CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 273

Communicating Vertical Hierarchies: the Adverse Selection Case

Salvatore Piccolo

February 2011

University of Naples Federico II | University of Salerno | Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unisa.it

CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 273

Communicating Vertical Hierarchies: the Adverse Selection Case

Salvatore Piccolo*

Abstract

I study the rationale for information sharing in a model where two principals, which exert production externalities one on another, endogenously decide whether to exchange information about their exclusive agents. I show that one novel effect shapes communication decisions when agents are privately informed about production costs. This effect is absent under complete information and it turns out to be of first-order magnitude relative to those emerging in such benchmark. Roughly, what matters is how sharing information impacts contracting relationships within opponent organizations, and therefore its effect on equilibrium outputs. Information exchange induces strategies to be correlated via the distortions channel. Because of those distortions, the equilibrium value of communication depends on the interplay between the nature of upstream externalities and the sign of cost correlation. When upstream externalities and cost correlation have the same sign, there exists a unique symmetric equilibrium with no communication. By contrast, when upstream externalities and cost correlation have opposite signs there exists a unique symmetric equilibrium where both principals share information. I also show that, unlike in previous models, under asymmetric information principals might run into a prisoner dilemma when there is no communication at equilibrium. Information sharing is also shown to have an unambiguous negative effect on rents. Moreover, there exists a system of transfers such that the equilibrium outcome obtained when both principals share information is collusion-proof.

Keywords: Adverse selection, communication, information sharing, vertical hierarchies.

Acknowledgements: I thank Piero Gottardi, David Martimort, Alessandro Pavan and Francesca Stroffolini for useful discussions. I also thank seminar participants at University of Verona. Usual disclaimers apply.

* University of Naples Federico II and CSEF. Corresponding address: Department of Economics, Via Cintia Monte S. Angelo, Università di Napoli Federico II, I-80126 Napoli, Italy. E-mail: salvapiccolo@gmail.com

Table of contents

1 Introduction

Understanding the reasons why people share private information is a long-standing research topic in economics. Information sharing (IS) agreements are widespread in real life. Banks and financial intermediaries usually exchange information about borrowers; sellers often share with competitors their knowledge about demand and cost conditions; retailers commonly report information regarding the downstream market to their suppliers; large corporations, as well as many other complex organizations, regularly disclose their management performance to outside parties.

Stemming from Novshek and Sonnenschein (1982), Clarke (1983), and Vives (1984), numerous contributions have shown that IS agreements can either emerge for efficiency reasons, or to dampen competition in oligopolistic markets.[1] In the banking literature, lenders exchange information about borrowers to better screen investment projects (adverse selection) or to avoid the danger of opportunistic behavior by funded entrepreneurs (moral hazard) — see, e.g., Pagano and Jappelli (1993) and subsequent models.[2] The role of experts, which acquire and disclose information to trading counterparts, has been studied in the intermediation literature — see, e.g., Lizzeri (1999) and Gromb and Martimort (2007). The related work on consumers' privacy considered, instead, environments where sellers can use information on individual purchasing history to engage in product customization and price discrimination — see, e.g., Acquisti and Varian (2005), Dodds (2002), and Taylor (2004). More recently, the role of strategic communication has also been studied in the 'networks' literature, which analyzes the reasons why people in the same network exchange private information — see, e.g., Calvó et al. (2009) and Hagenbach and Koessler (2010).

This literature offers a detailed picture of the reasons for the emergence of IS agreements. But, all these papers model *communicators* as *black-boxes* and are thus silent on the interplay between information exchange and agency conflicts *within* and, perhaps more importantly, *across* organizations. One recent exception is Calzolari and Pavan (2006) who study the costs and benefits of information transmission in a sequential common agency set-up.[3] The key difference between the mechanism design approach taken in Calzolari and Pavan (2006) and the earlier IS literature is that they consider situations where principals learn through costly contracting and then share the elicited information with rivals — i.e., games in which players acquire private information via the contracting interaction with common parties, and create new private information by taking decisions that affect both rivals and contractual counterparts.

In these games, IS brings out novel effects that are intimately related to the way the information disclosed by one player affects the contractual relationships between other players in the game, and therefore equilibrium choices. Calzolari and Pavan (2006), however, mainly focus on non-exclusive contracting, and are silent on the effects of communication when deals are exclu-

[1] See, e.g., Gal-Or (1986), Shapiro (1986), Raith (1996), Vives and Khun (2001), among others.

[2] See, e.g., Padilla and Pagano (1997)-(2000), Jappelli and Pagano (2000), Manove et al. (2001), Carlin and Rob (2009), Bennardo et al. (2010) among many others.

[3] The same idea has been developed under moral hazard by Bennardo et al. (2010) and Maier and Ottaviani (2009).

sive, which is the key feature of my model. Exclusivity clauses are common in many real markets, and are therefore worth to analyze — see, e.g., Caillaud et al. (1995). Several employment relationships are, by their own nature, exclusive (e.g., because of labor natural indivisibility); supply and franchising contracts in the manufacturing industry are often exclusive (i.e., retailers distributing a given brand are prevented from dealing with competing brands); procurement, regulatory and, to some extent, also financial contracting feature forms of exclusivity. Moreover, there exists substantial evidence showing that IS agreements are widespread in industries where exclusive deals are rather frequent.[4] For instance, the growth of information intensive channels is often seen as a mean to facilitate the dissemination of information not only within a given organization, but also among competing ones — see, e.g., Stern et al. (1996).[5] What are the drivers of IS decisions in these contexts? How does the exchange of information interplay with rent extraction on the one hand, and with horizontal externalities across organizations on the other? What are the resulting effects of this interaction?

These issues are addressed in a model where players interact strategically not only within the same organization but also with members of different, and potentially competing organizations. There are two independent principals that exert production externalities one on another. Each principal delegates production to an exclusive agent, which is privately informed about his marginal cost of production (type). Principals do not observe costs and need to elicit such information through the design of incentive compatible contracts. Moreover, they can (credibly) commit to share the (private) reports made by their agents at the contracting stage. IS decisions are taken simultaneously and non-cooperatively at the outset of the game. I allow both for positive and negative cost correlation. But, to keep the analysis tractable, I assume that upstream externalities are small and abstract from the possibility of information manipulation on the part of active communicators.[6]

I show that one main effect shapes principals' communication decisions when agents own privileged information. This effect is absent under complete information, and it turns out to be of first-order magnitude relative to the effects that drive communication decisions in such benchmark. Roughly, what matters is how sharing information impacts contracting relationships within opponent organizations, and therefore its resulting effect on equilibrium outputs. When there is adverse selection within and across organizations, IS induces strategies to be correlated, but mainly via the distortions channel. And, because of those distortions, the equilibrium value of communication depends only on the interplay between the nature of upstream externalities,

[4] According to Briley et al. (1994), for instance, this seems to be the established praxis in business format franchising where the mandatory disclosure of franchising contracts required by the Federal Trade Commission since 1979 allows firms to have free access to some of their rivals' characteristics — e.g., costs.

[5] For instance, partners that invest in bundles of sophisticated information technology like telecommunication and satellite linkages, bar coding and electronic scanning systems, database management systems etc.

[6] While the non-manipulability assumption is strong, it is usually imposed in the IS literature and is not implausible for certain types of communication — e.g., communication which entails the exchange of information about certifiable contractual agreements.

which can be either positive or negative, and the sign of cost correlation.

When upstream externalities and cost correlation have the same sign — i.e., they are either both positive or both negative — there exists a unique equilibrium in dominant strategies with no communication. To see why, suppose that upstream externalities as well as cost correlation are negative. By revealing her agent cost, a principal induces the rival to distort more (resp. less) output in the state where the first principal faces a high- (resp. low-) cost agent. This is because costs are negatively correlated, and therefore higher distortions will be forced in those states that are (conditionally) less likely. But, with negative externalities, this reduces the first principal's (expected) profit because reaction functions are downward sloped — i.e., each principal gains from expanding (resp. reducing) own production when the rival is inefficient (resp. efficient). Next, suppose that both upstream externalities and cost correlation are both positive. By revealing her agent's cost, a principal induces the rival to distort more (resp. less) output in the state where the first principal faces a low- (resp. high-) cost agent (because costs are positively correlated). But, this is detrimental to the first principal because, with positive externalities, reaction functions are upward sloped — i.e., each principal benefits from increasing production as a response to an expansion of the rival's output.

By contrast, when upstream externalities and cost correlation have opposite signs there exists a unique symmetric equilibrium in dominant strategies where both principals share information. To see why, suppose first that upstream externalities are negative and that costs are positively correlated. By disclosing her agent's cost, a principal induces the rival to distort more (resp. less) output in the state where the first principal deals with a low- (resp. high-) cost agent (because costs are positively correlated). And this increases the first principal's (expected) profit because reaction functions are downward sloped. Next, suppose that upstream externalities are positive and costs are negatively correlated. By revealing her agent's cost, a principal induces her opponent to distort more (resp. less) output in the state where the first principal deals with a high- (resp. low-) cost agent (again because costs are negatively correlated). And, this is beneficial to the first principal because reaction functions are upward sloped.

I also show that, unlike the complete information analysis, where the equilibrium outcome is always efficient, principals may run into a *prisoners' dilemma* when eliciting truthful information is costly. More precisely, when there is no communication at equilibrium, expected profits are higher when both principals share information. The intuition for this result is straightforward. As long as types are correlated, communication creates an *informational externality* that reduces the expected rents each principal needs to give up in order to elicit truthful revelation of types. This is because, when agents' contracts are contingent on the rivals' types, cost correlation generates a *relative performance evaluations* effect that relaxes the rent-extraction efficiency trade-off by allowing principals to span distortions over a broader set of contingencies — see, e.g., Riordan and Sappington (1988). When upstream externalities are small, this effect outperforms the strategic effect due to correlation among distortions.

The analysis is then extended to allow for agents' implicit collusion. Indeed, a potential drawback of communication is that, when both principals exchange information, the expected utility of each agent is affected by his opponent's report because contractual offers depend on such contingency. Therefore, in principle, it could be possible that the equilibrium allocation characterized in the regime where both principals share information, and each agent tells the truth expecting the rival to do the same, might lead to a collusive equilibrium of the message game where efficient agents always lie to reap higher rents at the expense of principals. However, it turns out that, within my framework where production externalities only affect upstream profits, there exists a system of transfers such that the equilibrium obtained under IS regime is collusion-proof if there are no side-payments across agents.

Although I have developed the formal arguments in an abstract principal/agent framework, the scope of my conclusions is much broader and it seems relevant for many applications in economics and finance. The results derived throughout the paper apply basically to any vertical hierarchies model involving horizontal externalities among principals, be it procurement contracting, manufacturers/retailers deals, executive compensations, patent licensing, insurance or credit relationships, to name only a few.

The paper is organized as follows. The game is introduced in Section 2. Section 3 describes the equilibrium outcome under complete information. Section 4 introduces asymmetric information and characterizes the equilibrium outputs in all possible information sharing regimes — i.e., when there is no communication, when both principals share information and when only one principal reveals her private information. Section 5 presents the equilibrium characterization. Section 6 considers the possibility of (implicit) collusion on the agent's side. Section 7 concludes. All proof are in the Appendix.

2 The model

Players. The game involves two vertical hierarchies. There are two (female) principals, P_1 and P_2, and two (male) exclusive agents, A_1 and A_2. Agent A_i ($i = 1, 2$) produces output q_i in P_i's behalf. All players are risk neutral. Principal P_i's utility from production is

$$V^i(q_i, q_j, t_i) = S^i(q_i, q_j) - t_i \quad i = 1, 2,$$

where q_i (resp. q_j) is the output produced by A_i (resp. A_j), and t_i (resp. t_j) is the monetary transfer flowing from P_i to A_i (resp. from P_j to A_j). Agent A_i's utility is

$$U^i(t_i, q_i, \theta_i) = t_i - \theta_i q_i \quad i = 1, 2.$$

The parameter $\theta_i \in \Theta_i \equiv \Theta$ ($i = 1, 2$) measures hierarchy i's marginal cost of production and is

A_i's private information. Hence, it can be learned by P_i only through a revelation mechanism, hereafter denoted C_i, and by P_j and A_j only by way of an IS agreement.

