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Microscopic Models of Financial Markets

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Abstract. This review deals with several microscopic models of financial markets which have been studied by economists and physicists over the last decade: Kim-Markowitz, Levy-Levy-Solomon, Cont-Bouchaud, Solomon-Weisbuch, Lux-Marchesi, Donangelo-Sneppen and Solomon-Levy-Huang. After an overview of simulation approaches in financial economics, we first give a summary of the Donangelo-Sneppen model of monetary exchange and compare it with related models in economics literature. Our selective review then outlines the main ingredients of some influential early models of multi-agent dynamics in financial markets (Kim-Markowitz, Levy-Levy-Solomon). As will be seen, these contributions draw their inspiration from the complex appearance of investors' interactions in real-life markets. Their main aim is to reproduce (and, thereby, provide possible explanations) for the spectacular bubbles and crashes seen in certain historical episodes, but they lack (like almost all the work before 1998 or so) a perspective in terms of the universal statistical features of financial time series. In fact, awareness of a set of such regularities (power-law tails of the distribution of returns, temporal scaling of volatility) only gradually appeared over the nineties. With the more precise description of the formerly relatively vague characteristics (e.g. moving from the notion of fat tails to the more concrete one of a power-law with index around three), it became clear that financial markets dynamics give rise to some kind of universal scaling laws. Showing similarities with scaling laws for other systems with many interacting subunits, an exploration of financial markets as multi-agent systems appeared to be a natural consequence. This topic was pursued by quite a number of contributions appearing in both the physics and economics literature since the late nineties. From the wealth of different flavors of multi-agent models that have appeared by now, we discuss the Cont-Bouchaud, Solomon-Levy-Huang and Lux-Marchesi models. Open research questions are discussed in our concluding section.

1 Introduction

Physicists not only know everything, they also know everything better. This indisputable dogma does not exclude, however, that some economists published work similar to what physicists now celebrate as "econophysics", only much earlier, like Nobel laureate Stigler [162]. Are econophysicists like Christopher Columbus, rediscovering something which others had found earlier, and also getting things somewhat wrong, but nevertheless changing human history? As the team of authors of this survey collects scientists from both disciplines, we do not attempt to give a definite answer to this question, but simply review some influential models by both physicists and economists, to allow a fair comparison.

Stylized facts is the economist's name for universal properties of markets, independent of whether we look at New York, Tokyo, or Frankfurt, or whether we are concerned with share markets, foreign exchange markets or derivative markets. The following is a collection of those "stylized facts" that are now almost universally accepted among economists and physicists:

(i) There is widespread agreement that we cannot predict whether the price tomorrow will go up or down, on the base of past price trends or other current information. (ii) If today the market had been very volatile, then the probability for observing a large change (positive or negative) tomorrow is also higher than on average (volatility clustering). (iii) The probability to have a large change in the market, by at least $x\%$, decays with a power law in $1/x$. Fact (iii) has first been discovered by Mandelbrot [121] who proposed the Levy stable model for financial returns. Over the recent years, the majority opinion among researchers in the field has, however, converged to the view that the tails of the cumulative distribution of returns are characterized by a power-law with exponent around 3. The underlying data would, hence, possess finite variance in contradiction to the Levy stable model. (iv) The q-th moments of the distribution of price changes are multifractal, i.e., their exponent is not a linear function of this index q (a rather new observation). Facts (i) to (iii) can be found in surveys on the econometrics of financial markets, cf. de Vries [169] and Pagan [131]. Fact (iv) has been first partially documented in Ding *et al.* [52] and has meanwhile also obtained the status of an universal feature of all markets in the empirical finance literature (Lobato and Savin [111]). Similar research on multiscaling (multifractality), albeit with different analytical tools, was conducted in numerous econophysics papers, starting with Mandelbrot *et al.* [122], Vandewalle and Ausloos [166].

After the pioneering microscopic market models from economists like Stigler [147], numerous such models were published in the physics literature since 1992. We concentrate here on those models which have raised enough interest to be also investigated by others than the original authors themselves. These include the models of (i) Kim and Markowitz [91], of (ii) Levy, Levy, Solomon [109,103,104,108,107,105,106], of (iii) Solomon, Levy and Huang [80], of (iv) Cont and Bouchaud [48], of (v) Solomon and Weisbuch [151] (see also [152]), and of (vi) Lux and Marchesi [119,120], all conceived as models for modern (financial) markets, as well as the model for ancient barter and self-organization of monetary exchange by Donangelo and Sneppen [54] (see also [53,16,17].

We start with the latter one since it (in its literal interpretation) refers to prehistoric times. We neglect the now (in physics circles) widespread Minority

Games, as they arose from the question when best to visit the El Farol bar in Santa Fe to avoid overcrowding. The weighted majority of the present authors prefers to drink experimentally instead of simulating drinks, and thus we leave these minority games to another review in this book.

2 Overview

Research applying microscopic simulations in economics and finance stems from several sources. First, a number of authors in the economics "mainstream" have resorted to some type of microscopic simulation in the course of their work on certain economic problems and models. The framework of Kim and Markowitz [91], explored in detail in section 4, may serve as a prominent example. Like many economists of that time, the authors were interested in explaining the sudden drop of the U.S. stock market on 17th October 1987. A widespread explanation for this event was the automatic overreaction of computer-based "dynamic hedging" strategies that had become popular strategies of institutional investors in the years before. However, models including the market interactions of many investors following such strategies are clearly hard to solve in an analytical manner. Therefore, Kim and Markowitz decided to investigate the destabilizing potential of dynamic hedging strategies via Monte Carlo simulations of a relatively complicated model of price formation in an "artificial" financial market (cf. Markowitz [124]). They were, however, not the first to rely on simulations of economic processes. During the fifties, the well-known economist A. W. Phillips -who first recovered the so-called Phillips curve (i.e., the inverse relationship between unemployment and inflation rate)- used a hydraulic machine for simulation of macroeconomic processes (Phillips [133], see also [129]). Even earlier, we can find simulations via electronic circuits published in economics journals (Morehouse *et al.* [126]).

However, the first simple Monte Carlo simulation of a financial market appeared in Stigler [162], who generated trading orders as random variables. Two decades later, simulations of different trading mechanisms played an important role in the literature on the "microstructure" of financial markets (Cohen *et al.* [46]). The interest here was mainly in questions of efficiency and stability of different forms of market organization and regulation as well as the impact of introducing computer-assisted trading. Like in the approach of Kim and Markowitz a few years later, the sheer complexity of the models, because of the aim to reproduce many features of real-life market, necessitated a simulation approach. Interestingly, the microstructure literature later moved on to other questions, namely, analysis of asymmetric information among traders. Luckily, Bayesian learning methods allowed to tackle large classes of asymmetric information models in a rigorous mathematical manner. As a consequence, the leading textbook of the nineties, "Market Microstructure

Theory" by O'Hara [130], only reviews theoretical work and lacks any reference to microscopic simulations.

Of course, it was only a matter of time, until models became so complicated that they could not be solved analytically anymore and had to be supported by numerical analysis. In the asymmetric information literature, interesting recent contributions coming close to microscopic simulations deal with learning in financial markets (de Fontnouvelle [65], Routledge [139,140]). Using different variants of adaptive learning mechanisms, these authors study how agents learn to use signals about future market prices and how to make inferences from these signals. The key interest is in whether or not the learning dynamics converges to a time-invariant equilibrium that would obtain under "rational" (i.e., correct) expectations.

With its focus on the extraction of information from imperfect signals by fully rational or learning investors, the dominant branch of models in financial economics neglected some of the most striking observations in real financial markets. Namely, there was no role at all in these models for features like chartist strategies (i.e., strategies looking for patterns in the plots of past prices) or herd behavior among traders. In some sense, all traders in traditional microstructure models behave like fundamentalists in that they try to infer the correct "fundamental" value of an asset from the limited amount of information they have. However, the existence of both chartists and fundamentalists in real markets is too obvious to be neglected and a long tradition of modeling the interaction of these two types of traders exists in economics literature. In fact, we can find interesting papers on this subject back in the fifties (Baumol [21]) showing the destabilizing potential of chartist strategies in a rigorous analytical analysis. The chartist versus fundamentalist topic was later dropped because of the seeming lack of "rationality" of agents' behavior in this models, that means, the apparent *ad-hoc* nature of the description of individual behavior. Nevertheless, we can still find some contributions to this strand of literature in the seventies and eighties (Zeeman [181], Beja and Goldman [22]) and as of the early nineties starting with Day and Huang [50] the chartists-fundamentalists interaction regained its place as an important research topic. The literature of the nineties has an abundant diversity of interacting agent models incorporating these features in one or the other way. An early application to foreign exchange markets is Frankel and Froot [66,67] who combine a standard monetary model of open economy macroeconomics with a chartist-fundamentalist approach to expectation formation (replacing the usual assumption of "rational" expectations in earlier models). Their aim is to provide a possible explanation of the well-known episode of the dollar bubble over the first half of the eighties. They show that a deviation from the fundamental value can set into motion a self-reinforcing interplay between forecasts and actual development: the initial deviation between price and fundamental value will trigger the switch of some agents from fundamentalist to chartist behavior. However, the more the market composition changes in favor of the chartist group, the less pressure will exist for prices to revert to their fundamental anchor values.

An important subsequent variation on Frankel and Froot's theme is the more elaborate model by DeGrauwe *et al.* [51] who show that this type of dynamics can lead to chaotic behavior of exchange rates. Their model is one of the first able to explain some stylized facts other than the mere deviation from the fundamental value. In particular, they show that their chaotic dynamics is hard to distinguish from a pure random walk process and that it helps to explain the forward premium puzzle (the finding, that forward rates are a poor and biased predictor for subsequent exchange rate movements).

Chaotic dynamics derived from the interaction of agents with different prediction functions for future price movements are the topic of a comprehensive research project on "adaptive belief systems" starting with Brock and Hommes [30] and extended in Brock and Hommes [29,31,32], Gaunersdorfer [68], Gaunersdorfer and Hommes [69], Gaunersdorfer *et al.* [70], and Chiarella *et al.* [44] (see also Hommes [77] for a review). While the early papers of this literature are mainly concerned with various bifurcation routes of chaotic attractors in such systems, the recent papers by Gaunersdorfer and Hommes [69] and Gaunersdorfer *et al.* [70] are concerned with a possible mechanism for volatility clustering emerging from this theoretical set-up. They show that co-existence of different attractors (e.g., a fixed point and a cycle or chaotic attractor) in a deterministic dynamics will lead to repeated switches between these attractors when small amounts of noise are introduced. Since different attractors are characterized by different degrees of volatility of prices, their varying influence on the overall time series generates a perplexingly realistic picture of switches of the market from tranquil to volatile phases and *vice versa*. Gaunersdorfer and Hommes [69] show that estimates of GARCH models can produce quite similar results as with empirical data.

The adaptive belief dynamics has agents switching between predictors according to their past performance. A group of alternative learning models have used modern computer-learning techniques as models of human adaptation. The best-known variant in the context of financial markets is surely the Santa Fe Artificial Stock Market (Arthur *et al.* [14], LeBaron *et al.* [102], Palmer *et al.* [132]),the authors of which included a statistical physicist. In this model, traders are equipped with a set of classifiers basically consisting of simple chartist and fundamentalist rules. Particular forecasts of future returns are connected with certain combinations of classifiers. Classifiers and forecasts are subjected to genetic operations (selection, cross-over, mutation). Over time, successful combination of rules (classifiers) should be maintained, whereas poor ones should be skipped in favor of better ones. The set-up of this and similar models is notably different from most other applications of machine learning techniques: whereas usually classifier systems, genetic programming, and neural networks are used to recover regularities in data sets that are independent from their own learning activity, the artificial financial market application deals with interacting agents, who naturally influence the performance of each others' attempt at learning the market's rules. The main finding of the early work at the Santa Fe Institute was that the dominance of either chartist or fundamentalist classifiers depends on the frequency of activation of the genetic operations. With more frequent activation, chartist behavior was found to be dominating. LeBaron *et al.* [102] showed that the model reproduces some empirical features like leptokurtosis of returns and correlation between volume and volatility. Other artificial markets include Chen and Yeh [42,43], who instead of classifiers systems use genetic programs as evolving models of their agents and also can show consistency of simulated data with some empirical findings. Cincotti *et al.* [45] construct a more general framework that is designed to accommodate various learning devices. Related research using genetic algorithm learning in prominent economic models can be found in Arifovic [10], Arifovic and Masson [11], Dawid [49], Szpiro [163] and Georges [71]. Le Baron [100,101] has models closely related to the SFI model, but with learning via neural networks and the interesting addition of variable memory length of the agents (cf. the Levy-Levy-Solomon model reviewed in section 5).

