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Abstract

Why do employed persons in large firms earn more than employed persons in small firms, even after controlling for observable characteristics? Complementary to previous results, this paper proposes a mechanism that gives an answer to this question. In the model, individuals accumulate human capital and are exposed to the risk of losing some of their human capital as they change jobs, voluntarily or involuntarily. The model, calibrated to the United States and Canada, accounts for one-third of the firmsize wage premium. Regarding the earnings gap between Canada and the United States, the model finds that it is solely due to differences in labor market uncertainty.

JEL classification: J24, J31
Bank classification: Economic models; Labour markets; Productivity

Résumé

Pourquoi les salariés des grandes entreprises sont-ils mieux rémunérés que les travailleurs des petites entreprises, même si l'on tient compte des caractéristiques observables? En complément des résultats antérieurs, les auteurs proposent un mécanisme qui permet de répondre à la question. Dans le modèle décrit, les salariés accumulent du capital humain, au risque d'en perdre une partie s'ils changent d'emploi, volontairement ou non. Le modèle, qui est étalonné en fonction des économies américaine et canadienne, parvient à expliquer le tiers de la prime salariale liée à la taille des entreprises. Quant à l'écart de rémunération entre le Canada et les États-Unis, il est uniquement imputable, d'après le modèle, aux différences dans les niveaux d'incertitude sur le marché du travail.

Classification JEL : J24, J31
Classification de la Banque : Modèles économiques; Marchés du travail; Productivité

1. Introduction

Why do employed persons in large firms earn more than employed persons in small firms, even after controlling for observable characteristics? This has largely been an open question for some time. It is not a purely academic question as the gap between large and small firms is substantial. Oi and Idson (1999) state that the size of the wage gap between large and small firms is comparable to the male-female wage gap and larger than the wage gap between whites and blacks. A number of theories to explain the gap have been put forward, but none have proven to be satisfactory.[1] Data limitations were initially cited as possible reasons for the failure to account for the size-wage gap, but the size-wage gap has persisted even in studies using the more recently available longitudinal and matched worker-firm data. After using one of these matched worker-firm data sets, Troske (1999) concludes that a large unexplained size-wage premium remains. The sorting of more skilled workers into larger firms/establishments accounts for 20 per cent of the premium, while the addition of firm/establishment characteristics such as the capital-labour ratio increases the fraction explained to 45 per cent. Troske (1999) suggests that part of the gap could be related to the possibility that large firms not only hire, but produce more skilled workers. However, Troske (1999) does not offer a mechanism through which this would be realized.

This paper proposes and evaluates the importance of such a mechanism in explaining the size-wage gap within a structural model. Individuals accumulate human capital over their working life, but are exposed to the possible risk of losing some of their human capital

[1] Oi and Idson (1999) review the empirical evidence testing theories such as: higher monitoring costs in large firms, efficiency wage models, rent sharing, differences in work organizations, compensating differentials, and complementarities between capital and labour. They do not find conclusive evidence supporting any of these hypotheses.

as they change jobs. The probability of job separation is higher in small firms and this greater uncertainty lowers the expected returns of investing in human capital when employed in a small firm.

Human capital accumulation is one of the two main theoretical sources for wage growth, the other is on-the-job search. However, recent studies by Bagger et al. (2006) and Yamaguchi (2007) suggest that human capital accumulation is the more dominant of the two.[2] As a result, this paper focuses on the human capital channel and abstracts from the search channel.

The idea that job uncertainty explains part of the size-wage gap is not entirely new. Mayo and Murray (1991) and Winter-Ember (2001) show that 100 per cent and 50 per cent of the size-wage gap, respectively, can be accounted for when measures of employment risk are added to wage regressions. However, Mayo and Murray (1991) do not offer an explanation for this empirical finding, and Winter-Ember (2001) suggests that the increased displacement risk for workers in small firms is a proxy for the heterogeneous quality of workers as less able and inherently more unstable workers sort themselves into less stable jobs in small firms.

In contrast to the two papers mentioned above, this paper presents a model that draws the link between uncertainty and human capital accumulation. The model is then calibrated using Canadian and U.S. data in the 1996-2001 time period, and the importance of uncertainty in explaining the size-wage gap through human capital accumulation is then evaluated. The model is found to account for roughly one-third of the average wage differen-

[2] Bagger et al. (2006) and Yamaguchi (2007) develop models that allow wages to grow via human capital and on-the-job search. They also include mechanisms where a worker that finds a better outside offer can use it to increase his wages at his current job. Their estimated models show that human capital accumulation accounts for roughly 70 per cent of the wage growth in the first ten years of a worker's career

tial between firms/establishments with more than 1000 employees and firms/establishments with 1-19 employees in both Canada and the United States. This is roughly the same amount accounted for by the sorting of workers in Troske (1999) and other studies. The model is also able to broadly match other aspects of the data: the median wage differential between firms/establishments sizes, median wages lower than the mean wage, higher tenure in larger firms/establishment, and the ordering of wages between firm and establishment sizes. Finally, by gradually changing the parameter values of the model from the Canadian to U.S. values, it is determined that higher degrees of job uncertainty in Canada also accounts for the bulk of the Canada-U.S. wage gap.

The next section of the paper presents Canadian and U.S. evidence on the size-wage gap. In section 3, the model is presented. In section 4, the calibration of the model is discussed. The results are presented in Section 5 and concluding remarks are contained in section 6.

2. Main facts

This section presents the main facts accounted for in this paper, differences in wages and tenures by firm/establishment. For Canada, the data come from the Survey of Labour and Income Dynamics (SLID) 1996-2001. The SLID is a series of six-year overlapping panels that began in 1993 and is representative of all individuals in Canada.[3] Other Canadian data sources, such as the Labour Force Survey, contains information on wage, tenure and firm/establishment size, but the advantage of panel data is that they allow the estimation

[3] The most recent panel, 1999-2004, is not used because of high non-response rates to the firm size question in later years of the SLID. Among wage-employed workers, non-response to the firm size question went from 2 per cent in 1993 to 11 per cent in 2005.

of job separation rates and transitions rates used later in the model.

For the United States, the data come primarily from the National Longitudinal Study of Youth (NLSY) 1979. The NLSY 1979 follows a sample of youths aged 14 to 22 in 1979 through to 2005. A limitation of the NLSY is that it studies a particular cohort. Therefore, when Canada-U.S. comparisons are made, the SLID is limited to individuals aged 31 to 39 in the year 1996. Other U.S. data were considered, but the NLSY was the only one where both firm/establishment size and tenure is collected in the same survey.[4] Another limitation of the NLSY is that firm size is observed with some error. The NLSY collects information on establishment size, and it asks whether more or less than 1000 workers are employed at the employer's other locations. Workers can be divided between firms with more or less than 1000 workers using these two pieces of information, but people in large firms will be under counted. For example, a person working in a establishment with 999 employees and with an employer that has less than 1000 employees in other locations would still be counted as working in a firm with less than 1000 employees. This limitation will tend to lower the wage differential between firms of different sizes. Finally, hourly wages in the NLSY do not include overtime, tips and commissions, but they are included in the SLID. Since a higher fraction of workers in large firms/establishments are found to have this type of income in the NLSY, the wage differential between large and small will be understated in the US relative to Canada.[5]

[4] The Panel Study of Income Dynamics collects firm size in only a limited number of years. In its annual March demographic supplement, the Current Population Survey (CPS) collects firm size information for the individuals longest job of the previous year, but tenure is collected in infrequent supplements (1996, 1998, and 2000) in February for jobs held at the time of the survey. Data from the CPS is not used because of this difference in the reference period.

[5] The NLSY asks whether workers received overtime, tips and commissions. A slightly higher fraction of workers in large firms report earning this type of income; 34 per cent versus 30 per cent.

The mean wage by firm and establishment size for Canada in the SLID is presented in [TABLE 1]. The size categories are small (1-19 employees), medium (20-999 employees) and large (1000 and more employees). The wages are presented in 1998 US dollars.[6] The wage differentials already take into account the non-random sorting of workers into size categories according to age, age squared, educational attainment, gender, industry and occupation.[7] [TABLE 1] shows that the wage differential between large and small size categories is substantial. Workers in large firms earn 25.3 per cent more than workers in small firms, and workers in large establishments earn 37.7 per cent more than workers in small establishments. [TABLE 1] also shows that both firm and establishment size matter. Workers in the small firms and by necessity small establishments earn the least at $11.55 per hour, and workers in small establishments who could be part of a larger firm earn slightly more at $11.73 per hour. The pattern is the same at the top end. Workers in large establishments who must also be part of large firms earn the most at $16.15 per hour, and workers in large firms who could be in any size of establishment earn $14.47 per hour.

[TABLE 2] shows the median wage by size category. [TABLE 2] suggests that the mean wage differentials are not driven by a few top managers in large firms and establishments. While the mean wage is greater than the median wage in the large size categories, this is also the case in the smaller size categories. As a result, the median wage differential are similar to the mean wage differentials.

[TABLE 3] presents the standard deviation of the wage by size category.[8] Similar to

[6] Wages are deflated using the consumer price index, and Canadian wages are converted to U.S. dollars using the1998 purchasing power parity factor of 0.85 from Statistics Canada.

