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# External Debt, Adjustment, and Growth

Delano P. Villanueva and Roberto S. Mariano

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## 6.1 Introduction

High ratios of external debt to GDP in selected Asian countries have contributed to the initiation, propagation, and severity of the financial and economic crises in recent years, reflecting runaway fiscal deficits and excessive foreign borrowing by the private sector. More importantly, the servicing of large debt stocks has diverted scarce resources from investment and long-term growth. Applying and calibrating the formal framework proposed by Mariano and Villanueva (2005) to Philippine data, we explore the joint dynamics of external debt, capital accumulation, and growth. The relative simplicity of the model makes it convenient to analyze the links between domestic adjustment policies, foreign borrowing, and growth. We estimate the optimal domestic saving rate that is consistent with maximum real consumption per unit of effective labor in the long run. As a by-

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product, we estimate the steady-state ratio of net external debt to GDP that is associated with this optimal outcome.

The framework is an extension of the standard neoclassical growth model that incorporates endogenous technical change and global capital markets. The steady-state ratio of the stock of net external debt to GDP is derived as a function of the real world interest rate, the spread and its responsiveness to the external debt burden and market perception of country risk, the propensity to save out of gross national disposable income, rates of technical change, and parameters of the production function.

Being concerned primarily with the long-run interaction between external debt, growth, and adjustment, our nonstochastic paper is not about solvency or liquidity per se. However, a continuous increase in the foreign debt to GDP ratio will, sooner or later, lead to liquidity and, ultimately, solvency problems. Steady-state ratios of external debt to GDP belong to the set of indicators proposed by Roubini (2001, 6): “. . . a non-increasing foreign debt to GDP ratio is seen as a practical sufficient condition for sustainability: a country is likely to remain solvent as long as the ratio is not growing.” Cash-flow problems, inherent in liquidity crises, also emerge from an inordinately large debt ratio that results from an unabated increase over time. In our proposed analytical framework, we allow debt accumulation beyond the economy's steady-state growth rate as long as the expected net marginal product of capital exceeds the effective real interest rate in global capital markets. When the return-cost differential disappears, net external debt grows at the steady-state growth of GDP, and the debt ratio stabilizes at a constant level, a function of structural parameters specific to a particular country. Among all such steady state debt ratios, we estimate an optimal debt ratio that is associated with the value of the domestic saving rate that maximizes consumer welfare.

The main results of the extended model are as follows.

1. The optimal domestic saving rate is a fraction of the income share of capital (the standard result is that the optimal saving rate is equal to capital's income share).

2. Associated with the optimal saving rate and maximum welfare is a unique steady-state net foreign debt to GDP ratio.

3. The major policy implications are that fiscal consolidation and the promotion of private saving are critical, while overreliance on foreign saving (net external borrowing) should be avoided, particularly in an environment of high cost of external borrowing that is positively correlated with rising external debt.

4. For debtor countries facing credit rationing in view of prohibitive risk spreads, even at high-expected marginal product of capital and low risk-free interest rates, increased donor aid targeted at expenditures on education, health, and other labor-productivity enhancing expenditures

would relax the external debt and financing constraints while boosting per capita GDP growth.

The plan of this paper is as follows. Section 6.2 describes the structure of our open-economy growth model with endogenous technical change. We begin with a brief review of the relevant literature and incorporate some refinements to the closed-economy model. First, Gross National Disposable Income (GNDI) instead of GDP is used, since net interest payments on the net external debt use part of GDP, while positive net transfers add to GDP, leaving GNDI as a more relevant variable in determining domestic saving.<sup>1</sup> Second, the marginal real cost of external borrowing is the sum of the risk-free interest rate and a risk premium, which is an increasing function of the ratio of the stock of net external debt to the capital stock. That is to say, *inter alia*, as the proportion of external debt rises, the risk premium goes up, and so does the effective cost of external borrowing, even with an unchanged risk-free interest rate. Third, via enhanced learning by doing, technical change is made partly endogenous.<sup>2</sup> On the balanced growth path, we then derive the optimal value of the domestic saving rate that maximizes the steady-state level of real consumption per unit of effective labor. Section 6.3 applies the optimal growth framework to the Philippines. Section 6.4 draws some implications for fiscal policy and external debt management. Section 6.5 concludes.

## 6.2 The Formal Framework

### 6.2.1 Brief Survey of the Literature

The Solow-Swan (1956) model has been the workhorse of standard neoclassical growth theory. It is a closed-economy growth model in which exclusively domestic saving finances aggregate investment. In addition, the standard model assumes that labor-augmenting technical change is exogenous, which determines the equilibrium growth of per capita output.

There have been two developments in aggregate growth theory since the Solow-Swan (1956) model appeared. First, technical change was made partly endogenous and partly exogenous. Conlisk (1967) was the first to introduce endogenous technical change into a closed-economy neoclassical growth model, in which the saving rate was assumed fixed. This was followed by the recent endogenous growth literature using endogenously and optimally derived saving-rate models (Romer 1986, Lucas 1988, Becker, Murphy, and Tamara 1990, Grossman and Helpman 1990, and Rivera-Batiz and Romer 1991, among others). Among all classes of closed-economy

1. In the Philippines, workers' remittances included in private transfers average \$7 to \$8 billion per year, or some 12 percent of GDP.

2. See Villanueva (1994).

growth models, the steady-state properties of fixed (Villanueva 1994) and optimally derived saving-rate models are the same.<sup>3</sup>

The second development was to open up the Conlisk (1967) model to the global capital markets. An early attempt was made by Otani and Villanueva (1989), followed by Agénor (2000) and Mariano and Villanueva (2005). The fixed saving-rate models of Otani and Villanueva (1989) and Mariano and Villanueva (2005) are variants of Conlisk's (1967) endogenous-technical change model and Arrow's (1962) *learning-by-doing* model, wherein experience (measured in terms of either output or cumulative past investment) plays a critical role in raising productivity over time.

In Mariano and Villanueva (2005), the aggregate capital stock is the accumulated sum of domestic saving and net external borrowing (the current account deficit). At any moment of time, the difference between the expected marginal product of capital, net of depreciation, and the marginal cost of funds<sup>4</sup> in the international capital market determines the proportionate rate of change in the external debt-capital ratio. When the expected net marginal product of capital matches the marginal cost of funds at the equilibrium capital-labor ratio, the proportionate increase in net external debt (net external borrowing) is fixed by the economy's steady-state output growth, and the external debt/output ratio stabilizes at a constant level. Although constant in long-run equilibrium, the steady-state external debt ratio shifts with changes in the economy's propensity to save out of national disposable income, the marginal cost of funds in world capital markets, the depreciation rate, the growth rates of the working population and any exogenous technical change, and the parameters of the risk-premium, production, and technical change functions.

