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FRINGE FIRMS: ARE THEY BETTER OFF IN A HETEROGENEOUS MARKET?

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Fringe firms: Are they better off in a heterogeneous market?

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This paper analyzes a market with three firms. One of them is the dominant firm and the two others are fringe firms. The formulation of demand allows a comparison between price competition with heterogeneous and homogeneous products. Because a parameterization is required to assure that market size is the same in both scenarios, no general conclusions can be drawn. But it can be shown that in large markets with relatively inelastic demand for the fringe firms' products and a cost advantage of the dominant firm, the fringe firms are better off if they produce a heterogeneous product.

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1. Introduction

In many markets large differences in the market shares of suppliers exist. Often a group of dominant firms serve the better part of the buyers.[1] In this article the extreme case is analyzed, namely a market with one firm with a large market share and fringe firms as small rivals.

There are indeed markets where one dominant firm has a long-lasting advantage over its smaller competitors – like Microsoft. Two prominent examples in Germany are the markets for soup seasoning and baking ingredients. The brand "Maggi" has been the market leader for decades and its smaller competitors have never reached a noteworthy market share although large supermarkets also sell less known substitutes. The same is true for Dr. Oetker's baking powder and pudding. Although the competitor's products are cheaper by far, Dr. Oetker remains the market leader. In both cases the name of the products is associated with approved quality, and in relation to the entire meal the costs are small. Customers believe that their home-made meals are tastier if they purchase the well-known brands. As in the model of Schmalensee (1982) the risk of failure hinders the consumers to replace a reliable product.[2] A less prominent example is the market for city maps. Falk city maps are dominant in Germany, but there are many small producers of cheaper and smaller maps that are sufficient for short trips.

The situation of the market leader with a high price and a large market share is comfortable. But what is the position of the fringe firms? Is it optimal for them to be small competitors in a heterogeneous market or would they be better off if the products were homogeneous? To answer this question we need a model that discriminates between these two types of markets with a dominant firm. Hence, with identical prices of all competitors the market has to be of the same size. In case of homogeneity the classical model is that of Forchheimer. This dominant firm model with a homogeneous product can be found in every industrial organization textbook. While the entry of fringe firms and the consequences for the position of the dominant firm have been widely discussed (see e. g. Berck and Perloff 1988, Cherry 2000 and

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[1] This may be the result of a sequential Stackelberg game (see e. g. Anderson/Engers 1992, Eaton/Ware 1987, Bongard/Wied-Nebbeling 2005).
[2] For an overview of all sorts of advantages a firm can exploit, see Geroski and Jacquemin (1984).

the literature cited there), I found no application to differentiated products.[3] There are many models for oligopolies with horizontally differentiated heterogeneous products.[4] But a spatial model like the circular city or a Hotelling line cannot be used here, because in case of homogeneity both spaces would shrink to a point, and the outcome in equilibrium would be Bertrand (price equals marginal costs). Other oligopoly models with heterogeneous firms refer to markets with symmetric demand (see e.g. Wang/Zhao, 2007). But all of these models are not consistent with the Forchheimer model of a dominant firm with a competitive fringe.

Therefore a system of demand functions is introduced here that allows for different market shares and heterogeneous preferences. Nevertheless, adding up the demand for the equally priced products gives total demand that can also be used for the Forchheimer model. By calculating the equilibriums in both markets and comparing the producer surpluses of the fringe firms, we can see in which scenario – homogeneous or heterogeneous – they are better off.

The paper is organized as follows: Section 2 sets out the basic assumptions of the models. Sections 3 and 4 are devoted to the price equilibrium in the heterogeneous and the homogeneous market with the focus on the outcome for the fringe firms. In section 5 the producer surpluses in both scenarios are analyzed, using a comparative static approach. Section 6 contains concluding remarks.

2. Assumptions

For tractability reasons the analysis is restricted to one dominant and two fringe firms. To compare the heterogeneous with the homogeneous market the model has to fulfill the following characteristics:

(I) In any case, the market share of the dominant firm must be considerably higher than the share of any fringe firm.

