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Author: W. Erwin Diewert,

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Comment W. Erwin Diewert

Introduction

Moyer, Reinsdorf, and Yuskavage address a number of important and interesting issues in their chapter. They first review the fact that (nominal) GDP can in theory be calculated in three equivalent ways:

W. Erwin Diewert is a professor of economics at the University of British Columbia and a research associate of the National Bureau of Economic Research.

- By summing final demand expenditures
- By summing value added[1] over all industries
- By summing over all sources of income received

However, the authors go beyond this well-known fact[2] and show that under certain conditions, *real GDP that is constructed by aggregating over the components of final demand is exactly equal to real GDP that is constructed by aggregating over the components of each industry's gross outputs less intermediate inputs,* provided that the Laspeyres, Paasche, or Fisher (1922) ideal formula is used in order to construct the real quantity aggregates.[3] This index number equivalence result is the most important result in the chapter.[4]

When the BEA calculates the rate of growth of GDP using a chained Fisher ideal index, it also provides a sources of growth decomposition; that is, it provides an additive decomposition of the overall growth rate into a number of subcomponents or contributions of the subcomponents to the overall growth rate. Thus, the growth contributions of $C + I + G + X - M$ add up to the overall growth of GDP.[5] However, many analysts are interested in the contributions to overall GDP growth of *particular industries* as opposed to the contributions of particular components of final demand. The index number equivalence result derived by Moyer, Reinsdorf, and Yuskavage means that if their conditions for the result to hold are satisfied, *then industry contributions to growth can be calculated that will exactly add up to total GDP growth,* provided that the Fisher formula is used.

What are the authors' conditions for the equivalence result to hold? Some of the more important conditions are

- Accurate industry value data on gross outputs and intermediate inputs that sum up to the components of final demand in value terms must be available for the two periods in the index number comparison.
- For each commodity produced or used as an intermediate input in the

1. Value added is defined as the value of gross outputs produced over the reference period minus the value of intermediate inputs used during the period. An intermediate input is defined as an input that has been produced by some other domestic or foreign producer.

2. See, for example, Hicks (1952).

3. When calculating Fisher, Laspeyres, or Paasche price or quantity indexes of value added for an industry, all prices are entered as positive numbers but the corresponding quantities are positive or negative numbers depending on whether the particular commodity is being produced as an output (entered as a positive quantity) or being used as an input (entered as a negative quantity).

4. Their result is generalized somewhat in Reinsdorf and Yuskavage (2004).

5. The particular Fisher decomposition formula being used by the BEA is due originally to Van Ijzeren (1987, 6). This decomposition was also derived by Dikhanov (1997), Moulton and Seskin (1999, 16), and Ehemann, Katz, and Moulton (2002). An alternative additive decomposition for the Fisher index was obtained by Diewert (2002) and Reinsdorf, Diewert, and Ehemann (2002). This second decomposition has an economic interpretation; see Diewert (2002). However, the two decompositions approximate each other fairly closely; see Reinsdorf, Diewert, and Ehemann (2002).

economy, the price faced by final demanders and by suppliers of that commodity must be the same for all demanders and suppliers.
- Commodity taxes are small enough in magnitude that they can be ignored.

The authors note that in practice, the first condition listed above is not satisfied for various reasons. We will not focus our discussion on this particular assumption. However, in the next section, we will attempt to find a counterpart to the Moyer, Reinsdorf, and Yuskavage equivalence result when commodity prices are *not* constant across demanders and suppliers of a particular commodity. In the upcoming section, we assume that there are no commodity taxes to worry about, but in the following section we again attempt to find a counterpart to the authors' equivalence result when there are commodity taxes on final outputs and possibly also on intermediate inputs. The final section concludes by looking at some of the implications of our results for statistical agencies and their data collection and presentation strategies.

Input-Output Accounts with No Commodity Taxes

In this section, we will address some of the problems associated with the construction of input-output tables for an economy, in both real and nominal terms. We will defer the problems that the existence of commodity taxes causes until the next section. However, in the present section, we will allow for a complication that makes the construction of input output tables somewhat difficult and that is the existence of *transportation margins.* The problem occurs when real input-output tables are constructed. Moyer, Reinsdorf, and Yuskavage note that the industry method for constructing real GDP will coincide with the usual final-demand method for constructing real GDP, provided that the deflator for any commodity is the same wherever that commodity is used or produced. In fact, in their empirical work, they make use of this assumption since independent deflators for all of the cells of the use and make matrices are generally not available and hence final demand deflators or selected gross output deflators are used as proxy deflators throughout the input-output tables. However, when an industry produces a commodity, its selling price will be less than the purchase price for the same commodity from final and intermediate demanders of the good, due to the costs of shipping the good from the factory gate to the geographic location of the purchasing unit. In addition, there may be various marketing and selling costs that need to be added to the manufacturer's factory gate price.

In the present section, we will address the problem of accounting for transportation margins in the simplest possible model of industry structure where there will be one industry (industry M) that produces a good (commodity 1), one industry (industry S) that produces a service (commodity 2), and one industry (industry T) that transports the good to final demanders[6] or to the service industry.[7] The transportation service will be regarded as commodity 3. We assume that the service output does not require transportation inputs to be delivered to purchasers of services.

Table 7C.1 combines the make and use matrices for the value flows in this highly simplified economy into a single input-output table. The industry M, S, and T columns list the sales of goods and services (plus signs are associated with sales) and the purchases of intermediate inputs (minus signs are associated with purchases) for each of the three inputs. The *final demand column* gives the total of the industry sales less industry intermediate input purchases for rows 1 to 4 over the three industries in the economy. Row 5 in table 7C.1 sums all of the transactions in the industry M, S, and T columns and thus is equal to *industry value added* (the value of gross outputs produced less the value of intermediate inputs used by the industry). The entry in row 5 of the final demand column is nominal GDP, and it is equal to both the sum of the final demands above it and to the sum of the industry M, S, and T value added along the last row of the table.

Rows 1 to 3 of table 7C.1 lists the transactions involving the manufactured good, commodity 1. We will explain these transactions and the associated notation row by row. In the industry M row 1 entry, we list the value of manufactured goods sold to the service sector, $p_1^{\mathrm{MS}}q_1^{\mathrm{MS}}$, where q_1^{MS} is the number of units sold to the service sector and p_1^{MS} is the average sales price.[8] Also in the industry M row 1 entry, we list the value of manufactured goods sold to the final demand sector, $p_1^{\mathrm{MF}}q_1^{\mathrm{MF}}$, where q_1^{MF} is the number of units sold to the final demand sector and p_1^{MS} is the corresponding average sales price. Note that p_1^{MS} will usually not equal p_1^{MF}; that is, for a variety of reasons, the average selling price of the manufactured good to the two sectors that demand the good will usually be different.[9] Now $p_1^{\mathrm{MS}}q_1^{\mathrm{MS}} + p_1^{\mathrm{MF}}q_1^{\mathrm{MF}}$ is the total revenue received by industry M during the period under consideration, but it will not be the total cost paid by the receiving sectors due to the existence of transport costs. Thus in row 1 of table 7C.1, we show the transportation industry as purchasing the goods from industry M, which explains the entry $-p_1^{\mathrm{MS}}q_1^{\mathrm{MS}} - p_1^{\mathrm{MF}}\, q_1^{\mathrm{MF}}$. The sum of the row 1 entries across the three entries is 0, and so the row 1 entry for the final demand column is left empty and corresponds to a 0 entry. Turning now to the row 2 entries,

6. In this highly simplified model, we will have only one final demand sector and we neglect the problems posed by imported goods and services. The transportation industry can be thought of as an aggregate of the transportation, advertising, wholesaling, and retailing industries.