[7] A wage regression was performed using these explanatory variables and firm size dummies. A predicted hourly wage was then calculated for each size category with the characteristics of the overall average worker.

[8] The standard deviation of wages in the data reported in this table is the standard deviation of the error

the mean, the standard deviation increases with size. However, the coefficient of variation declines slightly by size. The coefficient of variation between the small and medium size categories are nearly identical, while the coefficient of variation of wages in large firms relative to small firms is 0.95 and the coefficient of variation of wages in large establishments relative to small establishments is 0.92.

If jobs are less stable in large versus small firms/establishments, this should be manifested in average years of tenures by size. Indeed, [TABLE 4] shows that individuals in larger firms/establishments do achieve substantially higher tenures.[9] Furthermore, the pattern of mean tenures mirrors that of mean wages. Workers in small firms have the lowest tenures, followed by workers in small establishments, and workers in large establishments have the longest tenures, followed by workers in large firms.

[TABLE 5] and [TABLE 6] present Canada-U.S. comparisons of wages by size. The data used for Canada in these tables are consistent with those for the United States in their focus on individuals aged 31 to 39. In the case of wages by firm, the size categories are small (less than 1000 employees) and large (1000 or more employees). Within each size category U.S. wages are higher than Canadian wages, but size is still important as the workers in larger Canadian firms/establishment still earn more than U.S. workers one size category down. Somewhat unexpectedly, the wage-size relationship is found to be steeper in Canada than the United States, but this likely due to the imprecise size definition and the omission of overtime earnings etc in the NLSY data.[10] Finally, the Canadian wage differentials shown in

term by firm size from the wage regressions used to generate the wage differential. Thus, the wage dispersion due to age, education, gender, industry and occupation has already been removed.

[9] Similar to the tables that present the wage differentials by size, the tenure differentials shown here are derived from a tenure regression with controls for firm/establishment size, age, education, gender, industry and occupation.

[10] An analysis of U.S. CPS data reveals a similar sized wage-size relationship between Canada and the

[TABLE 6] are similar to the ones shown in [TABLE 1]. This is despite the fact that [TABLE 6] is for individuals aged 31 to 39 in 1996 and [TABLE 1] is for all ages. This is not entirely surprising as the average 31 to 39 year old is similar to the average individual overall.[11] This suggests Canada-U.S. comparisons with the smaller sample should be indicative of Canada-U.S. differences more broadly.

3. Model

This section describes the model. We first derive the main equilibrium condition and then use a numerical example to provide intuition for our later results. Since our main aim is to highlight a mechanism, we do not attempt to provide a very general model, but rather use a small model that has only the ingredients needed to make our point, namely that labor market uncertainty is a main factor in the determination of the firm size-wage premium.

Individuals live for N periods. During that time they enjoy leisure, $1 - l$, and a consumption goods, c. The good is bought in a competitive final goods market. The time they do not spend on leisure can be either sold in the labor market or spent on human capital accumulation, x.[12] We restrict our attention to an economy in which only one type of human capital is accumulated. This assumption is not essential for our results and helps to make the model more transparent.[13]

A worker can work for a type z_i firm/establishment, where the number of types is

United States. Again, CPS data are not used here because firm size and tenure information are collected in different months and refer to jobs in different years.

[11] The average 31-39 year old has a wage 7 per cent higher than the average worker in each of the size categories.

[12] Adding a savings opportunity into the model does not matter as long as the main source of income remains labor supplied to the market and the savings opportunity does not undo the labor market uncertainty.

[13] We have also considered a model in which two different types of human capital are accumulated of which only one is exposed to risk. The results are very similar to the ones found below.

finite and given by M. The types are taken as indicators of the size of the firm/establishment in terms of the number of employees and ordered from smallest to largest. Workers have the option of enhancing their human capital, h. For each worker, there exists a tenure and firm/establishment size specific probability of retaining their job. If a worker is separated from her job, then she looses a fraction of her human capital. We capture this by stating that she retains η per cent of her human capital. Here we assume that a job is associated with the firm/establishment.[14] We are not considering a promotion within a firm as a job change.

The problem of a person of age a at workplace z_i and with tenure t is:

$$v_a(h_a, z_i, t) = \max u(c, l) + \beta \sum_{j=1}^{M} p_{i,j}[\pi_i(t)\, v_{a+1}(h_{a+1}, z_j, t+1) + (1-\pi_i(t)) v_{a+1}(\eta h_{a+1}, z_j, 1)]$$

$s.t.$

$$\begin{aligned} c &= w h_a (l - x) \\ h_{a+1} &= (1-\delta) h_a + B (h_a x)^{\phi} \end{aligned}$$

We use the following notational conventions: the variables z_i, z_j stand for different firm size types and can take values from 1 to M. The index t stands for different possible tenure

[14]In our model, a firm/establishment is characterized by two transition processes. One that guides the probability of staying at a given firm/establishment type and one that determines the probability of staying with a job conditional on firm/establishment type and tenure.

durations at a given age a. For obvious reasons, it is impossible to have a tenure larger than one's age. While this seems trivial, it helps to reduce the state space. There are two Markov processes governing the stochastics of the economy. One that determines the probability of increasing one's tenure and retaining all the human capital is denoted by $\pi_i(t) = \Pr($staying at the same job given tenure $t)$. The other one governs the probability of moving from one firm/establishment size type to another, $p_{i,j} = \Pr($being at firm/establishment size z_j | being at a firm/establishment size $z_i)$. We assume that the economy is in steady state and thus the wage rate, w, is constant.[15] Tomorrow's human capital is the undepreciated part of today's human capital, $(1-\delta)$, and today's production of human capital. The production of human capital depends on the current level of human capital, time spent on human capital investment today, and the parameters of the human capital production function, B and ϕ.

Before we start analyzing the problem, there are couple of things that should be pointed out regarding our model. If either $\eta = 1$ or $\pi_i(t) = \pi$, for all z_i, t, then all individuals accumulate the same amount of human capital and wages do not depend on the firm/establishment size one works at. Thus all our results later on will rely on the probabilities of loosing tenure and the human capital retention rate η after a tenure loss. The approach we are taking abstracts on purpose from firm size specific opportunities to accumulate human capital and from selection issues. We realize that there is evidence suggesting that large firms promote human capital development more actively than small firms. This is

[15] We realize that the steady state assumption is very strong, but to the extent that the wage per unit of human capital supplied to the market is identical accross different groups in the economy the steady state assumption does not matter for our main results since all persons (independent of the workplace size) will be hit in the same way by a change in the wage, which is the only way the steady state assumption enters our results. A recent paper by Bowlus and Robinson (2005) suggests, for different education groups, that the wage per unit of human capital is roughly the same.

most visibly reflected in the number of hours per year devoted to further education.[16] As you shall see below, our model suggests that, for relevant parameters, employees at larger firms would invest more in human capital making it consistent with the observation. Regarding the sorting, we take the stance that a lot of this is captured by controlling in the data for educational sorting and thus has been considered previously in the literature and will be taken care of in our calibration.

Note that the future value of human capital in the last period of working life is independent of firm size given by $v_{N+1}(h', \star, \star) = 0$. This in turn implies that in the last period $x_N = 0, h_{N+1} = g(h_N, \star, \star) = (1-\delta) h_N$ and $v_N(h, s) = \max_{l \in [0,1]} u(whl, l)$. For simplicity, we assume that when born all workers have the same human capital, h_0. This is not an assumption that influences our overall results and can be relaxed. A relaxation will just add more dispersion to the ultimate wage distribution.

We solve the dynamic programming problem by recursively finding the policy functions given a wage rate w. We use the special functional form for the utility function:

[16] Using U.S. data, Black et al. (1999) find that larger firms and establishments offer more formal training than smaller firms and establishments, regardless of whether training is measured by duration or intensity. Furthermore, Dotsie and Montmarquette (2007) conclude that research on Canadian data generally finds that large establishments tend to offer more training opportunities than smaller establishments.

$u(c,l) = \alpha \log(c) + (1-\alpha)\log(1-l)$, which then leads to the further simplification:

$$
\begin{aligned}
v_a(h_a, z_i, t) &= \max_{x\in[0,1]} u(c,l) + \beta \sum_{j=1}^{M} p_{i,j}[\pi_i(t)\, v_{a+1}(h_{a+1}, z_j, t+1) + (1-\pi_i(t)) v_{a+1}(\eta h_{a+1}, z_j, 1)] \\
& \quad s.t. \\
c &= \alpha w h_a (1-x) \\
l &= 1 - (1-\alpha)(1-x) \\
x &= \left(\frac{h_{a+1} - (1-\delta) h_a}{B (h_a)^{\phi}}\right)^{1/\phi}
\end{aligned}
$$

In simplified notation filling in all the conditions the problem reduces to:

$$
v_a(h_a, z_i, t) = \max f(h_a, h_{a+1}) + \beta \sum_{j=1}^{M} p_{i,j}[\pi_i(t)\, v_{a+1}(h_{a+1}, z_j, t+1) + (1-\pi_i(t)) v_{a+1}(\eta h_{a+1}, z_j, 1)]
$$

where

$$
f(h,y) = (w\alpha)^{\alpha} (1-\alpha)^{1-\alpha} h^{\alpha} \left(1 - \left(\frac{y - (1-\delta) h}{B\ h^{\phi}}\right)^{1/\phi}\right),
$$

with y representing tomorrow's human capital stock.