Another strand of economic literature proposed to cope with the diversity of behavioral variants using a statistical approach cf. Kirman [93], Aoki [6,8], Ramsey [137], Lux [113], Foley [64] and Kaizoji [87–89]. Only part of this work is concerned with financial applications. A wealth of applications of statistical physics tools to other branches of economics can be found in Aoki's books. As concerns finance, perhaps the first attempt at a microscopic approach with stochastic features guided by work in statistical physics is Landes and Loistl [99]. Later work includes Youssefmir *et al.* [180], who reconsider the destabilizing potential of trend-following behavior, and Kirman [92] combining the statistical modeling of herding among speculators with an expectation formation à la Frankel and Froot. Similarly, Farmer and Joshi [63] reconsider the impact of several frequently used trading strategies in price formation, and Carvalho [35] shows that in a particular simplified variant of their model, emergence of a power-law for extreme returns can be rigorously demonstrated. Another highly relevant contribution is Aoki [9] who deals with a stochastic framework for market participation with infinitely many strategies or trading rules. Deriving the partition vector (the number of types or clusters of agents) from a rather general specification of the entry and exit dynamics, he shows that often the sum of the fractions of agents in the two largest groups will be close to 1. This may provide a theoretical rationale for the confinement to two trader groups in many models of speculative dynamics.

Another example of a statistical approach towards interacting agent dynamics in finance is the work by Lux and Marchesi, reviewed below (section 9). The latter group of models is, in fact, not too far from those proposed in the physics literature. Prominent early examples are the threshold dynamics (in the form of trigger values for agents' buy or sell decisions) by Takayasu *et al.* [164] and Bak *et al.* [18]. Their analysis is also concerned with scaling behavior of the resulting price dynamics and reports some interesting features. A somewhat related model leading to intermittent bursts of activity is Ponzi and Aizawa [135]. Later additions to that literature include the Cont-Bouchaud percolation model (reviewed in section 7), and related lattice-based set-ups by Iori [84] and Bornholdt [26]. Interestingly, contributions in this vein have recently also been applied to other financial phenomenon like contagion of bankruptcies and systemic risk in the inter-bank lending system (Heymann *et al.* [76], Iori and Jafarey [85], Aleksiejuk and Holyst [2]).

3 The Dynamics of Monetary Exchange

Before money was invented, exchange of goods would have required barter between agents with coincident endowments and wants. However, at a more advanced level of division of labor, one may trade by getting something one does already possess but which, as judged from past experience, one will be able to sell later easily to others. Donangelo and Sneppen [54] in this sense started with traders who initially have a random endowment of products. They then try to fill the gaps in their inventories by bartering with other traders, and keep in mind how often some specific product was asked from them. In the case a trader has something to sell but already has the product which the partner offers for barter, the first trader may opt to get a product already in his/her inventory. This is done with a probability proportional to the number of times this product was asked from this trader in recent times, and this product then plays the role of *money*: we cannot eat the money we earn, but we hope to buy food from it later.

For a suitable range of the number of units per trader and the number of differentiated products available, traders have enough holes in their inventories to barter, but after some time also trades involving money (in the above sense) play an important role; and sometimes no trade at all is possible in an encounter of two randomly selected traders. Which product evolves as the most desired "money" thus depends on the random dynamics of the market, without outside interference and without any special property of that product at the beginning. This result conforms to physics ideas that "everything" can be described by randomness, whether it is Boltzmann statistics for thermodynamics, the built-up of social hierarchies [25], or the value of the European currency. Economists may regard this view as over-simplified.

For one variant of the model, the time-dependence could be quantified: A stationary state is reached if every trader had several chances to trade with every possible other trader. The distribution of times for which one currency stays on top, then appears to follow a stretched exponential [160]. Other models for the "statistical mechanics of money" are surveyed in [74].

From the economists' point of view, the informational content of some of these studies is somewhat questionable as there are practically no measurements of the corresponding quantities in real economies. It is nevertheless interesting to note that quite similar models have been brought up by economists some time ago. Looking up contributions like the work by Jones [86] or the seminal paper by Kiyotaki and Wright [94], one finds almost the same structure as in the Donangelo and Sneppen approach. This is not too surprising insofar as - although Donangelo and Sneppen do not quote the rich literature that emerged from Kiyotaki's and Wright's search model - their work can, in fact, be traced back to these sources. A careful reading reveals that they draw their inspiration from an earlier paper in the physics literature, Yasutomi [177], who studied a model along the lines of Kiyotaki and Wright. It might have been useful to consult the by now voluminous literature on search-equilibrium models in economics rather than start from scratch with a similar pursuit. Be that is it may, the style of analysis in the early papers by economists was clearly different from that of Donangelo and Sneppen. Following the then prevalent style of reasoning in their subject they were theoretical investigations into the nature of equilibria in an economy with a large number of goods rather than truely dynamic model of the emergence of money. The question pursued was under what conditions one would find a "monetary" equilibrium in which one of the available goods emerges as a medium of exchange and under what conditions the economy remains stuck in a situation of barter trade. Like in many other areas in economics, the demonstration of existence of multiple equilibria (barter vs. monetary equilibrium, as well as different monetary equilibria) pointed to the necessity of investigating out-of-equilibrium dynamics.

To give the reader a feeling of the typical approach pursued in economics, we give a short sketch of the basic ingredients of the seminal Kiyotaki and Wright model that has stimulated a whole branch of recent economics literature. The set-up by Kiyotaki and Wright is, in fact, more that of an example than a general model of a multi-good economy. In particular, it is assumed that there are three commodities in the economy which are called goods 1, 2, and 3. There is also an infinite number of individuals who are specialized in both production and consumption: type i $(i = 1, 2, 3)$ agents derive pleasure (utility) only from consumption of good i, and are able to produce only good $i' \neq i$. A typical example used in many of the pertinent contributions has the following structure of consumption and production: $\left|\begin{array}{l} i \;\; 1\; 2\; 3 \\ \hline i' \; 2\; 3\; 1 \end{array}\right.$.

This implies that there is no double "coincidence of wants" in the economy. Therefore, intermediate trading of goods by agents who do not desire them as consumption goods is required for the satisfaction of the need of these agents. It is furthermore assumed that in every period there is a random matching process that assigns every agent to a pair with one other agent within the economy. Pairs of agents then have the chance to trade with each other (exchange their goods). In the theoretical papers on this subject, the focus is on the detection and characterization of steady state Nash equilibria: sets of trading strategies of each type of agents together with the steady state distribution of goods resulting from these strategies, so that each individual maximizes its expected utility under full information (rational expectations) about the strategies pursued by other individuals. There are also storage costs per period for goods that are not consumed by their owners. The distribution of both the instantaneous utilities derived from consumption and the storage costs are crucial for the types of Nash equilibria that exist in this model. A particular interesting situation is co-existence of so-called "fundamental" and "speculative" equilibria. In the former, only goods with lower storage costs are accepted by the agents (and, hence, they can be said to concentrate on fundamentals in their trading decisions) while in the latter case also some low-storage costs are traded against high-storage commodities. The motivation for this at first view unattractive exchange is higher marketability of the high-cost good. Accepting high-storage costs in the hope of higher chances to exchange these goods against their preferred one, the agents could be said to act out of a speculative motivation. This second case is the more interesting one as it corresponds to the "emergence of money": certain goods are not traded because of their intrinsic values, but purely because they are accepted by other agents. To solve for steady state equilibria requires to consider the development of expected life time utility for each group of agents:

$$E\sum_{t=0}^{\infty}\beta^t[I_i^u(t)U_i - I_{i'}^D(t)D_i - I_{ij}^C(t)c_{ij}] \tag{1}$$

where U_i is instantaneous utility from consumption, D_i instantaneous disutility from production (i.e., production costs), and c_{ij} the storage costs of good j for type i. $\beta < 1$ is the discount factor and I_i^u, I_i^D and I_{ij}^c are indicator functions assuming the value 1 at any period t in which consumption, production or exchange take place and 0 otherwise. Bellman's approach to dynamic programming allows to express this problem in terms of value functions of certain states. For example,

$$V_i(j) = -c_{ij} + \max \beta E[V_i(j')|j] \tag{2}$$

could be used to denote the value for an individual of group i to currently own one unit of good j. The value, $V_i(j)$, of this scenario consists of an instantaneous disutility, $-c_{ij}$, the negative storage costs incurred by this agent plus the discounted value of the expected change of its situation in the next period, $E[V_i(j')|j]$. Here j' could be identical to j (if he does not accept the exchange possibilities offered in the next period), to i (if he is offered his preferred good), or some $j' \neq i$ and $j' \neq j$ (if he accepts another good offered to him). Although this formalism greatly facilitates the analysis, rigorous derivation of the type of Nash equilibria sketched above is still a combinatorial nightmare. Of course, having demonstrated the potential of this kind of model to generate speculative equilibria as steady state solutions, the question emerges whether agents could detect these profitable trading possibilities. A number of authors have taken up the question of whether reasonable dynamics could lead to a self-organization of the Kiyotaki and Wright economy converging to either a fundamental or speculative equilibrium. Contributions to the dynamics of exchange economies made use of classifier systems [123] or genetic algorithms in order to describe the evolution of conventions and self-organization of monetary exchange within an ensemble of uncoordinated agents.

Besides articles with a computational approach of artificial and boundedly rational agents one can also find contributions with real agents in controlled laboratory environments being rewarded with real money in dependence on their utility gains [33,57]. To the surprise and dissappointment of some authors, both in experiments with artificial agents [123] and human subjects one often [33,56] only finds emergence of fundamental equilibria. Strangely enough a kind of fundamental steady state even appeared in some set-ups in which the "speculative" scenario is the unique equilibrium. Somewhat more favorable results concerning the "emergence of money" are obtained in a recent paper by Basci [20] who allows for imitative behavior.

Duffy [56] tried to combine artificial, agent-based simulations with laboratory experiments with a view to the above mentioned problem. He uses results of preliminary laboratory experiments for his computational approach which leads to an improvement with respect to the speed of learning compared with earlier experimental Kiyotaki-Wright environments [94].

Furthermore, Aoki [7] uses tools from statistical mechanics in his re-investigations of the Kiyotaki-Wright approach. In this perspective, the Donangelo and Sneppen approach appears to fit well into an established line of economics research, on the intriguing question: how could agents develop the idea of "money"? The early stage of the study of out-of-equilibrium dynamics in this context warrants that a great deal of collaborative work could still be done in this area in the future.

4 The First Modern Multi-Agent Model: Kim-Markowitz and the Crash of '87

After this digression into very fundamental questions of economic theorizing, we turn to the major playground of multi-agent models in economics: artificial economic life in the sense of computer-based stock or foreign exchange markets. Besides some early Monte Carlo simulations like Stigler [162] or Cohen *et al.* [46], the first "modern" multi-agent model is the one proposed by Kim and Markowitz [91]. The major motivation of their microsimulation study was the stock market crash in 1987 when the U.S. stock market decreased by more than twenty percent. Since this dramatic decrease could not be explained by the emergence of significant new information, ensuing research concentrated on factors other than information-based trading in determining stock price volatility (cf. [144]). But although hedging strategies, and portfolio insurance in particular, have been blamed to have contributed to the crash by increasing volatility [47], the theoretical work on the link between portfolio insurance and stock market volatility was rather limited at that time (e.g., [28]). In their simulation analysis, Kim and Markowitz, therefore, tried to explore the relationship between the share of agents pursuing portfolio insurance strategies and the volatility of the market.

4.1 The Model

The simulated market contains two types of investors, "rebalancers" and "portfolio insurers", and two assets, stocks and cash (with interest rate equal to 0). The wealth w of each agent at time t is given as

$$w_t = q_t p_t + c_t \tag{3}$$

where q_t is the number of stocks the agent holds at time t, p_t is the price of the stock at time t and c_t denotes the cash holdings of the agent at time t. *Rebalancers* aim at keeping one half of their wealth in stocks and the other half in cash, i.e.

$$\text{target of rebalancers} : q_t p_t = c_t = 0.5 w_t. \tag{4}$$

Thus, the rebalancing strategy has a stabilizing effect on the market: increasing prices induce the rebalancers to raise their supply or reduce their demand; decreasing prices have the opposite effect. *Portfolio insurers*, on the other hand, follow a strategy intended to guarantee a minimal level of wealth (the so-called "floor" f) at a specified insurance expiration date. They use the "Constant Proportion Portfolio Insurance" (CPPI) method proposed by Black and Jones [24]. The method can be described as keeping the value of the risky asset in a constant proportion to the so-called "cushion" s, which is the current portfolio value less the floor, i.e.

$$\text{target of portfolio insurers} : q_t p_t = k s_t = k(w_t - f_t) \tag{5}$$

where the CPPI multiple k is chosen greater than 1. Setting the multiple above 1 allows the investor to choose his exposure to the risky asset in excess of the cushion, and hence to extend his gains if prices increase. In case of falling prices, the cushion also decreases and the stock position is reduced accordingly. Given a more or less continuous revaluation of the portfolio structure, the floor is therefore (quite) safe. In this way, the Black-Jones formula imitates the effect of put options often applied in dynamic hedging strategies. Contrary to the rebalancing strategy, the portfolio insurance strategy implies a procyclical and therefore potentially destabilizing investment behavior: when prices fall, portfolio insurers will strive to protect their floor by reducing their stock position, and conversely, if prices increase, they will try to raise their stock position in order to realize additional gains.

