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INTERNATIONAL PORTFOLIO DIVERSIFICATION: SHORT-TERM FINANCIAL ASSETS AND GOLD

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ABSTRACT

Using a continuous-time finance-theoretic framework, this paper presents the optimal portfolio rule of an international investor who consumes N national composite goods and who holds N domestic-currency-denominated assets with known nominal interest rates in an environment where prices of goods, assets and exchange rates follow geometric Brownian motion. It is shown that the currency portfolio rule described in Macedo (1982a) is applicable to the case where there are N assets with a known price and one asset, gold, with a random price in terms of the numeraire.

Under these assumptions, it is found that the optimal portfolio of an investor consuming goods from all major industrialized countries (according to their weight in total trade) would be dominated in March 1981 by long positions in U.S. dollars (25%), yen (17%), D. marks (16%), French francs (15%), and pounds sterling (10%). An investor consuming only U.S. goods, by contrast, would hold 96% of his optimal portfolio in U.S. dollars. Because of the covariance of exchange rates and gold, the exclusion of the latter generates substantial reshuffling.

The analysis of the evolution of portfolios over time shows that shares changed dramatically at the beginning of the period and did not begin to approach their March 1981 values until the end of 1976. In the case of the yen and the pound there were oscillations throughout the period. With respect to the dollar share in the optimal portfolio of the U.S. and international investor, it is found that, in the period between late 1974 and mid-1976, a period in which the dollar is considered to have been "strong", a large decline in its optimal share took place.

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INTRODUCTION

The last decade showed a significant increase in trading in international financial markets, in face of increased uncertainty about prices, exchange rates and interest rates. In this paper we discuss this phenomenon in the context of international investment diversification, by individuals, firms and government agencies. We investigate optimal portfolio diversification by a class of risk-averse agents, who consume in fixed proportions goods produced in various countries. They are able to continuously reshuffle the composition of their wealth, made up of assets with known nominal returns denominated in different currencies. When prices of goods, prices of assets, and exchange rates are uncertain and consumption preferences are such that there is no "safe asset" in real terms, the best combination of risk and real return is provided by the portfolio rule presented here.

The theory of international portfolio diversification is an extension of the classic mean-variance framework of Markowitz (1959) and Tobin (1965). When continuous trading is possible, Merton (1969, 1971) spelled out under which conditions intertemporal maximization of expected utility would allow the separation of the portfolio rule from the consumption rule. In particular, he showed that, if asset prices are generated by stationary and lognormally distributed continuous-time stochastic processes (geometric Brownian motion) and if the instantaneous utility function of the agent is homothetic with constant relative risk aversion, a time-invariant portfolio rule could be derived. Furthermore, this rule would be the same as the one obtained if the agent was maximizing period by period a linear function of mean real return and the variance of return.

There have been several applications of the Tobin-Merton framework to international finance. In addition to those surveyed by Adler and Dumas (1982), one might refer here to the recent contributions of Stulz (1980), Dornbusch (1980a), Krugman (1981), Nairay (1981), Bortz (1982) and Meerschwam (1982) while bearing in mind the skeptical remarks of Tobin (1982). This paper contains a version of the international portfolio diversification model where the international investor is allowed to hold assets with uncertain prices, such as gold. The paper shows in Section I how the currency diversification rules derived by Kouri and Macedo (1978) and Macedo (1979 and 1982b) emerge as special cases of the portfolio rule derived here. Section II is devoted to the computation of optimal portfolios of gold and short-term financial assets over the period April 1973 to March 1981, using monthly data and quarterly holding periods. Drawing on the work of Goldstein (1982), the evolution of optimal portfolios over this period is also discussed. The conclusion outlines the main results and also contains some topics for future research in this area.

I OPTIMAL PORTFOLIO RULES

In this section, we present the optimal diversification rule for an agent who consumes fixed proportions of N composite goods produced in N countries and who holds a portfolio (that can be continuously reshuffled) of M assets with known nominal returns in domestic currency. The prices of the N goods, the prices of the M assets and the N-1 exchange rates are uncertain and are specified as continuous stochastic processes. As a result, real wealth accumulation, equal to the difference between the real rate of return on the portfolio and the rate of real consumption, is described by a stochastic differential equation. Given this flow budget constraint, at each moment in time the agent chooses a portfolio of assets and a consumption bundle. The optimal portfolio rule is thus one of the outcomes of the intertemporal constrained maximization of the expected utility of consumption from time 0 to time T[1/]. Since we are interested in the problem of an individual agent rather than in the determination of goods and assets prices and exchange rates in general equilibrium, we can assume that prices are stationary and lognormally distributed.[2/] For convenience, we specify prices in terms of the numeraire - arbitrarily defined as the currency of country N - and set M = N.[3/] Then, for i=1, ..., N, we have

$$\frac{dG_i}{G_i} = \pi_i dt + \sigma_i dz_i \tag{1}$$

$$\frac{dP_i}{P_i} = \mu_i dt + \delta_i du_i$$

where G_i is the price of the asset i expressed in terms of the numeraire, so that $G_i = G_i^d/S_i$ and $G_N^d = G_N$, G_i^d being the domestic currency price of asset i and S_i the price of currency i in terms of the numeraire;

P_i is the price of the good produced in country i expressed in terms of the numeraire, so that $P_i = P_i^d/S_i$ and $P_N^d = P_N$, P_i^d being the domestic currency price of good;

$\pi_i(\mu_i)$ is the instantaneous conditional mean proportional change per unit of time of $G_i(P_i)$;

$\sigma_i^2(\delta_i^2)$ is the instantaneous conditional variance per unit of time of $G_i(P_i)$, σ_{ij}, δ_{ij}, θ_{ij} being the instantantaneous conditional covariances per unit of time between G_i and G_j, P_i and P_j and G_i and P_j respectively; and

dz_i and du_i are Wiener processes with zero mean and unit variance, and instantaneous correlation coefficients ρ_{ij} (between dz_i and dz_j) and $\tilde{\rho}_{ii}$ (between dz_i and du_i).

It is convenient to measure (positive or negative) asset holdings as a proportion of real wealth, W. The share of wealth held in asset i is defined as:

$$(2) \qquad x_i = \frac{N_i Q_i}{W} \qquad i=1, \ldots, N;$$

where N_i are the holdings of asset i

$Q_i = G_i / \prod_j^N P_j^{\alpha_j}$ is the purchasing power of asset i over the N goods,[4/] α_j being the share of X_j in total expenditure and X_j the amount of good j consumed.

Utility is a strictly concave function of the instantaneous rate of consumption X_j of the N goods with constant expenditure share α_j and constant relative risk aversion $1-\gamma$. Given the state of the system, described by real wealth, we use the method of dynamic stochastic programming in order to find the optimal paths of the control variables x_i and X_j. Hence, we define the value function:

$$(3) \qquad J(W) = \max E_t \int_t^T \frac{1}{\gamma} \prod_j^N X_j(\tau)^{\alpha_j \gamma} \, d\tau$$

where E_t denotes expectation conditional upon information available at time t. From intertemporal utility maximization subject to the wealth accumulation constraint and the unity constraint on asset shares, we obtain first order conditions from which the consumption and portfolio rules can be derived.[5/] Stacking the M first order conditions on portfolio shares, we obtain:

$$(4) \qquad r + (1-\gamma)\Theta\alpha - (1-\gamma)Gx + (\lambda/J_W)e = \underline{0}$$

where $J_W = W\partial J/\partial W$;

$1-\gamma = -(\partial^2 J/\partial W^2)(W\partial J/\partial W)$;

λ is the Lagrange multiplier;

r is the vector of real returns;

α is the vector of expenditure shares;

x is the vector of portfolio shares;

e is a N column vector of ones;

$\underline{0}$ is a N column vector of zeros;

$G = \{\sigma_{ij}\}$ is the N by N variance-covariance matrix of changes in asset prices expressed in terms of the numeraire;

and $\Theta = \{\theta_{ij}\}$ is the N by N covariance matrix of changes in asset prices and changes in goods prices both expressed in terms of the numeraire.

Note that the expected real return on each asset is obtained by adding the expected proportional change in the purchasing power of the asset to its known nominal return in domestic currency:

$$r_i = R_i + dQ_i/Q_i; \qquad i = 1, \ldots, N .$$

Using the unity constraint on the portfolio shares (multiplied by $\gamma-1$), we augment (4) by another row, to get:

$$(5) \quad \begin{bmatrix} x \\ \hline \lambda/J_W(1-\gamma) \end{bmatrix} = \left[\begin{array}{c|c} G & e \\ \hline e' & 0 \end{array}\right]^{-1} \left\{ \left[\begin{array}{c|c} \Theta & 0 \\ \hline 0' & 0 \end{array}\right] \begin{bmatrix} \alpha \\ \hline 0 \end{bmatrix} + \frac{1}{1-\gamma} \begin{bmatrix} r \\ \hline 1-\gamma \end{bmatrix} \right\} .$$

Now we invert the augmented G matrix in (5):

$$(5') \quad \left[\begin{array}{c|c} G & e \\ \hline e' & 0 \end{array}\right] = \left[\begin{array}{c|c} G^{-1}K & y \\ \hline y' & -1/e'G^{-1}e \end{array}\right]$$

where $y = G^{-1}e/e'G^{-1}e$

and $K = I-ey'$, I being the identity matrix of order N.

