Volume Title: Annals of Economic and Social Measurement, Volume 2, number 4

Volume Author/Editor: Sanford V. Berg, editor

Volume Publisher:

Volume URL: http://www.nber.org/books/aesm73-4

Publication Date: October 1973

Chapter Title: Time-Varying Parameter Structures: An Overview

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Chapter URL: http://www.nber.org/chapters/c9932

Chapter pages in book: (p. 375 - 379)

Annals of Economic and Social Measurement, 2/4, 1973

TIME-VARYING PARAMETER STRUCTURES: AN OVERVIEW

BY DAVID A. BELSLEY AND EDWIN KUH

This paper summarizes the results of a NBER symposium on time-varying parameter structures and describes three areas of current research: random-coefficients models, systematic-variation models, and Kalman-filter models.

On January 26, 1973, the National Bureau of Economic Research. Computer Research Center for Economics and Management Science. hosted a symposium on the problem of time-varying parameter structures.[1] The participants. who are listed in the appendix, made formal presentations of papers and engaged in informal discussion of the state-of-the-art. Due to the wide interest in this topic. it was decided to publish revised versions of the papers which were prepared in connection with the symposium. This special issue of the *Annals of Economic and Social Measurement* brings together papers which deal with the three areas of current research into the time-varying parameter problem:

(1) Random-coefficients models
(2) Systematic (non-random) variation models
(3) Kalman-filter models.

This issue, therefore, affords a glimpse of the full width of the problem of estimating time-varying parameters. While this issue is clearly not exhaustive. its bibliographies offer an extensive key to the existing literature.

The problem of time-varying parameters has received increased attention because of an ever-growing body of evidence that the usual regression assumption of stable parameters often appears invalid. For instance. the apparent benefits for improved forecasts resulting from "adjusting the constant term" in regression equations is an indication that more variability is present than can be captured by ordinary autoregressive error terms. It seems. therefore. that more attention should be devoted to modeling regression processes where the regression parameters themselves are subject to various sorts of perturbations.

The rationales for time-varying parameter models are several. For one. the "true" coefficients themselves can often be viewed directly as the outcome of a stochastic process; e.g.. some sort of autocorrelated random event with a given mean. A related model assumes a cross-section of individuals who possess the same regression regime across time. but whose individual behavior at a given point in time can be viewed as a random sample from a population of coefficients. again with a given mean. This view gives rise to a cross-sectionally random coefficient model. Second, even when the underlying parameters are stable. situations arise in which a time-varying coefficient approach will prove to be effective. Such is the case when there are specification errors. such as excluded variables or linear

[1] We gratefully acknowledge the support of the National Science Foundation in providing travel funds for the symposium reported here, and for NSF support of the NBER Computer Research Center for Economics and Management Science which made the symposium possible.

approximations of curvelinear forms. These can often be more appropriately modeled as random coefficients rather than the simpler additive error term. Another difficulty that lends support to certain time-varying parameter models arises when the underlying data are aggregates. Shifts in aggregation weights among the micro components will often cause trends and/or other forms of auto-correlated behavior to appear in the macro-regression equation parameters.

The three main theoretical structures adopted in current research are listed above and described in greater detail in the next section. They arise as a result of econometricians' efforts to deal with specific limitations of the standard regression model. The Quandt–Goldfeld model, for example, postulates that there are one or more discrete "jumps" in the true parameter values. Here the econometrician seeks to estimate the points when the changes occur and the associated sets of coefficients. This problem stands in contrast to the more nearly continuous parameter variations assumed in the random coefficient models adopted in papers by Swamy, Rosenberg, Cooley and others. Related electrical engineering models using Kalman filtering have been applied to this problem in ways that increasingly appeal to econometricians. A major limitation of the Kalman filter is its frequent reliance on knowledge of the parameters of the stochastic process associated with the random coefficients. While engineers are often able to specify these parameters from direct physical information, econometricians are seldom so fortunate, and the identification and estimation problems are much more severe in an economic context.

We believe that this collection of papers, together with the references cited, will provide economists with a useful status report on where matters now stand and where they will be going. Clearly, this is only a beginning; a beginning that in our opinion represents a substantial increase in the realism with which econometricians can model economic behavior.

Fortunately it is unnecessary to survey the field of time-varying parameter structures, for this job is admirably accomplished in the lead article by Barr Rosenberg. However, it will prove useful to describe briefly the three main categories mentioned above and place the papers that follow in context.