Communication rules. At the outset of the game, principals *independently* and *simultaneously* decide whether to exchange the information acquired at the contracting stage. If principal P_i decides to disclose her private information about θ_i, principal P_j's contractual offer C_j to agent A_j can be conditioned on such contingency.

Announced disclosure policies cannot be renegotiated once principals learn their agents' costs — see, e.g., Vives (2001, Ch. 8) and Raith (1996) for a similar approach.[7]

Contracts. Contracts are secret: neither P_j nor A_j can observe C_i.[8] I use the Revelation Principle to characterize the equilibrium of the game — see, e.g., Laffont and Martimort (2002, Ch. 2) and Caillaud et al. (1995). Hence, P_i offers a direct revelation mechanism C_i to A_i, who then makes a private report (message) $m_i \in \Theta$ about his cost $\theta_i \in \Theta$.

When principal P_j does not share her private information about θ_j, principal P_i offers a mechanism

$$C_i \equiv \{t_i(m_i), q_i(m_i)\}_{m_i \in \Theta},$$

mapping A_i's report m_i into a monetary transfer $t_i(m_i)$ and an output $q_i(m_i)$.

When, instead, principal P_j discloses her private information about θ_j, contract C_i specifies an allocation contingent *also* on such information — i.e., on A_j's report to P_j. Therefore,

$$C_i \equiv \{t_i(m_i|m_j), q_i(m_i|m_j)\}_{(m_i,m_j) \in \Theta^2}.$$

The report made by A_i to P_i can be credibly shared with P_j, and then transmitted by P_j to A_j — i.e., there is no moral hazard issue on the principals' side (see Dequiedt and Martimort, 2010, for an analysis in this spirit). In other words, exchanged information is verifiable.

Timing. A two stage game is considered in which principals first choose simultaneously and independently their communication behavior; uncertainty realizes, contracts are offered and then communication, if any, takes place. The timing of the game (thereafter $\mathcal{G}$) is as follows:

- (**T=0**) Principals independently and simultaneously decide whether to share information.
- (**T=1**) Uncertainty realizes and agents privately observe their costs. All players learn the IS decisions made at **T=0.**
- (**T=2**) Contracts are offered.

[7] Ziv (1993) shows that, without commitment, there is no communication at equilibrium.

[8] In a regulatory environment, Iossa and Stroffolini (2010) argue that, by making procurement contracts public, regulators' might signal information about demand or costs to potential competitors of regulated firms. Secret contracts rule out this signaling issue in my framework.

- (**T=3**) Communication, if any, takes place.
- (**T=4**) Agents produce and payments are made.

Equilibrium concept. The equilibrium concept is Perfect Bayesian equilibrium, with the added *passive beliefs* refinement — i.e., when an agent is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the other agent (see, e.g., Caillaud et al., 1995, and Martimort, 1996).

Assumptions. The analysis is developed under the following assumptions:

A1 The type-space Θ is discrete, with $\Theta \equiv \{\underline{\theta}, \overline{\theta}\}$. The vector of random variables $\boldsymbol{\theta} = (\theta_1, \theta_2)$ is drawn from a joint cumulative distribution function with: $\Pr(\underline{\theta}, \underline{\theta}) = \nu^2 + \alpha$, $\Pr(\underline{\theta}, \overline{\theta}) = \Pr(\overline{\theta}, \underline{\theta}) = \nu(1-\nu) - \alpha$ and $\Pr(\overline{\theta}, \overline{\theta}) = (1-\nu)^2 + \alpha$. The marginal distribution entails: $\Pr(\underline{\theta}) = \nu$ and $\Pr(\overline{\theta}) = 1-\nu$. Posteriors are computed through the Bayes rule: $\Pr(\underline{\theta}|\underline{\theta}) = \nu + \frac{\alpha}{\nu}$, $\Pr(\underline{\theta}|\overline{\theta}) = \nu - \frac{\alpha}{1-\nu}$, $\Pr(\overline{\theta}|\underline{\theta}) = 1 - \nu - \frac{\alpha}{\nu}$ and $\Pr(\overline{\theta}|\overline{\theta}) = 1 - \nu + \frac{\alpha}{1-\nu}$.

The parameter α is equal to the correlation index between the two random variables θ_1 and θ_2— i.e., $\Pr(\underline{\theta}, \underline{\theta}) \Pr(\overline{\theta}, \overline{\theta}) - \Pr(\underline{\theta}, \overline{\theta})^2 = \alpha$. Hence, $\alpha > 0$ (resp. $<$) means positive (resp. negative) correlation between types.

A2 The surplus function $S^i(.)$ is symmetric — i.e., $S^i(x, y) = S^j(x, y) = S(x, y)$ for all (x, y) — and quadratic, with

$$S(q_i, q_j) = \kappa + \beta q_i - q_i^2 + \delta q_i q_j \quad i, j = 1, 2. \tag{1}$$

The quadratic set-up is the *workhorse* benchmark developed in the earlier IS literature — see, e.g., Raith (1996) and Vives (2001, Ch. 8). Under **A2**, $S^i(.)$ is concave in q_i so as to have single peaked profit functions. Moreover, $\beta > \overline{\theta} > \underline{\theta} > 0$ and $\Delta\theta \equiv \overline{\theta} - \underline{\theta}$ small to ensure positive outputs and rule out shut-down solutions in all cases. Finally, δ measures the extent of (strategic) complementarity ($\delta > 0$) or substitutability ($\delta < 0$) between outputs. To emphasize the contribution of the paper in the clearest possible way, throughout I assume that δ is small. Hence, whenever possible, expected profits will be computed through Taylor approximations.

A3 Non-negative probabilities:

(i) $\Pr(\underline{\theta}, \overline{\theta}) = \Pr(\overline{\theta}, \underline{\theta}) \geq 0 \Longleftrightarrow \nu(1-\nu) \geq \alpha$ if $\alpha \geq 0$;

(ii) $\min\{\Pr(\overline{\theta}, \overline{\theta}), \Pr(\underline{\theta}, \underline{\theta})\} \geq 0 \Longleftrightarrow \min\{(1-\nu), \nu\} \geq \sqrt{|\alpha|}$ if $\alpha < 0$.

To make the problem interesting for my purposes, I assume that agents must get a non-negative utility in each *contractible* state — i.e., A_i's utility must be non-negative in each state m_i if P_j does not disclose her private information about θ_j; and it must be non-negative in all states $\mathbf{m} = (m_i, m_j)$ when P_j discloses her information about θ_j.

A4 Limited liability on the agents' side — i.e.,

$$U(t_i, q_i, \theta_i) = t_i - q_i\theta_i \geq 0 \quad \forall (t_i, q_i, \theta_i) \in \Re^2 \times \Theta.$$

This hypothesis is standard in the screening literature, and plays an important role here: it makes it impossible for P_i to leave A_i with no surplus when P_j shares her private information about θ_j.[9]

Following Vives (1985) and Raith (1996), I will restrict attention to communication equilibria where principals follow the *all-or-none* sharing rule — i.e., disclosure policies are deterministic.

A5 Principals either fully commit to disclose their agents' costs, or they keep this information secret.

3 The complete information benchmark

Before studying asymmetric information, it is important to briefly describe the equilibria of game $\mathcal{G}$ under complete information. When costs are common knowledge within each hierarchy, agents are left with no rents irrespective of principals' communication decisions. Hence, it is as if the two hierarchies were vertically integrated. The structure of game $\mathcal{G}$ is then similar to Shapiro (1986), who analyzes a model where firms that compete à la Cournot exchange cost information.

In order to understand the key economic forces driving the (equilibrium) communication decisions under complete information, it is useful to state two properties of expected outputs and profits. Let s_i be the contingency upon which principal P_i is able to condition the contract offered to agent A_i given P_j's communication choice. Under **A5**, $s_i = \theta_i$ if P_j does not disclose her cost θ_j and $s_i = (\theta_1, \theta_2)$ if P_j commits to disclose it. Moreover, denote by $q_i^*(s_i)$ the output chosen by P_i in equilibrium.

Lemma 1 *Irrespective of P_j's communication choice, P_i's expected profit is*

$$V_i^* = \kappa + [\underbrace{\mathrm{E}_{s_i}(q_i^*(s_i)|\theta_i)}_{\text{Average of } q_i^*(s_i)}]^2 + \underbrace{\mathrm{E}_{s_i}(q_i^*(s_i) - \mathrm{E}_{s_i}(q_i^*(s_i)|\theta_i)|\theta_i)^2}_{\text{Variance of } q_i^*(s_i)}.$$

Expected output is the same irrespective of principals' communication decisions — i.e.,

$$q^* \equiv \mathrm{E}_{\theta_i}(q_i^*(\theta_i)) = \mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(q_i^*(\theta_i, \theta_j)|\theta_i) = \frac{\beta - \theta}{2-\delta} - \frac{1-\nu}{2-\delta}\Delta\theta.$$

[9] The full-surplus extraction result has been shown by Riordan and Sappington (1988) in a isolated principal-agent framework. In the auction literature, this result was developed by Cremer and McLean (1985). The same approach was then extended to a vertical hierarchies framework under no limited liability by Bertoletti and Poletti (1996).

In equilibrium, principals benefit from a larger scale of production and, because indirect profit functions are convex, they also enjoy more volatile outputs. Moreover, under **A2** (state contingent) outputs are linear in costs. Therefore, IS decisions do not affect expected output — see, e.g., Vives (2001, Ch. 8) among many others.

In sum, Lemma 1 implies that, when choosing her communication strategy, each principal simply picks the one that maximizes own equilibrium output volatility (given the opponent's communication behavior). The argument is as follows. By allowing P_j to learn θ_i, P_i can influence the distribution of P_j's equilibrium output, and therefore also her own output volatility. This is because, under assumption **A2**, reaction functions are linear. Hence, ceteris paribus, if $q_j^*(s_j)$ becomes more volatile, also the variance of $q_i^*(s_i)$ increases.

Proposition 1 *Suppose that δ is small. Under complete information game $\mathcal{G}$ has following equilibrium features:*

- *For $\delta = 0$ or $\Pr(\overline{\theta}, \underline{\theta}) = 0$, there are two symmetric, pay-off equivalent equilibria: one where both principals share information, and the other where they do not communicate.*
- *For $\delta \neq 0$, there exists a unique symmetric equilibrium in dominant strategies where both principals share information if $\alpha > 0$ or if $|\alpha| < \nu(1-\nu)$ when $\alpha < 0$. Otherwise, there exists a unique equilibrium in dominant strategies with no communication.*

To explain why, under complete information, principals communicate only when costs are positively or not too negatively correlated, suppose that P_j commits to disclose θ_j, and consider P_i's incentive to reveal θ_i. Sharing information has both direct and indirect effects on the equilibrium distribution of outputs. First, ceteris paribus, by allowing P_j to condition contract C_j on θ_i expands the set of contingencies upon which A_j's output can be conditioned. Hence, the volatility of output increases, everything else being kept equal: a *direct* effect of communication. Second, depending on the degree of cost correlation, revealing θ_i also affects the degree of correlation among equilibrium outputs: an *indirect* effect. If costs are positively correlated, when both principals share information there is a higher probability of being at the extreme values of the distribution of outputs — i.e., either in state $(\underline{\theta}, \underline{\theta})$ or $(\overline{\theta}, \overline{\theta})$. In those symmetric states, both firms produce the same output, which is either very large (when both are efficient) or very small (when they are both inefficient). Hence, in this case, communication also spurs volatility indirectly. If, instead, costs are negatively correlated, the most likely states are the intermediate ones — i.e., $(\overline{\theta}, \underline{\theta})$ and $(\underline{\theta}, \overline{\theta})$. In these asymmetric states A_j's equilibrium output when both principals share information is more concentrated around its mean. Therefore, in this case, communication has an indirect negative impact on volatility.