Substituting (5') into (5) and ignoring the N+1 row (which is the definition of λ) we obtain an expression for the vector of N optimal portfolio shares:

$$(6) \qquad x = y + G^{-1}K\Theta\alpha + \frac{1}{1-\gamma}G^{-1}Kr$$

The optimal portfolio decomposes into a capital position y, such that $e'y = 1$, and two zero-net-worth portfolios. The latter are constructed by comparing the mean and variance of the real return on the particular asset (respectively involving r and $\Theta\alpha$) with the mean and variance of the real return on the capital position. This is done through the "comparison matrix" K, such that $e'G^{-1}K = \underline{0'}$. We refer to $y + G^{-1}K\Theta\alpha$ as the minimum variance portfolio, x^m, and to $G^{-1}Kr/1-\gamma$ as the speculative portfolio, x^s.[6/]

To interpret (6) further, it is convenient to decompose the N by N variance-covariance matrix of changes in numeraire prices of assets (G) and the N by N covariance matrix of changes in numeraire prices of assets and goods (Θ), viz:

$$(7) \qquad G = G^d + \hat{S} - \hat{E} - \hat{E}'$$
$$\Theta = H + \hat{S} - \hat{E} - \hat{\Psi}$$

where G^d (= g_{ij}) is the N by N variance covariance matrix of changes in the domestic currency price of assets;

$\hat{S}$ is the N-1 by N-1 variance-covariance matrix of exchange rate changes, $S(=\tilde{\sigma}_{ij})$, bordered by zeros;

$\hat{E}$ is the N by N-1 covariance matrix between changes in domestic currency prices of assets and bilateral exchange rates, $E = (\varepsilon_{ij})$, augmented by a column vector of zeros;

H (= η_{ij}) is the N by N covariance matrix of changes in domestic currency prices of assets and goods;

and $\hat{\Psi}$ is the N-1 by N covariance matrix between changes in bilateral exchange rates and domestic goods prices, $\Psi(= \Psi_{ij})$ augmented by a row vector of zeros.

Next consider the case where the Nth asset has a known domestic currency price, so that it is essentially a short bond or deposit denominated in the numeraire currency. Then the G and Θ matrix can be rewritten as:

$$(8) \qquad G = \left[\begin{array}{c|c} \underset{\sim}{G} & \underset{\sim}{0} \\ \hline \underset{\sim}{0}' & 0 \end{array}\right] \quad ;$$

and

$$\Theta = \left[\begin{array}{c} \underset{\sim}{\Theta} \\ \hline \underset{\sim}{0}' \end{array}\right]$$

where $\underset{\sim}{0}$ is a N-1 column vector of zeros.

Substituting (8) into (4), the last row becomes:

$$(9) \qquad r_N + \lambda/J_W = 0$$

Using (9) to eliminate λ/J_W from (5), we now solve for $\underset{\sim}{x}$, the N-1 column vector of portfolio shares:

$$(10) \qquad \underset{\sim}{x} = \underset{\sim}{G}^{-1}\underset{\sim}{\Theta}\alpha + \frac{1}{1-\gamma}\underset{\sim}{G}^{-1}(\underset{\sim}{r}-\underset{\sim}{e}r_N)$$

where $\underset{\sim}{r} = (r_1 \quad r_{N-1})'$

and $\underset{\sim}{e}$ is a N-1 column vector of ones.

To obtain x_N, we use the unity constraint:

$$(10') \qquad x_N = 1 - \underset{\sim}{e}'\underset{\sim}{x} \quad .$$

Denoting the identity matrix of order N-1 by $\underset{\sim}{I}$, the rule for the N assets is then written as:

$$(11) \qquad x = \Gamma\alpha + \frac{1}{1-\gamma}\,\Sigma r$$

where $\Gamma = \begin{bmatrix} \underset{\sim}{G}^{-1}\underset{\sim}{\Theta} \\ \hline e'(\underset{\sim}{I}-\underset{\sim}{G}^{-1}\underset{\sim}{\Theta}) \end{bmatrix}$ is such that $e'\Gamma = e'$;

$\Sigma = \begin{bmatrix} \underset{\sim}{G}^{-1} & -\underset{\sim}{G}^{-1}\underset{\sim}{e} \\ \hline -\underset{\sim}{e}'\underset{\sim}{G}^{-1} & \underset{\sim}{e}'\underset{\sim}{G}^{-1}\underset{\sim}{e} \end{bmatrix}$ is such that $e'\Sigma = \underline{0}'$ and $\Sigma e = \underline{0}$.

Comparing (6) to (11), it is clear that, when one asset has a known price in terms of the numeraire, the minimum variance portfolio ($\Gamma\alpha$) cannot be decomposed into a capital position depending on asset price uncertainty and a zero-net-worth hedge portfolio determined by the covariance of changes in assets and goods prices in terms of the numeraire, weighted by preferences ($G^{-1}K\Theta\alpha$). Also, the zero-net-worth speculative portfolio is computed in terms of real returns relative to the Nth asset ($\Sigma r/1-\gamma$), rather than relative to the capital position ($G^{-1}Kr/1-\gamma$).[7/]

When all asset prices are known, G^d, E and H in (7) vanish and the $\underset{\sim}{G}$ and $\underset{\sim}{\Theta}$ matrices can be written as:

$$(7') \qquad \underset{\sim}{G} = S$$

$$\underset{\sim}{\Theta} = \tilde{S} - \Psi$$

where $\tilde{S} = [S \quad \underset{\sim}{0}]$

The Σ matrix used to weight real returns in (11) now becomes the augmented inverse of the variance-covariance matrix of exchange-rate changes. The Γ matrix used to weight consumption preferences in (11) decomposes further, so that the minimum variance portfolio for the N-1 assets can be written as:

(12) $\underset{\sim}{x}^m = (\tilde{I} - S^{-1}\Psi)\alpha$

where $\tilde{I} = S^{-1}\tilde{S}$ is the (N-1 by N) matrix obtained by augmenting $\underset{\sim}{I}$ by a N-1 column vector of zeros.

Using the unity constraint to obtain x_N we can express the minimum variance and the speculative portfolios as:

(13a) $x^m = (I - \Phi)\alpha$

where $\Phi = \begin{bmatrix} S^{-1}\Psi \\ \text{-------} \\ -e'S^{-1}\Psi \end{bmatrix}$ is such that $e'\Phi = \underline{0}'$;

(13b) $x^s = \frac{1}{1-\gamma}\Sigma r$.

It is clear from (13) that the capital position is given by the expenditure shares so that the minimum variance portfolio reduces to α when goods prices are known.[8/] Also, we again have the two zero-net-worth portfolios of (6), one hedging against the covariance of changes in domestic currency prices of goods and in exchange rates $(-\Phi\alpha)$, the other, x^s, based on real returns relative to the Nth currency.

Finally, consider the problem of the investor who holds currencies and one asset with an uncertain price in terms of the numeraire. In this case a rule in the form of (13) still applies, as shown in the Appendix. The reason for this remarkable equivalence is that the asset with an uncertain price has the same effect on the portfolio rule as the currency of a country whose good is not consumed by the investor.

Before we proceed to apply the rule in (13) to actual data on eight major currencies and gold, mention should be made of the special case of purchasing power parity. In that case there are no relative price changes, so that there is only one random domestic currency good price, say in the Nth currency and $P_i = P_N$ for all i in (1) above. Then the Θ matrix in (7') can be expressed as

(7") $\Theta = -\Psi_N e'$

where Ψ_N is the Nth column of Ψ.

Using (7") in the minimum variance portfolio, we see that preferences drop out and that the capital position is all in the Nth asset:[9/]

(14) $x^m = \underline{1}_N - \Phi_N$

where $\underline{1}_N$ is a N column vector with zeros in the first N-1 rows and one in the Nth row.

and $\Phi_N = \left[\dfrac{S^{-1}}{-\underset{\sim}{e}'S^{-1}}\right]\Psi_N$

The rule in (14) is applicable to the case where $P_j^d = P_i S_j$ is the only random price and $\Theta = -\Psi_j e'$ and also to an investor who only consumes the jth good because then $I-\Phi$ reduces to $\underline{1}_j - \Phi_j$.[10/]

II OPTIMAL PORTFOLIOS OF SHORT-TERM FINANCIAL ASSETS AND GOLD COMPUTED

1. Overview

In this section, we apply the time-invariant portfolio rule derived in Section I to investors holding financial assets with three-month maturities denominated in eight major currencies; the U.S. dollar (abbreviated to \$), used as the numeraire currency, the Canadian dollar (C\$), the French franc (FF), the German mark (DM), the Italian lira (IL), the Japanese yen (¥), the Swiss franc (SF) and the pound sterling (£) as well as gold (GO). The short-term financial assets are such that their domestic currency price is assumed to be known. Gold, in turn, is an asset with a zero nominal return and an uncertain price, G_o, in terms of the numeraire. Since there is only one uncertain asset price and there are N-1 bilateral exchange rates, S_i, defined in (1) as units of domestic currency per dollar, it is more convenient to express the price of gold in ounces per dollar or as $1/G_o$.[11/] The investors are assumed to have static expectations about the rate of change of exchange rates, the price of gold and numeraire prices of the goods entering their consumption basket. As defined above, real returns are equal to the (certain) nominal return in domestic currency plus the proportional rate of change of the purchasing power of the currency (or of gold) over the previous three months.