Stock price and trading volume evolve endogenously according to demand and supply. However, trading does not proceed continuously but at discrete points in time. Each investor reviews his portfolio at random intervals. He rates his asset positions using an individual price forecast computed according to the current demand and supply situation in the following way:

1. If only asks (i.e. buy orders) exist, the investor estimates the price at 101% of the highest ask price,
2. if only open bids (i.e. sell orders) exist, the investor estimates the price at 99% of the lowest bid price,
3. if both open asks and bids exist, the investor assumes that the price agreed upon by buyers and sellers will be placed somewhere between open bid and ask prices. More precisely it is assumed that his estimate of the new price is the average between the highest ask and the lowest bid price of the previous period, and
4. if neither asks nor bids exist, the investor assumes next period's price to equal the previous trading price.

Summarizing, the above assumptions amount to:

$$p^i_{est,t} = \begin{cases} 1.01\max(p^1_{ask,t},\cdots,p^n_{ask,t}), \\ \quad \text{if} \quad p^i_{bid,t}=0 \quad \text{for all} \quad i=1,...,n \\ \quad \text{and} \quad p^i_{ask,t}\neq 0 \quad \text{for at least one} \quad i, \\ \\ 0.99\min(p^1_{bid,t},\cdots,p^n_{bid,t}) \quad \text{for} \quad p^i_{bid}>0, \\ \quad \text{if} \quad p^i_{ask,t}=0 \quad \text{for all} \quad i=1,...,n \\ \quad \text{and} \quad p^i_{bid,t}\neq 0 \quad \text{for at least one} \quad i, \\ \\ 0.5\left[\max(p^1_{ask,t},\cdots,p^n_{ask,t})+\min(p^1_{bid,t},\cdots,p^n_{bid,t})\right] \\ \quad \text{for} \quad p^i_{bid}>0, \\ \quad \text{if} \quad p^i_{ask,t}\neq 0 \quad \text{for at least one} \quad i \\ \quad \text{and} \quad p^i_{bid,t}\neq 0 \quad \text{for at least one} \quad i, \\ \\ p_{t-1}, \quad \text{if} \quad p^i_{ask,t}=0 \quad \text{and} \quad p^i_{bid,t}=0 \quad \text{for all} \quad i=1,...,n, \end{cases} \tag{6}$$

where i denotes the agent and n is the number of investors. In case the estimated ratio between stocks and assets (relevant for rebalancers) or between stocks and cushion (relevant for portfolio insurers) is higher than the target ratio (0.5 or k for rebalancers and portfolio insurers respectively), the investor will place a sale order with $p^i_{bid,t}=0.99p^i_{est,t}$ (i.e. $p^i_{ask,t}=0$). Conversely, he will place a buy order if the evaluated ratio is smaller than the target ratio with $p^i_{ask,t}=1.01p^i_{est,t}$ (i.e. $p^i_{bid,t}=0$).[1] If matching counteroffers exist, incoming buy or sell orders are executed immediately (at the

[1] Strictly speaking, agents allow for deviations from the target value within a certain which they tolerate.

price of the particular counter-offer). Otherwise, they are put on a list and may be filled later during the trading day if suitable offers are made by other agents. Agents whose orders are open until the end of the day have the possibility to re-evaluate their portfolio structure the next day and to place a new order. A trading day is over when every agent who reviewed his portfolio has had the chance to place an order and to trade. At the end of each day agents who have lost their complete wealth (i.e., their cash plus the value of stocks rated at the closing price) are eliminated and, thus, excluded from any further trading activities.

4.2 Results

Every agent starts the simulation with the same value of his portfolio (i.e., 100,000 \$), half of it in stocks and half in cash. The price level at the start of the simulation is 100 \$. The CPPI multiple k and the insurance level g (i.e., the proportion of floor to initial assets) are chosen in a way that portfolio insurance agents start with their portfolio structure in equilibrium. The parameters for the insurance plans are set at $g = 0.75$ (i.e., at expiration date the losses should not exceed 25 % of the initial wealth) and $k = 2$. The duration of the insurance plans is 65 trading days for every plan and each portfolio insurer. Exogenous market influences are modeled by deposits and withdrawals of cash occurring at randomly determined points in time (exponentially distributed with a mean time of 10 trading days) and in random amounts (uniformly distributed between -8,000 and +8,000 \$) for each investor. The time intervals between the portfolio reviews are also determined by random draws for each investor (exponentially distributed with a mean time of 5 trading days).

In the following, we provide the details of simulations in which we have replicated and extended the results of Kim and Markowitz. Figure 1 and 2 show the daily development of (closing) prices and trading volume for 0, 50 and 75 CPPI agents, respectively, out of a total of 150 agents for the first 800 trading days. Compared with no CPPI agents, both trading volume and price fluctuations are generally higher in the cases of 50 and 75 CPPI investors. However, the time series for 50 and 75 CPPI agents exhibit a cyclical behavior inconsistent with empirical data. We have also studied the standard deviation of daily returns per trading period each consisting of 65 trading days. As can be seen in Figure 3, for the first periods the volatility for 75 CPPI agents is much higher than for 0 or for just 25 CPPI agents. But after about 15 trading periods the volatility for 75 CPPI agents declines remarkably. We have observed a similar decline of volatility in the case of 50 CPPI agents (not displayed in the figure). The reason, however, for this strong decrease in volatility in case of a high proportion of CPPI agents is simply that a significant number of agents become bankrupt in the course of the simulations.

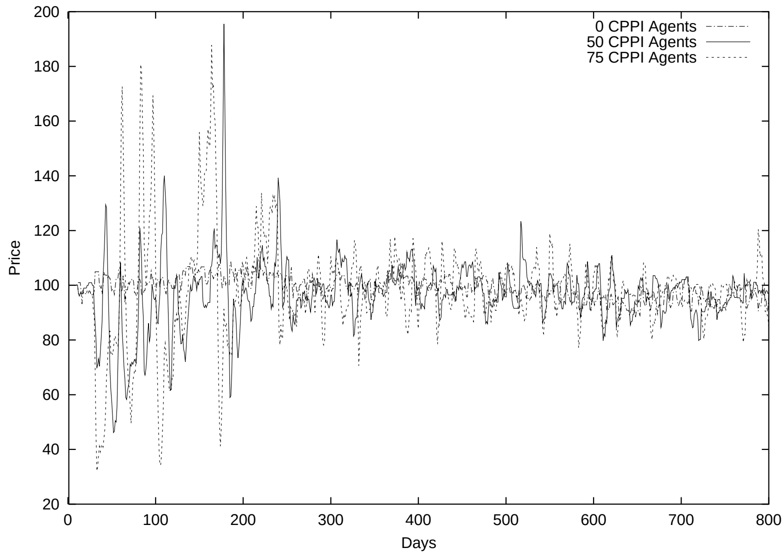

Fig. 1. The daily development of prices (total number of agents: 150)

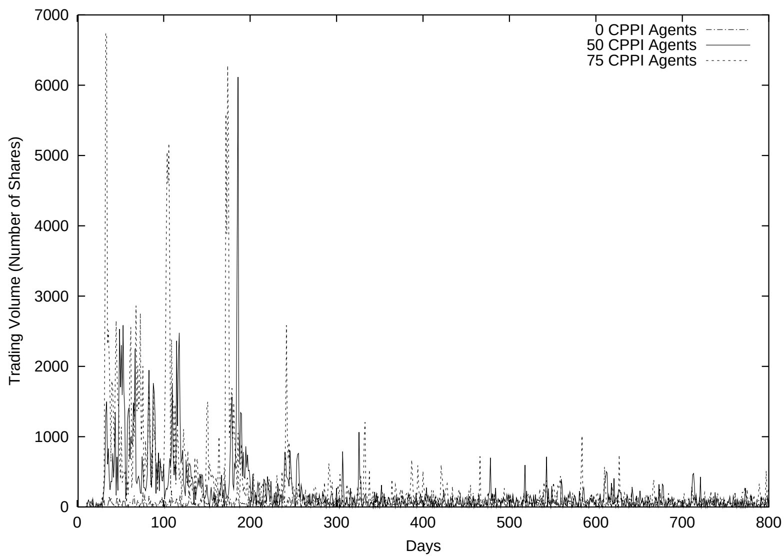

Fig. 2. The daily development of trading volume (total number of agents: 150)

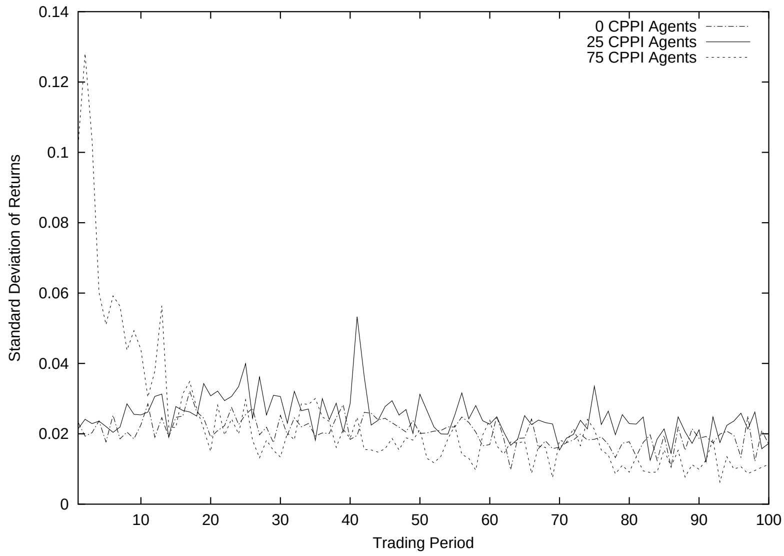

Fig. 3. The standard deviation of daily returns per trading period (total number of agents: 150)

Figure 4 shows a positive relationship between the proportion of bankrupt agents and the initial share of CPPI agents. Moreover, there is also a positive relation between the share of bankrupt CPPI agents in the total number of bankrupt agents and the initial rate of CPPI agents. Thus, in the case of 25 CPPI agents we had 13 bankrupt CPPI and 50 bankrupt rebalancing agents after 100 trading periods (i.e., 6500 days), whereas, in the case of 75 CPPI agents the ratio was 67 CPPI agents to 11 rebalancers (Figure 4, upper panel). The number of bankrupt investors is generally lower if we raise the total number of agents to 1500 (Figure 4, lower panel). Thus, it appears to be a kind of finite-size effect. Nevertheless, in this setting we still observe a reduction of volatility in the case of a CPPI agents' proportion equal to one half (i.e., 750 CPPI agents, cf. Figure 5). Compared to the previous setting, the level of volatility is now significantly higher with CPPI agents (both 250 and 750 CPPI agents) than without CPPI agents. From these experiments we conjecture that the impact of the portfolio insurance strategy on market volatility generally increases with growing market size.

Nevertheless, given that the model is intended to study the influence of portfolio insurance on the market, the strong reduction in the number of active market participants and, especially, the positive dependence of bankruptcies on the initial share of CPPI agents, constitutes a serious deficit of the model design. Presumably however, the quality of the results could be im-

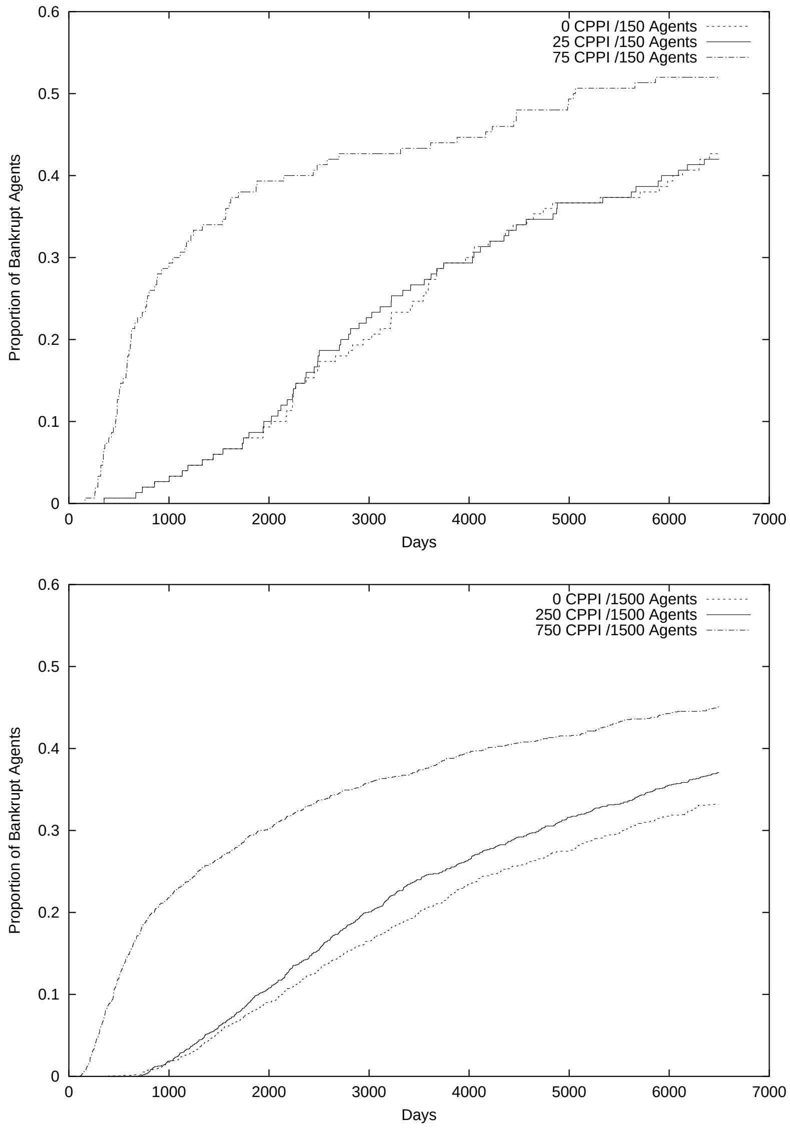

Fig. 4. The proportion of bankrupt investors to the total number of agents (150 and 1500 respectively)

proved by allowing bankrupt agents to be replaced by new solvent agents.[2] For a further set of simulations we replaced the individual bid and ask prices