In sum, with positive or not too negative cost correlation, there exists an equilibrium with communication because the direct effect of IS outperforms the indirect one; which, instead, prevails when correlation is negative and large in absolute value. Of course, when there are no upstream externalities, P_i's pay-off is unaffected by P_j's output. Learning θ_j has, therefore, no value. The same is true in the limiting case where costs are perfectly correlated — i.e., $\Pr(\overline{\theta}, \underline{\theta}) = 0$ — where the output volatility is the same irrespective of principals' communication decisions.

Armed with the equilibrium characterization of game $\mathcal{G}$, one may wonder whether these outcomes are efficient — i.e., if the maximize principals' expected profit. The next corollary makes this point.

Corollary 1 *Suppose that δ is small. Under complete information, the equilibrium of game $\mathcal{G}$ is always efficient — i.e., expected profits are higher when both principals share information than when they do not communicate if $\alpha > 0$ or if $|\alpha| < \nu(1-\nu)$ when $\alpha < 0$. The converse obtains otherwise.*

Under complete information, the equilibrium of game $\mathcal{G}$ is always efficient: a prediction that is in line with the general analysis of IS in oligopoly provided by Raith (1996). The interpretation of this result rests again on the tension between the direct and indirect effects of communication just discussed above. However, it is worth noting that, whereas earlier IS models mainly allow for independent or positively correlated types, negative correlation here may change the result. For instance, while Shapiro (1986) argues that, with substitutes, IS unambiguously increases profits, Corollary 1 shows that this is not necessarily true if α is negative and large in absolute value.

4 Asymmetric information

Consider now asymmetric information. Before sharing information, principals must learn their own agents' costs through costly contracting — i.e., they must give up an information rent in order to screen types. And, the minimization of these rents leads to equilibrium outcomes that are distorted away from efficiency. These distortions will, of course, depend on the information disclosure regime, which will in turn affect the strategic interaction between principals and, therefore, the value of communication.

Since IS decisions are public, I will solve game $\mathcal{G}$ with a standard backward induction argument. Hence, the first step will be to characterize equilibrium contracts within each of the following subgames: the case of no communication — i.e., the subgame where principals do not share information; the case of bilateral information sharing — i.e., the subgame where both principals disclose their private information; and, finally, the case of unilateral information sharing — i.e., the subgame where only one principal reveals her private information.

4.1 No communication

Suppose that principals do not exchange information. For a separating equilibrium to exist, principal P_i must offer a contract C_i that satisfies the following incentive and participation constraints

$$U_i(\theta_i) = t_i(\theta_i) - \theta_i q_i(\theta_i) \geq 0 \quad \forall\, \theta_i \in \Theta,$$

$$U_i(\theta_i) \geq t_i(m_i) - \theta_i q_i(m_i) \quad \forall\ (\theta_i, m_i) \in \Theta^2.$$

As standard in the screening literature, only the incentive constraint of the efficient type and the participation constraint of the inefficient type matter — see, e.g., Laffont and Martimort (2002, Ch. 2). Hence,

$$U_i(\underline{\theta}) \geq U_i(\overline{\theta}) + \Delta\theta q_i(\overline{\theta}), \tag{2}$$

$$U_i(\overline{\theta}) \geq 0. \tag{3}$$

Let $q^e(\theta_j)$ be agent A_j's output in a (symmetric) separating equilibrium. Principal P_i solves the following mechanism design problem

$$\max_{\{q_i(\theta_i), U_i(\theta_i)\}_{\theta_i \in \Theta}} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(S(q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i)|\theta_i) - \mathrm{E}_{\theta_i}(U_i(\theta_i)), \text{ subject to (2)-(3).}$$

It is easy to verify that, at the optimum, both (2) and (3) bind. Hence, principal P_i's (relaxed) optimization problem is

$$\max_{\{q_i(\theta_i)\}_{\theta_i \in \Theta}} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(S(q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i)|\theta_i) - \nu\Delta\theta q_i(\overline{\theta}). \tag{4}$$

Because agents' costs are correlated, when maximizing her expected profit, P_i's posterior on θ_j depends on A_i's report. Unlike the complete information case, however, P_i must now grant a costly rent $\Delta\theta q_i(\overline{\theta})$ to A_i in order to elicit a truthful report.

Optimizing, the necessary and sufficient first-order conditions are[10]

$$\mathrm{E}_\theta(S_1(q^e(\underline{\theta}), q^e(\theta))|\underline{\theta}) = \underline{\theta}, \tag{5}$$

$$\mathrm{E}_\theta(S_1(q^e(\overline{\theta}), q^e(\theta))|\overline{\theta}) = \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta. \tag{6}$$

Low-cost types' output is chosen so as to equalize (expected) marginal revenues to marginal costs: the efficient rule in the Bayesian sense. High-cost types are, instead, forced to produce a downward distorted output for rent extraction reasons.

Proposition 2 *Suppose that principals do not communicate. The unique symmetric separating PBE of this*

[10]With a slight abuse of notation I will denote by $S_1(.)$ the partial derivative of $S(q_i, q_j)$ with respect to q_i.

subgame features outputs linear in θ and $\Delta\theta$, with

$$q^e(\underline{\theta}) = q^*(\underline{\theta}) - \frac{\delta\nu(\nu(1-\nu)-\alpha)}{(2-\delta)(2\nu(1-\nu)-\alpha\delta)}\Delta\theta \qquad q^e(\overline{\theta}) = q^*(\overline{\theta}) - \frac{2\nu-\delta\left(\nu^2+\alpha\right)}{(2-\delta)(2\nu(1-\nu)-\alpha\delta)}\Delta\theta.$$

Moreover, the following properties hold:

- $q^e(\underline{\theta}) \geq q^*(\underline{\theta})$ *(resp. $<$) if $\delta \leq 0$ (resp. $>$).*
- $q^e(\overline{\theta}) < q^*(\overline{\theta})$.
- $\Delta q^e = q^e(\underline{\theta}) - q^e(\overline{\theta}) > 0$ *for $\Delta\theta \neq 0$ and $\nu \neq 0$, with*

$$\Delta q^e = \frac{\nu\Delta\theta}{2\nu(1-\nu)-\alpha\delta}.$$

- *Expected output is downward distorted — i.e.,*

$$\mathrm{E}_{\theta_i}(q^e(\theta_i)) = \frac{\beta-\overline{\theta}}{2-\delta} < q^*.$$

Albeit being set with an efficient rule, the equilibrium output in the low-cost state still features some distortion. This is because when principals do not share information, they must form expectations about the opponent's output, which in a separating equilibrium is distorted at bottom.[11] When $\delta = 0$ the output in the low-cost state is the efficient one — i.e., the first-best level that would emerge in a single hierarchy model. When $\delta \neq 0$, instead, the distortion of A_j's output affects A_i's output in the low cost state through the upstream externalities: if goods are substitutes (resp. complements) a lower A_j's output in the high-cost state calls for a higher (resp. lower) A_i's output in the low-cost state. Finally, expected output is lower under asymmetric information than under complete information because, with privately informed agents, rent extraction requires a downward distortion in the high cost state.

The next result helps understanding the equilibrium relationship between δ and α:

Corollary 1 *sign*$\frac{\partial\Delta q^e}{\partial\alpha}$ =*sign*δ.

The effect of increased correlation on the (equilibrium) output spread Δq^e depends solely on the sign of δ. The argument is as follows. A higher α implies that when one agent's cost is high, his opponent's cost is more likely to be high as well. Hence, for $\delta < 0$, it is less profitable for a principal to distort the output of a high cost type — whereby increasing the difference Δq^e. This is because, with strategic substitutes, each principal gains from expanding output when her rival is expected to under-produce. Differently, for $\delta > 0$, a higher α leads to an increase in Δq^e because

[11] See also Cella and Etro (2010) on this *two-way* distortion result.

of strategic complementarities: principals benefit from coordinating distortions in such a way to jointly increase production when they both deal with low cost agents and cut it back when they both deal with an inefficient type.

4.2 Bilateral information sharing

Suppose now that both principals exchange information. Consider a pure strategy, symmetric separating equilibrium where agent A_i truthfully reports his type to principal P_i — i.e., $m_i = \theta_i$ for all $i = 1, 2$ — who then discloses this information to the pair $P_j - A_j$. Since contract C_i specifies an allocation contingent on the aggregate state $\boldsymbol{\theta} = (\theta_1, \theta_2)$ and agents are protected by limited liability, in addition to the ex ante participation constraints,

$$\mathrm{E}_{\theta_j}\left(U_i\left(\theta_i|\theta_j\right)|\theta_i\right) \geq 0 \quad \forall\, \theta_i \in \Theta, \tag{7}$$

the following *limited liability* constraints must hold for C_i to be accepted

$$U_i\left(\theta_i|\theta_j\right) = t_i\left(\theta_i|\theta_j\right) - \theta_i q_i\left(\theta_i|\theta_j\right) \geq 0 \quad \forall\, (\theta_i, \theta_j) \in \Theta^2.$$

Clearly, when this inequality is satisfied in all states, (7) becomes redundant.

As for the truth-telling condition, recall that reports are made before information is eventually exchanged. Hence, A_i does not know A_j's cost when making his report. This leads to the following Bayesian incentive compatibility constraint

$$\mathrm{E}_{\theta_j}\left(U_i\left(\theta_i|\theta_j\right)|\theta_i\right) \geq \mathrm{E}_{\theta_j}\left(t_i\left(m_i|\theta_j\right) - \theta_i q_i\left(m_i|\theta_j\right)|\theta_i\right) \quad \forall\ (m_i, \theta_i) \in \Theta^2. \tag{8}$$

Principal P_i's mechanism design problem can be then solved in the standard way. First, it is easy to show that the relevant limited liability constraints are those of the high-cost type — i.e.,

$$U_i(\overline{\theta}|\theta_j) \geq 0 \quad \forall\, \theta_j \in \Theta, \tag{9}$$

while the relevant incentive constraint is that of the low-cost type

$$\mathrm{E}_{\theta_j}(U_i(\underline{\theta}|\theta_j)|\underline{\theta}) \geq \mathrm{E}_{\theta_j}(t_i(\overline{\theta}|\theta_j) - \underline{\theta} q_i(\overline{\theta}|\theta_j)|\theta_i).$$

Using a standard change of variables, this inequality rewrites as

$$\mathrm{E}_{\theta_j}(U_i(\underline{\theta}|\theta_j)|\underline{\theta}) \geq \mathrm{E}_{\theta_j}(U_i(\overline{\theta}|\theta_j)|\underline{\theta}) + \Delta\theta \mathrm{E}_{\theta_j}(q_i(\overline{\theta}|\theta_j)|\underline{\theta}). \tag{10}$$

Let $q^e(\theta_i|\theta_j)$ denote the equilibrium output. Principal P_i then solves

$$\max_{\{q_i(\theta_i|\theta_j),U_i(\theta_i|\theta_j)\}_{(\theta_i,\theta_j)\in\Theta^2}} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(S\left(q_i\left(\theta_i|\theta_j\right), q^e\left(\theta_j|\theta_i\right)\right) - \theta_i q_i\left(\theta_i|\theta_j\right) - U_i\left(\theta_i|\theta_j\right)|\theta_i\right),$$

subject to (9)-(10).