In Section I we assumed that the investor consumed a basket composed of goods produced in the various countries, whose prices in terms of U.S. dollars are denoted by P_j in (1), with weights given by the constant expenditure shares. We refer to these goods by the country name: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JA), Switzerland (SZ), the United Kingdom (UK), and the United States (US). For empirical purposes, however, we identify each one of these national goods with the consumer price index of the country in question. As a consequence of this simplification, we refer to an investor consuming the goods including the consumer price index of, say, Germany as the "German investor" even though it imported goods.[12/] In terms of the utility function in (5) above, the "national investor" of country j is defined as having $\alpha_j = 1$ and $\alpha_i = 0$ for $i \neq j$. This contrasts with the "international consumer-investor" who weights national consumer price indices by the share of each country in total trade and can thus be thought of as a weighted average of national investors.[13/] Then, the role of preferences in optimal portfolios is assessed by comparing different national investors to the international investor.

We report in subsection 2 optimal portfolios over the entire sample period, April 1973 to March 1981. We emphasize the total portfolios of the U.S. and the international investors but the total portfolios of other national investors can immediately be computed, because the computed speculative portfolios do not depend on consumption preferences and the minimum variance portfolios of all national investors are reported. Subsection 3 investigates the evolution of these portfolios since September 1974 as investors revise their estimates of variances and covariances at the end of every quarter by including the new observations on the risk and return characteristics of each asset.

2. Optimal Portfolios, April 1973 - March 1981

In Table 1 we present the pattern of correlations and covariances between exchange rate (and gold price) changes which underlies the computation of the speculative portfolio as well as the computation of the minimum variance portfolios of the different investors. The elements of the upper triangular matrix give estimates of the S matrix (including the price of gold). Since mean changes in exchange rates are expressed in number per quarter, we multiply their variances and covariances by 100 and refer to the units as percentages.

TABLE 1

EXCHANGE RATES AND GOLD: CORRELATIONS AND COVARIANCES

APRIL 1973 - MARCH 1981

	GO (ounces/$)	C$ (Canadian dollars/$)	FF (French francs/$)	DM (DM/$)	IL (Lira/$)	¥ (Yen/$)	SF (Swiss francs/$)	£ (pounds/$)
GO	2.281	.032	.326	.461	.245	.042	.411	.308
C$	0.1	.044	-.011	*	.023	.023	*	-.011
FF	0.4	-0.1	.292	.264	.204	.119	.257	.138
DM	0.5	*	0.8	.372	.198	.134	.332	.156
IL	0.3	-0.2	0.7	0.6	.292	.089	.184	.154
¥	*	-0.2	0.4	0.4	0.3	.303	.187	.112
SF	0.4	*	0.7	0.8	0.5	0.5	.462	.173
£	0.4	-0.1	0.5	0.5	0.5	0.4	0.5	.260

Note: * Less than 0.05 in absolute value

Upper triangular matrix is $\underset{\sim}{G}_o = \{\sigma_i \sigma_j \rho_{ij}\}$, defined in equation (A1) of the Appendix (in number per quarter squared times 100).

Lower triangular matrix reports ρ_{ij}.

Since variances and covariances are not directly comparable (because the variables have different means), correlation coefficients are reported in the diagonal elements of the lower triangle. It is clear from the table that the correlation coefficients between "Ecu area" currencies - including the Swiss franc but excluding the pound sterling - are uniformly higher than all other correlation coefficients. The lowest of the Ecu area correlations, between the lira and the D.Mark, is 0.6. The table also shows that the correlation coefficients between the Canadian dollar and the other currencies are the lowest (and negative). Between these two extremes, we find the correlation coefficients of gold, the yen, and the pound with the other currencies. The highest variance is the variance of the price of gold. The ranking of the variances of dollar exchange rate changes, on the other hand, is lowest for the Canadian dollar. The two "hard currencies" of Europe (DM and SF) exhibit a somewhat higher variance than the other currencies.

As mentioned in Section I, the speculative portfolio is based on the inverse of S, each element of which shows the effect of an increase in the return differential relative to the U.S. dollar on the speculative demand of all investors for a particular currency or gold. The elements of S^{-1}, therefore, provide estimates on the degree of substitutability (negative entries) and complementarity (positive entries) between assets. To obtain the own and cross effects of an increase in the real return of a given asset on the speculative position of that asset for an investor with unitary risk aversion (γ=0), S^{-1} is augmented by a row (column) equal to minus the sum of the elements of all other columns (rows). The resulting matrix, which we denoted above by Σ, is reported in Table 2 using an ordering of the assets which emphasizes the strength of the substitutability (-) and complementarity (+) effects among assets showing what might be called "currency blocs."

TABLE 2

OWN AND CROSS EFFECTS

(%, APRIL 1973 TO MARCH 1981)

	SF	DM	FF	IL	C$	$	¥	£	GO
SF	2.0	-1.5	-0.4	*	-0.3	0.6	-0.7	0.2	-0.1
DM	-1.5	3.2	-1.6	*	-0.5	0.3	0.2	-0.1	-0.1
FF	-0.4	-1.6	3.7	-1.2	0.7	-1.0	*	-0.2	*
IL	*	*	-1.2	2.0	0.7	-1.1	0.1	-0.5	*
C$	-0.3	-0.5	0.7	0.7	6.7	-7.4	0.5	-0.2	-0.1
$	0.6	0.3	-1.0	-1.1	-7.4	9.7	-1.1	-0.3	0.1
¥	-0.7	0.2	*	0.1	0.5	-1.1	1.4	-0.5	0.2
£	0.2	-0.1	-0.2	-0.5	-0.2	-0.3	-0.5	1.8	-0.2
GO	-0.1	-0.1	*	*	-0.1	0.1	0.2	-0.2	0.2

Notes: Σ_o matrix defined by equation (A4) in the Appendix

* less than 0.05%

Columns and rows may not add to zero due to rounding.

It is clear from Table 2 that, over and above the strong substitutability between the U.S. and the Canadian dollar - and, to a lesser degree, between the D.Mark and the Swiss franc - there are two, partly overlapping, "currency blocs": the "Ecu bloc" and the "dollar bloc," where the criterion for a bloc is a cross-effect of at least 1%. While the French franc and the lira belong to both blocs, the pound does not belong to either one, all its cross effects being less than or equal to .5% in absolute value. Table 2 also shows that the assumption of separability between gold and currencies mentioned in the Appendix is approximately correct and that the own effect on gold is quite small.

The Canadian and U.S. dollar own effects far exceed those of other currencies. In the Canadian dollar case, this is largely the result of the fact that, as noted, it exhibits the lowest variance of exchange rate changes. The high value of the U.S. own effect is observed here because it equals the sum of all elements of the S^{-1} matrix. The own effects are much greater than the absolute value of the cross effects. The single exception is the cross-effect between U.S. and Canadian dollar assets which exhibits, by far, the highest degree of substitutability. A one per cent increase in the real rate of return on one asset decreases the other's share in the speculative portfolio by 7.4% of the initial share (when $\gamma=0$). Contrary to the presumption in two-country models, we find that the U.S. dollar and D.Mark as well as U.S. dollar and Swiss franc are complements in the speculative portfolio.[14/] Also, with the exception of the observed complementarity between lira and the pound and the Swiss franc, the cross effects between all other European currencies are negative.

The estimates of the degree of substitutability and complementarity among assets that are provided by the Σ matrix reported in Table 2, together with the covariances between changes in exchange rates and domestic currency prices of national goods weighted by consumption preferences determine the inflation-hedge portfolio. In Table 3, we report the correlation coefficients between changes in dollar exchange rates (and in the price of gold) and national inflation rates, which we denoted in (1) above by $\tilde{\rho}_{ij}$. It is evident that these correlations are generally small. Note that the negative correlations in the Canadian row imply that the Canadian dollar appreciates relative to the U.S. dollar not only when foreign consumer prices rise but also when Canadian prices increase. Similarly, a rise in U.S. prices is associated with a depreciation vis-a-vis the dollar of the French franc, the D.Mark, the yen and the Swiss franc.

While the low values of the elements of Table 3 (particularly the underlined ones), indicate little correlation between domestic price and exchange rate movements, they do not, by themselves, imply the rejection of the relative purchasing power parity hypothesis. By equating the Θ matrix (7') and (7") in Section I, however, we can derive the correlation coefficients which would obtain if purchasing power parity prevailed. In all cases, they are vastly different from those reported in Table 3.

