Discussion of "The Leverage Cycle" by John Geanakoplos*

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John Geanakoplos deserves credit for having blazed the trail on the analysis of leverage and collateral, and their crucial impact on the workings of the financial system. His 1997 paper "Promises, Promises" was a milestone in the way that it brought together the institutions and practices that underpin modern capital markets with rigorous general equilibrium theory. It is also famous as an engrossing autobiography of the twists and turns in the route by which a theorist became a mortgage hedge fund principal. The discussion reflects the authoritative insights of a market professional as well as that of an economic theorist - a rare combination indeed. He has subsequently developed and refined the approach in several additional contributions. His 2000 Econometric Society World Congress lecture in Seattle (published as Geanakoplos (2003)) anticipates many of the ingredients of the current paper, further developed to incorporate shifts in belief and endowments in a dynamic economy in Fostel and Geanakoplos (2008).

In developing these ideas, he has been way ahead of the pack. Indeed, until recently, there was no pack. He was so far ahead of the curve that his early contributions did not attract the attention of the broader profession (outside the group of general equilibrium theorists) until the real world caught up with the theory. The financial crisis has changed everything, and his work is now center stage, as it deserves.

The theoretical framework developed by Geanakoplos reflects the intellectual pedigree from general equilibrium theory, especially the field of general

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equilibrium with incomplete markets (GEI). However, as elegant and rigorous as it is, this pedigree is also double-edged. At the same time as serving as a source of inspiration and a showcase for theoretical rigor, it can also be an impediment to more robust and intuitive modeling approaches that would find wide adoption and use by an applied audience in corporate finance and macroeconomics. The general results are very general, and address issues such as existence of equilibrium and their constrained efficiency. Although these general results give some guidance on potential sources of inefficiency for welfare analysis purposes, they are not easy to utilize in applied modeling. The more suggestive applied discussion is mainly weaved through a series of ingeniously crafted examples that have been constructed to illustrate a particular point. However, in contrast to the very general theoretical results, these examples are stark and specialized. They beg the inevitable question of how robust they are to rough handling in applied modeling.

The theoretical pedigree from GEI analysis also serves to divert attention away from the crucial role played by banks and other financial intermediaries. This crisis, as with many others, is difficult to explain fully without placing the role of financial intermediaries at the center. Securitization was meant to disperse credit risk to those who were better able to absorb losses. Financial intermediaries were meant to play their role in dispersing credit risk. In fact, in the current crisis the risks were concentrated in the financial intermediary sector itself. As leveraged institutions, they were the most vulnerable to losses on their assets, as they were in danger of having their equity wiped out, as many have found to their cost.

In this commentary, I make two broad points. First, the "narrative by examples" approach to applied modeling practised in Geanakoplos's paper can be highly enlightening but they can fall short of providing robust off-the-shelf models that can be used directly by applied researchers. One of the reasons for this gap is the very feature that makes general equilibrium theory so appealing - its solid foundations in terms of the preferences and beliefs of the agents. Alternative staging posts for the analysis that use intermediate categories such as balance sheet classifications and institutional "frictions" may be easier to work with and equally illuminating. I give an example below.

Secondly, there is still a need for a theoretical framework that gives a role to financial intermediaries that is commensurate with their importance in practice. In spite of the advances made by John Geanakoplos in this and in previous papers, much still remains to be done in bringing financial

intermediaries into the analysis of financial booms and busts.

Recasting the Main Insights

A key ingredient in the Geanakplos paper is a division of roles where agents are divided into the natural buyers of an asset (whether of houses or mortgages) and those who could potentially hold these assets, but normally end up as lenders to the natural buyers, instead. The natural buyers are those with the most optimistic beliefs about the asset's future value, and they are enabled to hold a larger position in these assets than they could based on their own resources by the credit supplied by the less optimstic agents.

The collateral requirment and the discounts (haircuts) arise from the need to satisfy the less optimistic agents that the loan is safe. But following bad news for the asset, there is a redistribution of wealth away from the optimists toward the pessimists. Some of the most overstretched optimists will have their equity wiped out altogether by the price change. The marginal buyer is therefore likely to be someone who is less optimistic or less rich than would have been the case if the asset had not been purchased on margin, and the wealth redistribution not been so adverse. For all these reasons, the shock to the asset price is amplified through changes in the wealth distribution and the identity of the marginal buyer.

