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INTERNATIONAL INTEREST-RATE AND PRICE-LEVEL LINKAGES UNDER FLEXIBLE EXCHANGE RATES: A REVIEW OF RECENT EVIDENCE

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Abstract

In an open economy, the scope for activist stabilization policy depends on the nature of the linkages between domestic and international markets for goods and assets. Two important relationships--purchasing power parity and uncovered interest-rate parity--have received extensive empirical attention in recent years and are fundamental building blocks of several empirical exchange rate models. This paper reviews and extends recent econometric findings on these two classical parity relationships and on their corollary, the international equality of expected real interest rates.

Econometric tests assuming rationality of expectations are on the whole unfavorable to the classical parity relationships: with few exceptions, they are strongly rejected. A central theme in the review of empirical work is the conditional heteroskedasticity of inflation and exchange rate forecast errors and the bias this statistical problem may impart to tests of international parity relationships. The paper proposes and implements a test for conditional heteroskedasticity which in many cases produces strong evidence that the problem is indeed important.

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International linkages between goods and asset markets are the key factors in exchange rate determination. The scope for activist stabilization policy depends on both the nature of the equilibrium implied by these linkages and the speed with which equilibrium is attained. Two important relationships--purchasing power parity, which links the exchange rate to relative national price levels, and uncovered interest-rate parity, which links the expected future path of the exchange rate to relative nominal interest rates--have received extensive empirical attention in recent years and are main building blocks of several empirical exchange rate models. 1/2 The purpose of this paper is to review and extend recent empirical evidence on these classical parity relationships within a rational-expectations framework.

When an economy is small and both classical parity relations hold even in the short run, monetary policy cannot influence the ex ante real rate of interest. Insofar as the ex ante real rate is an important determinant of saving and investment decisions, an important channel for stabilization policy disappears. In theoretical models of Dornbusch (1976) and Mussa (1982), temporary price-level stickiness allows money to influence the real interest rate in the short run even though uncovered parity holds exactly. Portfolio-balance models of exchange-rate determination (such as those of Branson (1979) and Girton and Henderson (1977)), stress imperfect substitution between bonds of different currency denomination. In these models, central banks can influence real interest rates if they can alter relative outside debt supplies.

As emphasized by Roll and Solnik (1979), among others, the classical

parity relations need not hold in a setting of uncertainty and risk aversion, even when prices are fully flexible and agents efficiently exploit all welfare-augmenting arbitrage opportunities. 3/ Unless at least one parity relationship fails, monetary policy cannot affect the expected real rate of interest; but the invalidity of a parity condition does not, in itself, imply that monetary policy has this power (see Henderson (this volume) and Obstfeld (1982b)). Thus, the series of tests performed below is at best a single component of a more extensive inquiry into the role of monetary policy in the open economy.

A central theme in our review of empirical work is the conditional heteroskedasticity of inflation and exchange rate forecast errors, and the bias this econometric problem may impart to tests of international parity relationships. Below, we propose and implement a test for conditional heteroskedasticity which in many cases produces strong evidence that the problem is indeed important.

The paper is organized as follows. Section I reviews the classical parity conditions and examines the recent behavior of bilateral ex post real interest rate differentials between the United States and the United Kingdom, Germany, Switzerland, Canada, and Japan. Section II carries out bilateral tests of ex ante real interest rate equality between the U.S. and these countries. Section III is devoted to empirical tests of uncovered interestrate parity. Finally, Section IV tests the hypothesis that relative purchasing power parity has held ex ante during the recent era of exchange rate flexibility.

I. Classical Parity Relationships and Real Interest Rates

To facilitate formal discussion of the classical parity relations, we

introduce the following notation:

 P_t = price level in the "home" country at the end of period t;

 P_t^* = price level in the "foreign" country at the end of period t;

S_t = the exchange rate at the end of period t, defined as the homecurrency price of foreign currency;

 $R_{k,t} = ln(1 + I_{k,t})$, where $I_{k,t}$ is the home-country k-period nominal interest rate at the end of period t;

 $R_{k,t}^{\star} = \ln(1 + I_{k,t}^{\star})$, where $I_{k,t}^{\star}$ is the foreign-country k-period nominal interest rate at the end of period t;

 $E_t(\cdot)$ = Conditional expectation operator, based on information available at the end of period t.

Purchasing power parity (PPP), in its relative form, states that the rate at which the relative price of two currencies changes over time must equal the difference between the national inflation rates. The doctrine of PPP has a long intellectual history, which is surveyed by Frenkel (1976, 1978). Using the foregoing notation, the PPP relation may be written as

$$\ln(S_{t}/S_{t-1}) = \ln(P_{t}/P_{t-1}) - \ln(P_{t}^{*}/P_{t-1}^{*}). \tag{1}$$

An implication of (1) is that relative PPP must be expected to hold ex ante, that is, for any k,

$$E_{t}[\ln(S_{t+k}/S_{t})] = E_{t}[\ln(P_{t+k}/P_{t}) - \ln(P_{t+k}^{*}/P_{t}^{*})].$$
 (2)

The ex ante relative PPP condition (2) is weaker than (1), of course. Magee (1978)

and Roll (1979) have suggested an "efficient markets" interpretation of ex ante PPP for a world with low transport costs.

Uncovered interest-rate parity (UIP) states that the nominal interest differential between similar bonds denominated in different currencies must equal the expected change in the logarithm of the exchange rate over the holding period. This explanation of international differences in nominal interest rates is associated with Fisher (1930). UIP implies that for any k,

$$R_{k,t} - R_{k,t}^* = E_t[\ln(S_{t+k}/S_t)].$$
 (3)

Condition (3) must hold when bonds differing only in their currencies of denomination are perfect substitutes in investors' portfolios.

Define the expected or ex ante k-period real interest rates for the home and foreign countries by

$$r_{k,t} = R_{k,t} - E_{t}[\ln(P_{t+k}/P_{t})], \qquad (4a)$$

$$r_{k,t}^* = R_{k,t}^* - E_{t}[\ln(P_{t+k}^*/P_{t}^*)].$$
 (4b)

By combining (2) and (3) with (4a) and (4b), we find that

$$r_{k,t} = r_{k,t}^* \tag{5}$$

Thus, under ex ante relative PPP and uncovered interest-rate parity, ex ante real rates of interest must be equalized internationally. The classical parity relationships imply that policymakers in a small open