Note further that each vector $S^{-1}\Psi_i$ has a simple interpretation: it gives the shares of the N-1 currencies in the inflation hedge portfolio of the national investor of country i.[15/] The dollar share of the inflation hedge portfolio is then obtained residually. Subtracting this portfolio from the expenditure share of the national investor of country i (given by a vector with 1 in row i and zeros elsewhere) we obtain the minimum variance portfolios of the national investor of country i. These portfolios are stacked together and reported in Table 4. They form what we denoted in Section I as the I - Φ matrix (expressed in percent). For example, the minimum variance portfolio of the German investor

TABLE 3

THE CORRELATION MATRIX OF EXCHANGE RATES

AND NATIONAL CONSUMER PRICE INDEXES

1973;4 - 1981;3

Asset \ Good	CA	FR	GE	IT	JA	SZ	UK	US
GO	0.1	-0.1	-0.1	-0.4	-0.3	*	*	-0.2
C$	-0.1	-0.3	-0.3	-0.1	-0.3	-0.2	*	-0.2
FF	0.1	0.1	0.2	0.1	0.1	0.2	-0.1	0.1
DM	0.3	0.2	0.2	*	*	0.3	*	0.2
IL	*	0.1	0.3	0.3	0.2	0.2	-0.1	-0.1
¥	*	0.2	0.3	0.3	0.2	0.4	0.1	0.2
SF	0.2	0.1	0.3	0.1	*	0.3	0.1	0.2
£	*	*	*	0.1	0.2	0.1	0.1	-0.2

Note: * less than 0.05 in absolute value

Ψ_o matrix defined after equation (A1) of the Appendix

TABLE 4

THE MINIMUM VARIANCE PORTFOLIO

OF NATIONAL INVESTORS

(%, APRIL 1973 TO MARCH 1981)

Holdings of \ Investor from	CA	FR	GE	IT	JA	SZ	UK	US
GO	-0.1	0.7	0.8	4.4	4.8	0.1	0.5	1.2
C$	105.1	7.6	8.1	-4.9 (-2.5)	21.0	6.7	8.4	8.5
FF	3.6	100.6	4.6	4.0	-5.8	3.3	19.7	1.6
DM	-6.5	-3.3	98.4	1.6 (2.9)	-4.0 (-2.5)	-10.4	-6.6	-6.0
IL	2.2	0.3	5.3	86.8	-3.3	-0.5	-0.3	5.0
¥	0.9	-0.8	-1.3	-4.3 (-8.4)	98.5 (94.0)	-7.2	-2.9	-0.9 (-2.1)
SF	-0.1	0.3	-3.0	-1.3	8.3 (11.8)	96.6	-5.8	-3.0
£	0.4	0.8 (1.6)	2.3	-0.8 (4.2)	-9.6 (-4.1)	3.6	95.7	3.9 (5.3)
$	-5.5	-6.2	-4.5	14.6 (11.6)	-9.8 (-13.1)	1.1	-8.7	89.7

I-Φ matrix defined by equation (A3) in the Appendix times 100.

Notes: Numbers in parentheses refer to the corresponding element in the I-Φ matrix without gold (noted only when significantly different).

Columns may not add to 100 due to rounding.

(3rd column of Table 4) would include long positions in DM (98%), Canadian dollars (8%), French francs (5%), pounds (2%) and gold (1%), and short positions in lire (3%), and yen (1%). We find that inflation risk is minimized for most national investors by holding gold, pound, French franc, and Canadian dollar assets, while borrowing in U.S. dollars, Swiss francs, yen, and DM.

The underlined elements in each one of the columns of Table 4 may also be interpreted as the extent to which a long position in the domestic currency of a given national investor is chosen in the construction of the inflation-hedge portfolio. This is consistent with the domestic currency being a "preferred monetary habitat" and is thus supported for those currencies whose "diagonal" element in Table 4 is greater than 100, i.e. Canada, France, and Switzerland.[16/] Hence, a "preferred local currency habitat" may be observed as a result of the inflation-hedging portfolio provided by one's domestic money, even in the absence of transaction or information costs.

The last column of Table 4 is of particular interest because, as noted at the end of Section I, if relative prices between national goods do not change, consumption preferences do not enter the minimum variance portfolio. In this context, relative purchasing power parity would imply that uncertainty with respect to the N national goods prices collapses into uncertainty about the price of a single national good, e.g. the good produced in the country of the numeraire currency.[17/] With the U.S. dollar chosen as the numeraire, the minimum variance portfolio of the U.S. investor would also be the "universal" minimum variance portfolio under purchasing power parity. The portfolio is dominated by a long position in U.S. dollars (90%). The U.S. (cum-universal-PPP) investor holds less than his consumption share in home currency in order to maintain an 8.9% long position in Canadian dollar assets while D.Mark, yen, and Swiss franc denominated borrowing supports short-term investments in gold, French franc, lire and pound assets.

Contrasting the last column of Table 4 with the "diagonal" elements of the other columns reveals that relative price changes were important, particularly in the cases of Italy and Japan. Specifically, we find that the Japanese investor's minimum variance portfolio differs significantly from the "universal-PPP" portfolio. Of particular note are the sign and magnitude of positions in Canadian dollar, French franc, lira, Swiss franc, and pound sterling assets. The last row of Table 4, which reports the residually determined shares of the U.S. dollar, also reflects the significance of relative price changes. Note that the 89.7% dollar share in the "universal" minimum variance portfolio stems from -10.3% dollar share in the "universal" inflation-hedge portfolio. It is thus smaller than the dollar share in the minimum variance/inflation-hedge portfolio of all national investors, especially those of the Italian and Swiss investors. In sum, this analysis shows that, since national inflation rates are not fully anticipated and relative prices change, even investors who only consume domestic goods (and are infinitely risk averse) will not hold a portfolio consisting only of home currency denominated claims. Rather, national investors exploit inflation risk-minimizing gains to diversification as provided by the variance-covariance structure of exchange rate changes relative to the covariance of exchange rate and domestic price changes.

Having presented and interpreted the Σ and $I-\Phi$ matrices, we are now in a position to report the components of the total portfolio, for given assumptions about consumption preferences and risk aversion. This is done in Table 5 for the U.S. investor (left panel) and an international investor (right panel). The speculative portfolio in the center column, common to both investors, is computed under the assumption that they are Bernouilli investors ($\gamma=0$).[18/]

As expected, the U.S. investor's minimum variance portfolio differs significantly from the same portfolio for the international investor (columns

Table 5

Optimal portfolio shares and their components

(%, April 1973 to March 1981)

Asset	U.S. investor				International investor		
	Total Portfolio (1)	Minimum Variance Portfolio: capital position (2a)	Minimum Variance Portfolio: inflation hedge (2b)	Speculative Portfolio (R.A.=1) (3)	Minimum Variance Portfolio: inflation hedge (4b)	Minimum Variance Portfolio: capital position (4a)	Total Portfolio (5)
GO	4	0	1	3	2	0	5
C$	0(3)	0	8	-8(-6)	8	8	8(11)
FF	1(1)	0	2	-1(-1)	4	12	15(15)
DM	-5(-4)	0	-6	1(2)	-4	19	16(17)
IL	0(-1)	0	5	-5(-6)	-1	8	2(2)
¥	4(0)	0	-1	5(2)	-2	14	17(12)
SF	-3(-1)	0	-3	0(2)	-1	3	2(6)
£	3(9)	0	4	-1(3)	0	11	10(15)
$	96(94)	100	-10	6(4)	-6	25	25(22)
Total	100(100)	100	0	0(0)	0	100	100(100)

Notes: col. (1) = col. (2a) + col. (2b) + col. (3)
col. (5) = col. (4a) + col. (4b) + col. (3)
numbers in parentheses in cols. 1, 3 and 5 refer to the optimal shares when gold is excluded

(2a + 2b) vs. columns (4a + 4b)). With the exception of the lira, we find, however, that the sign of the difference between expenditure shares and the minimum variance portfolio shares is invariant to consumption preferences. For example, both U.S. and international investors have greater holdings of gold, Canadian dollars and French francs than implied by the capital position (i.e. inflation-hedge portfolio shares greater than zero). On the other hand, the zero-net-worth inflation-hedge portfolio decreases the share of the D.Mark, yen, Swiss franc and U.S. dollar assets in the minimum variance portfolio.

The relationship between the minimum variance portfolio and consumption preferences can be illustrated further by multiplying each element ij of the $I-\Phi$ matrix by the ratio of the expenditure share j (column) to the minimum variance portfolio share i (row). We then obtain a matrix of elasticities of the shares of the international investor's minimum variance portfolio with respect to shares in expenditure. For example, the "own" elasticity for the U.S. dollar is 1.16. A ten per cent increase in the international investor's share of expenditure on U.S. goods would increase the dollar component of the minimum variance portfolio from 19% to 21% (=19 x 1.116). Other countries with "own" elasticity greater than one are Germany, Italy, Japan, Switzerland, and the U.K. Sizable cross-elasticities with respect to an increase in the U.S. expenditure share are on holdings of D.Marks (-.08) and lire (.06).

The speculative portfolio, dependent on own and cross effects between assets and real return differentials with the U.S. dollar includes long positions in U.S. dollars (6%), yen (5%), and gold (3%) and short positions in Canadian dollars (-8%) and lira (-5%). The relatively large positive share for the U.S. dollar is attributable less to its mean real return (-1.9%) than to its substitutability with Canadian dollar, French franc, and lira assets and to its complementarity with the Swiss franc. Return differentials with the dollar largely explain the attractiveness of yen assets (4% return differential) and gold (20% differential) and the short position in lira (-1.5% differential). Note that while the return differential for the Swiss franc was the same as for the yen, its share is zero rather than 5%. The reason in found in Table 2, where it can be seen that the yen is a strong substitute for the dollar compared with the weak compelementarity between the dollar and the Swiss franc. The high degree of substitutability between the Canadian and U.S. dollars is reflected by the fact that a relatively small difference in mean real returns results in a long position in U.S. dollar assets financed by Canadian dollar liabilities.