The major shortcoming of the Mariano and Villanueva (2005) model is its inability to pin down the steady-state external debt ratio that is consistent with maximum consumer welfare. We correct this shortcoming in the present paper. On the balanced growth path, if consumption per unit of effective labor (or any monotonically increasing function of it) is taken as a measure of the social welfare of society, we choose the domestic saving rate that maximizes social welfare by maximizing long-run consumption per unit of effective labor. Consistent with this optimal outcome is a steady-state ratio of net external debt to total output. Using parameters for the Philippines to calibrate the extended model, we show that it is locally stable, with a steady-state solution characterized by a constant capital/

3. Lucas (1988) specifies the effective labor  $L = hN$ , where  $h$  is human capital per head, and  $N$  is working population. His  $h$  variable is our variable  $A$  in  $L = AN$  (the variable is  $T$  in Otani and Villanueva [1989] and Villanueva [1994]; see equation (12) of our present paper), interpreted as a labor-augmenting technology or labor productivity multiplier.

4. Risk-free interest rate plus a risk premium. The LIBOR, U.S. Prime Rate, U.S. Federal Funds Rate, or U.S. Treasury, deflated by changes in an appropriate price index in the United Kingdom or United States, typically represents the risk-free interest rate. The risk premium is country specific and a positive function of a country's external debt burden and other exogenous factors capturing market perceptions of country risk.

effective labor ratio, an optimal domestic saving rate, and a unique external debt/capital ratio.<sup>5</sup> The latter interacts with long-run growth and domestic adjustment and is determined jointly with other macroeconomic variables, including a country's set of structural parameters.

### 6.2.2 The Extended Model

Our model can be summarized as follows:<sup>6</sup>

- (1)  $Y = Lk^\alpha$  (GDP)<sup>7</sup>
- (2)  $GNDI = Y - NFP + NTR$  (GNDI)
- (3)  $CAD = S^f = C + I - GNDI$  (Current account deficit)
- (4)  $C = cGNDI$  (Consumption function)
- (5)  $NFP = rD$  (Net factor payments)
- (6)  $NTR = \tau Y$  (Net transfers)
- (7)  $\dot{D} = CAD$  (Net debt issue)
- (8)  $d = \frac{D}{K}$  (Debt-capital ratio)
- (9)  $\frac{\dot{d}}{d} = \alpha k^{\alpha-1} - \delta - r$  (External borrowing function)
- (10)  $r^e = r^f + \phi d$  (Effective interest rate)
- (11)  $\dot{K} = I - \delta K$  (Capital growth)
- (12)  $L = AN$  (Effective labor)
- (13)  $\dot{N} = nN$  (Working population growth)
- (14)  $\dot{A} = \theta \left( \frac{K}{N} \right) + \lambda A$  (Technical change function)

5. For empirical external debt research using various statistical techniques, see Manasse, Roubini, and Schimmelpfennig (2003); Reinhart, Rogoff, and Savastano (2003); Kraay and Nehru (2004); Patillo, Poirson, and Ricci (2004); and Manasse and Roubini (2005). For a survey, see Kraay and Nehru (2004).

6. The numeraire is the foreign price of the investment good. Thus,  $P^d/eP^f$  is multiplied by residents' saving (in constant dollars), where  $P^d$  is the price of domestic output,  $e$  is the exchange rate in quantity of local currency units per unit of foreign currency, and  $P^f$  is the price of the investment good in foreign currency. Foreign saving denominated in foreign currency is deflated by  $P^f$  to get the real value. Similarly, the marginal real cost of external borrowing is the sum of the world interest rate and risk premium in foreign currency less the rate of change in  $P^f$ . Since model simplicity is our primary concern, we abstract from the effects of movements of these variables by arbitrarily assigning unitary values to these price and exchange rate indices without loss of generality. Incorporation of these variables in the extended model is straightforward and is done in Otani and Villanueva (1989).

7. Any production function will do, as long as it is subject to constant returns to scale. See Inada (1963).

$$(15) \quad k = \frac{K}{L} \quad (\text{Capital/effective labor ratio})$$

Here,  $Y$  is real GDP;  $K$  is physical capital stock;  $L$  is effective labor (in efficiency units, man-hours or man-days);  $A$  is labor-augmenting technology (index number);  $N$  is working population;  $k$  is the capital/effective labor ratio; GNDI is gross national disposable income; NFP is net factor payments; NTR is net transfers; CAD is external current account deficit;  $S^f$  is saving by nonresidents;  $C$  is aggregate consumption;  $I$  is gross domestic investment;  $D$  is net external debt<sup>8</sup>;  $d$  is the net external debt/capital ratio;  $r$  is the marginal real cost of net external borrowing;  $r^e$  is the effective world interest rate;  $r^f$  is the risk-free interest rate;  $\tau$ ,  $\delta$ ,  $\phi$ ,  $n$ ,  $\lambda$ , and  $\alpha$  are positive constants; and  $\theta$  is the learning coefficient, as in Villanueva (1994). In a closed economy (when  $D = 0$ ,  $S^f = 0$ ) with technical change partly endogenous ( $\theta > 0$ ), the model reduces to the Villanueva (1994) model; additionally, if technical change is completely exogenous ( $\theta = 0$ ), the model reduces to the standard neoclassical (Solow-Swan) model.

Consumption in the extended model reflects the openness of the economy—consumption is gross national disposable income plus foreign saving less aggregate investment.<sup>9</sup> Here,  $s = 1 - c$  is the propensity to save out of gross national disposable income. After we solve for the balanced growth path, we choose a particular value of  $s$  that maximizes social welfare (long-run consumption per unit of  $L$ ).<sup>10</sup>

The transfers/grants parameter  $\tau$  may be allowed to vary positively with the domestic savings effort  $s$ . Donors are likely to step up their aid to countries with strong adjustment efforts. Finally, donor aid  $\tau$  earmarked for education, health, and other labor-productivity enhancing expenditures is expected to boost the learning coefficient  $\theta$ .

Foreign saving is equivalent to the external current account deficit, which is equal to the excess of domestic absorption over national income or, equivalently, to net external borrowing (capital plus overall balance in the balance of payments)—noted in equations (3) and (7).

The derivation of the effective cost,  $r$ , of net external debt,  $D (= D^{\text{gross}} - A)$  is as follows: Assume a linear function for the effective interest rate  $r^e = r^f + \phi d$ , where  $0 < \phi < 1$  (equation 10); the second term is the spread that is increasing in  $d$ .<sup>11</sup> Net interest payments on net external debt

8.  $D$  is defined as external liabilities minus external assets; as such, it is positive, zero, or negative as external liabilities exceed, equal, or fall short of, external assets.