Figure 1

Ex Post U.S.-U.K. One-Month Real Interest Differential

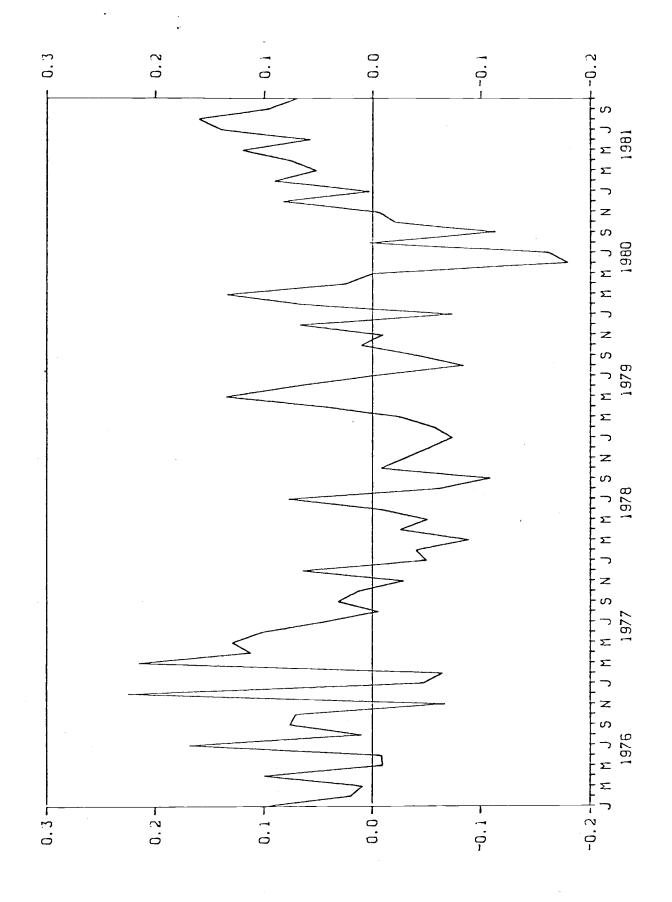


Figure 2

Ex Post U.S.-Germany One-Month Real Interest Differential

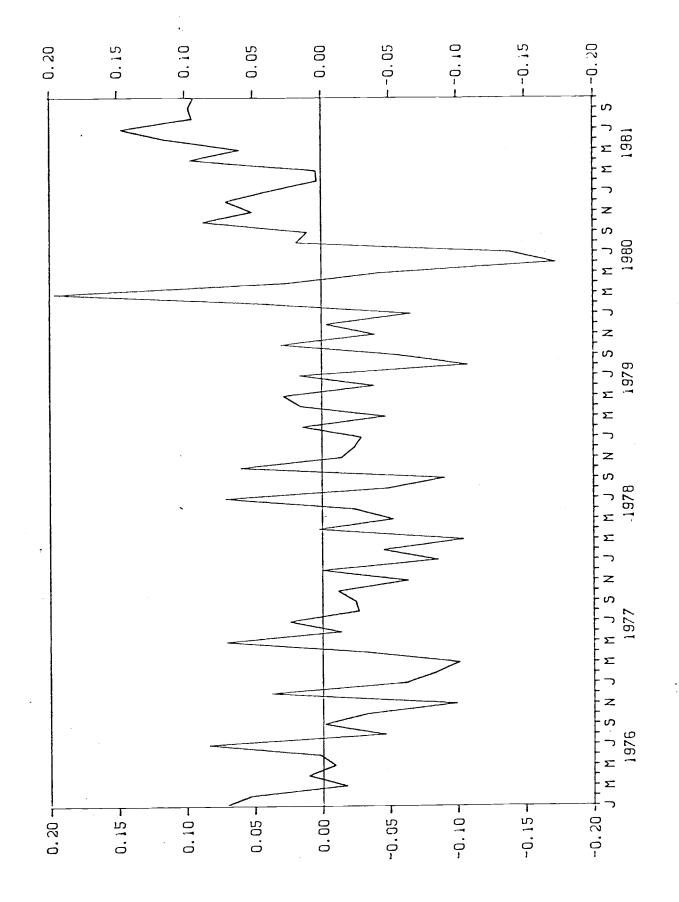


Figure 3

Ex Post U.S.-Switzerland One-Month Real Interest Differential

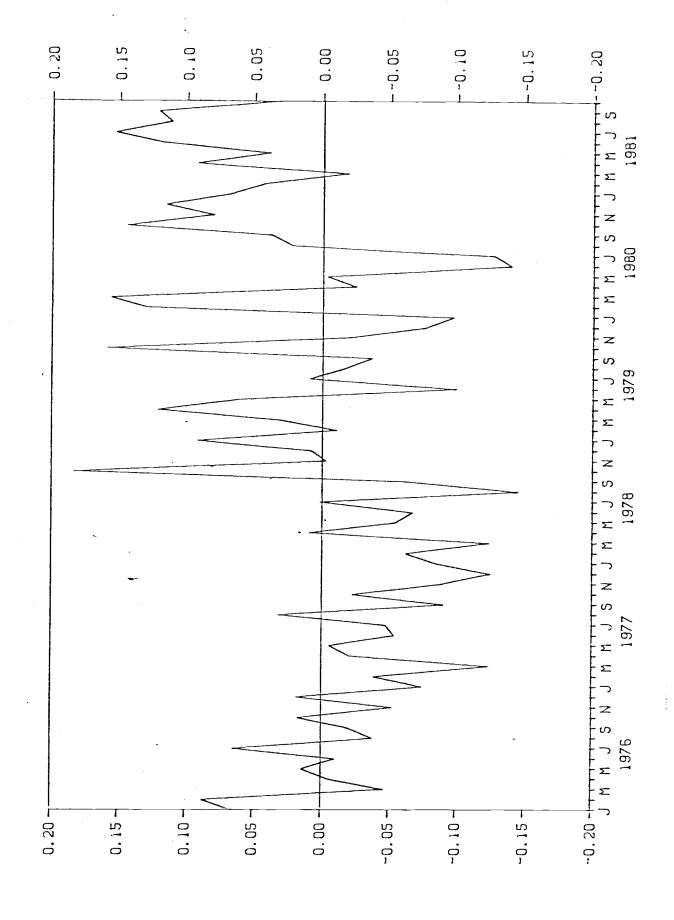


Figure 4
Ex Post U.S.-Canada One-Month Real Interest Differential

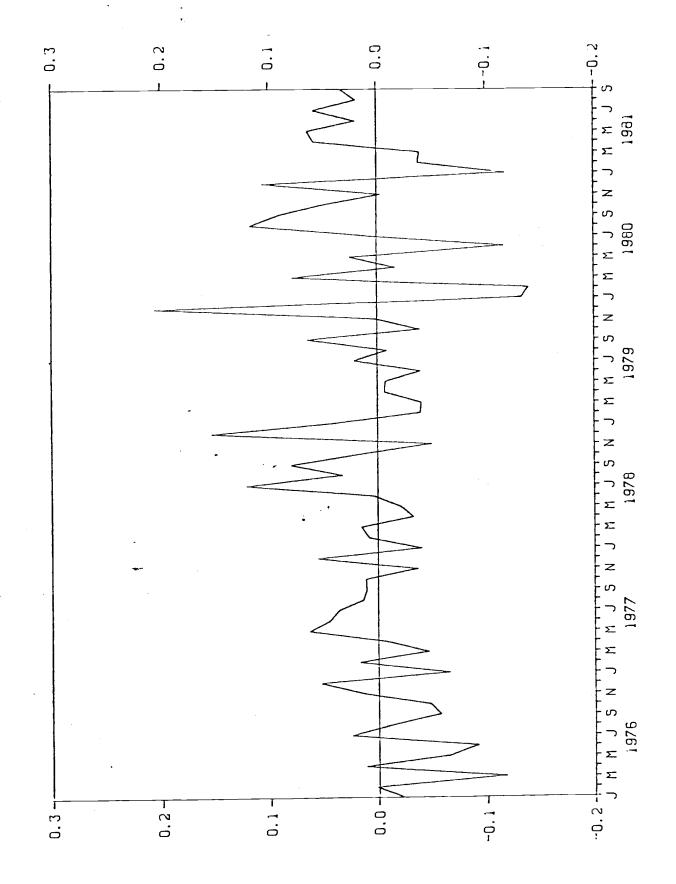
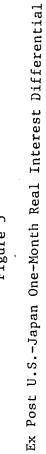
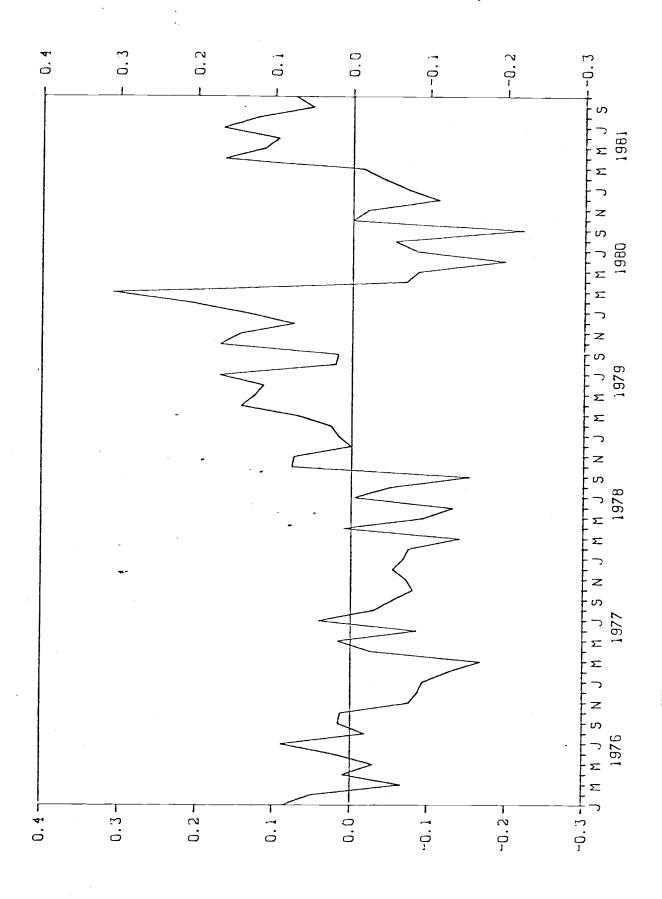


Figure 5





measures aimed at influencing the expected real interest rate.

Figures 1 through 5 plot monthly series of ex post one-month real interest rate differentials between the United States and the United Kingdom, Germany, Switzerland, Canada, and Japan. The series begin in January 1976 and are based on wholesale price index inflation rates and one-month Eurocurrency deposit rates.

Because the figures use nonoverlapping monthly data involving one-monthahead forecasts, the deviations from ex post real rate equality should be serially uncorrelated and trendless if agents' expectations are rational and real rates are equal across countries ex ante. All five figures suggest some degree of both serial dependence and trend, however. Ex post real rates in both the U.K. and Germany, for example, appear to have been on the whole above those in the U.S. over the period lasting from roughly July 1977 to December 1979. Between early 1976 and mid-1978, Swiss and Japanese ex post real rates were persistently above those in the U.S. The figures show a pronounced rise in U.S. ex post real rates relative to those in the five other countries beginning around the end of 1980.

While the figures are suggestive of the existence of ex ante real interest rate differentials over the period since January 1976, conclusive evidence can be provided only by econometric tests. We now turn to these.

II. The Equality of Ex Ante Real Interest Rates

The equality of ex ante real interest rates across countries has been tested in papers by Hodrick (1979) and Mishkin (1982). Hodrick (1979), using monthly data on three-month rates, performs bilateral tests to compare ex ante real rates in the United States and four other OECD countries over the period of generalized floating. He concludes that the empirical record, though mixed, is not inconsistent with the validity of condition (5). Mishkin (1982) carries out multilateral tests of equality using quarterly data for the U.S. and six other OECD countries. Over both the 1967:II to 1979;II and 1973:II to 1979:II sample periods, he obtains strong rejections of the hypothesis that ex ante real interest rates in the seven countries were equal.

In this section we test equation (5) taking into account the possible dependence of the conditional covariances of relative inflation forecast errors on nominal interest differentials. Such dependence induces a heteroskedasticity problem which invalidates hypothesis tests unless standard errors are estimated in an appropriate manner. Below, we establish the presence of a conditional heteroskedasticity problem and then use appropriate estimators to conduct a test similar to one of Hodrick's (1979). The results, based on monthly data, are on the whole unfavorable to the hypothesis that expected real interest rates have been equalized internationally in recent years.