(II) The two fringe firms $i = 2,3$ are identical. They produce with quadratic costs: $C_i = eq_i^2$ and thus have upward sloping marginal costs. Increasing marginal costs are

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[3] The model of Blank et al. is an exception. This model is not applicable because it cannot be transferred to the homogeneous case.
[4] See e. g. Perloff/Salop (1985) and Anderson/Palma (1988).

a necessary assumption for the equilibrium in the homogeneous market. The dominant firm produces with constant marginal cost, c. We refrain from fixed costs.

(III) With identical prices of the three firms, the quantity in both markets is the same. Otherwise, we would have incomparable market sizes.

The next assumptions refer to the demand in the heterogeneous market:

(IV) We make the conventional and necessary assumption that the demand of each firm reacts stronger to changes of its own price than to changes of a competitor's price.

(V) To handle the system analytically, the market shares at zero prices have to be prefixed.

(VI) Demand of the three firms must add up to total demand. This requires some prefixing of the reaction to changes in price according to (IV).

3. Price equilibrium in the heterogeneous market

Total demand depends on the prices of the dominant firm and the two fringe firms. It can be written as:

$$Q = \alpha - \beta p_1 - \gamma p_2 - \gamma p_3 \ . \tag{1}$$

The formulation of the demand function is quite general[5], whereas for the functions of the three individual firms some specifications are needed. According to assumptions (IV) to (VI), the demand equations of the three firms shall be:

$$q_1 = 0.6\alpha - 3\beta p_1 + \gamma p_2 + \gamma p_3 \quad \text{with } 3\beta > \gamma \tag{2}$$

$$q_2 = 0.2\alpha - 3\gamma p_2 + \beta p_1 + \gamma p_3 \quad \text{with } 3\gamma > \beta \tag{3}$$

$$q_3 = 0.2\alpha - 3\gamma p_3 + \beta p_1 + \gamma p_2 \quad \text{with } 3\gamma > \beta \ . \tag{4}$$

Adding up equations (2) to (4) gives total demand [equation (1)] as required. The market shares at zero prices are in accordance with assumption (I). The fixing of this market shares is somewhat disturbing, but otherwise it would not be possible to guarantee that the sum of the market shares adds up to one without complicating the analytical handling severely. Since the values of the parameters β and γ are not appointed by themselves, the reaction of demand to price variations of the dominant

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[5] It is compatible with the inverse demand schedule $p_i = a - q_i - b \sum\limits_{j \neq i} q_{-i}$, that can be derived as the result of consumers maximizing their utility (see Bloch, 1995).

firm and the fringe firms can vary within the limits on the right hand side of equations (2) to (4). We will consider these variations in detail in section 5.

We consider a simultaneous price game where the three firms want to maximize profits.[6] Since the market is heterogeneous, fringe firms do not have to adopt the price of the dominant firm but are able to calculate the profit-maximizing price by themselves. As usual in simultaneous price games, firms take the prices of the competitors as given and they can estimate demand correctly.

Hence, the dominant firm maximizes $\Pi_1 = (p_1 - c)q_1$ with respect to p_1. With $d\Pi_1/dp_1 = 0$ and solving for p_1 we get its reaction function:

$$p_1 (p_2,p_3) = \frac{3\alpha + 15\beta c + 5\gamma (p_2 + p_3)}{30\beta} \ . \tag{5}$$

Since the fringe firms $i = 2,3$ produce with quadratic costs, their profit function is $\Pi_i = (p_i - eq_i)q_i$. The first order condition gives:

$$p_i (p_1,p_j) = \frac{(6\gamma e + 1)(\alpha + 5(\beta p_1 + \gamma p_j))}{30\gamma(3\gamma e + 1)} \quad i, j = 2,3; \ i \neq j \ . \tag{6}$$

In equilibrium, prices of the fringe firms are identical because demand and costs are the same for both. Therefore, we can substitute p_i for p_j. This results in the reaction function of one fringe firm in response to alternative prices of the dominant firm 1:

$$p_i (p_1) = \frac{(6\gamma e + 1)(\alpha + 5\beta p_1)}{5\gamma(12\gamma e + 5)} \quad i = 2,3 \ . \tag{7}$$

In (5) we substitute p_2 and p_3 with the right side of (7) and get the equilibrium price of the dominant firm:

$$p_1^* = \frac{\alpha(48\gamma e + 17) + 15\beta c(12\gamma e + 5)}{20\beta(15\gamma e + 7)} \ , \tag{8}$$

and from (7) we derive the price of the fringe firms:

$$p_i^* = \frac{3(6\gamma e + 1)(3\alpha + 5\beta c)}{20\gamma(15\gamma e + 7)} \ . \tag{9}$$

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[6] There are two reasons for choosing a simultaneous game: first, it is consistent with the homogeneous case. Second, we avoid difficulties to accept the role of a price leader as mentioned in Tasnàdi (2004).