From this, we get by combining the FOC's with the envelope condition of the dynamic program:

$$
f_2(h_a, h_{a+1}) + \beta \sum_{j=1}^{M} p_{i,j}\left[\pi_i(t)\, f_1(h_{a+1}, h_{a+2}) + (1-\pi_i(t))\, \eta f_1(\eta h_{a+1}, h_{a+2})\right] = 0 \qquad (1)
$$

This is a functional problem of the form $F(h, g_a(h), g_{a+1}(h)) = 0$ with the terminal condition $h_{N+1} = g(h_N, \star, \star) = (1-\delta) h_N$. As such it can be solved backwards.

In the appendix, we derive the functional form, which is:

$$1/\beta = E_t\left[\frac{h_a}{h_{a+1}}\frac{1-x_a}{1-x_{a+1}}\left(\frac{h_a}{h_{a+1}}\right)^{\phi-1}\left(\frac{x_a}{x_{a+1}}\right)^{\phi-1}\left(1-\delta+\phi B\left(h_{a+1}x_{a+1}\right)^{\phi-1}\left(\alpha+\left(1-\alpha\right)x_{a+1}\right)\right)\right]$$

$$x_r = \left(\frac{h_{r+1}-\left(1-\delta\right)h_r}{Bh_r^{\phi}}\right)^{1/\phi};r=a,a+1$$

As already stated this problem can be solved backwards. To solve it, we use the collocation method with cubic splines as our approximation functions. We proceed as follows:

First we solve for the optimal policy at the end of the working life: $h_{N+1} = g_N\left(h_N, z, t\right) = \left(1-\delta\right)h_N$ with $x_N = d\left(h_N, z, t\right) = 0$. Given this solution, we then start iterating backward using the last functional equation 1 that represents the final decisions of an individual and solving at each step for the functions $h_{a+1} = g_a\left(h_a, z, t\right)$ and $x_{a+1} = d_a\left(h_a, z, t\right)$.

Explicitly for a given period a, we have to solve the equation:

$$1/\beta = E_t\left[\frac{h_a}{h_{a+1}}\frac{1-x_a}{1-x_{a+1}}\left(\frac{h_a}{h_{a+1}}\right)^{\phi-1}\left(\frac{x_a}{x_{a+1}}\right)^{\phi-1}\left(1-\delta+\phi Bh_{a+1}^{\phi-1}x_{a+1}^{\phi-1}\left(\alpha+\left(1-\alpha\right)x_{a+1}\right)\right)\right],$$

where:

$$h_{a+1} = g\left(h_a, z_i, t\right);\forall i=1,...,M,\forall t\leq a-1$$

$$x_a = d\left(h_a, z_i, t\right) = \left(\frac{g_a\left(h_a, z_i, t\right)-\left(1-\delta\right)h_a}{Bh_a^{\phi}}\right)^{1/\phi};\forall i=1,...,M,\forall t\leq a-1$$

$$
\begin{aligned}
x_{a+1} &= d\left(g\left(h_a, z_i, t\right), z_j, t\right) \\
&= \left(\frac{g_{a+1}\left(g_a\left(h_a, z_i, t\right), z_j, t+1\right) - (1-\delta)\, g_a\left(h_a, z_i, t\right)}{B g_a\left(h_a, z_i, t\right)^{\phi}}\right)^{1/\phi} ; \forall i, j = 1, ..., M, \forall t \leq a-1.
\end{aligned}
$$

Remark 1. Uncertainty and the loss of human capital after a job loss are key for the firm size-wage premium.

1. If the probability of a human capital loss is identical across workplace sizes, then all persons will accumulate the same amount of human capital and size does not matter.
2. If nobody ever leaves her initial workplace size, then the size gap is determined by the probability of switching a job and loosing some human capital.

We now consider a simple numerical experiment with two workplace sizes and four periods of life. The parameters except for the probability matrices are the ones for the Canada calibration of the model. The firm size transition matrix is symmetric with a 80% probability of staying in the same workplace.

In the first example, we gave both firm-types the same probability to retain their employees. Figure 1 shows the resulting policy functions. The main aspect to note is that policy functions are such that younger individuals have a higher steady state than older individuals and that as expected both firm types will have the same policy functions. Next in Figure 2, we show the policy functions with unequal retention probabilities across firm sizes. What is visible is that the firm-type with the higher probability to retain a worker has a uniformly upward shifted set of policy functions relative to the other firm-type. This means that independent from the initial human capital stock, workers who are less at risk of separating from their job will accumulate on average more human capital than more at risk workers. This upward shift is only driven by the retention probability matrix, though it would not be there if job changes had no negative impact on wages.

4. Calibration

To use the model for quantitative work, we need to determine the function parameters. For some of them this is easy, for others this is difficult. We focus on employed persons age 15 to 64 and divide the working life into 5 year periods. This is mostly done for computational purposes, but also to have enough data points per period available. We consider two benchmark cases: one with three types of firms and one with three types of establishments. Both firms and establishments are considered to be small, if they have less than 19 employees, of medium size if the number of employees is between 20 and 999, and they are large if they have 1000 or more employees. We have one special case, where we have to deviate from this size convention and that is for the firms in the United States. Here we only have two size categories 1 to 999 employees and more than 1000 employees. So, we change our definition of a small to mean the first size grouping and of a large firm to mean the latter grouping. All persons in the model discount time at an annual rate of 1/1.04. This reflects the fact that the annual real interest rate is roughly 4 per cent over the last decades. The basic parameters are collected in [TABLE 7].

For the model, of more importance are the following parameters which we take from the data: the transition matrix that determines the movement across firm sizes, Γ, and the probability of loosing one's job at a given firm and moving on to another firm, Π. The transition probabilities are obtained by estimating a multinomial choice model that takes into account the non-random sorting into firm size categories. The model is estimated one time for each firm size category. The sample is divided according to the firm size category individuals belong to in beginning of 1996, and the "choice" variables are the firm size categories individuals can possibly belong to in the beginning of 2001. Let the utility of being in firm size i for individual k be U_{ki}. Individual k chooses to be in firm size category i if $U_{ki} > U_{kj}$ for all $i \neq j$, where U_{ki} is parameterized as follows:

$$U_{ki} = \boldsymbol{\gamma}_i^{'}\mathbf{X}_{ki} + \varepsilon_{ki}, \quad for\ i = 1, 2, ..., M-1,$$

$$U_{kM} = \varepsilon_{kM,} \quad for\ i = M,$$

where $\mathbf{X}_k$ is the same vector of explanatory variables that were used in the wage regressions and ε_{ki} is a random shock that affects individual k's chance of being in firm size i. Allowing the errors to be distributed according to a multivariate normal leads to a multinomial probit model.[17] The probability of being in firm size category M conditional on characteristics of the average person is then:

$$\Pi(size = M|\overline{X}) = \int_{-\infty}^{\gamma_1{}'\overline{X}} ... \int_{-\infty}^{\gamma_{M-1}^{'}\overline{X}} f(\varepsilon_1^*, \varepsilon_2^*, ..., \varepsilon_{M-1}^*)\partial\varepsilon_1^*\partial\varepsilon_2^*...\partial\varepsilon_{M-1}^*,$$

where $f(.)$ is the probability density function of the multivariate normal distribution and $\overline{\mathbf{X}}$ is the vector of average characteristics for all workers in the estimating sample.

[17] Another common distributional assumption is the extreme value distribution that leads to a multinomial logit. The multinomial logit, however, does not allow correlation of the error terms across alternatives like the multinomial probit.

The probability of a job separation by firm size and tenure are obtained by estimating a continuous accelerated fail time model.[18] For example, in the case of two firm sizes, the model is as follows:[19]

$$\ln T_k = B_0 + \mathbf{B}_1^{'}\mathbf{X}_k + B_2 Small_k + e_k,$$

where T_k is the survival time, or completed tenure length, of individual k, $\mathbf{X}_k$ is the same vector of explanatory variables that were used in the wage regressions, $Small_k$ is a binary variable equal to one if individual k is employed in a small firm, and e_k is an independent error term that follows a generalized gamma distribution.[20] The survival time is obtained by following individuals whom are employed at the beginning of 1996 until they leave their job. Following the estimation of fail time model, the survival function - the probability having a job spell greater than time t - is calculated for the average individual by each firm size. For example, the survival function, $S(t)$, for individuals working in a small firm is:

$$\begin{aligned} S(t|\overline{X}, Small &= 1) = P(T > t|\overline{\mathbf{X}}, Small = 1) \\ &= P(\ln T > \ln(t|\overline{\mathbf{X}}, Small = 1)) \\ &= P(e > \ln(t) - B_0 - \mathbf{B}_1^{'}\overline{\mathbf{X}} - B_2). \end{aligned}$$

[18] Alternatively, exit rates have been estimated using cross-sectional data via the formation of synthetic cohorts. See Heisz (2002) and Neumark et al. (1999) for example. This approach is not followed here because it is not possible to obtain exit rates by firm size with this methodolgy. While it is possibile to obtain exit rates for individuals with tenure t by counting individuals with tenure t in one year and tenure $t+1$ in the following year, it is not possible to do so for by firm size because individuals can freely move across firm sizes.