9. From the national income identity (3).

10. Thus, the saving ratio  $s = 1 - c$  will be chosen endogenously.

11. An increase in  $d$  raises the credit risk and thus the spread. The parameter  $\phi$  is likely to be negatively correlated with the domestic saving effort. Countries with high domestic saving rates appear to enjoy low spreads (for given debt ratios) because of the quality of their adjustment policies—good fiscal policy, conservative monetary policy, and the like.

$$\begin{aligned}
 &= r^e D^{\text{gross}} - r^f A, \\
 &= (r^f + \phi d) D^{\text{gross}} - r^f A \\
 &= r^f (D^{\text{gross}} - A) + \phi d D^{\text{gross}} \\
 &= r^f (D^{\text{gross}} - A) + \phi \left( \frac{D}{K} \right) D^{\text{gross}} \\
 &= r^f D + \phi \left( \frac{D}{K} \right) D^{\text{gross}}
 \end{aligned}$$

Dividing both sides by  $D$ ,

$$(16) \quad r = r^f + \frac{\phi D^{\text{gross}}}{K} = r^f + \frac{\phi(D + A)}{K} = r^f + \phi d + \phi \left( \frac{A}{Y} \right) k^{(\alpha-1)}$$

$$r = r^f + \phi [d + \varepsilon k^{(\alpha-1)}].$$

where  $D^{\text{gross}}$  is gross external liabilities,  $A$  is gross external assets,  $D = D^{\text{gross}} - A$ , and  $A/Y = \varepsilon$ . Assume that the gross external assets,  $A$ , are a constant (minimum) fraction of GDP:  $A = \varepsilon Y$ .<sup>12</sup> In the case of the Philippines,  $\varepsilon = 0.214$  at present.

The optimal decision rule for net external borrowing is specified in equation (9)—at any moment of time, net external borrowing as percent of the total outstanding net stock of debt is undertaken at a rate equal to the growth rate of the capital stock plus the difference between the expected marginal product, net of depreciation, and the marginal real cost of funds,  $r$ .<sup>13</sup> A more general law of motion for external capital is:

$$(9') \quad \frac{\dot{d}}{d} = \beta [\alpha k^{(\alpha-1)} - \delta - r],$$

where  $\beta > 0$  measures the speed of adjustment of external capital to the discrepancy between capital's expected net marginal product and the world real interest rate. In his discussion of the Villanueva (1994) model in the context of open global capital markets, Agénor (2000, 594) obtains the key result that the steady-state values of capital intensity and the debt-to-capital ratio are locally stable if and only if the *adjustment speed of external capital is sufficiently large* (Appendix A of the present paper

12. A rule of thumb is that the variable  $A$  represents three to four months of imports.

13. Equations (7) and (9) equate net foreign saving with net foreign borrowing, which is not strictly true. Net foreign saving (sum of capital, financial, and overall accounts in the balance of payments) includes debt (bonds and loans) and nondebt-creating flows (equities and foreign direct investment); both flows use up a portion of GDP, with the latter as dividends and profit remittances abroad. Our variables  $D$  and  $r$ , respectively, should be interpreted broadly to include equities and foreign direct investment, as well as dividends and profit remittances.



demonstrates local stability of long-run equilibrium), that is,  $\beta > ([\eta + s\{1 - \eta\}]/[1 - \alpha])d_0$ , where  $\eta$  is the ratio of tax revenue to national income,  $s$  is the ratio of domestic saving to national income,  $\alpha$  is the elasticity of output with respect to the capital stock, and  $d_0$  is the initial debt-to-capital ratio.<sup>14</sup>

When the expected yield-cost differential is zero and  $k$  is at its steady-state value  $k^{*15}$ , the net external debt as ratio to output stabilizes at a constant level.<sup>16</sup> However, this constant debt level may not necessarily be optimal in the sense of being associated with maximum consumer welfare. For it to be so, it has to be associated with a particular value of the domestic saving rate that maximizes long-run consumption per effective labor.

### 6.2.3 Reduced Model

By successive substitutions, the extended model reduces to a system of two differential equations in  $k$  and  $d$ .<sup>17</sup>

$$(17) \quad \frac{\dot{k}}{k} = \left\{ \frac{s[(1 + \tau)k^{\alpha-1} - rd]}{(1 - d)} \right\} + \left[ \frac{(\alpha k^{\alpha-1} - \delta - r)d}{(1 - d)} \right] - \left[ \frac{\delta}{(1 - d)} \right] - \theta k - n - \lambda$$

$$= H(k, d)$$

$$(18) \quad \frac{\dot{d}}{d} = \alpha k^{\alpha-1} - \delta - r = J(k, d)$$

where  $r$  is a function of  $k$  and  $d$ —given by equation (16).

Long-run equilibrium is obtained by setting the reduced system (17) and (18) to zero, such that  $k$  is constant at  $k^*$  and  $d$  is constant at  $d^*$ . It is characterized by balanced growth:  $K$ ,  $L$ , and  $D$  grow at the same rate  $\theta k^* + n + \lambda$ . It also implies the condition  $\alpha k^{*\alpha-1} - \delta - r(d^*, k^*; r^f) = 0$ , which is the optimal rule for external net borrowing to cease at the margin.<sup>18</sup>

14. In the application of our framework to the Philippines, fully described in section 6.3, using  $s = 0.188$ ,  $\eta = 0.186$ ,  $d_0 = 0.13$  (historical averages from 1993 to 1998), and  $\alpha = 0.4$ , the adjustment speed  $\beta$  should be at least 0.073. Our assumed unitary value is much larger than this minimum.

15. An asterisk denotes steady-state value of any variable.

16. The steady-state current account balance may be positive (deficit), zero (in balance), or negative (surplus). This follows from the steady-state solution  $(\dot{D}/Y)^* = g^* d^*/k^{*\alpha-1}$ , where  $g^*$  is the steady-state growth rate of output,  $d^*$  is the steady-state debt-capital ratio, and  $k^*$  is the steady-state capital-effective labor ratio. As mentioned in footnote 8 and defined by equation (8), the variable  $d^*$  is the ratio of net external debt (external liabilities minus external assets) to the capital stock and can be positive, zero, or negative. More precisely,  $-1 < d^* < 1$ , depending on whether the accumulated sum of domestic savings is less than, equal to, or greater than the aggregate capital stock (accumulated sum of aggregate investments).

17. It can be seen that in a closed economy,  $d = \tau = 0$ , equation (18) drops out and, thus, equation (17) is identical to the Villanueva (1994) model (equation 9, p. 7). Further, with  $\theta = 0$ , equation (17) reduces to the Solow-Swan model.

18. When the yield-cost differential is zero, net external borrowing as percent of the outstanding net stock of debt proceeds at the steady-state growth rate of output.

The steady-state solutions for  $k^*$ ,  $d^*$ , and  $r^*$  are:

$$(19) \quad \left\{ \frac{s[(1 + \tau)k^{*(\alpha-1)} - r^*d^*]}{(1 - d^*)} \right\} - \frac{\delta}{(1 - d^*)} - \theta k^* - n - \lambda = 0$$

$$(20) \quad d^* = \frac{[(\alpha - \phi\epsilon)K^{*(\alpha-1)} - \delta - r^f]}{\phi}$$

$$(16') \quad r^* = r^f + \phi d^* + \phi\epsilon k^{*(\alpha-1)}$$

Long-run equilibrium is defined by point  $Q(d^*, k^*)$  in figure 6.1.<sup>19</sup> In regions  $N$  and  $W$ , the dynamics force  $d$  to increase, and in regions  $S$  and  $E$ , the dynamics force  $d$  to decrease. In regions  $N$  and  $E$ , the dynamics force  $k$  to decrease, and, in regions  $S$  and  $W$ , the dynamics force  $k$  to increase. Any initial point, like point  $A$ , leads to a movement toward the equilibrium point,  $Q$ , with a possible time path indicated by the line  $AQ$ .