The price of the fringe firms is increasing in α, which is a parameter of market size, in c, the MC of the dominant firm, and in β, the strength of the demand reaction to price changes of the dominant firm. The derivative dp_i^*/de is also positive, as expected. The stronger the increase in the MC of the fringe firms, the higher their profit maximizing price must be. The derivative $dp_i^*/d\gamma$ is negative – the more demand reacts to price changes of the fringe firms, the lower p_i^*.

The equilibrium quantity is established by the prices of the dominant firm and the fringe firms. Inserting prices in (3) or (4) yields:

$$q_i^* = \frac{9(3\alpha+5\beta c)}{20(15\gamma e+7)} \, . \tag{10}$$

In accordance with p_i^*, the quantity of the fringe firms increases with market size (α), the strength of the demand reaction to price changes of the dominant firm (β), and the MC of the dominant firm (c). The quantity decreases with the demand reaction to their own price changes (γ). With respect to the MC of the fringe firms, however, the derivative dq_i^*/de is negative, as expected.

The producer surplus of a fringe firm in the heterogeneous market is quoted as:

$$PR_{i,\,het} \equiv \Pi_i = (p_i^* - eq_i^*)q_i^*$$

$$PR_{i,\,het} = \frac{27(3\gamma e+1)(3\alpha+5\beta c)^2}{400\gamma(15\gamma e+7)^2} \, . \tag{11}$$

The profit of a fringe firm in the heterogeneous market reacts analogously to its output; hence: $d(PR_{i,\,het})/d\alpha > 0$, $d(PR_{i,\,het})/d\beta > 0$, $d(PR_{i,\,het})/dc > 0$, $d(PR_{i,\,het})/d\gamma < 0$ and $d(PR_{i,\,het})/de < 0$.

4. Equilibrium in the homogeneous market

In case of homogeneity we use the traditional Forchheimer model. Thus, the dominant firm maximizes its profit with respect to residual demand q_r, with

$$q_r = Q - \sum_{i=2}^{3} q_i \, . \tag{12}$$

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The fringe firms take the price of the dominant firm as given and maximize their profits by equating price to marginal cost.

The demand in the homogeneous market is:

$$Q = \alpha - zp \quad \text{with} \quad z = \beta + 2\gamma . \tag{13}$$

We take z because there is only one price and therefore a unique demand reaction to changes of that price. To be comparable with (1), z has to be the sum of $\beta + 2\gamma$. Only then demand is the same, provided that the prices of the three firms in the heterogeneous market are identical to each other and to the price in the homogeneous setting. Again, the horizontal intercept is α, while 1/z is the slope of the inverse demand curve.

The supply of each fringe firm corresponds to its MC (= $2eq_i$) for every given price of the dominant firm. Hence, the supply of both fringe firms is[7]

$$q_{f,\,agg} = 2 \cdot q_f = \frac{1}{e}p_d . \tag{14}$$

From (12) and (13) follows as residual demand:

$$q_r = \alpha - zp_d - \frac{1}{e}p_d = \alpha - \left(z + \frac{1}{e} \right)p_d . \tag{15}$$

The dominant firm maximizes

$$\Pi_d = (p_d - c)q_r$$

with respect to its price. With $d\Pi_d/dp_d = 0$ and solving for p_d we get the price of the dominant firm:

$$p_d{}^* = \frac{\alpha e + c + cez}{2(ez+1)} . \tag{16}$$

At this price, taken as given by a fringe firm, each of them supplies:

$$q_f{}^* = \frac{\alpha e + c + cez}{4e(ez+1)} . \tag{17}$$

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[7] To discriminate between the homogeneous and the heterogeneous market we now use the subscript f for the fringe firms and d for the dominant firm.