The basic scenario painted above (both on the way up, and on the way down) could be told in a simpler, static model where everyone has the same beliefs. Set today's date to 0. A single risky security is traded today in anticipation of its realized payoff next period (date 1). The risky security's payoff is a random variable $\tilde{w}$, with expected value $q > 0$, and uniformly distributed over the interval $[q - z, q + z]$ for small $z > 0$. The uniform density enables risk-free debt contracts to be written, as in Geanakoplos's paper. The variance of $\tilde{w}$ is $\sigma^2 = \frac{z^2}{3}$. There is also a risk-free security, cash, that pays an interest rate of zero.

Let p be the price of the risky security. For an investor with equity e who holds y units of the risky security, the payoff of the portfolio is the random variable:

$$W \equiv \tilde{w}y + (e - py) \qquad (1)$$

Now, in the same spirit as in Geanakoplos's paper, introduce two groups of investors - passive investors and active investors. The passive investors can be thought of as non-leveraged investors such as pension funds and mutual

3

funds, while the active investors can be interpreted as leveraged institutions such as banks and securities firms who manage their balance sheets actively.

Suppose passive investors have mean-variance preferences, and maximize $U = E(W) - \frac{1}{2\tau}\sigma_W^2$ where $\tau > 0$ is the investor's "risk tolerance" and σ_W^2 is the variance of W. The passive investor chooses y to maximize

$$U(y) = qy + (e - py) - \frac{1}{6\tau}y^2 z^2 \tag{2}$$

The optimal holding of the risky security is

$$y_P = \begin{cases} \dfrac{3\tau}{z^2}(q-p) & \text{if } q > p \\ \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

These linear demands can be summed to give the aggregate demand. If τ_i is the risk tolerance of the ith investor and $\tau = \sum_i \tau_i$, then (3) gives the aggregate demand of the passive sector as a whole.

Now turn to the portfolio decision of the active (leveraged) investors. These active investors are risk-neutral but face a Value-at-Risk (VaR) constraint, as is commonly the case for banks and other leveraged institutions. The general VaR constraint is that the capital cushion be large enough that the default probability is kept below some benchmark level. Consider the special case where that benchmark level is zero. Then, the VaR constraint boils down to the conditiion that leveraged investors issue only risk-free debt, as in Geanakoplos's model.

Denote by VaR the Value-at-Risk of the leveraged investor. The constraint is that the investor's capital (equity) e be large enough to cover this Value-at-Risk. The optimization problem is:

$$\max_y E(W) \quad \text{subject to VaR} \leq e \tag{4}$$

If the price is too high (i.e. when $p > q$) the investor holds no risky securities. When $p < q$, then $E(W)$ is strictly increasing in y, and so the Value-at-Risk constraint binds. The optimal holding of the risky security can be obtained by solving VaR $= e$. To solve this equation, write out the balance sheet of the leveraged trader as

Assets	Liabilities
securities, py	equity, e
	debt, $py - e$

The Value-at-Risk constraint stipulates that the debt issued by the investor be risk-free. For each unit of the security, the minimum payoff is $q - z$. In order for the investor's debt to be risk-free, y should satisfy $py - e \leq (q - z)\,y$, or

$$py - (q - z)\,y \leq e \tag{5}$$

The left hand side is the Value-at-Risk (the worst possible loss), which must be met by equity e. Since the constraint binds, the optimal holding of the risky securities for the leveraged investor is

$$y = \frac{e}{z - (q - p)} \tag{6}$$

and the balance sheet is

Assets	Liabilities	
securities, py	equity, e	(7)
	debt, $(q - z)\,y$	

Since (6) is linear in e, aggregate demand of the leveraged sector has the same form as (6), when e is the *aggregate capital* of the leveraged sector as a whole. Denoting by y_A the holding of the risky securities by the active investors and by y_P the holding by the passive investors, the market clearing condition is

$$y_A + y_P = S \tag{8}$$

where S is the total endowment of the risky securities. Figure 1 illustrates the equilibrium for a fixed value of aggregate capital e. For the passive investors, their demand is linear with the intercept at q. The demand of the leveraged sector can be read off from (6). The solution is fully determined as a function of e. In a dynamic model, e can be treated as the state variable (Danielsson, et al. (2009)).

