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## Comment Daniel McFadden

The rise of $401(\mathrm{k})$ plans as a channel for providing retirement incomes to employees makes enrollment in these plans increasingly important for the welfare of future retirees. The authors utilize persuasive natural experiments to quantify behavioral response to key $401(\mathrm{k})$ plan features: whether the default is automatic enrollment unless the employee opts out or nonenroll-

[^0]ment unless the employee opts in, whether there is employer matching of contributions or an unconditional employer contribution, and the default contribution rate for enrollees. Classical theory of rational life cycle savings implies that response to tax-qualified savings plans will depend critically on circumstances and intertemporal preferences (with matching or unconditional employer savings contributions inducing income and intertemporal substitution effects), but rational consumers should display no partial or incomplete adjustment that leaves final choices dependent on defaults. The authors' findings of significant default effects reinforces an extensive behavioral literature that shows that consumers are inconsistent in their transactions that trade certain consumption today for uncertain consumption in the future. These inconsistencies may be due to separate "mental accounts" for current and future consumption, difficulty in consistently evaluating risky prospects, time-inconsistent hyperbolic discounting, or breakdowns in decision making due to lack of attention and procrastination.

Natural questions for social planners dealing with default-sensitive consumers is what aspects of savings instruments are significant in influencing retirement savings behavior, and how defaults and other savings plan features can be designed to minimize the regret that consumers have at retirement when they review their past savings behavior. Note that inattention and incomplete optimization are bad for consumers who would otherwise be intertemporally consistent, but can be protective in circumstances where defaults avoid the excesses of faulty expectations and instant gratification. The authors find that employer matching of contributions to a $401(\mathrm{k})$ plan and unconditional employer $401(\mathrm{k})$ contributions both increase participation rates, but their effect on participation is small relative to the effect of opt-in versus opt-out. Then, adding or dropping matching or unconditional contributions to a $401(\mathrm{k})$ plan are not as critical as the opt-in or opt-out default. An additional question that would be interesting to answer if the authors' data permitted would be whether total $401(\mathrm{k})$ savings from both employer and employee contributions, and total savings taking into account all consumer intertemporal transactions, including purchase and financing of housing and other assets, increase as the result of employer matching or unconditional $401(\mathrm{k})$ contributions. Lack of consumer adjustment from defaults suggests that the answer would be affirmative. Then, to the extent that people undersave for retirement, employer 401(k) matching or unconditional contributions will increase consumer welfare more than the same payment in current wages.

## Rational Saving Behavior

At retirement, a large majority of consumers state that they regret that they did not save more. Also, consumption typically falls upon retirement, although whether this is inconsistent with rational life cycle planning depends on the nature of preferences for goods versus leisure. These observa-
tions are consistent with a tacit assumption in the chapter that most consumers do undersave. However, in defining the baseline against which various tax-qualified savings plans and consumer responses are to be judged, it is worth reviewing what a textbook Fisherian analysis of savings says about participation in such plans, and their impact on total savings.

The instruments for intertemporal allocation available to consumers are purchase of consumer durables (e.g., home ownership) with secured (e.g., mortgage) financing; credit card debt and other unsecured borrowing; and purchase of financial assets through conventional and tax qualified saving. Each channel for borrowing or saving carries its own rate of return (RoR), credit restrictions, and risk characteristics. Rational saving behavior is the result of optimal life cycle planning in the face of income endowments and the available instruments for intertemporal transactions. Then, rational savings will depend on current wealth and future income expectations, age and mortality expectations, family status (e.g., other wage earners, children), the available transaction instruments, and the applicable marginal RoR.

## The Intertemporal Budget Set

Consider a simple model with a working period and a retirement period, each thirty years long, and consider annual consumption at the representative ages forty-five and seventy-five for work and retirement. Abstract from buffer stock and diversification motives induced by risk, liquidity motives induced by transactions costs, and asset portfolio and consumption adjustments in response to life event shocks within the working and retirement eras. This is a gross oversimplification, but enough remains to clarify some key features of rational saving. Let $Y_{0}$ and $Y_{1}$ denote, respectively, endowments of working income now and retirement income in the future. Retirement consumption will be determined by $Y_{1}$ and by voluntary saving and borrowing. Let $t$ denote the marginal income tax rate, and assume for simplicity that it is independent of income level or period, and that capital gains are taxed the same as ordinary income. Let home ownership during the working years represent the single available consumer durable. Let $C_{0}$ and $C_{1}$ denote, respectively, working period consumption and retirement period consumption net of housing services and taxes.