The quantity of the fringe firms in the homogeneous market also increases with market size: $dq_f^*/d\alpha > 0$, and with the MC of the dominant firm: $dq_f^*/c > 0$. The derivative dq_f^*/dz is negative because as (16) shows, the price of the dominant firm declines with z. From (14) follows that the fringe firms produce a smaller quantity in total. Since there are only two fringe firms, the output of each firm must shrink as well. As expected, the derivative dq_i^*/de is negative; the steeper the cost function of the fringe firms, the smaller their optimal output is.

The producer surplus of a fringe firm in the homogeneous market is composed as before:

$$PR_{f,\,hom} \equiv \Pi_f = (p_d^* - eq_f^*)q_f^* \ .$$

Substitution of p_d^* and q_f^* gives:

$$PR_{f,\,hom} \ = \ \frac{(\alpha e + c + cez)^2}{16e(ez+1)^2} \ . \tag{18}$$

The profit of a fringe firm in the homogeneous market increases with market size and the MC of the dominant firm, and it decreases with a stronger demand reaction to price variations. The development of the producer surplus with respect to the slope of the cost function (e) is ambiguous, because the price set by the dominant firm increases with e, while the output of a fringe firm decreases.

5. Producer surplus of the fringe firms compared

Now we can evaluate if it is better to be a fringe firm with a heterogeneous instead of a homogeneous product. Since, with the exception of the parameter e, profit in both markets reacts in the same direction if demand or cost parameters shift, it is not possible to derive general conclusions. Thus, we have to calculate the reaction of profits to individual changes of the parameters, given the restrictions of the demand functions [equations (2) to (4)].

We start with a heterogeneous market where the products are substitutes, but not very close ones. The parameters are set as follows:

$\alpha = 100; \ \beta = 1/8; \ \gamma = 1/8; \ c = 20; \ e = 2.$

In the heterogeneous case the price of the dominant firm is $p_1^* = 119.1$, that of the fringe firms is $p_i^* = 87.2$. The price of the fringe is significantly lower, as expected. The dominant firm produces $q_1^* = 37.2$ and each fringe firm $q_i^* = 13.1$. The dominant firm has a market share of 58.7 percent and earns a profit of $\Pi_1^* = 3680.6$, while the producer surplus of a fringe firm is only $\Pi_i^* = 798.6$.

In the homogeneous scenario the price set by the dominant firm is $p_d^* = 67.1$. This price is distinctly lower than the prices of the differentiated products. The dominant firm's output amounts to $q_d^* = 41.25$ and the output of each fringe firm to $q_f^* = 16.8$. The market shares are 55.1 and 22.4 percent respectively. The profit of the dominant firm accounts for $\Pi_d^* = 1944.6$ and the one of a fringe firm for $\Pi_f^* = 563.5$. With $\Pi_i^* > \Pi_f^*$, the fringe firms in the heterogeneous market are thus better off with the initial parameter values (despite their smaller market shares).[8] This does not always have to be the case if the parameter values change.

Market size

PROPOSITION 1: The producer surplus in both markets increases with market size. It is larger in the heterogeneous market if demand is above a critical level.

PROOF: The derivatives of (11) and (18) with respect to α are both positive; hence profit increases with market size. With the other parameter values fixed and α variable, the profit of a fringe firm in the heterogeneous market is higher than with a homogeneous product, if

$$PR_{i,\,het} > PR_{f,\,hom}$$

and therefore

$$(6\alpha + 25)^2 > \frac{46225\,(2\alpha + 35)^2}{9261}\ .$$

This inequality is fulfilled for every $\alpha > 34.73$. In the homogeneous market, a fringe firm is only better off if the market is very small.

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[8] This result corresponds with the findings of Borenstein (1991).

Demand reaction to price changes of the dominant firm

PROPOSITION 2: If the products in the heterogeneous market are weak substitutes, the producer surplus of a fringe firm in the heterogeneous case is higher whatever the demand reaction to price changes of the dominant firm.