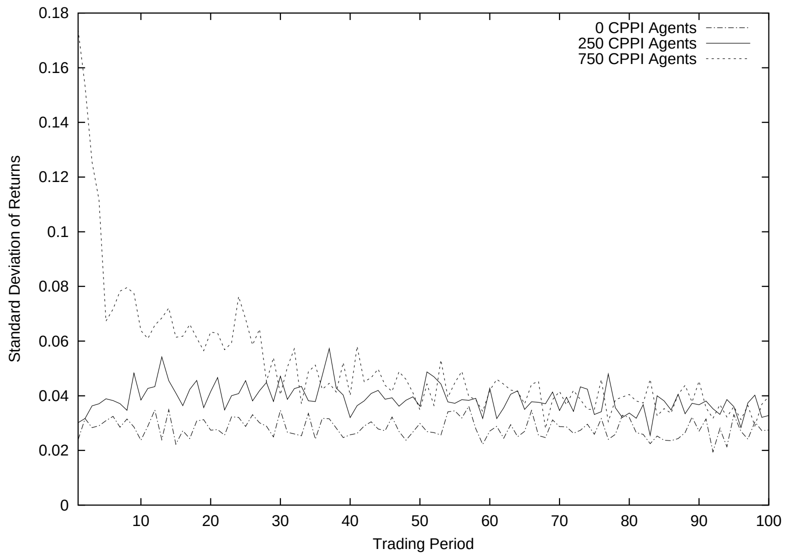

Fig. 5. The standard deviation of daily returns per trading period (total number of agents: 1500)

by one uniform market price which is set by a market maker reacting on the difference between supply and demand.[3] Thus, in case of excess demand (supply) prices rise (fall) proportional to the ratio of excess demand (supply) to the total number of shares with proportionality factor β:

$$p_t = p_{t-1}(1 + \beta \frac{ED_t}{ST_t}) \tag{7}$$

where ED is the excess demand and ST the total number of stocks in the market. As shown in Figure 6, after about 15 trading periods, the volatility in case of 75 CPPI agents for a price adjustment speed $\beta = 4$ hardly differs from the case with no CPPI agents. By increasing the price adjustment speed to $\beta = 8$ the volatility generally tends to increase (for both 0 and 75 CPPI agents). As in the previous setting, in this modified setting the strong decline

[2] Another way to prevent a large number of bankrupt agents is to choose an asymmetric distribution for the amounts withdrawn and deposited on the accounts of the agents. Actually, by just extending the limit of deposits from 8000 to 9000$ (and keeping the limit for withdrawals at -8000$) we discovered a strong reduction in the number of bankrupt agents.

[3] A similar modification of the model is described in Egenter *et al.* [60].

of volatility in the case of 75 CPPI agents is again due to the large number of bankrupt agents. Also similar to the original setting, we find almost cyclical

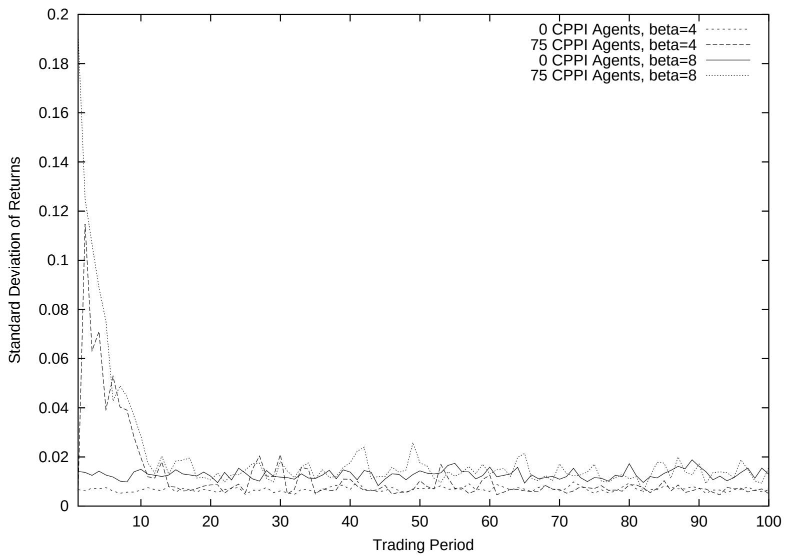

Fig. 6. The standard deviation of daily returns per trading period for $\beta = 4$ and $\beta = 8$ (total number of agents: 150)

price movements for a high proportion of CPPI agents among our market participants (Figure 7).

4.3 Conclusions

Deviating from our parameter setting, the original simulations by Kim and Markowitz start with the rebalancers' portfolio structure in disequilibrium, i.e., rebalancers initially have either too many or too few stocks. Additionally, in their setting, deposits are higher on average than withdrawals. The basic result of this approach is the demonstration of the destabilizing potential of portfolio insurance strategies. Kim and Markowitz, therefore, provide a theoretical foundation for the academic discussions on the sources of the 1987 crash. Their model, of course, was not designed to address other puzzles in empirical finance, like the "stylized facts" summarized in the introduction. A comprehensive simulation study and statistical analysis of model-generated data, in fact, showed that the time series characteristics exhibit hardly any similarities with empirical scaling laws [142]. Taking into account the pioneering character of this model and the intention of the authors to provide a partial explanation of the crash of October '87, the stakes should however not be set too high.

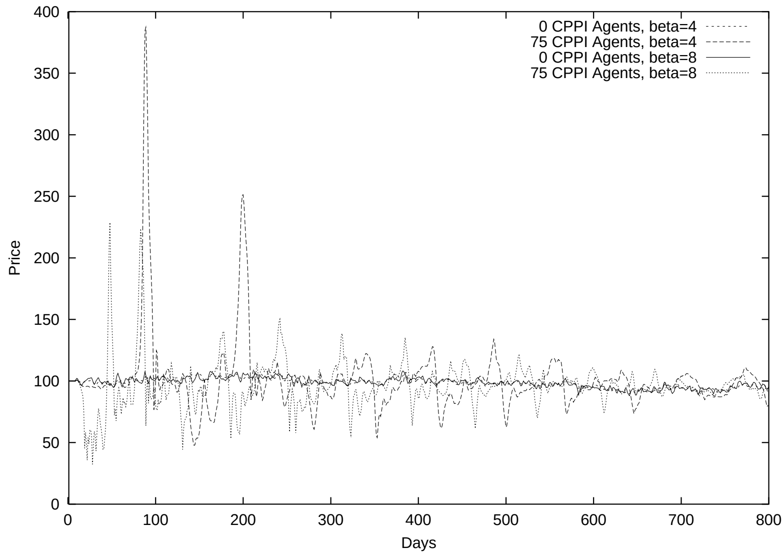

Fig. 7. The daily development of prices for $\beta = 4$ and $\beta = 8$ (total number of agents: 150)

5 An Early 'Econophysics' Approach: Levy-Levy-Solomon

Kim and Markowitz obviously tried to simulate a market populated by traders who pursue strategies found in real-life markets, and, therefore, gave a quite detailed description of activity at the microscopic level. In contrast to this highly specific set-up, more recent models deal with much more stylized and simple descriptions of traders' behavior. Historically, one of the first of these approaches is a collaboration of a group at Hebrew University including both economists and physicists. The first publication of their approach appeared in an economics journal in 1994 [103] which was followed later by more detailed reports in physics and computer science journals as well as a book [104,108,107,105,106].

5.1 The Model Set-Up

The model contains an ensemble of interacting speculators whose behavior is derived from a rather traditional utility maximization scheme. At the beginning of every period each investor i needs to divide up his entire wealth $W(i)$ into shares and bonds. Cash, credit, or short sales of stocks are not allowed. With $X(i)$ denoting the share of stocks in the portfolio of investor i, his wealth can be decomposed as follows:

$$W_{t+1} = \underbrace{X(i)W_t(i)}_{\text{sum of shares}} + \underbrace{(1-X(i))W_t(i)}_{\text{sum of bonds}} \tag{8}$$

with superimposed boundaries $0.01 < X(i) < 0.99$.

Additionally, the model assumes that the number of investors n as well as the supply of shares N_A are fixed. In addition to an identical utility function $U(W_{t+1})$, investors at the beginning also possess the same wealth and the same amount of stocks. Whereas the bond, assumed to be riskless, earns a fixed interest rate r, the stock return H_t is composed of two components (bonds are riskless in economics just like planets are point masses in the first physics lectures). On the one hand, either capital gains or losses can be the results of price variations p_t. On the other hand, the shareholder receives a daily or monthly [4] dividend payment D_t which grows by a constant rate over time:

$$H_t = \frac{p_t - p_{t-1} + D_t}{p_{t-1}} \tag{9}$$

In the base-line model, the preferences of investors are given by a logarithmic utility function $U(W_{t+1}) = \ln W_{t+1}$. This function fulfills the usual characteristics of a positively diminishing marginal utility. The consequence is an absolutely diminishing risk aversion, so that the amount of money invested in stocks increases with the wealth of an investor. The so-called "relative risk aversion" is constant and the share of stocks, therefore, remains constant.

Investors are assumed to form their expectations of future returns on the basis of their recent observations. Their memory span contains the past k total stock returns H_t. All investors with the same memory length k form an investor group G. They expect that the returns in question will reappear in the next period with a probability of $1/k$. The corresponding expected utility function $EU(X_G(i))$ has to be maximized with respect to the share of stocks X_G:

$$EU = \frac{1}{k}\left[\sum_{j=t}^{t-k+1} \ln\left[(1-X_G(i))W_t(i)(1+r) + X_G(i)W_t(i)(1+H_j)\right]\right] \tag{10}$$

$$f(X_G(i)) = \frac{\partial EU(X_G(i))}{\partial X_G(i)} = \sum_{j=t}^{t-k+1} \frac{1}{X_G(i) + \frac{1+r}{H_j - r}} = 0 \tag{11}$$

Like in most models, neither short-selling of assets nor buying financed by credit is allowed to the agents, so that the space of admissible solutions is restricted to a share of stocks in the interval $[0,1]$. Levy, Levy, Solomon, furthermore, impose minimum and maximum fractions of shares equal to 0.01

[4] The notion of the underlying time steps differs in the available publications.

and 0.99 in cases where the optimal solution of the optimization problem would imply a lower (higher) number. We, hence obtain either inner or outer solutions for $X_G(i)$ which are depicted in Table 1.

Table 1. Inner and outer solutions

$f(0)$	$f(1)$	$X_G(i)$
< 0	$-$	0.01
> 0	< 0	$0.01 < X(i) < 0.99$
> 0	> 0	0.99

When the optimum share of stocks is calculated for an investor group $X_G(i)$, a normally distributed random number ε_i is added to the result in order to derive each individual investor's demand or supply. This stochastic component may be interpreted as capturing the influence of idiosyncratic factors or of individual deviations from utility maximization from the economists' point of view. However, in the original papers it is motivated from a physics perspective as the influence of the "temperature" of the market. From aggregation of the stochastic demand functions of traders, the new stock price (and therefore, the total return H_t), can be calculated as an equilibrium price. One now eliminates the "oldest" total return from the investors' memory span and adds the "new" entry when the simulation process is finished for period t.