At the optimum, the transfer $t_i(\overline{\theta}|\theta_j)$ is such that the high-cost type gets no rent irrespective of his opponent's cost — i.e., $t_i(\overline{\theta}|\theta_j) = \overline{\theta}q_i(\overline{\theta}|\theta_j)$ for all θ_j. Moreover, it is also easy to verify that the incentive constraint (10) is binding — i.e.,

$$\mathrm{E}_{\theta_j}(U_i(\underline{\theta}|\theta_j)|\underline{\theta}) = \Delta\theta\mathrm{E}_{\theta_j}(q_i(\overline{\theta}|\theta_j)|\underline{\theta}). \tag{11}$$

Substituting this constraint into P_i's objective function, the (relaxed) optimization is

$$\max_{\{q_i(\theta_i|\theta_j)\}_{(\theta_i,\theta_j)\in\Theta^2}} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(S\left(q_i\left(\theta_i|\theta_j\right), q^e\left(\theta_j|\theta_i\right)\right) - \theta_i q_i\left(\theta_i|\theta_j\right)|\theta_i\right) - \nu\Delta\theta\mathrm{E}_{\theta_j}(q_i(\overline{\theta}|\theta_j)|\underline{\theta}). \tag{12}$$

Even if under IS principals can condition contracts on the opponent's costs, full surplus extraction is impossible. This is because agents must get a non-negative utility in every contractible state, so they are still able to command some information rents from their private information.

The symmetric equilibrium output is determined by the following necessary and sufficient first-order conditions

$$S_1\left(q^e(\underline{\theta}|\theta), q^e(\theta|\underline{\theta})\right) = \underline{\theta} \quad \forall\, \theta \in \Theta, \tag{13}$$

$$S_1(q^e(\overline{\theta}|\theta), q^e(\theta|\overline{\theta})) = \overline{\theta} + \frac{\nu}{1-\nu}\frac{\Pr(\theta|\underline{\theta})}{\Pr(\theta|\overline{\theta})}\Delta\theta \quad \forall\, \theta \in \Theta. \tag{14}$$

Again, there is no distortion at the top and downward distortion at the bottom — i.e., low-cost types produce according to an efficient rule, while high-cost types' output is distorted for rent extraction reasons. However, equation (14) implies that, under IS, the distortion increases with $\frac{\Pr(\theta|\underline{\theta})}{\Pr(\theta|\overline{\theta})}$. This ratio is an index of informativeness that signal $\overline{\theta}$ provides on the state θ relative to signal $\underline{\theta}$. Essentially, principals require a higher distortion in the rival's cost state that is (conditionally) less likely. For instance, if state $\theta_i = \overline{\theta}$ signals to P_i a higher probability of state $\theta_j = \overline{\theta}$ relative to $\theta_j = \underline{\theta}$, then P_i prefers to distort more $q_i(\overline{\theta}|\underline{\theta})$ than $q_i(\overline{\theta}|\overline{\theta})$, everything else being equal. Formally, this can be seen by taking the difference $\phi(\nu,\alpha) \equiv \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\underline{\theta}|\overline{\theta})} - \frac{\Pr(\overline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\overline{\theta})}$, which yields

$$\phi(\nu,\alpha) = \frac{\alpha\Pr(\overline{\theta})}{\Pr(\underline{\theta}|\overline{\theta})\Pr(\overline{\theta}|\overline{\theta})\Pr(\underline{\theta})} \geq 0 \quad (\text{resp. } < 0) \quad \Longleftrightarrow \quad \alpha \geq 0\,(\text{resp. } < 0).$$

Hence, the distortion of a high-cost agent's output is larger (resp. lower) when his opponent has a high (resp. low) cost too if types are negatively (resp. positively) correlated. This is because, with positive correlation ($\alpha > 0$) a principal dealing with a high-cost type anticipates that the opponent's agent is likely to have a high cost too, and therefore requires a higher distortion in the less likely state where the opponent has a low cost — i.e., $\phi(\nu, \alpha) > 0$. For $\alpha < 0$, the same type of argument explains why the distortion is higher when the opponent is inefficient.

Let $q^*(\theta_i|\theta_j)$ be agent A_i's equilibrium output with bilateral information sharing under complete information (See Appendix).

Proposition 3 *Suppose that both principals share information. The unique symmetric PBE of this subgame entails outputs linear in θ and $\Delta\theta$, with*

$$q^e(\underline{\theta}|\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) \qquad q^e(\underline{\theta}|\overline{\theta}) = q^*(\underline{\theta}|\overline{\theta}) - \frac{\delta(\nu^2+\alpha)}{(4-\delta^2)\left(\nu\left(1-\nu\right)-\alpha\right)}\Delta\theta,$$

$$q^e(\overline{\theta}|\underline{\theta}) = q^*(\overline{\theta}|\underline{\theta}) - \frac{2\left(\nu^2+\alpha\right)}{(4-\delta^2)\left(\nu\left(1-\nu\right)-\alpha\right)}\Delta\theta \qquad q^e(\overline{\theta}|\overline{\theta}) = q^*(\overline{\theta}|\overline{\theta}) - \frac{\nu\left(1-\nu\right)-\alpha}{(2-\delta)((1-\nu)^2+\alpha)}\Delta\theta.$$

Moreover, the following properties hold:

- $q^e(\underline{\theta}|\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta})$.
- $q^e(\overline{\theta}|\theta) < q^*(\overline{\theta}|\theta)$ *for all* $\theta \in \Theta$.
- $q^e(\underline{\theta}|\overline{\theta}) \geq q^*(\underline{\theta}|\overline{\theta})$ *(resp.* $<$*) if and only if* $\delta \leq 0$ *(resp.* $>$*).*
- *Expected output is the same when both principals share information and when they do not communicate — i.e.,*

$$\mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(q^e(\theta_i,\theta_j)|\theta_i) = \frac{\beta-\overline{\theta}}{2-\delta} < q^*.$$

Under IS there is a strategic linkage between the distortion required by one principal and the opponent's (equilibrium) output profile — i.e., the output that P_i requires from A_i depends on A_j's cost, and therefore it is correlated with the distortion that P_j is expected to impose on A_j's equilibrium output. This explains why the sign of δ induces over- or under-production for A_i relative to the complete information benchmark when $\theta_j = \overline{\theta}$. Finally, the fact that expected outputs are the same when there is no communication and when both principals share information is again due to linearity of outputs with respect to costs. Again, the fact that expected output is the same with and without communication is due to the quadratic specification in **A2.**

4.3 Unilateral information sharing

Suppose now that only one principal, say P_i with no loss of generality, commits to disclose her private information. Accordingly, let $q_i^e(\theta_i)$ and $q_j^e(\theta_j|\theta_i)$ be the corresponding equilibrium outputs. By using the same techniques developed above, it is easy to show that principal P_i's optimization problem is

$$\max_{\{q_i(\theta_i)\}_{\theta_i\in\Theta}} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(S(q_i(\theta_i), q_j^e(\theta_j|\theta_i)|\theta_i) - \mathrm{E}_{\theta_i}(\theta_i q_i(\theta_i)) - \nu\Delta\theta q_i(\overline{\theta}),$$

while principal P_j solves

$$\max_{\{q_j(\theta_j|\theta_i)\}_{(\theta_i,\theta_j)\in\Theta^2}} \mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(S(q_j(\theta_j|\theta_i), q_i^e(\theta_i)) - \theta_j q_j(\theta_j|\theta_i)|\theta_j) - \nu\Delta\theta\mathrm{E}_{\theta_i}(q_j(\overline{\theta}|\theta_i)|\underline{\theta}).$$

Given her IS decision, P_i has to take into account the fact that P_j will be able to condition A_j's output on A_i's report. Differently, P_j must take into account the fact that her choice of not sharing information will force P_i to condition the menu C_i only on A_i's report.

Optimizing, the necessary and sufficient first-order conditions of P_i's program are

$$\mathrm{E}_{\theta_j}(S_1(q_i^e(\underline{\theta}), q_j^e(\theta_j|\underline{\theta}))|\underline{\theta}) = \underline{\theta}, \tag{15}$$

$$\mathrm{E}_{\theta_j}(S_1(q_i^e(\overline{\theta}), q_j^e(\theta_j|\overline{\theta})|\overline{\theta}) = \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta. \tag{16}$$

The necessary and sufficient first-order conditions of P_j's program are

$$S_1(q_j^e(\underline{\theta}|\theta_i), q_i^e(\theta_i)) = \underline{\theta} \quad \forall\, \theta_i \in \Theta,$$

$$S_1(q_j^e(\overline{\theta}|\theta_i), q_i^e(\theta_i)) = \overline{\theta} + \frac{\nu}{1-\nu}\frac{\Pr(\theta_i|\underline{\theta})}{\Pr(\theta_i|\overline{\theta})}\Delta\theta \quad \forall\, \theta_i \in \Theta.$$

Low-cost types produce according to an efficient rule, while high-cost types have their output distorted for rent-extraction reasons. Moreover, because P_j does not share information, P_i can condition her contractual offer only on A_i's report, while P_j is able to implement a flexible production plan by conditioning A_j's output on θ_i. This asymmetry conveys to P_j a competitive advantage relative to P_i insofar as it allows to require a higher distortion in the states of nature that are (conditionally) less likely.

Proposition 4 *Suppose that only principal P_i shares information. The unique symmetric PBE of this subgame entails outputs linear in θ and $\Delta\theta$, with,*

$$q_j^e(\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{\delta}{4-\delta^2}\Delta\theta \qquad q_j^e(\overline{\theta}) = q_j^e(\underline{\theta}) - \frac{2}{(1-\nu)\left(4-\delta^2\right)}\Delta\theta,$$

$$q_i^e(\underline{\theta}|\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{\delta^2}{2(4-\delta^2)}\Delta\theta \qquad q_i^e(\underline{\theta}|\overline{\theta}) = q_i^e(\underline{\theta}|\underline{\theta}) - \frac{\delta}{(1-\nu)(4-\delta^2)}\Delta\theta,$$

$$q_i^e(\overline{\theta}|\underline{\theta}) = q_i^e(\underline{\theta}|\underline{\theta}) - \frac{\nu}{2\left(\nu(1-\nu)-\alpha\right)}\Delta\theta \qquad q_i^e(\overline{\theta}|\overline{\theta}) = q_i^e(\underline{\theta}|\overline{\theta}) - \frac{1-\nu}{2\left(\alpha+(1-\nu)^2\right)}\Delta\theta.$$

Moreover, the following properties hold:

- *$q_j^e(\underline{\theta}) > q_j^e(\overline{\theta})$.*

- *$q_i^e(\underline{\theta}|\underline{\theta}) > q_i^e(\underline{\theta}|\overline{\theta})$ (resp. $\geq$) if and only if $\delta > 0$ (resp. $\leq$).*

- *$q_i^e(\underline{\theta}|\underline{\theta}) > q_i^e(\overline{\theta}|\underline{\theta})$ and $q_i^e(\underline{\theta}|\overline{\theta}) > q_i^e(\overline{\theta}|\overline{\theta})$.*

- *Expected output is the same for both hierarchies and it is equal to the expected output obtained when there is no communication and when both principals share information — i.e.,*

$$\mathrm{E}_{\theta_j}\left(q_j^e\left(\theta_j\right)\right) = \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q_i^e\left(\theta_i|\theta_j\right)|\theta_i\right) = \frac{\beta - \overline{\theta}}{2-\delta} < q^*.$$

The economic interpretation of this result rests on the same arguments developed above. Hence, it will be omitted.