A Test of the Hypothesis

The assumption of rational expectations yields a simple bilateral test of the hypothesis that ex ante real rates are equal across countries. Let π_{t+k} and π_{t+k}^{\star} denote the realized inflation rates in the home and foreign

countries between the end of period t and the end of period $t\!+\!k$. Then

$$\pi_{t+k} = E_{t}[\ln(P_{t+k}/P_{t})] + u_{t+k}, \tag{6a}$$

$$\pi_{t+k}^{*} = E_{t}[\ln(P_{t+k}^{*}/P_{t}^{*})] + u_{t+k}^{*}, \tag{6b}$$

where u_{t+k} and u_{t+k}^* are mean-zero inflation forecast errors uncorrelated with any variables observed by the market by the end of period t. Because subsequent forecast errors are not part of that information set, $E(u_{t+k}u_{t+k-j})$, $E(u_{t+k}^*u_{t+k-j}^*) \neq 0$ for j < k even though $E(u_{t+k}u_{t+k-j}^*) = E(u_{t+k}^*u_{t+k-j}^*) = 0$ for $j \geq k$. Combining (4a), (4b), and (5) with (6a) and (6b), we obtain the relation

$$\pi_{t+k} - \pi_{t+k}^* = R_{k,t} - R_{k,t}^* + u_{t+k} - u_{t+k}^*$$
 (7)

Because the composite forecast error $e_{t+k} \equiv u_{t+k} - u_{t+k}^*$ is uncorrelated with $R_{k,t}$ and $R_{k,t}^*$ (both of which are known to agents at the end of period t), the parameters a and b in the regression equation

$$\pi_{t+k} - \pi^*_{t+k} = a + b(R_{k,t} - R^*_{k,t}) + e_{t+k}$$
 (8)

may be estimated consistently by ordinary least squares (OLS). A test of the hypothesis [a b] = [0 1] is a test of the hypothesis that expected real interest rates are equal in the home and foreign country. $\frac{4}{}$

While OLS is consistent when applied to equation (8), it is generally

inefficient relative to an instrumental-variables estimator of the type discussed by Cumby, Huizinga, and Obstfeld (1982) and by Hansen (1982). $\frac{5}{}$ Because e_{t+k} is orthogonal to any variables in agents' information set at time t, many instrumental variables are available. Below, we use third-country interest rates as additional instruments to estimate the parameters of (8) by the two-step two-stage least squares (2S2SLS) technique described by Cumby, Huizinga, and Obstfeld (1982). $\frac{6}{}$

Let Q_t denote the row vector $[1 (R_{k,t} - R_{k,t}^*)]$ and stack the Tobservations on (8) to obtain the regression model $\pi - \pi^* = Qd + e$, where $d \equiv [a b]$. Let X_t be a row vector of instrumental variables (including Q_t), all of which are uncorrelated with e_{t+k} . Then the 2S2SLS estimate of d can be written as

$$\hat{\mathbf{d}} = (Q^{-1}X^{-1}X^{-1}Q^{-1}Q^{-1}X^{-1}X^{-1}X^{-1}(\pi - \pi^{*}),$$
 (9)

where $\hat{\Omega}$ is a consistent estimate of $\Omega = \lim_{T \to \infty} (1/T) E(X'ee'X)$. Under standard regularity conditions (which include covariance stationarity of all series), $\sqrt{T}(\hat{d} - d)$ converges to a normal random vector with mean zero and asymptotic covariance matrix

$$plim(Q^-X\Omega^{-1}X^-Q)^{-1}$$
.

When X = Q, \hat{d} reduces to the OLS estimator $(Q^{\gamma}Q)^{-1}Q^{\gamma}(\pi - \pi^*)$.

Computation of \hat{d} and its asymptotic covariance matrix requires a consistent estimate of Ω . If we assume that for all j, the conditional covariance

$$E(e_{t+k}e_{t+k-j}|X_{t},...,X_{t-j}) = \sigma_{j},$$
(10)

a constant, then d may be written as

$$\hat{d} = [Q \cdot X(X \cdot \hat{\Sigma} X)^{-1} X \cdot Q]^{-1} Q \cdot X(X \cdot \hat{\Sigma} X)^{-1} X \cdot (\pi - \pi^*)$$
(11)

where $\hat{\Sigma}$ is an estimate of the variance-covariance matrix E(ee'), formed using the residuals from a first-step, consistent estimation of (8) (by OLS, say). The matrix

$$T[Q^{T}\chi(X^{T}\hat{\Sigma}X)^{-1}X^{T}Q]^{-1}$$
(12)

provides a consistent estimate of the asymptotic covariance matrix of $\sqrt{T}(\hat{d}-d)$ in this special case. (The usual textbook formula for the asymptotic covariance matrix of the two-stage least squares estimator (see Dhrymes (1974)) is based on assumption (10) and the assumption that $\sigma_j = 0$ for j > 0.)

Formula (12) is used by Hodrick (1979) to calculate the asymptotic confidence ellipse for OLS estimates of (8). But (12) is not justified, even in the OLS case, unless the conditional covariances of forecast errors with respect to lagged interest differentials are constants. $\frac{7}{}$ Condition (10) would be valid if the variables included in X_t were all strictly exogenous; but that is certainly not the case here. $\frac{8}{}$ The validity of (10) is thus an issue of considerable importance in constructing hypothesis tests concerning the coefficients of (8). Below, we describe and implement a test of (10).

When (10) fails, estimation of the matrix Ω is more involved. Hansen (1982) suggests the following procedure. As before, generate estimates $\hat{\mathbf{e}}_t$ of the residuals of (8) using some consistent (but not necessarily efficient) estimation procedure, for example, OLS. Then, calculate a consistent estimate $\hat{\mathbf{s}}(\xi)$ of the spectral density matrix of the vector stochastic process $\{X_t\hat{\mathbf{e}}_t\}$,

$$s(\xi) = \frac{1}{2\pi} \sum_{\ell=-\infty}^{\infty} \exp(-i\xi\ell) \left(X_{t} \hat{e}_{t-\ell} X_{t-\ell} \right). \tag{13}$$

A consistent estimate of Ω is provided by $2\pi s(0)$. This heteroskedasticity-consistent covariance matrix estimator is convenient, as it does not require detailed specification of either the nature of the heteroskedasticity or the nature of the serial correlation in the residuals of (8).

A Test of Conditional Homoskedasticity

To determine the appropriate estimator for the matrix Ω in (9), the empirical validity of assumption (10) must be examined. Here, we test (10) for the case j = 1. In that case, (10) asserts that

$$E(e_{t+k}^2|X_t) = \sigma^2, \tag{14}$$

a constant, so that the forecast error e_{t+k} is <u>conditionally homoskedastic</u> with respect to time-t values of the instrumental variables. Rejection of (14) is clearly a sufficient indication that formula (12) is inappropriate and may lead to faulty inferences.

Since our ultimate goal is to test whether a=0 and b=1 in (8), it is reasonable to test for conditional heteroskedasticity under the tentative assumption that the null hypothesis of ex ante real interest rate equality is valid. That assumption implies that e_{t+k} is simply the composite forecast error π_t - π_t^* - R_k , t+ R_k^* , which is obervable. By the properties of conditional means, the random variable

$$\eta_{t+k} \equiv e_{t+k}^2 - E(e_{t+k}^2 | X_t)$$

has unconditional mean zero and is uncorrelated with any variable in the information set generated by X_t . If (14) is valid, $n_{t+k} = e_{t+k}^2 - \sigma^2$,

and so (14) can be tested by estimating an equation of the form

$$e_{t+k}^{2} = \alpha + \beta(R_{k,t} - R_{k,t}^{*}) + \gamma(R_{k,t} - R_{k,t}^{*})^{2} + \eta_{t+k}.$$
 (15)

A test of the hypothesis $\beta=\gamma=0$ is a test of conditional homoskedasticity. Because η_{t+k} is uncorrelated with the regressors in (15) (all of which are included in the information set generated by instrumental variables dated to or earlier), OLS yields consistent parameter estimates. But 2S2SLS again yields an efficiency gain in general. Any variables in the information set generated by χ_t may be used as instrumental variables. $\frac{10}{}$

The foregoing test is similar in spirit to one proposed by White (1980) for cross-sectional estimation environments. White suggests regressing estimated equation residuals on cross-products of regressors. His procedure thus imposes no a priori coefficient constraints. The present setting, however, is one in which a simple null hypothesis is to be tested. Absence of conditional heteroskedasticity when the null is imposed is clearly necessary if formula (12) is to lead to valid inferences.