5.2 Previous Results

Models with only one investor group show periodic stock price developments (Figure 8) whose cycle length depends on the memory span k. This price development can be explained as follows: Let us assume that, at the beginning of the simulation, a random draw of the k previous total stock returns H_t occurs that encourages investors to increase the proportion of shares held in their portfolio. The larger total demand, then, causes an increase in price and therefore a new positive total return results. According to the updating of data the oldest total return will be dropped. This positive return causes the investor group to raise their stock shares successively up to a maximum of 99%. At this high price level the price remains almost constant for a little longer than k periods until the extremely positive return of the boom period drops from the investors' memory span. As explained above, the total return is composed of the capital gains or losses and of the dividend. Since the real dividend is relatively small because of the considerably high stock price, a relatively small (negative) total stock return (caused by the noise term ε_i) suffices to make the riskless bond appear more attractive. The desired share of stocks and with it the stock price, then, break down. If such a crash happens with an ensuing extremely negative total return the desired share of

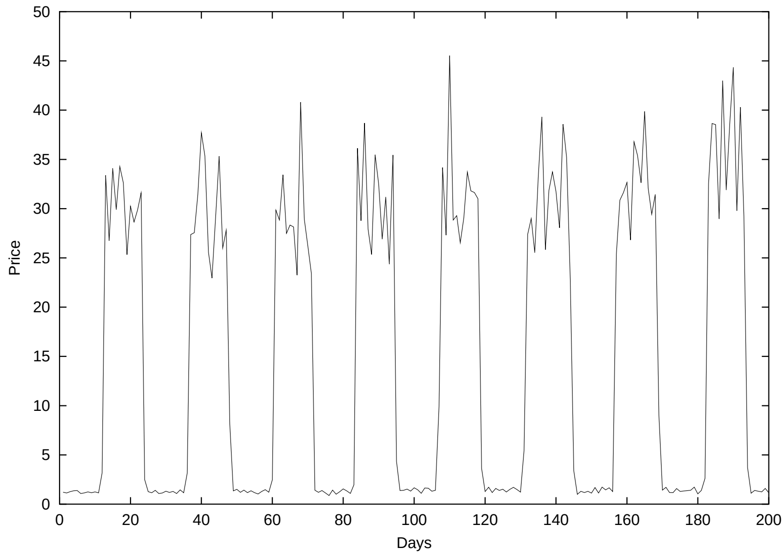

Fig. 8. With only one type of traders and a logarithmic utility function, the typical outcome of the Levy, Levy, Solomon model is a cyclic development of stock prices with periodic booms and crashes. Our own simulations produced all the visible patterns emphasized in [103,104,107].

stocks drops to a minimum of 1%. Again, it takes another k periods for the investors to forget about this extremely negative entry. Because of the then available high real dividend rate, investing in shares becomes more attractive compared to bonds. The total demand and the stock price start rising again and a new price cycle begins. If two groups with different memory spans are considered, strict periodicity still remains a possible outcome. However, depending on the choice of the memory spans, other dynamic patterns can appear. Looking at the distribution of total wealth, a dominating influence on the share price development by one group then does not necessarily mean that it also gains a dominant share of total wealth.

The model outcome becomes more irregular with three (and more) investor groups. For example, for the combination $k = 10, 141, 256$ and $n = 100$, Levy and Solomon claimed to have found chaotic motion in stock prices. However, Hellthaler [75] has shown that if the number of investors is increased from $n = 100$ to, for example, $n = 1000$, these chaotic stock price developments are replaced by periodic motion again. This effect persists if more than one type of stocks is traded [95]. Furthermore, Zschischang and Lux [184] found that the results concerning the wealth distribution for $k = 10$, 141 and 256 are not stable. While Levy, Levy and Solomon argued that the group with $k = 256$ usually turned out to be the dominant one, it is also possible that the investor group $k = 141$ will achieve dominance (Figure 9).

This shows an interesting extreme type of dependence of the model outcome on initial conditions brought about by seemingly minor differences within the first few iterations: depending solely on the random numbers drawn as the "history" of the agents at $t = 0$, we get totally different long-run results for the dynamics.

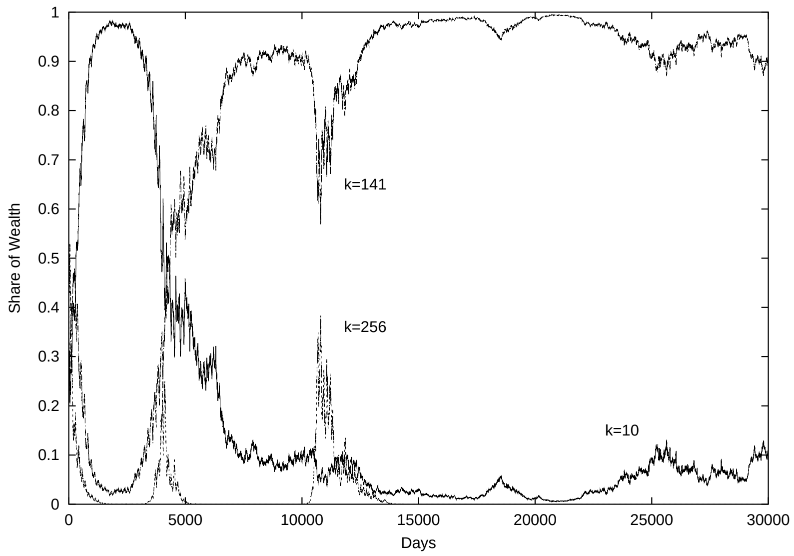

Fig. 9. Development of the distribution of wealth with three groups characterized by $k = 10$, 141, and 256, respectively. Depending on the initial conditions, either the group with $k = 256$ or the group with $k = 141$ as in the present case may happen to dominate the market

Of course, one would like to have microscopic models to provide an explanation of the power-law behavior of both large returns and the time-dependence in various powers of absolute returns. However, when investigating the statistical properties of Levy, Levy and Solomon's model, the outcome is as disappointing as with the Kim and Markowitz framework: none of the empirical scaling laws can be recovered in any of our simulations (see Zschischang [183] who investigates about 300 scenarios with different utility functions, memory spans and varying number of groups). As exemplified in Figure 10, models which are claimed to have a chaotic price development often have stock returns that appear to follow a Normal distribution (Figure 10) and do not account for "clustered volatilities" (Figure 11). These results are supported by standard tests of the Normal distribution and for the absence of correlations of stock returns. Zschischang and Lux argue that in these cases the Levy, Levy, Solomon model, instead of giving rise to low-dimensional

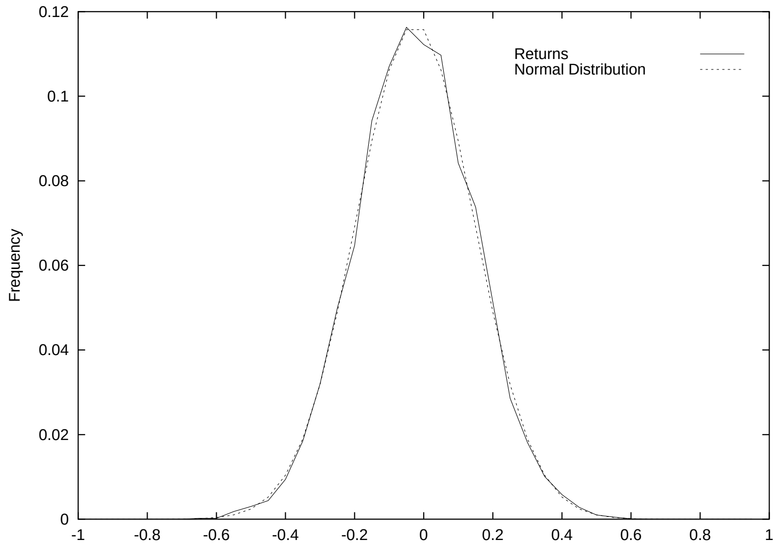

Fig. 10. Distribution of returns form a simulation with 6 groups characterized by memory spans $k = 10$, 36, 141, 193, 256, and 420. This is an example with a stock price development described as "chaotic" in [107]. However, it seems that the result is rather similar to pure randomness. The histogram is drawn for 20,000 observations after an initial transient of 100,000 time steps. The close similarity to the Normal distribution is confirmed by statistical measures: Kurtosis is 0.043 and skewness is -0.003. This yields a Jarque-Bera statistic of 1.55 which does not allow to reject the Normal distribution (significance is 0.46%)

chaotic dynamics and strange attractors, can effectively be viewed as a random number generator [184].

The original papers are not entirely clear about the lengths of the time increments of the model: they are sometimes denoted by "days" and sometimes by "months". Since at low frequencies, returns in real markets seem to approach a Gaussian distribution, in such an interpretation, the Normality of returns generated from the model might even appear to be a realistic feature. However, the mechanism for the emergence of a Gaussian shape is still different from its origin in monthly returns in reality. The latter seems to be the consequence of the aggregation of high-frequency returns whose distribution is within the domain of attraction of the Normal distribution (because of its power-law exponent above 2). In the LLS model, on the other hand, the Gaussian shape seems to originate from the aggregation of random demand functions *within* the same period.

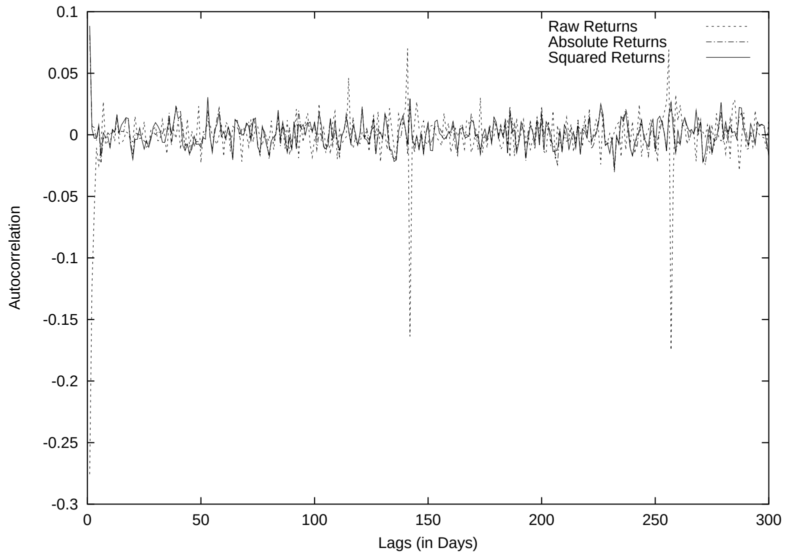

Fig. 11. Autocorrelations of raw, absolute and squared returns from the simulation of a "chaotic" case of the Levy-Levy-Solomon model. As can be seen, dependence in absolute and squared returns (as typical proxies for market volatility) is as weak as with raw returns themselves. The underlying scenario is the same as with Figure 10

6 Financial Markets and the Distribution of Wealth: Solomon-Levy-Huang

Again, the unrealistic time series characteristics of both the Kim and Markowitz and the Levy, Levy, Solomon approach should not be taken too seriously: both models are among the first attempts at microscopic simulations of financial markets and their aims were more to provide mechanisms for bubbles and crashes than to look at statistical features of the so generated time series. At least in the case of Levy, Levy, Solomon, the authors initially were not aware at all of the scaling laws characterizing financial markets (personal communication by Sorin Solomon). However, later on their model served as inspiration and starting point for the analysis of statistical properties of simulated data. As an interesting example, the inherent wealth dynamics in Levy, Levy, Solomon inspired a more thorough analysis of the development of traders' wealth in some kind of generalized Lotka-Volterra systems.

This extension is based on a 1996 model [109,149] which was re-investigated more recently. The pertinent results have been presented in a series of recent papers by Huang, Solomon and others. We thus call it the SLH model. Its mechanics can be described as a random multiplicative process for the wealth of each trader, with different traders coupled through their average wealth somewhat similar to predator-prey models.

Assume that all traders start with the same wealth but later each of them speculates differently on the market and gains or loses amounts proportional to his current wealth:

$$w_i(t+1) = \lambda w_i(t) \quad (i = 1, 2, \ldots, N) \tag{12}$$

where λ is a number fluctuating in a small interval centered about unity. This random multiplicative process has been discussed before. The new ingredient in SLH is the "welfare state": Nobody is allowed to fall below some poverty level $w_i = qW$ where $W = W(t) = \sum_i w_i(t)/N$ is the average wealth per trader at that time. Thus this model is very simple, but nevertheless possesses many realistic properties. Physicists can identify it with a random walk on a logarithmic wealth scale with one repelling boundary.[5] Instead of this cut-off, the authors also investigate the rule

$$w_i(t+1) = \lambda w_i(t) + aW(t), \tag{13}$$

which represents a rich society engaging in even redistribution of a certain fraction of overall wealth.

In the infinite N limit the same relative wealth distribution

$$P\left(\frac{w_i}{W}\right) \propto \left(\frac{w_i}{W}\right)^{-2-2a/D} \exp\left[\frac{-aW}{Dw_i}\right] \tag{14}$$

is obtained [138] for a more general and realistic model:

$$w_i(t+1) = \lambda w(i) + aW(t) - c(W,t)w_i(t) \tag{15}$$

where the arbitrary function $c(W,t)$ parameterizes the general state of the economy: time periods during which $-c(W,t)$ is large and positive correspond to boom periods while periods during which it is negative correspond to recessions. Complementarily, if one thinks of w_i as the real wealth (as opposed to the nominal number of dollars which could increase solely because of inflation) of each individual, an increase of the total amount of dollars in the system $W(t)$ means that an agent with individual wealth w_i will suffer from a real loss due to inflation in an amount proportional to the increase in average wealth and proportional to one's own wealth: $-c(W,t)w_i(t)$.