7. Service industries generally require some materials in order to produce their outputs.

8. Hence this price will be a unit value price over all sales of commodity 1 to the service sector.

9. Even if there is no price discrimination on the part of industry M at any point in time, the price of good 1 will usually vary over the reference period, and hence if the proportion of daily sales varies between the two sectors, the corresponding period average prices for the two sectors will be different.

Table 7C.1 Detailed input-output table in current dollars with no taxes

Row No.	Industry M	Industry S	Industry T	Final demand
1	$p_1^{MS}q_1^{MS} + p_1^{MF}q_1^{MF}$		$-p_1^{MS}q_1^{MS} - p_1^{MF}q_1^{MF}$	
2			$(p_1^{MF} + p_3^{MF})q_1^{MF}$	$(p_1^{MF} + p_3^{MF})q_1^{MF}$
3		$-(p_1^{MS} + p_3^{MS})q_1^{MS}$	$(p_1^{MS} + p_3^{MS})q_1^{MS}$	
4	$-p_2^{SM}q_2^{SM}$	$p_2^{SM} + q_2^{SM} + p_2^{SF}q_2^{SF}$		$p_2^{SF}q_2^{SF}$
5	$p_1^{MS} + q_1^{MS} + p_1^{MF}q_1^{MF}$ $- p_2^{SM}q_2^{SM}$	$p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$ $- (p_1^{MS} + p_3^{MS})q_1^{MS}$	$p_3^{MF}q_1^{MF} + p_3^{MS}q_1^{MS}$	$(p_1^{MF} + p_3^{MF})q_1^{MF}$ $+ p_2^{SF}q_2^{SF}$

Note: Blank cells signify a 0 entry.

the industry T row 2 entry shows the transportation industry selling commodity 1 to the final demand sector and getting the revenue $(p_1^{MF} + p_3^{MF})q_1^{MF}$ for this sale. This revenue consists of the initial cost of the goods delivered at the manufacturer's gate, $p_1^{MF}q_1^{MF}$, plus revenue received by the transportation sector for delivering good 1 from the manufacturing plant to the final demand sector, $p_3^{MF}q_1^{MF}$. Thus we are measuring the quantity of transportation services in terms of the number of goods delivered to the final demand sector, q_1^{MF}, and the corresponding average delivery price is p_3^{MF}, which can be interpreted as a transportation markup or margin rate.[10] Turning now to the row 3 entries, the industry T row 3 entry shows the transportation industry selling commodity 1 to the service sector and getting the revenue $(p_1^{MS} + p_3^{MS})q_1^{MS}$ for this sale. This revenue consists of the initial cost of the goods delivered at the manufacturer's gate, $p_1^{MS}q_1^{MS}$, plus revenue received by the transportation sector for delivering good 1 from the manufacturing plant to the service sector, $p_3^{MS}q_1^{MS}$. Thus we are measuring the quantity of transportation services in terms of the number of goods delivered to the services sector, q_1^{MS}, and the corresponding average delivery price is p_3^{MS}, which again can be interpreted as a transportation markup or margin rate. There is no reason to expect the transportation margin rates p_3^{MS} and p_3^{MF} to be identical since the costs of delivery to the two purchasing sectors could be very different.

Row 4 of table 7C.1 lists the transactions involving services, commodity 2. The industry S row 4 entry, $p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$, lists the value of services output delivered to the manufacturing industry, $p_2^{SM}q_2^{SM}$, plus the value of services output delivered to the final demand sector, $p_2^{SF}q_2^{SF}$. The quantities delivered to the two sectors are q_2^{SM} and q_2^{SF}, and the corresponding average prices are p_2^{SM} and p_2^{SF}. As usual, there is no reason to expect that these two service prices should be identical. The term $-p_2^{SM}q_2^{SM}$ appears in row 4 of the

10. Actually, p_3^{MF} should be interpreted more broadly as a combination of transport costs and selling costs, which would include retailing and wholesaling margins.

industry M column, since this represents the cost of services to the M sector. Similarly, the term ${}^{\mathrm{SF}}_{2}q_2^{\mathrm{SF}}$ appears in row 4 of the final demand column, since this represents the value of services delivered to the final demand sector, and this amount is also equal to the sum of the M, S, and T entries for row 4.

Note that every transaction listed in rows 1–4 of table 7C.1 has a separate purchaser and seller, and so the principles of double-entry bookkeeping are respected in this table.[11]

The entries in row 5 for the M, S, and T columns are the simple sums of the entries in rows 1–4 for each column and are equal to the corresponding industry value added. Thus, the industry M value added is equal to $p_1^{\mathrm{MS}}Q_1^{\mathrm{MS}} + p_1^{\mathrm{MF}}q_1^{\mathrm{MF}} - p_2^{\mathrm{SM}}q_2^{\mathrm{SM}}$, the value of manufacturing output at factory gate prices less purchases of services. The industry S value added is equal to $p_2^{\mathrm{SM}}q_2^{\mathrm{SM}} + p_2^{\mathrm{SF}}q_2^{\mathrm{SF}} - (p_1^{\mathrm{MS}} + p_3^{\mathrm{MS}})q_1^{\mathrm{MS}}$, the value of services output less the value of materials purchases but at prices that include the transportation margins. The industry T value added is equal to $p_3^{\mathrm{MF}}q_1^{\mathrm{MF}} + p_3^{\mathrm{MS}}q_1^{\mathrm{MS}}$, which is the product of the transportation margin rate times the amount shipped, summed over the deliveries of transport services to the final demand sector, $p_3^{\mathrm{MF}}q_1^{\mathrm{MF}}$, and to the services sector, $p_3^{\mathrm{MS}}q_1^{\mathrm{MS}}$. Finally, the entry in row 5 of the last column is equal to both the sum of industry value added over industries or to the sum of commodity final demands, $(p_1^{\mathrm{MF}} + p_3^{\mathrm{MF}})q_1^{\mathrm{MF}} + p_2^{\mathrm{SF}}q_2^{\mathrm{SF}}$. Note that the final demand price for the good (commodity 1) is $p_1^{\mathrm{MF}} + p_3^{\mathrm{MF}}$, which is equal to industry M's factory gate price, p_1^{MF}, plus the transportation margin rate, p_3^{MF}, that is, the final demand price for the good has imbedded in it transportation (and other selling) costs.

Looking at table 7C.1, it can be seen that there are three ways that we could calculate a Laspeyres quantity index of net outputs for the economy that the table represents:

- Look at the nonzero cells in the 4×3 matrix of input output values of outputs and inputs for the economy represented by rows 1–4 and columns M, S, and T and sum up these nonzero cells into ten distinct p_nq_n transactions.
- Look at the row 5, column M, S, and T entries for the industry value added components and sum up these cells into eight distinct transactions.
- Look at rows 1–4 of the final demand column and sum up the nonzero cells into two distinct p_nq_n transactions.[12]

11. Our notation is unfortunately much more complicated than the notation that is typically used in explaining input-output tables because we do not assume that each commodity trades across demanders and suppliers at the same price. Thus, our notation distinguishes three superscripts or subscripts instead of the usual two: we require two superscripts to distinguish the selling and purchasing sectors and one additional subscript to distinguish the commodity involved in each transaction. This type of setup was used in Diewert (2004b).

12. The first p_nq_n is $(p_1^{\mathrm{MF}} + p_3^{\mathrm{MF}})q_1^{\mathrm{MF}}$ and the second p_nq_n is $p_2^{\mathrm{SF}}\ q_2^{\mathrm{SF}}$.