Table I contains the homoskedasticity test results based on monthly data. Five countries—the United Kingdom, Germany, Switzerland, Canada, and Japan—are compared with the United States in the tests of ex ante real interest rate equality carried out below. Choosing an appropriate price index and interest rate is in itself an issue of considerable importance. Thus, the tests are performed for both consumer price index (CPI) and wholesale price index (WPI) inflation rates and for three nominal interest rates, the one-month and three-month Eurocurrency rates and a domestic three-month money-market rate. $\frac{11}{}$ All the resulting possibilities are represented in Table I. $\frac{12}{}$

The results illustrate the empirical relevance of the conditional hetero-

Table 1

Conditional Homoskedasticity of Inflation Forecast Errors

Countries	Interest Rate	Price Index	Test Statistic
U.S./U.K.	1-month Euro	CPI	3.42
u.s./u.ĸ.	1-month Euro	WPI	2.58
u.s./u.ĸ.	3-month Euro	CPI	7.21*
u.s./u.ĸ.	3-month Euro	WPI	7.53*
u.s./u.ĸ.	3-month money market	CPI	8.24*
u.s./u.ĸ.	3-month money market	WPI	9.40**
U.S./Germany	1-month Euro	CPI	6.20*
U.S./Germany	1-month Euro	WPI	4.23
U.S./Germany	3-month Euro	CPI	16.54**
U.S./Germany	3-month Euro	WPI	11.00**
U.S./Germany	3-month money market	CPI	32.35**
U.S./Germany	3-month money market	WPI	72.07**
U.S./Switzerland	1-month Euro	CPI	4.45
U.S./Switzerland	1-month Euro	WPI	11.97**
U.S./Switzerland	3-month Euro	CPI	74.08**
U.S./Switzerland	3-month Euro	WPI	42.38**
U.S./Switzerland	3-month money market	CPI	58.22**
U.S./Switzerland	3-month money market	WPI	11.64**
U.S./Canada	1-month Euro	CPI	2.56
U.S./Canada	1-month Euro	WPI	4.22
U.S./Canada	3-month Euro	CPI	21.49**
U.S./Canada	3-month Euro	WPI	9.93**
			(CONTINUED)

Table 1 (CONTINUED)

Countries	Interest Rate	Price Index	Test Statistic
U.S./Canada	3-month money market	CPI	2.19
U.S./Canada	3-month money market	WPI	5.95
U.S./Japan	1-month Euro	CPI	2.14
U.S./Japan	1-month Euro	WPI	2.55
U.S./Japan	3-month Euro	CPI	127.83**
U.S./Japan	3-month Euro	WPI	6.23*
U.S./Japan	3-month money market	CPI	9.58**
U.S./Japan	3-month money market	WPI	35.85**

Note: Data for tests using one-month interest rates run from January 1976 to September 1981. Data for tests using three-month interest rates run from January 1976 to July 1981. The test statistic is distributed asymptotically as $\chi^2(2)$. * = rejection at the 5 percent level; ** = rejection at the 1 percent level.

skedasticity problem in tests of real interest rate equality. In twenty of the thirty tests, the hypothesis of conditional homoskedasticity can be rejected at the 5 per cent level. In five of the remaining cases, the hypothesis can be rejected at the 20 percent level. Taken together, these results contradict the simplifying assumptions under which formula (12) is a consistent estimator of the asymptotic covariance matrix. Accordingly, a heteroskedasticity-consistent covariance matrix estimator is used to obtain the test results analyzed below.

Empirical Results

Tables 2a, 2b, and 2c report the results of bilateral tests of equality between the U.S. real interest rate and those of the U.K., Germany, Switzerland, Canada, and Japan. 13/ Except in the U.K. and Japanese cases, equality is strongly rejected for all combinations of price index and interest rate. The rejections in tests using onshore money-market interest rates (Table 2c) may in some cases be plausibly ascribed to the existence or prospect of capital controls. However, the rejections are almost equally strong when Eurocurrency interest rates are used in place of money-market rates; and arbitrage between differently denominated Eurocurrency deposits has not been restricted. 14/ On the whole, it seems difficult to explain the rejections of real interest rate equality by appealing to institutional factors that hinder international movements of capital.

In the case of the U.K., the evidence is on the whole very favorable to the hypothesis that ex ante real rates in the U.S. and U.K. have been equal during the recent years of floating exchange rates. While the U.S./U.K. test statistic lies in the 5 percent critical region in one

Table 2a

Equality of Ex Ante Real Interest Rates: One-Month Eurocurrency Rates

(January 1976 - September 1981)

Countries	Price Index	â	ĥ	Test Statistic
U.S./U.K.	CPI	0119 (.0086)	.7362 (.2351)	2.22
u.s./u.ĸ.	WPI	0216 (.0093)	.8197 (.2713)	5.34
U.S./Germany	CPI	.0278 (.0095)	.5031 (.2264)	9.13*
U.S./Germany	WPI	.0484 (.0148)	1371 (.3529)	11.21**
U.S./Switzerland	CPI	.0350 (.0125)	.3708 (.1970)	10.25**
U.S./Switzerland	WPI	.0844 (.0178)	3187 (.2655)	25.16**
U.S./Canada	CPI	.0010 (.0054)	.4043 (.1915)	12.61**
U.S./Canada	WPI	0111 (.0070)	.0317 (.3429)	8.01*
U.S./Japan	CPI	0028 (.0177)	.9623 (.2902)	.24
U.S./Japan	WPI	.0379 (.0125)	.0467 (.2350)	16.81**

Table 2b

Equality of Ex Ante Real Interest Rates: Three-Month Eurocurrency Rates

(January 1976 - July 1981)

Countries	Price Index	â	ĥ	Test Statistic
U.S./U.K.	CPI	0156 (.0084)	.7464 (.2135)	3.47
u.s./u.k.	WPI	0165 (.0093)	1.0665 (.1544)	4.15
U.S./Germany	CPI	.0380 (.0075)	.2997 (.1520)	26.02**
U.S./Germany	WPI	.0488 (.0122)	0972 (.2690)	17.68**
U.S./Switzerland	CPI	.0335 (.008 5)	.2945 (.1436)	25.32**
U.S./Switzerland	WPI	.0815 (.0137)	2740 (.1883)	46.04**
U.S./Canada	CPI	.0076 (.0040)	.3302 (.1238)	62.72**
U.S./Canada	WPI	0091 (.0039)	.2541 (.1816)	17.35**
U.S./Japan	CPI	.0060 (.0107)	.8323 (.1806)	1.40
U.S./Japan	WPI	.0446	1133 (.2223)	26.06**

Table 2c

Equality of Ex Ante Real Interest Rates: Domestic Money Market Rates

(January 1976 - July 1981)

Countries	Price Index	â	b	Test Statistic
u.s./u.ĸ.	CPI	0134 (.0074)	.7554 (.2400)	3.34
u.s./u.ĸ.	WPI	0153 (.0102)	1.1464 (.1974)	6.34*
U.S./Germany	CPI	.0379 (.0043)	.3137 (.1276)	78.85**
U.S./Germany	WPI	.0355	.1643 (.2569)	16.44**
U.S./Switzerland	CPI	.0352	.3451 (.1438)	23.11**
U.S./Switzerland	WPI	.0707 (.0108)	1144 (.1461)	58.37**
U.S./Canada	CPI	.0018 (.0049)	.2721 (.1015)	73.85**
U.S./Canada	WPI	0056 (.0046)	.2942 (.2032)	14.60**
U.S./Japan	CPI	.0180 (.0077)	.7229 (.1822)	5.47
U.S./Japan	WPI	.0385 (.0097)	5492 (.2290)	48.06**

case and is quite high in the others, the large size of the estimated constant term (a) relative to its estimated standard error is often the cause. In contrast, the estimated slope coefficient (b) is, in half the cases, within a standard deviation of unity. This evidence is consistent with the existence of a constant ex ante real interest differential between the U.S. and the U.K. The evidence therefore suggests that real interest rates in the two countries, though possibly different, are closely linked.

Tests for Japan using CPI inflation rates and Eurocurrency interest rates support the hypothesis of real interest rate equality. When WPI inflation rates are used in defining real interest rates, however, the hypothesis is easily rejected. Use of the CPI inflation rate together with the domestic money-market nominal interest rate yields a chi-square statistic that is quite close to the critical value of 5.99.

An interesting feature of the results is that nominal interest differentials have significant explanatory power in equations with the CPI inflation differential as the dependent variable, but do not usually help in forecasting relative WPI inflation rates. The U.K. is again an exception in this respect: Nominal U.S.-U.K. interest differentials are significant (and relatively unbiased) predictors of CPI and WPI inflation rates. The greater importance of the interest differential in CPI regressions is not surprising, for the expected future CPI is probably a better measure of the anticipated future "real" value of money to consumers than is the expected WPI. 15/

The tests demonstrate that ex ante real interest rate equality is often rejected decisively over the recent floating exchange rate period. In an attempt to shed light on the reasons for rejection, we now examine the two components of the hypothesis, uncovered interest parity and ex ante purchasing power parity.

III. Expectations and Nominal Interest Differentials

The hypothesis that expected exchange-rate movements offset nominal interest differentials so as to equalize expected nominal yields internationally has been tested extensively. Work in this area by Frenkel (1981) generally supports the view that uncovered interest-rate parity (UIP) has held quite closely over the period of generalized floating. However, a number of other studies reject the same hypothesis quite strongly (see Bilson (1981), Cumby and Obstfeld (1981), Geweke and Feige (1979), Hakkio (1981), Hansen and Hodrick (1980, 1983), Hsieh (1982), and Longworth (1981), among others).

We discuss below some econometric issues that arise in tests of UIP.

Among these, once again, is the problem of conditional heteroskedasticity, which is found to be important in the recent data. Tests of UIP which take this problem into account are performed, and these provide strong evidence against that hypothesis.

A Test of the Hypothesis

In the absence of default risk or transaction costs, covered interest arbitrage equates the forward premium on foreign exchange to the nominal interest differential between home- and foreign-currency bonds. Keynes (1923) provides the classic exposition. Denoting by $F_{k,t}$ the k-period forward price of foreign exchange, the covered interest parity condition may be written as

$$R_{k,t} - R_{k,t}^* = \ln(F_{k,t}) - \ln(S_t).$$
 (16)

Empirical studies such as Frenkel and Levich (1975,1977,1981), McCormick (1979), and Marston (1976) show that (16) holds quite closely in the Eurocurrency market, where the interst-bearing assets being compared have identical default and political risk characteristics.