5 Communication at equilibrium?

Building on the characterization developed above, I now turn to study the equilibrium communication outcome. Principals' profits when they choose to share information (I) or not to share it (N) are given by

$\mathcal{G}$:

P_1 \ P_2	I	N
I	V_I^e, V_I^e	$V_{I,N}^e$, $V_{N,I}^e$
N	$V_{N,I}^e$, $V_{I,N}^e$	V_N^e, V_N^e

Where, V_I^e and V_N^e are the (expected) profits that principals make when they both share information and when they do not communicate, respectively. Moreover, $V_{I,N}^e$ is the (expected) profit that a principal obtains when she commits to reveal her agent's cost when the opponent principal does not disclose her agent's cost. Similarly, $V_{N,I}^e$ is the (expected) profit that a principal obtains when she does not share information given that her opponent has committed to reveal her agent's cost.

An equilibrium where both principals share information exists if and only if the following condition holds

$$V_I^e \geq V_{N,I}^e. \tag{17}$$

By contrast, an equilibrium with no communication exists if and only if the following condition holds

$$V_N^e \geq V_{I,N}^e.$$

Note that the incentive to share information hinges on the effect that principals' communication decisions have on the rivals' behavior. This is because, each principal chooses her best communication behavior given the IS decision of the opponent. Therefore, there is one key effect that shapes the equilibrium communication decisions under asymmetric information. The argument is as follows. Since under IS principals tailor outputs to the opponent's type, distortions are correlated at equilibrium, and this might by profitable to the extent that it 'softens' upstream competition. Essentially, while outputs are independently distributed with no communication, under IS principals may whish to *synchronize* or *disharmonize* distortions for strategic reasons: a *correlated distortions* effect.

Proposition 5 *Suppose that δ is small enough. Under asymmetric information game $\mathcal{G}$ has the following equilibrium features:*

- *For $\delta = 0$, there are two symmetric, pay-off equivalent equilibria: one with IS and one without IS.*
- *For $\delta \neq 0$ and $\delta\alpha \leq 0$, there exists a unique symmetric equilibrium in dominant strategies where both principals share information.*
- *For $\delta\alpha > 0$, there is a unique symmetric equilibrium in dominant strategies without communication.*

The incentive of a principal to disclose her agent's cost depends only on the way this information affects the rival's equilibrium output via the correlated distortions channel. If goods are substitutes (resp. complements), and sharing information induces rivals to expand (resp. cut back) output in the most likely states, then principals have no incentive to unilaterally disclose their agents' costs. Similarly, if goods are substitutes (resp. complements), and sharing information induces rivals to cut back (resp. expand) output in the most likely states, then communication will be worthwhile.

Clearly, when $\delta = 0$ communication plays no role because there is no strategic interaction between principals: disclosing information does not affect rivals' behavior. When δ is different than zero (but small), instead, the correlated distortions effect kicks in and its impact on the value of communication depends only on the sign of $\delta\alpha$. To understand why, suppose that P_j commits to disclose θ_j, and consider P_i's incentive to reveal θ_i. The argument is as follows. Consider first $\delta\alpha < 0$ and suppose that $\delta < 0$ and $\alpha > 0$ — i.e., outputs are strategic substitutes and costs are positively correlated. By disclosing θ_i, P_i enables P_j to condition C_j on such contingency. Hence, when A_j's cost is high, P_j will distort more (resp. less) output in the state where A_i's cost is low (resp. high) because costs are positively correlated. But this increases P_i's profits because goods are strategic substitutes — i.e., reaction functions are negatively sloped — and each principal gains from reducing (resp. expanding) her production as a response to an expansion (resp. reduction) of the rival's (expected) output. Next, suppose that $\delta > 0$ and $\alpha < 0$ — i.e., outputs are strategic complements and costs are negatively correlated. By revealing θ_i, P_i induces P_j to distort more (resp. less) A_j's output in the state where θ_i is high (resp. low) because costs are negatively correlate. And this is beneficial to P_i because, with strategic complements, principals gain from choosing outputs that are positively correlated — i.e., reaction functions are positively sloped.

Consider now $\delta\alpha > 0$. In this case, the correlated distortions effect makes communication unprofitable. The argument is as follows. Suppose first that that $\delta < 0$ and $\alpha < 0$ — i.e., outputs are strategic substitutes and costs are negatively correlated. By revealing θ_i, P_i induces P_j to distort more (resp. less) A_j's output in the state where θ_i is high (resp. low) because costs are negatively correlated. But this reduces upstream profits because goods are substitutes and each principal gains from expanding (resp. reducing) own production when the rival is inefficient (resp. efficient). Next, suppose that $\delta > 0$ and $\alpha > 0$ — i.e., outputs are strategic complements and costs are positively correlated. By revealing θ_i, P_i induces P_j to distort more (resp. less) A_j's output in the state where θ_i is low (resp. high) because costs are positively correlated. But, this reduces P_i's expected profits because, with strategic complementarities, principals would like to increase production as a response to an expansion of the rival's output.

Note also that the correlated distortion effect is of first-order magnitude relative to the effects shaping communication decisions in the complete information analysis, where only the sign and magnitude of α matters in determining the value of communication at equilibrium.[12]

In the next proposition I compare the equilibrium profits when both principals share information and where they both do not share it.

Proposition 6 *Suppose that δ is small. Principals' expected profits are larger when they both share information than with no communication.*

The intuition for this result is straightforward. As long as types are correlated, communication creates an *informational externality* that, ceteris paribus, reduces the expected rents each principal needs to give up to elicit truthful revelation of types. This is because, when agents' contracts are contingent on the rivals' types, cost correlation generates a *relative performance evaluations* effect that relaxes the rent-extraction efficiency trade-off — see Riordan and Sappington (1988). For δ small this effect outperforms the strategic effect due to correlated distortions because, in this case, upstream externalities are negligible relative to the need for rent extraction, whereby making a bilateral IS sharing agreement jointly profitable for principals.

Hence, while under complete information the outcome of the non-cooperative game is always efficient from the principals' perspective, with asymmetric information a Prisoners' Dilemma may emerge.

Lemma 3 *Suppose that $\delta \neq 0$ but small and that $\alpha\delta > 0$. Then, game $\mathcal{G}$ entails no communication at equilibrium, but this is inefficient for the principals.*

[12]In the proof of Proposition 1 it is shown that under complete information, for δ small, only terms of second-order magnitude matter in signing the difference between expected profits with and without information sharing.

In sum, unlike in the complete information analysis, private and collective incentives to share information may diverge in a vertical hierarchies model with privately informed agents.

Finally, because of the *relative performance evaluations* effect, communication has an unambiguous impact on expected rents.

Proposition 7 *Suppose that δ is small. Agents' expected rents are larger under no communication than under IS.*

6 Agents' (implicit) collusion

So far, I have characterized separating equilibria featuring Nash behavior by agents at the revelation stage — i.e., incentive constraints have been constructed in such a way that, when making his report, A_i believes that A_j is telling the truth. However, under IS the expected utility of each agent is affected by his opponent's report because principals offer contractual rules that depend on such contingency. Therefore, in principle, the allocation characterized in this regime might not be robust to *implicit* collusion by the agents. I rule out side payments for obvious reasons. In fact, were I considering collusion among agents enforced through side transfers — see, e.g., Laffont and Martimort (1997)-(2000) — there would be no a priori reasons for ruling out side transfers also between principals. But, in this case, the analysis would be equivalent to Laffont and Martimort (2000) where a single principal — i.e., the coalition formed by P_1 and P_2. — deals with two colluding agents with correlated types — i.e., A_1 and A_2.

Hence, the issue is that there could exist an equilibrium of the continuation game where efficient agents (jointly) lie after receiving an offer C^e. In the absence of side transfers, this type of equilibrium would emerge when

$$t^e\left(\overline{\theta}|\overline{\theta}\right) - \underline{\theta}q^e(\overline{\theta}|\overline{\theta}) > t^e\left(\underline{\theta}|\overline{\theta}\right) - \underline{\theta}q^e\left(\underline{\theta}|\overline{\theta}\right). \tag{18}$$

That is, an efficient agent finds it profitable to lie instead of telling the truth given that his efficient rival lies as well. Inequality (18) clearly depends on how equilibrium transfers are constructed under IS. Hence, to investigate whether the equilibrium outcome characterized in Proposition 3 is undermined by the threat of collusion, one needs first to identify the equilibrium transfers and then check (18). However, under IS transfers are indeterminate in some states. This is because agents are asked to make their reports before the information about rivals is transmitted. Hence, the number of constraints that bind in a truthful equilibrium is smaller than the number of instruments available to each principal.

To pin down transfers, note that the limited liability constraints of the inefficient types imply $t^e(\overline{\theta}|\theta) = \overline{\theta}q^e(\overline{\theta}|\theta)$ for all θ. Hence, (18) rewrites as:

$$\Delta\theta q^e(\overline{\theta}|\overline{\theta}) > t^e\left(\underline{\theta}|\overline{\theta}\right) - \underline{\theta}q^e\left(\underline{\theta}|\overline{\theta}\right). \tag{19}$$

This implies that collusion is not viable if principals — actually even only one of them — implement a transfer $t^e\left(\underline{\theta}|\overline{\theta}\right)$ such that: (i) agents still tell the truth when rivals are expected to do so; (ii) limited liability constraints are satisfied in all states, and (iii) inequality (19) is not met. It turns out that this is possible in the framework at hand.

Proposition 8 *The equilibrium allocation characterized in Proposition 3 is robust to the threat of implicit collusion by the agents — i.e., (18) cannot hold as long as:*

$$t^e\left(\underline{\theta}|\underline{\theta}\right) = \underline{\theta}q^e(\underline{\theta}|\underline{\theta}),$$

and

$$t^e(\underline{\theta}|\overline{\theta}) = \underline{\theta}q^e(\underline{\theta}|\overline{\theta}) + \Delta\theta\frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\underline{\theta})}q^e(\overline{\theta}|\underline{\theta}) + \Delta\theta q^e(\overline{\theta}|\overline{\theta}).$$

Implicit collusion is therefore not an issue in the vertical hierarchies model at hand. Of course, this is partly due to the fact that agents do not impose production externalities one on another — see, e.g., Martimort (1996) for a model with this feature. In such setting communication would not only create informational externalities among agents but also generate direct production externalities that may make implicit collusion self-enforceable, an issue that I plan to tackle in future research.

7 Concluding remarks

Already in 1974 Arrow argued that the essence of an organization is based on the trade-off between the costs and benefits of communication among its members. This idea has been explored in many settings. But, so far, no paper has considered the exchange of private information among complex organizations. The objective of my analysis has been precisely to set up a model where two independent principals, which exert production externalities one on another, and contract with exclusive privately informed agents, endogenously decide whether to exchange information. The analysis has identified one novel effect of IS. This effect emerges only when agents are privately informed, and it turns out to be of first-order magnitude relative to the effects that drive communication decisions in the complete information benchmark. The key driver of IS decisions under asymmetric information is intimately linked with the distortions generated by the need for eliciting truthful information revelation. Essentially, IS induces distortions to be correlated. And, because of those correlation, the equilibrium value of communication depends only on the interplay between the nature of upstream externalities and the sign of cost correlation. The analysis has shown that there exists a unique equilibrium in dominant strategies with no communication when upstream externalities and cost correlation have the same sign. By contrast, there exists a unique symmetric equilibrium in dominant strategies where both principals share information emerges when upstream externalities and cost correlation have opposite signs. I have also show that, unlike the complete information analysis, where the equilibrium outcome is always efficient, principals may run into a *prisoners' dilemma* when agents own privileged information about their costs. Finally, I argued that the characterization under IS is robust to implicit collusion by the agents.

8 Appendix

Proof of Lemma 1. In order to show the result I must first characterize equilibrium outputs within the three communication regimes where: (i) both principals share information; (ii) they do not communicate; (iii) only one principal shares information.