Denote the ten-dimensional **p** and **q** vectors that correspond to the first detailed cell method of aggregating over commodities listed above as $\mathbf{p}^{\mathrm{IO}}$ and $\mathbf{q}^{\mathrm{IO}}$ respectively, denote the eight-dimensional **p** and **q** vectors that correspond to the second value-added method of aggregating over commodities listed above as $\mathbf{p}^{\mathrm{VA}}$ and $\mathbf{q}^{\mathrm{VA}}$ respectively and denote the two-dimensional **p** and **q** vectors that correspond to the third aggregation over final demand components method of aggregating over commodities listed above as $\mathbf{p}^{\mathrm{FD}}$ and $\mathbf{q}^{\mathrm{FD}}$ respectively.[13] Add a superscript t to denote these vectors evaluated at the data pertaining to period t. Then it is obvious that the inner products of each of these three period-t price and quantity vectors are all equal since they are each equal to period-t nominal GDP; that is, we have

$$\mathbf{p}^{\mathrm{IO}t} \cdot \mathbf{q}^{\mathrm{IO}t} = \mathbf{p}^{\mathrm{VA}t} \cdot \mathbf{q}^{\mathrm{VA}t} = \mathbf{p}^{\mathrm{FD}t} \cdot \mathbf{q}^{\mathrm{FD}t}; \quad t = 0, 1. \tag{1}$$

What is not immediately obvious is that the inner products of the three sets of price and quantity vectors are also equal if the price vectors are evaluated at the prices of one period and the corresponding quantity vectors are evaluated at the quantities of another period; that is, we also have, for periods 0 and 1, the following equalities:[14]

$$\mathbf{p}^{\mathrm{IO}1} \cdot \mathbf{q}^{\mathrm{IO}0} = \mathbf{p}^{\mathrm{VA}1} \cdot \mathbf{q}^{\mathrm{VA}0} = \mathbf{p}^{\mathrm{FD}1} \cdot \mathbf{q}^{\mathrm{FD}0} \tag{2}$$

$$\mathbf{p}^{\mathrm{IO}0} \cdot \mathbf{q}^{\mathrm{IO}1} = \mathbf{p}^{\mathrm{VA}0} \,\mathrm{dot}\, \mathbf{q}^{\mathrm{VA}1} = \mathbf{p}^{\mathrm{FD}0} \cdot \mathbf{q}^{\mathrm{FD}1} \tag{3}$$

Laspeyres and Paasche quantity indexes that compare the quantities of period 1 to those of period 0 can be defined as follows:

$$Q_L^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO}0}, \mathbf{p}^{\mathrm{IO}1}, \mathbf{q}^{\mathrm{IO}0}, \mathbf{q}^{\mathrm{IO}1}) \equiv \frac{p^{\mathrm{IO}0} \cdot q^{\mathrm{IO}1}}{p^{\mathrm{IO}0} \cdot q^{\mathrm{IO}0}}; \tag{4}$$

$$Q_L^{\mathrm{VA}}(\mathbf{p}^{\mathrm{VA}0}, \mathbf{p}^{\mathrm{VA}1}, \mathbf{q}^{\mathrm{VA}0}, \mathbf{q}^{\mathrm{VA}1}) \equiv \frac{p^{\mathrm{VA}0} \cdot q^{\mathrm{VA}1}}{p^{\mathrm{VA}0} \cdot q^{\mathrm{VA}0}};$$

$$Q_L^{\mathrm{FD}}(\mathbf{p}^{\mathrm{FD}0}, \mathbf{p}^{\mathrm{FD}1}, \mathbf{q}^{\mathrm{FD}0}, \mathbf{q}^{\mathrm{FD}1}) \equiv \frac{p^{\mathrm{FD}0} \cdot q^{\mathrm{FD}1}}{p^{\mathrm{FD}0} \cdot q^{\mathrm{FD}0}};$$

$$Q_P^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO}0}, \mathbf{p}^{\mathrm{IO}1}, \mathbf{q}^{\mathrm{IO}0}, \mathbf{q}^{\mathrm{IO}1}) \equiv \frac{p^{\mathrm{IO}1} \cdot q^{\mathrm{IO}1}}{p^{\mathrm{IO}1} \cdot q^{\mathrm{IO}0}}; \tag{5}$$

$$Q_P^{\mathrm{VA}}(\mathbf{p}^{\mathrm{VA}0}, \mathbf{p}^{\mathrm{VA}1}, \mathbf{q}^{\mathrm{VA}0}, \mathbf{q}^{\mathrm{VA}1}) \equiv \frac{p^{\mathrm{VA}1} \cdot q^{\mathrm{VA}1}}{p^{\mathrm{VA}1} \cdot q^{\mathrm{VA}0}};$$

$$Q_P^{\mathrm{FD}}(\mathbf{p}^{\mathrm{FD}0}, \mathbf{p}^{\mathrm{FD}1}, \mathbf{q}^{\mathrm{FD}0}, \mathbf{q}^{\mathrm{FD}1}) \equiv \frac{\mathbf{p}^{\mathrm{FD}1} \cdot \mathbf{q}^{\mathrm{FD}1}}{\mathbf{p}^{\mathrm{FD}1} \cdot \mathbf{q}^{\mathrm{FD}0}}.$$

13. All prices are positive, but if a quantity is an input it is given a negative sign.
14. The proof follows by a set of straightforward computations.

Using equations (1) and (3) and the definitions in equation (4), it can be seen that all three Laspeyres indexes of real output are equal; that is, we have

$$(6)\qquad Q_L^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}}) = Q_L^{\mathrm{VA}}(\mathbf{p}^{\mathrm{VA0}}, \mathbf{p}^{\mathrm{VA1}}, \mathbf{q}^{\mathrm{VA0}}, \mathbf{q}^{\mathrm{VA1}}) = Q_L^{\mathrm{FD}}(\mathbf{p}^{\mathrm{FD0}}, \mathbf{p}^{\mathrm{FD1}}, \mathbf{q}^{\mathrm{FD0}}, \mathbf{q}^{\mathrm{FD1}}).$$

Using equations (1) and (2) and the definitions in equation (5), it can be seen that all three Paasche indexes of real output are equal; that is, we have

$$(7)\qquad Q_P^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}}) = Q_P^{\mathrm{VA}}(\mathbf{p}^{\mathrm{VA0}}, \mathbf{p}^{\mathrm{VA1}}, \mathbf{q}^{\mathrm{VA0}}, \mathbf{q}^{\mathrm{VA1}}) = Q_P^{\mathrm{FD}}(\mathbf{p}^{\mathrm{FD0}}, \mathbf{p}^{\mathrm{FD1}}, \mathbf{q}^{\mathrm{FD0}}, \mathbf{q}^{\mathrm{FD1}}).$$

Since a Fisher ideal quantity index is the square root of the product of a Laspeyres and Paasche quantity index, it can be seen that equations (6) and (7) imply that all three Fisher quantity indexes, constructed by aggregating over input-output table cells or by aggregating over industry value added components or by aggregating over final demand components, are equal; that is, we have

$$(8)\qquad Q_F^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}}) = Q_F^{\mathrm{VA}}(\mathbf{p}^{\mathrm{VA0}}, \mathbf{p}^{\mathrm{VA1}}, \mathbf{q}^{\mathrm{VA0}}, \mathbf{q}^{\mathrm{VA1}}) = Q_F^{\mathrm{FD}}(\mathbf{p}^{\mathrm{FD0}}, \mathbf{p}^{\mathrm{FD1}}, \mathbf{q}^{\mathrm{FD0}}, \mathbf{q}^{\mathrm{FD1}}).$$

The above results extend to more complex input-output frameworks provided that all transactions between each pair of sectors in the model are accounted for in the model. Thus, we have extended the results of Moyer, Reinsdorf, and Yuskavage to input-output models where prices are not constant across industries.[15]