The total portfolios of the international and the U.S. investors (reported in Table 5) are computed under the assumption of unitary risk aversion. Of course, the higher the degree of risk aversion, the smaller the contribution of the speculative to the total portfolio. At the limit, when risk aversion is infinite, the speculative portfolio disappears so that the minimum variance and total portfolios are the same and optimal shares are independent of returns. It is clear from Table 5, column 5, that the total portfolio of the international investor is dominated by the minimum variance portfolio. The long positions of gold, D.Marks, yen, and U.S. dollars in the latter are reinforced by the speculative portfolio.

We now analyze the effect of excluding gold from the available menu of assets, reported in parentheses in Tables 4 and 5. The elements of the Σ matrix are not sensitive to the exclusion of gold, as expected from the low own effect in Table 2. We first note from Table 3 that the price of gold has the largest correlation with the Italian and Japanese consumer price indices (respectively -0.4 and -0.3). Accordingly, the exclusion of gold results in significant changes in the minimum variance portfolio of the Italian and

Japanese investors (columns 4 and 5 of Table 4). These differences do not affect the international investor, however, as can be seen in column (4b) of Table 5 while the last column of Table 4 suggested little change in the U.S. (cum-universal-PPP) investor minimum variance portfolio.

In fact, larger effects can be seen in the speculative portfolio. Excluding gold, the asset with the highest mean return, leads to an increase in the share of the D.Mark, the Swiss franc, the pound, the Canadian dollar, and the French franc totalling 13% (to 49%) and a decline of the share of the yen and the dollar totalling 8% (to 39%), the difference being the (5%) share in gold. These shifts illustrate the interaction of the change in the variance-covariance structure and of the change in return differentials on the speculative portfolio, a topic to which we return at the end of the next subsection.

3. The evolution of optimal portfolios over time

Table 5 reported minimum variance, speculative, and total optimal portfolios calculated with data from the whole sample period, April 1973 to March 1981. In this subsection, we study the evolution of these optimal portfolios for interim periods and assess whether changes in optimal portfolios were a response to changes in expected real return differentials or a response to changes in the observed variance-covariance structure. If, as we have assumed, the variance-covariance structure was stationary and investors had perfect knowledge of this true underlying structure, the inflation-hedge portfolio would not change over time and speculative portfolios would change only as a consequence of changes in real returns.

In Table 6, we report the U.S. dollar share in the inflation-hedge portfolios of the different national investors as well as of the international investor. The inflation-hedge portfolio share of the dollar is the minimum

Table 6

THE U.S. DOLLAR SHARE IN THE INFLATION HEDGE PORTFOLIO
OF DIFFERENT NATIONAL INVESTORS (%)

From April 1973 to March of:	Investor Consuming Only the Good of								International Investor[1/]
	Canada	France	Germany	Italy	Japan	Switzerland	U.K.	U.S.	
1975	-2	-33	-4	-28	-43	20	-70	-7	-22
1976	21	-6	-2	-6	-50	29	-33	6	-8
1977	-2	-7	-11	30	-47	-2	-9	-13	-11
1978	-1	-6	-13	23	-45	2	-20	-15	-13
1979	-9	-11	-10	15	-33	-4	-8	-17	-12
1980	-7	-8	-6	10	-13	3	-7	-14	-8
1981	-6	-6	-5	15	-10	1	-9	-10	-6

[1]Weighted sum of national investor's inflation-hedge portfolio where weights are given by the capital position in Table 3, column (4a).

variance portfolio share for all but the U.S. and the international investors. In the case of the U.S. investor (international investor), the minimum variance portfolio share of the dollar is obtained by adding the capital position of 100 (25) to the inflation-hedge portfolio share. It should also be recalled that movements in U.S. dollar shares are implied by changes in the sum of all other inflation-hedge portfolio shares since dollar shares are determined residually. It is clear from Table 6 that the dollar shares of all investors change substantially from year to year. Some patterns, however, do emerge. Since 1978, the short positions in U.S. dollars of both the international (column 9) and the German investor decline. The reduction in the Japanese investor's short position in dollars begins in 1976. The decline in the long position in dollars held by the Italian investor begins in 1977 but is reversed in 1981. This strengthening in the inflation-hedge demand for the dollar (smaller short positions and larger long positions) in 1981 is evidenced in all minimum variance portfolios except those of the Swiss and the U.S. investors. Over the entire period, we observe like movements in the minimum variance dollar shares of the U.S. and international investors. Although it is only roughly reflected in Table 6, we also found that the change over time in the share of many of the assets in the minimum variance portfolio is similar regardless of the choice of expenditure weights.

Next, we turn to Table 7 which summarizes the evolution of the own and cross effects of changes in the rate of return on the U.S. dollar. Specifically, this table reports the last row of Σ. It is determined residually so that each element of this row is minus the sum of the column elements of the inverse of the variance-covariance matrix of exchange rate (and gold price) changes. The sum of all the elements of this matrix is equal to the element in the U.S. column (own effect) of Table 7. In the last column of this table we report the U.S. dollar share in the speculative portfolio.

TABLE 7

Cross and Own Effects with the U.S. Dollar (%)
and the U.S. Dollar Share in the Speculative Portfolio

1973;4 to 3 of:	GO	CA	FR	GE	IT	JA	SZ	UK	US	US Dollar Share[1]
1975	0.6	-33.5	-8.2	4.2	-2.5	2.7	-0.9	4.51	33.0	33.0
1976	-0.3	-27.8	-7.0	4.4	-4.3	0.8	0.9	4.00	29.4	-14.6
1977	-0.4	-10.5	-2.6	1.3	-1.7	-1.5	1.9	0.5	13.1	7.1
1978	-0.5	-10.8	-2.5	2.2	-2.0	-2.7	1.1	0.3	14.8	7.6
1979	-0.2	-9.1	-1.4	0.7	-1.8	-1.0	0.9	-0.3	12.2	10.0
1980	0.0	-8.0	-1.3	0.5	-1.4	-1.3	0.7	0.1	10.9	3.8
1981	0.1	-7.4	-1.0	0.3	-1.1	-1.1	0.6	-0.3	9.7	6.4

[1] Sum of the element in each column times the mean real return (in % p.a.) on the respective asset, equals the share of the U.S. dollar in the speculative portfolio.

Except for a slight increase in 1978, there has been a steady and substantial decline in the own effect of an increase in the real return on the U.S. dollar denominated asset on its speculative share. Similarly, the magnitude of the cross effects of changes in dollar asset returns on the speculative shares of other assets has generally declined over the sample period. This pattern is most apparent in the Canadian and French columns. In all cases, the reduction in the size of own and cross effects of changes in U.S. real returns on speculative portfolios shares is associated with the observed pattern of increased variances and covariances of exchange rate and gold price changes. Between December 1975 and March 1981, the observed variance of exchange rate changes increased for all currencies except the German mark and French franc. We also found that the own and cross effects of changes in all other assets real returns have generally declined over the sample period. The cross effects between the European currencies have exhibited the greatest stability over time, both with respect to sign and magnitude.

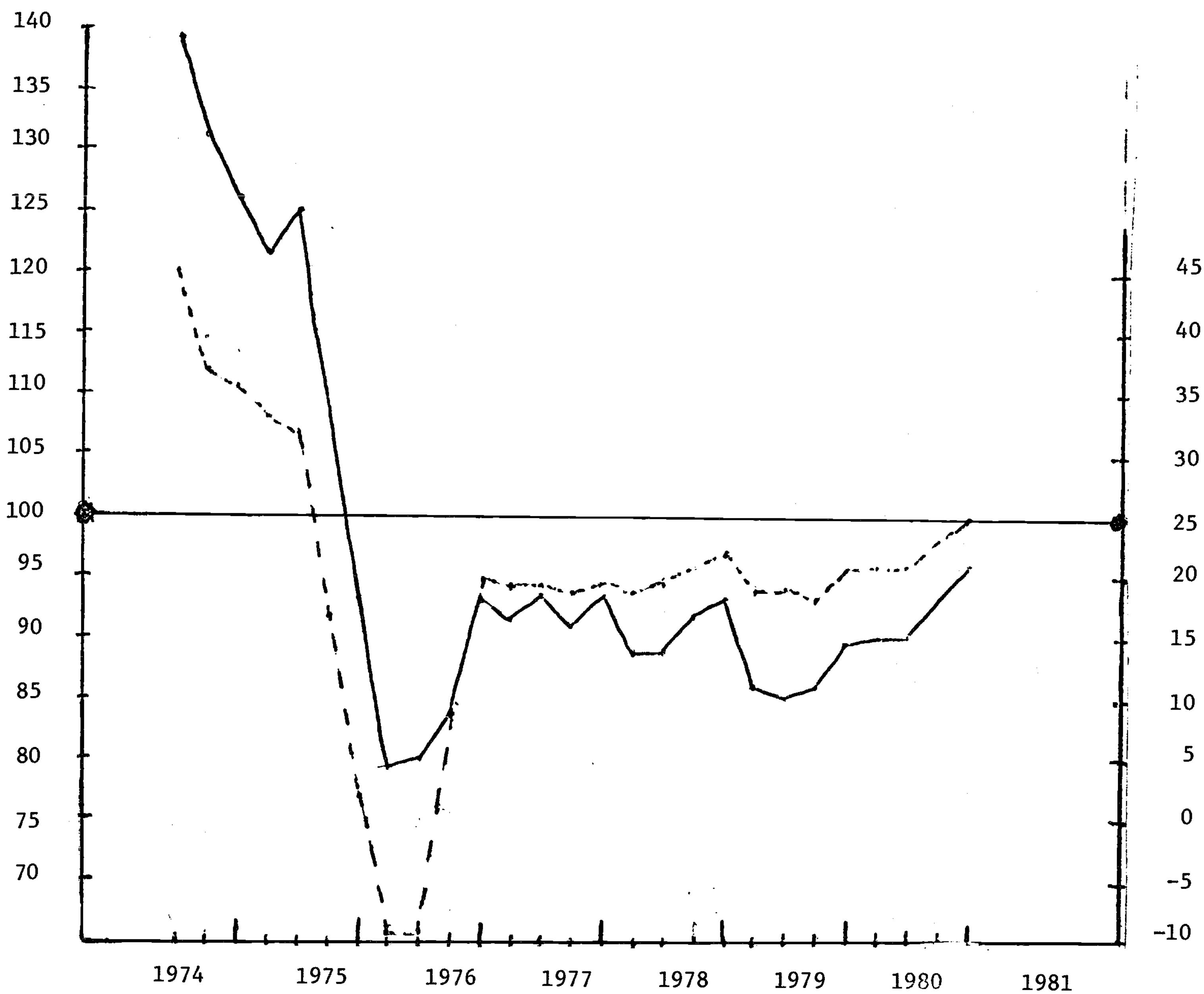
As noted in the previous subsection, the elements of Σ indicate the degree of substitutability and complementarity between assets. We thus interpret the first eight columns of Table 7 as reporting the evolution of the substitutability/complementarity relationships of all assets with the U.S. dollar. The consistently strong substitution effects between the Canadian and U.S. dollars, noted above, are evident in their negative sign and high absolute values. For example, in the late 1970's, they were close to 10%, showing that a 10% increase in the return on U.S. dollar assets decreases the speculative demand for Canadian dollars by 1%. For the pound sterling, the strong complementarity before the dramatic mid-1976 depreciation is followed by a very weak and erratic relationship. The degree of dollar-DM complementarity has significantly diminished over time. The increasing weakness in this relationship became particularly pronounced following the decline in the value of the U.S. dollar in late 1978.