### 6.2.4 Restrictions on External Financing

Using the definition of  $d$  (equation [8]), the law of motion for external capital as specified in equation (9) can be restated as:

$$(9') \quad \frac{\dot{D}}{D} = \frac{\dot{K}}{K} + \alpha k^{(\alpha-1)} - \delta - r.$$

Using the definition of  $k$  (equation [15]) and substituting equations (12) through (14) into equation (17), we obtain  $\dot{K}/K = \theta k + n + \lambda + H(k, d)$ . Substituting this into equation (9'),

$$(9'') \quad \frac{\dot{D}}{D} = \theta k + n + \lambda + H(k, d) + \alpha k^{(\alpha-1)} - \delta - r, \text{ where } r = r(d, k; r^f).$$

Equation (9'') says that at any moment of time, the amounts of external financing vary with levels of  $k$  and  $d$ . In long-run equilibrium (i.e., in the steady state), external financing as percent of the debt stock is equal to the GDP growth  $g^*$  (noting that  $H[k, d] = 0$  and  $\alpha k^{(\alpha-1)} - \delta - r = 0$ , and that, by the assumption of constant returns, GDP grows at the same rate as capital and effective labor),

$$(9''') \quad \left( \frac{\dot{D}}{D} \right)^* = \left( \frac{\dot{K}}{K} \right)^* = \left( \frac{\dot{Y}}{Y} \right)^* = g^*(k^*, d^*)$$

In other words, in the short run, there are no limits on the absolute level of the debt stock or on its increment. External financing is ruled in the short run by equation (9'') (a function of  $k$  and  $d$ , given  $r^f$ ). In the long run, external debt grows at the same rate as GDP, given by equation (9''') (a function of  $k^*$  and  $d^*$ ).

In figure 6.1, the speed of adjustment to long-run equilibrium (charac-

19. For the derivation of the slopes of the curves shown in figure 6.1, see appendix A.

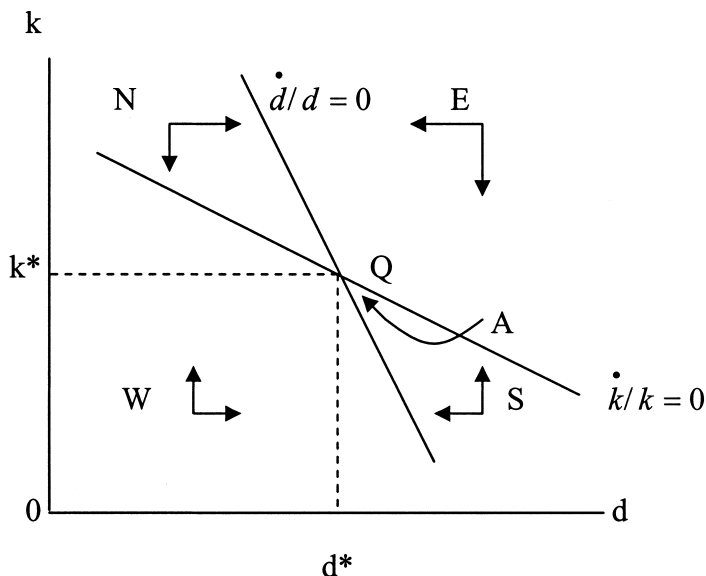


Fig. 6.1 Phase diagram of the extended model

terized by, among other conditions, a zero return-cost differential in global capital markets) varies with the initial values of  $k$  and  $d$ . In growth modeling, the speed of adjustment of state variables like  $d$  and  $k$  usually refers to the number of periods (e.g., years) it takes for the variables to adjust to their long-run equilibrium values. In figure 6.1, an adjustment trajectory may be that, if initially,  $d < d^*$ , but  $k > k^*$ ,  $d$  increases at first slowly, then accelerates toward  $d^*$ , aided by  $k$  falling to  $k^*$  and thus raising capital's net marginal product, which in turn induces higher capital inflows. Or, if initially both  $d < d^*$  and  $k < k^*$ ,  $d$  monotonically increases toward  $d^*$  (as does  $k$  toward  $k^*$ ), with more or less the same speed each period. There are other trajectory paths with different adjustment speeds in the north, east, and south quadrants of figure 6.1. Adjustment speeds of our model have been numerically solved by Chen (2006), confirming different adjustment speeds depending on the initial values of output growth (a function of initial values of  $k$  and  $d$ ), and on whether the adjustment starts from above or below  $g^*$ . A finite albeit slow adjustment of external capital is what is observed empirically, both domestically and especially for foreign capital.

### 6.2.5 Optimal Growth

In the steady state, output per unit of effective labor is  $y^* = k^{*\alpha}$ . If  $y^*$  is considered a measure of the standard of living, and since  $dy^*/dk^* > 0$ , it is possible to raise living standards by increasing  $k^*$ . This can be done by adjusting the domestic saving rate  $s$ , for example, by raising the public sector

saving rate and assuming imperfect Ricardian equivalence.<sup>20</sup> If consumption per unit of effective labor is taken as a measure of the social welfare of the society, the saving rate  $s$  that maximizes social welfare by maximizing long-run consumption can be determined. Phelps (1966) refers to this path as the *Golden Rule of Accumulation*.

From equation (3), steady-state consumption per unit of effective labor is

$$\begin{aligned} \left(\frac{C}{L}\right)^* &= \left(\frac{GNDI}{L}\right)^* + \left(\frac{S^f}{L}\right)^* - \left(\frac{I}{L}\right)^* = (1 + \tau)k^{*\alpha} - r^*d^*k^* \\ &+ \left\{ \left[ \frac{(dK/dt)}{K} \right]^* + \alpha k^{*[\alpha-1]} - \delta - r^* \right\} d^*k^* \\ &- \left\{ \left[ \frac{(dK/dt)}{K} \right]^* + \delta \right\} k^*, \end{aligned}$$

and since  $\alpha k^{*(\alpha-1)} - \delta - r^* = 0$ ,

$$= (1 + \tau)k^{*\alpha} - r^*d^*k^* - (1 - d^*)k^* \left[ \frac{(dK/dt)}{K} \right]^* - \delta k^*$$

where

$$r^* = r^f + \phi[d^* + \epsilon k^{*(\alpha-1)}].$$

Also in the steady state,

$$\left[ \frac{(dK/dt)}{K} \right]^* = \theta k^* + n + \lambda.$$

Thus,

$$(21) \quad \left(\frac{C}{L}\right)^* = (1 + \tau)k^{*\alpha} - r^*d^*k^* - (1 - d^*)(\theta k^* + n + \lambda)k^* - \delta k^*.$$