Random-Coefficients Models

Random-coefficients models are simply those in which some or all of the parameters are considered to be random variates. The history of this notion extends back at least to the work by H. Rubin in the Cowles Commission Monograph [5]. More recently C. R. Rao derived the principal results obtained from introducing random coefficients into the standard linear regression problem [4]. In this context, the usual model, $y(t) = x'(t)\beta + \varepsilon(t)$, becomes $y(t) = x'(t)\beta(t) + \varepsilon(t)$, where $x(t)$ is a k-vector of observations at time t and $\beta(t)$ is a k-variate stochastic function of time. Such models arise quite naturally in economics and management science. An obvious generalization, for example, of individual consumption behavior would assume

$$c_{it} = a_i(t) + b_i(t)y_{it} + \varepsilon_{it},$$

where c_i and y_i are the consumption and income of individual i, respectively, and $a_i(t)$ and $b_i(t)$ are random coefficients across time. The specification of $b_i(t)$ as random indicates that the individual's marginal propensity to consume is not constant in successive periods, but rather is a probabalistic value. Random-coefficients models, thus, offer a natural means of combining time-series and cross-section data. Barr Rosenberg's paper "The Analysis of a Cross-Section of Time Series by a Stochastically Convergent Parameter Regression" exploits this fact.

Recognition that $a_i(t)$, the "constant term", may be stochastic leads one to consider error terms $(a_i(t) + \varepsilon_{it})$ with several, hopefully separable, components. The paper, "Error Component Regression Models and their Applications," by Swarnjit S. Arora, deals with a much more interesting and sophisticated problem of this sort. Two other papers further develop the theory of random-coefficients models. P. A. V. B. Swamy's excellent paper, "Criteria, Constraints and Multicollinearity in Random Coefficient Regression Models" both summarizes and continues his path-breaking work in this area. And specific examples of the applicability of the random-coefficient model to economic problems are afforded by the paper by Thomas Cooley and Edward Prescott, "Varying Parameter Regression: A Theory and Some Applications," which is an off-shoot of their recent work [1].

Systematic Parameter-Variation Models

In the initial stages of econometric model building, the investigator usually takes a pragmatic point of view, purposely ignoring some influences that are not directly involved, but which may nevertheless impinge on the outcome. Such would be the case, for example, in an optimal inventory-investment model for the firm that ignored the modulating influences of a business cycle. The parametric structure in this instance could vary, not as a random variate, but systematically with the influence of the omitted effects. A well known special case of this problem of systematic parameter variation is the Quandt [3] study in which, after some time period T, the linear model assumed to generate the observed data changes discontinuously.

Since "shifting regressions" often plague economists and management scientists, it is useful to have techniques that test for shifts and which estimate the nature of the shift when it takes place. The papers of this section offer some procedures. Many other suggested techniques can be found in the references in the bibliographies.

The paper by Stephen Goldfeld and Richard Quandt, "The Estimation of Structural Shifts by Switching Regressions" extends Quandt's earlier work to a more general switching framework. David Belsley presents a method for identifying those outside variates whose influences were ignored in "On the Determination of Systematic Parameter Variations in the Linear Regression Model." Belsley also presents a test for the presence of parameter variation in the next paper, "A Test for Systematic Variation in Regression Coefficients."

Kalman Filter Models

The Kalman filter is a technique that has only recently begun to spill over to economics from the engineering literature on optimal control systems where the object is to estimate the parametric "state" of a control model at various points in time so that corrective action in its time path may be taken. Quite loosely, Kalman techniques arise when one attempts to find an optimal (least mean squared error) estimator for the "state" $\beta(t)$ in a linear model $y = x'(t)\beta(t) + \varepsilon(t)$ in which the parametric state is assumed to obey a first-order transition equation $\beta(t + 1) = A\beta(t) + U(t)$, where $U(t)$ is some appropriately specified stochastic term and A is a matrix of constants. The problem, as it turns out, is quite closely related to linear regression and serves to extend its range of application.

An enlightening example of the Kalman filter as applied to a rocket control problem was presented to the Symposium by Dr. David Duncan [2]. Unfortunately for this volume, his paper forms the basis of an article to be published elsewhere and cannot be included here. However, the paper by Alexander H. Sarris, "A Bayesian Approach to Estimation of Time-Varying Regression Coefficients," explains the Kalman filter and a pertinent application. J. Phillip Cooper, in "Time Varying Regression Coefficients: A Mixed Estimation Approach and Operational Limitations of the General Markov Structure," provides an alternative basis for some of Sarris's results. In the concluding note, the usefulness of the Kalman filter for investigating systematic parameter variation is explored by David Belsley, "On the Applicability of the Kalman Filter to the Determination of Systematic Parameter Variation."

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Appendix: Symposium Participants

Visiting Participants

Thomas Cooley, Tufts University
David Duncan, The Johns Hopkins University
Stephen Goldfeld, Princeton University
Clifford G. Hildreth, University of Minnesota
Richard E. Quandt, Princeton University
Barr Rosenberg, University of California, Berkeley
P. A. V. B. Swamy, The Ohio State University

Participants from the Center

Swarnjit Arora, University of Wisconsin at Milwaukee
David Belsley, Boston College
Edwin Kuh, Massachusetts Institute of Technology
John R. Meyer, Yale University
Aleco Sarris, Massachusetts Institute of Technology
Roy E. Welsch, Massachusetts Institute of Technology