[19] The actual estimation also takes into account censoring - job spells that have not ended by the end of the survey - and truncation - the non-randomness of the sample when the model is estimated using a set of workers that are currently employed at the beginning of the survey. On the other hand, in line with the wage regresssions presented earlier in the paper, unobserved heterogeneity is not taken into account.

[20] The generalized gamma distribution encompasses other commonly used distributional assumptions such as the exponential, Weibull and log-normal.

The probability of staying at a small firm by five-year tenure groups, $\Gamma(t_j)$, are then obtained as follows:

$$\begin{aligned} \Gamma(t_j) &= 1 - P(t_{j-1} < T < t_j | T > t_{j-1}) \\ &= 1 - \frac{S(t_{j-1}) - S(t_j)}{S(t_{j-1})}, \end{aligned}$$

where $t_j = 5,\ 10, ...,\ 65$.

We summarize the size-transition matrix and the job loss probability matrices for Canada in [TABLE 8][21] and for the United States in [TABLE 9].

Furthermore, we use the literature to determine the rate at which human capital is lost after a change of job, $1 - \eta$, and find it to be 30 per cent. This number is based primarily on the research of Morissette et al. (2007). Using Canadian administrative data between 1983-2002, Morissette et al. (2007) calculate the earnings losses of workers from firm closures and mass layoffs as a percentage of pre-displacement earnings. Based on our calculations from the results provided in Morissette et al. (2007), the loss in annual earnings one year after displacement is 42 per cent of annual earnings one year before displacement.[22] The 42 per cent average loss is pulled up somewhat by the losses of individuals with high seniority, but individuals with more than 5 years of tenure only make up roughly 10 per cent of all displaced workers over the period studied. Morissette et al. (2007) focuses on the

[21] In order to determine to what extend the sample restrictions for the United States matter in a significant way for our results, we also determined the probability matrices for the restricted sample for Canada. They can be found in Table 15. We will come to this issue later.

[22] This number is based on our own calculations from the tables presented in Morissette et al. (2007). Calculations using data from the year of the displacement are misleading because a displaced person could have been unemployed for a large part of that year. Hence, a large loss in that year might be due to less weeks worked and not the loss of human capital. The numbers for the year after displacement could be contaminated in the same way. However, the loss in annual earnings two years after displacement is still 32 per cent of annual earnings one year before displacement, higher than the 30 per cent used in this paper.

long-term earnings losses of workers, so their headline numbers are less than 42 per cent. In our model, individuals that are separated from their jobs tend to invest more in human capital than those that were not separated. Although the initial loss is 30 per cent, the difference in earnings between those that faced job separation and those that did not will have diminished to roughly 22 per cent in the following period, similar to the lower range of long-term earnings losses provided in Morissette et al. (2007).

Possibly more problematic is the fact not all job separations are due to firm closures or mass layoffs. In particular, it is unlikely that much human capital is lost when on-the-job search leads to a job-to-job transition. Furthermore, job changes due to spousal relocation or time off to take care of a parent may not lead to the same human capital loss as ones resulting from a firm closure or layoff. To address these issues, the wage losses by type of job separations are analyzed using the SLID. The earnings growth of individuals that did not change their jobs over the entire panel is compared to the earnings growth of individuals that had the same job in the first two years, changed their jobs, and were employed in the last year of the panel.[23] It is found that while individuals that changed their job because they found a new job made wage gains in excess of the control group, other job changers fared worse than the control group. In addition, the wages losses of workers within this other job changers category were not significantly different by reason of job separation.[24] Intuitively, this is not a surprising result because regardless of whether the job separation is due to a

[23] This "difference-in-differences" approach controls for the possibility that individuals that changed their job may have systematically lower (or higher) wages than individuals that did not change their job. A further differencing would allow for differences in trend wage growth between job changers and stayers, but at the cost of loss of information. Indeed, when this is done the difference between job stayers and all types of job changers becomes statistically insignificant.

[24] Covariates such as age, education, indusry, occupation, and gender were also included in the wage regression.

relocation of a spouse or a layoff, the individual's reservation wage after the job separation is the same. In contrast, workers that already have jobs would move only if their situations would be improved. Job separations because workers found a new job accounts for 18 per cent of all job separations in the SLID and 25 per cent of all job separations in the NLSY.[25] In order to account for these job-to-job transitions without specifically modelling them, the estimate of 42 per cent earnings losses is lowered to 30 per cent.[26]

In calibrating their models featuring worker displacement risk to the United States, Rogerson and Schindler (2002) and Krebs (2007) use 30 per cent and 15 per cent, respectively, as the long-term in earnings loss when displacement occurs. However, they do not have a mechanism whereby workers that have lost human capital can catch up by investing more in human capital. Thus a 30 per cent initial loss is not out of line with what is being used elsewhere in the literature.

The remaining parameters left to be determined are the relative attractiveness of consumption versus leisure, the initial level of human capital, and the parameters in the law of motion governing human capital accumulation. To determine these remaining parameters of the model we minimize the distance between the labour supply ($n = l - x$) and wage per hour (wh) series generated in the steady state of the model and the age-wage per hour ($\widehat{\hat{w}h}$) and age-hours worked ($\hat{n}$) profiles in the data.[27]

[25] It is also found that the fraction of job separations due to job-to-job transitions does not differ by size.

[26] Morissette et al. (2007) is patterned after Jacobson et al.'s (1993) study using U.S. data. Jacobson et al. (1993) finds workers with six or more years of tenure lose 25 per cent of earnings when displaced. They do not, however, study the losses of low seniority workers.

We do not rely soley on estimates of wage loss from the SLID because they are based on a relatively small sample of 5000 individuals, whereas Morissette et al. (2007) have a 10 per cent random sample of all Canadian workers.

[27] For Canada, these cross-sectional profiles come from the SLID. For the United States, the data come from the CPS. Since the NLSY follows a particular cohort, only partial age-wage per hour and age-hours worked profiles can be calculated.

$$\min_{\{\delta,\alpha,B,\phi,h_0\}} \left(\frac{1}{N}\lambda \sum_{a=1}^{N} \left(w_a h_a \frac{\hat{w}_1}{w_1} - \hat{w}_a \right)^2 + \frac{1}{N-2}\left(1-\lambda\right) \sum_{a=3}^{N} \left(n_a - \hat{n}_a\right)^2 \right)$$

$s.t.$ $\{w_a h_a, n_a\}$ are solutions to the individual's problem in steady state given the parameters.

Given the different magnitude of the series, the weight, λ, is set to 0.01 so that the two series get equal weight in the problem. We aim at the working time series only starting at age 30 and onwards since before that persons may not be not working or working very little and focusing rather on full-time or part-time studies. In our model this is not feasible since we abstract from student loans, parent subsidies, or other ways of smoothing consumption while not working.[28] The minimization problem is solved in two steps for the probability structure found for establishments of the respective country.

In the first step, we vary δ over a grid with stepsize 0.04 on $[0.01, 0.37]$ and for each given δ we solve the minimization problem over the remaining parameters using a Mead-Nelder algorithm. We do this because the problem is highly non-linear in δ, which implies that even for small changes in δ we might lose convergence of the underlying individual decision problem and the Mead-Nelder algorithm is too local in scope to do well on a global scale. With the obtained results we then determine the δ close to which we wish to search more rigorously.

Next, we start the full minimization problem using a Mead-Nelder algorithm at the

[28] We also used our calibration proceedure for the firms structure in the respective countries and for our benchmark cases their is not much of a difference, either in the found parameters or the fit of the model to the data.

initial point found in step one.

We report the results from this procedure in [TABLE 10]. To illustrate the success of the calibration, we show the plots of wages and hours worked for both countries, comparing the model with the data. This is done for Canada and the United States respectively in Figures 3 and 4. As already indicated above, the model is not able to replicate the hours worked for either the early or the late period, but it does fairly well for the age groups 25 to 55. Furthermore, the model fits the wage pattern in the data in particular for Canada very well.

Before we move on to the results of interest, we would like to emphasize that the probability matrices and the human capital retention rate after a job loss are by far the main parameters for all that follows. The other parameters have only a minor impact on the outcome of our analysis. Even for large variations of the other parameters our results remain the same.

5. Analyzing the size-wage gap

A. Analyzing the Canadian case

[TABLE 11] compares the size-wage premiums in the data and the model. The model accounts for 45 per cent of the mean wage differential between large and medium firms, and 35 per cent of the size premium between large and small firms. It performs as well in accounting for the mean wage differentials between establishment sizes. It accounts for 38 per cent of the mean wage differential between large and medium establishments and 37 per cent between large and small.