Maximizing  $(C/L)^*$  with respect to  $s$ , and noting that

$$d^* = \frac{[(\alpha - \phi\epsilon)K^{*(\alpha-1)} - \delta - r^f]}{\phi} \text{ and } r^* = r^f + \phi[d^* + \epsilon k^{*(\alpha-1)}],$$

$$\begin{aligned} (22) \quad \frac{\partial \left(\frac{C}{L}\right)^*}{\partial s} &= \left\{ (1 + \tau)\alpha k^{*(\alpha-1)} - r^*d^* - \delta - \left(\frac{1}{\phi}\right) \right. \\ &\cdot [(r^* + \phi d^*)(\alpha - \epsilon\phi)](\alpha - 1)k^{*(\alpha-1)} - (1 - d)\theta k^* \\ &- [2(\alpha - \epsilon\phi) + \epsilon](\alpha - 1)d^*k^{*(\alpha-1)} \\ &\left. - \left[ 1 - d^* - (\alpha - \epsilon\phi)(\alpha - 1)\left(\frac{1}{\phi}\right)k^{*(\alpha-1)} \right] g^* \right\} \frac{\partial k^*}{\partial s} = 0 \end{aligned}$$

20. There is ample empirical evidence that, at least for developing countries, the private sector saving rate does not offset one to one the increase in the public sector saving rate.

Since  $\partial k^*/\partial s > 0$ , the Golden Rule condition, is<sup>21</sup>

$$\begin{aligned} (1 + \tau)\alpha k^{*(\alpha-1)} - r^*d^* - \delta &= [1 - d^* - (\alpha - \varepsilon\phi)(\alpha - 1)\left(\frac{1}{\phi}\right)k^{*(\alpha-1)}]g^{**} \\ &+ [2(\alpha - \varepsilon\phi) + \varepsilon](\alpha - 1)d^*k^{*(\alpha-1)} \\ &+ \left(\frac{1}{\phi}\right)[(r^* + \phi d^*)(\alpha - \varepsilon\phi)](\alpha - 1)k^{*(\alpha-1)} \\ &+ (1 - d)\theta k^* \end{aligned}$$

or,

$$M = sUZ + V + W + X - \frac{U\delta}{(1 - d^*)},$$

or,

$$(23) \quad s = \frac{M - V - W - X + \frac{U\delta}{1 - d^*}}{UZ}.$$

where,

$$M = (1 + \tau)\alpha k^{*(\alpha-1)} - r^*d^* - \delta;$$

$$U = [1 - d^* - (\alpha - \varepsilon\phi)(\alpha - 1)\left(\frac{1}{\phi}\right)k^{*(\alpha-1)}];$$

$$V = [2(\alpha - \varepsilon\phi) + \varepsilon](\alpha - 1)d^*k^{*(\alpha-1)};$$

$$W = \left(\frac{1}{\phi}\right)[(r^* + \phi d^*)(\alpha - \varepsilon\phi)](\alpha - 1)k^{*(\alpha-1)};$$

$$X = (1 - d^*)\theta k^*; \text{ and}$$

$$Z = \frac{[(1 + \tau)k^{*(\alpha-1)} - r^*d^*]}{(1 - d^*)}.$$

$M$  is capital's net (of depreciation) marginal product less interest payments on the stock of net external debt. The first-order condition (22) says that for social welfare to be maximized the domestic saving rate should be raised to a point where the net return to capital is a multiple of the long-run growth rate of output.  $X$  is nothing more than the open-economy<sup>23</sup>

21. The second-order condition for a maximum,  $\partial^2(C/L)^*/\partial s^2 < 0$ , is a tediously long algebraic expression that, when evaluated at the steady-state values solved for the Philippines, yields a value of  $-0.01847$ . See last page of appendix A.

22. This expression for the optimal  $s$  uses the relation  $g^* = \theta k^* + n + \lambda = \{s[(1 + \tau)k^{(\alpha-1)} - r^*d^*] - \delta\}/(1 - d^*)$ , substituting it for  $g^*$  in equation (22).

23. Reduced by a factor  $(1 - d^*)$ . When  $d = 0$ , this term becomes  $\theta k^*$  in Villanueva (1994).

version of the endogenous component of labor-augmenting technical change—the component of  $(dA/dt)/A$  induced by learning that occurs at a higher level of capital intensity, which, in turn, is caused by a higher domestic saving rate. If there are no learning ( $\theta = 0$ ), no net external debt ( $d^* = 0$ ), and no net transfers ( $\tau = 0$ ), equation (22) reduces to  $\alpha k^{*(\alpha-1)} - \delta = \lambda + n$ , which is the familiar Golden Rule result from standard neoclassical growth theory (i.e., the optimal net rate of return to capital equals the natural growth rate). If there is learning ( $\theta > 0$ ) and there are no net external debt ( $d^* = 0$ ) and no net transfers ( $\tau = 0$ ), equation (22) reduces to the Villanueva (1994) Golden Rule result,  $\alpha k^{*(\alpha-1)} - \delta = g^* + \theta k^*$ , where  $g^* = \theta k^* + \lambda + n$ . The effect of opening up the economy to global capital and labor markets is to raise the optimal net rate of return to capital beyond  $\lambda + n$  or even beyond  $g^* + \theta k^*$  when  $\theta > 0$ —when there is learning by doing—because of four factors.

First, when the domestic saving rate  $s$  is raised, the equilibrium growth rate  $g^*$  will be higher than  $\lambda + n$ , by the amount of  $\theta \partial k^* / \partial s$ . Second, capital should be compensated for the effect on equilibrium output growth through the induced learning term  $\theta k^*$ . Third, when the domestic saving rate is raised, the equilibrium debt stock  $d^*$  will be lower, releasing resources toward more capital growth; the effective interest rate  $r^*$  also will be lower *pari passu* with a lower spread, further increasing domestic resources for investment and growth. Fourth, the availability of foreign saving to finance capital accumulation enhances long-run growth, up to a point.

An alternative interpretation of the previous Golden Rule can be given. A standard neoclassical result is that the optimal saving rate  $s$  should be set equal to the income share of capital,  $\alpha$ . To see this, set  $d^* = \tau = \theta = 0$ , and since  $\partial k^* / \partial s > 0$ , the Golden Rule condition is the standard relation

$$\alpha k^{*(\alpha-1)} - \delta = n + \lambda,$$

that is, the net (of depreciation) return to capital equals the steady-state natural growth rate. Since  $s k^{*(\alpha-1)} - \delta = n + \lambda$ ,

$$s k^{*(\alpha-1)} - \delta = \alpha k^{*(\alpha-1)} - \delta,$$

or,

$$s = \alpha.$$

If  $d^* = \tau = 0$  and technical change is partly endogenous ( $\theta > 0$ ), the modified Golden Rule is

$$\alpha k^{*(\alpha-1)} - \delta = g^* + \theta k^*,$$

which is Villanueva's result (1994).