It is well known that the Laspeyres and Paasche quantity indexes are consistent in aggregation. Thus, if we construct Laspeyres indexes of real value added by industry in the first stage of aggregation and then use the resulting industry prices and quantities as inputs into a second stage of Laspeyres aggregation, then the resulting two-stage Laspeyres quantity index is equal to the corresponding single-stage index, $Q_L^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}})$. Similarly, if we construct Paasche indexes of real value added by industry in the first stage of aggregation and then use the resulting industry prices and quantities as inputs into a second stage of Paasche aggregation, then the resulting two-stage Paasche quantity index is equal to the corresponding single-stage index, $Q_P^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}})$. Unfortunately, the corresponding result does not hold for the Fisher index. However, the two-stage Fisher quantity index usually will be quite close to the corresponding single-stage index, $Q_F^{\mathrm{IO}}(\mathbf{p}^{\mathrm{IO0}}, \mathbf{p}^{\mathrm{IO1}}, \mathbf{q}^{\mathrm{IO0}}, \mathbf{q}^{\mathrm{IO1}})$.[16]

15. The exact index number results in equation (8) were also derived by Diewert (2004b, 497–507) in an input-output model with no commodity taxes but with transportation margins and hence unequal prices.

16. See Diewert (1978, 889).

Table 7C.2 **Consolidated current-dollar table with transportation detail**

Row No.	Industry M	Industry S	Industry T	Final demand
1	$p_1^{MS}q_1^{MS} + p_1^{MF}q_1^{MF}$	$-p_1^{MS}q_1^{MS}$		$p_1^{MF}q_1^{MF}$
2	$-p_2^{SM}q_2^{SM}$	$p_2^{SM}q_2^{SM} + p_2^{SF}q_1^{SF}$		$p_2^{SF}q_2^{SF}$
3		$-p_3^{MS}q_1^{MS}$	$p_3^{MS}q_1^{MS} + p_3^{MF}q_1^{MF}$	$p_3^{MF}q_1^{MF}$

We are not quite through with table 7C.1. In the remainder of this section, we provide some consolidations of the entries in table 7C.1 and derive some alternative input output tables that could be useful in applications.

Table 7C.2 represents a consolidation of the information presented in table 7C.1. First, we sum the entries in rows 1 to 3 of table 7C.1 for each industry column. Recall that the entries in rows 1 to 3 represent the transactions involving the output of industry M. Second, we separate out from the sum of the entries over rows 1–3 all of the transactions involving the transportation price p_3 and put these entries in a separate row, which is row 3 in table 7C.2. The sum of the row 1–3 entries in table 7C.1 less row 3 in table 7C.2 is row 1 in table 7C.2. Row 2 in table 7C.2 is equal to row 4 in table 7C.1 and gives the allocation of the service commodity across sectors.

Table 7C.2 resembles a traditional input-output table. Rows 1 to 3 correspond to transactions involving commodities 1–3, respectively, and each industry gross output is divided between deliveries to the other industries and to the final demand sector. Thus the industry M row 1 entry in table 7C.2 gives the value of goods production delivered to the service sector, $p_1^{MS}q_1^{MS}$, plus the value delivered to the final demand sector, $p_1^{MF}q_1^{MF}$. Note that these deliveries are at the prices actually received by industry M; that is, transportation and selling margins are excluded. Similarly, the industry S row 2 entry gives the value of services production delivered to the goods sector, $p_2^{SM}q_2^{SM}$, plus the value delivered to the final demand sector, $p_2^{SF}q_2^{SF}$. Finally, the industry T row 3 entry gives the value of transportation (and selling) services delivered to the services sector, $p_3^{MS}q_3^{MS}$, plus the value delivered to the final demand sector, $p_3^{MS}q_3^{MF}$. If we summed the entries in rows 1–3 for each column in table 7C.2, we would obtain row 5 in table 7C.1, which gives the value added for columns M, S, and T and GDP for the last column. Thus, the new table 7C.2 does not change any of the industry value added aggregates listed in the last row of table 7C.1.

Although table 7C.2 looks a lot simpler than table 7C.1, there is a cost to working with table 7C.2 compared to table 7C.1. In table 7C.1, there were two components of final demand, $(p_1^{MS} + p_3^{MF})q_1^{MF}$, and $p_2^{SF}q_2^{SF}$. These two components are deliveries to final demand of goods at final demand prices (which include transportation margins) and deliveries of services to final demand. In table 7C.2, the old goods deliveries to final demand component is broken up into two separate components, $p_1^{MF}q_1^{MF}$ (deliveries of goods to final demand at factory gate prices), and $p_3^{MF}q_1^{MF}$, the transport costs of shipping the goods from the factory gate to the final demander. Thus, table 7C.2 requires that information on transportation margins be available; that is, information on both producer prices and margins be available whereas GDP could be evaluated using the last column in table 7C.1, which required information only on final demand prices.[17]

Looking at table 7C.2, it can be seen that it is unlikely that commodity prices are constant along the components of each row. This is unfortunate since it means that *in order to construct accurate productivity statistics for each industry, it generally will be necessary to construct separate price deflators for each nonzero cell in the input-output tables.*

Table 7C.2 allows us yet another way that real GDP for the economy can be constructed. For this fourth method for constructing Laspeyres, Paasche, and Fisher output indexes for the economy, we could use the nine nonzero p_nq_n values that appear in the nonzero components of rows 1–3 and the M, S, and T columns of table 7C.2 and use the corresponding p and q vectors of dimension 9 as inputs into the Laspeyres, Paasche, and Fisher quantity index formulae. It is easy to extend the string of equations (6), (7), and (8) to cover these new indexes. Thus we have a fourth method for constructing a Fisher output index that will give the same answer as the previous three methods.

The *real* input-output table that corresponds to the *nominal value* input-output table 7C.2 is table 7C.3.

The entries in row 1 of table 7C.3 are straightforward: the total production of goods by industry M, $q_1^{MS} + q_1^{MF}$, is allocated to the intermediate input use by industry S (q_1^{MS}) and to the final demand sector (q_1^{MF}). Similarly, the entries row 2 of table 7C.3 are straightforward: the total production of services by industry S, $q_2^{MS} + q_2^{SF}$, is allocated to the intermediate input use by industry M (q_2^{SM}) and to the final demand sector (q_2^{SF}). However, the entries in row 3 of table 7C.3 are a bit surprising in that they are essentially the same as the entries in row 1. This is due to the fact that we have measured transportation services in quantity units that are equal to the number of units of the manufactured good that are delivered to each sector.

We conclude this section by providing a further consolidation of the nominal input-output table 7C.2. Thus in table 7C.4, we aggregate the transportation industry with the goods industry and add the entries in row 3 of table 7C.2 to the corresponding entries in row 1; that is, we aggregate the transportation commodity with the corresponding good commodity that is being transported.

Row 1 in table 7C.4 allocates the good across the service industry and the final demand sector. Thus, the value of goods output produced by industry

17. Of course, in order to evaluate *all* of the cells in the input output tables represented by tables 7C.1 or 7C.2, we would require information on transportation margins in any case.