Assume that the rental rate on housing available to both workers and retirees is $R$. Assume that all retirees rent, and that workers either rent, or buy and occupy a house at purchase price $H$ with a mortgage $M$. Owners amortize their mortgage over the working years, and sell when they retire. Assume that a minimum house price $H^{*}$ is required to provide the same services as a rental, and $H>H^{*}$ is a consumer durable investment. There are three other channels for intertemporal transactions: conventional savings $(S)$, tax-qualified savings $(Q)$ in a $401(\mathrm{k})$ plan, and unsecured borrowing $(B)$. Let $r_{S}, r_{M}$, and $r_{B}$ denote the respective annual interest rates on saving, secured mortgage borrowing, and unsecured borrowing, and let $r_{H}$
denote the annual housing rate of capital gains less maintenance. Let $a_{k} \equiv$ $r_{k} /\left[1-\left(1+r_{k}\right)^{-30}\right]$ denote an annual payment over thirty years that has unit present value at interest rate $r_{k}$. Note that one unit of an asset in period 0 that has an annual interest rate $r_{k}$ yields a future value $\left(1+r_{k}\right)^{30}$ in period 1 , and the present value of a unit annual payment over 30 years at interest rate $r_{k}$ is $p_{k}=1 / a_{k}$. In the absence of risk or tax factors that induce an equity premium or discount, rent $R$ in housing market equilibrium when landlords discount at the interest rate $r_{S}$ satisfies $H^{*}=R / a_{S}+H^{*}\left(1+r_{H}\right)^{30} /\left(1+r_{S}\right)^{30}$.

Empirically, working consumers have high discount rates; to reflect this, we assume that consumers discount at rate $r_{B}$. This implies that consumers facing a market interest rate lower than $r_{B}$ will choose to consume more when working than when retired. For the two-period representation of the consumer's problem, assume that the initial equity $H-M$ in a house is annualized within period 0 to $(H-M) a_{B}$, and that the receipts $\left(1+r_{H}\right)^{30} H$ from a house sale in period 1 are annualized within period 1 to $\left(1+r_{H}\right)^{30} \mathrm{Ha}_{B}$. A mortgage $M$ that is amortized over the span of period 0 has annual payments $a_{M} M$, with $1-\left(1+r_{M}\right)^{\mathrm{T}-30}$ giving the fraction of $a_{M} M$ that is mortgage interest in year $T$. The annualized value at the rate $r_{B}$ of the mortgage interest payments is $a_{M}\left[1-a_{B}\left(1+r_{M}\right)\left(\left(1+r_{M}\right)^{-30}-\left(1+r_{B}\right)^{-30}\right) /\left(r_{B}-r_{M}\right)\right] M \equiv$ $a_{1} M$; this is a deduction from taxable income. Unsecured annual borrowing of $a_{B} B /\left(1+r_{B}\right)^{30}$ during the working period leads to a balance $B$ repaid with annual payments $a_{B} B$ during retirement.
If a $401(\mathrm{k})$ plan has a unconditional employer contribution $Q^{\#}$ and a matching rate $\lambda$, then the employee with an annual contribution Q in the working period receives an annual taxable return of $\left(Q^{\#}+Q(1+\lambda)\right)\left(1+r_{S}\right)^{30}$ in the retirement period. Credit constraints are an income-determined bound ( $B \leq y Y_{0}$ ) on unsecured borrowing, income and house value bounds ( $M \leq \alpha Y_{0}$ and $M \leq \beta H$ ) on a mortgage, and a cap $\left(Q \leq Q^{*}\right.$ ) on tax-qualified savings. The case that a $401(\mathrm{k})$ plan is not available will correspond to $Q^{\#}=$ $Q^{*}=0$. This notation is summarized in table 11C.1.