Since  $s k^{*(\alpha-1)} - \delta = g^*$ ,

$$\alpha k^{*(\alpha-1)} = s k^{*(\alpha-1)} + \theta k^*,$$

or,

$$s = \alpha - \theta k^{*(2-\alpha)}.$$

In general, Villanueva (1994) shows that for any income share of capital  $\pi = k^* f'(k^*)/f(k^*)$ , where  $f(\cdot)$  is the intensive form of the production function,  $s = \sigma\pi$ , where the fraction  $\sigma = (g^* + \delta)/(g^* + \delta + \theta k^*)$ .<sup>24</sup> Here,  $g^* + \delta + \theta k^* = f'(k^*)$  is the gross social marginal product of capital, inclusive of the positive externalities arising from the learning associated with capital accumulation in an endogenous growth model. Equivalently put, income going to capital as a share of total output should be a multiple of the amount saved and invested in order to compensate capital for the additional output generated by endogenous growth and induced learning. A value of  $\pi$  equal to  $s$ , implicit in the standard model, would undercompensate capital and thus be suboptimal from a societal point of view.

The open economy's optimal domestic saving rate, given by equation (23), is higher than  $\alpha - \theta k^{*(2-\alpha)}$  given by Villanueva (1994), reflecting the inherent risks involved in foreign borrowing.<sup>25</sup>

In general, the existence, uniqueness, and stability of the steady-state equilibrium are not guaranteed. However, appendix A shows that for a Cobb-Douglas production function, linear *learning-by-doing* and risk-premium functions, and values of the parameters for the Philippines, the extended model's equilibrium is locally stable in the neighborhood of the steady state.<sup>26</sup>

### 6.3 Application to the Philippines

Developments in fiscal policies and in access to external sources of capital in East Asia (as elsewhere) often raise important external debt issues. This section presents an illustrative numerical example<sup>27</sup> using representative parameters for the Philippines:  $\alpha = 0.4$ ,  $\delta = 0.04$ ,  $\tau = 0.07$ ,  $n = 0.025$ ,  $\lambda = 0.02$ ,  $r^f = 0.03$ ,  $\phi = 0.41$ , and  $\theta = 0.005$ .<sup>28</sup> Using Microsoft Excel's "Goal Seek" tool, the solution values are  $d^* = 0.07$  and  $k^* = 6.8$ . Note that  $d^*$  and  $k^*$  are functions of  $s$  in the reduced model (equations 16, 17, and

24. For derivation, see Villanueva (1994, 7–17).

25. In the calibration of the model to Philippine data in the next subsection, the optimal  $s$  estimated for the open-economy and closed-economy versions are 0.34 and 0.30, respectively. Both values are less than the share of income going to capital, equal to 0.40, consistent with the result first shown by Villanueva (1994) in an endogenous growth model.

26. Outside the neighborhood of the steady state, multiple equilibria, jumps, and the like are theoretically possible.

27. The next section discusses the linkages between fiscal policy and the management of external debt in the Philippines. The discussion in this section is specific to the Philippine case and, in particular, does not cover a situation in which the state is part of a federal currency union.

28.  $\phi = \phi^* k^{*(1-\alpha)}$  where  $\phi^* = 0.13$  was estimated using averages of the ratio of changes in the risk spread to changes in the ratio of external debt to GDP. See appendix B.

18), while the (optimal)  $s$  is a function of  $d^*$  and  $k^*$  (equation 23). Therefore, iterations were performed to obtain a unique value for  $s$  that satisfies equations (16), (17), (18), and (23). The resulting optimal value of the domestic saving rate  $s = 0.3429$ .

Steady-state per capita GDP growth is 5.4 percent per year, of which the endogenous component is 3.4 percent per year.<sup>29</sup> The steady-state risk premium is 288 basis points, steady-state gross external debt is 43.5 percent of GDP, and steady-state net external debt/GDP ratio is 22.1 percent of GDP.<sup>30</sup> Steady-state interest payments are 2.6 percent of GDP. Using the relation,

$$\left(\frac{\dot{D}}{D}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \left(\frac{\dot{L}}{L}\right)^* = g^* = \left(\frac{\dot{Y}}{Y}\right)^* \text{ (in the steady state),}$$

or,

$$\begin{aligned} (24) \quad \left(\frac{\dot{D}}{Y}\right)^* &= \left(\frac{\dot{Y}}{Y}\right)^* \left(\frac{D}{Y}\right)^* \\ &= \frac{g^* d^*}{k^{*(\alpha-1)}} \\ &= \frac{(0.079)(0.07)}{(6.8)^{-0.6}} = 0.017; \end{aligned}$$

the steady-state external current account deficit is 1.7 percent of GDP.<sup>31</sup>

The previous calculations are based on a 0.13 average ratio of changes in spread to changes in the external debt/GDP ratio estimated over the period 2000 to 2003. Table 6.1 shows the sensitivity of the results to alternative values of this ratio.

The estimated optimal domestic saving rate, steady-state per capita GDP growth rate, and the number of years it would take for per capita GDP to double are robust to alternative values of the ratio of changes in spread to changes in the external debt/GDP ratio of 0.10 to 0.15. However, as expected, the steady-state gross external debt/GDP ratio declines from 53 percent to 39 percent, and the steady-state net external debt/GDP ratio from 31 percent to 17 percent, as the sensitivity of the spread to the debt ratio rises from 0.10 to 0.15.<sup>32</sup>

In the Philippines, optimal long-run growth requires raising the domes-

29. It would take about thirteen years for per capita GDP to double.

30.  $(D/Y)^* = d^*/k^{*(\alpha-1)}$ .

31. Recall that  $-1 < d^* < 1$ . When  $-1 < d^* < 0$ , equation (24) solves for the external current account surplus (e.g., Singapore). When  $d^* = 0$ , the long-run current account is in balance.

32. If the sensitivity parameter is 0.15, it means that a 1 percent increase in the net external debt/GDP ratio is associated with an increase of 15 basis points in the spread.



**Table 6.1** Sensitivity calculations

$\phi^* = \Delta \text{ Spread}/\Delta (\text{Debt}/\text{GDP})$	0.10	0.13	0.15
Optimal domestic saving rate	0.3519	0.3429	0.3372
Gross external debt/GDP	0.5274	0.4353	0.3883
Net external debt/GDP	0.3134	0.2213	0.1743
Per capita GDP growth	0.0558	0.0540	0.0532
Years to double per capita income	12.54	12.96	13.13

tic saving rate from the historical average of 18.8 percent of GNP (IMF 1999, table 5) during 1993 to 1998 to a steady state 34 percent over the long term. This is necessary to achieve external viability while maximizing long-run consumption per effective labor. The savings effort should center on fiscal consolidation and adoption of incentives to encourage private saving, including market-determined real interest rates. From the national income identity (3), the external current account deficit CAD is equal to the excess of aggregate investment  $I$  over domestic saving  $S$  ( $= \text{GNDI} - C$ ), or  $\text{CAD} = I - S$ . Decomposing  $I$  and  $S$  into their government and private components,  $\text{CAD} = (I_g - S_g) + (I_p - S_p)$ , where the subscripts  $g$  and  $p$  denote government and private, respectively. The first term is the fiscal balance, and the second term is the private-sector balance. Fiscal adjustment is measured in terms of policy changes in  $S_g$  (government revenue less consumption) and in  $I_g$  (government investment). Given estimates of the private sector saving-investment balance and its components, the optimal government saving-investment balance may be derived as a residual; from this, the required government-saving ratio can be calculated because the optimal growth model implies a government-investment ratio.