In Figure 1, we show the evolution of the optimal U.S. dollar share in the total portfolio for the Bernouilli (γ=0) U.S. and international investors. These shares correspond to the sum of the appropriate column of Table 6 plus the last column of Table 7, to which we add the capital position (100 for the U.S. investor and 25 for the international investor). The similarity of the evolution in these shares is apparent. We noted above that the choice of expenditure weights did not greatly affect inflation-hedge portfolio shares. This is also evident when comparing movements in the U.S. and international investor's minimum variance portfolios. Further, as noted, there is no significant difference in the composition of the speculative portfolio when computed with real rates of return relevant to the international investor compared with real returns relevant to national investors. This is a consequence of the fact that own and cross-effects in the Σ matrix are far greater in magnitude than differences in national versus international investor's real rates of return. In fact, the composition of and changes in the speculative portfolio are invariant to the choice of real returns versus nominal interest rates adjusted for exchange rates changes.

Figure 1 reveals that the sharp decline in the attractiveness of the dollar between 1974 and mid-1976 was partly reversed in 1976 and that, since 1977, rather stable shares obtained. Over the late 1975 to early 1977 period, both the precipitous decline and the subsequent increase in the total optimal share of the U.S. dollar were the result of similar movements in the speculative portfolios. In the period prior to September 1975, we found that the U.S. dollar held the dominant share in the speculative portfolio. After that

Figure 1

Total Optimal U.S. Dollar Shares
%, from April 1973 to end of quarter of year indicated



——— U.S. consumer-investor (left scale, capital position)

----- international consumer-investor (right scale, capital position)

Source: Table 6

time, no asset clearly dominated this portfolio. Finally, it should be noted that the increase in the total dollar share for both the U.S. and the international investor in the 1980-1981 period was caused by like movements of the dollar share in both the inflation-hedge portfolio (becoming less negative) and the speculative portfolio.

Table 8 reports mean real returns on both U.S. dollar assets and optimal portfolios computed with different degrees of relative risk aversion. It is evident that the mean real return on the U.S. dollar is consistently negative and less than the return on the minimum variance portfolio (and a fortiori less than the return on the speculative and total portfolios). We also found that the return on the speculative portfolio is always lower than the mean real return on gold (the lowest return on gold between 1975 and March 1981 being 10.3%) and on the Swiss franc (the return on which ranged from 7.3% to 2.3% over this period). The yield on the speculative portfolio was also less than the return on the DM asset in all reported periods except March 1976 and March 1981. As a result, the return on the total portfolio for the Bernouilli investor is relatively low. As expected the return on the total portfolio is even lower when we increase the degree of relative risk aversion (e.g. $\gamma=-1$).

As noted above, changes in speculative shares were, in many periods, the dominant factor in the determination of movements in the total optimal portfolio. Clearly, observed changes in the speculative portfolio were a consequence of changes in both real returns and in the inverse of the augmented variance-covariance matrix of gold price and exchange rate changes, Σ. In Table 9, we report real return differentials with U.S. dollar assets observed in March of each year from 1975 to 1981. The importance of capital gains on gold, which bears no interest, is evident. The consistently positive yield differentials in favor of French francs, DM, yen, and Swiss franc dominated assets are also

TABLE 8

MEAN REAL RETURN ON THE U.S. DOLLAR AND ON THE OPTIMAL PORTFOLIOS[1/]

(% p.a.)

From April 1973 to March of:	Return on US Dollar	Minimum Variance Portfolio	Speculative Portfolio	Total Portfolio		
				γ=0[2/]	γ=-1[3/]	γ=1/2[4/]
	(1)	(2)	(3)	(4a)	(4b)	(4c)
1975	-6.4	-2.1	3.2	1.1	-0.5	4.3
1976	-2.9	-1.9	3.1	1.2	-0.2	4.3
1977	-2.5	-1.4	1.3	-0.1	-0.7	1.2
1978	-3.7	-1.5	2.1	0.6	0.2	2.7
1979	-3.8	-1.0	1.3	0.3	-0.3	1.6
1980	-3.1	-0.5	1.1	0.6	0.1	1.6
1981	-1.9	-0.3	0.9	0.6	0.3	1.5

[1]These returns are computed for the international investor.

Notes: [2]Column 4a = column (2) + column (3)

[3]Column 4b = column (2) + column (3)1/2

[4]Column 4c = column (2) + [2 + column (3)]

TABLE 9

REAL RETURN DIFFERENTIALS WITH THE U.S. DOLLAR[1/]

(% p.a.)

From April 1973 to March of:	GO	C$	FF	DM	IL	¥	SF	£
1975	40.7	-0.7	8.9	13.9	-0.2	-0.9	13.7	2.1
1976	15.3	0.8	4.1	5.4	-5.5	0.7	8.1	-3.3
1977	12.8	0.3	2.0	5.6	-3.9	2.7	6.4	-4.3
1978	14.1	-0.8	3.4	7.1	-1.1	5.8	10.5	-1.1
1979	15.6	-1.3	4.2	6.8	0.0	6.5	10.1	0.4
1980	26.4	-0.7	3.6	5.6	0.2	2.5	7.4	4.1
1981	20.2	-0.7	1.3	2.8	-1.4	4.2	4.2	2.4

[1]Mean real return on asset in column minus mean real return on U.S. dollar (see Table 12)

apparent. It is interesting to note that while the real return differential between Canadian and U.S. dollar assets is low, we have observed large movements in the speculative shares of these assets in response to small changes in a return differential as a consequence of their high degree of substitutability.

In Table 10, we record the percentage of the year over year change in a given asset's speculative portfolio share attributable to changes in real return differentials. That is, we decompose the relative effects of changes in real returns and in the observed variance-covariance matrix of exchange rate (and gold price) changes on movements over time in speculative portfolio shares of all assets. It should be emphasized that under the assumption that the variance-covariance structure of exchange rate (and gold price) changes is stationary and known with certainty by the investor, movements in speculative portfolio shares would be entirely due to changes in real returns. This would imply that investor's estimates of the true stationary Σ matrix are not subject to sampling error. In this case, all of the elements in Table 10 would be 100%, indicating that changes in speculative portfolios are fully attributable to real returns. In those cases where the reported percentage is between 0 and 100%, changes in the observed variance-covariance structure were found to reinforce the effect of changes in the real return differentials on (positive or negative) moveeements in speculative shares. Alternatively, elements greater than 100% imply that changes in the observed variance-covariance structure were a counterveiling influence. A negative element in the table indicates that the movement in the speculative share was dominated by changes in the observed Σ matrix while the counterveiling influence became the change in the real return vector.

Table 10

Percentage Change in Speculative Portfolio Shares
Due to changes in Real Returns 1/

12 month change in portfolio in March of:	GO	CA	FR	GE	IT	JA	SZ	UK	US
1976	67.3	79.2	76.3	99.6	-84.8	107.5	61.8	72.4	86.4
1977	-5.1	16.1	40.0	51.9	431.5	73.4	24.6	-8.0	16.6
1978	324.9	179.0	186.3	144.0	330.4	141.3	114.9	-304.1	445.8
1979	96.0	-90.3	131.4	-23.0	-20.0	-16.2	17.5	44.3	-22.4
1980	73.9	50.1	87.7	53.5	88.4	86.4	79.4	49.1	39.8
1981	70.8	-39.2	52.0	23.9	28.1	75.8	91.7	68.0	-39.0

1/ $\Sigma(r - r_{-12})$ as a percentage of $(\Sigma r - \Sigma_{-12} r_{-12})$, the change in Bernoulli investor speculative portfolio changes in the previous 12 month period.