If UIP holds, then (3) and (16) imply that

$$E_{t}[ln(S_{t+k})] = ln(F_{k,t})$$
 (17)

or, equivalently, that

$$ln(S_{t+k}) = ln(F_{k,t}) + v_{t+k},$$
 (18)

where v_{t+k} , the k-period forecast error $\ln(S_{t+k})$ - $E_t[\ln(S_{t+k})]$, has mean zero and is uncorrelated with information available at the end of period t. According to (18), the logarithm of the forward rate is an unbiased predictor of the future spot rate, and one-period ahead forecast errors (k = 1) are serially uncorrelated. When UIP fails, (17) becomes

$$E_{t}[ln(S_{t+k})] = ln(F_{k,t}) + \phi_{t}$$
 (19)

where ϕ_{t} is a risk premium which may fluctuate through time and may be serially correlated. Recent theoretical work shows that when asset holders are risk averse, market efficiency is consistent with the existence of a nonzero, possibly time-varying, risk premium (see, e.g., Frankel (1979b), Grauer, Litzenberger, and Stehle (1976), Hodrick (1981), Kouri (1977), Stockman (1978), and Stulz (1981)). When a nonzero risk premium exists, bonds denominated in different

currencies are imperfect substitutes in portfolios. The empirical implications of imperfect asset substitutability are that $\ln(F_{k,t})$ is not in general an unbiased predictor of $\ln(S_{t+k})$ and that the forward forecast error $\ln(S_{t+k})$ - $\ln(F_{k,t})$ need not be uncorrelated with information available to the market at time t.

Frenkel (1981) tests UIP by estimating the parameters of the equation

$$ln(S_{t+1}) = a + b ln(F_{1,t}) + v_{t+1}$$
 (20)

using monthly data (sampled from June 1973 to July 1979) on the spot and one-month forward dollar prices of the pound sterling, the French franc, and the Deutschemark. $\frac{16}{}$ A test of the hypothesis [a b] = [0 1] is a test of the UIP condition. Frenkel finds that the results of estimation are "broadly consistent" with the hypothesis that nominal interest differentials can be explained entirely by expected exchange rate movements.

A problem with the foregoing test, pointed out by Hansen and Hodrick and by Meese and Singleton (1982), (1980)/ is that the stochastic processes generating the logarithms of spot and forward exchange rates may be nonstationary. Even though least squares estimates of a and b in (20) will often be consistent in a nonstationary estimation environment, the usual asymptotic theory invoked to construct hypothesis tests becomes inapplicable. Mussa's (1979) observation that the logarithms of exchange rates seem to follow approximately a random walk is supported by statistical tests implemented by Meese and Singleton (1982). These tests, which involve the U.S. dollar's exchange rate against the Canadian dollar, the Swiss franc, and the Deutschemark, cannot reject the hypothesis that unit roots are present in the univariate autoregressive representations of the logarithms of spot and forward rates. The Meese-Singleton

findings suggest that the possibility of nonstationarity needs to be taken seriously in designing and evaluating hypothesis tests involving exchange rates.

A procedure that often avoids the unit-root problem is to test whether a = 0 and b = 1 in the equation

$$ln(S_{t+k}/S_t) = a + b ln(F_{k,t}/S_t) + v_{t+k}.$$
 (21)

Under the hypothesis of UIP, (21) is equivalent to (20), and states that the k-period forward premium is the market's expectation of the change in the logarithm of the spot rate over the next k periods. Like the tests cited above as rejecting UIP, the test just described works in terms of first differences rather than levels. Thus, the asymptotic theory used in testing is more likely to be justifiable.

Equation (21) is estimated below, and the hypothesis that a=0 and b=1 is tested. The tests are bilateral (unlike Bilson's (1981)), but expand Frenkel's (1981) information set by using third-currency forward premia observed at time t (which are uncorrelated with the disturbance v_{t+k}) as instrumental variables in forming 2S2SLS estimates of [a b]'. This yields parameter estimates more efficient than those produced by OLS, and so a more stringent test of the null hypothesis. Like Hansen and Hodrick (1980), we use weekly data on three-month forecasts.

A Test of Conditional Homoskedasticity

Tests of UIP have almost universally assumed that the conditional covariances of forecast errors do not depend on lagged forward premia. $\frac{17}{}$ Because

the forward premium is not a strictly exogenous variable, this assumption may be false, in which case the customary standard-error estimators have no asymptotic justification. As in the previous section, it is therefore of interest to test the conditional homoskedasticity assumption formally under the null hypothesis that UIP holds.

This can once again be done by estimating the equation

$$v_{t+k}^2 = \alpha + \beta \ln(F_{k,t}/S_t) + \gamma \ln(F_{k,t}/S_t)^2 + \epsilon_{t+k}$$
 (22)

Under conditional homoskedasticity, the expected value of v_{t+k}^2 conditional on forward premia observed at time t is a constant. Thus, we should find that $\beta = \gamma = 0$ in (22). As before, any variable in the conditioning set may be used as an instrumental variable in forming 2S2SLS estimates of (22).

Table 3 reports the results of testing the conditional homoskedasticity of three-month forward rate forecast errors. 18/ The tests involve the U.S. dollar's exchange rate against the pound sterling, the Deutschemark, the Swiss franc, the Canadian dollar, and the Japanese yen. Weekly data running from 7 January 1976 to 24 June 1981 are employed. The data are aligned to account for timing problems caused by bank holidays and weekends. 19/

In four of five cases, the null hypothesis of conditional homoskedasticity is strongly rejected. For the Canadian dollar, there is weak evidence against conditional homoskedasticity. The results suggest that a heteroskedasticity consistent covariance matrix estimator should be used in conducting hypothesis tests on the coefficients of equation (21).

TABLE 3

Conditional Homoskedasticity of Forward-Rate Forecast Errors

(Weekly Data, January 1976 - June 1981)

Exchange Rate	Test Statistic	
U.S./U.K.	3 08.13**	
U.S./Germany	26.38**	
U.S./Switzerland	13.20**	
U.S./Canada	2.57	
U.S./Japan	141.05**	

Note: The test statistic is distributed asymptotically as $\chi^2(2)$. * = rejection at the 5 percent level; ** = rejection at the 1 percent level.

Empirical Results

Results of estimating (21) and testing UIP appear in Table $4.\frac{20}{}$ In all cases save that of the dollar-Deutschemark exchange rate, the null hypothesis of UIP can be rejected at the 5 per cent level. In the case of Canada, however, rejection is entirely due to the large size of \hat{a} relative to its estimated standard error. As the estimated slope coefficient \hat{b} is quite close to unity, the rejection in the Canadian case cannot be considered very strong.

In four of five cases, the three-month forward premium has on average mispredicted the direction of movement of the subsequently observed spot rate. In the remaining case (that of Canada), the slope coefficient, while of the correct sign, is insignificantly different from zero. The test results are on the whole inconsistent with UIP, and they also suggest that forward premia contain little information regarding subsequent exchange-rate fluctuations. As emphasized by Dornbusch (1978, 1980), Frenkel (1981), and Mussa (1979), exchange rate changes over the recent period of floating seem to have been largely unanticipated.

An Additional Test

As a check on the validity of the conclusions reached above, an additional test, suggested by Geweke and Feige (1979) and by Hansen and Hodrick (1980), was performed. If UIP holds, then with weekly data and three-month forward rates, the forward forecast error \mathbf{v}_{t+13} must be uncorrelated with any information dated t or earlier. In particular, if \mathbf{v}_{t+13} is regressed on a constant, on \mathbf{v}_t , and on the time-t forward forecast errors for the other four currencies, one should not be able to reject the hypothesis that all

TABLE 4

Tests of Uncovered Interest Parity

(Weekly Data, January 1976 - June 1981)

Exchange Rate	â	б	Test Statistic
u.s./u.K.	.0086 (.0156)	2881 (.9741)	16.16**
U.S./Germany	.0214 (.0113)	7815 (1.1579)	3. 59
U.S./Switzerland	.0481 (.0214)	-2.2145 (1.1177)	9.11*
U.S./Canada	0076 (.0023)	.8285 (.7922)	12.44**
U.S./Japan	.0311 (.0097)	-2.8316 (.6740)	41.58**

coefficients equal zero. The results of this test are reported in Table 5. The equations were estimated by OLS, but the standard errors were calculated using a heteroskedasticity-consistent technique.

Rejection at the 5 percent level again occurs in all cases except that of Germany. Thus, the results of the present test are quite similar to those of Table 4. In addition, most of the estimated constant terms (Canada is the exception) are quite insignificant. None of the rejections in Table 5 appears to be caused exclusively by the large size of an estimated constant term relative to its standard error. Note that while the present tests are unable to reject UIP for dollar and Deutschemark deposits, tests by Hansen and Hodrick (1980) using a different data sample do reject that hypothesis.

While the two tests performed above cast considerable doubt on the hypothesis of perfect asset substitutability, their results should be interpreted with caution. First, political uncertainties may have introduced an element of default risk into forward transactions during the sample period. A second issue is the so-called "peso problem" (Krasker 1980), which is essentially a problem of finite-sample inference. If agents, over some significant time period, expect a major central bank intervention which does not materialize, nonoverlapping forward forecast errors will be correlated in the sample even if the expectation of intervention is rational in the light of past central bank behavior. While agents would be correct on average given an infinite sample containing infinitely many such episodes, the econometrician has only a finite history at her disposal. The dramatic Federal Reserve-Bundesbank interventions in the fourth quarters of 1978 and 1979 are examples of the type of event which, if incorrectly anticipated ex post, may give rise to a spurious correlation in nonoverlapping forecast errors.