Now consider a possible scenario following an improvement in the fundamentals of the security where the expected payoff of the risky securities rises
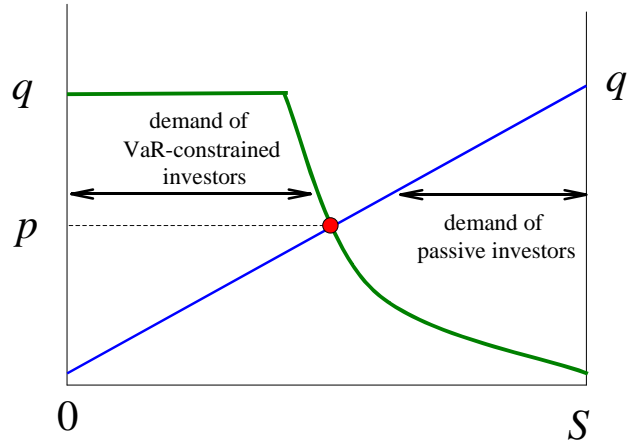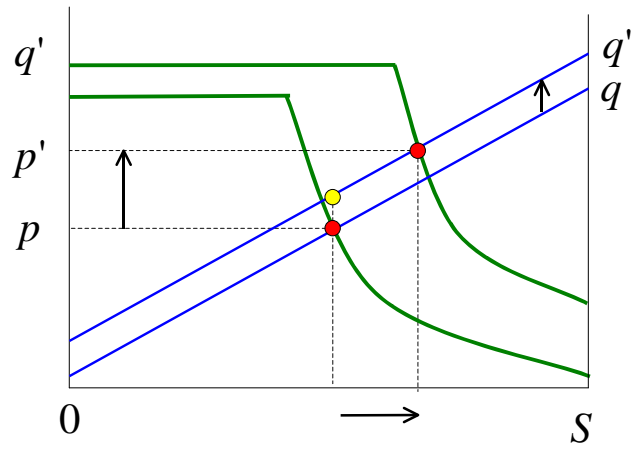
Figure 1: Market Clearing Price

Figure 2: Amplified response to improvement in fundamentals q
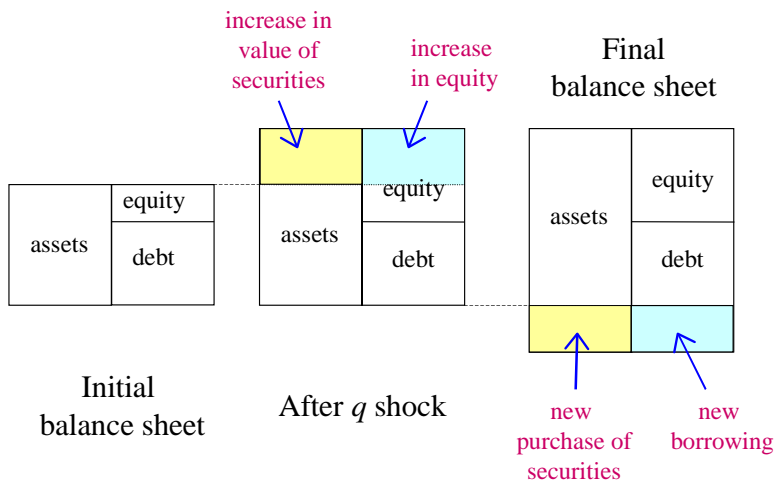
6

Figure 3: Balance sheet expansion from q shock

from from q to q'. Figure 2 illustrates the scenario. The improvement in the fundamentals of the risky security pushes up the demand curves for both the passive and active investors, as illustrated in Figure 2. However, there is an amplified response from the leveraged sector as a result of mark-to-market gains on their balance sheets.

From (7), denote by e' the new equity level of the leveraged investors that incorporates the capital gain when the price rises to p'. The initial amount of debt was $(q-z)\,y$. Since the new asset value is $p'y$, the new equity level e' is

$$e' = (z + p' - q)\,y \tag{9}$$

Figure 3 breaks out the steps in the balance sheet expansion. The initial balance sheet is on the left, where the total asset value is py. The middle balance sheet shows the effect of an improvement in fundamentals that comes from an increase in q, but before any adjustment in the risky security holding. There is an increase in the value of the securities without any change in the debt value, since the debt was already risk-free to begin with. So, the increase in asset value flows through entirely to an increase in equity. Equation (9) expresses the new value of equity e' in the middle balance sheet in Figure 3.

The increase in equity relaxes the Value-at-Risk constraint, and the lever-

7

aged sector can increase its holding of risky securities. The new holding y' is larger, and is enough to make the VaR constraint bind at the higher equity level, with a higher fundamental value q'. That is,

$$e' = (z + p' - q') y' \qquad (10)$$

After the q shock, the investor's balance sheet has strengthened, in that capital has increased without any change in debt value. There has been an erosion of leverage, and excess capacity appears on the balance sheet. Equity is now larger than is necessary to meet the Value-at-Risk. In order to utilize the slack in balance sheet capacity, the investor takes on additional debt to purchase additional risky securities. The demand response is upward-sloping. The new holding of securities is now y', and the total asset value is $p'y'$. Equation (10) expresses the new value of equity e' in terms of the new higher holding y' in the right hand side balance sheet in Figure 3. From (9) and (10), we can write the new holding y' of the risky security as

$$y' = y \left(1 + \frac{q' - q}{z + p' - q'} \right) \qquad (11)$$

From the demand of passive investors (3) and market clearing,

$$p' - q' = \frac{z^2}{3\tau} (y' - S)$$

Substituting into (11),

$$y' = y \left(1 + \frac{q' - q}{z + \frac{z^2}{3\tau} (y' - S)} \right) \qquad (12)$$