Also, the wages generated from the model preserve almost perfectly the ordering over firm and establishment sizes if compared with the data. In the data, workers in large establishments earn the highest wages followed by workers in large firms, medium establishments, medium firms, small establishments and small firms. The ordering is the same in the model as in data except for the fact that workers in small firms earn more than workers in small establishments. At $0.19, the wage differential between small firms and small establishments is small, but the probability of staying at small establishments is larger at all tenures than at small firms. Differences between establishment and firm size transition matrices may be cause the reverse ordering as the probability of moving from a small to large firm is 10 times higher than the probability of moving from a small to large establishment. This difference might be enough to lower the expected return of accumulating human capital in a small establishment relative to a small firm. If this is the case, a model incorporating a more flexible transition matrix between all firm-establishment size combinations might overturn the model-data difference in the ordering of wages between small firms and establishment.

With respect to the median wage differentials, the model does nearly as well as in the case of mean wage differentials. The model accounts for 38 per cent of the gap between the median wages of large and medium firms, 37 per cent of the gap between large and small firms, 32 per cent between large and medium establishments and 35 per cent of the gap between large and small establishments. As in the case of the mean wages, the ordering of the median wages generated by the model with respect to firm and establishment sizes is the same as in the data, except for small firms and establishments. Another way in the which the model matches the data is that the median wage is less than the mean wage in each of the size categories.

[TABLE 12] compares other aspects of the model and the data. The model accounts for a large fraction of the difference in the standard deviation of wages across firm and establishment sizes, especially between the large and medium size categories. The entire difference between large and medium firms is accounted for, while 71 per cent of the difference between large and medium establishments is explained. While the model does less well in accounting for the gap between large and small, the explained portions are still large, 35 per cent in the case of large and small firms and 51 per cent in the case of large and small establishments. In contrast to the data, where a slightly declining coefficient of variation by size is observed, the coefficient of variation in the model does not change by size. Not surprisingly, the standard deviation of wages in the data is higher than in the model. The model only accounts for wage dispersion due to job stability,[29] while many other sources of dispersion, such as search frictions and differing initial levels of human capital, are still

[29] Aging also contributes to the wage dispersion in the model, but this effect is removed by looking at the residual variation after controlling for age and age squared in a regression. Looking at the residual variation does not affect the comparison of dispersion across firm sizes in the data generated by the model because the difference in the job separation rates by size category used in the calibration are constructed such that they are independent of age.

present in the data.

The model does not account for the differences in tenure across size categories as well as it does for differences in wages. It accounts for 11 per cent of the tenure gap between large and medium firms, 14 per cent between large and medium establishments, 7 per cent between large and small firms and 12 per cent between large and small establishments. One reason for this poorer performance is that the job separation rates used in the calibration are calculated using flow data, while the tenure distribution in the data is drawn from the stock. The flow data capture the job separation rates exhibited in the 1996-2001 period, while the tenure distribution is the result of job separation rates that prevailed as far back as when the oldest person in the sample entered the labour force. Given the perception that the probability of having a "job for life" has declined over time, it is not surprising that more recent separation rates cannot generate as long average tenures. In the data, individuals in large firms and establishments have on average 9.3 and 10.5 years of tenure, respectively. This is compared to 6.8 and 7.0 years of tenure in the model. The same stock-flow argument would apply to the model's inability to explain the entire size-wage premium. However, in this case, the declining returns to tenure and experience commonly exhibited in the data would account for why more of the wage differential can be explained.[30]

Another reason for the poorer performance of the model in accounting for the tenure differential is related to the choice of a five-year model period. In the data, workers in small firms and establishments have on average 5.4 and 5.7 years of tenure, respectively, but in

[30] It is possible to back out job separations that would match the observed distribution of tenures, but this would tend to give an underestimate of the actual job separation rates. This is because the tenure distribution is calculated from a sample of workers conditional on having a job at the time of a survey, and so low tenure workers are less likely to appear in the sample than high tenure workers.

the model, workers have at least five years of tenure. Hence, workers in the model have on average higher tenures, at 6.4 years, for both small firms and establishments. A finer tenure grid was not used because of computational restrictions, but if one were to be used a larger fraction of the tenure differential and possibly the wage differential could be accounted for.

Finally, the distribution of employment across size categories in the model broadly matches that in the data. Differences here are entirely due to the fact that the transition matrices used in the calibration are being calculated using flow data, while the employment distribution is derived from the stock data.

B. Establishing similar results for the USA

[TABLE 12] presents the wage premia for Canada and the United States. Recall that the main difference between the results here and the previous two tables are that those are based on job separation rates and transition matrices calculated using individuals aged 31 to 39 in the year 1996, and that the firm size categories have been reduced to two. The model is able to explain a large fraction of the U.S. firm size wage premium; it accounts for 70 per cent of the average wage premium and 59 per cent of the median wage premium.[31] The model accounts for a smaller, but still significant portion of the establishment wage premium. It accounts for roughly 30 per cent of both the mean and median wage premiums between large and small establishments. The results for the United States are also similar to the ones for Canada in other ways: the median wage is less than the mean, and differences in tenure are reflected in differences in the average wage

[31] These larger fractions are not due to the underestimation of individuals in large firms mentioned earlier because the misclassification affects both the wage premium in the data and the job separation rates by firm size that drive the wage premium in the model. It could be related to the omission of overtime and commission income from the hourly wage data.

A way in which the results for the two countries differ is that the model predicts a steeper establishment size and firm size-wage relationship in Canada than in the United States. This is consistent with the data and is driven by the fact that the difference between the job separation rates between establishment sizes is larger in Canada than in the United States.

It is also interesting to compare the results for Canada in [TABLE 12] with the previous results for Canada, especially for the case of establishments where the size categories have remained the same. The size-wage relationships implied by the model using the broader sample is close to the one using the narrower sample. With the broader sample, there is a 5.8 per cent gap between large and medium establishments and a 13.5 per cent gap between large and small establishments. In the narrower sample, there is a 7.4 per cent gap between large and medium and a 15.7 per cent gap between large and small. As previously mentioned, the similarities should not be surprising as the average individual in the broader sample should be similar to the average person in the narrower sample. Thus, Canada-U.S. comparisons with establishment sizes using the smaller samples should reflect the same differences that would be found in a Canada-U.S. comparison using larger samples.

C. Uncertainty and the earnings gap between the United States and Canada

As indicated earlier, the parameters driving the size-wage premia are the job separation rates, the transition matrix and the rate of human capital retention when a job separation occurs. It is also informative to ascertain which parameters are driving the Canada-U.S. differences in the average wage. [TABLE 14] shows the results of an experiment that addresses that question. Starting with the establishment version of the model with all Canadian parameter values, we change sets of parameters one at a time to their U.S. counterparts until we reach the case with all U.S. parameter values. From this exercise we get an indication of which parameter values move Canada closest to the United States.

The average wage in Canada and the United States is \$13.1 and \$15.0, respectively. At \$13.14 for Canada and \$14.93 for the United States, the calibrated models match the data closely. When the Canadian transition matrix is replaced with the one estimated for the United States, the average wage rises to \$13.52; differences in the distribution of employment across establishment sizes accounts for 21 per cent of the Canada-U.S. wage gap.[32] Adding to this the difference in job separation rates moves the average wage up to \$15.24, which is above the observed U.S. wage. Changing the remaining parameters to their U.S. counterparts drops the average wage back to \$14.93. A similar pattern is followed when the standard deviation of earnings over the lifecycle or the standard deviation of cross-sectional earnings is examined.[33]

The above experiment suggests that the Canada-U.S. difference in job separation rates accounts for the Canada-U.S. difference in the mean wage. At first glance these results may seem to run counter to the common perception that the U.S. labour market as the more dynamic one. However, this is not the first paper to find more job instability in Canada than in the United States. Bowlus (1998) estimates labour market search models to examine why the unemployment rate in Canada was higher than in the United States in the late 1980s. Her estimation reveals a higher job destruction in Canada than the United States. More recently, Hobijn and Sahin (2007) find higher separations in Canada compared to

[32] Interestingly, this is nearly identical to the fraction of the wage gap that would be explained if one were to take the wage-size relationship in Canada and impose the U.S. employment distribution over firm sizes.

[33] Because of non-linearities, the ordering in which the changes occur could matter. However, experiments with different orderings also suggest the Canada-U.S. differences in the job separation rates is leading to the Canada-U.S. difference in the average wage.

the United States.[34] One hypothesis that helps explain the Canada-U.S. difference in job stability is the high and increasing use of temporary workers in Canada. Temporary workers include term and contract employees, casual workers and seasonal employees. Between 1997 and 2005, temporary employment grew 40 per cent in Canada, from 11.3 to 13.2 per cent of employment, with the bulk of the increase due to contract employees. In contrast, temporary employees accounted for only 4.6 per cent of employment in the United States in 1997, and 4.2 per cent in 2005.[35]

6. Conclusion

This paper introduces a parsimonious model that demonstrates how job uncertainty can play a role in accounting for the wage differential between large and small firms/establishments. Increased job uncertainty lowers the expected return of human capital accumulation because job changes generally entail some loss of human capital. Since the probability of a job separation is higher in small firms than in large firms, individuals in small firms tend to accumulate less human capital and consequently have on average lower wages. When the model is calibrated using Canadian and U.S. data, it is found that the model accounts for roughly one-third of the size-wage premium not already accounted for by the sorting of higher skilled individuals into larger firms. This paper adds to the literature by modelling the empirical finding of other researchers that uncertainty explains a large fraction of the size-wage premium, and it also builds upon another researcher's hypothesis that large firms

[34] Hobijn and Sahin (2007) find a 1.78 per cent monthly hazard rate for Canada over the 1992-2006 period, compared to a 1.06 per cent U.S. monthly hazard rate for the 2000-2006 period. Although the Canada-U.S. comparison is complicated by the differing time periods, compared to other OECD countries with data available over similar time periods, Canada is amongst those with the highest job separation rates.