No information sharing. Under complete information principals fully extract their agents' rents. The (symmetric) equilibrium market outcome, $q^*(\theta_i)$, is such that

$$q^*(\theta_i) = \arg\max_{q_i(\theta_i)} \mathrm{E}_{\theta_j}\left(S\left(q_i(\theta_i), q^*(\theta_j)\right) - \theta_i q_i(\theta_i) \,|\theta_i\right) \quad \forall\, \theta_i \in \Theta,$$

with the equilibrium transfer being set at the reservation level — i.e.,

$$U_i(\theta_i) = 0 \quad \forall\, \theta_i \in \Theta \quad \Longrightarrow \quad t^*(\theta_i) = \theta_i q^*(\theta_i) \quad \forall\, \theta_i \in \Theta.$$

A symmetric equilibrium then satisfies the following necessary and sufficient first-order conditions

$$\mathrm{E}_{\theta_j}\left(S_1\left(q^*(\theta_i), q^*(\theta_j)\right)|\theta_i\right) = \theta_i \quad \forall\, \theta_i \in \Theta. \tag{20}$$

Solving (20) under **A1**, the profile of outputs in the unique symmetric equilibrium of the game is

$$q^*(\underline{\theta}) = \frac{\beta - \underline{\theta}}{2-\delta} - \frac{\delta\left(\nu\left(1-\nu\right)-\alpha\right)}{\left(2-\delta\right)\left(2\nu\left(1-\nu\right)-\alpha\delta\right)}\Delta\theta \qquad q^*(\overline{\theta}) = q^*(\underline{\theta}) - \frac{\nu\left(1-\nu\right)}{2\nu\left(1-\nu\right)-\alpha\delta}\Delta\theta. \tag{21}$$

Bilateral information sharing. Consider now the regime where both principals share information. Again, because costs are common knowledge, agents are left with no rents. The transfer is set at the reservation level — i.e.,

$$U_i\left(\theta_i|\theta_j\right) = 0 \quad \forall\left(\theta_i, \theta_j\right) \in \Theta^2 \quad \Longrightarrow \quad t_i\left(\theta_i|\theta_j\right) = \theta_i q_i\left(\theta_i|\theta_j\right) \quad \forall\left(\theta_i, \theta_j\right) \in \Theta^2.$$

The equilibrium market outcome, $q^*\left(\theta_i|\theta_j\right)$, is such that

$$q^*\left(\theta_j|\theta_i\right) = \arg\max_{q_i(\theta_i|\theta_j)} S\left(q_i\left(\theta_i|\theta_j\right), q^*\left(\theta_j|\theta_i\right)\right) - \theta_i q_i\left(\theta_i|\theta_j\right) \quad \forall\left(\theta_i, \theta_j\right) \in \Theta^2.$$

The first-order necessary and sufficient conditions are

$$S_1\left(q^*\left(\theta_i|\theta_j\right), q^*\left(\theta_j|\theta_i\right)\right) = \theta_i \quad \forall\left(\theta_i, \theta_j\right) \in \Theta^2. \tag{22}$$

Solving (22) under **A1**, the profile of equilibrium outputs in the unique equilibrium is

$$q^*(\underline{\theta}|\underline{\theta}) = \frac{\beta - \underline{\theta}}{2-\delta} \qquad q^*(\underline{\theta}|\overline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{\delta}{4-\delta^2}\Delta\theta,$$

$$q^*(\overline{\theta}|\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{2}{4-\delta^2}\Delta\theta \qquad q^*(\overline{\theta}|\overline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{1}{2-\delta}\Delta\theta.$$

Unilateral information sharing. Finally, consider the case where one principal, say P_i, commits to disclose her agent cost, while her opponent, P_j does not share information. Principal P_i's optimization program is

$$\max_{q_i(\theta_i)} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(S(q_i(\theta_i), q_j^*(\theta_j|\theta_i)|\theta_i) - \mathrm{E}_{\theta_i}(\theta_i q_i(\theta_i)),$$

P_j's optimization program is

$$\max_{q_j(\theta_j|\theta_i)} \mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(S(q_j(\theta_j|\theta_i), q_i^*(\theta_i)) - \theta_j q_j(\theta_j|\theta_i)|\theta_j).$$

The system of first-order necessary and sufficient conditions that identify the equilibrium outputs are

$$\mathrm{E}_{\theta_j}(S_1(q_i^*(\theta_i), q_j^*(\theta_j|\theta_i))|\theta_i) = \theta_i \quad \forall\, \theta_i \in \Theta, \tag{23}$$

$$S_1(q_j^*(\theta_j|\theta_i), q_i^*(\theta_i)) = \theta_j \quad \forall\ (\theta_i, \theta_j) \in \Theta^2, \tag{24}$$

whose solutions under **A1** imply

$$q_i^*(\underline{\theta}) = \frac{\beta - \underline{\theta}}{2-\delta} - \frac{\delta(\nu(1-\nu)-\alpha)}{\nu(4-\delta^2)}\Delta\theta \qquad q_i^*(\overline{\theta}) = q_i^*(\underline{\theta}) - \frac{2\nu(1-\nu)+\alpha\delta}{\nu(1-\nu)(4-\delta^2)}\Delta\theta,$$

and

$$q_j^*(\underline{\theta}|\underline{\theta}) = q^*(\underline{\theta}|\underline{\theta}) - \frac{(\nu(1-\nu)-\alpha)\delta^2}{2\nu(4-\delta^2)}\Delta\theta \qquad q_j^*(\underline{\theta}|\overline{\theta}) = q^*(\underline{\theta}|\overline{\theta}) - \frac{(\alpha+(1-\nu)^2)\delta^2}{2(1-\nu)(4-\delta^2)}\Delta\theta,$$

$$q_j^*(\overline{\theta}|\underline{\theta}) = q^*(\overline{\theta}|\underline{\theta}) + \frac{(\nu^2+\alpha)\delta^2}{2\nu(4-\delta^2)}\Delta\theta \qquad q_j^*(\overline{\theta}|\overline{\theta}) = q^*(\overline{\theta}|\overline{\theta}) + \frac{(\nu(1-\nu)-\alpha)\delta^2}{2(1-\nu)(4-\delta^2)}\Delta\theta.$$

Characterization of expected profits. Using the first-order conditions (20), (22), (23) and (24) it is easy to show that expected profits are

$$V_i^* = \kappa + \mathrm{E}_{s_i}(q_i^*(s_i)|\theta_i))^2,$$

then from $\mathrm{E}(\tilde{x}^2) = [\mathrm{E}(\tilde{x})]^2 + \mathrm{E}(\tilde{x} - \mathrm{E}(\tilde{x}))^2$ it follows that

$$V_i^* = \kappa + [\mathrm{E}_{s_i}(q_i^*(s_i)|\theta_i)]^2 + \mathrm{E}_{s_i}(q_i^*(s_i) - \mathrm{E}_{s_i}(q_i^*(s_i)|\theta_i)|\theta_i)^2.$$

Consider now expected outputs. Let

$$q^* = q^*(\underline{\theta}|\underline{\theta}) - \frac{1-\nu}{2-\delta}\Delta\theta,$$

taking expectations it easily follows that

$$\mathrm{E}_\theta(q^*(\theta)) = \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q^*(\theta_i|\theta_j)|\theta_i) = \mathrm{E}_\theta(q_j^*(\theta)) = \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q_i^*(\theta_i|\theta_j)|\theta_i) = q^*,$$

which proves the result. ■

Proof of Proposition 1. Let V_I^* and V_N^* denote expected profits when both principals share information and when they do not communicate, respectively. Moreover, denote by $V_{N,I}^*$ principal P_i's profit and by $V_{I,N}^*$ principal P_j's profit in the regime where P_i does not share information while P_j shares information.

Suppose that δ is small but different than 0. A symmetric equilibrium where both principals share information exists if and only if

$$V_I^* \geq V_{N,I}^*.$$

Using a second-order Taylor approximation around the point $\delta = 0$ one has

$$V_{N,I}^* \approx \kappa + \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q_i^*(\theta_i|\theta_j)^2|\theta_i) + 2\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q_i^*(\theta_i|\theta_j)\frac{\partial q_i^*(\theta_i|\theta_j)}{\partial\delta}|\theta_i\right)\delta +$$
$$+\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q_i^*(\theta_i|\theta_j)\frac{\partial^2 q_i^*(\theta_i|\theta_j)}{\partial\delta^2} + \left(\frac{\partial q_i^*(\theta_i|\theta_j)}{\partial\delta}\right)^2\middle|\theta_i\right)\delta^2,$$

and, similarly,

$$V_I^* \approx \kappa + \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q_I^*(\theta_i|\theta_j)^2|\theta_i) + 2\delta\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^*(\theta_i|\theta_j)\frac{\partial q^*(\theta_i|\theta_j)}{\partial\delta}|\theta_i\right) +$$
$$+\delta^2\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^*(\theta_i|\theta_j)\frac{\partial^2 q^*(\theta_i|\theta_j)}{\partial\delta^2} + \left(\frac{\partial q^*(\theta_i|\theta_j)}{\partial\delta}\right)^2\middle|\theta_i\right),$$

Using the equilibrium outputs derived in Lemma 1 one has

$$\lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q_i^*(\theta_i|\theta_j)^2|\theta_i) = \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q^*(\theta_i|\theta_j)^2|\theta_i),$$

$$\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q_i^*(\theta_i|\theta_j)\frac{\partial q_i^*(\theta_i|\theta_j)}{\partial\delta}|\theta_i\right) = \lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^*(\theta_i|\theta_j)\frac{\partial q^*(\theta_i|\theta_j)}{\partial\delta}|\theta_i\right).$$

Hence,

$$V_I^* - V_{N,I}^* \approx \delta^2\left[\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^*(\theta_i|\theta_j)\frac{\partial^2 q^*(\theta_i|\theta_j)}{\partial\delta^2}\right) - \lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q_i^*(\theta_i|\theta_j)\frac{\partial^2 q_i^*(\theta_i|\theta_j)}{\partial\delta^2}\middle|\theta_i\right) +\right.$$
$$\left.+\lim_{\delta\to 0}\mathrm{E}_{\theta_j}\left(\left(\frac{\partial q^*(\theta_i|\theta_j)}{\partial\delta}\right)^2\right) - \lim_{\delta\to 0}\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(\left(\frac{\partial q_i^*(\theta_i|\theta_j)}{\partial\delta}\right)^2\middle|\theta_i\right)\right],$$

implying that

$$V_I^* - V_{N,I}^* \approx \frac{(\nu(1-\nu)+\alpha)\Pr(\overline{\theta},\underline{\theta})\delta^2\Delta\theta^2}{8\nu(1-\nu)}. \tag{25}$$

From equation (25) it is straightforward to show that, for δ small but different than 0, there exists a symmetric equilibrium where both principals share information as long as $\nu(1-\nu)+\alpha \geq 0$ — i.e., if $\alpha \geq 0$ or if $|\alpha| \leq \nu(1-\nu)$ when $\alpha < 0$.