Tests of Uncovered Interest Parity (Weekly Data, April 1976 - June 1981)

Exchange Rate	۰ در	$\hat{\mathfrak{b}}_1$	b ₂	b ₃	, b ₄	b, b _S	Test Statistic
U.S./U.K.	,0088	.1147	.5759 (.3178)	1979 (.2050)	.3059 (.3963)	.0054	22.91**
U.S./Germany	0049	0176 (.1509)	.0925 (.2750)	.2396 (.1695)	.2117 (.4463)	.0187 (.1785)	7.61
U.S./Switzerland	0071 (.0137)	1493 (.2137)	0304 (.3820)	.1529	.4762 (.6477)	.2333 (.2621)	13.00*
U.S./Canada	0093 (.0030)	0464 (.0462)	0231 (.1037)	.0089	2190 (.1479)	1060 (.0430)	50.97**
U.S./Japan	.0042	1462 (.2070)	.0066	0121 (.2530)	.5836	.4679	21.69**

Note: Standard errors appear in parentheses. The coefficient a represents a constant. The b_1 (i = 1,...,5) are the coefficients of the lagged forecast errors for the five currencies. b_1 = U.K., b_2 = Germany, b_3 = Switzerland, b_4 = Canada, and b_5 = Japan. The test statistic is distributed asymptotically as $\chi^2(6)$. rejection at the 5 percent level; ** = rejection at the 1 percent level.

IV. Exchange Rates and National Price Levels

The absolute version of the purchasing power parity (PPP) doctrine has not fared well in econometric tests on recent data, at least not in tests involving the U.S. (see, e.g., Frenkel (1981) and Krugman (1978)). Figures 6 through 10 display the time series of first differences of the real exchange rates of the U.K., Germany, Switzerland, Canada, and Japan against the U.S. The real exchange rate is defined as the dollar "value" of the foreign WPI divided by the U.S. WPI. The figures reveal that for all countries, the floating-rate period has been a period of much higher real exchange-rate variability vis-à-vis the U.S. than was the Bretton Woods era. 21/ The increase in the amplitude of deviations from PPP begins abruptly with the adoption of flexible rates. 22/

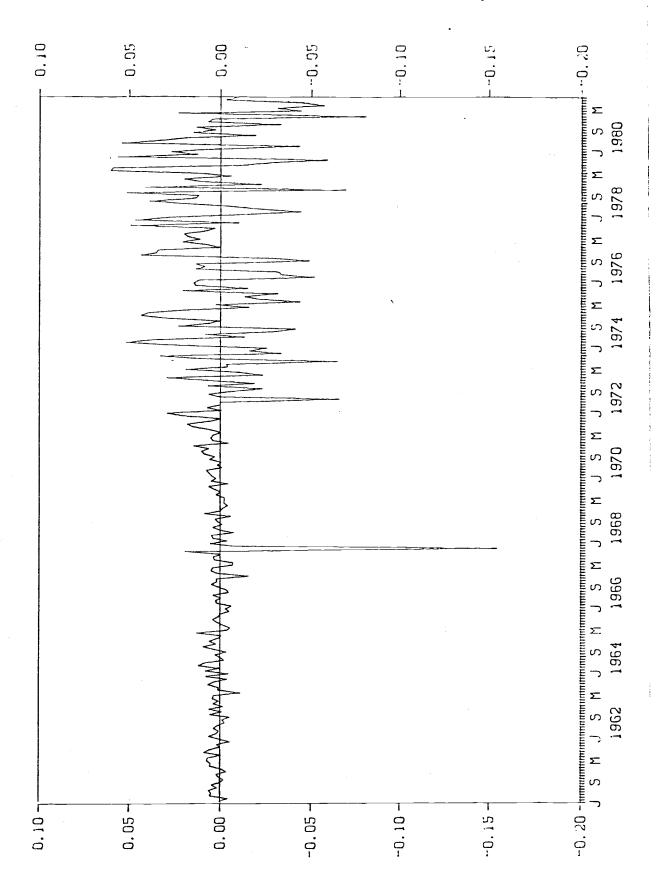
Here, we test whether relative PPP holds ex ante, that is, whether expected exchange-rate depreciation reflects the expected inflation differential between the home and foreign countries. If ex ante PPP does not hold, ex ante real interest rates will generally differ internationally. As Magee (1978) and Roll (1979) observe, ex ante PPP is under certain assumptions a consequence of the efficiency of international commodity markets. Both Roll (1979) and Frenkel (1981) present evidence that changes in real exchange rates are serially uncorrelated, and thus possess a key property of forecast-error series.

A Test of the Hypothesis

To design a test of ex ante relative PPP we return to equation (2). By combining (2) with (6a) and (6b) we obtain the equation

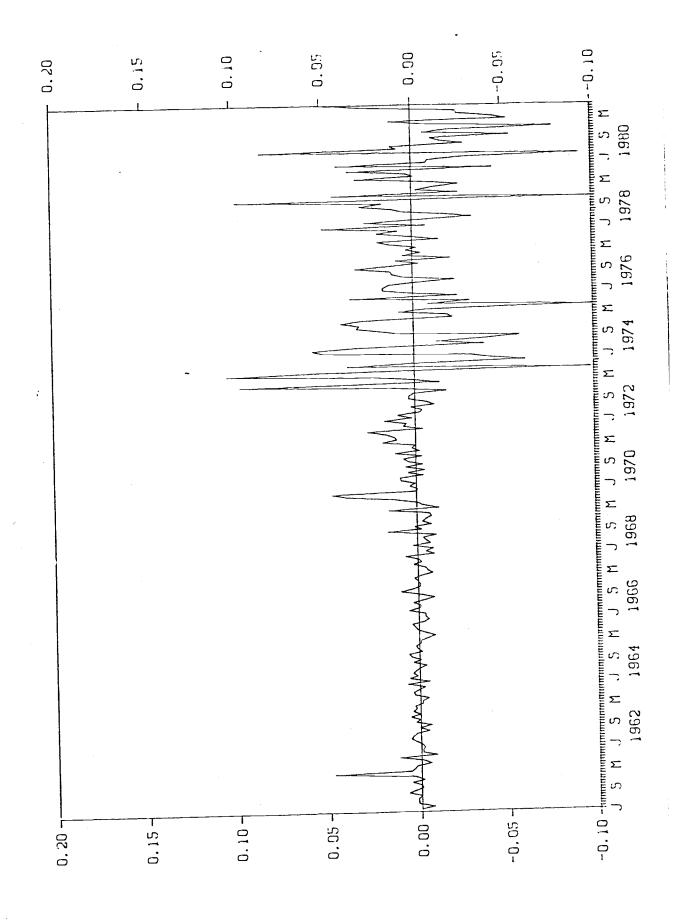
Figure 6

Change in the Real Exchange Rate between the United States and United Kingdom (Monthly Data)



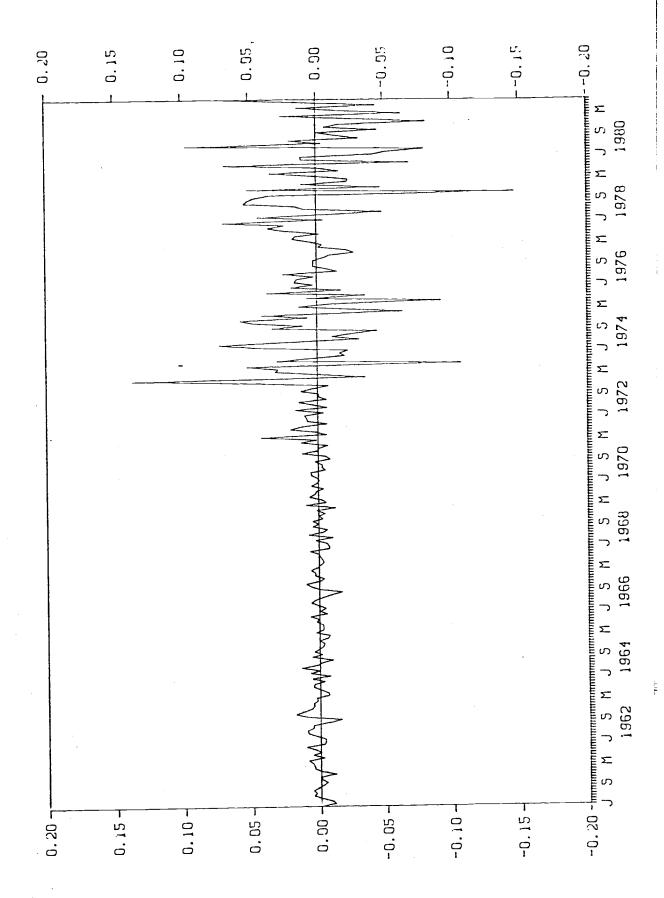
Change in the Real Exchange Rate between the United States and Germany Figure 7

(Monthly Data)