[35] OECD statistics (2007) are the source of these numbers.

can create more able workers.

The model is also used to determine which parameters can account for the Canada-U.S. wage gap. Given the parsimony of the model, the results need to be interpreted with caution, but the results do indicate that greater job uncertainty in Canada is an important contributing factor to the Canada-U.S. wage gap. It is more important than the distribution of employment across size categories.

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Appendix
A.1 Tables

Table 1: Mean Wage by Employment Size, Canada

	Firms		Establishments	
	Dollars	Small=1.0	Dollars	Small=1.0
Small	11.55	1.000	11.73	1.000
Medium	13.12	1.136	13.72	1.169
Large	14.47	1.253	16.15	1.377

Table 2: Median Wage by Employment Size, Canada

	Firms		Establishments	
	Dollars	Small=1.0	Dollars	Small=1.0
Small	11.06	1.000	11.20	1.000
Medium	12.42	1.123	13.02	1.162
Large	13.88	1.255	15.70	1.402

Table 3: Standard Deviation of Wage by Employment Size, Canada

	Firms		Establishments	
	Dollars	Small=1.0	Dollars	Small=1.0
Small	4.83	1.000	4.88	1.000
Medium	5.52	1.143	5.58	1.144
Large	5.73	1.185	6.16	1.264

Table 4: Mean Tenure by Employment Size, Canada

	Firms		Establishments	
	Years	Small=1.0	Years	Small=1.0
Small	5.4	1.000	5.7	1.000
Medium	7.0	1.304	8.0	1.388
Large	9.3	1.731	10.5	1.827

Table 5: Mean Wage by Firm Size, Canada and the United States

	Canada		United States	
	Dollars	Small=1.0	Dollars	Small=1.0
Small	13.40	1.000	14.65	1.000
Large	15.81	1.180	16.02	1.094

Table 6: Mean Wage by Establishment Size, Canada and the United States

	Canada		United States	
	Dollars	Small=1.00	Dollars	Small=1.00
Small	12.65	1.000	14.32	1.000
Medium	14.73	1.164	15.21	1.062
Large	17.13	1.353	17.71	1.237

Table 7: BASIC PARAMETERS

PARAMETERS	ASPECT REPRESENTED	VALUE
M	Number of firm/establishment types	2 or 3
	- two types	small $1-999$ large 1000+
	- three types	small $1-19$ medium $20-999$ large 1000+
N	# of working periods (a period is 5 years)	10
β	Time discounting	0.95^5

Table 8: CANADA: TRANSITION PROBABLITIES FOR FIRMS AND ESTABLISHMENTS, FULL SAMPLE.

	FIRMS				ESTABLISHMENTS			
	t\t+1	small	medium	large	t\t+1	small	medium	large
$\Gamma(t+1 \mid t)$	small	0.537	0.296	0.167	small	0.701	0.283	0.016
	medium	0.140	0.607	0.253	medium	0.118	0.845	0.037
	large	0.062	0.219	0.719	large	0.086	0.424	0.490
	tenure	small	medium	large	tenure	small	medium	large
$\Pi(staying \mid tenure\ s)$	1	0.134	0.177	0.229	1	0.14	0.201	0.272
	2	0.288	0.336	0.389	2	0.294	0.360	0.428
	3	0.360	0.408	0.458	3	0.367	0.431	0.494
	4	0.409	0.456	0.504	4	0.416	0.478	0.538
	5	0.447	0.492	0.538	5	0.453	0.513	0.570
	6	0.476	0.519	0.564	6	0.482	0.540	0.595
	7	0.495	0.542	0.585	7	0.506	0.562	0.615
	8	0.520	0.561	0.603	8	0.526	0.581	0.632
	9	0.538	0.578	0.618	9	0.544	0.597	0.647
	10	0.553	0.593	0.632	10	0.56	0.611	0.659

⋆ For these cells we do not have enough data to determine them, so we assume that they are the same as for the last year.

Table 9: UNITED STATES: TRANSITION PROBABLITIES FOR FIRMS AND ESTABLISHMENTS.

	FIRMS			ESTABLISHMENTS			
$\Gamma(t+1\|t)$	t\t+1	small	large	t\t+1	small	medium	large
	small	0.764	0.236	small	0.650	0.315	0.035
	large	0.218	0.782	medium	0.094	0.838	0.068
				large	0.062	0.310	0.628
$\Pi(staying\|$ $tenure\ s)$	tenure	small	large	tenure	small	medium	large
	1	0.165	0.249	1	0.172	0.212	0.264
	2	0.426	0.508	2	0.425	0.461	0.512
	3	0.540	0.601	3	0.529	0.561	0.604
	4	0.594	0.648	4	0.593	0.621	0.659
	5	0.638	0.687	5	0.637	0.662	0.697
	6	0.663	0.710	6	0.662	0.686	0.719
	7	0.546	0.629	7	0.544	0.588	0.644
	8	0.169	0.410	8	0.162	0.301	0.447
	9	∗0.169	∗0.410	9	∗0.162	∗0.301	∗0.447
	10	∗0.169	∗0.410	10	∗0.162	∗0.301	∗0.447

⋆ For these cells we do not have enough data to determine them, so we assume that they are the same as for the last year.

Table 10: HUMAN CAPITAL ACCUMULATION RELATED PARAMETERS, IN ANNUAL TERMS.

PARAMETERS	ASPECT REPRESENTED†	VALUE CANADA	USA
δ	Wage growth after $x_a = 0$	0.1046	0.1067
α	Labor supply n	0.3813	0.4178
B	Wage growth	1.9407	1.9182
ϕ	Wage growth	0.3029	0.2954
h_0	Wage level and growth	0.3431	0.4190
η	Human capital retention rate after job loss	0.7000	0.7000
w	Level of wages for age 15 to 19	17.8957	16.5632

† Given that the parameters are jointly determined it is not perfectly clear, what aspect of the data each parameter influences. The list below indicate s the aspect that the respective factor influences the most.

Table 11: FIRMSIZE AND ESTABLISHMENT SIZE-WAGE-PREMIUM FOR CANADA.

	FIRMS				
	Wage Per Hour			Wage Premium	
	small	medium	large	medium-large	small-large
Mean					
- Data	11.55	13.12	14.47	1.103	1.253
- Model	12.62	13.14	13.74	1.046	1.089
- % accounted for				44.7	35.2
Median					
- Data	11.06	12.42	13.88	1.117	1.255
- Model	11.53	12.09	12.62	1.044	1.094
- % accounted for				37.6	36.9

	ESTABLISHMENTS				
	Wage Per Hour			Wage Premium	
	small	medium	large	medium-large	small-large
Mean					
- Data	11.73	13.72	16.15	1.177	1.377
- Model	12.43	13.34	14.11	1.058	1.135
- % accounted for				32.8	35.8
Median					
- Data	11.20	13.02	15.70	1.207	1.402
- Model	11.39	12.19	13.00	1.066	1.141
- % accounted for				31.9	35.1

Table 12: DISTRIBUTIONAL FACTS FROM MODEL FOR FIRMS AND ESTABLISHMENTS.

	FIRMS				
	small	medium	large	medium-large	small-large
Fraction employed					
- Data	0.26	0.40	0.33		
- Model	0.17	0.38	0.45		
Mean tenure					
- Data	1.08	1.40	1.86	1.328	1.861
- Model	1.28	1.31	1.36	1.036	1.063
- % accounted for				11.0	7.3
Standard deviation of wage					
- Data	4.83	5.52	5.73	1.037	1.185
- Model	3.89	3.94	4.14	1.051	1.064
- % accounted for	80.5	71.4	72.3	137.8	34.6

	ESTABLISHMENTS				
	small	medium	large	medium-large	small-large
Fraction employed					
- Data	0.38	0.56	0.06		
- Model	0.28	0.66	0.06		
Mean tenure					
- Data	1.15	1.60	2.10	1.316	1.827
- Model	1.27	1.33	1.39	1.045	1.095
- % accounted for				14.2	11.5
Standard deviation of wage					
- Data	4.88	5.58	6.16	1.105	1.264
- Model	3.78	3.99	4.29	1.075	1.135
- % accounted for	77.5	71.5	69.6	71.4	51.1

Table 13: CANADA - US: FIRMSIZE AND ESTABLISHMENT SIZE WAGE PREMIUMS

		FIRMS	ESTABLISHMENTS	
		small-large	medium-large	small-large
Mean				
	Canada			
	- Data	1.180	1.163	1.353
	- Model	1.042	1.074	1.157
	- % accounted for	23.3	45.4	44.5
	United States			
	- Data	1.094	1.164	1.237
	- Model	1.066	1.037	1.075
	- % accounted for	70.2	22.6	31.6
Median				
	Canada			
	- Data	1.198	1.176	1.408
	- Model	1.044	1.094	1.185
	- % accounted for	22.2	53.4	45.3
	United States			
	- Data	1.142	1.199	1.322
	- Model	1.083	1.045	1.099
	- % accounted for	58.5	22.6	30.7

Table 14: EXPERIMENTS: IMPORTANCE OF UNCERTAINTY ON HUMAN CAPITAL ACCUMULATION.