The following results are obtained: For infinite markets, a power law $\propto 1/w^\alpha$ was obtained for the probability of traders with wealth larger than w. The exponent for this power law is given by the cut-off: $\alpha = 1/(1-q)$. Thus if $q \cong \frac{1}{3}$ we have $\alpha \cong 1.5$ in agreement with well-known empirical findings. It would be interesting to see if in real economies this exponent and

[5] This passage, having been contributed by D.S., obviously reflects the tendency of physicists to know everything better. In fact, the remaining authors (although they are only economists) see no reason why they should be unable to recognize a random walk with a reflecting boundary.

the analogous one for the price fluctuations depend on the lower cut-off for wealth: The more egalitarian or socialist a country is, the higher will be q, and the higher will be the exponent α, making extreme wealth inequalities and market fluctuations less probable.

The amount trader i invests on the stock market is proportional to the wealth w_i : George Soros produces more price changes than the present authors together (we expect this to change in the near future). Thus the fluctuations of the market price were at first thought to have the same tail exponent α as the wealth distribution. However, this is not true because the different traders are not statistically independent: the cut-off $w_i \geq qW$ introduces a coupling via the average wealth W.

Moreover, real markets are finite, and according to a 1999 review of microscopic market models [156], the majority of these models get unrealistic properties like periodic oscillations, if the market size goes to infinity. In short, a few hundred professional speculators and not the millions of non-speculative families dominate most of the market movements. The thermodynamic limit, so traditional in statistical physics where a glass of beer contains 10^{25} molecules and where 16×10^{12} sites were already simulated [165], may, therfore, be very unrealistic for markets.

Indeed, simulations of the SLH model for $10^2 \ldots 10^4$ traders gave effective exponents $\alpha \simeq 3$, i.e. close to the desired one for the price flcutuations (not the wealth distribution). These exponents are valid only in some intermediate range: For small wealth the cut-off is important, and nobody can own more wealth than is available in the whole market. We refer to the SLH papers for more details on this approach [80,148,81,23,82,150,138,112].

A somewhat related recent strand of literature has analysed simple monetary exchange models. The main question pursued in this area is emergence of inequality within a pool of agents due to the randomness of their fortunes in economic interactions. This line of research is represented, among others by [27], [38], [55]. The structure of all these models is very simple: agents are randomly matched in pairs and try to catch some of the other agents wealth in this encounter. A random toss decides which of both opponents is the winner of this match. The successful agent, then, leaves the battle field with his wealth having increased by a fraction of the other agent's previous belongings. The above papers show that this simple random exchange model (with only minor differences in the stochastic formalisation in the above papers) leads to an endogeneous emergence of inequality within an initially homogenous population. It is, however, worthwile to point out that exactly the same process had already been proposed in [3] by sociologist John Angle and has been extended in various ways in the pertinent literature over the years ([4,5]). Needless to say that physicists would have gained by first consulting the literature on the subject before starting to duplicate well-established lines of research. It might also be remarked that in the recent economics literature, a number of more realistic models of wealth formation and agend-based models exist (e.g. [146]). A more extensive discussion of this class of model can be found in [117].

7 Percolation Theory Applied to Finance: Cont-Bouchaud

Together with the random walk model of Bachelier [15] hundred years ago, and the random multiplicative traders of SLH, the Cont-Bouchaud model is one of the simplest models, having only a few free parameters (compared, e.g., to the "terribly complicated" Lux-Marchesi model reviewed below). Economists like biologists may bemoan this tendency of physicists, but the senile third author from the physics community likes it. Also, it is based on decades of percolation research in physics, chemistry and mathematics, just as Iori's random-field Ising model uses many years of physics experience in that area [83]. Obviously, with this type of models, "econophysicists" have introduced new tools of analysis to financial modeling. As recent research in economics has focused on communication and information among traders (e.g., [93,19]), the random-field and percolation models might be welcome means for the investigation of information transmission or formation of opinions among groups of agents.

In percolation theory, invented by the later chemistry Nobel laureate Paul Flory in 1941 to explain polymer gelation (cooking of your breakfast egg), and later applied by Broadbent and Hammersley to coal-dust filters, and by Stuart Kauffman to the origin of life, each site of a large lattice is either occupied (with probability p), or empty (with probability $1-p$).

Clusters are groups of occupied neighbors. A cluster is infinite if its mass s (the number of occupied sites belonging to that cluster) increases with a positive power of the lattice size (and not only logarithmically). When the density p increases from zero to one, at some sharp percolation threshold p_c for the first time (at least) one infinite cluster appears; for all $p > p_c$ we have exactly one infinite cluster for large and not too anisotropic lattices, filling a positive fraction p_∞ of the whole lattice. For all $p < p_c$ we have no infinite cluster. At $p = p_c$ we find incipient infinite clusters which are fractal. If p approaches p_c from above, the fraction p_∞ vanishes as $(p-p_c)^\beta$ with some critical exponent β varying between zero and unity for dimensionality going from one to infinity. The Hoshen-Kopelman and the Leath algorithm to produce and count percolation clusters are well documented with complete programs available from DS.

The average number n_s of clusters containing s sites each follows a scaling law for large s close to the threshold p_c

$$n_s = s^{-\tau} f[(p-p_c)s^\sigma] \tag{16}$$

where the exponent τ varies from 2 to 2.5 if d goes from one to infinity, and the exponent σ stays close to $1/2$ for $2 \le d \le \infty$. The previous exponent β equals

$(\tau - 2)/\sigma$. The details of the lattice structure do not matter for the exponents, only for the numerical value of $p_c (\cong 0.5927464$ for nearest neighbors on the square lattice). On a Bethe lattice (Cayley tree), $\tau = \frac{5}{2}$, $\sigma = \frac{1}{2}$, $\beta = 1$, and the above scaling function f for the cluster numbers is a Gaussian; these exponents are also found for $6 < d \leq \infty$ and for the case that each site is a neighbor to all other sites (i.e., a "random graph") [127].

The above model is called site percolation; one can also keep all sites occupied and instead break the links between neighboring sites with a probability $(1-p)$. This case is known as bond percolation and has the same exponents as site percolation. Computer programs to count percolation clusters were published by many, including the senile co-author [158,159]. All this knowledge was available already before percolation was applied to market fluctuations.

In the Cont-Bouchaud market model, originally restricted to the mathematically solvable random graph limit and later simulated, as reviewed in [157], on lattices with $2 \leq d \leq 7$, each occupied site is a trader. A cluster is a group of traders making joint decisions; thus the model simulates the herding tendency of traders. At each time step, each cluster either trades (with probability $2a$) or sleeps (with probability $1 - 2a$), and when it trades it either buys or sells an amount proportional to the size s of the cluster i; thus a usually is the probability for the member of a cluster to be a buyer. The market price is driven by the difference between the total supply and demand; the logarithm of the price changes proportionally to this difference (or later [161,182] to the square-root of the difference). The concentration p is either fixed or varies over the whole interval between zero and unity, or between zero and p_c. The results deteriorate [37,40] if the price change is no longer proportional to the difference between demand D and supply S, but to the relative difference $(D - S)/(D + S)$ or to a hyperbolic tangent $\tanh[\mathrm{const}\cdot(D - S)]$. However, the latter has been found to be a more realistic description of the price impact of demand variations [134]. Some results are: If the activity a increases from small values to near its maximum value 1/2, the histogram of the price fluctuations changes from an asymptotic power law to something close to a Gaussian, similar to the crossover in real markets when the observation time goes to infinity. For small activities, the cumulative probability to have a change by at least $x\%$, varies in its tail as $1/x^{\mu}$, with [161,157] $\mu = 2(\tau + \sigma - 1)$ if we use Zhang's square-root law and average over all p between 0 and p_c. This exponent μ varies from 2.9 and 3.3 to 4 if d increases from two and three to infinity. Thus in the realistic dimensions of a city or bank building, $d = 2$ or 3, we get the desired $\mu \cong 3$. Figure 12 shows simulations giving this power law, except for flattening at small x and a cut-off due to finite market sizes at large x. The curve through the data is the Student-t distribution following from Tsallis statistics [154].

Volatility clustering, positive correlations between trading volume and price fluctuations, as well as the observed asymmetry between sharp peaks and flat valleys is seen if the activity increases (decreases) in a time of increas-

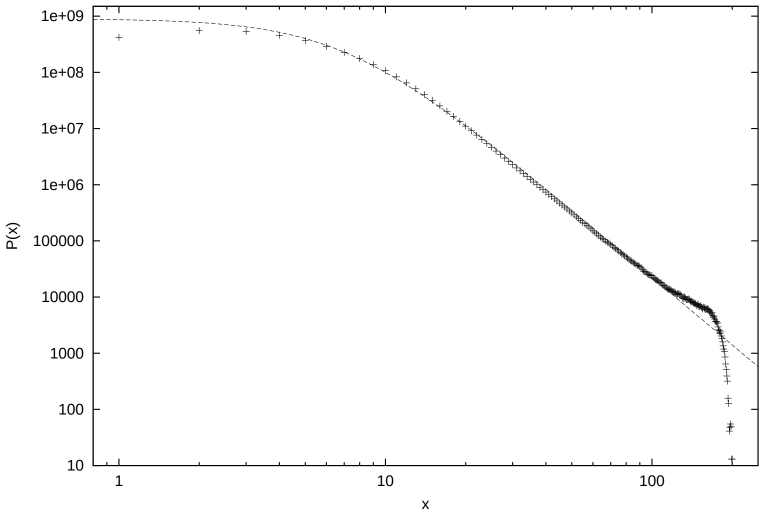

Fig. 12. Distributions of price changes from the Cont-Bouchaud percolation model. The figure also compares $P(x)$ with const$/(3+0.06*x^2)^2$. The underlying data are averages from many 301×301 square lattices

ing (decreasing) market prices. Nearly log-periodic oscillations are seen if a non-linear restoring force (buy if the price was low) is combined with some hysteria (buy if the price was rising). For more details we refer to the original papers following [48] reviewed in [157]. After this review, more effects were understood and variations were tried. The crossover towards a Gaussian return distribution for increasing activity was explained [97]. Instead of lattices of various dimensions, the Barabasi network was assumed as the home of the Cont-Bouchaud traders, with good results [98]. Thermodynamic Ising-like modifications [145], in the direction of the Iori model [83] were proposed and gave reasonable histograms of price fluctuations. The lack of time-reversal invariance was recovered by putting some risk aversion psychology into the buying and selling probabilities [39]. Multifractality was found [36] in the higher moments of the return distributions for different time intervals. Also a combination of these various modifications worked reasonably though not ideally [40]; see e.g. Figure 13.

Applications included triple correlations between Tokyo, Frankfurt and New York markets [143] and the effects of a small Tobin tax on all transactions [61,176]. Two physicists, Ehrenstein and Stauffer and an economist, Westerhoff first independently simulated how such a tax would influence the market. Depending on parameters, either the total tax revenue has a maximum in the region of up to one percent taxation, or it increases monotonically. Taking into account the tendency of governments to overexploit available

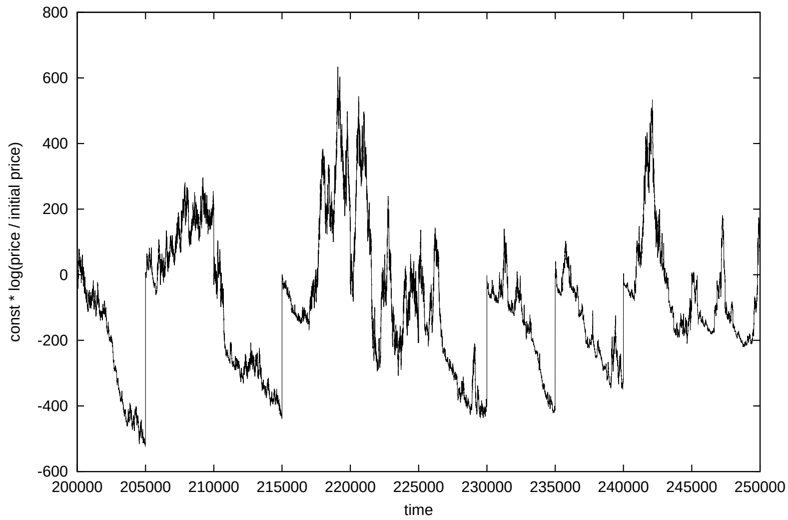

Fig. 13. Examples of price versus time in a modified Cont-Bouchaud model, showing sharp peaks and flat valleys [40]

sources of tax income, they recommend the Tobin tax only for the first case, not the second. It then would reduce the amount of speculation, but not by an order of magnitude [62]. Summarizing, it therefore appears that the Cont-Bouchaud models and the subsequent variations on their theme contributed by other authors have gone some way in explaining important stylized facts of financial markets. Nevertheless, economists often feel somewhat uneasy about this approach. The reason is that its starting point is known knowledge about the characteristics of certain graph-based dynamics (i.e., percolation models in statistical physics). The "explanation" of stylized facts in economics is, then, achieved to some extend via a mere relabeling of known physical quantities into new ones with an economic interpretation. Economists, of course, would like to start with basic facets of economics interaction of real-life markets rather than with a lattice-based architecture. Furthermore, the many attempts at "improvements" of the model outlined above show that realistic results are only obtained under extremely specific settings. Hence, it appears questionable whether this framework really allows an explanation of empirical findings that is "independent of microscopic details" as postulated in an econophysics manifesto (Stanley et al. [155]).