Table 7C.3 Consolidated constant-dollar table with transportation detail

Row No.	Industry M	Industry S	Industry T	Final demand
1	$q_1^{MS} + q_1^{MF}$	$-q_1^{MS}$		q_1^{MF}
2	$-q_2^{SM}$	$q_2^{SM} + q_2^{SF}$		q_2^{SF}
3		$-q_1^{MS}$	$q_1^{MS} + q_1^{MF}$	q_1^{MF}

Table 7C.4 Consolidated current-dollar table with no transportation detail

Row No.	Industry M + T	Industry S	Final demand
1	$(p_1^{MS} + p_3^{MS})q_1^{MS}$ $+ (p_1^{MF} + p_3^{MF})q_1^{MF}$	$-(p_1^{MS} + p_3^{MS})q_1^{MS}$	$(p_1^{MF} + p_3^{MF})q_1^{MF}$
2	$-p_2^{SM}q_2^{SM}$	$p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$	$p_2^{SF}q_2^{SF}$
3	$(p_1^{MS} + p_3^{MS})q_1^{MS}$ $+ (p_1^{MF} + p_3^{MF})q_1^{MF} - p_2^{SM}q_2^{SM}$	$p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$ $- (p_1^{MS} + p_3^{MS})q_1^{MS}$	$(p_1^{MF} + p_3^{MF})q_1^{MF} + p_2^{SF}q_2^{SF}$

M + T is $(p_1^{MS} + p_3^{MS})q_1^{MS} + (p_1^{MF} + p_3^{MF})q_1^{MF}$ and hence purchasers' prices are used in valuing these outputs. The value of deliveries to the services and final demand sectors are (including transportation margins) $(p_1^{MS} + p_3^{MS})q_1^{MS}$ and $(p_1^{MF} + p_3^{MF})q_1^{MF}$ respectively. The row 2 entries in table 7C.4, which allocate the service-sector outputs across demanders, are the same as the row 2 entries in table 7C.2. Row 3 in table 7C.4 gives the sum of the entries in rows 1 and 2 for each column. Thus the row 3, column (1) entry gives the value added of the combined goods producing and transportation industries while the row 3, industry S entry gives the value added for the services industry. The final demand entry in row 3 of table 7C.4 is the nominal value of GDP, as usual.

Looking at table 7C.4, it can be seen that it is unlikely that commodity prices are constant along the components of each row. Again, this is unfortunate since it means that *in order to construct accurate productivity statistics for each industry, it generally will be necessary to construct separate price deflators for each nonzero cell in the input-output tables.*

Table 7C.4 allows us yet another way that real GDP for the economy can be constructed. For this fifth method for constructing Laspeyres, Paasche, and Fisher output indexes for the economy, we could use the six nonzero $p_n q_n$ values that appear in rows 1 and 2 and columns (1) and (2) of table 7C.4 and use the corresponding **p** and **q** vectors of dimension 6 as inputs into the Laspeyres, Paasche, and Fisher quantity index formulae. It is easy to extend the string of equations (6), (7), and (8) to cover these new indexes. Thus we have a fifth method for constructing a Fisher output index that will give the same answer as the previous four methods.

The organization of production statistics that is represented by table

7C.4 is convenient for some purposes, in that outputs are valued consistently at final demand prices. However, it has the disadvantage that the transportation, retailing, and wholesaling industries have disappeared, which means that these margins have to be imputed to the goods-producing industries. Moreover, the primary inputs that are used by the transportation, retailing, and wholesaling industries would also have to be allocated to goods-producing industries. It is unlikely that users of industry production statistics would welcome these changes. Thus we conclude that organizing production statistics according to the layout in table 7C.2 would be preferable for most purposes.

In the following section, we introduce commodity taxes into our highly simplified model of the industrial structure of the economy.

Input-Output Tables When There are Commodity Taxes

Although governments in the United States do not impose very large commodity taxes on production as compared to many European countries, U.S. commodity taxes are large enough so that they cannot be ignored.

We return to the production model that corresponds to table 7C.1 in the previous section but we now assume that there is the possibility of a commodity tax falling on the output of each industry. In order to minimize notational complexities, we assume that each producing industry collects these commodity taxes and remits them to the appropriate level of government. Thus industry M collects the tax revenue $t_1^{MS}q_1^{MS}$ on its sales of goods to industry S and the tax revenue $t_1^{MF}q_1^{MF}$ on its sales to the final demand sector so that t_1^{MS} and t_1^{MF} are the specific tax rates that are applicable (across all levels of government) on sales of goods to the service industry and to the final demand sector respectively.[18] Similarly, industry S collects the tax revenue $t_2^{SM}q_2^{SM}$ on its sales of services to industry M and the tax revenue $t_2^{SF}q_2^{SF}$ on its sales to the final demand sector. Finally, industry T collects the tax revenue $t_3^{MS}q_1^{MS}$ on its sales of transportation services to industry S and the tax revenue $t_3^{MF}q_1^{MF}$ on its sales of transportation services to the final demand sector.

We now add the commodity taxes collected by each industry to the old industry revenues that appeared in table 7C.1. Thus the old revenue received by industry M listed in row 1 of table 7C.1, $p_1^{MS}q_1^{MS} + p_1^{MF}q_1^{MF}$, is replaced by $(p_1^{MS} + t_1^{MS})q_1^{MS} + (p_1^{MF} + t_1^{MF})q_1^{MF}$ in row 1 of table 7C.5. Similarly, the old revenue received by industry S listed in row 4 of table 7C.1, $p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$, is replaced by $(p_2^{SM} + t_2^{SM})q_2^{SM} + (p_2^{SF} + t_2^{SF})q_2^{SF}$ in row 4 of

18. Ad valorem tax rates can readily be converted into specific taxes that are collected for each unit sold. Usually, tax rates are lower for sales to industry purchasers compared to sales to final demand, but this is not always the case since exports are generally taxed at zero rates. In any case, usually t_1^{MS} will not be equal to t_1^{MF}. If sales to a particular sector are not taxed, then simply set the corresponding tax rate equal to zero. Product-specific subsidies can be treated as negative commodity taxes.

Table 7C.5 Detailed input-output table in current dollars with commodity taxes

Row No.	Industry M	Industry S	Industry T	Final demand
1	$(p_1^{MS}+t_1^{MS})q_1^{MS}$ $+ (p_1^{MF}+t_1^{MF})q_1^{MF}$		$-(p_1^{MS}+t_1^{MS})q_1^{MS}$ $-(p_1^{MF}+t_1^{MF})q_1^{MF}$	
2			$(p_1^{MF}+t_1^{MF})q_1^{MF}$ $+ (p_3^{MF}+t_3^{MF})q_1^{MF}$	$(p_1^{MF}+t_1^{MF})q_1^{MF}$ $+ (p_3^{MF}+t_3^{MF})q_1^{MF}$
3		$-(p_1^{MS}+t_1^{MS})q_1^{MS}$ $-(p_3^{MS}+t_3^{MS})q_1^{MS}$	$(p_1^{MS}+t_1^{MS})q_1^{MS}$ $+ (p_3^{MS}+t_3^{MS})q_1^{MS}$	
4	$-(p_2^{SM}+t_2^{SM})q_2^{SM}$	$(p_2^{SM}+t_2^{SM})q_2^{SM}$ $+ (p_2^{SF}+t_2^{SF})q_2^{SF}$		$(p_2^{SF}+t_2^{SF})q_2^{SF}$
5	$-t_1^{MS}q_1^{MS} - t_1^{MF}q_1^{MF}$	$-t_2^{SM}q_2^{SM} - t_2^{SF}q_2^{SF}$	$-t_3^{MF}q_1^{MF} - t_3^{MS}q_1^{MS}$	
6	$p_1^{MS}q_1^{MS} + p_1^{MF}q_1^{MF}$ $- p_2^{SM}q_2^{SM} - t_2^{SM}q_2^{SM}$	$p_2^{SM}q_2^{SM} + p_2^{SF}q_2^{SF}$ $-(p_1^{MS}+t_1^{MS})q_1^{MS}$ $-(p_3^{MS}+t_3^{MS})q_1^{MS}$	$p_3^{MF}q_1^{MF} + p_3^{MS}q_1^{MS}$	$(p_1^{MF}+t_1^{MF})q_1^{MF}$ $+ (p_3^{MF}+t_3^{MF})q_1^{MF}$ $+ (p_2^{SF}+t_2^{SF})q_2^{SF}$