		MEAN EARNINGS	% OF GAP ACCOUNTED FOR
CANADA	Data	13.10	
	Canadian model	13.14	
	+US Γ		
	Experiment 1	13.52	21.2
	+US Π		
	Experiment 2	15.24	117.3
	+US parameters		
	U.S. model	14.93	100.0
United States	Data	15.00	

A.2 Figures

Figure 1: POLICY FUNCTIONS WITH EQUAL JOB LOSS PROBABILITIES.

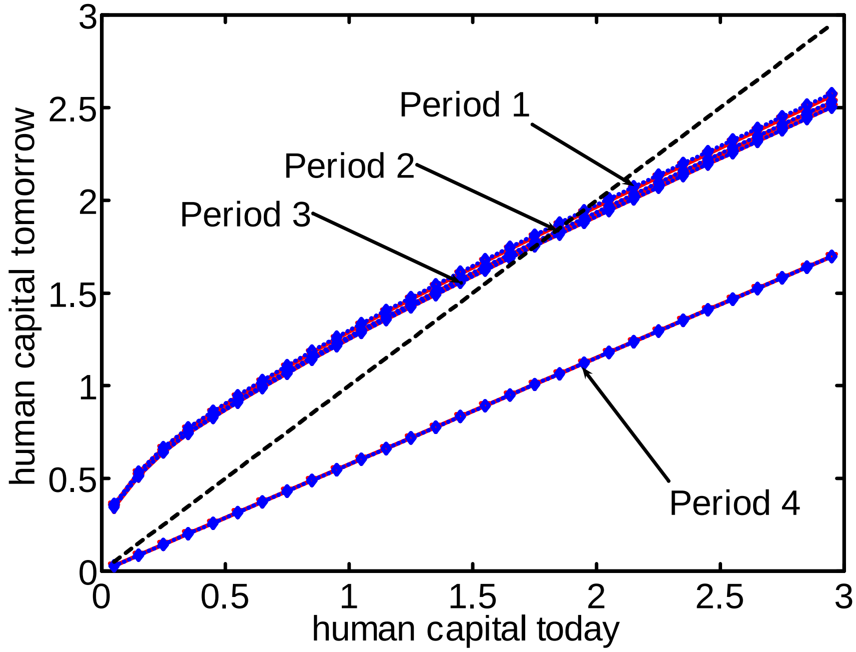


Figure 2: POLICY FUNCTIONS WITH UNEQUAL JOBLOSS PROBABILITIES.

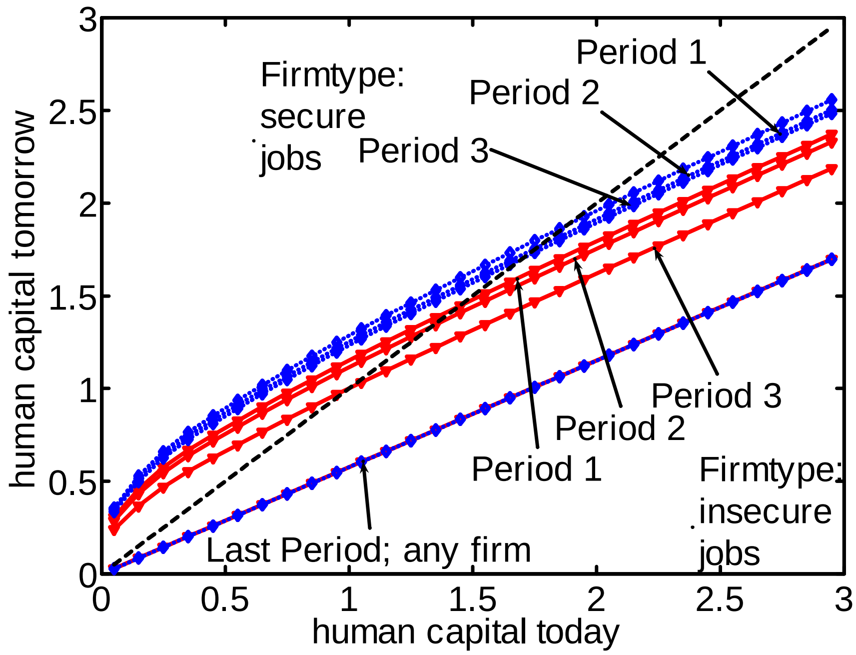


Figure 3: CALIBRATION RESULTS FOR CANADA, DATA(-) AND MODEL(-v).

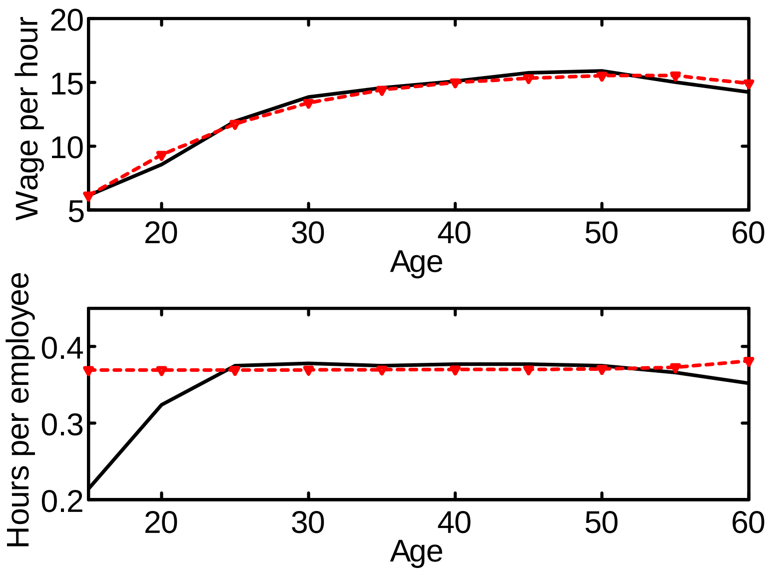


Figure 4: CALIBRATION RESULT UNITED STATES, DATA (-) AND MODEL(-v).

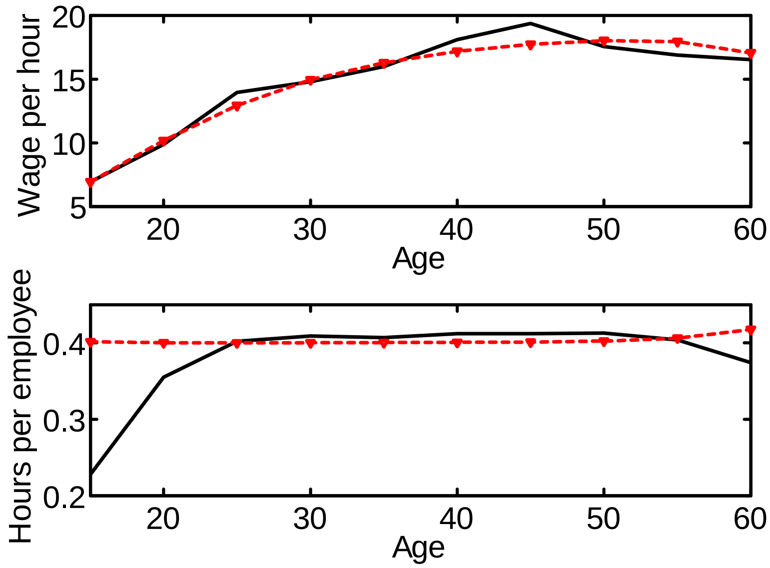
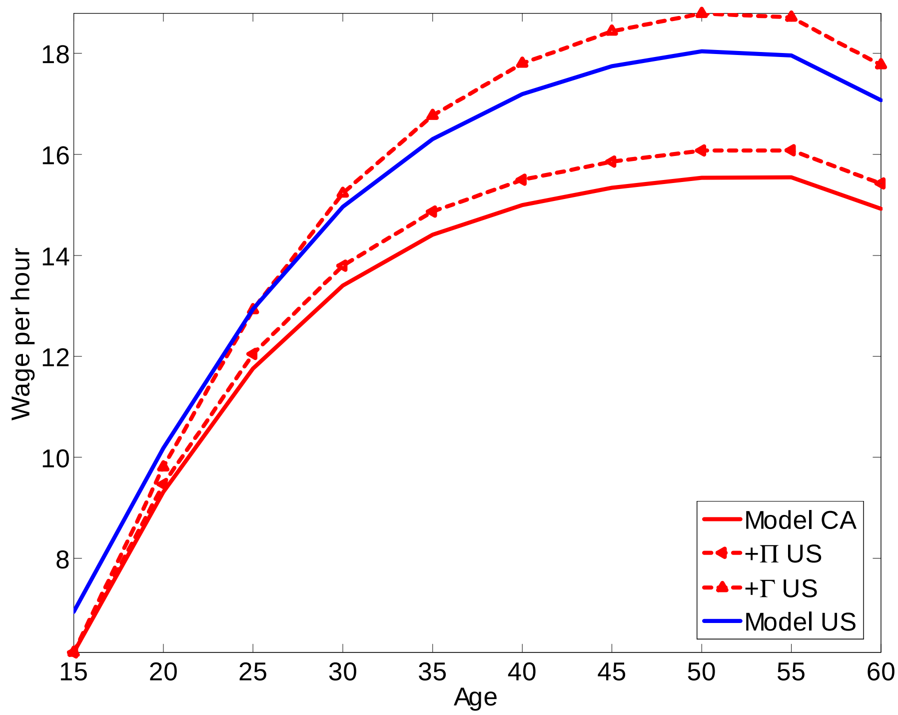


Figure 5: HOW IMPORTANT IS UNCERTAINTY FOR THE US - CANADA WAGE GAP?



