &= \pi'(m_i)u(d_i) [1 + \gamma_i u(d_i) (2\pi(m_i) - 1)] \end{aligned}$$

The first condition excludes the cases where (1)  $c_1 = c_2$ ,  $m_1 < m_2$  and  $d_1 > d_2$ , (2)  $c_1 = c_2$ ,  $m_1 \leq m_2$  and  $d_1 > d_2$ , (3)  $c_1 = c_2$ ,  $m_1 < m_2$  and  $d_1 \geq d_2$ . Moreover, the budget constraint excludes the cases where (1)  $c_1 = c_2$ ,  $m_1 \geq m_2$  and  $d_1 > d_2$  and (2)  $c_1 = c_2$ ,  $m_1 > m_2$  and  $d_1 \geq d_2$ . Hence, it must be the case that  $m_1 \geq m_2$  and  $d_1 < d_2$  or  $m_1 > m_2$  and  $d_1 \leq d_2$ .

The part (ii) can be proven by similar reasoning.

## B first-best

We rewrite (6) and (7) for each type under the assumption that  $\gamma_1 > 0$  and  $\gamma_2 = 0$ :

$$\begin{aligned} u'(d_1) [1 - 2\gamma_1 u(d_1) (1 - \pi(m_1))] &= \lambda \\ u'(d_2) &= \lambda \\ \pi'(m_1)u(d_1) \left[ 1 - \gamma_1 u(d_1) (1 - 2\pi(m_1)) - \lambda \frac{d_1}{u(d_1)} \right] &= \lambda \\ \pi'(m_2)u(d_2) \left[ 1 - \lambda \frac{d_2}{u(d_2)} \right] &= \lambda \end{aligned}$$

Combining first two equations, one finds that  $d_1 < d_2$ . Using two last equations we find that  $m_1 \leq m_2$ . In the specific case where  $\gamma_1$  is very high, such that

$$1 - \gamma_1 u(d_1) (1 - 2\pi(m_1)) - \lambda \frac{d_1}{u(d_1)} > 1 - \lambda \frac{d_2}{u(d_2)}$$

for any level of  $d_i$ , one has that  $m_1 > m_2$ .