PROOF: To compare the profits we have to take into account that β is included in z. In equation (18) z has to be substituted by $\beta + 2\gamma$. An increasing β in the heterogeneous market yields a larger z in the homogeneous one. The profit inequality is given by:

$$(\beta+3)^2 > \frac{1849(4\beta+23)^2}{12096(4\beta+3)^2} \quad .$$

This inequality holds for every $\beta > 0$. Whatever the reaction of demand to price changes of the dominant firm, a fringe firm has a higher profit in the heterogeneous market. However, this result depends on the values chosen. Assuming $\gamma = \frac{1}{4}$, which indicates closer substitutes, β would have to be bigger than 0.2 for profits to be higher in the heterogeneous market. If $\gamma = \frac{1}{2}$, the condition is $\beta > 0.59$. Hence, the fringe firms are only definitely better off if their own demand is relatively price-inelastic, in other words, if they sell a niche product. Interestingly, whatever the values for γ and β, a higher z due to an increasing β means that in the homogeneous market the profits drop, while in the heterogeneous market the fringe firms ameliorate.

Demand reaction to price changes of fringe firms

PROPOSITION 3: There is no definite relation between the ranking of the producer surplus and magnitude of the demand reaction to price changes of the fringe firms.

PROOF: As in the previous case, γ is part of z and z has to be substituted by $\beta + 2\gamma$ in (18). Solving for γ we get:

$$\frac{6\gamma+1}{\gamma(30\gamma+7)^2} > \frac{32(16\gamma+45)^2}{16875(16\gamma+5)^2} \quad .$$

For this inequality no definite solution can be found. Assuming $\alpha = 100$; $\beta = \frac{1}{8}$; c = 20 and e = 2, the profit in the heterogeneous market is larger if γ is smaller than 0.2. Again, the reaction of demand to price changes of a fringe firm has to be moderate if the inequality applies. With better substitutability of the heterogeneous products fringe firms were in a relatively better position if they produced totally

homogeneous products. In both cases the profits shrink with an increase in γ (respectively in z), but the decline is weaker with homogeneity.

MC of the dominant firm

PROPOSITION 4: The producer surplus of a fringe firm in the heterogeneous market is bigger than in the homogeneous one if the dominant firm does not produce too costly.

PROOF: It is always profitable for the fringe firms if the dominant firm is a high-cost producer. If the costs of the dominant firm are very high, a fringe firm benefits more in the homogeneous market, because:

$$(c+480)^2 > \frac{1849\,(7c+800)^2}{9261}$$

yields c < 57.6. Profits in the homogeneous market are lower with small MC of the dominant firm but they grow faster as the MC of the dominant firm increase and at some point they outrun the profits in the heterogeneous market. Since dominance is often combined with cost advantages, a fringe firm in the heterogeneous market will usually have the higher producer surplus.

Slope of the fringe firms' cost function

PROPOSITION 5: If the fringe firms do not produce with very low costs, the producer surplus in the heterogeneous case is larger.

PROOF: As shown in section 4, the profit in the homogeneous market does not always rise with a cheaper production of the fringe. In the example chosen, the maximum profit with homogeneity is combined with e = 1.838. Nevertheless, from

$$\frac{3e+8}{(15e+56)^2} > \frac{(43e+8)^2}{16875\,e\,(3e+8)^2}$$

follows: e > 0.03. With non-negligible costs, fringe firms make a higher profit in the heterogeneous market.

3. Concluding remarks

In this paper we present a model that allows us to compare the profits of fringe firms in a market with homogeneous to one with heterogeneous products. To make such a

comparison, some restrictive conditions are necessary that limit the generality of the linear demand system in the heterogeneous case. The producer surplus of the fringe firms in both scenarios is rising with market size and with the marginal costs of the dominant firm. It is declining with a stronger reaction to price variations. That is why profits can only be calculated through simulations with varying parameter values. These show that a fringe firm can achieve a higher producer rent in a heterogeneous market, if

– the size of the market is not very small

– demand for the products of the fringe firms is relatively inelastic

– the marginal costs of the dominant firm are not too high and

– the costs of the fringe firms are non-negligible.

Since these conditions are probably fulfilled in many cases, a fringe firm should be searching for a niche for a heterogeneous product[9] and set its own price instead of merely imitating the dominant firm's product and price even if market shares are smaller in the heterogeneous market. Opposing this could be the higher entry costs of a heterogeneous market.

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[9] A niche can also be made up of the supply of products that the dominant firms ceased to offer as in the case of the UK fertilizer market in the 1970s (see Shaw 1982, p. 240).

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