table 7C.5. The old revenue received by industry T for its deliveries of goods shipped to the final demand sector listed in row 2 of table 7C.1, $(p_1^{MF} + p_3^{MF})q_1^{MF}$, is replaced by $(p_1^{MF} + t_1^{MF})q_1^{MF} + (p_3^{MF} + t_3^{MF})q_1^{MF}$ in row 2 of table 7C.5. The term $(p_1^{MF} + t_1^{MF})q_1^{MF}$ is equal to $p_1^{MF}q_1^{MF}$ (the revenue that the manufacturer gets for its sales of goods to the final demand sector) plus $t_1^{MF}q_1^{MF}$ (the amount of commodity taxes collected by the manufacturing sector on its sales of goods to the final demand sector). The term $(p_3^{MF} + t_3^{MF})q_1^{MF}$ reflects the additional charges that final demanders of the good pay for delivery of the good to the final demand sector, and this term is equal to the sum of $p_3^{MF}q_1^{MF}$ (the transportation sector's revenue for shipping goods to the final demand sector) plus $t_3^{MF}q_1^{MF}$ (the amount of taxes collected by the transportation sector that fall on shipping services to the final demand sector). Finally, the old revenue received by industry T for its deliveries of goods shipped to the services sector listed in row 3 of table 7C.1, $(p_1^{MS} + p_3^{MS})q_1^{MS}$, is replaced by $(p_1^{MS} + t_1^{MS})q_1^{MS} + (p_3^{MS} + t_3^{MS})q_1^{MS}$ in row 3 of table 7C.5. The term $(p_1^{MS} + t_1^{MS})q_1^{MS}$ is equal to $p_1^{MS}q_1^{MS}$ (the revenue that the manufacturer gets for its sales of goods to the services sector) plus $t_1^{MS}q_1^{MS}$ (the amount of commodity taxes collected by the manufacturing sector on its sales of goods to the services sector). The term $(p_3^{MS} + t_3^{MS})q_1^{MS}$ reflects the additional charges that service sector demanders of the good pay for delivery of the good to the service sector, and this term is equal to the sum of $p_3^{MS}q_1^{MS}$ (the transportation sector's revenue for shipping goods to the services sector) plus $t_3^{MS}q_1^{MS}$ (the amount of taxes collected by the transportation sector that fall on shipping services to the services sector). With the addition of the commodity tax terms, it can be seen that the first four rows of table 7C.5 are exact counterparts to the first four rows of table 7C.1.

Row 5 in table 7C.5 is a new row that has been added to the rows of table 7C.1, and it lists (with negative signs) the commodity tax revenues raised by the industry on its sales of products to final demand and other industries. These tax payments to the government are costs and hence are listed with negative signs.

The cells in row 6 of table 7C.5 are the sums down each column of the entries in rows 1 to 5. Thus the entries in row 6 list the *value added* of each industry for industries M, S, and T.[19] The row 6 entry for the final demand sector is simply the sum of final demand purchases for goods, including all tax and transportation margins, $(p_1^{\mathrm{MF}} + t_1^{\mathrm{MF}} + p_3^{\mathrm{MF}} + t_3^{\mathrm{MF}})q_1^{\mathrm{MF}}$, plus final demand purchases of services, including indirect taxes on services, $(p_2^{\mathrm{SF}} + t_2^{\mathrm{SF}})q_2^{\mathrm{SF}}$.

We now come to an important difference between table 7C.1 and table 7C.5: the sum of the industry M, S, and T value added (the entries along row 6 of table 7C.5) is no longer equal to the sum of the final demands down rows 1 to 4 of the final demand column: we need to add the commodity tax payments made by the three industries to the industry value-added sum in order to get the sum of final demands at final demand prices. It is worth spelling out this equality in some detail. Thus, define the industry M, S, and T value added, v^{M}, v^{S}, and v^{T} respectively, as follows:

$$v^{\mathrm{M}} \equiv p_1^{\mathrm{MS}}q_1^{\mathrm{MS}} + p_1^{\mathrm{MF}}q_1^{\mathrm{MF}} - p_2^{\mathrm{SM}}q_2^{\mathrm{SM}} - t_2^{\mathrm{SM}}q_2^{\mathrm{SM}}; \tag{9}$$

$$v^{\mathrm{S}} \equiv p_2^{\mathrm{SM}}q_2^{\mathrm{SM}} + p_2^{\mathrm{SF}}q_2^{\mathrm{SF}} - (p_1^{\mathrm{MS}} + t_1^{\mathrm{MS}})q_1^{\mathrm{MS}} - (p_3^{\mathrm{MS}} + t_3^{\mathrm{MS}})q_1^{\mathrm{MS}}; \tag{10}$$

$$v^{\mathrm{T}} \equiv p_3^{\mathrm{MF}}q_1^{\mathrm{MF}} + p_3^{\mathrm{MS}}q_1^{\mathrm{MS}}. \tag{11}$$

Notice that for each industry, outputs are valued at producer prices that exclude the commodity taxes collected by the industry but intermediate inputs are valued at prices that the industry faces; that is, the intermediate input prices include the commodity taxes paid by the supplying industries. In summary, the prices for outputs and intermediate inputs that are in the definitions in equations (9)–(11) are the prices actually faced by the respective

19. Note that our definition of industry value added is the value of outputs sold at purchasers' prices less the value of intermediate inputs at purchasers' prices less commodity taxes collected for the government by that industry. The usual definition of industry value added does not net off industry commodity tax remittances to the government; that is, the usual definition of value added does not subtract off row 5 but rather adds these commodity tax remittances to primary input payments. The problem with this latter treatment of industry commodity tax payments is that it does not provide a suitable framework for measuring industry productivity growth performance. Thus, our suggested treatment of indirect commodity taxes in an accounting framework that is suitable for productivity analysis follows the example set by Jorgenson and Griliches (1972), who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output" (85). Put another way, commodity tax payments to the government cannot readily be regarded as a payment for the services of a primary input.

industry. This is the set of prices that is best suited to a set of productivity accounts.[20] Finally, define the value of final demands, v^F, and the value of commodity taxes, v^τ, as follows:

$$(12) \qquad v^F \equiv (p_1^{MF} + t_1^{MF} + p_3^{MF} + t_3^{MF})q_1^{MF} + (p_2^{SF} + t_2^{SF})q_2^{SF};$$

$$(13) \qquad v^\tau \equiv t_1^{MS}q_1^{MS} + t_1^{MF}q_1^{MF} + t_2^{SM}q_2^{SM} + t_2^{SF}q_2^{SF} + t_3^{MF}q_1^{MF} + t_3^{MS}q_1^{MS}.$$

Using the definitions in equations (9) to (13), it is straightforward to verify that the following identity holds:

$$(14) \qquad v^M + v^S + v^T = v^F - v^\tau;$$

that is, the sum of industry M, S, and T value added equals GDP (or the value of final demands at purchasers' prices) less the value of commodity taxes that fall on outputs and intermediate inputs.

In addition to adding row 5 (the industry commodity tax payments to the government) to the rows of table 7C.1, table 7C.5 has another important difference compared to table 7C.1: the principles of double-entry bookkeeping are not respected in the present version of table 7C.5. The problem is that the industry tax payments listed in row 5 of table 7C.5 are not transferred to another column in the table. However, this deficiency could be corrected by creating a government "industry" column where the industry tax payments could be received. A more complete model of the economy would decompose final demand into a government sector as well as the other traditional $C + I + X - M$ final demand sectors.