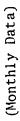


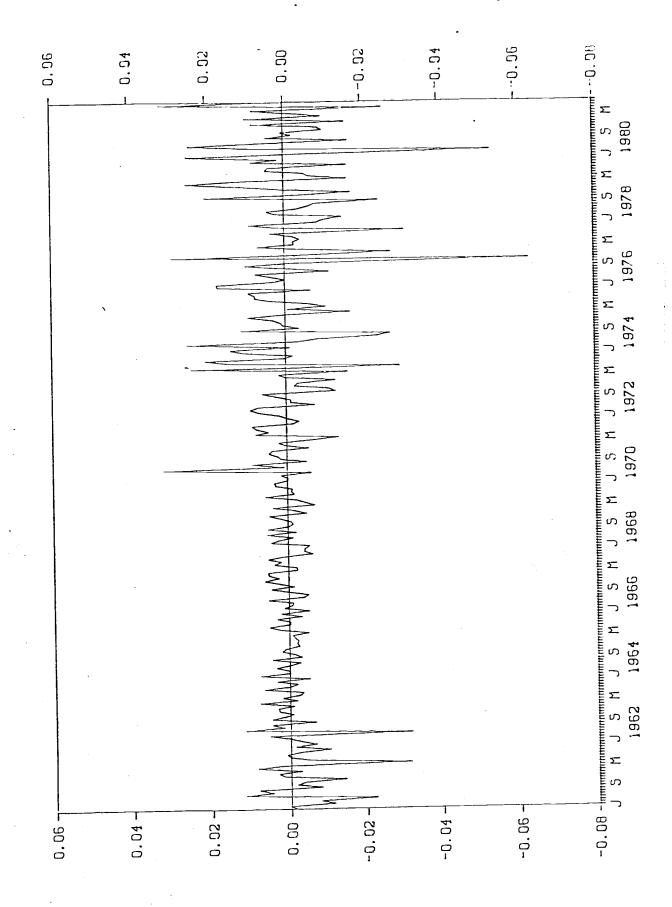
Change in the Real Exchange Rate between the United States and Switzerland Figure 8

(Monthly Data)



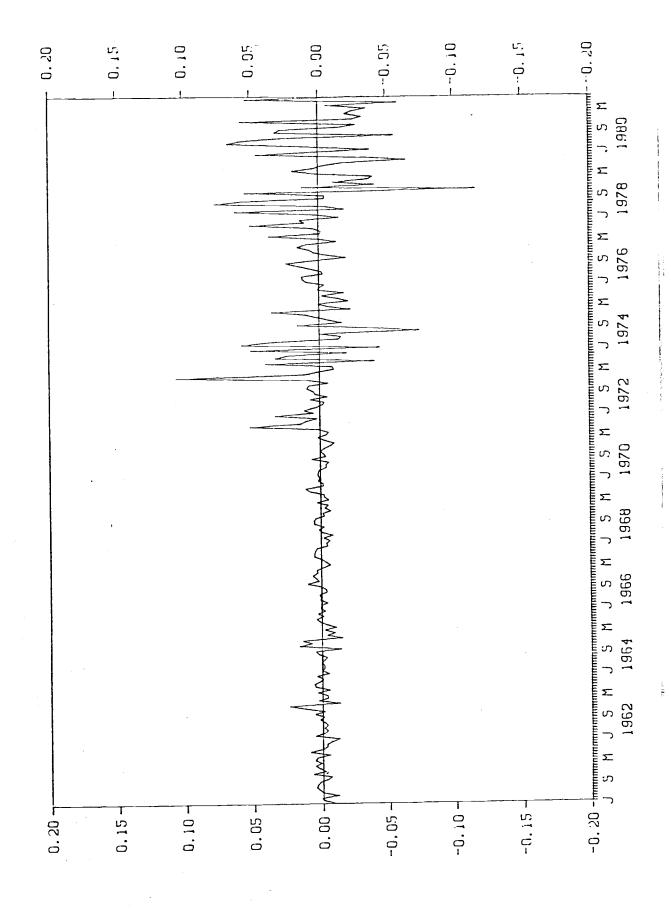
Change in the Real Exchange Rate between the United States and Canada





Change in the Real Exchange Rate between the United States and Japan Figure 10

(Monthly Data)



$$\pi_{t+k} - \pi_{t+k}^* = E_t[\ln(S_{t+k}/S_t)] + u_{t+k} - u_{t+k}^*.$$
 (23)

If $E_t[\ln(S_{t+k}/S_t)]$ were observable, a test of whether a=0 and b=1 in the equation

$$\pi_{t+k} - \pi_{t+k}^* = a + b E_t[\ln(S_{t+k}/S_t)] + e_{t+k}$$
 (24)

would be a test of ex ante relative PPP. Because the regressor in (24) is not observable, however, we must find a proxy variable. One possibility, following McCallum (1976), is to use the realized depreciation $\ln(S_{t+k}/S_t)$ as a proxy. With this substitution, (24) becomes

$$\pi_{t+k} - \pi_{t+k}^* = a + b \ln(S_{t+k}/S_t) + e_{t+k} - bv_{t+k}$$
 (25)

where $v_{t+k} = \ln(S_{t+k}/S_t) - E_t[\ln(S_{t+k}/S_t)]$. Because the independent variable in (25) is correlated with the composite disturbance $e_{t+k} - bv_{t+k}$, OLS is an inconsistent estimation procedure here. But an instrumental-variables estimator such as 2S2SLS can be used to estimate [a b] consistently. Since e_{t+k} and v_{t+k} are rational forecast errors, any relevant variables in the time-t information set may be used as instrumental variables.

Empirical Results

Results of estimating (25) over a one-month forecasting horizon with monthly data are reported in Table 6. $\frac{23}{}$ As in the previous tests, a heteroskedasticity consistent covariance matrix estimator was employed. Tests of

TABLE 6

Tests of Ex Ante PPP
(September 1975 - May 1981)

Countries	Price Index	â	б	Test Statistic
u.s./u.K.	CPI	0033 (.0010)	.1660 (.1205)	48.32**
u.s./u.K.	WPI	0048 (.0012)	1763 (.3415)	17.07**
U.S./Germany	CPI	.0033 (.0006)	.1902 (.0789)	166.21**
U.S./Germany	WPI	.0034 (.0009)	1707 (.1218)	94.25**
U.S./Switzerlan	d CPI	.0037 (.0007)	.1174 (.1111)	63.98**
U.S./Switzerlan	d WPI	.0073 (.0018)	2333 (.2255)	29.94**
U.S./Canada	CPI	0003 (.0005)	.0822 (.1395)	65.13**
U.S./Canada	WPI	0002 (.0007)	.1984 (.1786)	26.87**
U.S./Japan	CPI	.0007 (.0012)	.1523 (.1848)	21.46**
U.S./Japan	WPI	.0037 (.0008)	.0330 (.0725)	180.79**

Note: Standard errors appear in parentheses. The test statistic is distributed asymptotically as $\chi^2(2)$. * = rejection at the 5 percent level; ** = rejection at the 1 percent level.

the null hypothesis for a three-month forecasting horizon were also performed, but these are not reported as they only reinforce the message of Table 6.

That message is that expected exchange rate changes have been poor and biased predictors of relative inflation rates over the years of generalized floating. The hypothesis a=0 and b=1 is decisively rejected for all countries, regardless of the price index used. Further, the estimated slope coefficients are almost always insignificant and frequently of the wrong sign. The one exception to this occurs in the case of the dollar-Deutschemark rate, where we find that the expected depreciation rate does help forecast the U.S.-German CPI inflation differential.

Table 7 uses the adjusted Q statistic of Ljung and Box (1978) to test whether real exchange rate changes have been serially uncorrelated in recent years. The test statistics, which are computed for twelve lags using monthly data, confirm the Roll-Frenkel finding that real exchange rate changes are not serially correlated. Only in the Canadian case can the null hypothesis of no serial correlation be rejected at better than the 20 percent significance level. While the foregoing evidence is supportive of ex ante relative PPP, the results of Table 6 are strongly at variance with that hypothesis. On balance, it seems reasonable to conclude that the "efficient markets" version of relative PPP has not characterized the recent experience with floating rates.

TABLE 7
Tests for Serial Correlation of Real Exchange Rate Changes
(September 1975 - May 1981)

Countries	Price Index	Test Statistic	Marginal Significance Level
U.S./U.K.	CPI	7.79	.80
U.S./U.K.	WPI	10.63	.56
U.S./Germany	CPI	7.09	.85
U.S./Germany	WPI	7.73	.81
U.S./Switzerland	CPI	4.30	. 98
U.S./Switzerland	WPI	5.00	.96
U.S./Canada	CPI	17.62	.13
U.S./Canada	WPI	16.11	.19
U.S./Japan	CPI	11.88	.46
U.S./Japan	WPI	8.08	.78

Note: The test statistic is distributed asymptotically as χ^2 (12).

Conclusion

This paper has studied the interplay among price levels, interest rates, and exchange rates over the recent period of managed exchange-rate flexibility. Attention was focused on the two classical parity conditions that link prices and nominal interest rates internationally and on their corollary, the international equality of ex ante real rates of interest. Econometric tests of these propositions within a rational-expectations framework provided significant evidence against them. As a by-product of the investigation, we found that inflation and exchange-rate forecast errors appear to be conditionally heteroskedastic.

When monetary disturbances are dominant, the classical parity relationships may be a reliable guide to the comovements of nominal macro-variables. But the past decade has been characterized by moderate inflation coupled with substantial real disturbances. In such circumstances, the classical conditions appear to be too simple and aggregative to provide an adequate explanation of macroeconomic events in a world of differentiated commodities and assets.

Whether the failure of the parity relations has conferred monetary autonomy on small open economies is an entirely distinct question.

Further theoretical and empirical research is needed before a confident answer can be ventured.

Appendix: The Data

Section II

Prices: WPI's are taken from <u>International Financial Statistics</u> (IFS), line 63. CPI's come from IFS, line 64.