8 Social Percolation and Marketing: Solomon-Weisbuch

Nevertheless, scientists have a natural tendency to apply what they have learned to as many different problems as possible (maximizing thereby the number of their publications). Percolation theory seems to be one of the discoveries one can use in quite a number of fields. Besides financial markets, another application concerns "social percolation" and its use in marketing [151] which we review in the following (departing shortly from our central subject of financial markets).

As in the previous section, every site of a large lattice is randomly either occupied or empty, and a cluster is a set of occupied neighbors. Now we identify occupied sites with potential customers of one specific product, say, a Hollywood movie. Each site i has a certain quality expectation p_i, and the product has a certain quality q. The values of p_i are homogeneously distributed between zero and unity. Only those people visit this movie (or more generally, buy this product) who regard its quality as sufficient, i.e. who have $p_i < q$. We thus define as occupied a site with $p_i < q$, and then a site is occupied with probability q. All sites i in a cluster have $p_i < q$.

If all potential customers are immediately informed about the new product and its quality q, then a fraction q of all sites will buy, a trivial problem. But since we get so much advertising, we may mistrust it and consider buying a movie ticket only if we hear from a neighbor about its quality q. Thus a site i buys if and only if one of its nearest neighbors on the lattice has bought before, if i has not bought before, and if $p_i < q$ (customers are assumed to have the same perception of the quality of the product, i.e. the quality assessment q they tell their neighbors is the same for all customers.) Initially, all occupied sites on the top line of the lattice (top plane in three dimensions) are regarded as customers who have bought and who thus know the quality q. In this way, geometry plays a crucial role, and only those sites belonging to clusters which touch the upper line get informed about the movie and see it. If and only if one cluster extends from top to bottom of the lattice, we say that the cluster percolates or spans. And standard percolation theory then predicts a spanning cluster if the occupation probability (or quality) q is larger than some threshold p_c, which is about 0.593 on the square lattice [158,141,34]. (Instead of starting from the top and moving towards the bottom, one may also start with one site in the center and move outwards. The cluster then percolates if it touches the lattice boundary.)

In this way the decades-old percolation theory divides new products into two classes: Hits and flops [173], depending on whether or not the quality was high enough to produce a spanning cluster. For a flop, $q < p_c$, only sites near the initially occupied top line get informed that the new cluster exists, while for a hit, $q > p_c$, also customers "infinitely" far away from the starting line buy the movie ticket. In the case of a hit, except if q is only very slightly larger than p_c, nearly all customers with $p_i < q$ get the information and go to the movie; only a few small clusters are then isolated from the spanning network and have no chance to react. Figure 13 shows three examples, where one time step corresponds to one search through all neighbors of previously occupied sites (Leath algorithm [159]). In traditional marketing theories, as

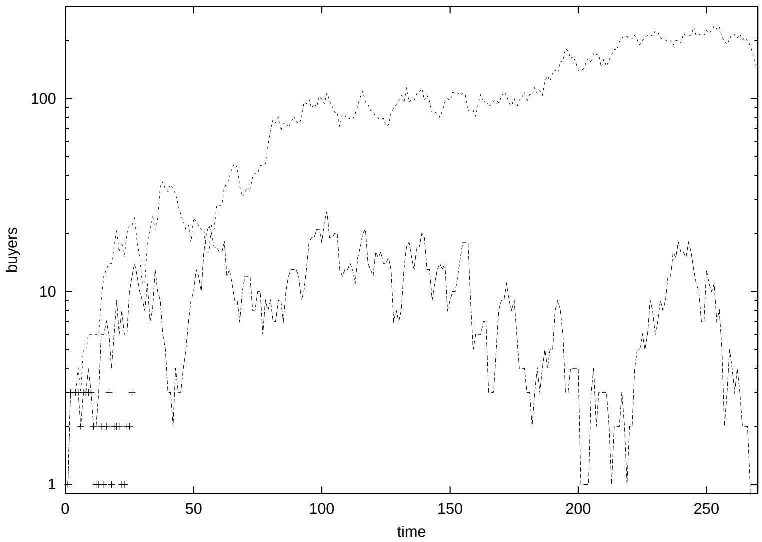

Fig. 14. Examples of social percolation starting with one central site occupied; $q = p_c - 0.05$ (plus signs), $q = p_c$ (middle line), $q = p_c + 0.05$ (upper line). We plot logarithmically the number of new buyers in each time interval [72]. For the first four time steps the three buying curves agree, since the same random numbers have been used

discussed in [72], one neglects the spatial structure, and a growing market has an exponential time dependence until saturation sets in. For averages over lattices with spanning clusters, instead we have power laws in time [72]. In reality, both cases have been observed, in addition to complicated behavior somewhat similar to the curve $q = p_c$ of Figure 13.

The first modification of this static percolation model is to assume that the quality q changes in time: When a movie was successful (i.e. when the cluster percolated), the producer lowers the quality of the next movie slightly; when it was a flop (no percolating cluster), q is slightly increased. With this dynamics, like $q \to q \pm 0.001$, the q automatically moves to the threshold p_c, an example of self-organized criticality. In addition, we may assume [151] that also the p_i change: p_i increases by a small amount if i just has seen a movie, and it decreases by the same amount if the agent did not see a movie previously (in the second case one has to distinguish whether the customer refused to buy because of $p_i > q$ or merely was not informed about the movie.) In this case also the p_i can move towards p_c , though slower than q, or they may be blocked at some intermediate value; also instabilities can occur where q and all p_i move towards infinity. These difficulties were clarified by Huang [79], who also applied this model to stock markets [78].

Information through advertising influences the percolative phase transition [136]. We refer to the literature cited above as well as to [73,1,175,172] [174,147] for further details and modifications.

There is (of course) also a large body of economic research dealing with similar problems. In fact, the analysis of *irreversible lock-in* and *path dependence* in the adaption of new goods or new technologies is often based on mass-statistical models. A prominent example is Arthur [12] who used nonlinear Polya urn models as an abstract model of such processes. The application of similar ideas as an explanation for geographical concentration of economic activity led to a remarkable revival of the formerly dormant field of regional economics over the nineties (cf. Arthur [13], Krugman [96]). Multi-agent approaches to "hits" and "flops" in the movie industry (using Bose-Einstein dynamics) with empirical applications can be found in De Vany and Walls [168] and De Vany and Lee [167].

With the next (and last) model we come back to financial markets.

9 Speculative Interaction and the Emergence of Scaling Laws: Lux-Marchesi

The model of Lux and Marchesi [119] has its roots in earlier attempts of economists at introducing heterogeneity into stochastic models of speculative markets. Inspired by the analysis of herd behavior in ant colonies [93] and earlier applications of statistical mechanics to various problems in sociology and political sciences (Weidlich and Haag [170,171]), a stochastic model of trading in financial markets has been developed in [113]. The basic ingredient of this contribution was a kind of mean-field dynamics for the opinion formation process among speculators together with a phenomenological law for the ensuing price adjustment in the presence of disequilibria. Using the Master equation formalism, it could be shown that the model is capable of generating "bubbles" with over- or undervaluation of the asset as well as periodic oscillations with repeated market crashes. A detailed analysis of the dynamics of second moments (variances and co-variances) was added in [116] where the potential explanatory power of multi-agent models for the typical time-variations of volatility in financial markets was pointed out.

The group interactions in this model have been enriched in [115] by allowing agents to switch between a chartist and fundamentalist strategy. This more complicated dynamics was shown to give rise to chaotic patterns in mean values of the relevant state variables (the number of agents in each group plus the market price). Numerical analysis of simulated chaotic attractors showed that they came along with leptokurtosis (fat tails) of returns, hence providing a possible explanation of one of the ubiquitous stylized facts of financial data.

Both microscopic simulations as well as more detailed quantitative analyses of the resulting time series appeared in Lux and Marchesi [119,120] and

Chen *et al.* [41]. The fact that these key issues were approached quite lately in the development of this market model to some extent reflects a broader trend in the related literature: As already pointed out above, almost all the early simulation models developed in economics had the initial goal of investigating the formation of expectations of economic agents in out-of-equilibrium situations (where it is hard to form "rational", i.e., correct expectations about the future) and analyzing the selection of equilibria in the presence of multiple consistent solutions of a static framework. Interestingly, a development similar to that of the Lux-Marchesi model can also be observed in the case of the Santa Fe Artificial Stock Market. Although the latter was constructed by a group of researchers from economics, physics, biology and computer science already in the eighties, an analysis of the statistical properties of the resulting time series only appeared recently (LeBaron *et al.*, [102]).

The dynamics of the "terribly complicated" (D.S.) Lux-Marchesi model is illustrated in Figure 15. It is a kind of feedback between group dynamics

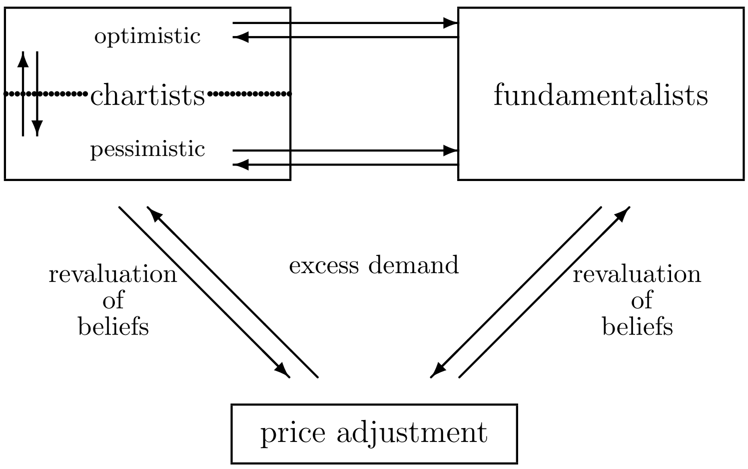

Fig. 15. Flowchart of dynamics of the Lux-Marchesi model: agents are allowed to switch between different behavioral alternatives. The number of individuals in these groups determines excess demand (the difference between demand and supply). Imbalances between demand and supply cause price adjustments which in turn affect agents' choices of strategies

and price adjustment in the presence of imbalances between demand and supply. Starting with basic definitions we denote by N the total number of agents operating in our artificial market, n_c the number of noise traders, n_f the number of fundamentalists ($n_c + n_f = N$), n_+ the number of optimistic noise traders, n_- the number of pessimistic noise traders ($n_+ + n_- = n_c$); p is the market price, p_f the fundamental value.

The dynamics of the model are composed of the following elements:

1. *noise traders' changes of opinion* from a pessimistic to an optimistic mood and *vice versa*: the probabilities for these changes during a small time increment Δt are given by $\pi_{+-}\Delta t$ and $\pi_{-+}\Delta t$ and are concretised as follows:

$$\begin{aligned} \pi_{+-} &= v_1 \frac{n_c}{N} \exp(U_1) \\ \pi_{-+} &= v_1 \frac{n_c}{N} \exp(-U_1) \\ U_1 &= \alpha_1 x + \frac{\alpha_2}{v_1} \frac{dp}{dt} \frac{1}{p} \end{aligned} \tag{17}$$

Here, the basic influences acting on the chartists' formation of opinion are the majority opinion of their fellow traders, $x = \frac{n_+ - n_-}{n_c}$, and the actual price trend, $\frac{dp}{dt}\frac{1}{p}$. Parameters v_1, α_1, and α_2 are measures of the frequency of revaluation of opinion and the importance of "flows" (i.e. the observed behaviour of others) and charts, respectively. Furthermore, the change in asset prices has to be divided by the parameter v_1 for the frequency of agents' revision of expectations since for a consistent formalization one has to consider the *mean* price change over the *average* interval between successive revisions of opinion. The transition probabilities are multiplied by the actual fraction of chartists (that means, it is restricted to such a fraction) because chartists are also allowed to interact with fundamental traders in the second component of the group dynamics that follows below.