Let $D$ denote a discrete variable that is one if the consumer buys a house, and zero if she rents. Taking into account the rules that the $401(\mathrm{k})$ contribution and mortgage interest expense are deductible from taxable income in period 0 , and all $401(\mathrm{k})$ saving is taxed when withdrawn, representative working period and retirement period annual consumption satisfy

$$
\begin{aligned}
C_{0}= & Y_{0}-t\left(Y_{0}-Q\right)+B a_{B} /\left(1+r_{B}\right)^{30}-S-Q \\
& -(1-D) R-D\left(a_{M} M+a_{B}(H-M)-t a_{1} M\right), \\
C_{1}= & (1-t)\left(Y_{1}+\left(1+r_{S}\right)^{30} S+\left(1+r_{S}\right)^{30}\left(Q^{\#}+Q(1+\lambda)\right)\right) \\
& +t S-R-a_{B} B+D a_{B} H\left((1-t)\left(1+r_{H}\right)^{30}+t\right) .
\end{aligned}
$$

A home purchaser will benefit from obtaining the maximum possible mortgage if

Table 11C. $1 \quad$ Channels for intertemporal transactions

| Channel | Quantity | Interest rate | Credit constraint |
| :--- | :---: | :---: | :---: |
| Conventional saving | $S$ | $r_{S}$ | none |
| House purchase | $H$ | $r_{H}$ | $H \geq H^{*}$ |
| Mortgage | $M$ | $r_{M}$ | $M \leq \min \left\{\beta H, \alpha Y_{0}\right\}$ |
| Revolving credit | $B$ | $r_{B}$ | $B \leq \gamma Y_{0}$ |
| Tax qualified 401(k) saving | $Q$ | $r_{S}$ | $Q \leq K$ |

$$
\partial C_{0} / \partial M=a_{B}+t a_{l}-a_{M}>0
$$

For example, at tax rate $t=0.3$ and real annual interest rates $r_{B}=0.10$ and $r_{M}=0.05$, one has $a_{B}=0.106, a_{M}=0.065, a_{l}=0.034$, and $\partial \mathrm{C}_{0} / \partial M=0.051$. The after-tax RoR for the intertemporal transaction instrument $S$ is $\mathrm{RoR}_{S}=$ $-\left(\partial \mathrm{C}_{1} / \partial \mathrm{S}\right) /\left(\partial \mathrm{C}_{0} / \partial \mathrm{S}\right)=(1-t)\left(1+r_{S}\right)^{30}+t$. Similarly, the after-tax RoR for $B$ and $Q$ are, respectively, $\operatorname{RoR}_{B}=\left(1+r_{B}\right)^{30}$ and $\operatorname{RoR}_{Q}=(1+\lambda)\left(1+r_{S}\right)^{30}$. The present value of postponing taxes implies $\mathrm{RoR}_{Q}>\mathrm{RoR}_{S}$. Continuing the numerical example with $\lambda=1$ gives $\operatorname{RoR}_{B}=17.45, \operatorname{RoR}_{Q}=4.85$ when $\lambda=1$ and $\operatorname{RoR}_{\mathrm{Q}}=2.43$ when $\lambda=0$, and $\operatorname{RoR}_{S}=2.00$. The housing after-tax RoR for $H>H^{*}$ with a maximum mortgage unconstrained by income is $\operatorname{RoR}_{H}=a_{B}\left[t+(1-t)\left(1+r_{H}\right)^{30}\right] /\left[\beta a_{M}+(1-\beta) a_{B}-\beta t a_{l}\right]$. When the mortgage is constrained by income $\left(H>\alpha Y_{0} / \beta\right.$ and $M=\alpha Y_{0}$ ), the housing RoR is $\operatorname{RoR}_{H 0}=t(1-t)\left(1+r_{H}\right)^{30}$. In the example, when $r_{H}=0.02$ and $\beta=0.7$, the RoR are $\mathrm{RoR}_{H}=2.88$ and $\mathrm{RoR}_{H 0}=1.57$. The ranking of $\mathrm{RoR}_{H}$ relative to $\operatorname{RoR}_{Q}$ and $\mathrm{RoR}_{S}$ is quite sensitive to the tax rate, the expected rate of capital gains less maintenance for housing, and $\lambda$. Note that unconditional employer contributions $Q^{\#}$ to a $401(\mathrm{k})$ plan are equivalent to a shift in retirement income.