Consider now a symmetric equilibrium with no communication. This equilibrium exists if and only if

$$V_N^* \geq V_{I,N}^*.$$

Using a second-order Taylor approximation around $\delta = 0$ one has

$$\begin{aligned} V_{I,N}^* &\approx \kappa + \lim_{\delta\to 0} \mathrm{E}_{\theta_j}(q_j^*(\theta_j)^2) + 2\delta \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\left(q_j^*(\theta_j)^2 \frac{\partial q_j^*(\theta_j)}{\partial \delta}\right) + \\ &\quad + \delta^2 \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\left(q_j^*(\theta_j)^2 \frac{\partial^2 q_j^*(\theta_j)}{\partial \delta^2} + \left(\frac{\partial q_j^*(\theta_j)}{\partial \delta}\right)^2\right), \end{aligned}$$

and, similarly,

$$\begin{aligned} V_N^* &\approx \kappa + \lim_{\delta\to 0} \mathrm{E}_{\theta_i}(q^*(\theta_i)^2) + 2\delta \lim_{\delta\to 0} \mathrm{E}_{\theta_i}\left(q^*(\theta_i) \frac{\partial q^*(\theta_i)}{\partial \delta}\right) + \\ &\quad + \delta^2 \lim_{\delta\to 0} \mathrm{E}_{\theta_i}\left(q^*(\theta_i) \frac{\partial^2 q^*(\theta_i)}{\partial \delta^2} + \left(\frac{\partial q^*(\theta_i)}{\partial \delta}\right)^2\right). \end{aligned} \tag{26}$$

Using the outputs characterized in Lemma 1, it follows that

$$\lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q_j^*(\theta_j)^2) = \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}(q^*(\theta_j)^2),$$

$$\lim_{\delta\to 0} \mathrm{E}_{\theta_j}\left(q_j^*(\theta_j)^2 \frac{\partial q_j^*(\theta_j)}{\partial \delta}\right) = \lim_{\delta\to 0} \mathrm{E}_{\theta_j}\left(q^*(\theta_j)^2 \frac{\partial q^*(\theta_j)}{\partial \delta}\right).$$

Hence,

$$\begin{aligned} V_N^* - V_{I,N}^* \approx \delta^2 \lim_{\delta\to 0} \Bigg[&\mathrm{E}_{\theta_j}\left(q^*(\theta_j)^2 \frac{\partial^2 q^*(\theta_j)}{\partial \delta^2}\right) - \mathrm{E}_{\theta_j}\left(q_j^*(\theta_j)^2 \frac{\partial^2 q_j^*(\theta_j)}{\partial \delta^2}\right) + \\ &+ \mathrm{E}_{\theta_j}\left(\left(\frac{\partial q^*(\theta_j)}{\partial \delta}\right)^2\right) - \mathrm{E}_{\theta_j}\left(\left(\frac{\partial q_j^*(\theta_j)}{\partial \delta}\right)^2\right)\Bigg], \end{aligned}$$

implying that

$$V_N^* - V_{I,N}^* \approx -\frac{(\nu(1-\nu)+\alpha)\Pr(\overline{\theta},\underline{\theta})\delta^2\Delta\theta^2}{8\nu(1-\nu)}. \tag{27}$$

From equation (27) it is straightforward to show that,for δ small, an equilibrium with no communication exists if and only if α is negative large in absolute value — i.e., $|\alpha| > \nu(1-\nu)$.

Note also that because the sign of (25) is always opposite to the sign of (27), the equilibria of game $\mathcal{G}$ are in dominant strategies for $\delta \neq 0$. Finally, it is easy to show that for $\delta = 0$ information sharing has no impact on principals' profits — i.e., $V_N^* - V_{I,N}^* = V_I^* - V_{N,I}^* = V_I^* - V_N^* = 0$. Hence, for $\delta = 0$, game $\mathcal{G}$ features multiple pay-off equivalent equilibria with and without IS. ■

Proof of Corollary 1. First, using Lemma 1 it is easy to show that $V_I^* = V_N^*$ for $\delta = 0$ and that

$$V_I^* - V_N^* = \frac{\nu(1-\nu)(12-\delta^2)\delta^2\Delta\theta^2}{4(2+\delta)^2(2-\delta)^2} > 0$$

for $\alpha = 0$. Suppose now that δ is different than 0 but small. Using the Taylor approximations for V_I^* and V_N^* derived in the proof of Proposition 1 it follows that

$$V_I^* - V_N^* \approx \delta^2 \lim_{\delta\to 0}\left[\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^*\left(\theta_i|\theta_j\right)\frac{\partial^2 q^*\left(\theta_i|\theta_j\right)}{\partial\delta^2}\middle|\,\theta_i\right) - \mathrm{E}_{\theta_i}\left(q^*\left(\theta_i\right)\frac{\partial^2 q^*\left(\theta_i\right)}{\partial\delta^2}\right) + \right.$$
$$\left. + \mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(\left(\frac{\partial q^*\left(\theta_i|\theta_j\right)}{\partial\delta}\right)^2\middle|\,\theta_i\right) - \mathrm{E}_{\theta_i}\left(\left(\frac{\partial q^*\left(\theta_i\right)}{\partial\delta}\right)^2\right)\right],$$

implying that

$$V_I^* - V_N^* \approx \frac{3\Pr(\overline{\theta},\underline{\theta})\left(\nu(1-\nu)+\alpha\right)\delta^2\Delta\theta^2}{16\nu\left(1-\nu\right)},$$

which immediately yields the result. ■

Proof of Proposition 2. To compute the equilibrium outputs under no IS, one simply solves the system of first-order conditions (5)-(6) for the quadratic specification in **A1**. The difference Δq^e follows immediately. ■

Proof of Corollary 1. Differentiating Δq^e with respect to α

$$sign\frac{\partial\Delta q^e}{\partial\alpha} = sign\frac{\delta\nu}{\left(2\nu\left(1-\nu\right)-\alpha\delta\right)^2},$$

which proves the result. ■

Proof of Proposition 3. To compute the equilibrium outputs with bilateral information sharing, one simply solves the system of first-order conditions (13)-(14) for the quadratic specification in **A1**. It then follows that

$$\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^e\left(\theta_i|\theta_j\right)|\theta_i\right) = \nu\left[(\nu+\frac{\alpha}{\nu})q^e(\underline{\theta}|\underline{\theta}) + (1-\nu-\frac{\alpha}{\nu})q^e(\underline{\theta}|\overline{\theta})\right] +$$
$$+ (1-\nu)\left[(\nu-\frac{\alpha}{1-\nu})q^e(\overline{\theta}|\underline{\theta}) + (1-\nu+\frac{\alpha}{1-\nu})q^e(\overline{\theta}|\overline{\theta})\right] = \frac{\beta-\overline{\theta}}{2-\delta},$$

and that

$$\mathrm{E}_{\theta_i}\left(q^e\left(\theta_i\right)\right) = \nu q^e(\underline{\theta}) + (1-\nu)\,q^e(\overline{\theta}) = \frac{\beta-\overline{\theta}}{2-\delta},$$

Hence, $\mathrm{E}_{\theta_j}\mathrm{E}_{\theta_i}\left(q^e\left(\theta_i|\theta_j\right)|\theta_i\right) = \mathrm{E}_{\theta_i}\left(q^e\left(\theta_i\right)\right)$. Moreover,

$$\mathrm{E}_{\theta_i}\left(q^e\left(\theta_i\right)\right) - \mathrm{E}_{\theta_i}\left(q^*\left(\theta_i\right)\right) = \frac{\beta-\overline{\theta}}{2-\delta} - \frac{\beta-\underline{\theta}-(1-\nu)\Delta\theta}{2-\delta} = -\frac{\nu\Delta\theta}{2-\delta} < 0.$$

The rest of the proof is straightforward. ■

Proof of Proposition 4. To compute the equilibrium outputs with unilateral information sharing, one simply solves the system of first-order conditions (15)-(16) for the quadratic specification in **A1.** The rest of the proof is straightforward. ■

Proof of Proposition 5. Suppose first that both α and δ are different than 0 and that δ is small. A symmetric equilibrium where both principal share information exists if and only if

$$V_I^e \geq V_{N,I}^e.$$

Using a second-order Taylor approximation around the point $\delta = 0$ one has

$$V_I^e \approx \kappa + \lim_{\delta \to 0} \mathrm{E}_{\theta_j} \mathrm{E}_{\theta_i} (q^e(\theta_i|\theta_j)^2 \, |\theta_i) + 2\delta \lim_{\delta \to 0} \mathrm{E}_{\theta_j} \mathrm{E}_{\theta_i} \left(q^e(\theta_i|\theta_j) \frac{\partial q^e(\theta_i|\theta_j)}{\partial \delta} \middle| \theta_i \right), \tag{28}$$

and

$$V_{N,I}^e \approx \kappa + \lim_{\delta \to 0} \mathrm{E}_{\theta_j} (q_j^e(\theta_j)^2) + 2\delta \lim_{\delta \to 0} \mathrm{E}_{\theta_j} \left(q_j^e(\theta_j) \frac{\partial q_j^e(\theta_j)}{\partial \delta} \right).$$

Hence,

$$\begin{aligned} V_I^* - V_{N,I}^* \approx \; & \lim_{\delta \to 0} \left[\mathrm{E}_{\theta_i} \mathrm{E}_{\theta_j} (q^e(\theta_j|\theta_i)^2 \, |\theta_j) - \mathrm{E}_{\theta_i} (q_j^e(\theta_j)^2) \right] + \\ & + 2\delta \lim_{\delta \to 0} \left[\mathrm{E}_{\theta_i} \mathrm{E}_{\theta_j} \left(q^e(\theta_j|\theta_i) \frac{\partial q^e(\theta_j|\theta_i)}{\partial \delta} \middle| \theta_j \right) - \mathrm{E}_{\theta_j} \left(q^e(\theta_j) \frac{\partial q^e(\theta_j)}{\partial \delta} \right) \right]. \end{aligned}$$

Using the outputs characterized in Propositions 3 and 4

$$V_I^* - V_{N,I}^* \approx -\frac{\alpha \delta \Delta \theta^2}{4(\alpha + (1-\nu)^2)}, \tag{29}$$

which shows that when δ is small and both δ and α are different than 0, there exists a symmetric equilibrium of the game where both principals share information if $\alpha\delta < 0$.

A symmetric equilibrium where principals do not share information exists if and only if

$$V_N^e \geq V_{I,N}^e.$$

Using a second-order Taylor approximation around the point $\delta = 0$ one has

$$V_N^e \approx \kappa + \lim_{\delta \to 0} \mathrm{E}_\theta (q^e(\theta)^2) + 2\delta \lim_{\delta \to 0} \mathrm{E}_\theta \left(q^e(\theta) \frac{\partial q^e(\theta)}{\partial \delta} \right), \tag{30}$$

and

$$V_{I,N}^e \approx \kappa + \lim_{\delta \to 0} \mathrm{E}_\theta (q_j^e(\theta)^2) + 2\delta \lim_{\delta \to 0} \mathrm{E}_\theta \left(q_j^e(\theta) \frac{\partial q_j^e(\theta)}{\partial \delta} \right).$$

Hence,

$$V_N^* - V_N^* \approx \lim_{\delta \to 0} \left[\mathrm{E}_\theta (q^e(\theta)^2) - \mathrm{E}_\theta (q_j^e(\theta)^2) \right] + $$
$$+2\delta \lim_{\delta \to 0} \left[\mathrm{E}_\theta \left(q^e(\theta) \frac{\partial q^e(\theta)}{\partial \delta} \right) - \mathrm{E}_\theta \left(q_j^e(\theta) \frac{\partial q_j^e(\theta)}{\partial \delta} \right) \right].$$

Using the outputs characterized in Propositions 2 and 4

$$V_N^* - V_{I,N}^* \approx \frac{\delta \alpha \Delta \theta^2}{4(1-\nu)^2}, \tag{31}$$

implying that when δ is small and both δ and α are different than 0, there exists a symmetric equilibrium of the game where both principals do not share information if $\delta\alpha > 0$.