This defines a quadratic equation in y'. The solution is where the right hand side of (12) cuts the 45 degree line. The leveraged sector amplifies booms and busts if $y' - y$ has the same sign as $q' - q$. Then, any shift in fundamentals gets amplified by the portfolio decisions of the leveraged sector. The condition for amplification is that the denominator in the second term of (12) is positive. But this condition is guaranteed from (11) and the fact that $p' > q' - z$ (i.e. that the price is higher than the worst possible realized outcome).

Amplification is increasing in leverage, seen from the fact that $y' - y$ is larger when z is small. Recall that z is the fundamental risk. When z

is small, the associated Value-at-Risk is also small, allowing the leveraged sector to maintain high leverage. The higher is the leverage, the greater is the marked-to-market capital gains and losses. Amplification is large when the leveraged sector itself is large relative to the total economy. Finally, note that the amplification is more likely when the passive sector's risk tolerance τ is high.

The amplifying mechanism works exactly in reverse on the way down. A negative shock to the fundamentals of the risky security drives down its price, which erodes the marked-to-market capital of the leveraged sector. The erosion of capital induces the sector to shed assets so as to reduce leverage down to a level that is consistent with the VaR constraint. Risk premium increases when the leveraged sector suffers losses, since $q - p$ increases. The two circular figures below depict the feedback from prices to actions to back to prices, both on the "way up" and on the "way down".[1]

Feedback "on the way up"　　　Feedback "on the way down"
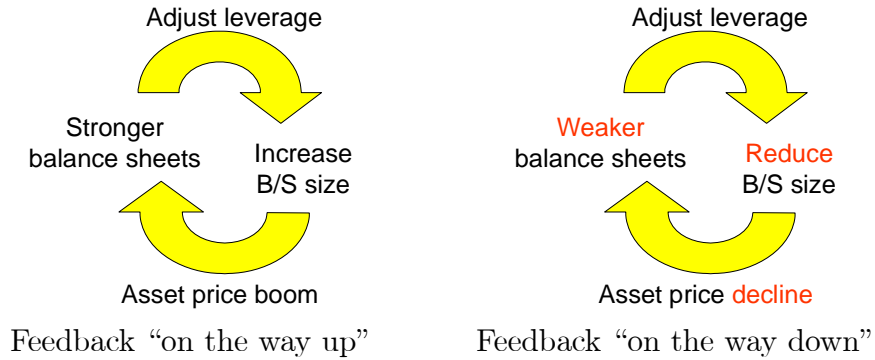
This example illustrates the amplifying effect of fundamental changes to asset values in the same spirit as the examples in Geanakoplos's paper. What the construction shows is how the main insights can be formalized fairly robustly by invoking the intermediate modeling device of balance sheet management. There is no need to rely on differences in beliefs or specific features of the time tree. Instead, the job is done by the upward-sloping demand responses of the leveraged traders, who operate with warped incentives, or constraints that reflect the anticipation of those warped incentives. Gromb and Vayanos (2002) examined balance sheet effects with leveraged traders, and Xiong's (2001) "wealth effects" have similar consequences to the mark-to-market capital gains of VaR-constrained investors. Brunnermeier and

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[1]Adrian and Shin (2007) discuss the empirical consequences of such balance sheet dynamics for the financial system as a whole.

Pedersen (2009) have examined the feedback through "margin spirals" of leveraged investors.

These alternative approaches (as well as my example above) have two possible advantages over Geanakoplos's more explicitly micro-founded approach. First, the active investors with warped incentives are very reminiscent of the banks and financial intermediaries who operate with risk constraints. The constraints need to be explained, but once they are assumed to be in place, many steps in the argument become more transparent. The constraints themselves come from outside the simple model. The most natural way to explain such constraints would be through agency frictions (see Adrian and Shin (2008)). Second, by illustrating the equilibrium both "on the way up" and "on the way down" in the same model, the mechanisms invoked are less tailored to the paricular scenario to be painted. The price to be paid is that we lose the rigor of the foundational elements of GEI general equilibrium theory. But for applied modeling purposes, some short-cuts may prove to be an advantage.

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