Because the various instruments for saving or borrowing have different RoR and restrictions, these equations in each of the two conditions $D=$ 0 and $D=1$ define a convex budget set with piecewise linear boundaries. The overall budget set is then the possibly nonconvex union of these two conditional budget sets. The rational consumer will seek the efficient boundary of the overall budget set, and will always save at the highest available post-tax RoR and borrow at the lowest available post-tax RoR. When the best available borrowing RoR exceeds the best available saving RoR, the rational consumer will gain by reducing borrowing and saving dollar for dollar until one is zero. This gives the Fisherian exclusionary rule that a rational consumer will never a borrower and lender be, except possibly for investment channels like housing that lever the purchase with mortgage borrowing, and give the combination favorable tax treatment. The optimal intertemporal allocation for the rational consumer will be at a node, or on a line segment between nodes at which only one intertemporal channel is not constrained at a boundary. Thus, consumers who maximize utility with
positive unsecured borrowing will have zero conventional saving, and will not respond to incremental new 401(k) savings channels whose RoR is lower than their effective RoR on debt. It is more efficient for them to pay down debt if an incremental change in tastes or income endowment leads them to want to postpone some consumption. Consumers who have positive conventional savings at their utility maximum will enroll in a 401(k) plan when it becomes available, and this portfolio shift will reduce conventional saving. It is also possible that consumers who have invested in owner-occupied housing beyond the level necessary for basic housing services will enroll in a 401(k) plan when it becomes available, and reduce their housing investment. Whether these offsets are partial or complete will depend on income and substitution effects. Only consumers at a node in the budget set where they are neither borrowers nor savers will have unambiguously higher savings after introduction of a $401(\mathrm{k})$ channel. In no case will the opt-in or opt-out default on a $401(\mathrm{k})$ plan influence the rational consumer's final position.

The clear-cut implications from the Fisherian exclusionary rule will be relaxed in reality when some instruments are used for liquidity to reduce transactions costs, and/or when instruments are risky and the benefits of diversification offset loss in expected return. However, it is difficult to avoid the implication that in general it is rational for borrowers facing high marginal RoR to pay down debt rather than carry both debt and voluntary 401(k) contributions. Then, 401(k) defaults that induce high participation rates may misdirect consumers who should instead be reducing debt, although the consumer who irrationally undersaves and does not offset induced taxdeferred saving with increased debt may benefit from the precommitment and be better off than with no savings at all.

## Rational Intertemporal Optimization

For further discussion within the simple Fisherian framework of savings behavior, and the income and substitution effects induced by $401(\mathrm{k})$ plans, it is useful to represent the budget set graphically. Figure 11C. 1 illustrates the overall budget set for typical values. ${ }^{1}$ The budget set for renters is the area southwest of the line segments through $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$. Node c corresponds to no tax-qualified or conventional savings and no borrowing. The segment c-d corresponds to unsecured borrowing to the maximum at d . The segment $\mathrm{b}-\mathrm{c}$ corresponds to $401(\mathrm{k})$ contributions to a maximum at b , and the segment $\mathrm{a}-\mathrm{b}$ corresponds to conventional saving. The budget set for owners is the area southwest of the line segments through e, $\mathrm{f}, \mathrm{g}, \mathrm{h}, \mathrm{i}$. Node h corresponds to a minimum house purchase with a maximum mortgage, and no other saving or borrowing. The segment h-i corresponds to unsecured borrowing

[^1]

Fig. 11C. 1 Fisherian budget set
to a maximum at i . The segment g - h corresponds to $401(\mathrm{k})$ contributions to a maximum at g . The segment $\mathrm{f}-\mathrm{g}$ corresponds to a consumer durable investment in housing, with maximum mortgage financing, to a maximum at f where the income constraint on a mortgage binds. The segment e-f corresponds to conventional saving. The union of the renter and owner budget sets is not convex - linear combinations of $b$ and $h$ are not attainable. Note that nodes a and i are interior to the budget set, as are various points reached by combinations of borrowing or saving at less than the best available RoR.

A population of consumers with varying rates of impatience will maximize preferences among the exposed nodes and line segments of the given overall budget set. The more impatient rent, and the more patient own. (Heterogeneity in wealth and the intertemporal distribution of income endowments will induce heterogeneity in overall budget sets, and this will also contribute to heterogeneity in optimal choices.) In the example in the figures, consumers who optimize as renters at the node c or on the segment $\mathrm{c}-\mathrm{d}$, or optimize as owners at $h$ or on the exposed part of the line segment $h-i$, will not contribute voluntarily to a $401(\mathrm{k})$ plan. Note that these conclusions are quite sensitive to the relative position of the node $h$ and the line segment $a-b$; increases in the $401(\mathrm{k})$ employer match rate or a higher contribution ceiling $Q^{*}$, or an increase in $r_{S}$, may shift h to the southwest of the a-b segment so that it is no longer efficient to be a homeowner without savings.