A.3 Sensitivity analysis

This section considers various issues related to data restriction and parameter choices.

Table 15: CANADA: TRANSITION PROBABLITIES FOR FIRMS AND ESTABLISHMENTS FOR RESTRICTED SAMPLE.

	TWO FIRMSIZES			THREE ESTABLISHMENT SIZES			
$\Gamma(t+1\|t)$	t\t+1	small	large	t\t+1	small	medium	large
	small	0.734	0.266	small	0.658	0.226	0.116
	large	0.333	0.667	medium	0.081	0.694	0.225
				large	0.029	0.180	0.791
$\Pi(leaving\|$ $tenure\ s)$	tenure	small	large	tenure	small	medium	large
	1	0.186	0.277	1	0.159	0.204	0.282
	2	0.387	0.462	2	0.360	0.401	0.462
	3	0.484	0.549	3	0.460	0.496	0.549
	4	0.546	0.604	4	0.524	0.557	0.604
	5	0.591	0.644	5	0.571	0.601	0.644
	6	0.625	0.674	6	0.607	0.634	0.674
	7	0.589	0.650	7	0.565	0.601	0.650
	8	0.308	0.465	8	0.236	0.340	0.465
	9	∗0.308	∗0.465	9	∗0.236	∗0.340	∗0.465
	10	∗0.308	∗0.465	10	∗0.236	∗0.340	∗0.465

⋆ For these cells we don't have enough data to determine them, so we assume that they are the same as the for the last year.

A.4 Theoretical derivations

This section derives the main functional equations used in this paper from the household problem.

$$
\begin{aligned}
v_a\left(h_a, z_i, t\right) &= \max_{x\in[0,1]} \alpha \log\left(c\right) + \left(1-\alpha\right)\log\left(1-l\right) + \\
&\quad \beta \sum_{j=1}^{M} p_{i,j}[\pi_i\left(t\right) v_{a+1}\left(h_{a+1}, z_j, t+1\right) + (1-\pi_i\left(t\right)) v_{a+1}\left(\eta h_{a+1}, z_j, 0\right)] \\
&\quad s.t. \\
c &= w h_a\left(l-x\right) : \lambda \\
h_{a+1} &= \left(1-\delta\right) h_a + B\left(h_a x\right)^{\phi} : \eta
\end{aligned}
$$

From this household problem, we get the following FOC and envelop conditions:

$$
\begin{aligned}
\alpha / c_a &= \lambda_a \\
\left(1-\alpha\right) / \left(1-l_a\right) &= w_a h_a \lambda_a \\
B\phi_2 h_a^{\phi-1} x_a^{\phi-1} \mu_a &= w_a \lambda_a \\
\mu &= \beta E_{t,i}\left(\pi_i\left(t\right) v_{1,a+1}\left(h_{a+1}, z_j, t+1\right) + (1-\pi_i\left(t\right))\eta v_{1,a+1}\left(\eta h_{a+1}, z_j, 0\right)\right)_a \\
v_1\left(h_{a+1}, z, t\right) &= \left(1-\delta+\phi B h_{a+1}^{\phi-1} x_{a+1}^{\phi-1}\left(\alpha+\left(1-\alpha\right)x_{a+1}\right)\right)\mu_{a+1}
\end{aligned}
$$

Based on these conditions, we derive the following equation system:

$$
\begin{aligned}
l_a &= \alpha + (1-\alpha)\, x_a \\
c_a &= w_a h_a \left(l_a - x_a\right) \\
h_{a+1} &= (1-\delta)\, h_a + B h_a^{\phi} x_a^{\phi} \\
1/\beta &= E_{t,i}\Bigg[\pi_i(t)\,\frac{c_a}{c_{a+1}}\frac{w_{a+1}}{w_a}\left(\frac{h_a}{h_{a+1}}\frac{x_a}{x_{a+1}}\right)^{\phi-1}\Big(1-\delta+\phi B h_{a+1}^{\phi-1} x_{a+1}^{\phi-1}\left(\alpha+(1-\alpha)\,x_{a+1}\right)\Big) \\
&\qquad + \left(1-\pi_i(t)\right)\eta \tfrac{c_a}{c_{a+1}}\tfrac{w_{a+1}}{w_a}\left(\tfrac{h_a}{h_{a+1}}\right)^{\phi-1}\left(\tfrac{x_a}{x_{a+1}}\right)^{\phi-1} \\
&\qquad \Big(1-\delta+\phi B\left(\eta h_{a+1}\right)^{\phi-1} x_{a+1}^{\phi-1}\left(\alpha+(1-\alpha)\,x_{a+1}\right)\Big)\Bigg]
\end{aligned}
$$

Which leads under the stationarity assumption to the system stated in the main text in Section 3:

$$
\begin{aligned}
1/\beta &= E_{t,i}\Bigg[\pi_i(t)\,\frac{c_a}{c_{a+1}}\left(\frac{h_a}{h_{a+1}}\frac{x_a}{x_{a+1}}\right)^{\phi-1}\Big(1-\delta+\phi B h_{a+1}^{\phi-1} x_{a+1}^{\phi-1}\left(\alpha+(1-\alpha)\,x_{a+1}\right)\Big) \\
&\qquad + \left(1-\pi_i(t)\right)\eta \tfrac{c_a}{c_{a+1}}\left(\tfrac{h_a}{h_{a+1}}\right)^{\phi-1}\left(\tfrac{x_a}{x_{a+1}}\right)^{\phi-1} \\
&\qquad \Big(1-\delta+\phi B\left(\eta h_{a+1}\right)^{\phi-1} x_{a+1}^{\phi-1}\left(\alpha+(1-\alpha)\,x_{a+1}\right)\Big)\Bigg] \\
c_t &= \alpha w_t h_t \left(1-x_t\right); t = a, a+1 \\
x_t &= \left(\frac{h_{t+1}-(1-\delta)\, h_t}{B h_t^{\phi}}\right)^{1/\phi}; t = a, a+1
\end{aligned}
$$

Here the stationarity assumption is equivalent to constant aggregate variables and thus as a result the wage rate is unchanged over time. This implies that $w_{a+1}/w_a = 1$.

To get a starting point from which to search for the general solution to the above functional equation we solve for the infinite horizon solution in a world without uncertainty. As before, we focus on the steady state problem, where aggregates are unchanged and thus

$w_{a+1}/w_a = 1$. We realize that this is not a close guess for our problem since the lifetime in our model is finite and there is not enough time to get to the infinite horizon steady state from an arbitrary stock of initial human capital. Still, it is a useful tool to give us an idea where the system would head if persons were to live very long and thus serves well as a first guess for our computational analysis. This leads us to the following two equations:

$$\begin{aligned} 1/\beta &= 1 - \delta + \phi B h^{\phi-1} x^{\phi-1} \left(\alpha + (1-\alpha) x\right) \\ h &= \left(\frac{B}{\delta}\right)^{1/(1-\phi)} x^{\phi/(1-\phi)} \end{aligned}$$

or

$$\begin{aligned} \frac{1/\beta - (1-\delta)}{\delta} x &= \alpha\phi + (\phi - \phi\alpha) x \\ x &= \frac{\alpha\phi\delta}{(-1+\alpha)\phi\delta + \left(\delta + \frac{1}{\beta} - 1\right)} \end{aligned}$$

which have the solution:

$$\begin{aligned} x &= \frac{\alpha\phi\delta}{\left(\delta + \frac{1}{\beta} - 1\right) - (1-\alpha)\phi\delta}, \\ h &= \left(\frac{B}{\delta}\right)^{1/(1-\phi)} \left(\frac{\alpha\phi\delta}{\left(\delta + \frac{1}{\beta} - 1\right) - (1-\alpha)\phi\delta}\right)^{\phi/(1-\phi)}. \end{aligned}$$