Assume, however, the following *hypothetical* worst-case scenario for the Philippines. For whatever reason (political, social, etc.), owing to the initial high level of the external debt, market perceptions reach a very high adverse level. Despite a high-expected marginal product of capital, the risk premium is prohibitively high at any level of the debt ratio and the risk-free interest rate, such that the Philippine public sector faces credit rationing.<sup>33</sup> In such circumstances, as Agénor (op. cit., 595–96) suggests, increased foreign aid targeted at investment broadly defined to include physical and human capital may benefit the Philippines, provided that economic policies are sound.

33. The credit risk is included in the risk premium. The higher is the credit risk assigned by international creditors/investors, the higher is the risk premium and, consequently, the higher is the effective real interest rate.

#### 6.4 Implications for Fiscal Policy and External Debt Management

The implications for fiscal policy and external debt management are clear for the Philippines. The first step is to launch an effective external debt-management strategy that will articulate the short- and long-run objectives of fiscal policy and debt management and ensure effective centralized approval and monitoring of primary debt issues to global financial markets, aided by (a) detailed electronic data on external debt, both outstanding and new debt, by borrowing institution, maturity, terms, and so on, and by (b) an interagency desk exclusively responsible for top quantitative and analytic work on external debt for the benefit of policy makers.

The level of external debt can be reduced only by cutting the fiscal deficit immediately and at a sustained pace over the medium term. In this context, the privatization of the National Power Corporation (NAPOCOR) is essential, since a big chunk of sovereign debt issues is on behalf of NAPOCOR.

Interest payments on total government debt currently eat up a significant share of government revenues, leaving revenue shortfalls to cover expenditures on the physical infrastructure and on the social sectors (health, education, and the like). With a successful and steady reduction of the stock of debt and the enhancement of domestic savings led by the government sector (via increases in  $S_g$ ), the sensitivity of the risk spread to the external debt would decrease, resulting in interest savings that would provide additional financing for the infrastructure and social sectors. Furthermore, there are clear implications for both revenue-raising and expenditure-cutting measures. On the revenue side, although the recently enacted and signed VAT bill is welcome, there remains low compliance on the VAT, resulting in very low collections. There is evidence of VAT sales being substantially underdeclared on a regular basis. Our concrete proposal would be to set up a computerized system of VAT sales wherein an electronic copy of the sales receipt is transmitted in real time by merchants, producers, and service providers to the Bureau of Internal Revenue (BIR). In this manner, total sales subject to the VAT submitted come tax time can be compared by the BIR against its own electronic receipts. It is estimated that if only 50 percent of total sales were collected from VAT, the current budget deficit (some P200+ billion) could be wiped out. This proposal easily beats current proposals to raise taxes because as they stand, marginal tax rates are already very high (resulting in tax evasion and bribes). The imposition of *sin* taxes (on cigarettes and liquor sales) would provide little relief. Individual and corporate tax reforms are also necessary—different tax brackets should be consolidated into a few, with significant reductions in marginal income tax rates; at the same time, the number of exemptions should be drastically reduced to widen the tax base. The whole customs tariffs structure should be reviewed with the aim of reducing average tariff rates

further, while eliminating many exemptions. The role of the customs assessor and collector should be severely restricted, with computerized assessment and collection being put in place, similar to our VAT proposal.

## 6.5 Conclusion

This paper has explored the joint dynamics of external debt, capital accumulation, and growth. In developing countries in East Asia and elsewhere, external debt issues are often associated with public policy decisions about fiscal policy. This has been especially relevant since the Asian financial crisis in the late 1990s. The relative simplicity of our model makes it convenient to analyze the links between domestic adjustment policies, foreign borrowing, and growth. We estimate the optimal domestic saving rate for the Philippines that is consistent with maximum real consumption per unit of effective labor in the long run. As a by-product, we estimate the steady-state ratio of net external debt to GDP that is associated with this optimal outcome. The framework is an extension of the standard neo-classical growth model that incorporates endogenous technical change and global capital markets. Utilizing this framework, the linkages between fiscal policy and external debt management are discussed in the context of a calibrated model for the Philippines. The major policy implications are that, in the long run, fiscal adjustment and the promotion of private saving are critical; reliance on foreign saving in a globalized financial world has limits; and when risk spreads are highly and positively correlated with rising external debt levels, unabated foreign borrowing depresses long-run welfare.

The obvious policy conclusions of the extended model are:

1. Fiscal consolidation and strong incentives for private saving are essential to achieving maximum per capita GDP growth;
2. The domestic saving rate should be set below the share of capital in total output, owing to positive externalities arising from learning by doing associated with capital accumulation. Equivalently put, income going to capital as a share of total output should be a multiple of the amount saved and invested in order to compensate capital for the additional output generated by endogenous growth and induced learning;
3. Reliance on foreign savings (external borrowing) has limits, particularly in a global environment of high interest rates and risk spreads;
4. When real borrowing costs are positively correlated with rising external indebtedness, the use of foreign savings is even more circumscribed; and
5. When risk spreads are prohibitively high despite high-expected marginal product of capital, there is a role for increased foreign aid earmarked for education and health, provided that economic policies are sound.

## Appendix A

### Stability Analysis

Partially differentiating text equations (16), (17), and (18) with respect to  $k$  and  $d$  and evaluating in the neighborhood of the steady state yield

$$(1) \quad a_{11} = H_k = [s(1 + \tau) - \varepsilon\phi d^*] \left[ \frac{1}{(1 - d^*)} \right] (\alpha - 1) k^{*\alpha-2} \\ + (\alpha - \varepsilon\phi) [(\alpha - 1) d^* k^{*\alpha-2}] \left[ \frac{1}{(1 - d^*)} \right] - \theta \\ = ?$$

$$(2) \quad a_{12} = H_d = -[s(1 - d^*)^{-2}] [(1 + \tau)(k^{*\alpha-1}) - r^* d^*] \\ - s(r^* + \phi d^*)(1 - d^*)^{-1} - \phi d^*(1 - d^*)^{-1} + \delta(1 - d^*)^{-2} \\ = ?$$

$$(3) \quad a_{21} = J_k = (\alpha - \varepsilon\phi)(\alpha - 1) k^{*\alpha-2} = ?$$

$$(4) \quad a_{22} = J_d = -\phi < 0$$

In the steady state, text equations (19) and (20) are equated to zero:

$$(5) \quad H(k, d) = 0$$

$$(6) \quad J(k, d) = 0$$

Totally differentiating (5) and (6) with respect to  $k$  and  $d$  yields,

$$(7) \quad \frac{H_k dk}{dd} + H_d = 0$$

$$(8) \quad \frac{J_k dk}{dd} + J_d = 0$$

The slope of the  $\dot{k}/k = 0$  curve is given by:

$$(9) \quad \frac{dk}{dd} \bigg|_{\dot{k}/k = 0} = 0 = -\frac{H_d}{H_k} = -\frac{a_{12}}{a_{11}} = ?$$