2. *switches between the noise trader and fundamentalist group* are formalised in a similar manner. Formally, one has to define four transition probabilities, where the notational convention is again that the first index gives the subgroup to which a trader moves who had changed her mind and the second index gives the subgroup to which she formerly belonged (hence, as an example, π_{+f} gives the probability for a fundamentalist to switch to the optimistic chartists' group):

$$\pi_{+f} = v_2 \frac{n_+}{N} \exp(U_{2,1}), \quad \pi_{f+} = v_2 \frac{n_f}{N} \exp(-U_{2,1}) \tag{18}$$

$$\pi_{-f} = v_2 \frac{n_-}{N} \exp(U_{2,2}), \quad \pi_{f-} = v_2 \frac{n_f}{N} \exp(-U_{2,2}). \tag{19}$$

The forcing terms $U_{2,1}$ and $U_{2,2}$ for these transitions depend on the difference between the (momentary) profits earned by using a chartist or fundamentalist strategy:

$$U_{2,1} = \alpha_3 \left\{ \frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p} - R - s \cdot \left| \frac{p_f - p}{p} \right| \right\} \tag{20}$$

$$U_{2,2} = \alpha_3 \left\{ R - \frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p} - s \cdot \left| \frac{p_f - p}{p} \right| \right\} \tag{21}$$

The first term of the U functions represents the profit of chartists from the n_+ group and n_- group. The second term is the profit of the fundamentalists. The parameters v_2 and α_3 are reaction coefficients for the frequency with which agents reconsider appropriateness of their trading strategy, and for their sensitivity to profit differentials, respectively. Excess profits (compared to alternative investments) enjoyed by chartists from the optimistic group are composed of nominal dividends (r) and capital gains due to the price change (dp/dt). Dividing by the actual market price gives the revenue per unit of the asset. Excess returns compared to other investment opportunities are computed by substracting the average real return (R) received by the holders of other assets in our economy. Fundamentalists, on the other hand, consider the deviation between price and fundamental value p_f (irrespective of its sign) as the source of arbitrage opportunities from which they may profit after a return of the price to the underlying fundamental value. As the gains of chartists are immediately realised whereas those claimed by fundamentalists occur only in the future (and depend on the uncertain time for reversal to the fundamental value) the latter are dicounted by a factor $s < 1$. Furthermore, neglecting the dividend term in fundamentalists' profits is justified by assuming that they correctly perceive the (long-term) real returns to equal the average return of the economy (i.e. $r/p_f = R$) so that the only source of excess profits in their view is arbitrage when prices are "wrong" ($p \neq p_f$). As concerns the second U-function, $U_{2,2}$ one considers profits from the viewpoint of pessimistic chartists who in order to avoid losses will rush out of the market and sell the asset under question. Their fall-back position by acquiring other assets is given by the average profit rate R which they compare with nominal dividends plus price change (which, when negative, amounts to a capital *loss*) of the asset they sell. This explains why the first two items in the forcing term are interchanged when proceeding from $U_{2,1}$ to $U_{2,2}$.

3. *price changes* are modelled as endogenous responses of the market to imbalances between demand and supply. Assuming that optimistic (pessimistic) chartists enter on the demand (supply) side of the market, excess demand (the difference between demand and supply) of this group is:

$$ED_c = (n_+ - n_-)t_c \tag{22}$$

with t_c being the average trading volume per transaction. Fundamentalists' sensitivity to deviations between market price and fundamental value leads to a law of the type:

$$ED_f = n_f \cdot \gamma \frac{p_f - p}{p} \tag{23}$$

with γ being a parameter for the strength of reaction. In order to conform with the general structure of our framework, we also formalise the price adjustment process in terms of (Poisson) transition probabilities. In particular, we use the following transition probabilities for the price to increase or decrease by a small percentage $\Delta p = \pm 0.001p$ during a time increment Δt:[6]

$$\pi_{\uparrow p} = max[0, \beta(ED + \mu)] \ , \ \pi_{\downarrow p} = -min[\beta(ED + \mu), 0] \tag{24}$$

where β is a parameter for the price adjutment speed and $ED = ED_c + ED_f$ is overall excess demand (the sum of excess demand by both noise traders and fundamentalists).

This probabilistic rule for price adjustments is, in fact, equivalent to the traditional Walrasian adjustment scheme. It can be shown that the mean value dynamics of the price can be depicted by the textbook differential equation for the dependence of price changes on overall excess demand:

$$\frac{dp/dt}{p} = \beta \cdot ED = \beta \cdot (ED_c + ED_f) \tag{25}$$

Note that these price changes feed back on agents' decisions to follow one or the other trading strategy: a price increase will reinforce optimistic beliefs and will make formerly pessimistic chartists join a bullish majority. Similarly, price changes might bring p closer to an assumed fundamental value, p_f, which strengthens fundamentalist beliefs, or they might lead to larger deviations from p_f which reinforces the position of chartists. All in all, the resulting confirmation or disappointment of agents' opinions together with changing profitability of strategies will lead to switches between groups altering the composition of the population and effecting excess demand of the following period. The model also allows for exogeneous changes of the fundamental value:

4. *changes of fundamental value*: in order to assure that none of the stylised facts of financial prices can be traced back to exogenous factors, we assume that the log-changes of p_f are Gaussian random variables: $\ln(p_{f,t}) - \ln(p_{f,t-1}) = \varepsilon_t$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. The Poisson type dynamics of asynchronous updating of strategies and opinions by the agents can only be approximated in simulations. In particular, one has to choose appropriately small time increments in order to avoid artificial synchronicity of decisions. In [35, 108, 109] a simulation program with some flexibility in the choice of the time increment is used. Namely, time increments $\Delta t = 0.01$ are used for "normal times", while during volatility bursts the precision of the simulations was automatically increased by a factor 5 (switching to $\Delta t = 0.002$) when the frequency of price changes became higher than average. This procedure requires that all the above Poisson rates be divided by 100 or 500, (depending on the precision of the simulation) in order to arrive at the probability for any single individual to change his behaviour during $[t,\, t + \Delta t]$. Similarly, it is assumed that the auctioneer adjusts the prevailing price by one elementary

[6] The increment Δp has been chosen as small as possible in order to avoid artificial lumpiness of price changes with concentration of the distribution of returns at a few values only.

unit (one cent or one pence) with probabilities $w_{\uparrow p}$ or $w_{\downarrow p}$ during one time increment. For the time derivative, dp/dt, the average of the prices changes during the interval $[t - 10\Delta t,\ t]$ has been used. Furthermore, occurence of the "absorbing states" $n_c = 0$ ($n_f = N$) and $n_c = N$ ($n_f = 0$) was excluded by setting a lower bound to the number of individuals in both the group of chartists and fundamentalists.

The overall results of this dynamics is easily understood by investigation of the properties of stationary states (cf. [120]), i.e., situations in which there are no predominant flows to one of the groups and the price remains constant. Such a scenario requires that there is a balanced disposition among (chartist) traders, i.e., we neither have a dominance of optimists over pessimists (nor *vice versa*) and that the price is equal to the fundamental value (which makes fundamentalists inactive). A little reflection reveals that in such a situation, there is no advantage to either the chartist or fundamentalist strategy: no misprizing of the asset nor any discernible trends exist. Hence, the composition of the population becomes *indeterminate* which implies that, in the vicinity of these stationary states, the group dynamics is governed only by stochastic factors. [7] Hence, to a first approximation one can abstract from the economic forces which apparently become relevant only in out-of-equilibrium situations. As detailed in [119,120], the stationary states described above may either be locally stable or unstable with the number of chartists acting as a bifurcation parameter. Simulations show that temporary deviations into the unstable region can be interpreted as intermittent behavior which generates clusters of volatility and numerically accurate power laws for the tail behavior of raw returns as well as long-term dependence in absolute and squared returns. Figure 16 illustrates the interplay between the dynamics of relative price changes and the development of the number of chartists among traders. As can be directly inferred from the graph, an increase of the number of chartists leads to intermittent fluctuations. Note also that the model incorporates self-stabilizing forces leading to a reduction of the number of chartists after a period of severe fluctuations. The reason is that large deviations of the price from its fundamental value lead to high potential profits of the fundamentalist strategy which induces a certain number of agents to switch away from chartism. Chen *et al.* [41] also show that the motion of the market price appears totally random (judged by standard tests for determinism and nonlinearity) in tranquil times but shows traces of non-linear structure during more volatile episodes [41]. This feature appears to be in harmony with findings for the U.S. stock market [110].

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[7] A similar indeterminacy in the number of agents in different groups has been found in a model of resource extraction (Youssefmir and Huberman [179]). They also emphasize that this indeterminacy can lead to burst of activity (temporary large fluctuations).

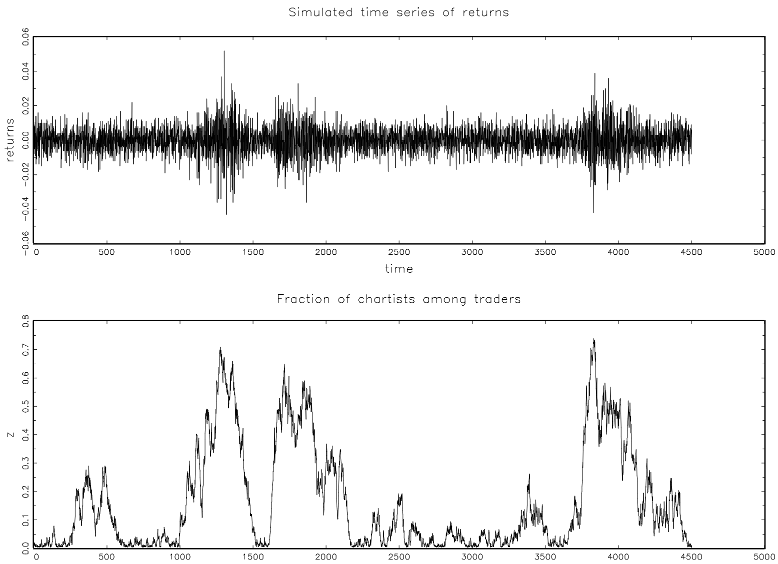

Fig. 16. Time series of returns (relative price changes, upper panel) and the fraction of chartists (lower panel) from a typical simulation of the Lux-Marchesi model

10 Discussion

While early attempts at microscopic simulations of financial markets appeared unable to account for the ubiquitous scaling laws of returns (and were, in fact, not devised to explain them), some of the recent models seem to be able to explain some of the statistical properties of financial data (usually denoted as "anomalies" in economics). Nevertheless, there is still a number of important topics left to future research: first, the recent surge of newly available data on the intra-daily level has opened a Pandora's box of new regularities on very small time scales (cf. Dunis and Zhou [58]). While the ubiquitous scaling laws found in all markets might be explained well by simple mechanisms beloved by physicists, the more delicate intra-daily patterns may require more detailed models (denoted as "monsters" in a workshop presentation of a recent paper by Maslov [125]). If physicists do not want to stop half-way in their contribution to economics, they may probably have to develop, as is typically done in economics, models with more institutional background.[8] Second, although we have a bunch of models for power-laws, their generality is still restricted in one very important respect: the "interesting" dynamics does only apply for a certain range of the population size N of speculators and in most cases does not survive for $N \to \infty$. This has been shown for the Kim and Markowitz and Lux and Marchesi models in Egenter *et al.* [60] and probably applies to most alternative approaches. A recent investigation of Chen and Yeh's artificial stock market also shows that their interesting results tend to vanish when the number of traders increases (cf. Yeh [178]). In Lux and Marchesi, the finite size effect immediately becomes apparent by realizing that the overall number of agents affects excess demand and, therefore, the right-hand side of the price-adjustment equation. However, although one might expect that this leads to more severe fluctuations with increasing N, the contrary is the case: fluctuations become dampened with higher N and finally die out altogether with a cross-over of returns to a Normal distribution. Of course, linear dependence of excess demand on N is not realistic. The task for future research is, therefore, to look for self-organizing forces in the market (maybe via the addition of wealth dynamics) which may lead to an effective confinement of the level of excess demand.

Have the econophysics papers reviewed here brought anything new to economics? Certainly they did not invent microscopic computer modeling of markets or empirical analysis of market fluctuations. But the large number of enonophysicists pushed these areas since physicists are more familiar with computer simulation and empirical analysis than many mainstream economists more interested in exact mathematical solutions. Of course, percolation and random field Ising models are clear physics contributions, and the introduction of multi-fractal processes as stochastic models of financial prices (a topic which is outside the scope of the present review) is conceived as an important innovation by many economists. Here again, we find that economists have been aware of the multi-scaling of returns for some time [52,114], but suffered from a lack of appropriate models in their tool-box (cf. [118] for more details on this issue).

Have econophysists made any predictions which were later confirmed? If we define as "prediction" something which has appeared in a journal on paper before the predicted event was over, we exclude all private communications or e-prints, and know only three cases: The warning published in September 1997 that a "krach" should occur before the end of November [59] (it occurred in October)[144]; the assertion that the Nikkei index in Tokyo should go up in 1999, which it did by roughly the predicted amount, and the prediction that the U.S. stock market should reach a lower turning point in early 2004 which did not happen [153]. Even if "successful" relatively vague predictions like the above are, of course, at best interpreted as anecdotical evidence, but are surely not significant from a statistical perspective.

[8] Similar probably to the development of statistical models of traffic flows, cf. Nagel *et al.* [128]

Have we become rich in this way? The senile co-author gained 50% in half a year by believing the above predictions, and similar anecdotal evidence exists from others. Interestingly, in this way the one contributer with a physics background seems to show show a better performance in private portfolio management than the three economists who rather concentrated on their academic careers. Of course success is often reported proudly while failures are kept as a secret. In this way, certain strategies might appear successful simply because of a bias in awareness of positive outcomes versus negative ones. More than half of a century ago, the prominent economist Nicholas Kaldor [90] explained the prevalence of chartist strategies by such a misperception of their track record. But more reliable are the flourishing companies like Prediction Company (New Mexico) or Science-Finance (France) founded by physicists Farmer and Bouchaud, respectively, together with economists, giving employment to 10^2 people. This seems quite a success for theoretical physicists.

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