Finally, it is easy to verify that $\lim_{\delta \to 0} \left(V_I^* - V_{N,I}^* \right) = \lim_{\delta \to 0} \left(V_N^* - V_{I,N}^* \right)$. While

$$\lim_{\alpha \to 0} \left(V_I^* - V_{N,I}^* \right) = \frac{\nu \left(8 - \delta^2\right) \delta^2 \Delta \theta^2}{4(1-\nu)(2+\delta)^2 (2-\delta)} > 0 \quad \lim_{\alpha \to 0} \left(V_N^* - V_{I,N}^* \right) = -\frac{\nu \left(8 - \delta^2\right) \delta^2 \Delta \theta^2}{4(1-\nu)(2+\delta)^2 (2-\delta)} < 0.$$

Note also that since the sign of (29) is always opposite to the sign of (31), the equilibria of game $\mathcal{G}$ are in dominant strategies. ■

Proof of Proposition 6. Suppose that both δ and α are different than 0 and that δ is small. Taking the difference between (28) and (30)

$$V_I^e - V_N^e \approx \lim_{\delta \to 0} \left[\mathrm{E}_{\theta_j} \mathrm{E}_{\theta_i} (q^e(\theta_i|\theta_j)^2 \,|\theta_i) - \mathrm{E}_\theta (q^e(\theta)^2) \right] + $$
$$+2\delta \lim_{\delta \to 0} \left[\mathrm{E}_{\theta_j} \mathrm{E}_{\theta_i} \left(q^e(\theta_i|\theta_j) \frac{\partial q^e(\theta_i|\theta_j)}{\partial \delta} \middle| \theta_i \right) - \mathrm{E}_\theta \left(q^e(\theta) \frac{\partial q^e(\theta)}{\partial \delta} \right) \right].$$

Using the outputs characterized in Propositions 2 and 3, it is easy to show that

$$V_I^e - V_N^e \approx \frac{\Delta \theta^2}{4(1-\nu)((1-\nu)^2 + \alpha)} \left[\frac{\alpha^2}{\nu(1-\nu) - \alpha} - \frac{\alpha\delta\left(\alpha + 2(1-\nu)^2\right)}{1-\nu} \right],$$

which immediately shows that $V_I^e > V_N^e$ for δ small enough and different than 0. Suppose now that $\alpha = 0$, then it is easy to verify that

$$\lim_{\alpha \to 0} (V_I^e - V_N^e) = \frac{\left(12 - \delta^2\right) \Delta \theta^2 \delta^2 \nu}{4(1-\nu)(2+\delta)^2 (2-\delta)^2} > 0,$$

which concludes the proof. ■

Proof of Lemma 3. The proof of this result follows immediately from Proposition 6 and Lemma 3. ■

Proof of Proposition 7. Agents' expected rents when there is no communication are equal to

$$\mathrm{E}_{\theta_i}(U^e(\theta_i)) = \nu\Delta\theta q^e(\overline{\theta}). \tag{32}$$

When instead both principals share information agents' expected rents are equal to

$$\mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(U^e(\theta_i|\theta_j)) = \nu\Delta\theta\mathrm{E}_{\theta_j}(q^e(\overline{\theta}|\theta_j)|\underline{\theta}). \tag{33}$$

Taking the difference between (32) and (33):

$$\mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j}(U^e(\theta_i|\theta_j)|\theta_i) - \mathrm{E}_{\theta_i}(U^e(\theta_i)) = \nu\Delta\theta[\mathrm{E}_{\theta_j}(q^e(\overline{\theta}|\theta_j)|\underline{\theta}) - q^e(\overline{\theta})].$$

First, note that

$$\nu\Delta\theta \lim_{\alpha\to 0}[\mathrm{E}_{\theta_j}(q^e(\overline{\theta}|\theta_j)|\underline{\theta}) - q^e(\overline{\theta})] = -\frac{\nu^2\delta^2\Delta\theta^2}{2(1-\nu)(2-\delta)(2+\delta)} < 0.$$

Suppose now that α is different than zero and that δ is small, using a first-order Taylor approximation

$$\nu\Delta\theta[\mathrm{E}_{\theta_j}(q^e(\overline{\theta}|\theta_j)|\underline{\theta}) - q^e(\overline{\theta})] \approx$$

$$\approx -\frac{\Delta\theta^2}{2(1-\nu)(\alpha+(1-\nu)^2)}\left[\frac{\alpha^2}{\nu(1-\nu)-\alpha} - \frac{\delta\alpha\left(\alpha\nu+(1-\nu)^2(1+\nu)\right)}{2(1-\nu)}\right]. \tag{34}$$

which immediately implies the result. ■

Proof of Proposition 8. To show the result one needs to verify that the maximal transfer $t^e(\underline{\theta}|\overline{\theta})$ compatible with the Bayesian incentive constraint (11) must satisfy the following properties: (i) break inequality (19), and (ii) satisfy the agent's limited liability constraint in state $(\underline{\theta},\overline{\theta})$. Developing (11):

$$t^e(\underline{\theta}|\overline{\theta}) = \underline{\theta}q^e(\underline{\theta}|\overline{\theta}) - \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\underline{\theta})}\left(t^e(\underline{\theta}|\underline{\theta}) - \underline{\theta}q^e(\underline{\theta}|\underline{\theta})\right) + \Delta\theta\frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\underline{\theta})}q^e(\overline{\theta}|\underline{\theta}) + \Delta\theta q^e(\overline{\theta}|\overline{\theta}).$$

Which implies

$$\hat{t}^e(\underline{\theta}|\overline{\theta}) = \max_{t^e(\underline{\theta}|\underline{\theta})}\left\{t^e(\underline{\theta}|\overline{\theta}) : t^e(\underline{\theta}|\underline{\theta}) - \underline{\theta}q^e(\underline{\theta}|\underline{\theta}) \geq 0\right\} = \underline{\theta}q^e(\underline{\theta}|\overline{\theta}) + \Delta\theta\frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\underline{\theta})}q^e(\overline{\theta}|\underline{\theta}) + \Delta\theta q^e(\overline{\theta}|\overline{\theta}). \tag{35}$$

Clearly, $\hat{t}^e(\underline{\theta}|\overline{\theta})$ satisfies the agent's limited liability constraint. Moreover, substituting (35) into (19) one gets:

$$0 > \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\overline{\theta}|\underline{\theta})}q^e(\overline{\theta}|\underline{\theta}),$$

which is the contradiction that delivers the result. ■

References

[1] ACQUISTI, A. AND VARIAN, H.R, (2005), "Conditioning Prices on Purchase History," *Marketing Science*, 24: 367-381.

[2] BENNARDO, A., PAGANO, M. AND PICCOLO, S., (2010) "Multiple-Bank Lending, Creditor Rights and Information Sharing", mimeo CSEF.

[3] BERTOLETTI, P. AND POLETTI, C., (1997), "X-Inefficiency, Competition and Market Information", *Journal of Industrial Economics*, 45: 359-375.

[4] BRILEY, M., HOMCHICK, R. AND SINGER, T., (1994), *Information Sharing Among Health Care Providers: Antitrust Analysis and Practical Guide*, Chicago, American Bar Association.

[5] CAILLAUD, B., JULLIEN, B. AND PICARD, P., (1995), "Competing Vertical Structures: Precommitment and Renegotiation", *Econometrica*, 63: 621-646.

[6] CALZOLARI, G. AND PAVAN, A., (2006), "On the Optimality of Privacy in Sequential Contracting", *Journal of Economic Theory*, 130: 168-204.

[7] CALVO'-ARMENGOLY, A., MARTI', J., AND PRAT, A., (2010), "Endogenous Communication in Complex Organizations", mimeo LSE.

[8] CARLIN, B. AND ROB, R., (2009), "A Welfare Analysis of Regulation in Relationship Banking Markets", *Review of Finance*, 64: 785-821.

[9] CELLA, M. AND ETRO, F., (2010), "Oligopolistic Screening and Two-Way Distortion", mimeo.

[10] CLARKE, R., (1983), "Collusion and the Incentives for Information Sharing", *Bell Journal of Economics*, 14: 383-394.

[11] CREMER, J. AND MCLEAN, R., (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions", *Econometrica*, 56: 1247-57.

[12] DEQUIEDT, V. AND MARTIMORT, D., (2010), "Mechanism Design with Bilateral Contracts", mimeo IDEI.

[13] DODDS, S., (2003), "Economic Aspects of Individual Privacy and Confidentiality," Department of Economics, Queen's University, Canada.

[14] GAL-OR, E., (1985), "Information Sharing in Oligopoly", *Econometrica*, 53: 329-343.

[15] GROMB, D. AND MARTIMORT, D., (2007), "Collusion and the Organization of Delegated Expertise", *Journal of Economic Theory*, 37: 271-299.

[16] HAGENBACH, J. AND KOESSLER, F., (2010), "Strategic Communication Networks", *The Review of Economic Studies*, 77: 1072-1099.

[17] IOSSA, E., AND STROFFOLINI, F., (2010), "Vertical Integration and Costly Demand Information in Regulated Network Industries", mimeo.

[18] KHUN, K. AND VIVES, X., "Information Exchanges among Firms and their Impact on Competition", Luxembourg, Office for Official Publications of the European Communities.

[19] LAFFONT, J.J. AND MARTIMORT, D., (1997), "Collusion Under Asymmetric Information", *Econometrica*, 65: 875-911.

[20] LAFFONT, J.J. AND MARTIMORT, D., (2000) "Mechanism Design with Collusion and Correlation", *Econometrica*, 68: 309-342.

[21] LAFFONT, J.J. AND MARTIMORT, D., (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

[22] LIZZERI, A., (1999), "Information Revelation and Certification Intermediaries," *Rand Journal of Economics*, 30: 214-231.

[23] MAIER, N. AND OTTAVIANI, M., (2009), "Information Sharing in Common Agency: When is Transparency Good?", *Journal of the European Economic Association*, 7: 162-187.

[24] MANOVE, M., PADILLIA, A.J. AND PAGANP, M., (2001), "Collateral vs. Project Screening: A Model of Lazy Banks", *Rand Journal of Economics*, 32: 726-744.

[25] MARTIMORT, D., (1996), "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory", *Rand Journal of Economics,* 27: 1-31.

[26] MAS-COLELL, A., M. WHINSTON and J. GREEN (1995), *Microeconomic Theory.* Oxford University Press, Oxford.

[27] NOVSHEK, W. AND SONNENSCHEIN, H., (1982), "Fulfilled Expectations Counot Duopoly with Information Acquisition and Release", *Bell Journal of Economics*, 13: 214-218.

[28] PAGANO, M. AND T. JAPPELLI, (1993), "Information Sharing in Credit Markets", *Journal of Finance,* 48: 1693-1718.

[29] PADILLA, A. AND PAGANO, M., (1997), "Endogenous Communication Among Banks and Entrepreneurial Incentives", *Review of Financial Studies,* 10: 205-236.

[30] PADILLA, A.J. AND PAGANO, M., (2000), "Sharing Default Information as a Borrower Discipline Device", *European Economic Review,* 44: 1951-1980.

[31] RAITH, M., (1996), "A General Model of Information Sharing in Oligopoly", *Journal of Economic Theory*, 71: 260-288.

[32] RIORDAN, M. AND SAPPINGTON, D., (1988), "Optimal Contracts with Public ex post Information", *Journal of Economic Theory*, 45: 189-199.

[33] SHAPIRO, C., (1986), "Exchange of Cost Information in Oligopoly", *Review of Economic Studies,* 53: 433-446.

[34] STERN, L., A. EL-ANSARY AND A. COUGHLAN, (1996), *Marketing Channels*. Upper Saddle River, NJ: Prentice Halt.

[35] TAYLOR, C., (2004), "Consumer Privacy and the Market for Customer Information", *Rand Journal of Economics*, 35: 631–51.

[36] VIVES, X., (1984), "Duopoly information equilibrium: Cournot and bertrand", *Journal of Economic Theory*, 34: 71-94.

[37] VIVES, X., (2003), "Information Sharing: Economics and Antitrust", mimeo, 2007.

[38] VIVES, X., (2000), *Oligopoly Pricing*. Cambridge (Mass.), MIT Press.

[39] VIVES, X., (2006), "Information Sharing among Firms", *The New Palgrave Dictionary of Economics*.

[40] ZIV, A, (1993), "Information Sharing in Oligopoly: The Truth-Telling Problem," *RAND Journal of Economics*, 24: 455-465.