The slope of the  $\dot{d}/d = 0$  curve is given by:

$$(10) \quad \frac{dk}{dd} \bigg|_{\dot{d}/d = 0} = 0 = -\frac{J_d}{J_k} = -\frac{a_{22}}{a_{21}} = ?$$

Let  $A$  be the matrix of partial derivatives defined by equations (1) through (4). For stability, a necessary and sufficient condition is that the

eigenvalues of  $A$  have negative real parts, and a necessary and sufficient condition for this is that:

$$(11) \quad \text{tr}(A) < 0,$$

and

$$(12) \quad |A| > 0.$$

Since the signs of equations (1) through (3) are ambiguous, both trace (11) and determinant (12) conditions are indeterminate. The trace condition is:

$$a_{11} + a_{22} < 0.$$

The determinant condition is:

$$a_{11}a_{22} - a_{12}a_{21} > 0.$$

Assuming values of parameters estimated for the Philippines and evaluating the matrix of partial derivatives in the neighborhood of the steady state,  $a_{11} = -0.4226$ ,  $a_{12} = -0.2516$ ,  $a_{21} = -0.0112$ , and  $a_{22} = -0.4107$ . Thus, the trace condition (11)  $a_{11} + a_{22} < 0$  is met. The determinant condition (12) is also met. The extended model's phase diagram shown in text figure 6.1 reflects these considerations.

The second-order condition for maximum consumption per unit of  $L$  is:

$$(13) \quad \partial^2 \left( \frac{C}{L} \right)^* \partial s = (1 + \tau)\alpha(\alpha - 1)k^{*(\alpha-2)} - r^* \left( \frac{dd^*}{dk^*} \right) \\ - d^* \left[ \left( \frac{\partial r^*}{\partial k^*} \right) + \left( \frac{\partial r^*}{\partial d^*} \right) \left( \frac{dd^*}{dk^*} \right) \right] - \left( \frac{1}{\phi} \right) [(r^* + \phi d^*)(\alpha - \phi \epsilon)] \\ \cdot [(\alpha - 1)k^{*(\alpha-2)}] - \left( \frac{1}{\phi} \right) (r^* + \phi d^*)(\alpha - \epsilon \phi)(\alpha - 1)(\alpha - 1)k^{*(\alpha-2)} \\ - \left( \frac{1}{\phi} \right) (\alpha - 1)k^{*(\alpha-1)} \left[ (\alpha - \epsilon \phi) \left( \frac{\partial r^*}{\partial k^*} + \frac{dd^*}{dk^*} \right) \right] - (1 - d^*)\theta \\ + \theta k^* \left( \frac{dd^*}{dk^*} \right) - [2(\alpha - \epsilon \phi) + \epsilon] \left[ d^*(\alpha - 1)k^{*(\alpha-2)} + k^{*(\alpha-1)} \left( \frac{dd^*}{dk^*} \right) \right] \\ - \left[ 1 - d^* - (\alpha - \epsilon \phi)(\alpha - 1) \left( \frac{1}{\phi} \right) k^{*(\alpha-1)} \right] \theta \\ + g^* \left[ \frac{dd^*}{dk^*} + (\alpha - \epsilon \phi)(\alpha - 1) \left( \frac{1}{\phi} \right) (\alpha - 1)k^{*(\alpha-2)} \right] < 0$$

where

$$\frac{dd^*}{dk^*} = (\alpha - \varepsilon\phi)(\alpha - 1)k^{*(\alpha-2)}\left(\frac{1}{\phi}\right)$$

$$\frac{\partial r^*}{\partial k^*} = \phi\varepsilon(\alpha - 1)k^{*(\alpha-2)}$$

$$\frac{\partial r^*}{\partial d^*} = \phi\left(\frac{\partial d^*}{\partial k^*}\right)$$

$$g^* = \theta k^* + \lambda + n$$

When evaluated at the steady state,  $\partial^2(C/L)^*\partial s = -0.01847 < 0$  and, thus, satisfies the second-order condition (13) for a maximum.

## Appendix B

### Data

#### Definitions

1. C: Deflated Consumption Expenditures
2. GNP: Deflated Gross National Product
3. GNDI: Deflated Gross National Disposable Income
4. CAB: Deflated Current Account Balance
5. JACI: JPMorgan Asia Credit Index on Asian U.S. dollar denominated bonds, containing more than 110 bonds, using their dirty prices and weights according to respective market capitalization. It includes sovereign bonds, quasi-sovereign bonds, and corporate bonds from those countries.

#### Data Sources

1. JACI Spread: JP Morgan Markets
2. U.S. GDP Deflator: International Financial Statistics (IFS)
3. U.S. CPI for all urban consumers: U.S. Bureau of Labor Statistics (USBLS)
4. Philippine External Debt: Bangko Sentral ng Pilipinas (BSP)
5. External Assets: Bangko Sentral ng Pilipinas (BSP)
6. Nominal GDP: IFS
7. Average Exchange Rates: BSP
8. Consumption, GNP, GNDI, CAB, Current Transfers, GDP Deflator: (IFS)

## Sample Period

Philippine JACI Spreads: 2000–2003

## Software Used

1. Philippine JACI Spreads: Microsoft Excel
2. Philippine Optimal Domestic Saving Rate: Microsoft Excel, “Goal Seek”

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## Comment Francis T. Lui

Villanueva and Mariano's paper provides us with a useful framework for identifying the conditions under which the external debt level is sustainable. The latter is defined as the existence of a steady state to which the external debt/GDP level converges, and at the same time the economy is on a balanced growth path. To achieve this objective, prudent fiscal policy and promotion of private saving are recommended. These policy implications make a lot of sense. The actual application of the model to the Philippines is also credible. Countries with governments that are overspending or with people who do not save enough should take the paper seriously. Philippine policy-makers may also find estimate of the Golden-Rule saving rate interesting.

The model, built on earlier papers by Villanueva (1994, 2003), consists of 15 equations, which include identities, laws of motion, a production function, and an equation governing the rate of change of technology. There is also an equation, the consumption function, that is behavioral. A major improvement of this model over its earlier version and Villanueva (2003) is that an explicit optimization problem has been incorporated. By solving for the steady state and maximizing per capita consumption, the model can generate Golden-Rule consumption and saving paths. This exercise is important because it provides the calibrated results with a more solid microfoundation. Without the maximization, one may easily cast doubt on whether coefficients in the model are robust to policy changes.

The objective function of the maximization problem in the paper is per capita consumption. While this has the advantage of making the model simple and easily interpretable, it is not the same as the more conventional utility function. Concavity in the latter can allow us to take into account