Table 7C.6 is the counterpart to table 7C.2 in the previous section and it represents a consolidation of the information presented in table 7C.5. The industry M, row 1 entry in table 7C.6 is the sum of the row 1 and row 5 entries in table 7C.5 (this consolidation nets out the commodity taxes on the manufacturing output) and the industry M, row 2 entry in table 7C.6 is equal to the industry M, row 4 entry in table 7C.5 (the services intermediate input allocation to industry M remains unchanged). The industry S, row 3 entry in table 7C.5 is split between rows 1 and 3 in table 7C.5 (this splits the total intermediate input cost for industry S into a goods component and a transportation component). The industry S, row 2 entry in table 7C.6 is equal to the sum of the industry S, rows 4 and 5 entries in table 5 (this consolidation nets out the commodity taxes on the service sector outputs). The industry T, row 3 entry in table 7C.6 is the sum of rows 1–5 for industry T in table 7C.5. The table 7C.5 final demand entry for row 2 is split into goods and transportation services components, which are allocated to rows 1 and 3 of table 7C.6. The final demand for services entry in row 4 of table 7C.5 is switched to row 2 of the final demand column in table 7C.6.

20. As noted earlier, these are the prices that were recommended by Jorgenson and Griliches (1972, 85) for productivity accounts.

Table 7C.6 **Consolidated input-output table in current dollars with commodity taxes**

R	Industry M	Industry S	Industry T	Final Demand
1	$p_1^{\mathrm{MS}} q_1^{\mathrm{MS}} + p_1^{\mathrm{MF}} q_1^{\mathrm{MF}}$	$-(p_1^{\mathrm{MS}} + t_1^{\mathrm{MS}}) q_1^{\mathrm{MS}}$		$(p_1^{\mathrm{MF}} + t_1^{\mathrm{MF}}) q_1^{\mathrm{MF}}$
2	$-(p_2^{\mathrm{SM}} + t_2^{\mathrm{SM}}) q_2^{\mathrm{SM}}$	$p_2^{\mathrm{SM}} q_2^{\mathrm{SM}} + p_2^{\mathrm{SF}} q_2^{\mathrm{SF}}$		$(p_2^{\mathrm{SF}} + t_2^{\mathrm{SF}}) q_2^{\mathrm{SF}}$
3		$-(p_3^{\mathrm{MS}} + t_3^{\mathrm{MS}}) q_1^{\mathrm{MS}}$	$p_3^{\mathrm{MF}} q_1^{\mathrm{MF}} + p_3^{\mathrm{MS}} q_1^{\mathrm{MS}}$	$(p_3^{\mathrm{MF}} + t_3^{\mathrm{MF}}) q_1^{\mathrm{MF}}$

Thus, row 1 of table 7C.6 allocates the production of goods across the sectors of the economy, row 2 allocates the flow of services and row 3 allocates the flow of transportation services. If we summed down each column of table 7C.6, we would obtain the value added of industry M, v^{M} defined by equation (9), the value added of industry S, v^{S} defined by equation (10), the value added of industry T, v^{T} defined by equation (11), and (nominal) GDP, v^{F} defined by equation (12). The constant-dollar input-output table that corresponds to the nominal input-output table 7C.6 is still table 7C.3 in the previous section.

Looking at table 7C.6, it can be seen that the existence of commodity tax wedges means it is unlikely that commodity prices are constant along the components of each row. Again, this is unfortunate since it means that *in order to construct accurate productivity statistics for each industry, it generally will be necessary to construct separate price deflators for each nonzero cell in the input-output tables.*

We conclude this section by again seeing if we can obtain a counterpart to the Moyer, Reinsdorf, and Yuskavage exact index number result in this more complicated model where there are commodity tax wedges. Looking at the identity in equation (14), it can be seen that since nominal GDP is not equal to the sum of industry value added, if the value of commodity tax revenue v^{τ} is not equal to zero, we will not be able to get an exact result unless we add a government commodity tax revenue "industry" to the M, S, and T industries. Thus, we now define the value added of the commodity tax "industry" as v^{τ}, and we rewrite the identity in equation (14) as follows:

$$(15) \qquad v^{\mathrm{F}} = v^{\mathrm{M}} + v^{\mathrm{S}} + v^{\mathrm{T}} + v^{\tau}.$$

Now we can repeat the analysis in the previous section with a few obvious modifications. Thus, looking at equation (15) and table 7C.6, it can be seen that there are three ways that we could calculate a Laspeyres GDP quantity index for the economy that the table represents:

- Look at the nonzero cells in the 3×3 matrix of input-output values of outputs and inputs for the economy represented by rows 1–3 and columns M, S, and T of table 7C.6 and sum up these nonzero cells into nine distinct $p_n q_n$ transactions. Add to these nine $p_n q_n$ transactions the six $t_n q_n$ tax transactions that are defined by the right-hand side of equation (13), which gives us fifteen distinct price × quantity transactions in all.

- Look at the row 5, column M, S, and T entries for the industry value-added components listed in table 7C.5 and sum up these cells into eight distinct p_nq_n transactions. Add to these eight p_nq_n transactions the six t_nq_n tax transactions that are defined by the right-hand side of equation (13), which gives us fourteen distinct price times quantity transactions in all.
- Look at rows 1–3 of the final demand column in table 7C.6 and sum up the nonzero cells into two distinct p_nq_n transactions.[21]

Denote the fifteen-dimensional **p** and **q** vectors that correspond to the first detailed cell method of aggregating over commodities listed above as p^{IO} and q^{IO} respectively, denote the fourteen-dimensional **p** and **q** vectors that correspond to the second value-added method of aggregating over commodities listed above as p^{VA} and q^{VA} respectively, and denote the two-dimensional **p** and **q** vectors that correspond to the third aggregation over final demand components method of aggregating over commodities listed above as p^{FD} and q^{FD} respectively. Add a superscript t to denote these vectors evaluated at the data pertaining to period t. Then it is obvious that the inner products of each of these three period-t price and quantity vectors are all equal since they are each equal to period-t nominal GDP; that is, we have

$$\mathbf{p}^{IOt} \cdot \mathbf{q}^{IOt} = \mathbf{p}^{VAt} \cdot \mathbf{q}^{VAt} = \mathbf{p}^{FDt} \cdot \mathbf{q}^{FDt}; \quad t = 0, 1. \tag{16}$$

Now the rest of the analysis can proceed as in the previous section; see equations (2)–(8) and repeat this analysis in the present context. As in the second section, it can be shown that all three Fisher quantity indexes, constructed by aggregating over input-output table cells or by aggregating over industry value-added components or by aggregating over final demand components, are equal; that is, we have

$$\begin{aligned} (16) \qquad Q_F^{IO}(\mathbf{p}^{IO0}, \mathbf{p}^{IO1}, \mathbf{q}^{IO0}, \mathbf{q}^{IO1}) &= Q_F^{VA}(\mathbf{p}^{VA0}, \mathbf{p}^{VA1}, \mathbf{q}^{VA0}, \mathbf{q}^{VA1}) \\ &= Q_F^{FD}(\mathbf{p}^{FD0}, \mathbf{p}^{FD1}, \mathbf{q}^{FD0}, \mathbf{q}^{FD1}). \end{aligned}$$

Thus, we have extended the results of Moyer, Reinsdorf, and Yuskavage to input output models where commodity tax distortions are present.[22] The usual BEA Fisher contributions to growth methodology can be used in order to decompose overall GDP growth into industry growth contributions plus a commodity tax change contribution (this is the contribution to GDP growth of the artificial commodity tax industry).

21. The first p_nq_n is $(p_1^{MF} + t_1^{MF} + p_3^{MF} + t_3^{MF})q_1^{MF}$ and the second p_nq_n is $(p_2^{SF} + t_2^{SF})q_2^{SF}$.