Now consider the impact of changes in features of the $401(\mathrm{k})$ plan. First, rational consumers treat an increase in an unconditional employer contribu-
tion to a $401(\mathrm{k})$ plan as a component of retirement income, with no substitution effect on discretionary saving. In the usual case that current and future consumption are both normal goods, the income effect of an increased 401(k) unconditional contribution will reduce period 0 voluntary savings, but increase total savings and period 1 consumption. Second, increasing the employer match rate on voluntary $401(\mathrm{k})$ contributions increases $\mathrm{RoR}_{Q}$, or the slopes of the a-h and b-c segments in the figures. The substitution effect will increase contribution rates of consumers whose optima are on the g -h or b-c line segments, and may induce participation of some consumers located at the nodes $h$ and $b$. The income effect in the normal goods case will offset some of the increased savings induced by the substitution effect. If the increase in the matching rate moves the budget sets around in a way that makes some nodes without $401(\mathrm{k})$ contributions inefficient, there could be a nonmarginal 401(k) participation response to a marginal increase in the matching rate.

The nature of heterogeneity of tastes and endowments among rational consumers, as well as tax rates and RoR, will determine the overall population impact of changes in $401(\mathrm{k})$ features. The example in the figures does not necessarily describe the dominant effect. However, the example serves to illustrate the point that universal participation in 401(k) plans, even with significant matching, is not necessarily rational, and that the overall impact of $401(\mathrm{k})$ plan parameters must be assessed in terms of overall life cycle consequences. The important next step after showing that consumers adjust only partially from defaults is to establish that it is second-best welfaremaximizing to select defaults that increase participation. Even if this step is true for most, it is a significant research problem to define welfare when consumer behavior does not reveal consistent preferences, to determine who the winners are when a policy changes defaults, and to determine how defaults should be targeted to consumer segments to approximate a secondbest welfare maximum.

## Conclusions

I conclude with some observations on sources and consequences of behavioral inconsistencies, their impact on "happiness" broadly defined, and the ease with which these errors can be controlled to increase consumer welfare through defaults, framing, and education. There seem to be two major categories of deviations from life cycle rationality. First, there are errors that arise from lack of attention, procrastination, faulty perceptions, and careless optimization. These are "mechanical" errors in preference maximization that may either be overcome through consumer education, or finessed through framing and defaults. The consumer may be aware of these shortcomings in her decision process, and recognize that defaults, precommitments, or other interventions facilitate her decisions and increase her happiness. In the case of $401(\mathrm{k})$ choices, the use of enrollment windows,
penalties for delay, and rewards for immediate response may focus attention and discourage procrastination. Consumers are also more likely to attend to employer unconditional contributions or matching if they are marketed as attention-getting "bargains" for the consumer, or framed to deflect attention from current cost.

Second, there are errors that arise from mistakes in anticipating the consumer's own tastes, particularly time-inconsistent impatience, asymmetric loss aversion, and genuine instabilities in tastes, including phenomena such as physical or emotional health-linked tastes that cannot be anticipated from the consumer's current state. Here, intervention is more difficult, both because the consumer may resist changes that appear to reduce current utility, and because in the absence of stable preferences it is difficult to make unambiguous judgments about consumer welfare. Further, interventions to encourage saving have to counter the active, sharp marketing practices of financial institutions that profit from exploiting consumers' intertemporal irrationality and drive for instant gratification through overselling of credit cards, subprime mortgages, and equity loans. Laboratory lottery experiments in which both the utility and the probability of payoffs are ambiguous seem to offer the best possibility of testing consumer acceptance of information or procedures that reduce ambiguity, encourage strategic planning, or build resistence to marketing appeals for instant gratification.

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[^1]:    1. Numerical values are $Y_{0}=150, Y_{1}=50, H^{*}=200, Q^{\#}=4, Q^{*}=4, t=0.3, \alpha=2, \beta=$ $0.7, \gamma=1$, and $\lambda=1$. The interest rates are $r_{B}=0.1, r_{M}=0.05, r_{S}=0.03$, and $r_{H}=0.02$.