Interest Rates: One- and three-month Eurocurrency deposit rates come from Data Resources, Inc. (for the U.K., Germany, and Switzerland) and from the Harris Bank of Chicago Weekly Review (for Canada and Japan). Three-month domestic money-market rates come from Morgan Guaranty's World Financial Markets, and are quoted at or near the end of the month. For the U.S., the rate on prime industrial paper is used. Interbank deposit rates are used for the U.K., Germany, and Switzerland. For Canada, the rate used is that on prime finance company paper. The interest rate on three-month repurchase agreements is used as the Japanese money-market rate.

Section III

Spot and ninety-day forward exchange rates are noon rates collected by the Federal Reserve System. Spot rates are matched to the maturity of the corresponding forward contract, as described by Riehl and Rodriguez (1977). Morgan Guranty's World Calendar of Holidays is used to account for bank holidays, weekends, etc.

Section IV

Prices: Same as Section II.

Exchange Rates: End-of-month rates taken from IFS, line ag.

Footnotes

- * We thank J. Frenkel, R. Hodrick, B. Loopesko, R. Meese, and F. Mishkin for helpful comments and discussions. Assistance from N. Killefer and J. Withers is acknowledged with thanks. Obstfeld's research was supported in part by a grant from the National Science Foundation.
- 1. Examples include Bilson (1979), Frankel (1979a), Frenkel (1976), Hodrick (1978), and Hooper and Morton (1982).
 - 2. In these circumstances, monetary policy also loses its power to systematically influence the terms of trade or real exchange rate, and a second avenue of demand management is thus closed. Even so, monetary policy can be effective if nominal wages are sticky (see Obstfeld (1982a)). But this possibility disappears as well when wages are fully and instantaneously indexed to the aggregate price level. While monetary policy may be ineffective, tax policy can always succeed in driving a wedge between home and foreign ex ante real rates. The discussion below abstracts from taxes. Also ignored is the possibility that changes in monetary growth rates might influence the terms of trade through real effects of the Tobin-Sidrauski sort.
 - 3. If there are no default risks, covered interest arbitrage is riskless (in home currency terms), and so <u>covered</u> interest parity must always hold exactly in the absence of transaction costs. In contrast, uncovered arbitrage involves home-currency risk in an essential way. The relation between covered and uncovered interest parity is discussed in Section III, below.

- 4. This test is suggested by Hodrick (1979). However, he uses the k-period forward premium rather than the k-period nominal interest differential on the right-hand side of (8). The two procedures should yield very similar results when Eurocurrency interest rates are being compared (see Section III).
- 5. The reason is that the latter uses more information. As noted in the next paragraph of the text, OLS is a special "just-identified" case of this type of instrumental-variables estimator.
- 6. When the forecast horizon k exceeds one period, e_{t+k} is serially correlated and, under the null hypothesis, has the covariance matrix of a moving average (MA) process. As Hansen and Hodrick (1980) note, two-step serial correlation corrections of the generalized least squares type are inconsistent, even though OLS is consistent. The inconsistency is due to the fact that the nominal interest differential is not a strictly exogenous variable. To see this, suppose that k=3, so that the hypothesis involves three-month interest rates observed monthly. Assume that the vector stochastic process $[(\pi_t \pi_t^*)(R_{3,t} R_{3,t}^*)]'$ is covariance stationary and has the indeterministic bivariate Wold representation

$$\pi_{t} - \pi_{t}^{*} = \sum_{i=1}^{\infty} \psi_{i} v_{t-i} + \sum_{i=1}^{\infty} \theta_{i} \omega_{t-i} + v_{t},$$

$$R_{3,t} - R_{3,t}^{*} = \sum_{i=1}^{\infty} \rho_{i} v_{t-i} + \sum_{i=1}^{\infty} \delta_{i} \omega_{t-i} + \omega_{t},$$

where $E_t(v_{t+j}) = E_t(\omega_{t+j}) = 0$ for j > 0 (see Sargent (1979), p. 257). Under the null hypothesis, $E_t(\pi_{t+3} - \pi_{t+3}^*) = \sum_{i=0}^{\infty} \psi_{i+3} v_{t-i} + \sum_{i=0}^{\infty} \theta_{i+3} w_{t-i} = R_3, t - R_3^*, t$ = $\sum_{i=1}^{\infty} \rho_i v_{t-i} + \sum_{i=1}^{\infty} \delta_i \omega_{t-i} + \omega_t$. Thus, if ex ante real interest rates are equal, i=1 $\psi_3 = 0$, $\theta_3 = 1$, $\rho_i = \psi_{i+3}$ and $\delta_i = \theta_{i+3}$. This implies that $\pi_{t+3} - \pi_{t+3}^* - R_3, t$

- * $R_{3,t}^{\star} = v_{t+3} + \psi_1 v_{t+2} + \psi_2 v_{t+1} + \theta_1 w_{t+2} + \theta_2 w_{t+1} = e_{t+3}$. Now e_{t+3} has the covariance matrix of an MA process and, by Granger's lemma (see Ansley, Spivey, and Wrobleski (1977)), can be written as an invertible second-order MA process, $e_{t+3} = \zeta_{t+3} + \lambda_1 \zeta_{t+2} + \lambda_2 \zeta_{t+1}$. But even though e_{t+3} is uncorrelated with the regressors in (8), ζ_{t+3} need not be; and therefore application of a generalized-least-squares transformation to (8) will generally induce a nonzero correlation between the filtered disturbance ζ_{t+3} and the filtered regressors. For a more detailed argument, see Cumby, Huizinga, and Obstfeld (1982). Hansen and Hodrick (1980) use the Wold theorem to provide a similar characterization of the form of the forward exchange rate forecast error when contract periods overlap in the data.
- 7. See Dhrymes (1974), pp. 183-4. A more recent discussion of the failure of assumption (10) in regression models with i.n.i.d. residuals appears in White (1980). For time series models, see Engle (1982) and Hansen (1982). It is important to note that even if (10) does not hold, the estimator given in (11) still yields consistent (but relatively inefficient) estimates of parameters.
- 8. The condition would also be valid if the instruments and disturbances were jointly normally distributed. Without the joint normality assumption, however, lack of correlation need not imply statistical independence.
- 9. One can of course obtain more efficient covariance matrix estimates by imposing such information if it is known. Cumby, Huizinga, and Obstfeld (1982) describe one way of doing this. Their method is implemented in obtaining the empirical results reported in this paper. White (1980) has proposed a

heteroskedasticity-consistent covariance matrix estimator in a cross-sectional context, along with a test of homoskedasticity. White's test is discussed further below.

- 10. Any product of instrumental variables is a legitimate regressor in (15), but we have excluded all but two in the a priori belief that the others are less likely to be significant in explaining e_{t+k}^2 . It is worth emphasizing that the possibility of conditional heteroskedasticity does not contradict the assumption that e_{t+k} follows a covariance-stationary process. The latter assumption requires only that the <u>unconditional</u> variance of e_{t+k} be constant over time.
- 11. In order to distinguish empirically between inflation risk and default risk, studies of U.S. real interest rates focus on U.S. treasury bills, which yield a riskless nominal return (Fama 1975, Mishkin 1981, Shiller 1980). As Mishkin (1982) observes, cross-country comparisons of real interest rates are most informative when the bonds being compared have the same default and political risk characteristics. This is true of Eurocurrency deposits denominated countries' in different currencies, but not of onshore bonds traded in different/financial centers. Thus, tests of real rate equality using domestic money-market interest rates should be interpreted with caution. Another cause for caution is the fact that the prices entering CPIs and WPIs are not all sampled every month in revising the previous month's index; indeed some prices are observed only once a year (see Fama(1977), Nelson and Schwert (1977), and Shiller (1980)). This means that over short periods, changes in the price indices correspond only imperfectly to actual price-level movements. Because the implied measurement errors are serially correlated, our tests of real interest rate equality are,

to some extent, biased. It would be of considerable interest to perform these tests on twleve-month interest and inflation rates.

- 12. The instrumental variables in these regressions were the time-t nominal interest differentials for all countries in the sample and the time-t nominal interest differentials squared. All data are described in the appendix.
- 13. The instrumental variables in the regressions were the time-t nominal interest differentials for all countries in the sample.
- 14. Further, any political risks attaching to Eurocurrency deposits are not denomination-specific, and thus should not influence ex ante real interest differentials in the Eurocurrency market (cf. footnote 11, above).
- 15. Fama (1975) uses the CPI inflation rate in his study of the predictive power of U.S. short-term interest rates.
- 16. Similar tests have been conducted by Frenkel (1976) (for the German experience of the 1920s), Frankel (1980), and Stockman (1978). Levich (1978, 1979) surveys the early literature in this area.
- 17. Hansen and Hodrick (1980) make this assumption explicitly. In a later paper, Hansen and Hodrick (1983) allow for conditional heteroskedasticity in testing a forward foreign exchange pricing model. Hsieh (1982) accounts for conditional heteroskedasticity in his tests, and obtains results similar to those reported in Table 5 below.

- 18. The instrumental variables were the time-t forward premia and squared forward premia for all countries in the sample.
- 19. See Meese and Singleton (1982) and Riehl and Rodriguez (1977).
- 20. The instrumental variables were the time-t forward premia for all countries in the sample.
- 21. There are two sharp jumps in the German series over the Bretton Woods period. These correspond to the Deutschemark revaluations of 1961 and 1969. The spike in the U.K. series corresponds to the sterling devaluation of 1967.
- 22. Genberg (1978) also notes this phenomenon.
- 23. Instruments were lagged inflation differentials vis-à-vis the U.S. for all countries in the sample.

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