22. Results analogous to equation (16) were derived by Diewert (2004a, 479–84) under more restrictive assumptions; that is, each sector was assumed to face the same vector of commodity prices except for the commodity tax distortions.

Conclusion

There are a number of important implications that emerge from the above discussion:

- With appropriate adjustments for commodity taxes, a Fisher index of value added growth by industry can be used to construct an independent estimate of real GDP growth.
- The existence of transportation and selling margins and commodity taxes means that the assumption that a *single* price deflator can be used for productivity measurement purposes to deflate all of the value cells long the row of an input-output table is likely to be a very rough approximation at best. In principle, each nonzero cell in a nominal input output table will require its own separate deflator.[23]
- The existence of commodity taxes that fall within the production sector poses special problems for statistical agencies. These taxes need to be identified by cell position in the input-output tables instead of just reported as a single sum for the industry as is done at present.

The last point requires a bit more explanation. Looking at table 7C.6, it can be seen that row 2 entry for industry M is $-(p_2^{SM} + t_2^{SM})q_2^{SM}$, which is (minus) the value of service intermediate inputs used by industry M, including the commodity tax portion, $t_2^{SM}q_2^{SM}$. Similarly, the row 1 entry for industry S is $-(p_1^{MS} + t_1^{MS})q_1^{MS}$, which is minus the value of materials intermediate inputs used by industry S, including the commodity tax portion, $t_1^{MS}q_1^{MS}$. The row 3 entry for industry S is $-(p_3^{MS} + t_3^{MS})q_1^{MS}$, which is (minus) the value of transportation services purchased by industry S, including the commodity tax on transport services, $t_3^{MS}q_1^{MS}$. It can be shown that the existence of these commodity tax distortions on intermediate input purchases by the private production sector leads to a loss of overall productive efficiency. Thus, even though industry M and industry S are operating efficiently so that they are on the frontiers of their production possibilities sets, the consolidated production sector is not operating efficiently. The explanation for this phenomenon was given by Gerard Debreu (1951, 285):[24] there is a loss of systemwide output (or waste, to use Debreu's term) due to the imperfection of economic organization; that is, different production units, while techni-

23. In the very simple model considered in the previous two sections, there was no aggregation bias in each cell of the various input-output tables that were constructed. However, in a real-life input-output table, we will not be able to classify commodities down to a very fine level of detail. Hence, there will be a mix of related commodity transactions in each cell of an empirical input-output table. Due to the differing mixes of micro commodities in each cell, it can be seen that each cell will require its own deflator and moreover, the entries along any row of the resulting deflated real input-output table will not in general add up to the corresponding total in the final demand column. Thus, forcing constant-dollar input-output tables to add up along rows will generally impose errors on the data.

24. See also Diamond and Mirrlees (1971).

cally efficient, face different prices for the same input or output, and this causes net outputs aggregated across production units to fall below what is attainable if the economic system as a whole were efficient. In other words, a condition for systemwide efficiency is that all production units face the same price for each separate input or output that is produced by the economy as a whole. Thus, the existence of commodity taxes that fall on intermediate inputs causes producers to face different prices for the same commodity, and if production functions exhibit some substitutability, then producers will be induced to jointly supply an inefficient economywide net output vector. The overall size of the loss of productive efficiency depends on the magnitudes of elasticities of substitution and on the size of the commodity tax distortions, t_2^{SM}, t_1^{MS}, and t_3^{MS}.[25] In order to obtain empirical estimates of this loss of productive efficiency, it is necessary to estimate production functions or dual cost or profit functions for each industry in the economy. Thus, for the economy represented by table 7C.6, it would be necessary to estimate three sectoral production functions (or their dual equivalents), and hence a time series of the price quantity data in each cell of the input-output table would need to be collected. For the econometric estimation, it would not be necessary for the statistical agency to provide information on the tax wedges; that is, only prices that include the tax wedges (along with the associated quantities) would need to be provided by the statistical agency.[26] However, in order to *calculate* the loss of productive efficiency induced by the tax wedges, t_2^{SM}, t_1^{MS}, and t_3^{MS}, the statistical agency would have to provide information on the size of these wedges.

The loss of productive efficiency due to the existence of taxes that fall within the production sector of the economy is of course not the total loss of efficiency that can be attributed to indirect tax wedges: there are additional losses of efficiency that are due to the taxes that fall on the components of final demand. Thus if we look down the three rows of the final demand column in table 7C.6, we see that each final demand price has a tax wedge included in it: t_1^{MF} is the final demand tax wedge for commodity 1 (the good), t_1^{SF} is the final demand tax wedge for commodity 2 (the service), and t_3^{MF} is the final demand tax wedge for commodity 3 (the transport service).[27] Each of these three tax wedges creates some additional losses of overall efficiency in the economy. In order to obtain empirical estimates of these efficiency losses or excess burdens, it will be necessary to estimate household preferences in addition to the production functions mentioned

25. To the accuracy of a second-order approximation, the size of the loss will grow quadratically in the tax rates t_2^{SM}, t_1^{MS} and t_3^{MS}; see Diewert (1983, 171).

26. For examples of econometric studies that estimate sectoral production functions or their dual equivalents, see Jorgenson (1998) or Diewert and Lawrence (1994, 2002).

27. Note that these three tax rates plus the three that appeared as taxes on intermediate inputs in the input-output table 7C.6 add up to the six commodity tax wedges in our model of the economy.

in the previous paragraph. For econometric estimation purposes, it is sufficient for the statistical agency to provide final demand prices and quantities demanded where the prices include the commodity tax wedges; that is, only the total prices, including commodity taxes, are required for econometric estimation. However, in order to calculate the deadweight loss generated by these commodity taxes, it will be necessary to have estimates of the tax wedges; that is, tax researchers will require estimates of t_1^{MF}, t_2^{SF}, and t_3^{MF}.[28] This information is required not only so that total excess burdens can be estimated but also so that *marginal excess burdens* of each tax can be estimated. The marginal excess burden of a tax rate is an estimate of the efficiency loss generated by a small increase in the tax rate divided by the extra revenue that the increase in the tax rate generates. If reasonably accurate information on marginal excess burdens could be made available to policymakers, this information would be very valuable in evaluating the consequences of either increasing or decreasing existing tax rates.[29] However, as indicated in this paragraph and the preceding one, it will not be possible to calculate estimates of these marginal excess burdens unless the statistical agency makes available information on the tax wedges and the associated quantities for each major indirect tax in the economy.

Thus, for purposes of modeling the effects of indirect commodity taxes, our conclusion is that the new architecture for an expanded set of U.S. accounts that is outlined in Jorgenson and Landefeld (2004) is not quite adequate to meet the needs of taxation economists. In addition to the tables that are presented in Jorgenson and Landefeld, we need an additional table that gives tax rates and the associated quantities (or revenues) for each cell where the tax appears. In terms of table 7C.6, we need not only price and quantity information for each of the nonzero cells in the table, but also price and quantity information for the six tax revenue flows in our model, namely t and q information for the tax flows $t_1^{MS}q_1^{MS}$, $t_1^{MF}q_1^{MF}$, $t_2^{SM}q_2^{SM}$, $t_2^{SF}q_2^{SF}$, $t_3^{MF}q_1^{MF}$, and $t_3^{MS}q_1^{MS}$. An additional benefit of making this information available is that this information is also required in order to reconcile the industry productivity accounts with the economy's final demand GDP accounts.

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28. For additional material on measuring deadweight losses, see Diewert (1981, 1983).

29. For examples of studies that estimate marginal excess burdens, see Jorgenson and Yun (2001) or Diewert and Lawrence (1994, 1995, 2002).

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