Only in 1976 and 1978 were year over year changes in the speculative share of the U.S. dollar dominated by changes in real return differentials. For example, between March 1977 and March 1978 the optimal dollar share increased by 0.5%. If the observed Σ matrix had remained constant over this period, however, the share of U.S. assets in the speculative portfolio would have increased by 2.1%. Alternatively, the March 1979 dollar share increased by 2.4% over the previous year. If the Σ matrix observed in March 1978 had prevailed, the dollar speculative share would have fallen by 0.5% as a consequence of increased gold, French franc, yen, and pound assets' return differentials (see Table 9). Thus, the increase in the share of the dollar over the year was entirely the consequence of favorable changes in its substitutability-complementarity relationships with other assets. Similarly, between March 1980 and March 1981, we observe a 2.7% increase in the optimal dollar share. Changes in return differentials alone would have resulted in a 1% decline in the optimal share. This effect, however, was overwhelmed by a 3.7% increase in the dollar share attributable to changes in the observed variance-covariance structure (i.e. the optimal dollar share would have increased by 3.7% if real return differentials had remained constant at their March 1980 level).

In contrast to the case of the U.S., changes in DM speculative portfolio shares were, in most period, principally the result of movements in real return differentials. In 1976, for example, the 14.1% drop in the optimal DM share was entirely the consequence of changes in real return differentials (e.g. between March 1975 and March 1976, the return differential in favor of DM assets declined from 13.9% to 5.4%). In 1978, the 3.4% decline in the optimal DM share was fully attributable to changes in the vector of real returns. In this instance, however, changes in the observed variance-covariance structure seved to reduce the magnitude of this effect.

In Table 10, we also observed a similarity in the relative contribution of changes in real return differntials across assets in a given year. That is, in 1977 and 1978, changes in the observed Σ matrix played a significant role in the determination of changes in most speculative shares. In comparison, in March of 1976, 1980, and 1981, the importance of movements in real return differentials were of relatively greater importance in the reshuffling of the observed speculative portfolios.

CONCLUSION

Using a continuous-time finance-theoretic framework, Section 1 of this paper presented the optimal portfolio rule of an international investor who consumes N national composite goods and who holds N domestic-currency-denominated assets with known nominal interest rates in an environment where prices of goods, assets and exchange rates follow geometric Brownian motion. It was shown that the optimal portfolio decomposes into a capital position and two zero-net worth portfolios. The capital position depends only on the relative variances and covariances of changes in asset prices in terms of the numeraire. The first zero-net-worth portfolio depends on expenditure shares and on a comparison of the covariance between the changes in prices of goods and assets. The other zero-net-worth portfolio, scaled by risk aversion, depends on a comparison of mean real return to the return on the capital position. Also, the currency portfolio rule described in Kouri and Macedo (1978) was shown to be applicable to the case where one asset has a known price in terms of the numeraire. In the Appendix, it is shown that the currency portfolio rule described in Macedo (1982a) is applicable to the case where there are N assets with a known price and one asset, gold, with a random price in terms of the numeraire. An extension of the framework which allows for a richer menu of assets, along the lines of the equity and currency portfolios of Adler and Dumas (1982), is in Meerschwam (1982).

Under the assumptions of Section I, optimal portfolios were computed and presented in Section II. These portfolios are based on the inflation hedging portential provided by short-term financial assets denominated in different currencies (and gold) as well as on the substitutability/complementarity relationships among these assets.

In general, optimal diversification involves departures from both the "preferred monetary habitat" hypothesis, according to which portfolio shares would match expenditure shares, and the "purchasing power parity" hypothesis, according to which preferences would not affect the minimum variance portfolio. Specifically we found that the optimal portfolio of an investor consuming goods from all major industrialized countries (according to their weight in total trade) would be dominated in March 1981 by long positions in U.S. dollars (25%), yen (17%), D. marks (16%), French francs (15%), and pounds sterling (10%). An investor consuming only U.S. goods, by contrast, would hold 96% of his optimal portfolio in U.S. dollars. The inflation-hedge portion of this portfolio reveals that inflation risk is minimized for both the international and U.S. investor by holding Canadian dollars, French francs, and gold, and by borrowing U.S. dollars, marks, Swiss francs, and yen. In addition, the U.S. investor would hold lire and pounds, while the international investor would borrow lire.

In March 1981, the optimal speculative portfolio, maximizing mean real returns, would include long positions in U.S., German, and Japanese assets and in gold, and short positions in Canadian dollars, French francs, lire, and pounds. The analysis of the speculative portfolio reveals strong substitutability between U.S. and Canadian dollars. It likewise reveals substitutability of the U.S. dollar for French and Italian assets and weak complementarity of the U.S. dollar with D. mark and Swiss franc assets.

Because of the covariance of exchange rates and gold, the exclusion of the latter generates substantial reshuffling; the international investor would then have a long position in pounds (4%), Canadian dollars (3%), Swiss francs and French francs (2% each) and a short position in lira (6%), U.S. dollars and D. marks (2% each) and yen (1%).

The analysis of the evolution of portfolios over time showed, that shares changed dramatically at the beginning of period and did not begin to approach their March 1981 values until the end of 1976. In the case of the yen and the pound there were oscillations throughout the period. With respect to the dollar share in the optimal portfolio of the U.S. and international investor, we found that, in the period between late 1974 and mid-1976, a period in which the dollar is considered to have been "strong", a large decline in its optimal share took place. This shows the importance of the variability (and the associated uncertainty) of the changes in the value of the U.S. dollar, even when the currency itself is "strong". After the lows reached in mid-1976, the share increased again and stabilized in mid-1977 at levels well below those of before the end of 1974.

These oscillations over time are confirmed by the computation of optimal portfolios with constant (2-year) sample length but different base-periods, as in Goldstein (1982). Also, the existence of a relatively unexploited set of data on the foreign currency positions of U.S. commercial banks and nonbanks will allow for an explicit test of this framework, another topic being currently researched by Goldstein. Depending on the data availability, actual reserve diversification by central banks could also be contrasted with the results of the optimizing framework developed here, pursuing the line of research of Healy (1981).

In sum, work in this area should continue to be motivated by the need to analyze the microfoundations that underly the questions of macroeconomic policy in interdependent economies. Instead of constraining assets to be substitutes, as is done in the usual two-country macro literature, we believe that gains from portfolio diversification have to be analyzed in a multicurrency finance-theoretic framework such as the one presented in this paper.

APPENDIX

The optimal portfolio rule for short-term financial assets and gold.

Denote the proportion of gold in the optimal portfolio by x_o and its real return by r_o. In this case, the $\underset{\sim}{G}$ and $\underset{\sim}{\Theta}$ matrices are both (N by N) and can be partitioned as follows:

$$\text{(A1)} \qquad \underset{\sim}{G}_o = \left[\begin{array}{c|c} g & -g\varepsilon' \\ \hline -g\varepsilon & S \end{array}\right]$$

$$\underset{\sim}{\Theta}_o = \left[\begin{array}{c} -g\tilde{\varepsilon}' \\ \hline \tilde{S} \end{array}\right] - \Psi_o$$

where $g = g_{oo}$ is the variance of the price of gold in terms of currency N

$\varepsilon = (\varepsilon_{o1}/g_{oo} \; \varepsilon_{oN}-1/g_{oo})'$ is a N-1 column vector of covariances between the changes in the price of gold and in the N-1 exchange rates divided by the variance of gold

$\tilde{\varepsilon} = (\varepsilon' \mid 0)'$

$\eta = (\eta_{o1} \mid \eta_{oN})'$ is a N column vector of covariance between the changes in the price of gold and in the domestic currency prices of the N goods.

and $\Psi_o = \left[\begin{array}{c} -\eta' \\ \hline \Psi \end{array}\right]$

In fact, the portfolio for gold and the N-1 currencies is of the same form as (10):

$$\text{(10')} \qquad x_o = \underset{\sim}{G}_o^{-1} \underset{\sim}{\Theta}_o \alpha + \frac{1}{1-\gamma} \underset{\sim}{G}_o^{-1} (\underset{\sim}{r}_o - er_N)$$

where $\underset{\sim}{x}_o = (x_o \mid x')$ and $\underset{\sim}{r}_o = (r_o \mid \underset{\sim}{r}')'$ are N by 1.

Using (A1) we can now write

$$\underset{\sim}{G}_o^{-1} = \left[\begin{array}{c:c} 1/g + \varepsilon' S_o^{-1}\varepsilon & \varepsilon' S_o^{-1} \\ \hdashline S_o^{-1}\varepsilon & S_o^{-1} \end{array}\right]$$

$$\text{(A2)} \quad \underset{\sim}{G}_o^{-1}\underset{\sim}{\Theta}_o = \begin{bmatrix} \underline{0}' \\ \hdashline \tilde{I} \end{bmatrix} + \begin{bmatrix} \eta'/g - \varepsilon' S_o^{-1}\Psi_o \\ S_o^{-1}\tilde{\Psi}_o \end{bmatrix}$$

$$\text{where } S_o = (S - g\varepsilon\varepsilon') = \begin{bmatrix} \sigma_1^{\,2}(1-\rho_1^{\,2}) & \sigma_{1N-1}(1-\rho_1\rho_{N-1}/\rho_{1N-1}) \\ \sigma_{1N-1}(1-\rho_1\rho_{N-1}/\rho_{1N-1}) & \sigma_{N-1}^2\ (1-\rho_{N-1}^2) \end{bmatrix}$$

is the N-1 by N-1 variance covariance matrix of exchange rate changes, each term ij of which is corrected for the ratio of the product of the correlation coefficiencies of exchange rates i and j with gold and the correlation coefficient between exchange rates i and j $(\rho_i\rho_j/\rho_{ij})$

$$\text{and } \tilde{\Psi}_o = \Psi - \varepsilon\eta' = \begin{bmatrix} \Psi_{11}(1-\rho_1\tilde{\rho}_1/\tilde{\rho}_{11}) & \Psi_{1N}(1-\rho_1\tilde{\rho}_N/\tilde{\rho}_{1N}) \\ \Psi_{N-11}(1-\rho_{N-1}\tilde{\rho}_1/\tilde{\rho}_{N-11}) & \Psi_{N-1N}(1-\rho_{N-1}\tilde{\rho}_N/\tilde{\rho}_{N-1N}) \end{bmatrix}$$

is the N-1 by N matrix of covariance between goods prices in domestic currency and exchange rates, each term ij of which is corrected for the ratio of the product of the correlation coefficient of exchange rate i(ρ_i) and price of good j$(\tilde{\rho}_j)$ with gold and the correlation coefficient between exchange rate i and price of good j$(\tilde{\rho}_{ij})$.

To obtain x_N we use the constraint $x_N = 1 - e'\underset{\sim}{x}_o$ and we define a (N+1 by N) Φ matrix such that its columns sum to zero, $\underline{e}'\Phi = \underline{0}'$, $\underline{e}$ being a N+1 column vector of ones:

$$\text{(A3)} \quad \Phi_o = \begin{bmatrix} A \\ \hdashline -a' \end{bmatrix} S_o^{-1}\tilde{\Psi}_o + \begin{bmatrix} \eta'/g \\ \hdashline \underline{\underline{\theta}} \\ \hdashline -\eta'/g \end{bmatrix}$$

where $A = [\frac{\varepsilon}{I}]$, I being the identity matrix of order N-1

$a = \varepsilon + e = A'e$

and $\underline{0}$ is a (N-1 by N) matrix of zeros

We also define a (N+1 by N+1 Σ_o matrix, which has the same structure as the Σ matrix in (11) with $\tilde{G}_o$ replacing S and e replacing $\tilde{e}$. Using (A1) and (A2) we can express it in terms of S_o, ε and g:

$$\text{(A4)} \qquad \Sigma_o = \left[\begin{array}{c|c} \underline{\omega} + A'S_o^{-1}A & -\omega - A'S_o^{-1}a \\ \hline -\omega' - a'S_o^{-1}A & 1/g + a'S_o^{-1}a \end{array}\right]$$

where $\underline{\omega} = [\frac{\omega'}{\underline{\underline{0}}}]$

and $\omega = [\, 1/g \mid \tilde{0}' \,]'$

We then write instead of (13)

$$\text{(13')} \qquad x = (I-\Phi)_o\alpha_o + \frac{1}{1-\gamma}\Sigma_o r$$

where $\alpha_o = (0 \mid \alpha')'$ is a N + 1 vector obtained by augmenting α with a zero in the first row.

and I is the identity matrix of order N + 1.

The definitions of S_o and $\tilde{\Psi}_o$ in (A2) imply that if the price of gold is uncorrelated with exchange rates, so that ε vanishes, then $S = S_o$ and $\tilde{\Psi} = \Psi_o$ and the minimum variance portfolio can be expressed in terms of the Φ matrix defined in (13), corrected for the η vector. Then $A = \tilde{I}$ and $a = \tilde{e}$ in (A3) and (A4). The portfolio rule becomes:

$$\text{(A5)} \qquad x = \left(\left[\begin{array}{c} \eta'/g \\ \hline I \\ \hline -\eta'/g \end{array}\right] - \left[\begin{array}{c} 0' \\ \hline \Phi \end{array}\right] \right)\alpha + \frac{1}{1-\gamma}\Sigma_{oo} r$$

$$\text{where} \quad \Sigma_{oo} = \left[\begin{array}{c|c|c} 1/g & 0' & -1/g \\ \hline 0 & S^{-1} & -S^{-1}\tilde{e} \\ \hline -1/g & -\tilde{e}'S^{-1} & 1/g + \tilde{e}'S^{-1}\tilde{e} \end{array}\right].$$

NOTES

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1. A constant-elasticity bequest function and a constant discount rate could easily be introduced. Nairay (1981) allows for a variable discount rate and an infinite horizon.

2. For an endogenous determination of these processes see Nairay (1981). Applications to international finance are in Stulz (1982) and Bortz (1982). More general exogenous processes are used in Macedo (1982b), and Macedo, Goldstein and Meerschwam (1982), henceforth MGM.

3. More general cases, specifying prices in domestic currency are in Meerschwam (1982).

4. The purchasing power of a currency is the optimal price index when the indirect utility functions is separable. See more on the concept in Kouri and Macedo (1978) and Macedo (1982a). Work with more general utility functions has been done by Stulz (1980).

5. The derivations are in MGM.

6. Kouri (1975) referred to the "hedging demand for forward exchange which is proportional to the value of imported goods consumed" and to the "speculative demand" in a two-country model where national investors have different preferences. The decomposition between minimum variance and speculative portfolios for the international investor holding N currencies when prices and exchange rates are lognormally distributed is in Kouri and Macedo (1978).

7. Note that, by Itô's lemma, mean real return differentials depend on the variance of the exchange rate as well as on the covariance of prices and exchange rates, weighted by α. This implies that $\partial x_i/\partial\alpha_i > 0$ if $\gamma<0$, that is to say the individual is more risk-averse than the Bernouilli investor. See references in Macedo (1982b).

8. This result is emphasized by Adler and Dumas (1981).

9. In the models of Solnik (1973) and Kouri (1977), the assumption of purchasing power parity and no inflation in the Nth country eliminates hedging so that the minimum variance portfolio is all in the Nth currency, $x^m = \underline{1}$.

10. If, as pointed out in Adler and Dumas (1981), exchange rate changes are typically not passed on to prices, (14) is the relevant rule, making $\Phi_N = \Phi_{CPI}\beta$ where Φ_{CPI} captures the covariance between exchange rates and the components of the Nth country CPI and β are the CPI weights as in Macedo (1982a).

11. In the derivation of the Appendix, the price of gold is in units of currency N per ounce, which is why the covariance with goods prices and exchange rates enter with a negative sign.

12. See footnote (10) above

13. These weights are given as the simple average of the dollar value of imports and exports of the eight countries. The U.S. dollar share is 25%, which makes the comparison of the U.S. investor (with a share of 100% in the U.S. consumer price index) to the international investor particularly unsightful in attempts at bracketing the dollar share in optimal portfolios. See a discussion of weighing schemes in Macedo (1982a).

14. See for example, Dornbusch (1980b).

15. Notice that each element ij of the $S^{-1}\Psi$ matrix involves the ratio of the standard deviation of the change in the price of good j to the standard deviation of the change in the dollar exchange rate of currency i. These

ratios are in the 20-40% range for Italy, Japan, and the U.S. countries with a relatively high variance of inflation, and in the 10-20% range for the other countries. Thus, for example, when N = 3 the 1, 2 element of $S^{-1}\Psi$ would be $\Phi_{12} = \frac{\zeta_2}{\sigma_1} R_{12}$ where $R_{12} = \rho_{12} - \tilde{\rho}_{12}\tilde{\rho}_{22}/1-\rho_{12}^2$. When gold is included , we have instead

$$R_{12} = \frac{(1-\rho_1^2)\ (\tilde{\rho}_{12}-\rho_1\tilde{\rho}_2) - (\rho_{12}-\rho_1\rho_2)\ (\tilde{\rho}_{22}-\rho_2\tilde{\rho}_2)}{1-\rho_1^2 - \rho_2^2 - \rho_{12}^2 + 2\rho_{12}\rho_1\rho_2}$$

where $\rho_i(\tilde{\rho}_j)$ refers to the correlation of the price of gold with exchange rate i (price of good j).

16. Except for the U.S., this corresponds to a negative "diagonal" element in the $S^{-1}\Psi$ matrix. Using the expression in the previous footnote we see that the "own" inflation hedge in Table 4 of -3.4% for Switzerland corresponds to (ζ/σ) SZ = 14% and R_{SZ} = - 0.24 (whilst the underlined element in Table 3 was $\tilde{\rho}_{SZ}$= 0.3) and that the value of 13.2% for Italy corresponds to $(\zeta/\sigma)_{IT}$ = 28% and R_{IT} = 0.47 ($\tilde{\rho}_{IT}$ = 0.3).

17. If price indices in different countries were constructed using identical goods and weights, the composition of the universal-PPP minimum variance portfolio would be independent of the choice of of the numeraire. However, when goods and weights and hence price indices vary by country, the universal minimum variance portfolio is determined according to the choice of the numeraire. See footnote 10 above.

18. See footnote 7 above on this terminology.

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